HW #5 for Nonparametric Part, Fall, 2022

Due:

I. Do Monte Carlo simulations to compare the performances of the local linear and local constant estimations for the nonparametric regression function

$$Y_t = m(X_t) + u_t, \quad 1 \le t \le T \tag{1}$$

for different settings, where X_t is stationary. To make your own conclusions based on your simulations, please do the followings:

A. Choosing different sample sizes, different kernels, different bandwidths, and different bandwidth selection methods. Any conclusions and comments?

B. Compare the local linear method with local constant method. Any conclusions and comments?

To assess the performance of finite samples, for each setting, you need to compute the mean absolute deviation errors (MADE) for $\widehat{m}(\cdot)$, defined as

$$\text{MADE} = n_0^{-1} \sum_{k=1}^{n_0} \left| \widehat{m}(x_k) - m(x_k) \right|,$$

where $\widehat{m}(\cdot)$ is the nonparametric estimate of $m(\cdot)$ and $\{x_k\}$ are the grid points, taken to be arbitrary within the range of data. Note that you can choose any distribution for X_t and u_t to generate your samples for your simulation. Also, note that the choice of the grid points is not important so that they can be chosen arbitrarily. In general, the number of replications can be taken to be at least $n_{sim} = 1,000$. The question is how to report the simulation results. There are two ways of doing so. You can display the n_{sim} values of MADE either in a boxplot form (**boxplot() in R**) or in a table by presenting the *median* and *standard deviation* of the n_{sim} values of MADE. Either one is okay but the boxplot is preferred by most people.

II. Re-consider model (1), where X_t is nonstationary (say, I(1); that $x_t = x_{t-1} + u_t$, and nearly I(1); that $x_t = \rho\, x_{t-1} + u_t$ where $\rho = 1 - \theta/T$ for some $\theta > 0$) and re-do Parts A and B in Problem I for different values of θ. Please compare your results for nonstationary cases with those in Problem I. Also, make your own conclusions on what you find in your simulations.

III. Because of the existence of inverted yield curves in the term structure of interest rates, the spread of interest rates should be nonlinear. To verify this, consider the weekly U.S. interest rates of (a) Treasury 1-year constant maturity rate, denoted by r_{1t}, and (b) Treasury 3-year constant maturity rate, denoted by r_{3t}. The data span is from January 5, 1962 to now, which can be downloaded from the Federal Reserve Bank of St. Louis. Let $s_t = r_{3t} - r_{1t}$ be the spread in log interest rates. Please do the followings:

1. Is $\{s_t\}$ nonstationary? To answer this question, please find an appropriate test. Please draw the conclusion using 5% significance level.

2. Is $\{s_t\}$ nonlinear? To answer this question, please find an appropriate nonparametric model and conduct some nonlinear tests. Please draw the conclusion using 5% significance level.

3. Let $s_t^* = s_t - s_{t-1}$ be the change in interest rate spread. Is $\{s_t^*\}$ nonlinear? To answer this question, please find an appropriate nonparametric model and conduct some nonlinear tests. Please draw the conclusion using 5% significance level.

IV. Consider three real data sets for the U.S. Treasury bill (Secondary Market Rate): the **daily** 3-month Treasury bill from January 4, 1954 to today, the **weekly** 3-month Treasury bill from January 8, 1954 to today, and the **monthly** 3-month Treasury bill from January 1, 1934 to today.

1. Apply the nonparametric regression estimation methods to estimate the drift and diffusion functions for each series and higher moments such as skewedness and kurtosis.

2. Any conclusions and comments on three drift and diffusion functions? Also, think about your conclusions on whether a jump diffusion model is appropriate for the data; see Johannes (2004).

3. Compare your results with results in Aït-Sahalia (1996), Jiang and Knight (1997), and Stanton (1997).

♡ Note that the real data sets can be downloaded from the web site for Federal Reserve Bank of Saint Louis at *http://research.stlouisfed.org/fred2/categories/46*. You need to hand in all necessary materials (tables or graphs) to support your conclusions. If you need any help, please come to see me.

References

Aït-Sahalia, Y. (1996). Nonparametric pricing of interest rate derivative securities. *Econometrica*, **64**, 527-560.

Jiang, G.J. and J.L. Knight (1997). A nonparametric approach to the estimation of diffusion processes, with an application to a short-term interest rate model. *Econometric Theory*, **13**, 615-645.

Johannes, M.S. (2004). The statistical and economic role of jumps in continuous-time interest rate models. *Journal of Finance*, **59**, 227-260.

Stanton, R. (1997). A nonparametric model of term structure dynamics and the market price of interest rate risk. *Journal of Finance*, **52**, 1973-2002.