ME608 Fall 2017 - CFD

Homework 4

Unstructured Meshes

&

First Spatial Discretization

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1 Overview of Fluid Field

1.1 Set Up

The lid-triven cavity problem at Re=100 ($Re = \dfrac{U_{lid}L}{\nu}$) will be solved in this report, with U_{lid} = -1 (m/s), L= 1 m, ν =0.01 (m^2/s). The third mesh with ncv=2033 is selected as the main simulation and cfl=0.1 is selected, the reason of which will be explained later.

$$\begin{cases} \overrightarrow{\nabla} \cdot \overrightarrow{u} = 0 \\ \dfrac{\partial \overrightarrow{u}}{\partial t} + \overrightarrow{\nabla} p - \nu \nabla^2 \overrightarrow{u} = 0 \end{cases} \tag{1}$$

The convective term is neglected in this low Renolds number problem, as solved by Eq. 1. No -slip and no-penetration condition is added as boundary condition and the still status is taken as initial condition as Eq. 2.

$$\begin{cases} \text{t=0} \quad \overrightarrow{u}_{inside} = 0 \\ \text{t} \neq 0 \;\; v_{wall} = v_{lid} = u_{wall} = 0, u_{lid} = U_{lid} = -1 \end{cases} \tag{2}$$

1.2 Velocity Field

The contour and streamline figure in Fig. 1 and Fig. 2 match well with physics, especially for streamline, where secondary structure was captured. However, the contours didn't show such charicteristic, i.e. the opposite velocity. This may result from the interpolate method.

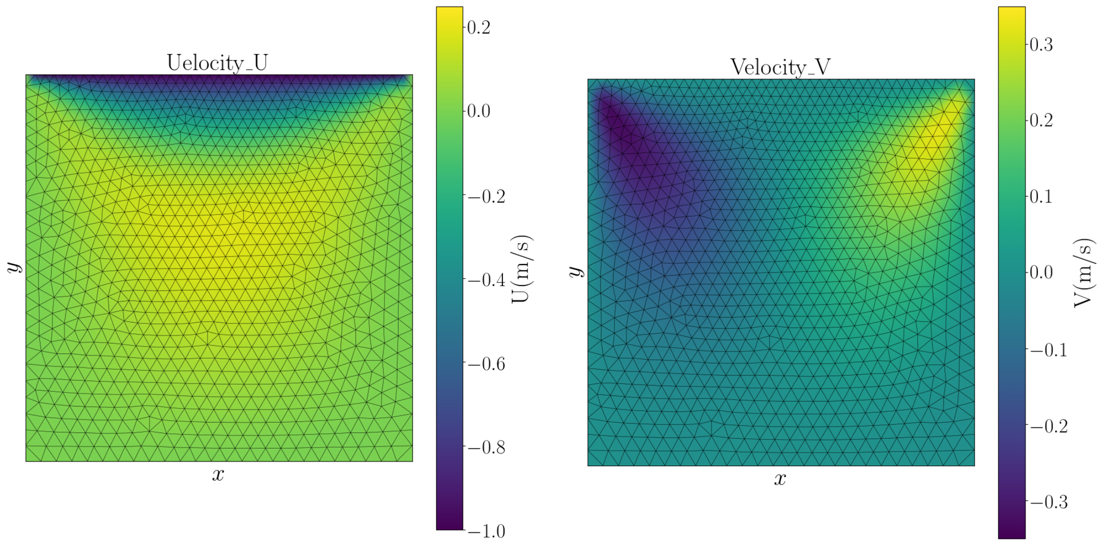

Figure 1: Velocity field for Mesh_3

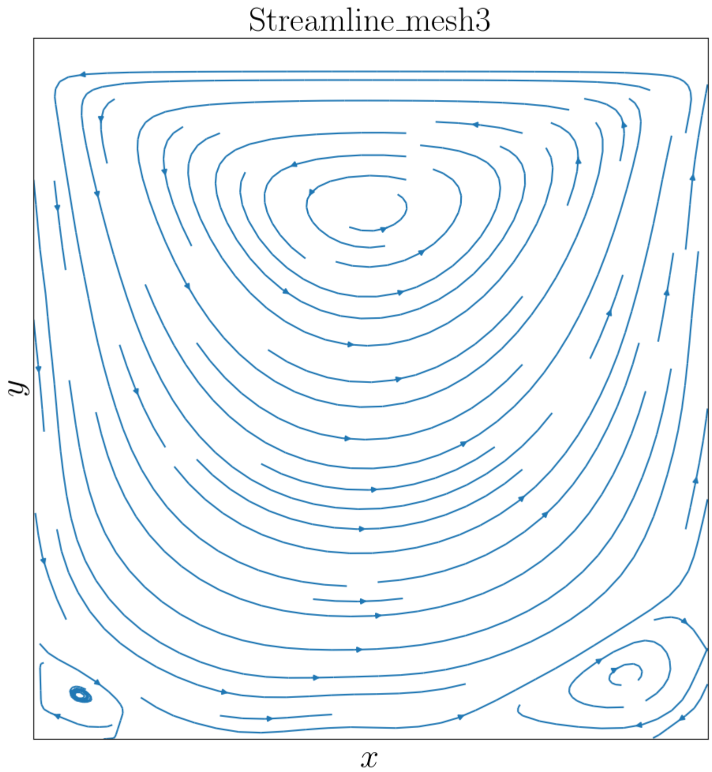

Figure 2: Streamline for Mesh_3

2 Spacial Discretion

Three sets of mesh with ncv=228,ncv=918 and ncv=2203 are tested in the Streamline figure, Fig. 3. With same CFL, the secondary structure is detected only in mesh_3. Therefore the mainly analizition in this report will be mainly about mesh_3.

For the operator, the method for constructing Laplas is same with homework 3, the twice dot product of the gradient operator, constructed via least square method, showing extremely good stability.

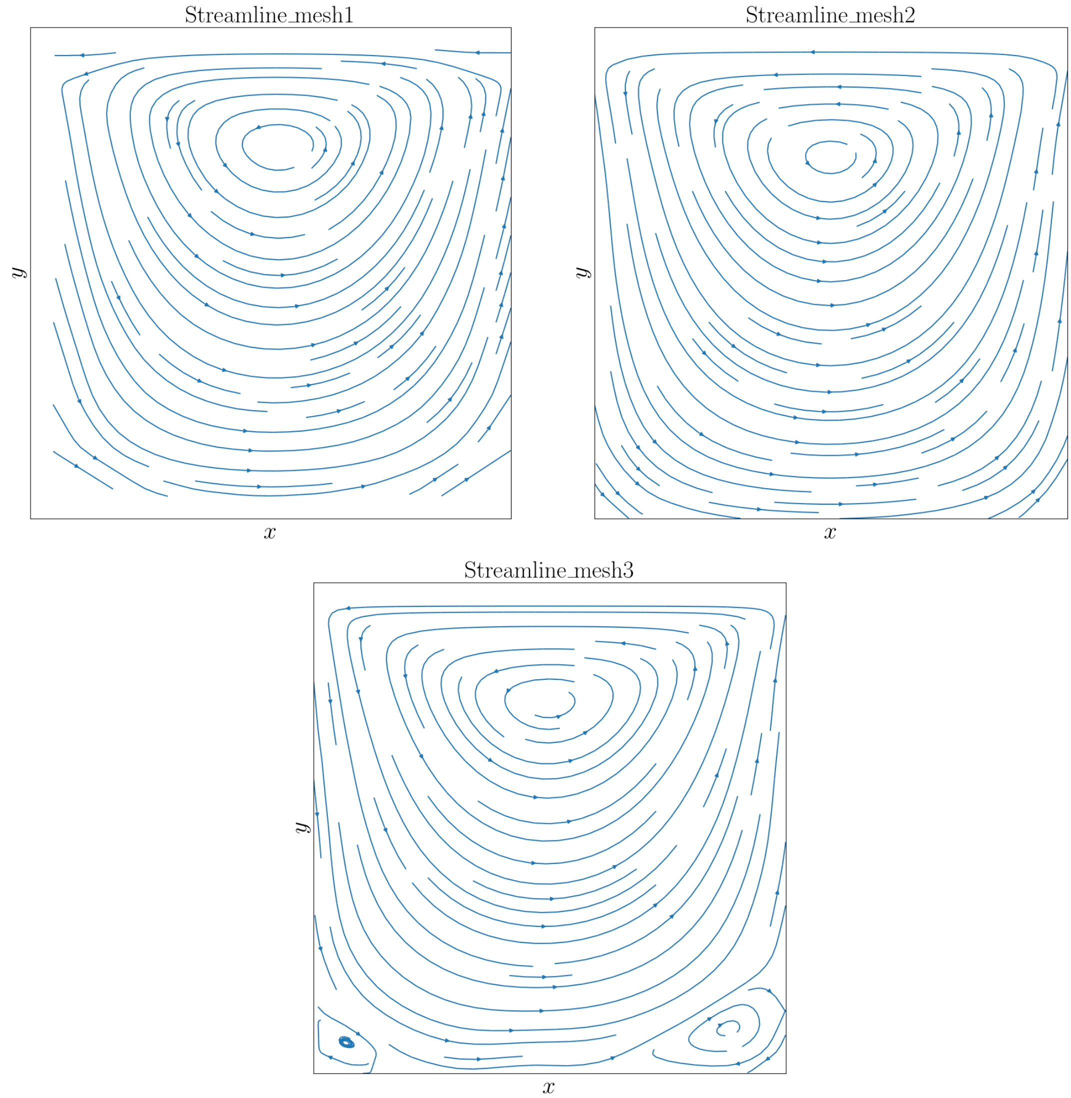

Figure 3: Streamline for three sets of mesh with the same CFL

3 Time Advancement

The Explicit Euler method is used in time advecement. The time step is selected corresping to CFL, defined as $CFL = max(4\alpha\triangle t/A_{cell})$. As mentioned in Homework 3, the special Laplas operator used in this study can get the converge result with CFL $\sim$ 10. Present study can also run with CFL $\approx$ 8, yet the result would be abnormal. Even with CFL=1 , the slightly reasonable solution could not be obtained. Fig. 4 shows the result with CFL = 1, 0.4, 0.1, and 0.05.

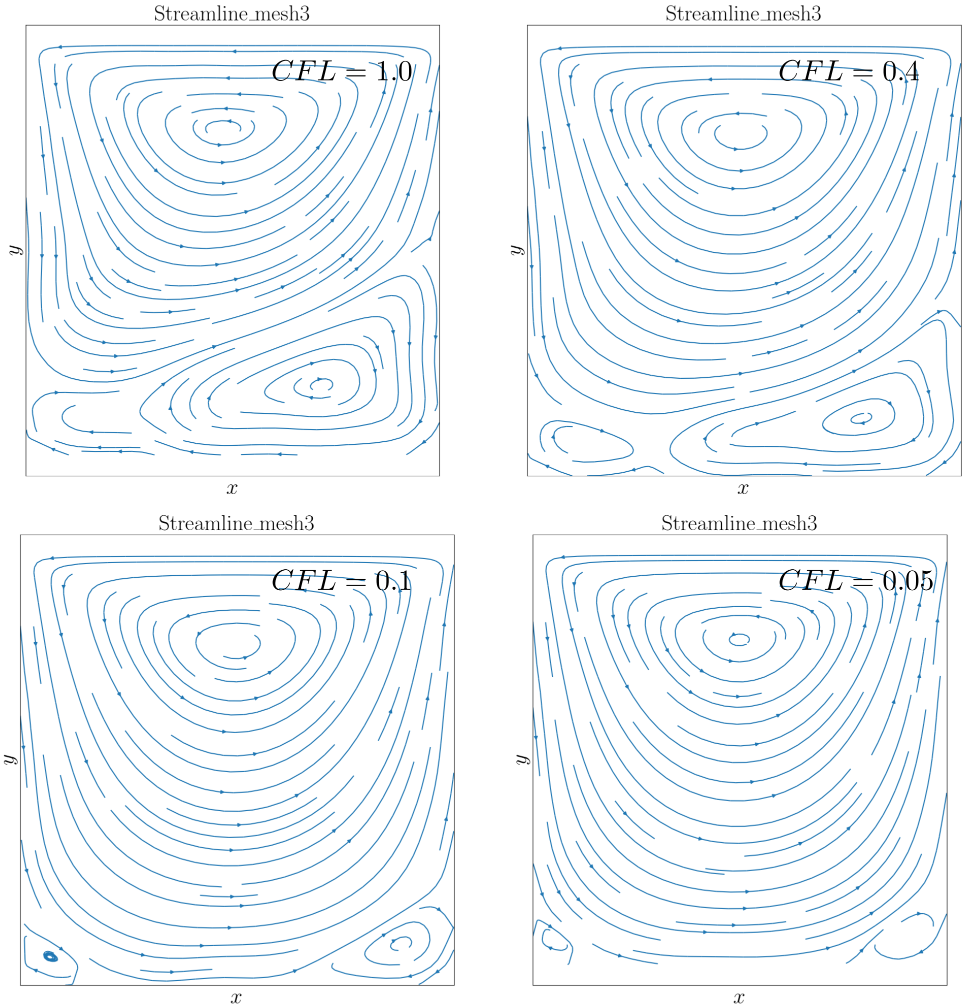

Figure 4: Streamline for mesh_3 with different CFL

4 Average Method and Discussion

As the pressure correction process is performed on the face, to enter next loop, the returning to node is neccessary. Three methods are used in this section.

1. estimate the velocity $\overrightarrow{u}_f^{n+1}$ at the nodal locations by taking an arithmetic average of the surrounding face values $\overrightarrow{u}_f^{n+1}$

2. formulate a new gradient operator for pressure going from cell-centers to nodal values and apply the newly defined gradient of p^{n+1} directly on the nodes, hence updating $\overrightarrow{u}^*$ to $\overrightarrow{u}^{n+1}$ directly at the nodal locations

3. evaluate the pressure gradient at the face locations consistently with (4) and estimate its value on the nodal loca- tions by simply taking an arithmetic average of it; use the newly defined averaged (hence, smooth) pressure gradient field to update $\overrightarrow{u}^*$ to $\overrightarrow{u}^{n+1}$ directly at the nodal location

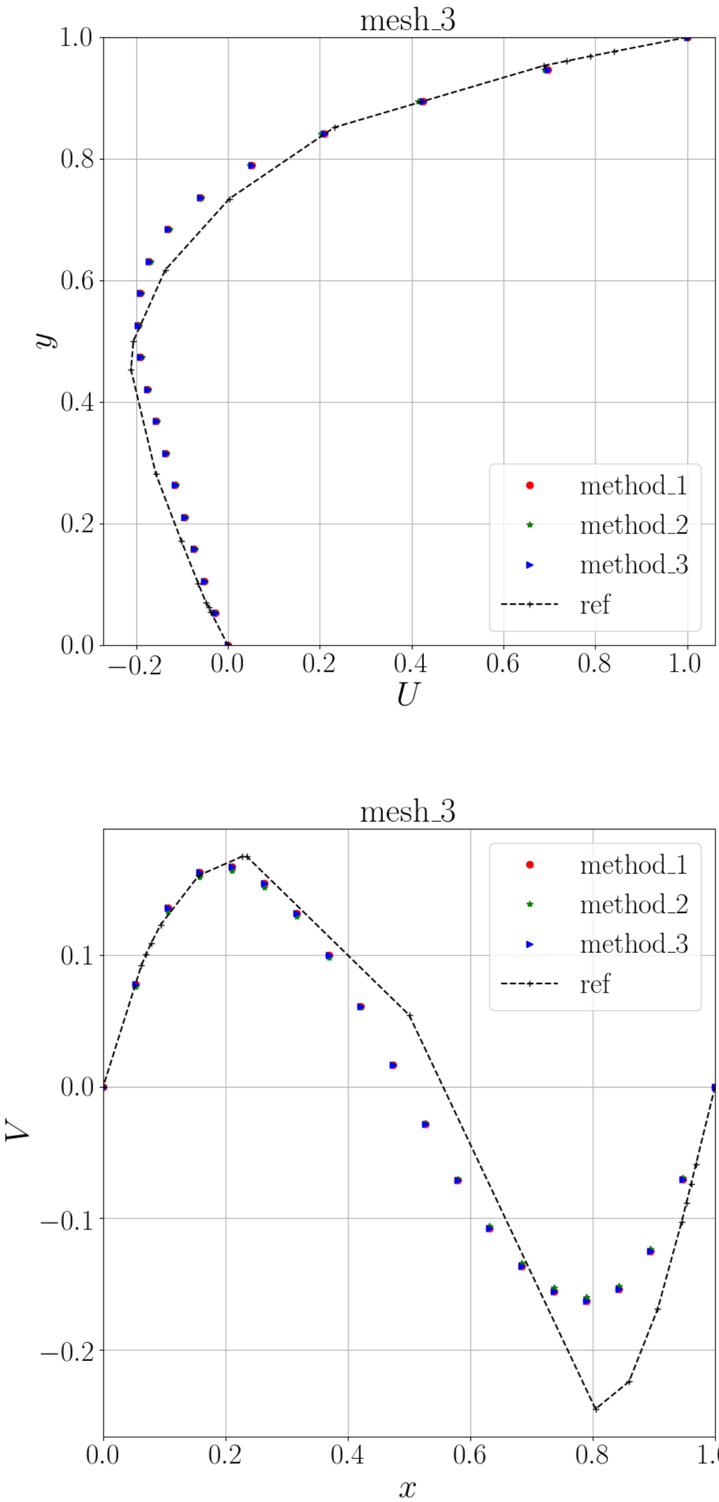

Figure 5: Comparison between three average method and U.Ghia 's results, noted as dash line'+–'.

Original data (initialed with U_{lid}= -1)in this study was timed (-1) to better compare the result with U.Ghia's (initialed with U_{lid}= 1).As shown in Fig. 5, the solution from these three method do not show much difference. This perhaps results from the method used in spacial operator. The gradient operator, used in all cases, no matter from cv to node or from face to node, all needs the participation of the center node. What we can get from the least square method directly is $u_i - u_c$, where u_i is surrounding node, i.e. the center of cv or face, and u_c is the center node. While the u_i as known (input), the other operator to evaluate u_c is needed to complete this process. Here the simple arithmetic average is adopted, and such gradient operator is then developed into the Laplas operator, both of them are used in the whole solving process. This may explain why developing new gradient operator (method 2)and the simply arithmetic average (method 1 and method 3) would get extremely similar results.

Meanwhile, the average process can also explain the stability as discussed above, and the lossing of the accuracy. U velocity along vertical certer line (x=0.5L) and V velocity along horizontal center line(y=0.5L) are compared with U.Ghia 's results.The V velocity profile doesn't capture the peak, which may result from too much averige process. The U velocity profile can capture the peak, yet show some offset from the paper's shape.

Another angle to check the profile is to check the integration along x-axis. Due to the conservition of mass, the positive and nagetive area under the curve should be equal to each other. For the V profile, the present study show similar peak and the zero point occurs at 0.5, approximately showing the integration as zero. The reference shows show lower peak and larger bottom line on positive part and vice versa.

For low Renolds number flow ($Re < 1$), symmetry should be an inportant chariceristic. However, in present study the Re is not low enough, while still solved by purely diffusive equition. The reference inclued the convection in the solution, which may explain the symmetry in present solution and the asymmetry in the reference.