Vector PDE	Nonzero Pattern	Description	Iterative Solver	Solver Tol. (2-norm)	Precisions Solver/PrecCalc/PrecStorage	#dof	Precond Tol.	number of V-cycle in an iteration	StructMG④	SysPFMG	BoomerAMG
Rhd-3T	3d7	Data are from radiation hydrodynamics [1]. The matrix is discretized from three-temperature equations and extracted from a timestep in the time-dependent simulation. It is **highly anisotropic** (see nonzero values distribution in Figure 2 in our paper and analysis in [1]).	CG (Sym.)	||r||/||b|| < 1e-9	FP64/FP32/FP16	6.30M	0.0	1	Enable ZeroGuess Coarsen: 3D full Smoother: sym PGS on all levels except for direct solve on coarsest level Interp: cell_3d8 Restrict: cell_3d8	Enable ZeroGuess Coarsen: 1D semi ① Relax type: 2 (Red-Black G.S.)	Coarsen type: 8 (PMIS) ② Strong threshold: 0.25 Interp: 6 (extended+i) Max rowsum: 0.9 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) Other:default
Oil-4C	3d7	Data are from petroleum reservoir simulation [2]. The matrix is discretized by the Fully Implicit Method (FIM) and extracted from a timestep in the time-dependent simulation. It is **highly anisotropic due to inhomogeneous permeability** [3]. Settings of SPE1 and SPE10 benchmarks are combined to generate large enough cases.	GMRES (Non-sym.)	||r||/||b|| < 1e-4	FP64/FP32/FP16	31.5M	0.0	1	Enable ZeroGuess Coarsen: 3D full Smoother: sym PGS on all levels Interp: cell_3d8 Restrict: cell_3d8	Not Applicable	NumFunctions: 4 SetNodal: 2 coarsen type: 6 (Falgout) Relax type: 26 (Sym G.S./Jacobi hybrid) Relax order: 1 (C-F) ② Other: default
Solid-3D	3d15	Data are from a linear elastic material model, characterized by its isotropic and homogeneous nature. The material exhibits a Poisson's ratio of 0.25. It is **a relatively well-conditioned system compared to other cases** in our paper. The elastic body is a cube, with each face subjected to fixed boundary conditions.	CG (Sym.)	||r||/||b|| < 1e-9	FP64/FP32/FP16	11.8M	0.0	1	Enable ZeroGuess Coarsen: 3D full Smoother: sym PGS on all levels except for direct solve on coarsest level Interp: cell_3d64 Restrict: cell_3d8	Enable ZeroGuess Coarsen: 1D semi ① Relax type: 2 (Red-Black G.S.)	**AMG Version in our paper (without InterpVector)** NumFunctions: 3 SetNodal: 4 Coarsen type: 10 (HMIS) Interp type: 6 (extended+i) Relax type: 26 (Sym G.S./Jacobi hybrid) Other: default **AMG Version from Reviewer#1 and #4 (with InterpVector)** NumFunctions: 3 Nodal: 4 Nodal Diag:1 Coarsen type: 8 (PMIS) Aggressive levels: 1 Interp type: 6 (extended+i) Pmax: 6 Relax type: 16 (Chebyshev) for up and down, 8 for coarsest InterpVecVariant: 2 (GM-2) InterpVecQMax: 4 Strong threshold: 0.25 ③ Other: default

Scalar PDE	Nonzero Pattern	Description	Iterative Solver	Solver Tol. (2-norm)	Precisions Solver/PrecCalc/PrecStorage	#dof	Precond Tol.	number of V-cycle in an iteration	StructMG④	SMG	PFMG	BoomerAMG
Rhd	3d7	**Rhd comes from the same problem with Rhd-3T**. Data are from radiation hydrodynamics [1]. By decoupling the radiation diffusion equation from the above Rhd-3T system through operator splitting, an **anisotropic** scalar linear system is derived.	CG (Symmetric Positive Definite)	||r||/||b|| < 1e-9	FP64/FP32/FP16	2.10M	0.0	1	Enable ZeroGuess Coarsen: 3D full Smoother: sym PGS on all levels except for direct solve on coarsest level Interp: cell_3d8 Restrict: cell_3d8	Enable ZeroGuess Coarsen: 1D semi ① Other: default	Enable ZeroGuess Coarsen: 1D semi ① RAP type: 0 (Galerkin) Relax type: 1 (weighted Jacobi) Other: default	Coarsen type: 8 (PMIS) ② Strong threshold: 0.25 Interp type: 6 (extended+i) Max rowsum: 0.9 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) other:default
Oil	3d7	**Oil comes from the same problem with Oil-4C.** Data are from petroleum reservoir simulation [2]. It decouples one component from the above Oil-4C system by the Implicit Pressure Explicit Composition (IMPEC) method. The derived matrices are still **highly anisotropic**.	GMRES (Non-sym.)	||r||/||b|| < 1e-7	FP64/FP32/FP16	31.5M	0.0	1	Enable ZeroGuess Coarsen: 3D full Smoother: sym PGS on all levels Interp: cell_3d8 Restrict: cell_3d8	Not Applicable	Not Applicable	Coarsen type: 10 (HMIS) Strong threshold: 0.3 ② Interp type: 6 (extended+i) Pmax elemts: 5 Max rowsum: 0.9 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) Relax weights: 1.1 ② Relax order: 1 (C-F) ② Other:default
Weather	3d19	Data are from the operational weather forecasting system of China Meteorological Administration [4]. This case is of 2km resolution of Chinese region. The matrices are month-dependent, and we use the one in December 2018. They are **highly anisotropic due to irregular earth topography and nonuniform latitudinal spacing**. The right-hand-side vectors are extracted from the time-dependent operational simulation.	GMRES (Non-sym.)	||r|| < 1e-5	FP32/FP32/FP16	637M	0.0	1	Enable ZeroGuess Coarsen: 2D semi Smoother: LGS or ILU (both are good) on finest level, and LGS on other coarser levels Interp: cell_2d16 Restrict: cell_2d4	Enable ZeroGuess Coarsen: 1D semi ① Other: default	Enable ZeroGuess Coarsen: 1D semi ① RAP type: 0 (Galerkin) Relax type: 1 (weighted Jacobi) Other: default	Coarsen type: 10 (HMIS) Strong threshold: 0.25 Interp: 6 (extended+i) Pmax elemts: 5 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) Other:default

[1] https://doi.org/10.1002/nla.2078
[2] https://www.spe.org/web/csp/datasets/set02.htm
[3] https://github.com/OpenCAEPlus/OpenCAEPoro/blob/main/examples/spe10 shows inhomogeneous permeability and porosity
[4] https://doi.org/10.1007/s11434-008-0494-z
①: SMG, PFMG, and SysPFMG could only support 1D semi-coarsening
② Application-specific tuning according to literature [1] for rhd and rhd-3T, and according to OpenCAEPoro (https://github.com/OpenCAEPlus/OpenCAEPoro/blob/main/examples/spe1a/csr.fasp) for oil and oil-4C.
③ Reviewer#4 suggests 0.5, and hypre document suggests 0.9 for elasticity problem, but we found 0.25 is better in term of total time.
④ StructMG uses cell-centered interpolation/restriction to handle anisotropy in these cases. See https://doi.org/10.1016/0021-9991(92)90168-X for details.

Other comments:

Software version: **hypre 2.25.0**

The types and values of "Solver Tol." are determined by the original applications. "Precond Tol." is set to 0 because multigrids are used as preconditioners.

All multigrid preconditioners are applied once (1 V-cycle) inside one iteration. Both pre- and post-smooth numbers are 1.

Experiment results of BoomerAMG with InterpVector in Solid-3D are displayed in the following figure.

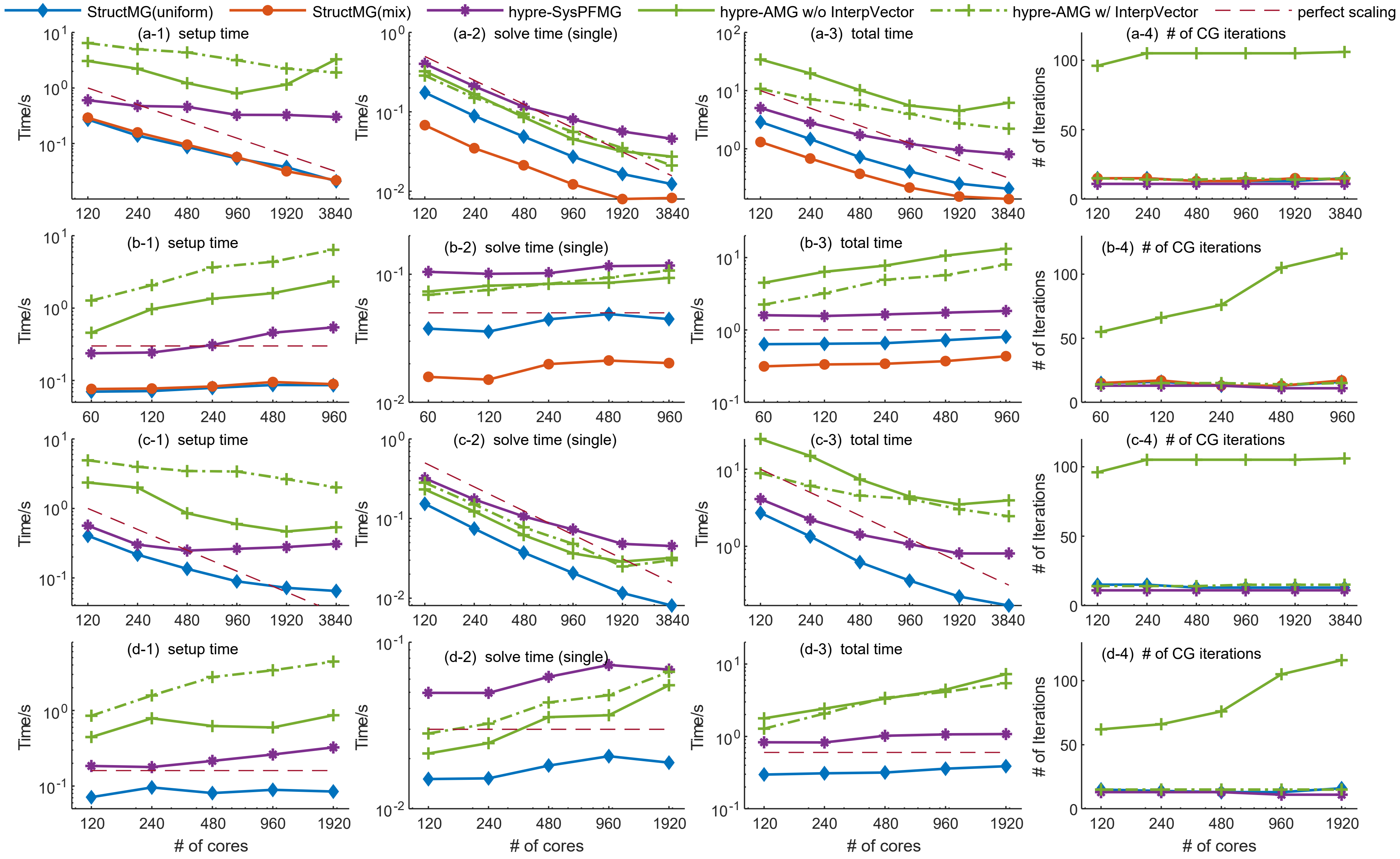

Figure: Elasticity problem experiment.
(a) Strong scaling test on ARM-based machines.
(b) Weak scaling test on ARM-based machines.
(c) Strong scaling test on X86 machines.
(d) Weak scaling test on X86 machines.

Conclusions:

1. BoomerAMG with InterpVector greatly reduces the number of iterations but brings about significant setup time.
2. BoomerAMG with InterpVector is still the slowest and least scalable in term of total time.
3. StructMG and hypre-SysPFMG do not require additional input of InterpVector, but are still faster than BoomerAMG with InterpVector.
4. StructMG could still obtain min/max speedups of 7.1x/18x on ARM-based and 3.3x/14x on X86 over BoomerAMG with InterpVector.