Scalar PDE	Nonzero Pattern	Description	Iterative Solver	Tol.	Precisions	#dof	number of V-cycle in an iteration	StructMG④	SMG	PFMG	BoomerAMG
Rhd	3d7	Data are from radiation hydrodynamics [1]. It decouples the radiation diffusion equation from the below Rhd-3T system.	CG (SPD.)	1e-9	FP64/FP32/FP16	2.10M	1	Enable ZeroGuess Coarsen: 3D full Smoother: PGS (LU at coarsest) Interp: cellwise 3d8 Restrict: cellwise 3d8	Enable ZeroGuess Coarsen: 1D semi ① Other: default	Enable ZeroGuess Coarsen: 1D semi ① RAP type: 0 (Galerkin) Relax type: 1 (weighted Jacobi) Other: default	Coarsen type: 8 (PMIS) ② Strong threshold: 0.25 Interp type: 6 (extended+i) Max rowsum: 0.9 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) other:default
Oil	3d7	Matrices data are from petroleum reservoir simulation [2]. It decouples one component from the below Oil-4C system by Implicit Pressure Explicit Composition (IMPEC) method.	GMRES (Non-sym.)	1e-7	FP64/FP32/FP16	31.5M	1	Enable ZeroGuess Coarsen: 3D full Smoother: PGS Interp: cellwise 3d8 Restrict: cellwise 3d8	N/A	N/A	Coarsen type: 10 (HMIS) Strong threshold: 0.3 ② Interp type: 6 (extended+i) Pmax elemts: 5 Max rowsum: 0.9 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) Relax weights: 1.1 ② Relax order: 1 (C-F) ② Other:default
Weather	3d19	Data are from operational weather forcasting system of China Meteorological Administration [3]. This case is of 2km resolution of Chinese region in Dec 2018. Matrices are highly anisotropic due to irregular earth surface and nonuiform latitudinal spacing.	GMRES (Non-sym.)	1e-5	FP32/FP32/FP16	637M	1	Enable ZeroGuess Coarsen: 2D semi Smoother:LGS/ILU ⑤ Interp: cellwise 2d16 Restrict: cellwise 2d4	Enable ZeroGuess Coarsen: 1D semi ① Other: default	Enable ZeroGuess Coarsen: 1D semi ① RAP type: 0 (Galerkin) Relax type: 1 (weighted Jacobi) Other: default	Coarsen type: 10 (HMIS) Strong threshold: 0.25 Interp: 6 (extended+i) Pmax elemts: 5 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) Other:default

Vector PDE	Nonzero Pattern	Description	Iterative Solver	Tol.	Precisions	#dof	number of V-cycle in an iteration	StructMG④	SysPFMG	BoomerAMG
Rhd-3T	3d7	Data are from radiation hydrodynamics [1]. Matrices discretized from three-temperature equations are highly anisotropic (see nonzero values distribution in Figure 2 in paper) and extracted from a timestep in time-dependent simulation.	CG (Sym.)	1e-9	FP64/FP32/FP16	6.30M	1	Enable ZeroGuess Coarsen: 3D full Smoother: PGS (LU at coarsest) Interp: cellwise 3d8 Restrict: cellwise 3d8	Enable ZeroGuess Coarsen: 1D semi ① Relax type: 2 (Red-Black G.S.)	Coarsen type: 8 (PMIS) ② Strong threshold: 0.25 Interp: 6 (extended+i) Max rowsum: 0.9 Aggressive levels: 1 Relax type: 6 (Sym G.S./Jacobi hybrid) Other:default
Oil-4C	3d7	Matrices data are from petroleum reservoir simulation [2] and discretized by Fully Implicit Method (FIM). Settings of SPE1 and SPE10 benchmarks are combined to generate large enough cases.	GMRES (Non-sym.)	1e-4	FP64/FP32/FP16	31.5M	1	Enable ZeroGuess Coarsen: 3D full Smoother: PGS Interp: cellwise 3d8 Restrict: cellwise 3d8	N/A	NumFunctions: 4 SetNodal: 2 coarsen type: 6 (Falgout) Relax type: 26 (Sym G.S./Jacobi hybrid) Relax order: 1 (C-F) ② Other: default
Solid-3D	3d15	Matrices data are from a linear elastic material model, characterized by its isotropic and homogeneous nature. The material exhibits a Poisson's ratio of \nu = 0.25. It is a relatively well conditioned system compared to other cases in our paper. The elastic body is a cube, where each face is subjected to fixed boundary conditions.	CG (Sym.)	1e-9	FP64/FP32/FP16	11.8M	1	Enable ZeroGuess Coarsen: 3D full Smoother: PGS (LU at coarsest) Interp: cellwise 3d64 Restrict: cellwise 3d8	Enable ZeroGuess Coarsen: 1D semi ① Relax type: 2 (Red-Black G.S.)	**Version in paper (without InterpVector)** NumFunctions: 3 SetNodal: 4 Coarsen type: 10 (HMIS) Interp type: 6 (extended+i) Relax type: 26 (Sym G.S./Jacobi hybrid) Other: default **Version from Reviewer#1 and #4 (with InterpVector)** NumFunctions: 3 Nodal: 4 Nodal Diag:1 Coarsen type: 8 (PMIS) Aggressive levels: 1 Interp type: 6 (extended+i) Pmax: 6 Relax type: 16 (Chebyshev) for up and down, 8 for coarsest InterpVecVariant: 2 (GM-2) InterpVecQMax: 4 Strong threshold: 0.25 ③ Other: default

[1] https://doi.org/10.1002/nla.2078
[2] https://www.spe.org/web/csp/datasets/set02.htm
[3] https://doi.org/10.1007/s11434-008-0494-z

①: SMG, PFMG, and SysPFMG could only support 1D semi-coarsening
② Application-specific tuning according to literature [1] for rhd and rhd-3T, and according to OpenCAEPoro (https://github.com/OpenCAEPlus/OpenCAEPoro/blob/main/examples/spe1a/csr.fasp) for oil and oil-4C.
③ Reviewer#4 suggests 0.5, and hypre document suggests 0.9 for elasticity problem, but we found 0.25 is better in term of total time
④ StructMG uses cell-centered interpolation/restriction to handle anisotropy in these cases. See https://doi.org/10.1016/0021-9991(92)90168-X for details.
⑤ Finest level smoother could use ILU or LGS (both are good), and coarser levels use LGS to achieve best performance

Other comments: All multigrid preconditioners are applied once (1 V-cycle) inside one iteration. Both pre- and post-smooth numbers are 1.