

10.4 Review of Hierarchical Framework for Model Comparison

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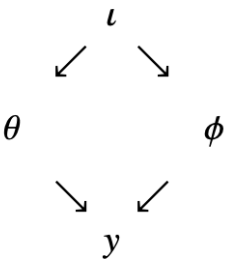
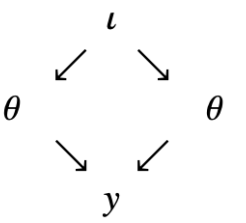
10.4 Review of Hierarchical Framework for Model Comparison

10.4.1 Comparing Methods for MCMC Model Comparison

Section 10.2.2 provided an example of model comparison using trans-dimensional MCMC with pseudopriors. Notice that the method yielded the relative posterior probabilities of the models without computing the individual values of $p(D|M1)$ and $p(D|M2)$.

Both methods for model comparison (i.e., trans-dimensional MCMC with pseudopriors and approximating $p(D|M)$ directly) require us to find an approximation to the posteriors.

Here we can see the hierarchical framework for model comparison

Dependency	Math Form	
	$p(\iota) = \text{dcat}(.5, .5)$	
	$p(\theta \iota=1) = \text{true prior}$ $p(\theta \iota=2) = \text{pseudo prior}$	$p(\phi \iota=1) = \text{pseudo prior}$ $p(\phi \iota=2) = \text{true prior}$
	$p(y \theta) = \dots$	$p(y \phi) = \dots$
Dependency	Math Form	
	$p(\iota) = \text{dcat}(.5, .5)$	
	$p(\theta \iota=1) = \text{true prior 1}$	$p(\theta \iota=2) = \text{true prior 2}$
	$p(y \theta) = \dots$	

Upper panel: When the models involve different parameters, hence different likelihood functions.

Lower panel: When the models involve the same likelihood function, and differ only in the prior on the parameter. The lower panel shows two copies of parameter θ merely for the purpose of comparison with the upper panel. There is really only one θ parameter in this case. For both panels, the 50-50 constants for the prior on the model index are arbitrary.

10.4.2 Summary

This chapter's main message regarding model comparison is that different models can be thought of as varying on a higher-level categorical parameter that indexes the models. Estimates of this indexical parameter provide an estimate of the posterior believability of the models. Thus, model comparison is “really” just estimation at a higher level. In BUGS, the model index can be set up as a random variable, and its probability can be approximated like any other variable.

Bayesian model comparison is especially useful when comparing models that have genuine prior believability. If we instead start with a model or models that have little if any prior believability, then the Bayesian model comparison is an exercise in meaninglessness, as the result will only reveal which of the unbelievable models is least unbelievable.

Bayesian model comparison is especially useful for comparing non-nested models, which involve distinct accounts of the data, using different parameters and different likelihood functions. The resulting Bayes factor can be very sensitive to the priors used within each model, however.

When comparing two models that have different parameters, the priors for the two models must be established using comparable criteria.

One way to establish appropriate priors for the two models is by using informed priors instead of uninformed, “automatic”, convenience priors.

Another approach is to use a set of previous data, or plausible and audience-agreeable fictitious data, that is small but large enough to overwhelm any vague, primordial prior.