10.3 Model Comparison and Nested Models

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The model comparison of the previous section indicated that a model with a single k_0 parameter for all four groups was about 8 times more believable than a model with distinct k_1, k_2, k_3, and k_4 parameters for the four groups.

In the author's mind, the best estimates of the distinct kappa parameters are the separate estimates, as shown in Figure 10.4. The posterior distribution on the four distinct parameters indicates a large degree of overlap, but that does not mean that the parameter values are literally the same, it means merely that the differences are small compared to our uncertainty about the values.

Instead of doing a model comparison between a model with four distinct k values and a model with one shared k value, we could just look at the posterior distribution of the four distinct k values. At every step in the chain, we compute the differences of the various group's k values, and examine whether the differences tend to be near zero.

The model involving a single shared k value is a restricted version of the full model involving four distinct k values. We get to the restricted model from the full model by demanding that all the distinct k values are equal: $k_1 = k_2 = k_3 = k_4 = k_0$. We say that the restricted model is **nested** within the full model, and that the model comparison of the previous section was a **nested model comparison**.

When the restricted model is genuinely believable for theoretical reasons. In this case, when we have a viable theory that asserts that a specific restriction is true, then it makes sense to test that restriction as a unique model. But if the restriction is merely a simplification of convenience, without genuine theoretical motivation, then it is more meaningful to examine the posterior distribution of the full model, rather than the restricted model that we do not actually believe.

Another way of expressing this distinction is formally. Denote the full model as M_F and denote the restricted model as M_R. We know from Bayes rule that

$$\frac{p(M_R|D)}{p(M_F|D)} = \underbrace{\frac{p(D|M_R)}{p(D|M_F)}}_{\text{BF}} \frac{p(M_R)}{p(M_F)}$$

In conclusion, model comparison for nested models should be undertaken only when it is truly meaningful to do so; it should not be undertaken routinely and automatically as "the" way to assess parameter values. And, when nested model comparison is conducted, the parameter estimates in the unrestricted models should be examined for coherence with the conclusion from the nested model comparison, because the unrestricted model might show credible differences among parameters even if the restricted model "wins" a model comparison.