13.2 Sample Size for a Single Coin

Zongyi Liu

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As our first worked-out example, in which data is from a single coin.

13.2.1 When the Goal is to Exclude a Null Value

Suppose that our goal is to show that the coin is biased, namely, we want to show that 0.5 is not among the credible values.

Because of previous research, we think that the true bias of the coin is very close to $\theta = 0.65$.

To generate simulated data from hypothetical coins that have biases, we can go through this replication process

First, select a value for the "true" bias in the coin, from the data-generating distribution that is narrowly centered on $\theta = .65$. Suppose that the selected value is .638. Second, simulate flipping a coin with that bias N times. The simulated data have z heads and $N - z$ tails. The proportion of heads, z/N, will tend to be around .638, but will be higher or lower because of randomness in the flips. Third, using the audience-agreed prior for purposes of data analysis, determine the posterior beliefs regarding θ if z heads in N flips were observed. Tally whether or not the 95% HDI excludes the null value of $\theta = .50$.

Table 13.1 shows the minimal sample size needed for the 95% HDI to exclude $\theta = .5$ when flipping a single coin. As an example of how to read the table, suppose we have a data-generating hypothesis that the coin has a bias very near $\theta = .65$. This hypothesis is implemented, for purposes of Table 13.1, as a beta distribution with shape parameters of $.65 * 2000$ and $(1 - .65) * 2000$.

Table 13.1: Minimal sample size required for 95% HDI to exclude 0.5, when flipping a single coin.

	Generating Mean θ						
Power	**.55**	**.60**	**.65**	**.70**	**.75**	**.80**	**.85**
.7	642	148	64	33	18	13	7
.8	866	191	82	43	26	18	10
.9	1274	264	111	59	36	24	16

In Table 13.1, that as the generating mean increases, the required sample size decreases. This makes sense intuitively: When the generating mean is large, the sample proportion of heads will tend to be large, and so the HDI will tend to fall toward the high end of the parameter domain. In other words, when the generating mean is large, it doesn't take a lot of data for the HDI to fall well above $\theta = .5$.

13.2.2 When the Goal is Precision

Not all research has as its goal the exclusion of a particular null value. Sometimes the goal is to establish a precise estimate of the parameter values. Other times, the research may have a null value of interest, but the data-generating hypothesis is too vague for the null value to be excluded most of the time, or, the goal may be to demonstrate that the HDI falls entirely within the ROPE. In these cases, the goal becomes precision of parameter estimation.

Example: Suppose we are interested in assessing the preferences of the general population regarding political candidates A and B. In particular, we would like to have high confidence in estimating whether the preference for candidate A exceeds $\theta = .5$. A recently conducted poll by a reputable organization found that of 10 randomly selected voters, 6 preferred candidate A and 4 preferred candidate B. If we use a uniform pre-poll prior, our post-poll estimate of the population bias is a beta(7,5) distribution. As this is our best information about the population so far, we can use the beta(7,5) distribution as a data-generating distribution for planning the follow-up poll. Unfortunately, a beta(7,5) distribution has a 95% HDI from $\theta = .318$ to $\theta = .841$, which means that $\theta = .5$ is well within the data-generating distribution. How many more people do we need to poll so that 80% of the time we would get a 95% HDI that falls above $\theta = .5$?

In this case, that we can never have a sample size large enough to achieve the goal of 80% of the HDIs falling above $\theta = .5$.

Consider what happens when we sample a particular value θ from the data-generating distribution, e.g., $\theta = .4$. We use that θ value to simulate a random sample of votes. Suppose N for the sample is huge, which implies that the HDI will be very narrow. What value of θ will the HDI focus on? Almost certainly it will focus on the value $\theta = .4$ that was used to generate the data.

The $beta(\theta, 7, 5)$ has only about 72% of the θ values are above .5. Therefore, even with an extremely large sample size, we can get at most 72% of the HDIs to fall above .5.

Table 13.2 shows the minimal sample size needed for the 95% HDI to have maximal width of 0.2.

Table 13.2: Minimal sample size required for 95% HDI to have maximal width of 0.2, when flipping a single coin.

	Generating Mean θ						
Power	**.55**	**.60**	**.65**	**.70**	**.75**	**.80**	**.85**
.7	91	91	89	86	80	69	58
.8	92	92	91	90	86	79	67
.9	93	93	93	92	91	88	79

In Table 13.2, as the desired power increases, the required sample size increases only slightly. For example, if the data-generating mean is .6, then as the desired power rises from .7 to .9, the minimal sample size rises from 91 to 93. This is because the distribution of HDI widths, for a given sample size, has a very shunted high tail, and therefore small changes in N can quickly pull the high tail across a threshold such as .2. On the other hand, as the desired HDI width decreases (not shown in the Table), the required sample size increases rapidly.