

7.6 Local Regression

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Local regression is a different approach for fitting flexible non-linear functions, which involves computing the fit at a target point x_0 using only the regression nearby training observations.

Note that in Step 3 of Algorithm 7.1, the weights K_{i0} will differ for each value of x_0 . In other words, to obtain the local regression fit at a new point, we need to fit a new weighted least squares regression model by minimizing the RSS_{cv} equation we formulated above for a new set of weights. Local regression is sometimes referred to as a memory-based procedure, because like nearest-neighbors.

Algorithm 7.1 Local Regression At $X = x_0$

1. Gather the fraction $s = k/n$ of training points whose x_i are closest to x_0 .
2. Assign a weight $K_{i0} = K(x_i, x_0)$ to each point in this neighborhood, so that the point furthest from x_0 has weight zero, and the closest has the highest weight. All but these k nearest neighbors get weight zero.
3. Fit a weighted least squares regression of the y_i on the x_i using the aforementioned weights, by finding $\hat{\beta}_0$ and $\hat{\beta}_1$ that minimize

$$\sum_{i=1}^n K_{i0}(y_i - \beta_0 - \beta_1 x_i)^2$$

4. The fitted value at x_0 is given by $\hat{f}(x_0) = \hat{\beta}_0 + \hat{\beta}_1 x_0$.

Local Regression

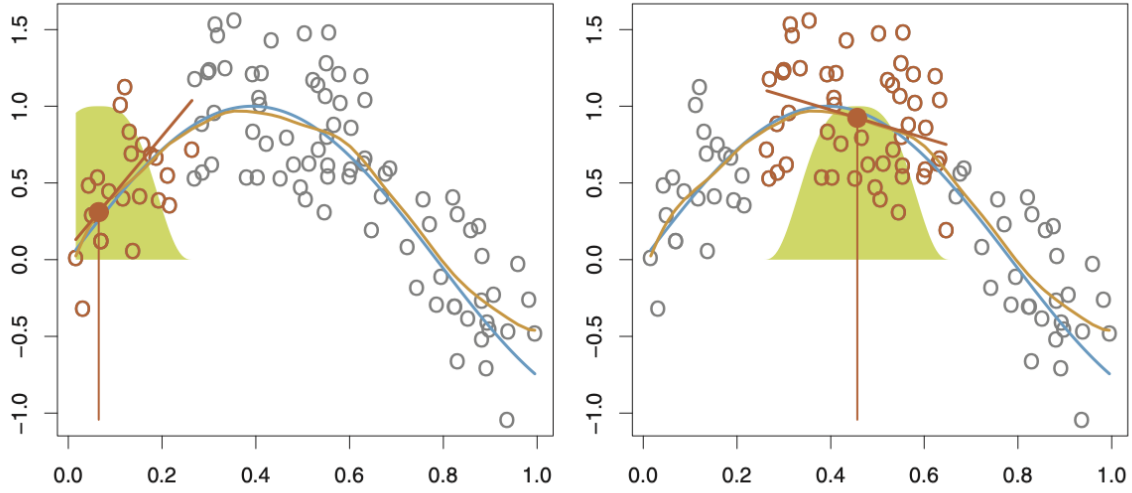


FIGURE 7.9. Local regression illustrated on some simulated data, where the blue curve represents $f(x)$ from which the data were generated, and the light orange curve corresponds to the local regression estimate $\hat{f}(x)$. The orange colored points are local to the target point x_0 , represented by the orange vertical line. The yellow bell-shape superimposed on the plot indicates weights assigned to each point, decreasing to zero with distance from the target point. The fit $\hat{f}(x_0)$ at x_0 is obtained by fitting a weighted linear regression (orange line segment), and using the fitted value at x_0 (orange solid dot) as the estimate $\hat{f}(x_0)$.