11.3 Confidence Interval and Highest Density Interval

Zongyi Liu

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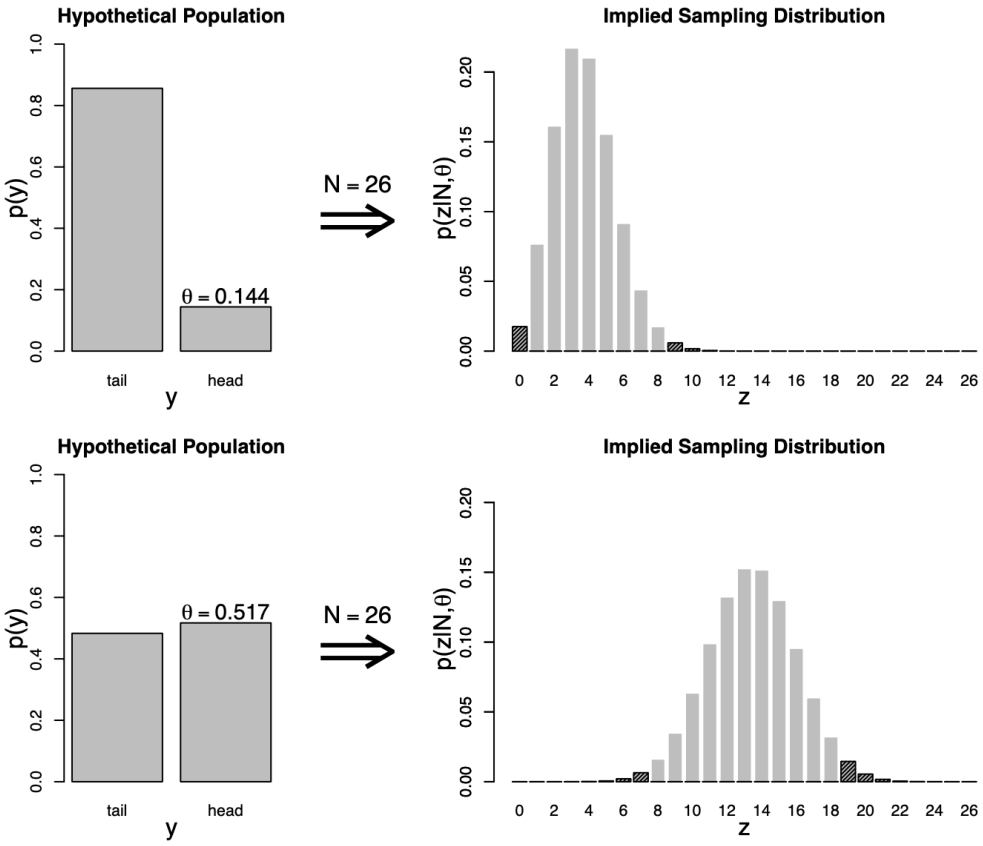
11.3 Confidence Interval and Highest Density Interval

11.3.1 NHST Confidence Interval

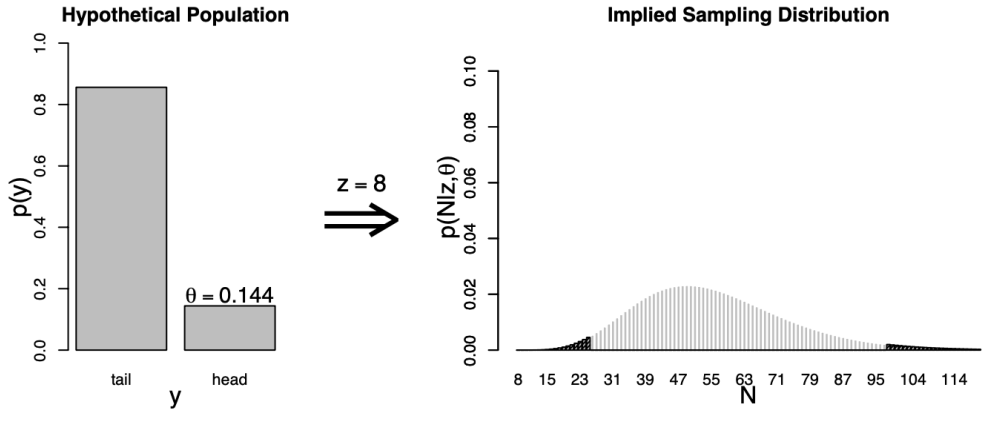
The primary goal of NHST is determining whether a particular "null" value of a parameter can be rejected. One can also ask what range of parameter values would not be rejected. This range of non-rejectable parameter values is called the confidence interval.

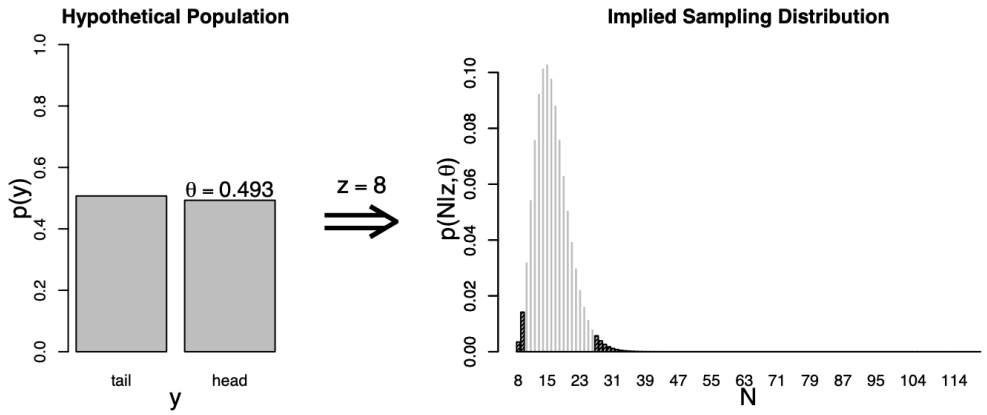
The 95% confidence interval consists of all values of θ that would not be rejected by a (two-tailed) significance test that allows 5% false alarms.

The confidence interval tells us something about the probability of extreme unobserved data values that we might have gotten if we repeated the experiment according to the covert intentions of the experimenter. But the confidence interval tells us little about the believability of any particular θ value, which is what we want to know.



Confidence interval when $z = 8$ and $N = 26$, with N fixed by the experimenter's intention. Upper row shows $\theta = .144$, which is the lowest for which $z = 8$ is not in the rejection tail. Lower row shows $\theta = .517$, which is the highest for which $z = 8$ is not in the rejection tail. The NHST confidence interval is, therefore, $\theta \in [.144, .517]$.





Confidence interval when $z = 8$ and $N = 26$, with z fixed by the experimenter's intention. Upper row shows $\theta = .144$, which is the lowest for which $N = 26$ is not in the rejection tail. Lower row shows $\theta = .493$, which is the highest for which $N = 26$ is not in the rejection tail. The NHST confidence interval is, therefore, $\theta \in [.144, .493]$.

11.3.2 Bayesian HDI

A concept in Bayesian inference, that is somewhat analogous to the NHST confidence interval, is the highest density interval (HDI), which was introduced in Section 3.3.5. The 95% HDI consists of those values of θ that have at least some minimal level of posterior believability, such that the total probability of all such θ values is 95%.

There are some advantages of the HDI over an NHST confidence interval.

First, the HDI has a direct interpretation in terms of the believabilities of values of θ. The HDI is explicitly about $p(\theta|D)$, which is exactly what we want to know. The NHST confidence interval, on the other hand, has no direct relationship with what we want to know; there's no clear relationship between the probability of rejecting the value θ and the believability of θ.

Second, the HDI has no dependence on the intention of the experimenter during data collection, because the likelihood has no dependence on the intention of the experimenter during data collection. The NHST confidence interval, in contrast, tells us about probabilities of data relative to what might have been if we replicated the experimenter's covert intentions.

Third, the HDI is responsive to the analyst's prior beliefs, as it should be. The Bayesian analysis indicates how much the new data should alter our beliefs. The prior beliefs are overt and publicly decided. The NHST analysis, on the contrary, is ignorant of, and unresponsive to, the accumulated prior knowledge of the scientific community.