7.1 Polynomial Regression

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Chapter 7 is mainly for the extension beyond linearity in fitting the model, such as:

- Polynomial regression. It raises predictors to a power, like X, X^2, X^3
- Step function. It cuts the range of a variable into K distinct regions in order to produce a qualitative variable. It is useful to fit a piece-wise constant function.
- Regression splines. It is more flexible than polynomials and step functions, and are an extension of the two.
- Smoothing splines.
- Local regression. It is similar to splines, but allows to overlap, and do so in a very smooth way.
- Generalized additive models. It extends the methods above to deal with multiple predictors.

7.1 Polynomial Regression

Previously, we express the linear regression as

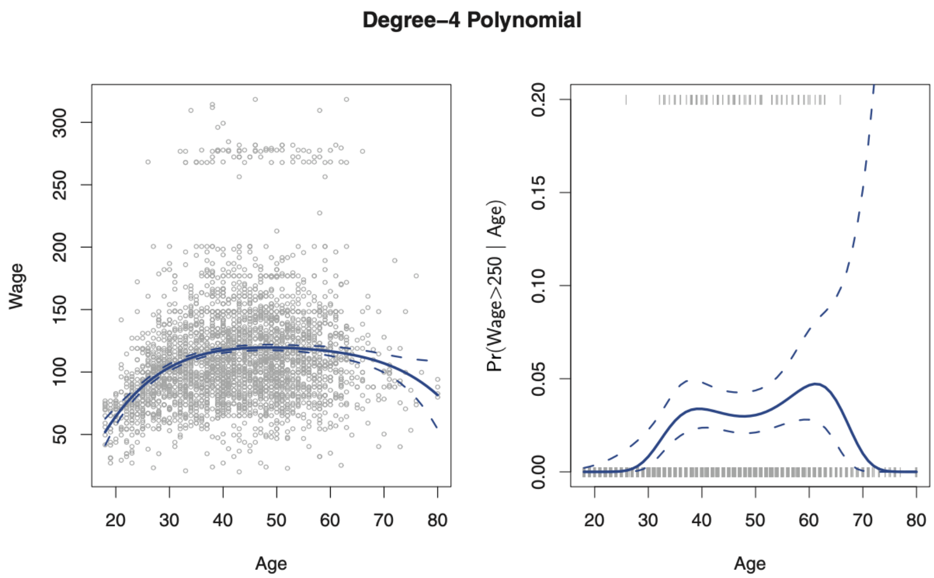
$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

but we can also extend it into a polynomial term

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \ldots + \beta_d x_i^d + \epsilon_i,$$

This approach is known as polynomial regression. It is not very common to use d larger than 3 or 4 empirically.

We can draw the polynomial regression in plots as below:



The left part of the figure can be expressed as

$$\hat{f}(x_0) = \hat{\beta}_0 + \hat{\beta}_1 x_0 + \hat{\beta}_2 x_0^2 + \hat{\beta}_3 x_0^3 + \hat{\beta}_4 x_0^4.$$

and the right part of the figure can be expressed as

$$\Pr(y_i > 250 | x_i) = \frac{\exp(\beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \ldots + \beta_d x_i^d)}{1 + \exp(\beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \ldots + \beta_d x_i^d)}.$$