6.6 Lab: Ridge Regression and the Lasso

Zongyi Liu

2023-05-31

6.6 Ridge Regression and the Lasso

Here we will use `glmnet()` package

`model.matrix()` is helpful, because it can - produce a matrix corresponding to predictors - utomatically transforms any qualitative variables into dummy variables

```
library(ISLR)
Hitters=na.omit(Hitters)
x=model.matrix(Salary~.,Hitters)[,-1]
y=Hitters$Salary
```

6.1.1 Ridge Regression

In the `glmnet()` function, there is an `alpha` argument, if `alpha=0` then a ridge regression model is fit, and if `alpha=1` then a lasso model is fit.

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 4.1-8
```

```
grid=10^seq(10,-2,length=100)
ridge.mod=glmnet(x,y,alpha=0,lambda=grid)
```

By default the `glmnet()` function performs ridge regression for an automatically selected range of λ values

```
dim(coef(ridge.mod))
```

```
## [1]  20 100
```

We expect the coefficient estimates to be much smaller, in terms of l_2 norm, when a large value of λ is used, as compared to when a small value of λ is used.

```
ridge.mod$lambda[50]
```

```
## [1] 11497.57
```

```
coef(ridge.mod)[,50]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
## 407.356050200   0.036957182   0.138180344   0.524629976   0.230701523
##           RBI         Walks         Years        CAtBat         CHits
##   0.239841459   0.289618741   1.107702929   0.003131815   0.011653637
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##   0.087545670   0.023379882   0.024138320   0.025015421   0.085028114
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
##  -6.215440973   0.016482577   0.002612988  -0.020502690   0.301433531
```

```
sqrt(sum(coef(ridge.mod)[-1,50]^2))
```

```
## [1] 6.360612
```

We can use the `predict()` function for a number of purposes, for example, we can obtain the ridge regression coefficients for a new value of λ, say 50.

```
predict(ridge.mod,s=50,type="coefficients")[1:20,]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
##  4.876610e+01 -3.580999e-01  1.969359e+00 -1.278248e+00  1.145892e+00
##           RBI         Walks         Years        CAtBat         CHits
##  8.038292e-01  2.716186e+00 -6.218319e+00  5.447837e-03  1.064895e-01
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##  6.244860e-01  2.214985e-01  2.186914e-01 -1.500245e-01  4.592589e+01
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
## -1.182011e+02  2.502322e-01  1.215665e-01 -3.278600e+00 -9.496680e+00
```

We can then split the samples into a training and a test set to test the errors of ridge regression and lasso.

```
set.seed (1)
train=sample(1:nrow(x), nrow(x)/2)
test=(-train)
y.test=y[test]
```

Then we fit a ridge regression model on the training set, and evaluate the MSE on it

```
ridge.mod=glmnet(x[train,],y[train],alpha=0,lambda=grid, thresh=1e-12)
ridge.pred=predict(ridge.mod,s=4,newx=x[test,])
mean((ridge.pred-y.test)^2)
```

```
## [1] 142199.2
```

If we have simply fit a model with just an intercept, we would have predicted each test observation using the mean of the training observations.

```
mean((mean(y[train])-y.test)^2)
```

```
## [1] 224669.9
```

We could also get the same result by fitting a ridge regression model with a very large value of λ:

```
ridge.pred=predict(ridge.mod,s=1e10,newx=x[test,])
mean((ridge.pred-y.test)^2)
```

```
## [1] 224669.8
```

So we can say that fitting a ridge regression model with $\lambda = 4$ leads to a much lower test MSE than fitting a model with just an intercept.

```
ridge.pred=predict(ridge.mod,s=0,newx=x[test,],exact=T,x=x[train,],y=y[train])
mean((ridge.pred-y.test)^2)
```

```
## [1] 168588.6
```

```
lm(y~x, subset=train)
```

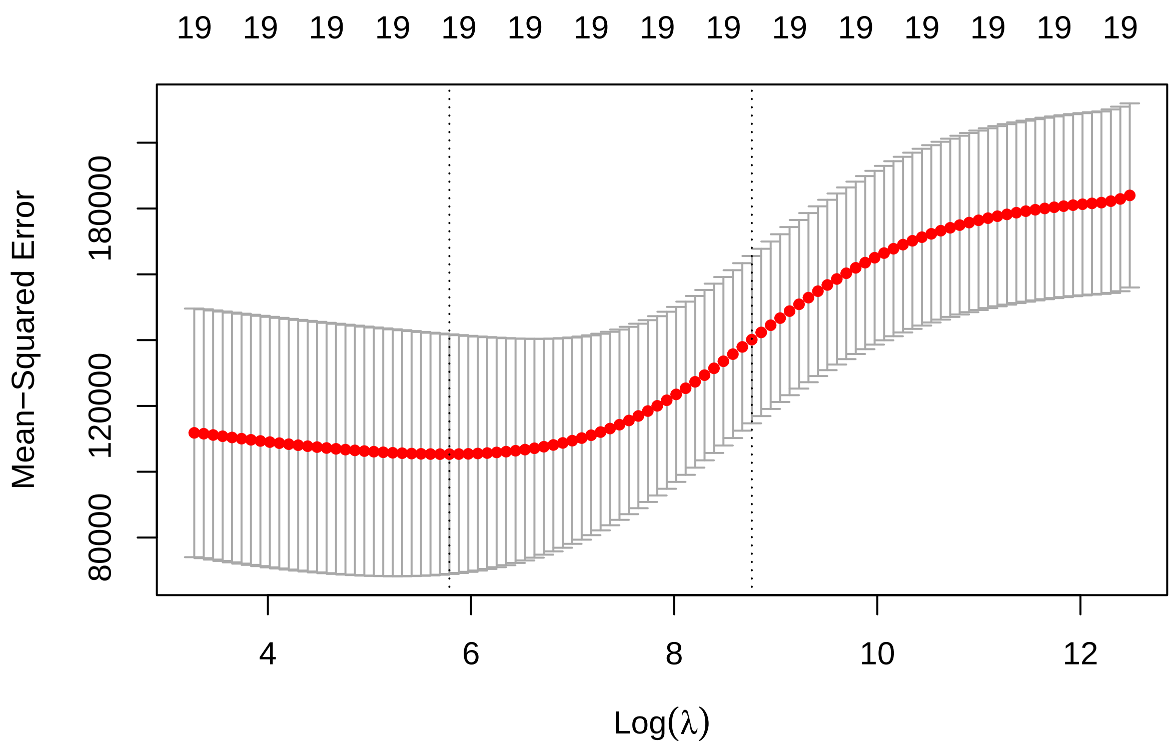
```
##
## Call:
## lm(formula = y ~ x, subset = train)
##
## Coefficients:
## (Intercept)       xAtBat        xHits       xHmRun        xRuns         xRBI
##    274.0145      -0.3521      -1.6377       5.8145       1.5424       1.1243
##      xWalks       xYears      xCAtBat       xCHits      xCHmRun       xCRuns
##      3.7287     -16.3773      -0.6412       3.1632       3.4008      -0.9739
##       xCRBI      xCWalks     xLeagueN   xDivisionW     xPutOuts     xAssists
##     -0.6005       0.3379     119.1486    -144.0831       0.1976       0.6804
##     xErrors  xNewLeagueN
##     -4.7128     -71.0951
```

```
predict(ridge.mod,s=0,exact=T,type="coefficients",x=x[train,],y=y[train])[1:20,]
```

```
##  (Intercept)        AtBat         Hits        HmRun         Runs          RBI
##  274.0200994   -0.3521900   -1.6371383    5.8146692    1.5423361    1.1241837
##        Walks        Years       CAtBat        CHits       CHmRun        CRuns
##    3.7288406  -16.3795195   -0.6411235    3.1629444    3.4005281   -0.9739405
##         CRBI       CWalks      LeagueN    DivisionW      PutOuts      Assists
##   -0.6003976    0.3378422  119.1434637 -144.0853061    0.1976300    0.6804200
##       Errors   NewLeagueN
##   -4.7127879  -71.0898914
```

Also, instead of arbitrarily choosing $\lambda = 4$, it would be better to use cross-validation to choose the tuning parameter λ. We can do this using the built-in cross-validation function, `cv.glmnet()`.

```
set.seed(1)
cv.out=cv.glmnet(x[train,],y[train],alpha=0)
plot(cv.out)
```




```r
bestlam=cv.out$lambda.min
bestlam
```

```
## [1] 326.0828
```

Here we can see that the value of λ associated with the smallest cv-error is 212, then we will try to find the test MSE.

```r
ridge.pred=predict(ridge.mod,s=bestlam,newx=x[test,])
mean((ridge.pred-y.test)^2)
```

```
## [1] 139856.6
```

This shows the improvement of our model, and we will choose it as our final model:

```r
out=glmnet(x,y,alpha=0)
predict(out,type="coefficients",s=bestlam)[1:20,]
```

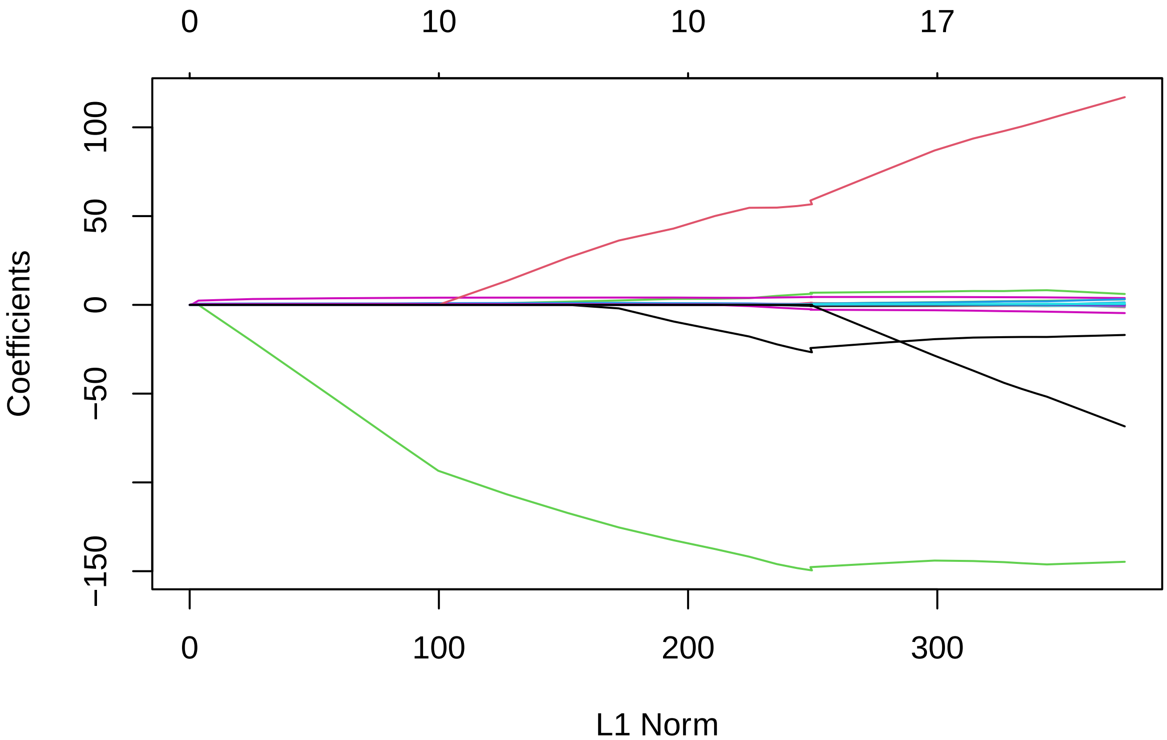
```
##  (Intercept)        AtBat         Hits        HmRun         Runs          RBI
##  15.44383120   0.07715547   0.85911582   0.60103106   1.06369007   0.87936105
##        Walks        Years       CAtBat        CHits       CHmRun        CRuns
##   1.62444617   1.35254778   0.01134999   0.05746654   0.40680157   0.11456224
##         CRBI       CWalks      LeagueN    DivisionW      PutOuts      Assists
##   0.12116504   0.05299202  22.09143197 -79.04032656   0.16619903   0.02941950
##       Errors   NewLeagueN
##  -1.36092945   9.12487765
```

6.6.2 Lasso Regression

We will use the same function to perform lasso regression

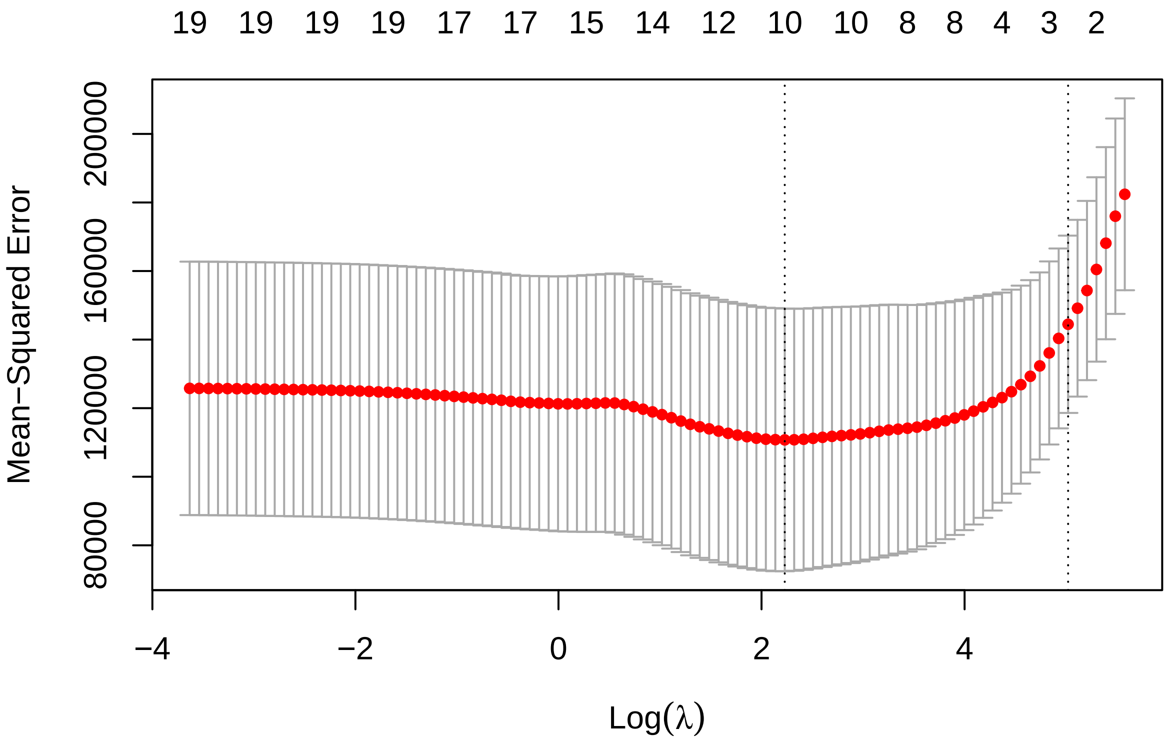
```
lasso.mod=glmnet(x[train,],y[train],alpha=1,lambda=grid)
plot(lasso.mod)
```

```
## Warning in regularize.values(x, y, ties, missing(ties), na.rm = na.rm):
## collapsing to unique 'x' values
```



Then we have

```
set.seed(1)
cv.out=cv.glmnet(x[train,],y[train],alpha=1)
plot(cv.out)
```




```
bestlam=cv.out$lambda.min
lasso.pred=predict(lasso.mod,s=bestlam,newx=x[test,])
mean((lasso.pred-y.test)^2)
```

```
## [1] 143673.6
```

This is substantially lower than the test set MSE of the null model and of least squares, and very similar to the test MSE of ridge regression with λ chosen by cross-validation.