2.1 Models of Observations and Models of Beliefs

Zongyi Liu

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0.0 Prologue

炎正中微，大盜移國。九縣飆回，三精霧塞。人厭淫詐，神思反德。

光武誕命，靈貺自甄。沈幾先物，深略緯文。尋邑百萬，貔虎為群。

長轂雷野，高鋒彗雲。英威既振，新都自焚。虔劉庸代，紛紜梁趙。

三河未澄，四關重擾。神旌乃顧，遞行天討。金湯失險，車書共道。

靈慶既啟，人謀咸贊。明明廟謨，赳赳雄斷。於赫有命，系隆我漢。

2.1 Models of Observations and Models of Beliefs

Suppose we have a coin from our friend the numistmatist1. We notice that on the ob- verse is embossed the head of Tanit (of ancient Carthage), and on the reverse side is embossed a horse. And we are considering if the coin is fair.

We have some assumptions about this coin. First, we have assumed that the coin has some inherent fairness or bias, that we can't directly observe. All we can actually observe is an inherently probabilistic effect of that bias, namely, whether the coin comes up heads or tails on any given flip.

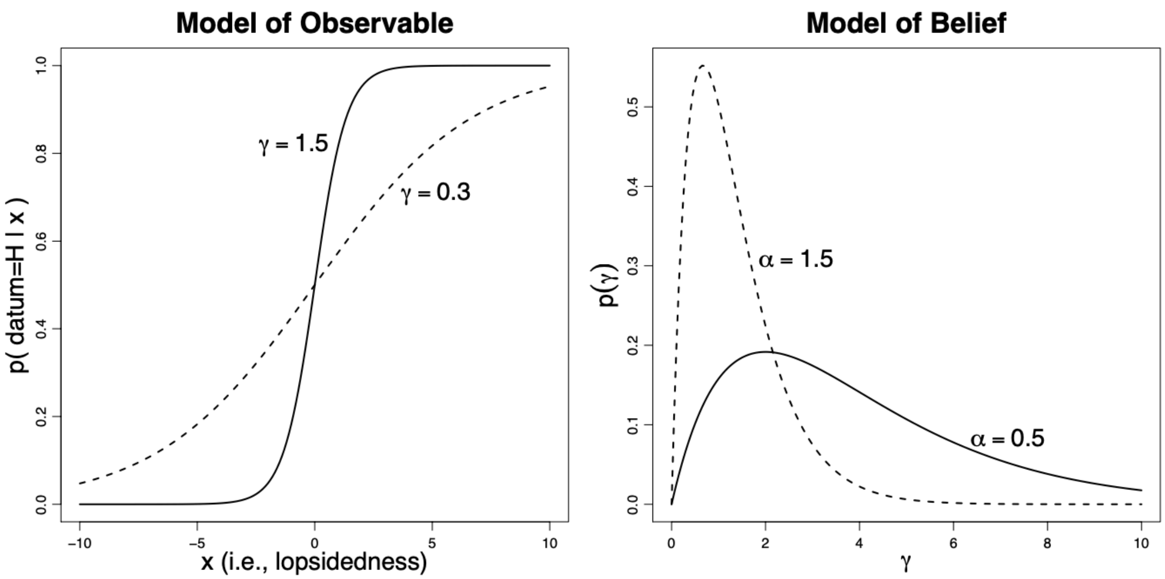
The second set of assumptions is about our beliefs regarding the bias of the coin.

When we want to get specific about our model assumptions, then we have to use mathematical descriptions. A "formal" model uses mathematical formulas to precisely describe something. In this book, we'll almost always be using formal models, and so whenever the term "model" comes up, you can assume it means a mathematical description.

2.1.1 Models have parameters

Consider a model of the probability that it will rain at a particular location. This model is a formula that generates a numerical probability as its output. The probability of rain depends on many things, but in particular it might depend on elevation above sea level. Thus, the probability of rain, which is the output of the model, depends on the location's elevation, which is a value that is input to the model. The exact relationship between input and output could be modulated by another value that governs exactly how much the input affects the output. This modulating value is called a **parameter**.

As another example, consider the probability that a coin comes up heads. We could model the probability of heads as a function of the lopsidedness of the coin. To measure lopsidedness, first consider slicing the coin like a bagel, exactly halfway between the head and tail faces. The lopsidedness is defined as the mass of the tail side minus the mass of the head side, measured in milligrams. Therefore, when lopsidedness is positive, the tail side is heavier, and heads are more likely to come up.



In summary, we can have a mathematical model of the probability that certain observable events happen. This mathematical model has parameters. The values of the parameters determine the exact probabilities generated by the model. Our beliefs regard the possible values of the parameters. We may believe strongly in some parameter values but less strongly in other values. The form of our beliefs about various parameter values can itself be expressed as a mathematical model, with its own (hyper-)parameters.

2.1.2 Prior and Posterior Beliefs

We could believe that the coin is fair, that is, that the probability of coming up heads is 50%. We could instead have other beliefs about the coin, especially if it's dated 350BCE, which no coin would be labeled if it were really minted BCE.

After flipping the coin and observing 2 heads in 10 flips, we will want to modify our beliefs. It makes sense that we should now believe more strongly that the bias is 20%, because we observed 20% heads in the sample.

Before observing the flips of the coin, we had certain beliefs about the possible biases of the coin. This is called a **prior** belief because it's our belief before taking into account some particular set of observations. After observing the flips of the coin, we had modified beliefs. These are called a **posterior** belief because they are computed after taking into account a particular set of observations. Bayesian inference gets us from prior to posterior beliefs.