7.3 Basis Functions

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7.3 Basic Functions

Polynomial and piecewise-constant regression models are special cases of a basis function approach. The idea is to have a family of functions or transformations that can be applied to a variable X: $b_1(X), b_2(X), \ldots, b_K(X)$, instead of fitting a linear model, we fit the model

$$y_i = \beta_0 + \beta_1 b_1(x_i) + \beta_2 b_2(x_i) + \beta_3 b_3(x_i) + \ldots + \beta_K b_K(x_i) + \epsilon_i.$$

Here the basic functions $b_1(\mathring{u})$, $b_2(\mathring{u}),\ldots,$ $b_K(\mathring{u})$ are fixed and unknown.

For polynomial regression, the basis functions are $b_i(x_j) = x_i^j$.

For piecewise constant functions, the basis functions are $b_j(x_j) = I(c_j \leq x_i < c_{j+1})$.