8.3 The Posterior via Grid Approximation

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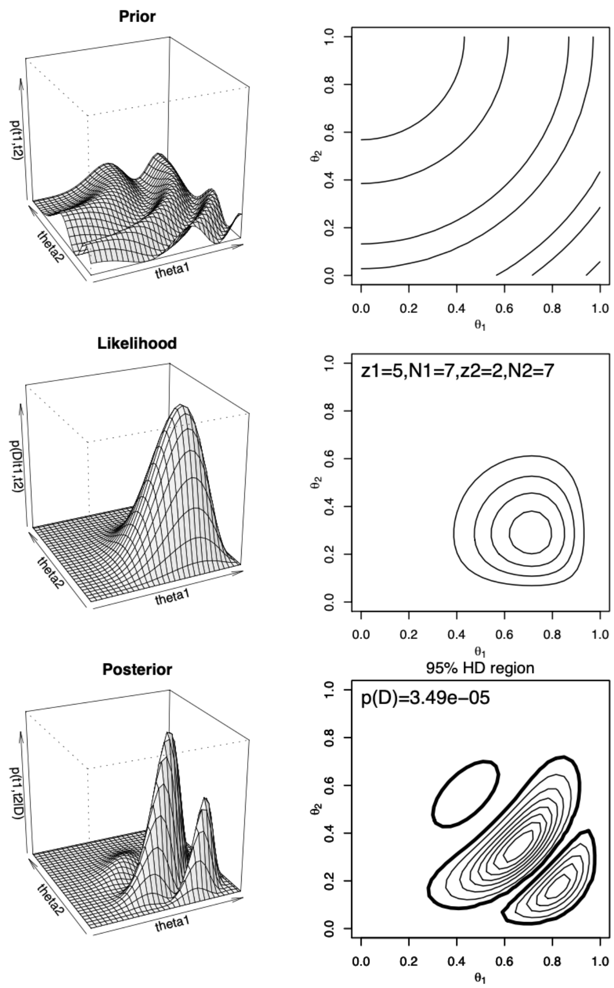
When the parameter space is samll enough, we can approximate the integral in the denominator of Bayes' rule by a sum over densely placed points in the parameter space. The continuous form of Bayes' rule would become:

$$\begin{aligned} p(\theta_1, \theta_2|D) &= p(D|\theta_1, \theta_2)p(\theta_1, \theta_2) \bigg/ \iint d\theta_1 d\theta_2 p(D|\theta_1, \theta_2)p(\theta_1, \theta_2) \\ &\approx p(D|\theta_1, \theta_2)p(\theta_1, \theta_2) \bigg/ \sum_{\theta_1} \sum_{\theta_2} p(D|\theta_1, \theta_2)p(\theta_1, \theta_2) \end{aligned}$$

There are two advantages of using a grid approximation:

1. We not rely on formal analysis to derive a specification of the posterior distribution, therefor we can specify any prior distribution we like, and still come up with an approximation of the posterior distribution.
2. A highest density region can be approximated for any posterior distribution, for finding a multidimensional highest density region by formal analysis can be challenging, to say the least, but approximating one from a grid approximation is easy.

With grid approximation, even unusual prior distributions can be used:


Prior
p(t1,t2)
theta2
theta1
θ2
θ1
Likelihood
p(D|t1,t2)
z1=5,N1=7,z2=2,N2=7
Posterior
p(t1,t2|D)
95% HD region
p(D)=3.49e−05