5.1 The Likelihood Function: Bernoulli Distribution

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2023-06-09

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In this section we introduced a new distribution: Bernoulli distribution, which can be expressed as

$$p(y|\theta) = \theta^y(1-\theta)^{(1-y)}$$

for y in the set $\{1, 0\}$ and θ in the interval $[0, 1]$. When $y = 1$, the righthand side of the equation above reduces to θ, and when $y = 0$, the righthand side of the equation above reduces to $1 - \theta$.

The equation above demonstrates the Bernoulli distribution. We can show the sum of the probability:

$$\sum_y p(y)$$

Another perspective on Equation 5.1 is to think of the data value y as fixed by an observation, and the value of θ as variable. Equation 5.1 then specifies the probability of the fixed y value if θ has some particular value. Different values of θ yield different probabilities of the datum y. When thought of in this way, Equation 5.1 is the likelihood function of θ.

When we flip the coin N times, we have a set of data, $D = \{y_1, ..., y_N\}$, where each y_i is 0 or 1. By assumption, each flip is independent of the others. Therefore, the probability of getting the set of N flips $D = \{y_1, ..., y_N\}$ is the product of the individual outcome probabilities:

$$p(\{y_1, ..., y_N\}|\theta) = \Pi_i p(y_i|\theta) = \Pi_i \theta^{y_i}(1-\theta)^(1-y_i)$$