Homework 2, STAT 5241

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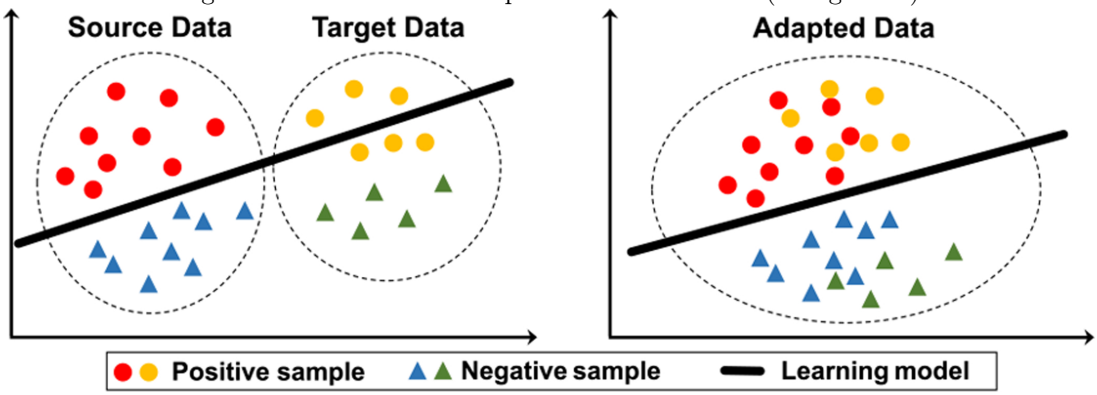
1 Questions

1.1 Question 1

The main problem for the team is predicting autism in a European population using a model trained on US data; they must design their ML process to account for the significant differences in key features such as ethnicity, demographics, and environmental factors, which are expected to be different in Europe. The first step involves thorough data preprocessing to align and normalize features across the two datasets, ensuring consistency and handling missing values through imputation. And the idea thing is to preprocess the data of Europe as much as like the data for US, as the paper by Mehrabi et al. mentioned suggested that biases might occur in many places.

I then did some search on the Internet and found it was called **domain adaptation**, where model is trained on one dataset and tested on another one. As illustrated in Wang et al.'s paper (2020).

Figure 1: Scenario which requires "Domain Shift" (Wang et al.)



In this paper, the author mentioned 4 methods, and I think would be helpful for this scenario:

- Instance Weighting, which assigns weights to source domain samples based on their relevance to the target domain, ensuring that the model focuses more on source instances that are similar to target instances.
- Feature Transformation: Techniques in this category aim to map data from both domains into a common feature space where their distributions are more aligned, facilitating better model performance across domains.
- Adversarial Training: Inspired by Generative Adversarial Networks (GANs), adversarial training methods involve a domain discriminator that encourages the feature extractor to learn representations indistinguishable between domains, promoting domain-invariant features.
- Reconstruction-Based Methods; they utilize autoencoders or similar architectures to reconstruct input data, ensuring that the learned features capture essential characteristics applicable across domains.

As for hyper-parameter tuning, it should be conducted using K-fold cross-validation on the US dataset (training set), with attention to ensuring the validation splits are representative. If a small subset of European data is available, it can be used for fine-tuning to improve generalization. Bayesian optimization or grid search can efficiently explore the hyperparameter space, focusing on key parameters such as regularization terms, learning rates, and model complexity. For evaluation, the primary metric should be the F1 score, which balances precision and recall and is particularly

suitable for imbalanced datasets. Secondary metrics like AUC-ROC, precision, recall, and confusion matrices provide additional insights into model performance, while calibration metrics such as the Brier score ensure probabilistic predictions are reliable.

During testing, the model should be evaluated on the European dataset without further tuning, or rather, **fine-tuning**, and performance metrics should be reported alongside cross-validation results from the US dataset. Error analysis is crucial to identify misclassified cases and understand whether errors stem from feature distribution differences or other factors. Additionally, the team must consider bias and fairness, ensuring the model does not disproportionately misclassify specific subgroups, and prioritize interpretability using methods like SHAP (SHapley Additive exPlanations) or LIME (Local Interpretable Model-agnostic Explanations) to align predictions with clinical knowledge.

By integrating these steps, the team can develop a model that generalizes effectively to the European population while trying to maximize the robustness, fairness, and transparency of their model.

1.2 Question 2

1.2.1 Part A

We need to pre-process the data. There are columns with many missing values, which should be considered to drop first. Then we can drop rows with missing values. If we simply drop rows with missing data, or drop the rows with missing data before deleting columns with significant number of missing data, it will distorted the regression results.

The result I got using self-coded K-fold CV from scratch is as below:

```
Best RBF Kernel Parameters: lambda=0.01, gamma=0.1
Best Polynomial Kernel Parameters: lambda=0.01, degree=2
RBF Kernel Test MSE: 0.014702546829905011
Polynomial Kernel Test MSE: 0.009525269150252024
```

1.2.2 Part B

And comparing the results of scratch code and `sklearn` code, the hyperparameters are the same:

```
Custom RBF Kernel Parameters: lambda=0.01, gamma=0.1
Custom Polynomial Kernel Parameters: lambda=0.01, degree=2
scikit-learn RBF Kernel Parameters: {'alpha': 0.01, 'gamma': 0.1}
scikit-learn Polynomial Kernel Parameters: {'alpha': 0.01, 'degree': 2}
```

The tuned parameters are the same for the built-in model of `sklearn` and the cross-validation I coded up by myself. Plugging those hyper-parameters back, we would get the same MSE as we did in Part A. For RBF Kernel, the best-tuned parameter are $\lambda = 0.01$ and $\gamma = 0.1$, and for Polynomial Kernel, the best-tuned parameters are $\lambda = 0.01$, $degree = 2$.

1.2.3 Part C

I redid the OLS model in Homework 1, and calculated the MSE using code below

```
    residuals = y_new - y_pred

    # Calculate MSE
    mse = np.mean(residuals**2)
```

The result is as below:

```
Mean Squared Error (MSE): 0.0165
```

Here the non-linear model performed better than the linear model on this dataset. First of all, linear model is a simple way to fit the data, which is highly likely to be influenced by the noise in the dataset, even though it might has lower MSE. It might cause the problem of overfitting. Whereas the CV can split the dataset into several subsets, and evaluate the model on multiple test and training sets, which would make it much generalizable, and overcome the problem of overfitting. And as we mentioned before, linear models just build the model based on data in the dataset, if the training set is not representative, it might not be a good model; but the CV is more generalizable, giving it ability to comprehensively reflect model performance.

Secondly, CV has hyper-parameters tuning, from which the optimal hyper-parameters can be selected, improving model performance.

Another aspect is that OLS lacks a regularization mechanism, making it susceptible to issues like high-dimensional data or multicollinearity. Where as we implemented regularization here, which helps control model complexity, prevent overfitting, and improve generalization. As for the efficiency to use data, the OLS just use one set of data, whereas CV divides the data multiple times, ensuring that all data points are used for both training and validation.

Finally, if we simply compare the MSE, which is the prediction errors given by different models, the MSE of cross-validated models are still smaller than those of the OLS model.

1.3 Question 3

1.3.1 Part A

Here I listed the confusion matrices, test accuracies given by python for 5 methods below. I did not generate the confusion matrix picture due to the restriction of page-count. Full reports, including `recall` rate, `f1-score`, etc, were inserted in the Appendix by me.

Firstly, for Logistic Regression in One-vs-Rest setting, I got the report:

```
Test Accuracy: 93.54%
Confusion Matrix:
[[1328   39   52]
[  54 1195   45]
[  39   50 1254]]
```

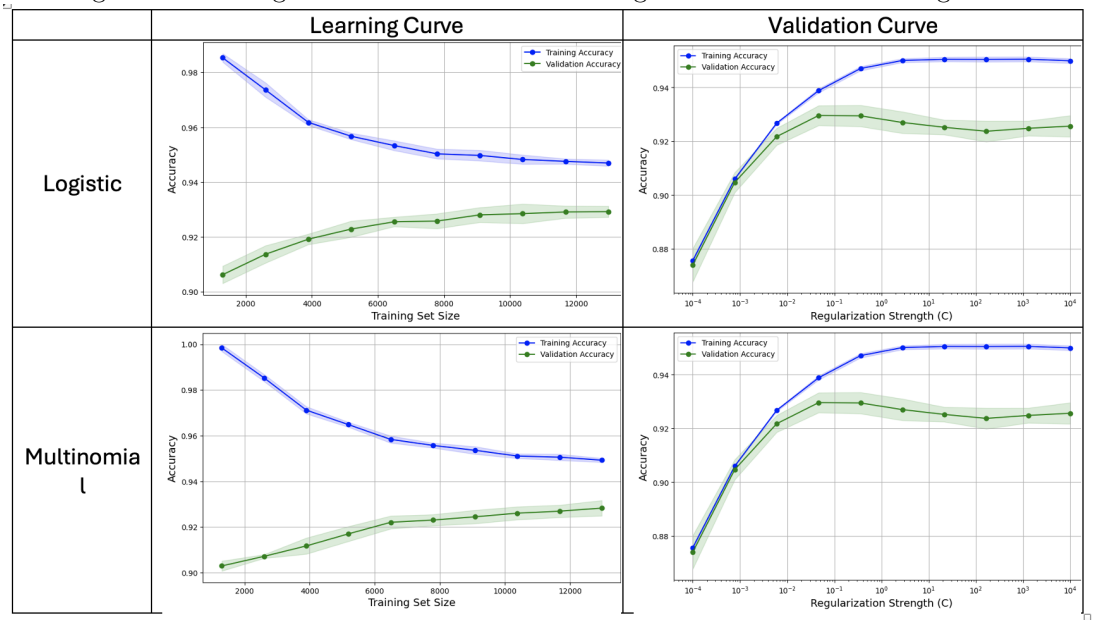
For Multinomial Regression:

```
Test Accuracy: 93.12%
Confusion Matrix:
[[1324   43   52]
[  51 1201   42]
[  39   52 1252]]
```

Because in this case, the dimension of the dataset is too high, which is 784, as the number of pixels in the set. So we can not print the decision boundary as the lab did without using Reducing Dimension methods, like Principle Component Analysis (PCA). I did PCA and put pictures in later part.

Here I plotted the learning and validation curves for logistic and multinomial regressions, in two plots, both training and validation errors are low and similar, which means the model is well-generalized. We can also use the validation curve to find that over 10^{-1}, there will be a problem of training accuracy increases while validation accuracy being low, and the tuned parameter should be chosen before the divergence of those two lines.

Figure 2: Learning and Validation Curves for Logistic and Multinomial Regression



For Naive Bayes (also not possible to plot the decision boundary of Naive Bayes without Dimension Reduction):

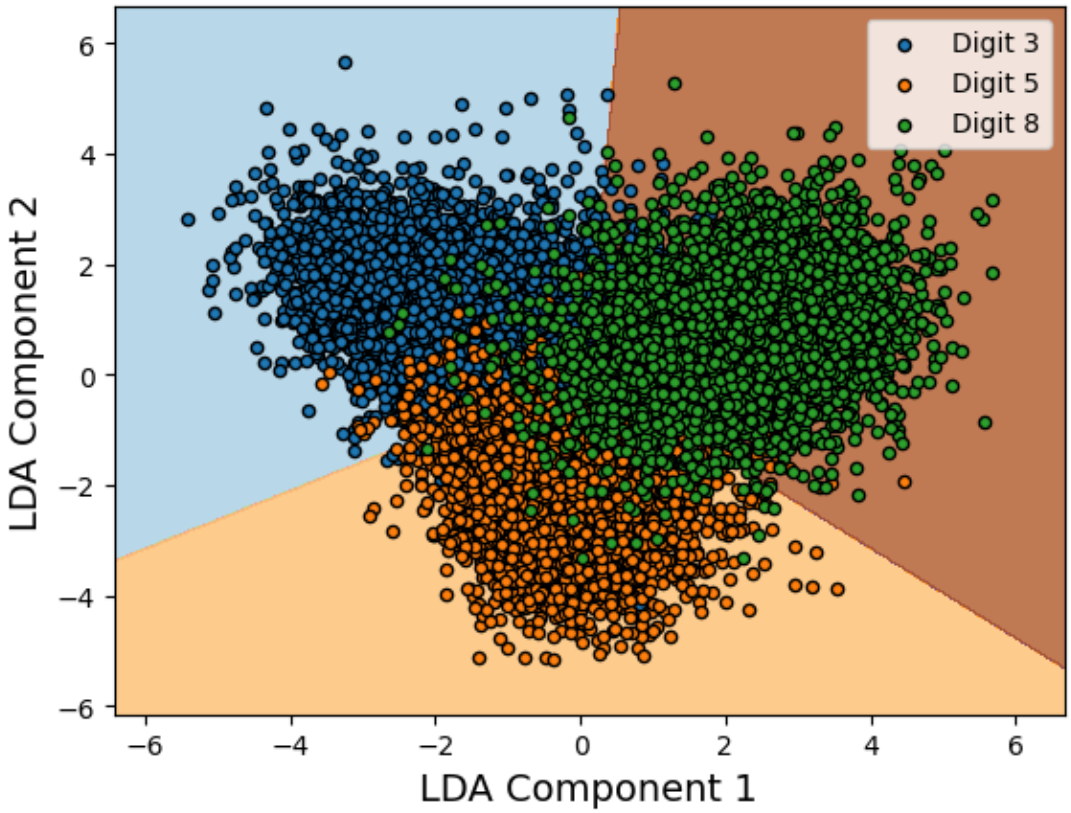
```
Test Accuracy: 50.22%
Confusion Matrix:
[[ 595   26  798]
[  95  138 1061]
[  18   21 1304]]
```

For Linear Discriminant Analysis:

```
Test Accuracy (LDA): 91.69%
Confusion Matrix (LDA):
[[1285   69   65]
[  47 1202   45]
[  34   77 1232]]
```

In LDA, we computes the between-class scatter matrix and within-class scatter matrix to find directions that maximize class separation, instead of the covariance matrix of the data and finds the principal components (eigenvectors) sorted by eigenvalues. Typically PCA are used for unsupervised tasks like data visualization, feature extraction, and noise reduction, whereas LDA mostly be used supervised tasks like classification and pattern recognition. Thus the decision boundary of LDA is linear, but PCA is not, which can be seen from two graphs below.

Figure 3: Decision Boundary after LDA

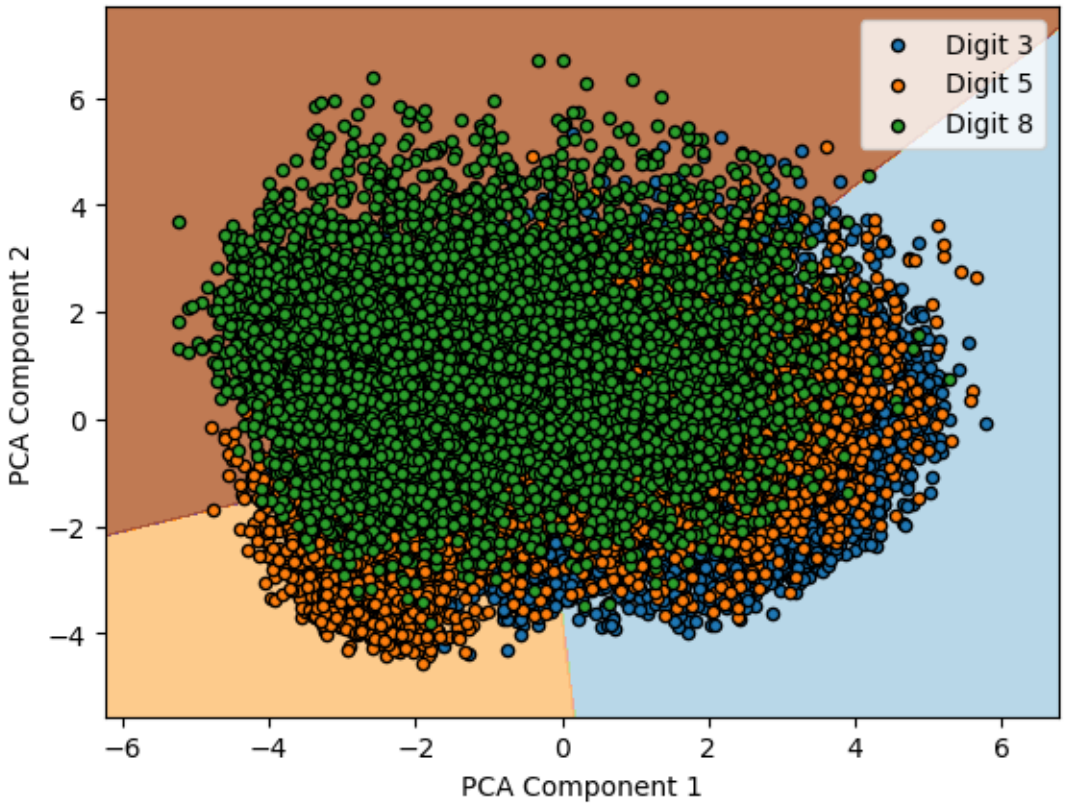


Finally, for Linear Support Vector Machine:

```
Test Accuracy (Linear SVM): 92.73%
Confusion Matrix (Linear SVM):
[[1319   44   56]
[  57 1193   44]
[  39   55 1249]]
```

If we want to plot the decision boundary of Linear SVM, we must first do a Principle Component Analysis (PCA).

Figure 4: Decision Boundary after PCA



Overall, if we directly compare the test accuracy evaluated by python, the logistic regression has the highest accuracy rate, whereas the Naive Bayes has the least accuracy.

In the python built-in function `confusion matrix()`, the vertical axis represents the true values, whereas the horizontal axis represents the predictive value. Thus we can added the misclassified terms accordingly. However, I noticed that there was a large proportion of data misclassified in Naive Bayes case, so I splitted them into two parts, firstly I calculated the total misinterpreted values for 4 methods other than Naive Bayes; here the most often misclassified digit is 3, and has 420 counts. And for 5 and 8, the numbers are equal, which is 385.

If we add up the NB method, overall, digit 5 is more likely to be misclassified, and this trend towards error reaches to peak when we use Naive Bayes method. The total number of misclassified 5 is 1542, and 3 is 1244, and 8 is 424.

Figure 5: Count of Misclassified Digits

	A	B	C	D	E	F	G	H	I
1		**miclassified**	**Logistic**	**Multinomial**	**LDA**	**LSVM**	**Total, except NB**	**Naïve Bayes**	**Total, with NB**
2		mis-classified as 5	39	43	69	44	195	26	221
3	**3**	mis-classified as 8	52	52	65	56	225	798	1023
4		total	91	95	134	100	420	824	1244
5		mis-classified as 3	54	51	47	57	209	96	305
6	**5**	mis-classified as 8	45	42	45	44	176	1061	1237
7		total	99	93	92	101	385	1157	1542
8		mis-classified as 3	39	39	34	39	151	18	169
9	**8**	mis-classified as 5	50	52	77	55	234	21	255
10		total	89	91	111	94	385	39	424
11									

1.3.2 Part B

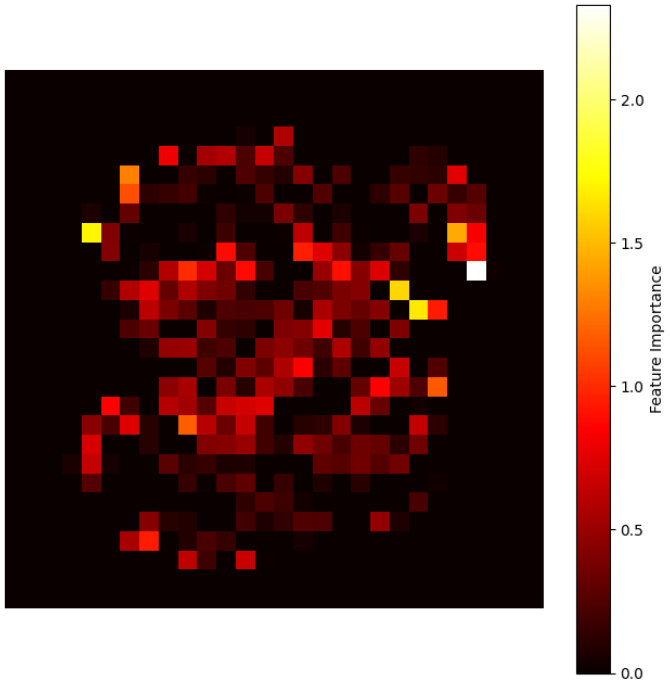
The group-lasso regularized multinomial logistic regression will help us to select features that separate the three digits,

We can flatten the picture with 28*28 pixels into 784 features, and use the regression select important features that separate 3, 5, and 8. I put the print-out in appendix, and there are overall 8 important features.

For visualization, we can end up getting a heatmap. In the heatmap, bright regions represent pixels that are highly important for distinguishing between the digits 3, 5, and 8. The model relies heavily on these pixels for making predictions.

And dark regions represent pixels that contribute little to the classification task. The model considers these pixels unimportant.

Figure 6: Headmap showing Features that Separate the Three Digits



2 Appendix

Supplementary Github Repo

2.1 Question 2

2.1.1 Setup

```
# Same as Homework 1

from ucimlrepo import fetch_ucirepo


# fetch dataset
communities_and_crime = fetch_ucirepo(id=183)

# data (as pandas dataframes)
X = communities_and_crime.data.features
y = communities_and_crime.data.targets

# metadata
print(communities_and_crime.metadata)

# variable information
print(communities_and_crime.variables)

# Inspect the shape of X and y
print(X.shape)  # Should be (1994, 127)
print(y.shape)  # Should be (1994, 1)

# Check for missing values
print(X.isnull().sum())  # Count of missing values per feature

# Inspect the first few rows of X and y
print(X.head())
print(y.head())
X = X.iloc[:, 5:]
print(X.dtypes) # There are object columns within the data. The object data type is
    the default type for columns containing text (strings) in a pandas DataFrame.
```

Drop columns and rows with missing values:

```
# Convert all values to numeric (force non-numeric to NaN)
X = X.applymap(pd.to_numeric, errors='coerce')

# Replace "?" with NaN
X.replace("?", np.nan, inplace=True)

# Check the number of missing values in each column
missing_counts = X.isnull().sum()

# Determine threshold for column removal (e.g., remove if >50% missing)
threshold = 0.5 * len(X)  # Adjust this threshold as needed
cols_to_drop = missing_counts[missing_counts > threshold].index

# Drop those columns
X_cleaned = X.drop(columns=cols_to_drop)

X_with_y = pd.concat([X_cleaned, y], axis=1)

# Remove rows containing missing values in the combined DataFrame
X_with_y_cleaned = X_with_y.dropna()

```

```
    X_with_y_cleaned

    # Separate X and y after cleaning
    X_final = X_with_y_cleaned.drop(columns=['ViolentCrimesPerPop'])
    y_final = X_with_y_cleaned['ViolentCrimesPerPop']
```

Standardize it:

```
    Y = y_final.to_numpy()

    # Standardize X (centered and scaled)
    scaler = StandardScaler(with_mean=True, with_std=True)
    X_standardized = scaler.fit_transform(X_final)
```

2.1.2 Split into Different Sets

```
    # Set a random seed for reproducibility
    rng = default_rng(1)

    # Define the proportions for the split
    train_prop = 0.6
    validation_prop = 0.2
    test_prop = 0.2

    # Calculate the number of observations for each split
    total_samples = X_with_y_cleaned.shape[0]
    train_size = int(train_prop * total_samples)
    validation_size = int(validation_prop * total_samples)
    test_size = total_samples - train_size - validation_size

    # Create a random permutation of row indices
    indices = rng.choice(np.arange(total_samples), size=(total_samples), replace=False)

    # Split the dataset into train, validation, and test sets
    y_train = Y[indices[:train_size]]
    y_val = Y[indices[(train_size + 1):(train_size + validation_size)]]
    y_test = Y[indices[(train_size + validation_size + 1):]]

    X_train = X_standardized[indices[:train_size]]
    X_val = X_standardized[indices[(train_size + 1):(train_size + validation_size)]]
    X_test = X_standardized[indices[(train_size + validation_size + 1):]]
```

2.1.3 Using K-fold CV from scratch to tune Kernel Ridge Regularization

```
    import numpy as np
    from sklearn.metrics import mean_squared_error

    # Define the RBF kernel function
    def rbf_kernel(X1, X2, gamma):
    pairwise_dists = np.sum(X1**2, axis=1).reshape(-1, 1) + np.sum(X2**2, axis=1) - 2 *
        np.dot(X1, X2.T)
    return np.exp(-gamma * pairwise_dists)

    # Define the Polynomial kernel function
    def polynomial_kernel(X1, X2, degree, c=1):
    return (np.dot(X1, X2.T) + c) ** degree

    # Kernel Ridge Regression
    def kernel_ridge_regression(X_train, y_train, X_test, kernel, lambda_, **
        kernel_params):
    """
```

```
    Perform Kernel Ridge Regression.

    Parameters:
    X_train: Training data (n_samples, n_features).
    y_train: Target values (n_samples,).
    X_test: Test data (n_samples_test, n_features).
    kernel: Kernel function (rbf_kernel or polynomial_kernel).
    lambda_: Regularization parameter.
    **kernel_params: Kernel-specific parameters (e.g., gamma for RBF, degree for
        Polynomial).

    Returns:
    y_pred: Predicted values for X_test.
    """
    # Compute the kernel matrix
    K_train = kernel(X_train, X_train, **kernel_params)
    K_test = kernel(X_test, X_train, **kernel_params)

    # Solve for the dual coefficients alpha
    n_samples = X_train.shape[0]
    alpha = np.linalg.inv(K_train + lambda_ * np.eye(n_samples)) @ y_train

    # Predict on the test set
    y_pred = K_test @ alpha
    return y_pred

    # K-Fold Cross-Validation
    def k_fold_cross_validation(X_standardized, Y, k, kernel, lambda_range, gamma_range=
        None, degree_range=None):
    """
    Perform K-fold cross-validation to select the best hyperparameters.

    Parameters:
    X: Input data (n_samples, n_features).
    y: Target values (n_samples,).
    k: Number of folds.
    kernel: Kernel function (rbf_kernel or polynomial_kernel).
    lambda_range: List of regularization parameters to try.
    gamma_range: List of gamma values for RBF kernel (optional).
    degree_range: List of degree values for Polynomial kernel (optional).

    Returns:
    best_lambda: Best regularization parameter.
    best_gamma: Best gamma value (for RBF kernel).
    best_degree: Best degree value (for Polynomial kernel).
    """
    fold_size = len(X_standardized) // k
    best_lambda = None
    best_gamma = None
    best_degree = None
    best_score = float('inf')

    # Grid search over hyperparameters
    for lambda_ in lambda_range:
    if kernel == rbf_kernel:
    # RBF kernel: only gamma is needed
    for gamma in (gamma_range if gamma_range is not None else [1.0]):
    scores = []
    for i in range(k):
    # Split into training and validation sets
    val_indices = range(i * fold_size, (i + 1) * fold_size)
    train_indices = np.setdiff1d(range(len(X)), val_indices)

```

```
    X_train, y_train = X[train_indices], y[train_indices]
    X_val, y_val = X[val_indices], y[val_indices]

    # Train and predict
    y_pred = kernel_ridge_regression(X_train, y_train, X_val, kernel, lambda_, gamma=
        gamma)

    # Compute the validation score (e.g., mean squared error)
    score = mean_squared_error(y_val, y_pred)
    scores.append(score)

    # Average score across folds
    avg_score = np.mean(scores)
    if avg_score < best_score:
    best_score = avg_score
    best_lambda = lambda_
    best_gamma = gamma

    elif kernel == polynomial_kernel:
    # Polynomial kernel: degree and optionally c are needed
    for degree in (degree_range if degree_range is not None else [2]):
    scores = []
    for i in range(k):
    # Split into training and validation sets
    val_indices = range(i * fold_size, (i + 1) * fold_size)
    train_indices = np.setdiff1d(range(len(X)), val_indices)

    X_train, y_train = X[train_indices], y[train_indices]
    X_val, y_val = X[val_indices], y[val_indices]

    # Train and predict
    y_pred = kernel_ridge_regression(X_train, y_train, X_val, kernel, lambda_, degree=
        degree)

    # Compute the validation score (e.g., mean squared error)
    score = mean_squared_error(y_val, y_pred)
    scores.append(score)

    # Average score across folds
    avg_score = np.mean(scores)
    if avg_score < best_score:
    best_score = avg_score
    best_lambda = lambda_
    best_degree = degree

    return best_lambda, best_gamma, best_degree

    # Example usage
    if __name__ == "__main__":
    # Generate synthetic data
    np.random.seed(42)
    X = np.random.rand(100, 2)  # 100 samples, 2 features
    y = 3 * X[:, 0] + 5 * X[:, 1] + np.random.randn(100) * 0.1  # Linear relationship
        with noise

    # Split data into train, validation, and test sets
    train_size = int(0.6 * len(X))
    val_size = int(0.2 * len(X))
    test_size = len(X) - train_size - val_size

    indices = np.random.permutation(len(X))
    X_train, y_train = X[indices[:train_size]], y[indices[:train_size]]
```

```
    X_val, y_val = X[indices[train_size:train_size + val_size]], y[indices[train_size:
        train_size + val_size]]
    X_test, y_test = X[indices[train_size + val_size:]], y[indices[train_size + val_size
        :]]

    # Define hyperparameter ranges
    lambda_range = [0.01, 0.1, 1, 10]
    gamma_range = [0.01, 0.1, 1]  # For RBF kernel
    degree_range = [2, 3, 4, 5, 6, 7]      # For Polynomial kernel

    # Perform K-fold cross-validation for RBF kernel
    best_lambda_rbf, best_gamma_rbf, _ = k_fold_cross_validation(
    X_train, y_train, k=5, kernel=rbf_kernel, lambda_range=lambda_range, gamma_range=
        gamma_range
    )
    print(f"Best RBF Kernel Parameters: lambda={best_lambda_rbf}, gamma={best_gamma_rbf}"
        )

    # Perform K-fold cross-validation for Polynomial kernel
    best_lambda_poly, _, best_degree_poly = k_fold_cross_validation(
    X_train, y_train, k=5, kernel=polynomial_kernel, lambda_range=lambda_range,
        degree_range=degree_range
    )
    print(f"Best Polynomial Kernel Parameters: lambda={best_lambda_poly}, degree={
        best_degree_poly}")

    # Evaluate on the test set with the best parameters
    y_pred_rbf = kernel_ridge_regression(X_train, y_train, X_test, rbf_kernel,
        best_lambda_rbf, gamma=best_gamma_rbf)
    y_pred_poly = kernel_ridge_regression(X_train, y_train, X_test, polynomial_kernel,
        best_lambda_poly, degree=best_degree_poly)

    print(f"RBF Kernel Test MSE: {mean_squared_error(y_test, y_pred_rbf)}")
    print(f"Polynomial Kernel Test MSE: {mean_squared_error(y_test, y_pred_poly)}")
```

2.1.4 Redo CV Using sklearn

```
    import numpy as np
    from sklearn.metrics import mean_squared_error
    from sklearn.model_selection import GridSearchCV, KFold
    from sklearn.kernel_ridge import KernelRidge

    # Define the RBF kernel function
    def rbf_kernel(X1, X2, gamma):
    pairwise_dists = np.sum(X1**2, axis=1).reshape(-1, 1) + np.sum(X2**2, axis=1) - 2 *
        np.dot(X1, X2.T)
    return np.exp(-gamma * pairwise_dists)

    # Define the Polynomial kernel function
    def polynomial_kernel(X1, X2, degree, c=1):
    return (np.dot(X1, X2.T) + c) ** degree

    # Kernel Ridge Regression
    def kernel_ridge_regression(X_train, y_train, X_test, kernel, lambda_, **
        kernel_params):
    """
    Perform Kernel Ridge Regression.

    Parameters:
    X_train: Training data (n_samples, n_features).
    y_train: Target values (n_samples,).
    X_test: Test data (n_samples_test, n_features).
```

```
    kernel: Kernel function (rbf_kernel or polynomial_kernel).
    lambda_: Regularization parameter.
    **kernel_params: Kernel-specific parameters (e.g., gamma for RBF, degree for
        Polynomial).

    Returns:
    y_pred: Predicted values for X_test.
    """
    # Compute the kernel matrix
    K_train = kernel(X_train, X_train, **kernel_params)
    K_test = kernel(X_test, X_train, **kernel_params)

    # Solve for the dual coefficients alpha
    n_samples = X_train.shape[0]
    alpha = np.linalg.inv(K_train + lambda_ * np.eye(n_samples)) @ y_train

    # Predict on the test set
    y_pred = K_test @ alpha
    return y_pred

    # K-Fold Cross-Validation
    def k_fold_cross_validation(X, y, k, kernel, lambda_range, gamma_range=None,
        degree_range=None):
    """
    Perform K-fold cross-validation to select the best hyperparameters.

    Parameters:
    X: Input data (n_samples, n_features).
    y: Target values (n_samples,).
    k: Number of folds.
    kernel: Kernel function (rbf_kernel or polynomial_kernel).
    lambda_range: List of regularization parameters to try.
    gamma_range: List of gamma values for RBF kernel (optional).
    degree_range: List of degree values for Polynomial kernel (optional).

    Returns:
    best_lambda: Best regularization parameter.
    best_gamma: Best gamma value (for RBF kernel).
    best_degree: Best degree value (for Polynomial kernel).
    """
    fold_size = len(X) // k
    best_lambda = None
    best_gamma = None
    best_degree = None
    best_score = float('inf')

    # Grid search over hyperparameters
    for lambda_ in lambda_range:
    if kernel == rbf_kernel:
    # RBF kernel: only gamma is needed
    for gamma in (gamma_range if gamma_range is not None else [1.0]):
    scores = []
    for i in range(k):
    # Split into training and validation sets
    val_indices = range(i * fold_size, (i + 1) * fold_size)
    train_indices = np.setdiff1d(range(len(X)), val_indices)

    X_train, y_train = X[train_indices], y[train_indices]
    X_val, y_val = X[val_indices], y[val_indices]

    # Train and predict
    y_pred = kernel_ridge_regression(X_train, y_train, X_val, kernel, lambda_, gamma=
        gamma)
```

```
    # Compute the validation score (e.g., mean squared error)
    score = mean_squared_error(y_val, y_pred)
    scores.append(score)

    # Average score across folds
    avg_score = np.mean(scores)
    if avg_score < best_score:
    best_score = avg_score
    best_lambda = lambda_
    best_gamma = gamma

    elif kernel == polynomial_kernel:
    # Polynomial kernel: degree and optionally c are needed
    for degree in (degree_range if degree_range is not None else [2]):
    scores = []
    for i in range(k):
    # Split into training and validation sets
    val_indices = range(i * fold_size, (i + 1) * fold_size)
    train_indices = np.setdiff1d(range(len(X)), val_indices)

    X_train, y_train = X[train_indices], y[train_indices]
    X_val, y_val = X[val_indices], y[val_indices]

    # Train and predict
    y_pred = kernel_ridge_regression(X_train, y_train, X_val, kernel, lambda_, degree=
        degree)

    # Compute the validation score (e.g., mean squared error)
    score = mean_squared_error(y_val, y_pred)
    scores.append(score)

    # Average score across folds
    avg_score = np.mean(scores)
    if avg_score < best_score:
    best_score = avg_score
    best_lambda = lambda_
    best_degree = degree

    return best_lambda, best_gamma, best_degree

    # Example usage
    if __name__ == "__main__":
    # Generate synthetic data
    np.random.seed(42)
    X = np.random.rand(100, 2)  # 100 samples, 2 features
    y = 3 * X[:, 0] + 5 * X[:, 1] + np.random.randn(100) * 0.1  # Linear relationship
        with noise

    # Split data into train, validation, and test sets
    train_size = int(0.6 * len(X))
    val_size = int(0.2 * len(X))
    test_size = len(X) - train_size - val_size

    indices = np.random.permutation(len(X))
    X_train, y_train = X[indices[:train_size]], y[indices[:train_size]]
    X_val, y_val = X[indices[train_size:train_size + val_size]], y[indices[train_size:
        train_size + val_size]]
    X_test, y_test = X[indices[train_size + val_size:]], y[indices[train_size + val_size
        :]]

    # Define hyperparameter ranges
    lambda_range = [0.01, 0.1, 1, 10]
```

```
    gamma_range = [0.01, 0.1, 1]  # For RBF kernel
    degree_range = [2, 3, 4]      # For Polynomial kernel

    # Perform K-fold cross-validation for RBF kernel (custom implementation)
    best_lambda_rbf_custom, best_gamma_rbf_custom, _ = k_fold_cross_validation(
    X_train, y_train, k=5, kernel=rbf_kernel, lambda_range=lambda_range, gamma_range=
        gamma_range
    )
    print("Custom RBF Kernel Parameters: lambda={}, gamma={}".format(
        best_lambda_rbf_custom, best_gamma_rbf_custom))

    # Perform K-fold cross-validation for Polynomial kernel (custom implementation)
    best_lambda_poly_custom, _, best_degree_poly_custom = k_fold_cross_validation(
    X_train, y_train, k=5, kernel=polynomial_kernel, lambda_range=lambda_range,
        degree_range=degree_range
    )
    print("Custom Polynomial Kernel Parameters: lambda={}, degree={}".format(
        best_lambda_poly_custom, best_degree_poly_custom))

    # Use scikit-learn's GridSearchCV for RBF kernel
    krr_rbf = KernelRidge(kernel='rbf')
    param_grid_rbf = {'alpha': lambda_range, 'gamma': gamma_range}
    grid_search_rbf = GridSearchCV(krr_rbf, param_grid_rbf, cv=KFold(n_splits=5), scoring
        ='neg_mean_squared_error')
    grid_search_rbf.fit(X_train, y_train)
    print("scikit-learn RBF Kernel Parameters:", grid_search_rbf.best_params_)

    # Use scikit-learn's GridSearchCV for Polynomial kernel
    krr_poly = KernelRidge(kernel='poly')
    param_grid_poly = {'alpha': lambda_range, 'degree': degree_range}
    grid_search_poly = GridSearchCV(krr_poly, param_grid_poly, cv=KFold(n_splits=5),
        scoring='neg_mean_squared_error')
    grid_search_poly.fit(X_train, y_train)
    print("scikit-learn Polynomial Kernel Parameters:", grid_search_poly.best_params_)

    # Compare results
    print("\nComparison of Custom and scikit-learn Hyperparameters:")
    print("RBF Kernel:")
    print(f"Custom: lambda={best_lambda_rbf_custom}, gamma={best_gamma_rbf_custom}")
    print(f"scikit-learn: {grid_search_rbf.best_params_}")

    print("\nPolynomial Kernel:")
    print(f"Custom: lambda={best_lambda_poly_custom}, degree={best_degree_poly_custom}")
    print(f"scikit-learn: {grid_search_poly.best_params_}")
```

2.1.5 Redo the OLS as in Homework 1

```
    # Step 1: Separate y and X
    X_new = X_final
    y_new = y_final

    # Step 2: Add a constant to X (for the intercept term)
    X_new = sm.add_constant(X_new)

    # Step 3: Fit the OLS model
    model = sm.OLS(y_new, X_new)

    results = model.fit()

    # Step 4: View the results
    print(results.summary())
```

OLS Regression Results

Dep. Variable:	ViolentCrimesPerPop	R-squared:	0.696
Model:	OLS	Adj. R-squared:	0.680
Method:	Least Squares	F-statistic:	43.26
Date:	Fri, 07 Mar 2025	Prob (F-statistic):	0.00
Time:	13:15:31	Log-Likelihood:	1261.1
No. Observations:	1993	AIC:	-2320.
Df Residuals:	1892	BIC:	-1755.
Df Model:	100		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.5504	0.203	2.712	0.007	0.152	0.948
population	0.1840	0.397	0.463	0.643	-0.595	0.963
householdsize	-0.0223	0.086	-0.259	0.796	-0.191	0.147
racepctblack	0.2049	0.051	4.008	0.000	0.105	0.305
racePctWhite	-0.0492	0.059	-0.837	0.403	-0.164	0.066
racePctAsian	-0.0144	0.034	-0.420	0.674	-0.082	0.053
racePctHisp	0.0609	0.053	1.139	0.255	-0.044	0.166
agePct12t21	0.1104	0.106	1.043	0.297	-0.097	0.318
agePct12t29	-0.2292	0.156	-1.467	0.143	-0.536	0.077
agePct16t24	-0.1302	0.164	-0.793	0.428	-0.452	0.192
agePct65up	0.0497	0.103	0.481	0.630	-0.153	0.253
numbUrban	-0.2964	0.387	-0.766	0.444	-1.055	0.462
pctUrban	0.0467	0.016	2.989	0.003	0.016	0.077
medIncome	-0.1998	0.173	-1.158	0.247	-0.538	0.139
pctWWage	-0.2016	0.089	-2.259	0.024	-0.377	-0.027
pctWFarmSelf	0.0488	0.020	2.422	0.016	0.009	0.088
pctWInvInc	-0.1731	0.068	-2.563	0.010	-0.306	-0.041
pctWSocSec	0.0762	0.107	0.712	0.477	-0.134	0.286
pctWPubAsst	0.0050	0.046	0.108	0.914	-0.085	0.095
pctWRetire	-0.0900	0.037	-2.445	0.015	-0.162	-0.018
medFamInc	0.2880	0.160	1.797	0.073	-0.026	0.602
perCapInc	0.0955	0.189	0.506	0.613	-0.274	0.465
whitePerCap	-0.3510	0.152	-2.303	0.021	-0.650	-0.052
blackPerCap	-0.0288	0.025	-1.131	0.258	-0.079	0.021
indianPerCap	-0.0357	0.019	-1.841	0.066	-0.074	0.002
AsianPerCap	0.0216	0.019	1.145	0.252	-0.015	0.059
OtherPerCap	0.0438	0.019	2.341	0.019	0.007	0.081
HispPerCap	0.0357	0.025	1.435	0.151	-0.013	0.085
NumUnderPov	0.1112	0.138	0.805	0.421	-0.160	0.382
PctPopUnderPov	-0.1721	0.063	-2.745	0.006	-0.295	-0.049
PctLess9thGrade	-0.0999	0.068	-1.474	0.141	-0.233	0.033
PctNotHSGrad	0.0525	0.096	0.548	0.584	-0.136	0.241
PctBSorMore	0.0504	0.077	0.651	0.515	-0.101	0.202
PctUnemployed	0.0045	0.041	0.111	0.911	-0.075	0.084
PctEmploy	0.2485	0.079	3.151	0.002	0.094	0.403
PctEmplManu	-0.0658	0.032	-2.054	0.040	-0.129	-0.003
PctEmplProfServ	-0.0267	0.041	-0.654	0.513	-0.107	0.053
PctOccupManu	0.0723	0.055	1.318	0.188	-0.035	0.180
PctOccupMgmtProf	0.1226	0.086	1.419	0.156	-0.047	0.292
MalePctDivorce	0.4585	0.248	1.851	0.064	-0.027	0.944
MalePctNevMarr	0.2267	0.068	3.339	0.001	0.094	0.360
FemalePctDiv	0.1627	0.309	0.526	0.599	-0.444	0.770
TotalPctDiv	-0.5619	0.519	-1.084	0.279	-1.579	0.455
PersPerFam	-0.1405	0.168	-0.834	0.404	-0.471	0.190
PctFam2Par	0.0186	0.160	0.117	0.907	-0.294	0.331
PctKids2Par	-0.3227	0.155	-2.080	0.038	-0.627	-0.018
PctYoungKids2Par	-0.0323	0.048	-0.670	0.503	-0.127	0.062
PctTeen2Par	-0.0029	0.043	-0.069	0.945	-0.087	0.081

2.2 Question 3

2.2.1 Read the Data

```
import numpy as np
from sklearn import datasets
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA

# Load MNIST dataset and filter digits 3, 5, and 8
from sklearn.datasets import fetch_openml

# Fetch MNIST from openml
mnist = fetch_openml('mnist_784', version=1, as_frame=False)
X, y = mnist["data"], mnist["target"]

mnist_df = pd.DataFrame(np.concatenate((mnist['target'].reshape(-1, 1), mnist['data'
    ]), axis=1),
columns= ['target'] +  mnist['feature_names'])

mnist_df
```

Then we keep only 3, 5, and 8.

```
import numpy as np
import pandas as pd
from sklearn.datasets import fetch_openml

# Fetch MNIST from OpenML
mnist = fetch_openml('mnist_784', version=1, as_frame=False)
X, y = mnist["data"], mnist["target"]

# Convert labels to integers
y = y.astype(int)

# Correct filtering: Keep only digits 3, 5, and 8
selected_digits = {3, 5, 8}
filter_mask = np.isin(y, list(selected_digits))

X_filtered = X[filter_mask]  # Keep only selected digits
y_filtered = y[filter_mask]  # Keep corresponding labels

# Print dataset size
print(f"Original dataset size: {X.shape[0]}")
print(f"Filtered dataset size (only 3, 5, 8): {X_filtered.shape[0]}")

# Convert to DataFrame (Optional)
mnist_df = pd.DataFrame(np.column_stack((y_filtered, X_filtered)),
columns=['target'] + mnist.feature_names)

# Display the dataframe
mnist_df
```

2.2.2 Logistic Regression with OvR

```
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import accuracy_score, confusion_matrix, classification_report
import numpy as np
```

```
    import matplotlib.pyplot as plt

    # 1. Load MNIST
    mnist = fetch_openml('mnist_784', version=1, as_frame=False)
    X, y = mnist.data, mnist.target
    y = y.astype(int)

    # filter 3, 5, 8
    selected_digits = [3, 5, 8]
    mask = np.isin(y, selected_digits)
    X_filtered, y_filtered = X[mask], y[mask]

    X_filtered = X_filtered / 255.0

    # Divide the sets
    X_train, X_test, y_train, y_test = train_test_split(X_filtered, y_filtered, test_size
        =0.2, random_state=42)

    # 2. Train the Logistic Regression (One-vs-Rest)
    log_reg_ovr = LogisticRegression(multi_class='ovr', solver='liblinear', max_iter=100,
         random_state=42)
    log_reg_ovr.fit(X_train, y_train)

    # 3. Evaluate
    y_pred = log_reg_ovr.predict(X_test)
    accuracy = accuracy_score(y_test, y_pred)
    print(f"Test Accuracy: {accuracy * 100:.2f}%")

    # Confusion Matrix
    conf_matrix = confusion_matrix(y_test, y_pred)
    print("Confusion Matrix:")
    print(conf_matrix)

    # Class Report
    class_report = classification_report(y_test, y_pred)
    print("Classification Report:")
    print(class_report)

    # 4. Visualize
    num_images = 5
    indices = np.random.choice(len(X_test), num_images, replace=False)

    plt.figure(figsize=(10, 5))
    for i, index in enumerate(indices):
    plt.subplot(1, num_images, i + 1)
    plt.imshow(X_test[index].reshape(28, 28), cmap='gray')
    plt.title(f"Pred: {y_pred[index]}\nTrue: {y_test[index]}")
    plt.axis('off')
    plt.show()
```

```
Test Accuracy: 93.12%
Confusion Matrix:
[[1328   39   52]
[  54 1195   45]
[  39   50 1254]]
Classification Report:
precision    recall  f1-score   support

3       0.93      0.94      0.94      1419
5       0.93      0.92      0.93      1294
8       0.93      0.93      0.93      1343

accuracy                           0.93      4056
macro avg       0.93      0.93      0.93      4056
weighted avg       0.93      0.93      0.93      4056
```

2.2.3 Multinomial Regression

```python
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import accuracy_score, confusion_matrix, classification_report
import numpy as np
import matplotlib.pyplot as plt

# Similar steps

mnist = fetch_openml('mnist_784', version=1, as_frame=False)
X, y = mnist.data, mnist.target
y = y.astype(int)

selected_digits = [3, 5, 8]
mask = np.isin(y, selected_digits)
X_filtered, y_filtered = X[mask], y[mask]

X_filtered = X_filtered / 255.0

X_train, X_test, y_train, y_test = train_test_split(X_filtered, y_filtered, test_size
    =0.2, random_state=42)

log_reg_multinomial = LogisticRegression(multi_class='multinomial', solver='lbfgs',
    max_iter=100, random_state=42)
log_reg_multinomial.fit(X_train, y_train)

y_pred = log_reg_multinomial.predict(X_test)
accuracy = accuracy_score(y_test, y_pred)
print(f"Test Accuracy: {accuracy * 100:.2f}%")

conf_matrix = confusion_matrix(y_test, y_pred)
print("Confusion Matrix:")
print(conf_matrix)

class_report = classification_report(y_test, y_pred)
print("Classification Report:")
print(class_report)

num_images = 5
indices = np.random.choice(len(X_test), num_images, replace=False)

plt.figure(figsize=(10, 5))
for i, index in enumerate(indices):
plt.subplot(1, num_images, i + 1)
```

```
    plt.imshow(X_test[index].reshape(28, 28), cmap='gray')
    plt.title(f"Pred: {y_pred[index]}\nTrue: {y_test[index]}")
    plt.axis('off')
    plt.show()
```

```
Test Accuracy: 93.12%
Confusion Matrix:
[[1324   43   52]
 [  51 1201   42]
 [  39   52 1252]]
Classification Report:
              precision    recall  f1-score   support

           3       0.94      0.93      0.93      1419
           5       0.93      0.93      0.93      1294
           8       0.93      0.93      0.93      1343

    accuracy                           0.93      4056
   macro avg       0.93      0.93      0.93      4056
weighted avg       0.93      0.93      0.93      4056
```

Do the Learning Curve

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split, learning_curve
from sklearn.linear_model import LogisticRegression

# 1. Load and filter the MNIST dataset
mnist = fetch_openml('mnist_784', version=1, as_frame=False)
X, y = mnist.data, mnist.target
y = y.astype(int)

# Filter out samples with labels 3, 5, 8
selected_digits = [3, 5, 8]
mask = np.isin(y, selected_digits)
X_filtered, y_filtered = X[mask], y[mask]

# Scale pixel values from [0, 255] to [0, 1]
X_filtered = X_filtered / 255.0

# Split the dataset into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X_filtered, y_filtered, test_size
    =0.2, random_state=42)

# 2. Define the Logistic Regression model
log_reg = LogisticRegression(solver='lbfgs', max_iter=100, random_state=42)

# 3. Compute the learning curve
train_sizes, train_scores, val_scores = learning_curve(
log_reg, X_train, y_train, cv=5, scoring='accuracy', train_sizes=np.linspace(0.1,
    1.0, 10)
)

# Calculate mean and standard deviation of training and validation scores
train_scores_mean = np.mean(train_scores, axis=1)
train_scores_std = np.std(train_scores, axis=1)
val_scores_mean = np.mean(val_scores, axis=1)
val_scores_std = np.std(val_scores, axis=1)

# 4. Plot the learning curve
plt.figure(figsize=(10, 6))
```

```
    plt.plot(train_sizes, train_scores_mean, label='Training Accuracy', color='blue',
        marker='o')
    plt.fill_between(train_sizes, train_scores_mean - train_scores_std, train_scores_mean
         + train_scores_std, alpha=0.15, color='blue')
    plt.plot(train_sizes, val_scores_mean, label='Validation Accuracy', color='green',
        marker='o')
    plt.fill_between(train_sizes, val_scores_mean - val_scores_std, val_scores_mean +
        val_scores_std, alpha=0.15, color='green')

    # plt.title('Learning Curve for Logistic Regression', fontsize=16)
    plt.xlabel('Training Set Size', fontsize=14)
    plt.ylabel('Accuracy', fontsize=14)
    plt.legend(loc='best')
    plt.grid(True)
    plt.show()
```

Do the Validation Curve

```
    from sklearn.model_selection import validation_curve

    # Define the range of hyperparameter C (inverse of regularization strength)
    param_range = np.logspace(-4, 4, 10)

    # Compute validation curve
    train_scores, val_scores = validation_curve(
    log_reg, X_train, y_train, param_name='C', param_range=param_range, cv=5, scoring='
        accuracy'
    )

    # Calculate mean and standard deviation of training and validation scores
    train_scores_mean = np.mean(train_scores, axis=1)
    train_scores_std = np.std(train_scores, axis=1)
    val_scores_mean = np.mean(val_scores, axis=1)
    val_scores_std = np.std(val_scores, axis=1)

    # Plot the validation curve
    plt.figure(figsize=(10, 6))
    plt.semilogx(param_range, train_scores_mean, label='Training Accuracy', color='blue',
         marker='o')
    plt.fill_between(param_range, train_scores_mean - train_scores_std, train_scores_mean
         + train_scores_std, alpha=0.15, color='blue')
    plt.semilogx(param_range, val_scores_mean, label='Validation Accuracy', color='green'
        , marker='o')
    plt.fill_between(param_range, val_scores_mean - val_scores_std, val_scores_mean +
        val_scores_std, alpha=0.15, color='green')

    # plt.title('Validation Curve for Logistic Regression', fontsize=16)
    plt.xlabel('Regularization Strength (C)', fontsize=14)
    plt.ylabel('Accuracy', fontsize=14)
    plt.legend(loc='best')
    plt.grid(True)
    plt.show()
```

2.2.4 Naive Bayes

```
    from sklearn.datasets import fetch_openml
    from sklearn.model_selection import train_test_split
    from sklearn.naive_bayes import GaussianNB
    from sklearn.metrics import accuracy_score, confusion_matrix, classification_report
    import numpy as np
    import matplotlib.pyplot as plt

```

```
    # 1. Load and filter the MNIST dataset
    mnist = fetch_openml('mnist_784', version=1, as_frame=False)
    X, y = mnist.data, mnist.target
    y = y.astype(int)

    # Filter out samples with labels 3, 5, 8
    selected_digits = [3, 5, 8]
    mask = np.isin(y, selected_digits)
    X_filtered, y_filtered = X[mask], y[mask]

    # Scale pixel values from [0, 255] to [0, 1]
    X_filtered = X_filtered / 255.0

    # Split the dataset into training and testing sets
    X_train, X_test, y_train, y_test = train_test_split(X_filtered, y_filtered, test_size
        =0.2, random_state=42)

    # 2. Train a Gaussian Naive Bayes model
    naive_bayes = GaussianNB()
    naive_bayes.fit(X_train, y_train)

    # 3. Evaluate the model
    y_pred = naive_bayes.predict(X_test)
    accuracy = accuracy_score(y_test, y_pred)
    print(f"Test Accuracy: {accuracy * 100:.2f}%")

    # Confusion matrix
    conf_matrix = confusion_matrix(y_test, y_pred)
    print("Confusion Matrix:")
    print(conf_matrix)

    # Classification report
    class_report = classification_report(y_test, y_pred)
    print("Classification Report:")
    print(class_report)

    # 4. Visualize the results
    num_images = 5
    indices = np.random.choice(len(X_test), num_images, replace=False)

    plt.figure(figsize=(10, 5))
    for i, index in enumerate(indices):
    plt.subplot(1, num_images, i + 1)
    plt.imshow(X_test[index].reshape(28, 28), cmap='gray')
    plt.title(f"Pred: {y_pred[index]}\nTrue: {y_test[index]}")
    plt.axis('off')
    plt.show()
```

```
Test Accuracy: 50.22%
Confusion Matrix:
[[ 595   26  798]
[  95  138 1061]
[  18   21 1304]]
Classification Report:
precision    recall  f1-score   support

3       0.84      0.42      0.56      1419
5       0.75      0.11      0.19      1294
8       0.41      0.97      0.58      1343

accuracy                           0.50      4056
macro avg       0.67      0.50      0.44      4056
weighted avg       0.67      0.50      0.45      4056
```

2.2.5 Linear Discriminant Analysis

```python
from sklearn.datasets import fetch_openml
from sklearn.model_selection import train_test_split
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.metrics import accuracy_score, confusion_matrix, classification_report
import numpy as np
import matplotlib.pyplot as plt

# 1. Load and filter the MNIST dataset
mnist = fetch_openml('mnist_784', version=1, as_frame=False)
X, y = mnist.data, mnist.target
y = y.astype(int)

# Filter out samples with labels 3, 5, 8
selected_digits = [3, 5, 8]
mask = np.isin(y, selected_digits)
X_filtered, y_filtered = X[mask], y[mask]

# Scale pixel values from [0, 255] to [0, 1]
X_filtered = X_filtered / 255.0

# Split the dataset into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X_filtered, y_filtered, test_size
    =0.2, random_state=42)

# 2. Train a Linear Discriminant Analysis (LDA) model
lda = LinearDiscriminantAnalysis()
lda.fit(X_train, y_train)

# 3. Evaluate the model
y_pred = lda.predict(X_test)
accuracy = accuracy_score(y_test, y_pred)
print(f"Test Accuracy (LDA): {accuracy * 100:.2f}%")

# Confusion matrix
conf_matrix = confusion_matrix(y_test, y_pred)
print("Confusion Matrix (LDA):")
print(conf_matrix)

# Classification report
class_report = classification_report(y_test, y_pred)
print("Classification Report (LDA):")
print(class_report)

# 4. Visualize the results
```

```
    num_images = 5
    indices = np.random.choice(len(X_test), num_images, replace=False)

    plt.figure(figsize=(10, 5))
    for i, index in enumerate(indices):
    plt.subplot(1, num_images, i + 1)
    plt.imshow(X_test[index].reshape(28, 28), cmap='gray')
    plt.title(f"Pred: {y_pred[index]}\nTrue: {y_test[index]}")
    plt.axis('off')
    plt.show()
```

```
Test Accuracy (LDA): 91.69%
Confusion Matrix (LDA):
[[1285   69   65]
[  47 1202   45]
[  34   77 1232]]
Classification Report (LDA):
precision    recall  f1-score   support

3       0.94      0.91      0.92      1419
5       0.89      0.93      0.91      1294
8       0.92      0.92      0.92      1343

accuracy                           0.92      4056
macro avg       0.92      0.92      0.92      4056
weighted avg       0.92      0.92      0.92      4056
```

2.2.6 Linear SVM

```
    from sklearn.datasets import fetch_openml
    from sklearn.model_selection import train_test_split
    from sklearn.svm import LinearSVC
    from sklearn.metrics import accuracy_score, confusion_matrix, classification_report
    import numpy as np
    import matplotlib.pyplot as plt

    # 1. Load and filter the MNIST dataset
    mnist = fetch_openml('mnist_784', version=1, as_frame=False)
    X, y = mnist.data, mnist.target
    y = y.astype(int)

    # Filter out samples with labels 3, 5, 8
    selected_digits = [3, 5, 8]
    mask = np.isin(y, selected_digits)
    X_filtered, y_filtered = X[mask], y[mask]

    # Scale pixel values from [0, 255] to [0, 1]
    X_filtered = X_filtered / 255.0

    # Split the dataset into training and testing sets
    X_train, X_test, y_train, y_test = train_test_split(X_filtered, y_filtered, test_size
        =0.2, random_state=42)

    # 2. Train a Linear SVM model with One-vs-Rest strategy
    linear_svm = LinearSVC(multi_class='ovr', max_iter=10000, random_state=42)  # Use One
        -vs-Rest
    linear_svm.fit(X_train, y_train)

    # 3. Evaluate the model
    y_pred = linear_svm.predict(X_test)
    accuracy = accuracy_score(y_test, y_pred)
```

```
    print(f"Test Accuracy (Linear SVM): {accuracy * 100:.2f}%")

    # Confusion matrix
    conf_matrix = confusion_matrix(y_test, y_pred)
    print("Confusion Matrix (Linear SVM):")
    print(conf_matrix)

    # Classification report
    class_report = classification_report(y_test, y_pred)
    print("Classification Report (Linear SVM):")
    print(class_report)

    # 4. Visualize the results
    num_images = 5
    indices = np.random.choice(len(X_test), num_images, replace=False)

    plt.figure(figsize=(10, 5))
    for i, index in enumerate(indices):
    plt.subplot(1, num_images, i + 1)
    plt.imshow(X_test[index].reshape(28, 28), cmap='gray')
    # plt.title(f"Pred: {y_pred[index]}\nTrue: {y_test[index]}")
    plt.axis('off')
    plt.show()
```

```
Test Accuracy (Linear SVM): 92.73%
Confusion Matrix (Linear SVM):
[[1319   44   56]
[  57 1193   44]
[  39   55 1249]]
Classification Report (Linear SVM):
precision    recall  f1-score   support

3       0.93      0.93      0.93      1419
5       0.92      0.92      0.92      1294
8       0.93      0.93      0.93      1343

accuracy                           0.93      4056
macro avg       0.93      0.93      0.93      4056
weighted avg       0.93      0.93      0.93      4056
```

2.2.7 Plot the Confusion Matrix

```
    from sklearn.metrics import confusion_matrix
    import seaborn as sns

    # Logistic Regression Confusion Matrix
    log_reg_pred = log_reg.predict(X_test)
    log_reg_cm = confusion_matrix(y_test, log_reg_pred)

    # Multinomial Regression Confusion Matrix
    multinomial_pred = multinomial_reg.predict(X_test)
    multinomial_cm = confusion_matrix(y_test, multinomial_pred)

    # Naive Bayes Confusion Matrix
    naive_bayes_pred = naive_bayes.predict(X_test)
    naive_bayes_cm = confusion_matrix(y_test, naive_bayes_pred)

    # LDA Confusion Matrix
    lda_pred = lda.predict(X_test)
    lda_cm = confusion_matrix(y_test, lda_pred)

```

```
    # SVM Confusion Matrix
    svm_pred = svm.predict(X_test)
    svm_cm = confusion_matrix(y_test, svm_pred)

    def plot_confusion_matrix(cm, class_names):
    plt.figure(figsize=(6, 5))
    sns.heatmap(cm, annot=True, fmt='d', cmap='Blues', xticklabels=class_names,
        yticklabels=class_names)
    plt.xlabel('Predicted')
    plt.ylabel('True')
    plt.title('Confusion Matrix')
    plt.show()

    class_names = ['3', '5', '8']

    # Plot confusion matrices
    plot_confusion_matrix(log_reg_cm, class_names)
    plot_confusion_matrix(multinomial_cm, class_names)
    plot_confusion_matrix(naive_bayes_cm, class_names)
    plot_confusion_matrix(lda_cm, class_names)
    plot_confusion_matrix(svm_cm, class_names)
```

2.2.8 Group-Lasso Regularized Multinomial Regression for Feature Selection

If we want to find out the important features here

```
    import numpy as np
    import matplotlib.pyplot as plt
    from tensorflow.keras.datasets import mnist
    from group_lasso import GroupLasso
    from sklearn.linear_model import LogisticRegression
    from sklearn.pipeline import make_pipeline
    from sklearn.preprocessing import StandardScaler
    from sklearn.metrics import accuracy_score, classification_report

    # 1. Load MNIST dataset and select digits 3, 5, 8
    (x_train, y_train), (x_test, y_test) = mnist.load_data()

    # Only select digits 3, 5, 8
    selected_digits = [3, 5, 8]
    train_mask = np.isin(y_train, selected_digits)
    test_mask = np.isin(y_test, selected_digits)

    # Extract data for selected digits
    x_train_selected = x_train[train_mask]
    y_train_selected = y_train[train_mask]
    x_test_selected = x_test[test_mask]
    y_test_selected = y_test[test_mask]

    # Flatten images into vectors (28x28 -> 784)
    x_train_selected = x_train_selected.reshape(x_train_selected.shape[0], -1)
    x_test_selected = x_test_selected.reshape(x_test_selected.shape[0], -1)

    # Normalize pixel values to [0, 1]
    x_train_selected = x_train_selected / 255.0
    x_test_selected = x_test_selected / 255.0

    # Map labels to 0, 1, 2 (for 3, 5, 8 respectively)
    label_map = {3: 0, 5: 1, 8: 2}
    y_train_selected = np.array([label_map[y] for y in y_train_selected])
    y_test_selected = np.array([label_map[y] for y in y_test_selected])

```

```
    # 2. Define Group-Lasso regularized multinomial logistic regression
    # Define feature groups (each pixel is a group)
    groups = np.arange(x_train_selected.shape[1])  # Each feature is a group

    # Create GroupLasso model
    group_lasso = GroupLasso(
    groups=groups,
    group_reg=0.1,  # Regularization strength for groups
    l1_reg=0.01,    # L1 regularization strength
    n_iter=1000,    # Number of iterations
    scale_reg="group_size",  # Scale regularization by group size
    supress_warning=True,
    )

    # Create multinomial logistic regression model
    logistic_regression = LogisticRegression(  # Fixed typo: Changed to
        logistic_regression
    multi_class="multinomial",  # Multinomial logistic regression
    solver="lbfgs",             # Optimization algorithm
    max_iter=1000,              # Maximum number of iterations
    )

    # Combine GroupLasso and logistic regression into a pipeline
    pipeline = make_pipeline(
    StandardScaler(),  # Standardize data
    group_lasso,       # Group-Lasso feature selection
    logistic_regression  # Multinomial logistic regression
    )

    # 3. Train the model
    pipeline.fit(x_train_selected, y_train_selected)

    # 4. Make predictions
    y_pred = pipeline.predict(x_test_selected)

    # 5. Evaluate the model
    print("Accuracy:", accuracy_score(y_test_selected, y_pred))
    print("Classification␣Report:\n", classification_report(y_test_selected, y_pred))

    # 6. Feature selection results
    # Get the selected features from Group-Lasso
    selected_features = group_lasso.sparsity_mask_  # Boolean mask, True for selected
        features

    selected_features
```

The printout will be:

```
Accuracy: 0.7659944367176634
Classification Report:
precision    recall  f1-score   support

0       0.77      0.82      0.79      1010
1       0.71      0.69      0.70       892
2       0.81      0.79      0.80       974

accuracy                           0.77      2876
macro avg       0.76      0.76      0.76      2876
weighted avg       0.77      0.77      0.77      2876
```

And

```
array([False, False, False, False, False, False, False, False, False,
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```

```
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       False, False, False, False, False, False, False, False, False,
       False, False, False, False, False, False, False, False, False,
       False])
```

Then for the visualization:

```python
# Load MNIST dataset
mnist = fetch_openml('mnist_784', version=1)
X, y = mnist["data"], mnist["target"]

# Convert labels to integers
y = y.astype(int)

# Extract samples for digits 3, 5, 8
digits = [3, 5, 8]
mask = np.isin(y, digits)
X_filtered = X[mask]
y_filtered = y[mask]

# Remap labels to 0, 1, 2 for easier multi-class classification
y_filtered = np.where(y_filtered == 3, 0, y_filtered)
y_filtered = np.where(y_filtered == 5, 1, y_filtered)
y_filtered = np.where(y_filtered == 8, 2, y_filtered)

# Normalize pixel values to [0, 1]
X_filtered = X_filtered / 255.0

# Split dataset into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X_filtered, y_filtered, test_size
    =0.2, random_state=42)

# Define Group Lasso regularized multinomial logistic regression
# Using L1 regularization (Lasso) as an approximation for Group Lasso
model = LogisticRegression(penalty='l1', solver='saga', multi_class='multinomial',
    max_iter=1000, C=0.1)

# Train the model
model.fit(X_train, y_train)

# Predict on the test set
y_pred = model.predict(X_test)

# Calculate accuracy
accuracy = accuracy_score(y_test, y_pred)
print(f"Test Accuracy: {accuracy:.4f}")

# Get the model's weight matrix (coef_)
weights = model.coef_  # Shape: (3, 784)

weights
```

```
43
44      # Calculate feature importance (sum of absolute weights)
45      feature_importance = np.sum(np.abs(weights), axis=0)
46
47      # Reshape feature importance into a 28x28 image
48      feature_importance_image = feature_importance.reshape(28, 28)
49
50      # Visualize the selected features
51      plt.figure(figsize=(8, 8))
52      plt.imshow(feature_importance_image, cmap='hot', interpolation='nearest')
53      plt.colorbar(label='Feature␣Importance')
54      # plt.title('Selected Features (Group Lasso Regularization)')
55      plt.axis('off')
56      plt.show()
```