

# A PVT-Independent Constant- $G_m$ Bias Technique Based on Analog Computation

Ching-Yun Chu and Yu-Jiu Wang, *Member, IEEE*

**Abstract**—This brief presents a new process, voltage, and temperature (PVT)-independent constant- $G_m$  bias technique for any  $I_{out}(V_{in})$ -monotonic convex (or concave) transconductors. In this brief, the conventional constant- $g_m$  biasing is formulated into an analog computation process that calculates the constant- $G_m$  bias voltage  $V_0$  from an input current. An analog computer measures the effective  $G_m$  from two matched transconductors and converges it to the inverse of a precision resistance by adjusting  $V_0$ . Through this, a well-defined large-signal constant- $G_m$  bias voltage can be found. The proposed technique eliminates the power-law or exponential-law assumptions on the  $I_{out}(V_{in})$  characteristics in a conventional design. An interpolation calculation follows to find the optimal small-signal constant- $g_m$  bias voltage  $V_{out}$ . The fundamental limitations of the proposed technique are the required monotonicity and convexity (or concavity) of the transconductor's  $I_{out}(V_{in})$ . Error sources include mismatches of paired devices and the input offset voltage and open-loop gain of the operation amplifier (OpAmp). These errors are analyzed in detail in this brief. Biasing circuits based on different transistor types are designed using the Taiwan Semiconductor Manufacturing Company (TSMC) 65-nm CMOS process. Computer simulations show that the proposed biasing circuitry can achieve  $\pm 0.22\%$   $g_m$  variations from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$ .

**Index Terms**—Constant  $G_m$ , process, voltage, temperature (PVT) variation.

## I. INTRODUCTION

IN ANALOG-INTEGRATED circuits such as radiofrequency (RF) amplifiers [1], analog front end [2], or  $g_m$ -C filters [3], [4], constant-transconductance biasing [5] is important in keeping their gain and bandwidth stable against process, voltage, and temperature (PVT) variations. Errors of conventional constant- $g_m$  biasing come from the following: 1) The transconductor  $I_{out}(V_{in})$  does not follow a simple power law, particularly in short-channel MOSFETs; 2) the precision of  $g_m$  is affected by transistors' secondary effects, including input current  $I_{in}$  (base current or gate leakage current), body effects, and output impedance  $R_{out}$  (early effect or channel-length modulation); and 3) an insufficient phase margin in the feedback loop can cause transient bias errors or even bias oscillation [6]. In addition to errors, circuits like power amplifiers need large-signal constant- $G_m$  biasing that cannot be readily derived using conventional biasing techniques.

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The authors are with the Department of Electronics Engineering, National Chiao Tung University, Hsin-Chu 30010, Taiwan (e-mail: ywang@faculty.nctu.edu.tw).

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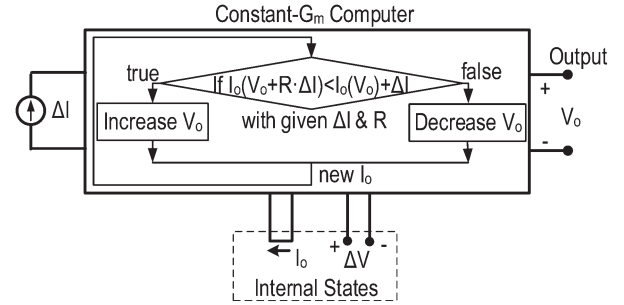


Fig. 1. Block diagram of the constant- $G_m$  analog computer for a monotonic and convex  $I_{out}(V_{in})$  without any output interpolator.

This brief presents a constant- $G_m$  bias technique for any  $I_{out}(V_{in})$  monotonic and convex (or concave) transconductors. The organization of this brief is as follows. Section II formulates the conventional constant- $g_m$ -biasing problem into an analog computation problem and uses it to examine the underlying fundamentals of conventional constant- $g_m$  biasing circuitry. Based on this, a constant- $G_m$  computer is derived from the definition of  $G_m$  with discussions on error models. The small-signal constant- $g_m$  bias voltage can be generated based on optimal interpolation. Section III discusses several design tradeoffs, including temperature, device sizing, etc., and the optimal interpolation to provide the optimal constant- $g_m$  bias voltage using computer simulations. A conclusion is drawn in the final section.

## II. ARCHITECTURE

### A. Formulation of the Constant- $G_m$ Biasing Problem

For any transconductor, output current  $I_{out}$  is a function of its input voltage  $V_{in}$ . The general  $G_m$  is defined in a difference sense by relating the output currents  $I_{out}(V_0) \equiv I_0$  and  $I_{out}(V_0 + \Delta V) \equiv I_0 + \Delta I$  between two input voltages  $V_0$  and  $V_0 + \Delta V$  with  $G_m \equiv \Delta I / \Delta V$ . To design a constant- $G_m$  biasing circuit is equivalent to designing an analog computer that continuously calculates the bias voltage  $V_0$  that keeps the ratio of  $\Delta I / \Delta V$  constant regardless of PVT drifts based on the input current  $\Delta I$  shown in Fig. 1. Therefore,  $\Delta I$  and  $V_0$  are the input and output to the analog computer, and  $I_0$  and  $\Delta V = \Delta I / G_m$  are the internal states.

The algorithm of this computation is as follows: 1)  $I_{out}(V_0) + \Delta I$  and  $I_{out}(V_0 + \Delta V)$  are constructed using two matched transconductors as shown in Fig. 2(a). Note that a monotonic and convex  $I_{out}(V_{in})$  is assumed in this figure; 2) these two currents are compared; and 3) when  $I_{out}(V_0 + \Delta I / G_m) < I_{out}(V_0) + \Delta I$ , the  $V_0$  should be increased until

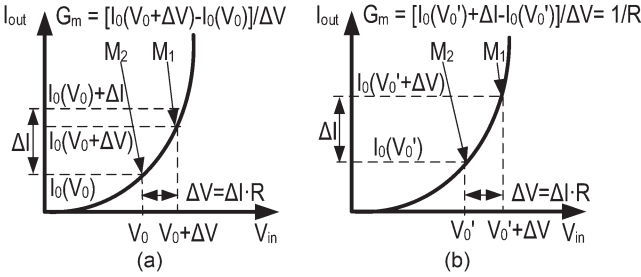


Fig. 2. Operating principles of the constant- $G_m$  analog computer. Instantaneous state (a) where  $I_{out}(V_0 + \Delta I/G_m) < I_{out}(V_0) + \Delta I$  for a monotonic and convex  $I_{out}(V_{in})$  and (b) when it is biased properly with  $G_m = 1/R$ . Note that  $I_0(V_0)$  and  $I_0(V_0 + \Delta V)$  are the output currents of matched transistors  $M_1$  and  $M_2$ , respectively.

they are equal as in Fig. 2(b). If  $I_{out}(V_{in})$  is concave,  $V_0$  should be decreased in the same scenario. This process can also be realized in digital with more complexity.

### B. Conventional Design as an Analog Computer

With the idea of analog computation in mind, how should the conventional constant- $g_m$  biasing circuit in Fig. 3(a) [7] be interpreted? In this circuit,  $M_1$  and  $M_2$  have a sizing ratio of  $1 : N$ . We can scale them to a  $1 : 1$  ratio without changing voltages in all nets using the circuit of Fig. 3(b). Note that the upper current sources  $M_3$  and  $M_4$  are scaled to an  $N : 1$  ratio, and the resistor is scaled to  $N \cdot R$  to maintain its voltage drop. An equivalent way to represent this circuit is to transform it into an equivalent circuit as shown in Fig. 3(c). Now,  $M_3$  and  $M_4$  are back to the same size with an input current  $\Delta I$  provided. It can be identified that the difference current  $\Delta I$  between the matched  $M_1$  and  $M_2$  is  $N - 1$  times the internal state  $I_0$  through feedback as shown in Fig. 3(d). In addition, the difference voltage  $\Delta V$  between the matched  $M_1$  and  $M_2$  is  $N \cdot R \cdot I_0$ .

Note that  $\Delta I$  and  $\Delta V$  in a conventional design are not related by  $\Delta V = \Delta I \cdot R$  as in Fig. 2. In Fig. 3(c), the gate-to-source voltage of  $M_2$  is equal to  $V_0$ , and its drain current is  $I_0$ . The gate-to-source voltage of  $M_1$  is equal to  $V_0 + \Delta V$ , and its drain current is  $I_0 + \Delta I$ . Since  $M_1$  and  $M_2$  are matched, the equivalent  $G_m$  at  $V_0$  can be derived by dividing  $\Delta I$  by  $\Delta V$  as described in Fig. 2. From Fig. 3(c),  $\Delta V = I_0 \cdot N \cdot R$ . Hence

$$G_m \equiv \frac{\Delta I}{\Delta V} \Big|_{\Delta I=(N-1)I_0} = \frac{\Delta I}{N \cdot R \cdot I_0} = \left( \frac{N-1}{N} \right) \frac{1}{R}. \quad (1)$$

This is to say that the conventional circuit in Fig. 3(a) is a constant- $G_m$  computer that calculates the bias voltage  $V_0$  based on input  $\Delta I = (N - 1) \cdot I_0$ . Note that  $\Delta I$  is iteratively defined based on  $I_0$ . This suggests that, when  $V_0$  is used to bias another transistor, its output ac current swing differs for different PVT conditions when the biased transistor is driven by an input ac voltage swing equal to  $\Delta V$ .

### C. Proposed Constant- $G_m$ Computer

The performance of the constant- $G_m$  computer in Fig. 3(a) can be improved by directly providing a constant input  $\Delta I$  without any internal-state feedback. The schematics of the

proposed constant- $G_m$  computer is shown in Fig. 4. The input  $\Delta I$  provides various design options. For small-signal or fixed input-swing application,  $\Delta I$  can be generated from a constant current reference. However, for applications that need high-dynamic-range output swings,  $\Delta I$  can be made proportional to the magnitude of the input signal.

The major difference between the circuits in Figs. 3(c) and 4 is how  $\Delta V$  is generated. In Fig. 3(c),  $\Delta V \propto I_0$ . However, in the proposed circuit,  $\Delta V \propto \Delta I$ , which is mirrored directly from the input. Under the scheme of analog computation, OpAmp  $A_1$  does not have any internal-state feedback to the input. In addition, when the device  $I_{out}(V_{in})$  changes due to PVT drifts, OpAmp  $A_1$  forces  $I_{out}(V_0 + \Delta I \cdot R)$  to be equal to  $I_{out}(V_0) + \Delta I$ . Hence

$$G_m \equiv \frac{\Delta I}{\Delta V} \Big|_{\Delta I} = \frac{\Delta I}{R \cdot \Delta I} = \frac{1}{R}. \quad (2)$$

No assumption is made for the device  $I_{out}(V_{in})$ , except for it being monotonically increasing and convex. If  $I_{out}(V_{in})$  of  $M_1$  and  $M_2$  in Fig. 4 is monotonically increasing but concave, the input terminals of  $A_1$  in Fig. 4 will need to be swapped.

### D. Small-Signal $g_m$ Bias Voltage

The circuit of Fig. 4 has a well-defined constant- $G_m$  bias voltage  $V_0$ . However, biasing the transistor using  $V_0$  will not result in constant- $g_m$  biasing. Since the  $I_{out}(V_{in})$  of  $M_1$  and  $M_2$  is convex,  $g_m(V_0) < G_m < g_m(V_0 + \Delta V)$ . An optimal small-signal constant- $g_m$  voltage  $V_{out}$  can be found by using an interpolated  $V_{out} \equiv V_0 + \xi \Delta V$  to bias  $M_0$ . This interpolation is accomplished through a resistive voltage divider shown in Fig. 4. The optimization of interpolation factor  $\xi$  will be discussed in Section III.

### E. Error Models and Analysis

The proposed circuit shown in Fig. 4 relies on the matching between the  $M_1 - M_2$  pair and the mirrored currents  $I_0$  and  $\Delta I$ . Mismatches between each pair will contribute to errors in the output voltage  $V_0$ . Assume that the currents of  $M_1$  and  $M_2$  are  $I_{out}(V_1) + ((\delta I_{out})/2)$  and  $I_{out}(V_2) - ((\delta I_{out})/2)$ , respectively.  $(\delta I_{out})/2$  is the mismatch current in the transconductor's average  $I_{out}(V_{in})$  characteristic. The drain currents of  $M_3$  and  $M_4$  are  $I_0 + \Delta I + ((\delta I_0)/2)$  and  $I_0 + \Delta I - ((\delta I_0)/2)$ , respectively, with  $(\delta I_0)/2$  mismatches.

Moreover, voltage  $V_1$  can be expressed as  $V_2 - V_{os} + \Delta I \cdot R$ .  $V_{os}$  is the residual voltage difference across the positive and negative terminals of OpAmp  $A_1$  as  $V_{os} = (V_{SG,3}/A_1) + V'_{os}$ . Here,  $V'_{os}$  is the input offset voltage of  $A_1$ , and  $V_{SG,3}$  is the source-to-gate voltage of  $M_3$ . For a 1-mV  $V'_{os}$ , a more than 60-dB  $A_1$  is sufficient to make the term  $(V_{SG,3}/A_1)$  insignificant.

Define the large-signal  $G_m$  as  $(I_{out}(V_2 + \Delta I \cdot R) - I_{out}(V_2))/\Delta I R$  and the reference  $G_{m0}$  as  $(1/R)$ , respectively. The error of  $G_m$  from  $G_{m0}$  can be calculated by

$$\frac{\Delta G_m}{G_{m0}} \equiv \frac{G_m - G_{m0}}{G_{m0}} \approx \left( -\frac{\delta I_{out}}{\Delta I} + \frac{\delta I_0}{\Delta I} + \frac{V_{os}}{\Delta I \cdot R} \right). \quad (3)$$

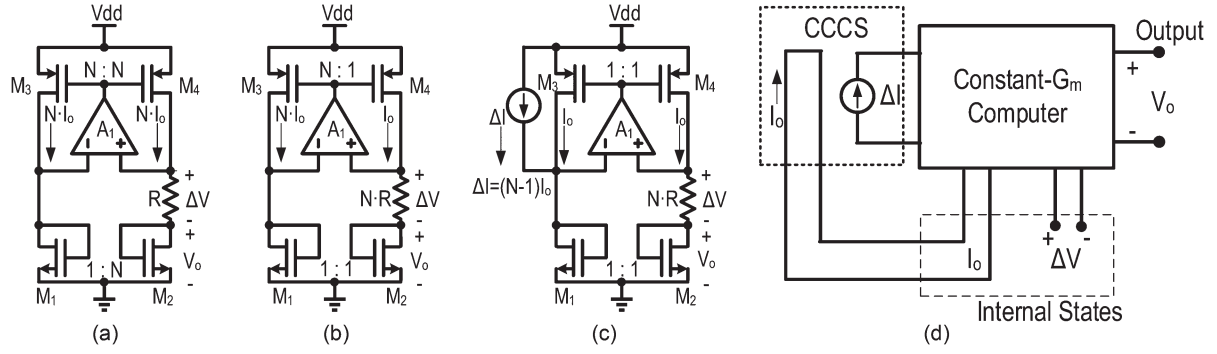


Fig. 3. Conversion of a conventional constant- $g_m$  biasing circuit into a conceptual analog computer: (a) Original circuit, (b) circuit after the right branch is scaled down, (c) circuit with the difference current  $\Delta I$  separated, and (d) block diagram of the analog computer.

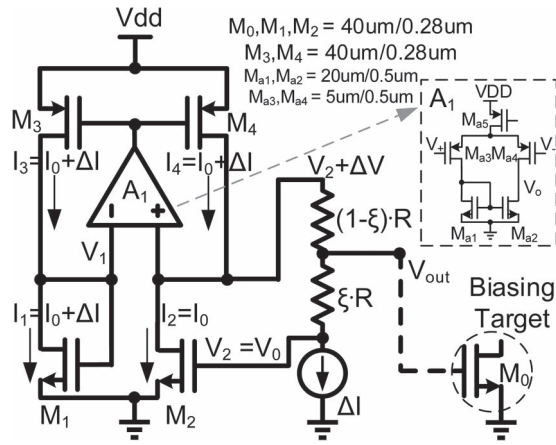


Fig. 4. Proposed architecture for a monotonically increasing and convex device  $I_{out}(V_{in})$ . A voltage interpolator is used to provide optimal constant- $g_m$  bias voltage for  $M_0$ .  $\Delta I$  is the only overhead current consumption.

Mismatches cause systematic error in  $G_m$ . Input current  $\Delta I$  should be made sufficiently large compared to all mismatch currents.

### III. SIMULATION RESULTS

Computer simulations of the circuit in Fig. 4 are presented in this section. All the simulations are based on the TSMC 65-nm CMOS process. The resistor  $R$  used in the simulation has zero temperature coefficient (TC) as the reference for  $G_m$ . In practice, an external low-TC precision resistor should be used. From Figs. 5–8, behavioral OpAmps with 120-dB open-loop gain and zero input offset voltage are used in the simulations and the optimizations of  $\xi$ . Both the gain and input offset voltage of the OpAmp have zero TC. Device sizes other than those of  $M_1$  and  $M_2$  are listed in Fig. 4. Since there is no channel-length modulation issue in this circuit, all devices have a minimum length.

DC and ac simulations are used to find operating points and to calculate the effective small-signal  $g_m$  for  $M_1$ ,  $M_2$ , and the  $V_{out}$ -biased equally sized transistor  $M_0$ . The effective  $g_m$  is not equal to the intrinsic  $g_{mi}$  in the transistor model due to parasitic resistors in all transistor terminals. In MOSFET,  $g_m \approx g_{mi} \cdot (1 - g_{mr_s})(1 - g_{md_r})$ , where  $r_s$  and  $r_d$  are the source and drain parasitic resistors, respectively.

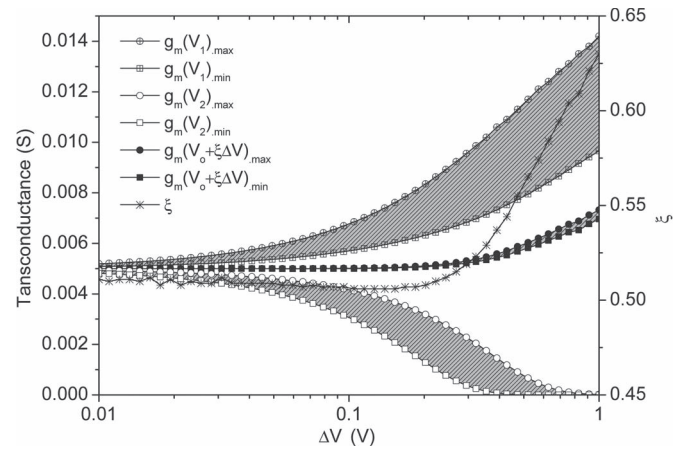


Fig. 5. Optimal  $\xi$ ,  $g_m(V_1)$ ,  $g_m(V_2)$ , and  $g_m(V_0 + \xi\Delta V)$  for  $\Delta V$  from 10 mV to 1 V with temperature from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$ . The  $W/L$  values of  $M_1$  and  $M_2$  are kept at  $40\ \mu\text{m}/0.28\ \mu\text{m}$ .

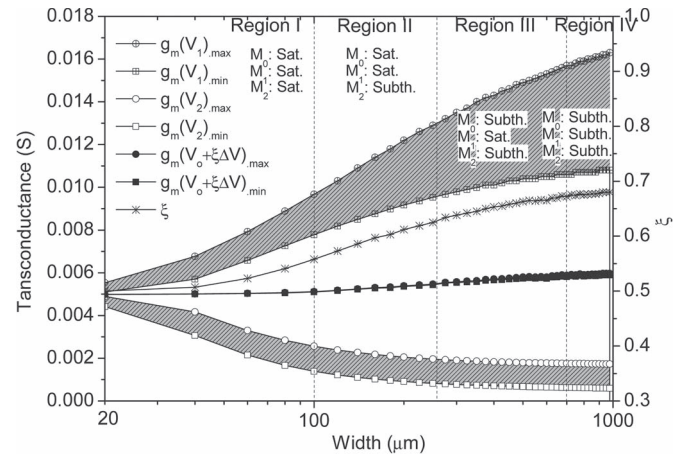


Fig. 6. Optimal  $\xi$  for  $W$  of  $M_1$  and  $M_2$  from 20 to  $1000\ \mu\text{m}$  with temperature from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$ , along with corresponding  $g_m(V_1)$ ,  $g_m(V_2)$ , and  $g_m(V_{out})$ . For all simulations, the lengths of  $M_1$  and  $M_2$  are chosen as  $0.28\ \mu\text{m}$ , and  $\Delta V$  is kept at 100 mV.

In all simulations, the effective small-signal transconductances of  $M_1$ ,  $M_2$ , and the  $V_{out}$ -biased transistor  $M_0$  are plotted and compared. The large-signal effective  $G_m$ 's are calculated based on  $G_m \equiv (I_{D1} - I_{D2}) / (V_{GS1} - V_{GS2})$ , where  $I_{D1}$  and  $I_{D2}$  and  $V_{GS1}$  and  $V_{GS2}$  are the drain currents and



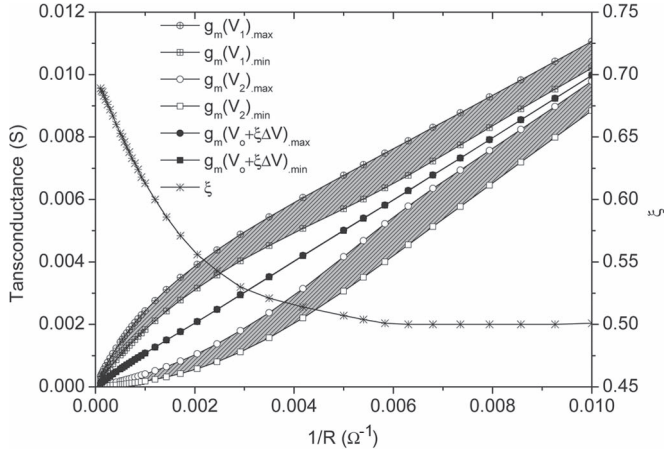


Fig. 7. Optimal  $\xi$ ,  $g_m(V_1)$ ,  $g_m(V_2)$ , and  $g_m(V_0 + \xi\Delta V)$  for a varying  $1/R$  with temperature from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$ . For all simulations,  $W/L$  values of  $M_1$  and  $M_2$  are chosen as  $40\ \mu\text{m}/0.28\ \mu\text{m}$ , and  $\Delta V$  is kept at  $100\ \text{mV}$ .

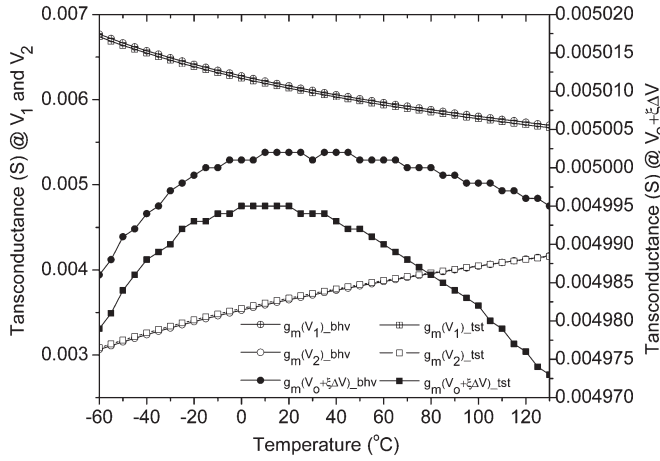


Fig. 8.  $g_m(V_1)$ ,  $g_m(V_2)$ , and  $g_m(V_0 + \xi\Delta V)$  from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$  for a behavioral and a 25-dB transistor-level OpAmp. For all simulations,  $W/L$  values of  $M_1$  and  $M_2$  are chosen as  $40\ \mu\text{m}/0.28\ \mu\text{m}$ , and  $\xi$  is kept at 0.503.

gate-to-source voltages of  $M_1$  and  $M_2$ , respectively. From Figs. 5–8,  $M_1$  and  $M_2$  use thick-gate-oxide MOSFETs.

The effects of  $\Delta V$  on the proposed circuit are first studied. In these simulations, the  $W/L$  values of  $M_1$  and  $M_2$  are kept at  $40\ \mu\text{m}/0.28\ \mu\text{m}$ , and the  $1/R$  is kept at  $5\ \text{mS}$ . The gate voltage difference  $\Delta V$  between  $M_1$  and  $M_2$  is swept by increasing the input current  $\Delta I$ . For each simulated  $\Delta V$ , its optimal  $\xi$  that presents the minimum  $g_m$  variations from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$ , is found. From Fig. 5, as  $\Delta V$  increases, the optimal  $\xi$  slightly increases. Also, the  $g_m$  variation at optimal  $\xi$  increases as  $\Delta V$  increases. This is due to the fact that  $g_m(V_2)$  is clipped to zero for some temperatures from  $\Delta V > 0.3\ \text{V}$ .

Then, the effects of transistors' operating regions are studied. In these simulations,  $\Delta V$  is kept at  $100\ \text{mV}$  for various widths of  $M_1$  and  $M_2$ , and the  $1/R$  is kept at  $5\ \text{mS}$ . As the device size increases, the operating region of the  $M_1 - M_2$  pair will move from the saturation region to the subthreshold region. For each simulated  $W/L$ , the optimal  $\xi$  to achieve minimum  $g_m$  variations from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$  is found and presented in Fig. 6. The operation regions for  $M_1$  and  $M_2$  will shift from Region I, where both transistors are in saturation, to Region IV, where both transistors are in the subthreshold region. From this

TABLE I  
SIMULATION RESULTS OF CONSTANT- $G_m$  ANALOG COMPUTER

Device	Optimal $\xi$	Temp.	$G_m^*$	$g_m(V_0)^*$
2.5V NMOSFET in sat. region $W/L = 40\ \mu\text{m}/0.28\ \mu\text{m}$	0.503	$-60^\circ\text{C}$	4.986	$5.004 \pm 0.361$
		$27^\circ\text{C}$	5.002	$5.017 \pm 0.311$
		$130^\circ\text{C}$	4.995	$5.009 \pm 0.283$
2.5V NMOSFET in subth. region $W/L = 800\ \mu\text{m}/0.28\ \mu\text{m}$	0.675	$-60^\circ\text{C}$	5.834	$5.847 \pm 0.301$
		$27^\circ\text{C}$	5.912	$5.922 \pm 0.234$
		$130^\circ\text{C}$	5.841	$5.848 \pm 0.182$
1.0V NMOSFET in sat. region $W/L = 5\ \mu\text{m}/60\ \text{nm}$	0.562	$-60^\circ\text{C}$	5.437	$5.446 \pm 0.383$
		$27^\circ\text{C}$	5.401	$5.417 \pm 0.322$
		$130^\circ\text{C}$	5.428	$5.443 \pm 0.295$
1.0V NMOSFET in subth. region $W/L = 100\ \mu\text{m}/60\ \text{nm}$	0.665	$-60^\circ\text{C}$	6.196	$6.218 \pm 0.327$
		$27^\circ\text{C}$	6.175	$6.193 \pm 0.251$
		$130^\circ\text{C}$	6.201	$6.220 \pm 0.199$

\* Units are in mS.

\*\*  $\Delta V$  is kept at  $100\ \text{mV}$  and  $1/R$  is equal to  $5\ \text{mS}$ .

\*\*\* Mismatches of all transistors pairs and  $M_0$  are considered.

figure, as the operation region moves from the saturation region to the subthreshold region,  $\xi$  increase from 0.50 to 0.67. This is due to the  $I_{\text{out}}(V_{\text{in}})$  of the devices shifting from the power-law to exponential function, as analyzed in Appendix A. Also, the  $g_m$  in the temperature range with optimal  $\xi$  increases slightly as  $W/L$  increases since  $g_m$  variations instead of  $g_m$  itself are minimized during  $\xi$  optimization.

Next, the effects of  $1/R$  on  $\xi$  are studied. In this simulation,  $\Delta V = 100\ \text{mV}$ , and the  $W/L$  of  $M_1$  and  $M_2$  is  $40\ \mu\text{m}/0.28\ \mu\text{m}$ . However, the reference resistance  $R$  is swept. For each simulated  $R$ ,  $\Delta I$  is adjusted to maintain a constant  $\Delta V$ . In addition, its optimal  $\xi$  to achieve minimum  $g_m$  variations from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$  is found and presented. From the simulation results shown in Fig. 7, as the  $1/R$  increases,  $\xi$  decreases to 0.50. Also, the  $g_m$  in the temperature range with optimal  $\xi$  increases as  $1/R$  increases.

Finally, an optimal and constant  $\xi = 0.503$  to provide minimum  $g_m$  variation in saturation mode is chosen. Transconductance simulation results for  $-60^\circ\text{C}$  to  $130^\circ\text{C}$  are presented in Fig. 8.  $g_{m1}$  and  $g_{m2}$  converge toward  $1/R$  as the temperature increases due to the fact that the  $I_{\text{out}}(V_{\text{in}})$  curve of  $M_1$  and  $M_2$  becomes more linear at higher temperatures. For the  $g_m$  of the biased transistor,  $\pm 0.16\%$  variations are found over the temperature range.

To verify that the proposed technique is independent of transconductor  $I_{\text{out}}(V_{\text{in}})$  characteristics, three other optimal and constant  $\xi$ 's to provide minimum  $g_m$  variation for different operating regions and devices are similarly found. The simulation results for all cases, including process variations, are summarized in Table I.

The performance of the 2.5-V NMOSFET in the saturation region case is chosen to compare with the performance of other reported works. The direct replacement of the behavioral OpAmp to a minimally implemented single-stage 25-dB OpAmp only slightly degrades its  $g_m$  variations from  $\pm 0.16\%$  to  $\pm 0.22\%$  as shown in Fig. 8 and Table II. From this table, this work achieves a promising performance in the simulation level covering a wider temperature range.

TABLE II  
PERFORMANCE SUMMARY OF THE CONSTANT- $G_m$  CIRCUITS

	this work*	[8]	[7]	[9]
Implementation Level	simulation	simulation	simulation	simulation
$G_m$ standard deviations	$\pm 0.22\%$	$\pm 0.76\%$	$\pm 1\%$	$\pm 1.5\%$
Temperature range	$-60$ to $130^\circ\text{C}$	$-30$ to $110^\circ\text{C}$	$-25$ to $125^\circ\text{C}$	$20$ to $80^\circ\text{C}$
Current consumption	2mA**	1.1mA(subth. region)	1.5mA(sat. region)	1.5mA(sat. region)

\* 2.5V NMOSFET in saturation region with a  $\Delta I$  as 500uA

\*\* OpAmp draws 100 $\mu$ A.

#### IV. CONCLUSION

Formulating the classical constant- $g_m$  biasing problem into an analog computation problem allows us to study it from a different angle. From this perspective, it is found that the conventional constant- $g_m$  biasing technique relies on iterative computation that obscures the definition of both  $G_m$  and  $g_m$ . This also makes biasing depend on device characteristics.

By removing the iterative path in analog computation,  $G_m$  in conventional biasing can be better defined. This results in a design independent of device characteristics. In addition, an optimal constant- $g_m$  bias voltage can be generated using optimal interpolation  $\xi$  as analyzed in Appendix A. From simulation results, high-precision  $G_m$  and  $g_m$  can be achieved from  $-60^\circ\text{C}$  to  $130^\circ\text{C}$  as summarized in Table II.

#### APPENDIX A

##### RELATIONSHIPS BETWEEN SMALL-SIGNAL $g_m$ AND $\xi$

Consider the exponential and power-law  $I_{\text{out}}(V_{\text{in}})$  for MOSFETs in the subthreshold region and saturation region, respectively.

For the exponential case,  $I_{\text{out}}(V_0)$  and  $I_{\text{out}}(V_0 + \Delta V)$  can be obtained based on

$$I_{\text{out}}(V_{\text{in}}) = I_s \cdot \left( e^{\frac{V_{\text{in}} - V_{\text{th}}}{\phi_t}} - 1 \right) \quad (\text{A.1})$$

where  $V_{\text{th}}$  is the threshold voltage and  $\phi_t$  is the effective thermal voltage. The transconductance that is biased at  $V_0 + \xi \Delta V$  can be expressed as

$$\frac{\partial I_{\text{out}}(V_0 + \xi \Delta V)}{\partial V_{\text{in}}} = \frac{I_s \cdot e^{\frac{V_0 - V_{\text{th}} + \xi \Delta V}{\phi_t}}}{\phi_t}. \quad (\text{A.2})$$

$\xi$  can be found through solving

$$\frac{I_{\text{out}}(V_0 + \Delta V) - I_{\text{out}}(V_0)}{\Delta V} = \frac{\partial I_{\text{out}}(V_0 + \xi \Delta V)}{\partial V_{\text{in}}}. \quad (\text{A.3})$$

After some rearrangements,  $\xi$  can be derived as

$$\xi = \frac{1}{X_e} \cdot \ln \left( \frac{e^{X_e} - 1}{X_e} \right) \quad (\text{A.4})$$

where  $X_e \equiv (\Delta V)/\phi_t$  to simplify the equation.

For a  $I_{\text{out}}(V_{\text{in}}) = C_0 \cdot (V_{\text{in}} - V_{\text{th}})^n$   $n$ th-order power-law device,  $\xi$  can be calculated similarly to

$$\xi = \left( \frac{(1 + X_p)^n - 1}{n \cdot (X_p)^n} \right)^{\frac{1}{n-1}} - \frac{1}{X_p} \quad (\text{A.5})$$

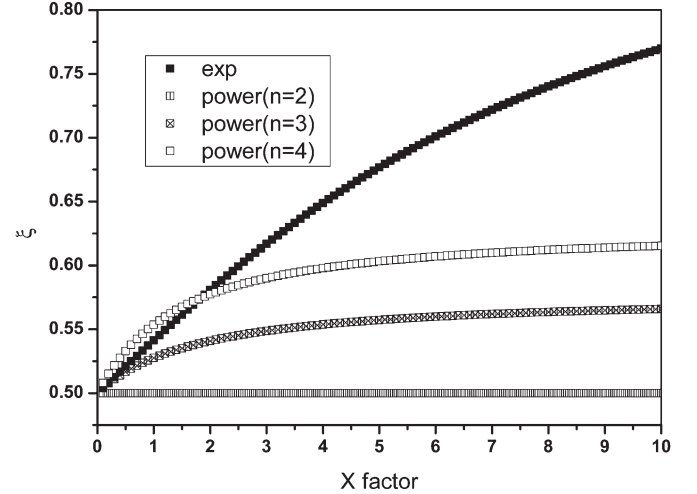


Fig. 9. Relationships between  $\xi$  versus  $X_e$  and  $X_p$  for exponential and power-law (order = 2, 3, 4) devices, respectively.

where  $X_p \equiv \Delta V/(V_0 - V_{\text{th}})$ . The relationships between  $\xi$  versus  $X_e$  and  $X_p$  are plotted in Fig. 9. In the proposed design, the  $X_e$  is approximately 4.93 for MOSFETs in the subthreshold region, and thus, the corresponding  $\xi$  is 0.65.

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