Astro 81 - Homework 7

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Question 1.

(a) Gravitational binding energy of the Sun and energy usage gives us an estimated lifetime.

$$\begin{aligned}
U &= \frac{3}{5}\frac{GM_\odot^2}{R} \\
U &= \frac{3}{5}\frac{6.67\times10^{-11}\,\mathrm{Nm^2/kg^2}\cdot 2\times10^{30}\,\mathrm{kg}^2}{6.96\times10^{8}\,\mathrm{m}} \\
U &= 2.30\times10^{41}\,\mathrm{J} \\
U &= t_{pred}\cdot L_\odot \\
t_{pred} &= 2.30\times10^{41}\,\mathrm{J}/3.8\times10^{26}\,\mathrm{J\,s^{-1}} \\
t_{pred} &= 6.05\times10^{14}\,\mathrm{s} \\
t_{pred} &= \boxed{2\times10^{7}\,\mathrm{yr}}
\end{aligned}$$

This would only last the sun 20 million years, not a few billion.

(b) Approximate the star to be 100% Hydrogen and constant luminosity till it is all burned up after its lifetime of 10×10^{10} years.

$$\begin{aligned}
\text{Efficiency} &= \frac{E_{actual}}{E_{potential}} \\
E_{actual} &= t_\odot\cdot L_\odot \\
&= 10\times10^{10}\,\mathrm{yr}\cdot 3.8\times10^{26}\,\mathrm{J\,s^{-1}} \\
&= 1.2\times10^{45}\,\mathrm{J} \\
E_{potential} &= mc^2 \\
&= 0.1\cdot 2\times10^{30}\,\mathrm{kg}\cdot(3\times10^{8}\,\mathrm{m\,s^{-1}})^2 \\
&= 1.8\times10^{46}\,\mathrm{J} \\
\text{Efficiency} &= \boxed{6.67\,\%}
\end{aligned}$$

(c) If it has to burn through the remaining 90% of hydrogen that's 9 times the previous amount of energy to burn through, with 100 times the rate of burning. The new time $t_{\odot,red} = 0.09t_{\odot,main}$ or $\boxed{9\,\%}$ the main sequence lifetime.

Question 2.

(a) Mean free path can be calculated with expected value of distance to the closest particle given a 3D Poisson distribution of particles. But the equation is

$$l_\gamma = \frac{1}{\kappa_\gamma \rho}$$
$$l_\gamma = \frac{1}{10\,\mathrm{m^2/kg} \cdot 10^5\,\mathrm{kg/m^3}}$$
$$l_\gamma = \boxed{10^{-6}\,\mathrm{m}}$$

(b) Same as the above but replace the mass absorption coefficient.

$$l_{neutrino} = \frac{1}{\kappa_\gamma \rho}$$
$$l_{neutrino} = \frac{1}{10^{-21}\,\mathrm{m^2/kg} \cdot 10^5\,\mathrm{kg/m^3}}$$
$$l_{neutrino} = \boxed{10^{16}\,\mathrm{m}}$$

(c) Use velocity of the photon with the mean free path.

$$ct_\gamma = l_\gamma$$
$$t_\gamma = \frac{10^{-6}\,\mathrm{m}}{3 \times 10^8\,\mathrm{m\,s^{-1}}}$$
$$t_\gamma = \boxed{3.3 \times 10^{-15}\,\mathrm{s}}$$

(d) Central pressure of a $10M_\odot$ star will be about $0.01P_\odot$ since central density is proportional to $1/M^2$. Since central pressure decreases, central density will too, and for an ideal gas we have $P = n\rho T$ so $P \propto \rho$ so the new central density is also about $0.01\rho_\odot$ from what we had before. $l_\gamma \propto 1/\rho$ and $t_\gamma \propto l_\gamma$ so $t_\gamma \propto 1/\rho$ so we get that the new mean free path time is $1/0.01$ or $\boxed{\text{100 times}}$ the answer in part c.

Question 3.

(a) Calculate absolute magnitude then luminosity which is in units of Sun luminosity.

$$\begin{aligned} m - M &= 5\log R - 5 \\ 7.5 - M &= 5\log 1000 - 5 \\ M &= -2.5 \\ -2.5\log(L/L_\odot) &= -2.5 \\ L &= \boxed{10L_\odot} \end{aligned}$$

(b) Mass-luminosity relation is approximately $L \propto M^3$ so if the luminosity is 10 times that of the sun the mass is $10^{1/3}$ times the mass of the sun, or $\boxed{2.15m_\odot}$.

(c) Since $t_{ms} \propto M/L$ we can get an approximate relation to get its age in relation to the Sun where the Sun's lifetime is $t_\odot = 10^{10}\,\text{yr}$, $t = 2.15/10t_\odot$ which is $\boxed{2.15 \times 10^9\,\text{yr}}$.