HW1: Physics 115A (Due 5:00 PM, 10/10, 2021)

1. **(50 points)** Consider the distribution of blackbody radiation. As we have shown in class, Planck's formula correctly describes the experimental data for the energy density per unit frequency $\rho(\nu, T)$, with

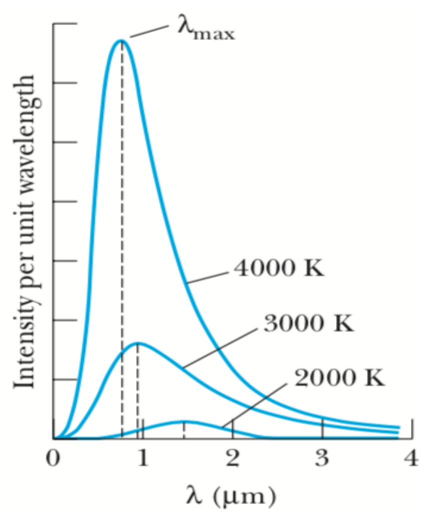
$$\rho(\nu, T) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/k_B T} - 1}, \tag{1}$$

where ν is the frequency, c is the speed of light, T is the temperature, $k_B = 1.380 \times 10^{-23}$ J/K is Boltzmann's constant, and $h = 6.626^{-34}$ J $\cdot$ s is the famous Planck's constant. Answer the following questions

(a) **(20 points)** Let $u(\lambda, T)$ be the energy density per unit *wavelength* with λ being the wavelength, from Eq. (1), derive start

$$u(\lambda, T) = \frac{8\pi ch}{\lambda^5}\frac{1}{e^{hc/\lambda k_B T} - 1}. \tag{2}$$

(b) **(10 points)** Experimentally one finds $u(\lambda, T)$ as a function of λ at different temperature T has the behavior shown in Fig. 1. Note that as T increases, the wavelength λ_{max} at which $u(\lambda, T)$ reaches a maximum shifts toward shorter wavelengths. Show that there is a general rela-



tionship between temperature T and λ_{max} stating that

$$T\lambda_{\text{max}} = \text{constant}. \tag{3}$$

(c) **(10 points)** Obtain a numerical value for the constant in Eq. (3).

(d) **(10 points)** Show that the total energy in the blackbody at temperature T is proportional to T^4, and evaluate the factor of proportionality.

2. (20 points) In general, quantum mechanics is relevant when the de Broglie wavelength of the particle in question (h/p) is greater than the characteristic size of the system (d). In thermal equilibrium at (Kelvin) temperature T, the average kinetic energy of a particle is

$$\frac{p^2}{2m} = \frac{3}{2}k_B T$$

(where k_B is Boltzmann's constant), so the typical de Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{3mk_B T}}.$$

The purpose of this problem is to anticipate which systems will have to be treated quantum mechanically, and which can safely be described classically.

(a) **Solids.** The lattice spacing in a typical solid is around $d = 0.3$ nm. Find the temperature below which the free[18] *electrons* in a solid are quantum mechanical. Below what temperature are the *nuclei* in a solid quantum mechanical? (Use sodium as a typical case.) *Moral:* The free electrons in a solid are *always* quantum mechanical; the nuclei are almost *never* quantum mechanical. The same goes for liquids (for which the interatomic spacing is roughly the same), with the exception of helium below 4 K.

(b) **Gases.** For what temperatures are the atoms in an ideal gas at pressure P quantum mechanical? *Hint:* Use the ideal gas law ($PV = Nk_BT$) to deduce the interatomic spacing. *Answer:* $T < (1/k_B)(h^2/3m)^{3/5}P^{2/5}$. Obviously (for the gas to show quantum behavior) we want m to be as *small* as possible, and P as *large* as possible. Put in the numbers for helium at atmospheric pressure. Is hydrogen in outer space (where the interatomic spacing is about 1 cm and the temperature is 3 K) quantum mechanical?

3. (15 points)

(a) (9 points) Please construct the energy level diagram of the Li^{2+} ion, whose Z =3.

(b) Four possible transitions for the Li^{2+} ion are listed here:

(A) $n_{\mathrm{i}} = 2;\, n_{\mathrm{f}} = 5$

(B) $n_{\mathrm{i}} = 5;\, n_{\mathrm{f}} = 3$

(C) $n_{\mathrm{i}} = 7;\, n_{\mathrm{f}} = 4$

(D) $n_{\mathrm{i}} = 4;\, n_{\mathrm{f}} = 7$

(b1) (2 points)Which transition emits the photons with the shortest wavelength?

(b2) (2 points)For which transition does the atom gain most energy?

(b3) (2 points)For which transition(s) does the atom lose energy?

4. (15 points)

A two-slit diffraction experiment is done with slits of unequal widths. When only one slit is open, the number of electrons reaching the screen per second is 25 times the number of electrons reaching the screen per second when only slit 2 is open. When both slits are open, an interference pattern shows up. Find the ratio of the probability of an electron arriving at an interference maximum to the probability of an electron arriving at an adjacent interference minimum.