Physics 115A - Homework 1

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Question 1.

Deriving the energy density for wavelength.

$$\begin{aligned}
u(\lambda, T) &= p(c/\lambda, T) \\
&= \frac{8\pi(c/\lambda)^2}{c^3}\frac{h(c/\lambda)}{e^{hc/\lambda kT}-1} \\
&= \frac{8\pi c^2}{\lambda^2}\frac{hc/\lambda}{e^{hc/\lambda kT}-1} \\
&= \frac{8\pi ch}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}
\end{aligned}$$

λ_{max} occurs when $\frac{\mathrm{d}u}{\mathrm{d}\lambda} = 0$.

$$\begin{aligned}
\frac{\mathrm{d}u}{\mathrm{d}\lambda} &= 0 \\
8\pi hc\left(-\frac{5}{\lambda^6\left(e^{\frac{hc}{k\lambda T}}-1\right)} + \frac{hce^{\frac{ch}{k\lambda T}}}{k\lambda^7 T\left(e^{\frac{ch}{k\lambda T}}-1\right)^2}\right) &= 0 \\
\left(\frac{1}{\lambda^6(e^{\frac{hc}{k\lambda T}}-1)^2}\right)\left(-5e^{\frac{hc}{k\lambda T}} + 5 + \frac{hc}{k\lambda T}e^{\frac{hc}{k\lambda T}}\right) &= 0 \\
5 - \left(5 - \frac{hc}{k\lambda T}\right)e^{\frac{hc}{k\lambda T}} &= 0 \\
5 - 5e^x + xe^x &= 0 \\
x = \frac{hc}{k\lambda T} &= 5 = const. \\
\lambda_{max}T = \frac{hc}{5k} &= const.
\end{aligned}$$

Getting Wien's constant.

$$\begin{aligned}
\frac{hce^5}{5k} &= \frac{(6.626\times 10^{-34}\,\mathrm{J\,s})(3\times 10^8\,\mathrm{m/s})e^5}{5(1.38\times 10^{-23}\,\mathrm{J/K})} \\
&= 2.88\times 10^{-3}\,\mathrm{m\,K}
\end{aligned}$$

Total energy is integral over all wavelengths.

$$\begin{aligned} E &= \int_0^\infty \frac{8\pi ch}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \, \mathrm{d}\lambda \\ &\propto T^4 \end{aligned}$$