Astronomy 82: Problem Set #6

Submit on Top Hat by midnight on May 16, 2021

1) The pattern speed (that is, the angular velocity) of the spiral structure in our Galaxy has been estimated to be 40 km/s per kpc.

a) If the Sun's orbital velocity around the center of the Galaxy is 240 km/s, and if the Sun is located at a galactocentric radius of 8 kpc, is the Sun inside or outside of the Galaxy's co-rotation radius? (Assume that the Galaxy has a flat rotation curve.) And therefore is the Sun catching up with the spiral structure and coming into the spiral arms from behind, or are the spiral arms overtaking the Sun, so that the Sun enters the front of the spiral arms?

b) How many times has the Sun passed through a spiral arm in its lifetime? (This is important for those who want to know whether cosmic events have influenced the evolution of life on Earth, because star formation and therefore supernovae are concentrated in spiral arms. Being inside of a supernova remnant could indeed have consequences for the level of background radioactivity, which of course could influence evolution by provoking mutations.)

2) The Sun has a peculiar velocity of about 20 km/s. That is, on top of pure circular rotation around the Galactic center, it is moving at that speed. Other stars in the solar neighborhood have similar peculiar speeds in random directions. If the local density of stars is 0.1 per pc^3, how often does the Sun come within 50 AU of another star? (50 AU being roughly the size of the planetary part of the solar system.)

3) The nuclear star cluster at the center of the Galaxy can be approximated as a spherically symmetric, constant-density core of radius 0.5 pc, beyond which the density of stars declines as r^{-2}, where r is the radial distance from the Galactic center. The central density of stars is 500,000 $M_\odot$ pc^{-3}. Calculate the radius of the gravitational domain of influence of the central supermassive black hole, which is the region inside of which the black hole's gravity is more important (stronger) than the gravity of all the stars inside that region. The black hole's mass is 4×10^6 $M_\odot$.

4) Assume that the Sun is orbiting the center of the Galaxy in a circular orbit at a speed V = 250 km/s and that the Sun is 8.2 kpc from the center. **(15 points)**

a) What is the mass interior to the solar orbit? (You can assume that the mass is spherically distributed.)

b) If the Galaxy's rotation curve is flat out to 30 kpc, and then becomes Keplerian beyond that ($V \propto r^{-1/2}$), what is the escape velocity from the Galaxy, starting on the solar circle?

5) **a)** Using the figure below as a guide, determine the *maximum* radial velocity that one would observe for a distant star along a line of sight in the Galactic plane at Galactic longitude *l*. Assume that the Galaxy has a flat rotation curve, with a velocity of 250 km/s, and evaluate the maximum velocity at longitude $l = 30^o$. In the figure, GC is the Galactic center, and R_o is the distance between the Sun and the GC.

b) Determine the distance of a star located at Galactic longitude 30^o if it has a radial velocity of +80 km/s, as observed by an observer in the solar system. Assume that neither the Sun nor the star being observed has a peculiar velocity on top of its Galactic rotation speed. (Careful, there may be more than one answer!)