Astronomy 82 – Problem Set #8

Due by Thursday, June 3 at midnight on Top Hat

1. The lifetime (actually, half-life) of a muon in its rest frame is 2.2 microseconds.
   a) how, then, can a muon travelling at nearly the speed of light, make it all the way from the top of the earth's atmosphere to the surface of the solid earth, a distance of about 100 km? At nearly the speed of light, the muon lifetime times its speed is less than a kilometer. (Relativistic muons are created when high-energy cosmic rays strike atomic nuclei in the upper atmosphere.) *Answer this from both the muon's perspective and our perspective from the ground.*
   b) How far would we typically see a muon travel if it was moving at 99.5% the speed of light?

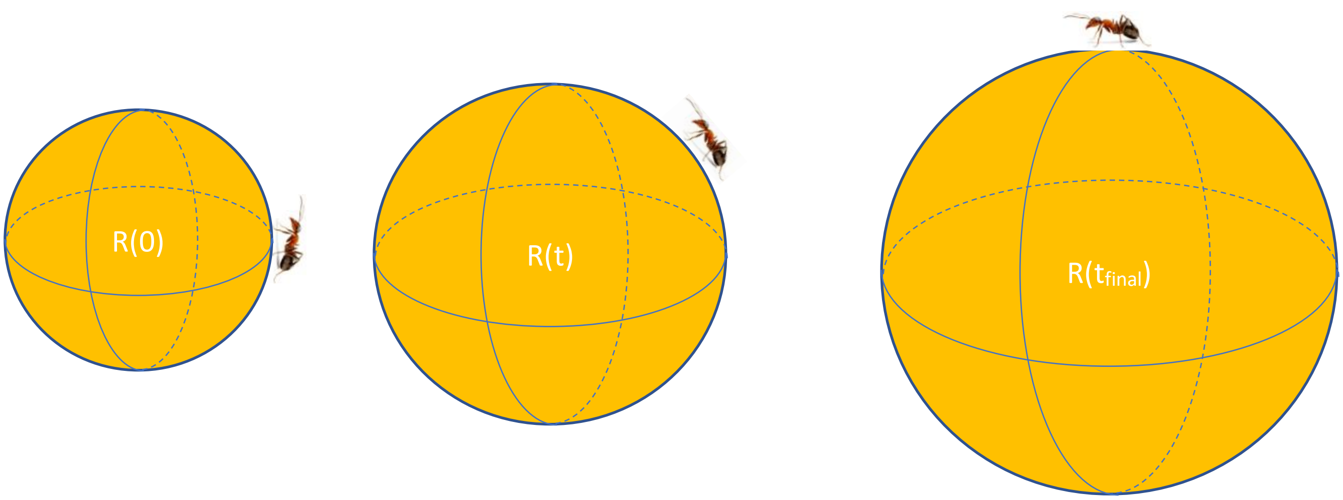
2. Photons in the cosmic microwave background were last scattered immediately prior to the recombination epoch, when the universe had cooled to a temperature at which electrons could combine with protons to create neutral hydrogen atoms. The temperature of the universe then was about 3000 K.
   a) What was the peak wavelength of cosmic background photons at that time, and what part of the electromagnetic spectrum were they mostly in?
   b) At what redshift did recombination occur (show how you got the answer)?
   c) What was the average mass-energy density of the universe at the recombination epoch? (The critical density of the universe is now $3H_o^2/(8\pi G) = 1.06 \times 10^{-29}$ gm cm^{-3}, for Hubble's constant H_o, which amounts to about 6 hydrogen atoms per cubic meter, but only a small fraction of the mass-energy is in the form of baryonic matter.)

3. The average temperature of the cosmic microwave background radiation is 2.728 K. However, there is a "dipole anisotropy" that makes the CMBR appear to be warmer in one direction (toward galactic coordinates of L = 264.4 degrees, b = 48.4 degrees) by 3.37 milliKelvin. (It is similarly colder by the same amount in the opposite direction.)
   a) How fast is the Sun moving through the Hubble Flow?
   b) Describe what might cause this motion (there are multiple contributions).

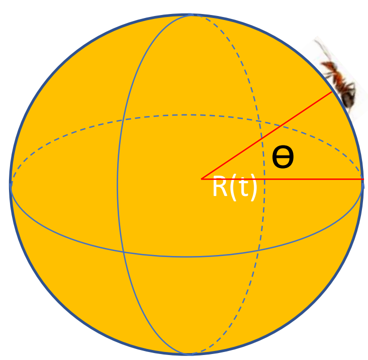
4. You observe a Type Ia supernova exploding at a redshift of $z = 1.5$. Using the relativistic formula relating velocity, v, to redshift, $1 + z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$, describe how the light curve of that supernova would be different from that of an identical Type Ia supernova located nearby.

5. One solution of Friedmann's equations for an expanding universe is a three dimensional sphere which is the surface of a 4-dimensional ball much like the surface of the Earth is a 2-dimensional sphere that is the surface of a 3-dimensional ball. But since the solution is dynamic, the sphere has an ever changing radius. During the present and past of our Universe that radius would be increasing (expanding). Although we don't believe this is

the universe we live in (ours in very close to flat), it is still among the easiest to wrap our brains around and demonstrate some of the properties of an expanding universe. We can further think about just a 2-dimensional slice of the Universe which would be an actual spherical surface like a globe that is increasing in size. So imagine a spherical surface that is expanding over time with a radius R that is a function of time R(t). Imagine a traveler, like an ant walking on a balloon that is being blown up, that moves at constant speed v while the radius expands. If the "ant" starts at the equator of the sphere at time 0 when the radius of the sphere is R(0) and walks towards the pole. If the initial radius is 10,000mm and the radius increases by 1mm every second, then when (t_{final}) will an ant walking at 5 mm/sec reach the pole? What was the original distance between the equator and the pole when the ant started? What is the distance between the equator and the pole when the ant arrives? How far did the ant walk? This situation is exactly like a photon that always travels at a fixed speed c in a Universe that is expanding.



Perhaps the easiest way to approach this problem is to think about the ant's angular progress as viewed from the center of the sphere.



At any given time t, the fixed speed of the ant, v, will be equal to a rate of change of the central angle in radians times the current radius of the sphere:

$$R(t)\frac{d\theta}{dt} = v$$

$$so \quad d\theta = \frac{v}{R(t)}dt$$

Substitute in the appropriate expression for R(t) and integrate both sides setting the initial angle to 0 degrees (equator) and the final angle to π/2 (pole). Make sure to use radians!!! And keep as many digits as possible through your calculations.