PHY115A HW#8

(Late policy: 0% credit)

1. (25 points)

Problem 3.5 The **hermitian conjugate** (or **adjoint**) of an operator $\hat{Q}$ is the operator $\hat{Q}^\dagger$ such that

$$\langle f|\hat{Q}g\rangle = \langle \hat{Q}^\dagger f|g\rangle \quad \text{(for all } f \text{ and } g\text{).} \qquad [3.20]$$

(A hermitian operator, then, is equal to its hermitian conjugate: $\hat{Q} = \hat{Q}^\dagger$.)

(a) Find the hermitian conjugates of x, i, and d/dx.

(b) Construct the hermitian conjugate of the harmonic oscillator raising operator, a_+ (Equation 2.47).

(c) Show that $(\hat{Q}\hat{R})^\dagger = \hat{R}^\dagger\hat{Q}^\dagger$.

2. (25 points)

Problem 3.6 Consider the operator $\hat{Q} = d^2/d\phi^2$, where (as in Example 3.1) ϕ is the azimuthal angle in polar coordinates, and the functions are subject to Equation 3.26. Is $\hat{Q}$ hermitian? Find its eigenfunctions and eigenvalues. What is the spectrum of $\hat{Q}$? Is the spectrum degenerate?

3. (25 points)

Problem 3.10 Is the ground state of the infinite square well an eigenfunction of momentum? If so, what is its momentum? If not, *why* not?

4. (25 points)

Problem 3.26 An **anti-hermitian** (or **skew-hermitian**) operator is equal to *minus* its hermitian conjugate:

$$\hat{Q}^\dagger = -\hat{Q}. \qquad [3.95]$$

(a) Show that the expectation value of an anti-hermitian operator is imaginary.

(b) Show that the commutator of two hermitian operators is anti-hermitian. How about the commutator of two *anti*-hermitian operators?