ASTR0 81 - Assignment #7

Note: This last problem set is just a few short problems to practice recent material, but does not provide a comprehensive review of the last two weeks of material. Due Friday before 5pm

1. The main sequence lifetime of the sun is roughly $t_\odot \sim 10^{10}$ yr. Once a star burns all of the Hydrogen *in its core* (which contains about 10% of the star's initial Hydrogen content), it will begin to evolve off of the main sequence. The mass of the sun is $M_\odot \sim 2 \,\text{x}\, 10^{30}$ kg , and the bolometric luminosity of the sun is $L_\odot \sim 3.8 \,\text{x}\, 10^{26}$ W.

   (a) During the 1800s, the leading theory was that the sun was radiating gravitational potential energy released as it contracted from an originally infinitely-large diffuse cloud. But by that time, geologic evidence already indicated the solar system was billions of years old. Show that the release of gravitational potential energy as the sun contracted is nowhere near enough to keep the sun shining that long.

   (b) We now know that nuclear fusion is actually the energy source that powers the sun. Given the information provided in this problem, calculate the efficiency of Hydrogen fusion, i.e. the percentage of energy burned during fusion that actually gets radiated away.

   (c) Once a solar mass star leaves the main sequence, it then expands into a red giant and its luminosity increases by a factor of ∼100. If burning the remaining Hydrogen in the star is the only source of energy production during the red giant stage (a very crude approximation, but a good order of magnitude demonstration), how long is the red giant stage for a solar mass star? Express your answer as a percentage of the main sequence lifetime of the sun.

2. The central density of the sun is $\sim 10^5 kg/m^3$. The mass absorption coefficient of a photon is $\kappa_\gamma = 10$ m^2 kg^{-1}. The mass absorption coefficient of a neutrino is $\kappa_{neutrino} = 10^{-21}$ m^2 kg^{-1}.

   (a) Calculate the average distance that a photon in the core of the sun can travel before being scattered or absorbed. Note: this distance is defined as the mean free path.

   (b) Calculate the average distance that a neutrino in the core of the sun can travel before being scattered or absorbed. How does this compare to the mean free path for photons?

   (c) For what length of time can a photon travel in the center of the sun before being scattered or absorbed?

(d) For what length of time can a photon travel in the center of a $10 M_\odot$ main sequence star before being scattered or absorbed? Express your answer as a fraction of the time calculated in part (b).

3. Say we observe a stellar cluster that is 1,000 pc away in which the brightest star still on the main sequence has an apparent bolometric magnitude of 7.5.

   (a) What is the luminosity of the brightest main sequence star in that cluster, in units of solar luminosity?

   (b) What is the mass of the brightest main sequence star in that cluster, in units of solar masses?

   (c) What is the age of the cluster?