hw 7

Problem 1

a) Trapezoidal approximation.

$$\int_1^2 x\ln x \, dx \approx \frac{1}{8}\left[(1\ln 1 + 1.25\ln 1.25) + (1.25\ln 1.25 + 1.5\ln 1.5) + (1.5\ln 1.5 + 1.75\ln 1.75) + (1.75\ln 1.75 + 2\ln 2)\right] = \boxed{0.6399}$$

b) $$\frac{1}{12}\left[f(1) + 4f(1.25) + f(1.5) + f(1.5) + 4f(1.75) + f(2)\right] = \boxed{0.6363}$$

Problem 2

a $$10^{-5} = \frac{h^2}{12}\left|f''(\mu)\right| = \frac{h^2}{12}\left|\frac{1}{\mu}\right| \Big\} \max \text{ at } 1$$

$$= h^2/12 \rightarrow \boxed{h \leq 0.01095}$$

$$n = \frac{2-1}{h} \geq \boxed{92}$$

b $$10^{-5} = \frac{h^4}{180}\left|f^{(4)}(\mu)\right| = \frac{h^4}{180}\left|\frac{2}{\mu^3}\right| \Big\} \max \text{ at } 1$$

$$= h^4/90 \rightarrow \boxed{\begin{matrix} h \leq 0.1732 \\ n \geq 6 \end{matrix}}$$

Problem 3

Should equal $f(x) = 1, x, x^2, x^3$

$$\int_{-1}^1 dx = 2 = a + b$$

$$\int_{-1}^1 x\,dx = \frac{x^2}{2}\Big|_{-1}^1 = 0 = -a + b + c + d$$

$$\int_{-1}^1 x^2 dx = \frac{2}{3} = a + b - 2c + 2d$$

$$\int_{-1}^1 x^3 dx = 0 = -a + b + 3c + 3d$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 2 \\ -1 & 1 & 3 & 3 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2/3 \\ 0 \end{bmatrix}$$

$$\boxed{a = 1,\ b = 1,\ c = \frac{1}{3},\ d = -\frac{1}{3}}$$

Problem 4

$$\sum w_i f(x_i) = \frac{1}{\sqrt{\pi}}\left[2(0.2364)f(0.9061) + 2(0.4786)f(0.5384) + (0.5688)f(0)\right]$$

$$= \boxed{0.84270}$$

Problem 5

a $$\boxed{u(x) = -1 + 2\frac{x-a}{b-a}}$$

b $$\frac{1}{2}(u+1) = \frac{x-a}{b-a}$$

$$\frac{b-a}{2}(u+1) + a = x, \quad dx = \frac{b-a}{2}du$$

$$\int_a^b f(x)\,dx = \boxed{\int_{-1}^1 f\left(\frac{b-a}{2}(u+1)+a\right)\frac{b-a}{2}\,du}$$

c $$\int_1^2 \ln x\,dx = \int_{-1}^1 \ln\left(\frac{1}{2}(u+1)+1\right)\frac{1}{2}du$$

$$= \int_{-1}^1 \frac{1}{2}\ln\left(\frac{1}{2}(u+3)\right)du$$

$$\approx \frac{1}{2}\ln\left(\frac{1}{2}\left(-\frac{1}{\sqrt{3}}+3\right)\right) + \frac{1}{2}\ln\left(\frac{1}{2}\left(\frac{1}{\sqrt{3}}+3\right)\right) = \boxed{0.3865}$$

actual # is 0.38629

relative error: $\boxed{.07\%}$

Problem 6

a $\langle x_1, x_1\rangle = 5$ / $\langle x_1, x_2\rangle = -2$

$\langle x_2, x_2\rangle = 2$ $\langle x_1, x_3\rangle = -1$

$\langle x_3, x_3\rangle = 10$ $\langle x_2, x_3\rangle = -3$

$$v_1 = (0, -1, 2)$$

$$v_2 = (1, 0, -1) - \frac{-2}{5}(0, -1, 2)$$

$$= (1, 0, -1) + \left(0, -\frac{2}{5}, \frac{4}{5}\right)$$

$$= \left(1, -\frac{2}{5}, -\frac{1}{5}\right)$$

$$v_3 = (-3, 1, 0) + \frac{1}{5}(1, 0, -1) + \frac{3}{2}\left(1, -\frac{2}{5}, -\frac{1}{5}\right)$$

$$= \left(-\frac{13}{10}, \frac{2}{5}, -\frac{1}{2}\right)$$

$$\boxed{\begin{aligned} q_1 &= \left(0, -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) \\ q_2 &= \left(\sqrt{\tfrac{5}{8}}, 0, -\sqrt{\tfrac{5}{8}}\right) \\ q_3 &= \left(\frac{-3\sqrt{10}}{\sqrt{21}}, \sqrt{\tfrac{10}{21}}, 0\right) \end{aligned}}$$

b $v_1 = 1$

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$q_1 = 1/\sqrt{\int_{-1}^{1} dx}$ $\boxed{\frac{1}{\sqrt{2}}}$

$v_2 = x - \frac{\cancel{\langle 1, x\rangle}\to 0}{\langle 1,1\rangle}\left(\frac{1}{2}\right) = x$

$\boxed{x}$

$q_2 = \frac{x}{\sqrt{\int_{-1}^{1} x^2 dx}} = \boxed{\sqrt{\frac{3}{2}}\, x}$

$v_3 = x^2 - \frac{\langle 1, x^2\rangle}{\langle 1,1\rangle}\frac{1}{\sqrt{2}} - \frac{\frac{3}{2}\cancel{\langle x, x^2\rangle}\to 0}{\frac{3}{2}\langle x, x\rangle}\sqrt{\frac{3}{2}}\, x$

$= x^2 - \frac{1}{3\sqrt{2}}$

$q_3 = \frac{x^2 - \frac{1}{3\sqrt{2}}}{\langle v_3, v_3\rangle} = \boxed{\frac{x^2 - \frac{1}{3\sqrt{2}}}{\frac{23}{45} - \frac{2\sqrt{2}}{9}}}$

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