Physics 115A HW 5

1. (25 points) Problem (2.12 2nd ed.; 2.12 3rd ed.) in "Introduction to Quantum Mechanics" by David J. Griffiths.

   $*$**Problem 2.12** Find $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$, and $\langle T \rangle$, for the nth stationary state of the harmonic oscillator, using the method of Example 2.5. Check that the uncertainty principle is satisfied.

2. (20 points) Problem (2.14 2nd ed.; see below 3rd ed.) in "Introduction to Quantum Mechanics" by David J. Griffiths.

   Problem 2.14 A particle is in the ground state of the harmonic oscillator with classical frequency ω, when suddenly the spring constant quadruples, so $\omega' = 2\omega$, without initially changing the wave function (of course, Ψ will now *evolve* differently, because the Hamiltonian has changed). What is the probability that a measurement of the energy would still return the value $\hbar\omega/2$? What is the probability of getting $\hbar\omega$? [*Answer:* 0.943.]

3. (15 points) Problem (2.15 2nd ed.; 2.14 3rd ed.) in "Introduction to Quantum Mechanics" by David

   J. Griffiths.

   Problem 2.15 In the ground state of the harmonic oscillator, what is the probability (correct to three significant digits) of finding the particle outside the classically allowed region? *Hint:* Classically, the energy of an oscillator is $E = (1/2)ka^2 = (1/2)m\omega^2 a^2$, where a is the amplitude. So the "classically allowed region" for an oscillator of energy E extends from $-\sqrt{2E/m\omega^2}$ to $+\sqrt{2E/m\omega^2}$. Look in a math table under "Normal Distribution" or "Error Function" for the numerical value of the integral.

4. (30 points) Problem (2.41 2nd ed.; 2.40 3rd ed.) in "Introduction to Quantum Mechanics" by David J. Griffiths.

   Problem 2.41 A particle of mass m in the harmonic oscillator potential (Equation 2.43) starts out in the state

   $$\Psi(x,0) = A\left(1 - 2\sqrt{\frac{m\omega}{\hbar}}x\right)^2 e^{-\frac{m\omega}{2\hbar}x^2},$$

   for some constant A.

   (a) What is the expectation value of the energy?

   (b) At some later time T the wave function is

   $$\Psi(x,T) = B\left(1 + 2\sqrt{\frac{m\omega}{\hbar}}x\right)^2 e^{-\frac{m\omega}{2\hbar}x^2},$$

   for some constant B. What is the smallest possible value of T?

5. (10 points) Problem (2.42 2nd ed.; 2.41 3rd ed.) in "Introduction to Quantum Mechanics" by David J. Griffiths.

Problem 2.42 Find the allowed energies of the *half* harmonic oscillator

$$V(x) = \begin{cases} (1/2)m\omega^2 x^2, & \text{for } x > 0, \\ \infty, & \text{for } x < 0. \end{cases}$$

(This represents, for example, a spring that can be stretched, but not compressed.) *Hint:* This requires some careful thought, but very little actual computation.