Problem 1

a) $G\frac{Mm}{r^{\cancel{2}}} = \frac{mv^2}{\cancel{r}} \rightarrow \frac{GM}{3R_\odot} = v^2$, note $\rho = \frac{M}{V} = \frac{N(2\,amu)}{6.022\times10^{23}/mol}$

$$v^2 = \frac{GM}{3R_\odot} = \frac{G\cdot\frac{4}{3}\pi(3R_\odot)^{\cancel{3}\,2}\cdot(10^4\cdot100^3\frac{1}{m^3})\cdot\left(\frac{.002016\ kg}{6.022\times10^{23}\ mol}\right)}{3\cancel{R_\odot}}$$

$$v = 2.02\times10^{-4}\ \frac{m}{s}\cdot\left(6.96\times10^8\ \frac{m}{rad}\right)^{-1} = \boxed{2.9\times10^{-13}\ \frac{rad}{s}}$$

b) $\omega = v/r = 240\ \frac{km}{s}\cdot\left(8\ kpc\cdot3.086\times10^{16}\ \frac{km}{kpc}\right)^{-1}$

$= \boxed{9.72\times10^{-16}\ \frac{rad}{s}}$ above is ~ 1000× faster

It's a bit faster which should be expected for a smaller object than the Milky Way.

Problem 2

Given $s^2 = 2R^2$ ... we get $2R^2 = R^2 + \frac{2GMR}{V_0^2}$

so $R^2 = \frac{2GMR}{V_0^2} \rightarrow \cancel{R} = \frac{2GM}{V_0^2} = \frac{2G(\frac{4}{3}\pi R^3)\rho}{V_0^2}$

$$R^2 = \frac{V_0^2}{\frac{8}{3}G\rho\pi} = \frac{[(.01)(2.98\times10^4\ \frac{m}{s})]^2}{\frac{8}{3}(6.67\times10^{-11}\ \frac{Nm^2}{kg^2})(\pi)(3\ \frac{g}{cm^3}\cdot\frac{1\ kg}{1000\ g}\cdot100^3\ \frac{cm^3}{m^3})}$$

$$\boxed{R = 2.3\times10^5\ m}$$

Scanned with CamScanner

Problem 3

Fraction of photons $\Rightarrow$ luminosity

$I = I_0 e^{-\tau}$ and $m - M = -2.5 \log\left(\frac{I}{I_0}\right) = 2.5 \log e^{-\tau}$

$30 = -2.5 \log\left(\frac{I}{I_0}\right) \rightarrow -12 = \log \frac{I}{I_0} \rightarrow \frac{I}{I_0} = \boxed{10^{-12}}$

$2.5 = -2.5 \log\left(\frac{I}{I_0}\right) \rightarrow -1 = \log \frac{I}{I_0} \rightarrow \frac{I}{I_0} = \boxed{0.1}$

Problem 4

a) note $\tau = n\sigma$ so $\frac{n_{12}}{n_{13}} \approx 65$ so $\frac{\tau_{12}}{\tau_{13}} = 65 \rightarrow \tau_{12} = 65\,\tau_{13}$

$10 \approx \frac{I_{12}(1-e^{-\tau_{12}})}{I_{13}(1-e^{-\tau_{13}})} \rightarrow 10 \approx \frac{1}{1-e^{-\tau_{13}}}$ assume $\tau_{12} >> 1$

$1 - e^{-\tau_{13}} = \frac{1}{10} \rightarrow e^{-\tau_{13}} = 0.9 \rightarrow \tau_{13} = 0.105$

$\tau_{12} = 65(.105) = \boxed{6.84}$

b) It may be different/higher because the Sun is fusing up to ^{12}C but not as much ^{13}C compared to the rest of the galaxy with higher mass stars that fuse more isotopes of carbon.