CS M146 - Homework 5

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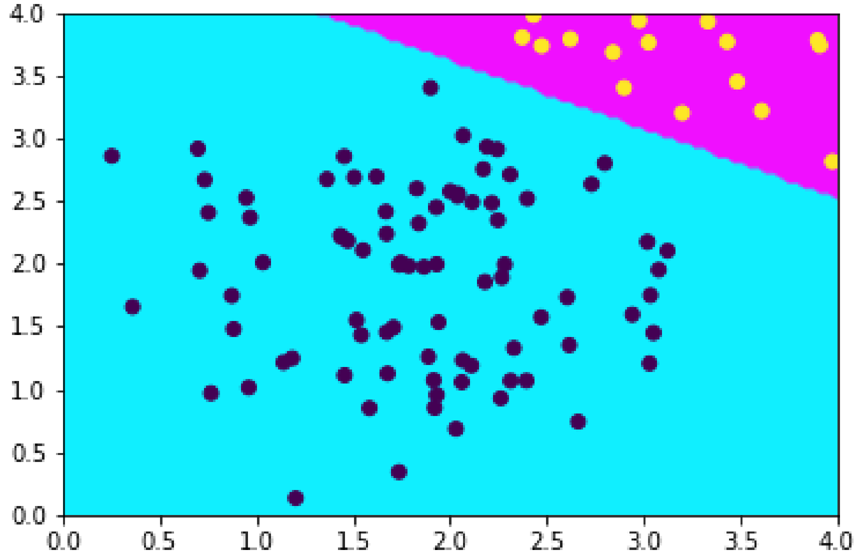
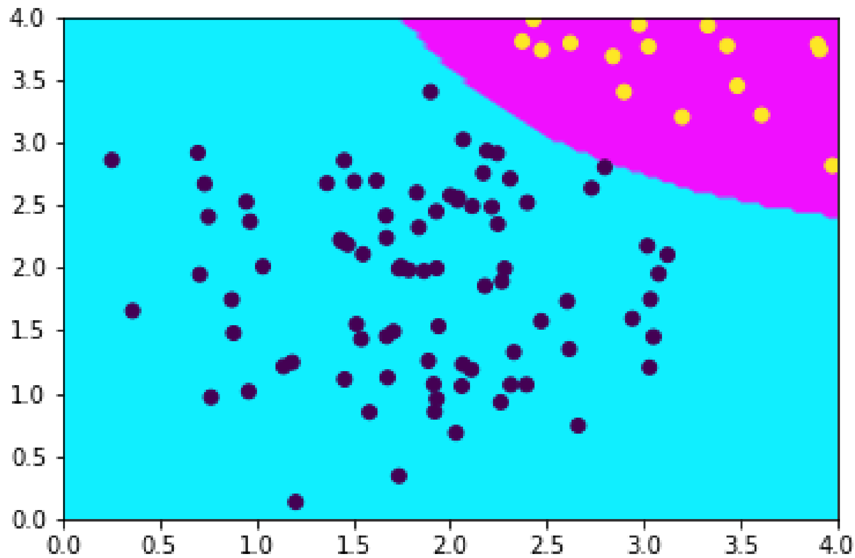
Question 1.

For the no-kernel kernel I got $w = (5.92, 3.24), b = -28$ which is the same as the results in HW1.

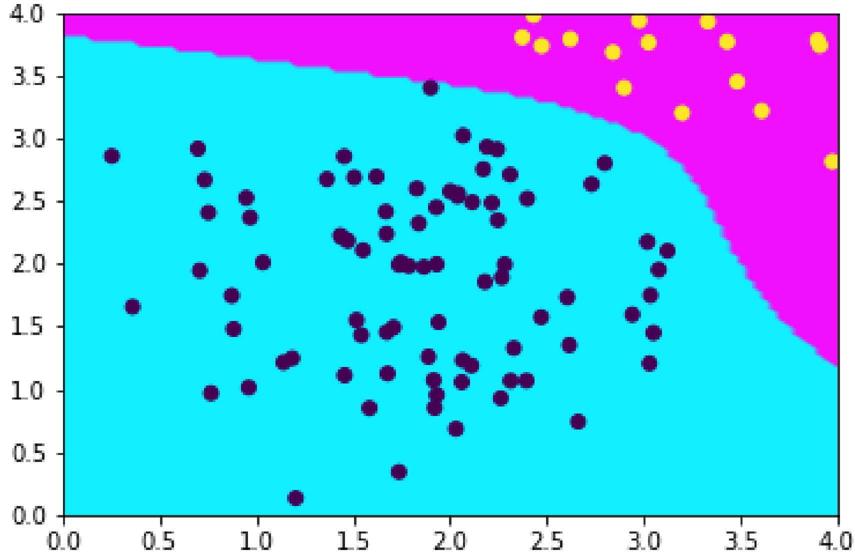
No-kernel kernel on 2031 dataset gives us accuracy of 1.0, data is linearly separated.



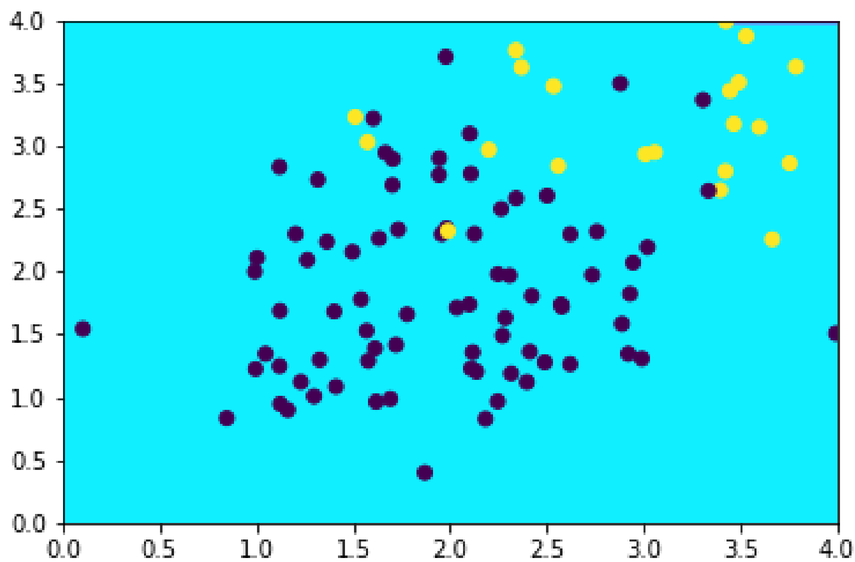
Polynomial kernel on 2031 dataset. The accuracy after training is 1.0.



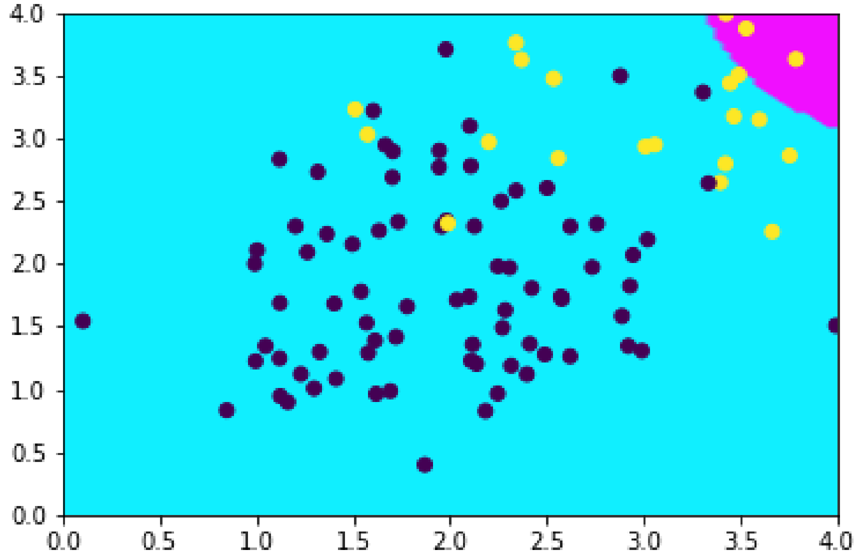
Gaussian kernel on 2031 dataset with $\sigma = 1$. The accuracy after training is 1.0.



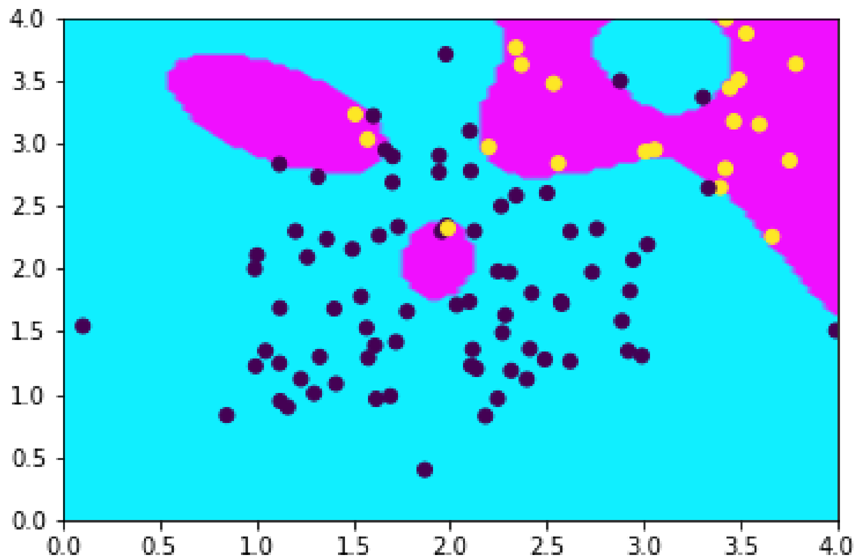
No-kernel kernel on 2030 dataset gives us accuracy of 0.79, data is linearly separated.



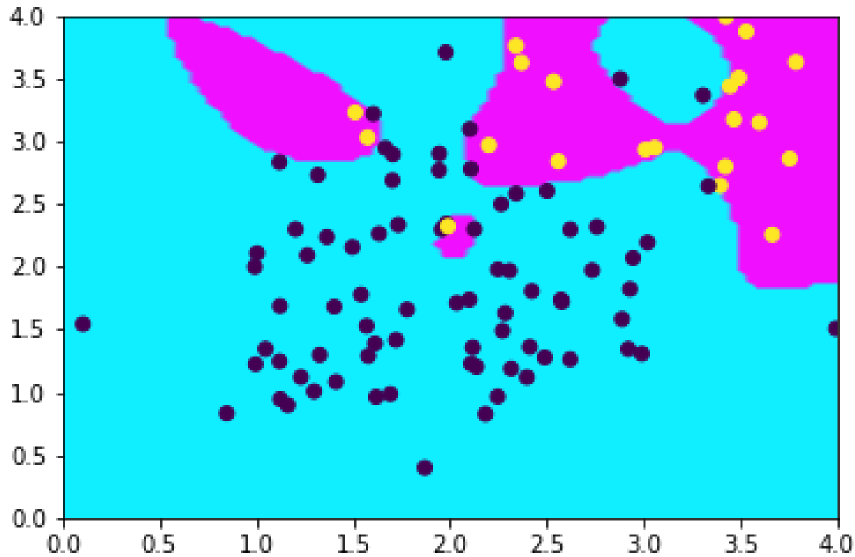
Polynomial kernel on 2030 dataset. The accuracy after training is 0.82.



Gaussian kernel on 2030 dataset. The accuracy after training is 0.98. Definitely a stranger-looking classifier but it works to cluster a bit.



Gaussian kernel on 2030 dataset with $\sigma = 3$. The accuracy after training is 1.0. Only this kernel was able to perfectly classify the training dataset on the 2030 data.



Question 2.

Conditional probabilities on $C = 0$.

$$\begin{aligned}
P(A|C=0) &= P(A=1|C=0) \\
&= 0.224 + 0.056 \\
&= 0.28 \\
P(B|C=0) &= P(B=1|C=0) \\
&= 0.024 + 0.056 \\
&= 0.08 \\
P(A,B|C=0) &= P(A=1,B=1|C=0) \\
&= 0.056
\end{aligned}$$

Conditional probabilities on $C = 1$.

$$\begin{aligned}
P(A|C=1) &= P(A=1|C=1) \\
&= 0.27 + 0.03 \\
&= 0.30 \\
P(B|C=1) &= P(B=1|C=1) \\
&= 0.03 + 0.03 \\
&= 0.06 \\
P(A,B|C=1) &= P(A=1,B=1|C=1) \\
&= 0.03
\end{aligned}$$

A and B are conditionally independent iff $P(A,B|C) = P(A|C)P(B|C)$. Let's see what it is for each value of C.

$$\begin{gathered}
P(A,B|C=0)?P(A|C=0)P(B|C=0) \\
0.056?0.28 * 0.08 \\
0.056 \neq 0.0224 \\
P(A,B|C=1)?P(A|C=1)P(B|C=1) \\
0.03?0.30 * 0.06 \\
0.056 \neq 0.018
\end{gathered}$$

A and B are not conditionally independent given C.

$$\begin{aligned}
P(A) &= 0.224 + 0.056 + 0.27 + 0.03 \\
&= 0.58 \\
P(B) &= 0.024 + 0.056 + 0.03 + 0.03 \\
&= 0.14
\end{aligned}$$

A and B are independent if $P(A,B) = P(A)P(B)$.

$$\begin{gathered}
P(A,B)?P(A)P(B) \\
0.056 + 0.03?0.58 * 0.14 \\
0.086 \neq 0.0812
\end{gathered}$$

A and B are not independent.

Question 3.

Calculate maximum likelihood estimates.

$$\begin{aligned}
P(G=1) &= 6/8 \\
P(O=1|G=1) &= 3/6 \\
P(B=1|G=1) &= 2/6 \\
P(C=1|G=1) &= 3/6 \\
P(A=1|G=1) &= 5/6 \\
P(O=1|G=0) &= 2/2 \\
P(B=1|G=0) &= 2/2 \\
P(C=1|G=0) &= 1/2 \\
P(A=1|G=0) &= 0/2
\end{aligned}$$

Sample 9 classification. Sample 9 is classified as a good restaurant.

$$\begin{aligned}
S_1 &= P(G=1)P(O=0|G=1)P(B=1|G=1)P(C=0|G=1)P(A=1|G=1 \\
&= (6/8)(3/6)(2/6)(3/6)(5/6) \\
&= 0.052 \\
S_0 &= P(G=0)P(O=0|G=0)P(B=1|G=0)P(C=0|G=0)P(A=1|G=0 \\
&= (2/8)(0/2)(2/2)(1/2)(0/2) \\
&= 0
\end{aligned}$$

Sample 10 classification. Sample 10 is classified as a good restaurant.

$$\begin{aligned}
S_1 &= P(G=1)P(O=1|G=1)P(B=1|G=1)P(C=1|G=1)P(A=1|G=1) \\
&= (6/8)(3/6)(2/6)(3/6)(5/6) \\
&= 0.052 \\
S_0 &= P(G=0)P(O=1|G=0)P(B=1|G=0)P(C=1|G=0)P(A=1|G=0) \\
&= (2/8)(2/2)(2/2)(1/2)(0/2) \\
&= 0
\end{aligned}$$

Calculate maximum likelihood estimates with Laplace smoothing.

$$
\begin{aligned}
P(G=1) &= 6/8 \\
P(O=1|G=1) &= 4/8 \\
P(B=1|G=1) &= 3/8 \\
P(C=1|G=1) &= 4/8 \\
P(A=1|G=1) &= 6/8 \\
P(O=1|G=0) &= 3/4 \\
P(B=1|G=0) &= 3/4 \\
P(C=1|G=0) &= 2/4 \\
P(A=1|G=0) &= 1/4
\end{aligned}
$$

Sample 9 smoothed classification. Sample 9 is classified as a good restaurant.

$$
\begin{aligned}
S_1 &= (6/8)(4/8)(3/8)(4/8)(6/8) \\
&= 0.0527 \\
S_0 &= (2/8)(1/4)(3/4)(2/4)(1/4) \\
&= 0.0058
\end{aligned}
$$

Sample 10 smoothed classification. Sample 10 is classified as a good restaurant, but the margin is a bit closer than before.

$$
\begin{aligned}
S_1 &= (6/8)(4/8)(3/8)(4/8)(6/8) \\
&= 0.0527 \\
S_0 &= (2/8)(3/4)(3/4)(2/4)(1/4) \\
&= 0.0175
\end{aligned}
$$

Question 4.

Joint probability of the data.

$$
\begin{aligned}
L &= \Pi_i P(x_{ij}, x_{ik}, y_i) \\
L &= \Pi_i P(y_i) P(x_{ij}|y_i) P(x_{ik}|y_i) \\
L &= \Pi_i (1-\theta_0)\mathbf{1_{y_i=1}} \left(\theta_{j,k|y=1}\theta_{j,s|y=1}\right) + (\theta_0)\mathbf{1_{y_i=0}} \left(\theta_{j,k|y=0}\theta_{j,s|y=0}\right) \\
L &= \Pi_{y_i=1}(1-\theta_0)(\theta_{j,k|y=1}\theta_{j,s|y=1}\Pi_{y_i=0}(\theta_0)(\theta_{j,k|y=0}\theta_{j,s|y=0}) \\
N_{y_i=0} &= mP(y_i=0) \\
N_{y_i=0} &= m\theta_0 \\
L &= \boxed{\left[(1-\theta_0)\theta_{j,k|y=1}\theta_{j,s|y=1}\right]^{m(1-\theta_0)} \left[\theta_0\theta_{j,k|y=0}\theta_{j,s|y=0}\right]^{m\theta_0}}
\end{aligned}
$$