CS 161 - Homework 5

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Question 1.

(a) The sentence is neither, it is invalid. "Smoke implies fire" does not imply "No smoke implies no fire" because "No smoke and fire" is not precluded by "Smoke implies fire."

Smoke	Fire	¬Smoke	¬Fire	Smoke $\implies$ Fire	¬Smoke $\implies$ ¬Fire	(Smoke $\implies$ Fire) $\implies$ (¬Smoke $\implies$ ¬Fire)
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

(b) The sentence is neither, it is invalid. "Smoke implies fire" means that all events in which there is smoke will imply fire. The event "Smoke or heat" may or may not include smoke, and in the case it does not include smoke, "Smoke implies fire" does not imply any relationship.

Smoke	Fire	Heat	Smoke $\implies$ Fire	Smoke$\vee$Heat	(Smoke$\vee$Heat) $\implies$ Fire	(Smoke $\implies$ Fire) $\implies$ ((Smoke$\vee$Heat) $\implies$ Fire)
T	T	T	T	T	T	T
T	F	T	F	T	F	T
F	T	T	T	T	T	T
F	F	T	T	T	F	F
T	T	F	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	T	T
F	F	F	T	F	T	T

(c) This is valid. As seen in the truth table, the starred propositions imply each other, as in they are each true iff the other is true.

Smoke	Fire	Heat	Smoke $\wedge$Heat	(Smoke$\wedge$Heat) $\implies$ Fire	Smoke $\implies$ Fire	Heat $\implies$ Fire	(Smoke $\implies$ Fire) $\vee$(Heat $\implies$ Fire)	original sentence
T	T	T	T	T*	T	T	T*	T
T	F	T	T	F*	F	F	F*	T
F	T	T	F	T*	T	T	T*	T
F	F	T	F	T*	T	F	T*	T
T	T	F	F	T*	T	T	T*	T
T	F	F	F	T*	F	T	T*	T
F	T	F	F	T*	T	T	T*	T
F	F	F	F	T*	T	T	T*	T

Question 2.

(a) First definitions:

is mythical $= A$	is mortal $= B$
is mammal $= C$	is horned $= D$
is magical $= E$	

Then sentences:

- If A, then not B.
- If not A, then (B and C).
- If (not B or C), then D.
- If D, then E.

(b) Sentences in CNF:

- $A \implies \neg B$
- $\neg A \implies (B \wedge C)$
- $(\neg B \vee C) \implies D$
- $D \implies E$

(c) (a) $\boxed{\text{No}}$, A and $\neg A$ are always premises and their implications are only premises for other propositions.
(b) $\boxed{\text{Yes}}$, E is implied by D which is in turn implied by $\neg B$ and C.
(c) $\boxed{\text{Yes}}$, D can be known if $\neg B$ or C is known.

Question 3.

- P(oil) = 0.5
- P(gas) = 0.2
- P(neither) = 0.3
- P(both) = 0
- If oil then P(positive) = 0.9 = P(positive|oil)
- If gas then P(positive) = 0.3 = P(positive|gas)
- If neither then P(positive) = 0.1 = P(positive|neither)

We want to find P(oil|positive). Use Bayes:

$$\begin{aligned} P(oil|positive) &= \frac{P(positive|oil)P(oil)}{P(positive)} \\ &= \frac{P(positive|oil)P(oil)}{P(positive|oil)P(oil) + P(positive|gas)P(gas) + P(positive|neither)P(neither)} \\ &= \frac{(0.9)(0.5)}{(0.9)(0.5) + (0.3)(0.2) + (0.1)(0.3)} \\ &= \frac{5}{6} \end{aligned}$$

The probability that oil is present is $\boxed{5/6}$.