# CS 146 - Homework 1

Zooey Nguyen

zooeyn@ucla.edu

April 5, 2021

**Question 1.**

(a)

$$\begin{bmatrix} 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} = -2 + 0 + 6 = \boxed{4}$$

(b) $\|x\|_1 = 2 + 2 + 3 = \boxed{7}$, $\|y\|_1 = -1 + 0 + 2 = \boxed{1}$

(c) $\|x\|_2 = \sqrt{4 + 4 + 9} = \boxed{\sqrt{17}}$, $\|y\|_2 = \sqrt{1 + 0 + 4} = \boxed{\sqrt{5}}$

(d) Use dot product.

$$x \cdot y = \|x\|_2 \|y\|_2 \cos \theta$$
$$4 = \sqrt{17 \cdot 5} \cos \theta$$
$$\cos \theta = \boxed{\frac{4}{\sqrt{85}}}$$

**Question 2.**

Note overall probability of defectiveness is $P(defective) = (0.25)(0.05) + (0.35)(0.04) + (0.40)(0.02) = 0.0345$

$$
\begin{aligned}
P(A|defective) &= P(defective|A)P(A)/P(defective) \\
&= (0.05)(0.25)/(0.0345) \\
&= \boxed{0.362} \\
P(B|defective) &= (0.35)(0.04)/(0.0345) \\
&= \boxed{0.405} \\
P(C|defective) &= (0.40)(0.02)/(0.0345) \\
&= \boxed{0.232}
\end{aligned}
$$

## Question 3.

(a)

$$E[X + Y] = \Sigma_x \Sigma_y x + y P(x,y)$$
$$= \Sigma_x \Sigma_y x P(x,y) + \Sigma_x \Sigma_y y P(x,y)$$
$$= \boxed{E[X] + E[Y]}$$

(b)

$$Var[X + Y] = \Sigma_x \Sigma_y [(x+y) - E[X+Y]]^2 P(x,y)$$
$$= \Sigma_x \Sigma_y [x + y - E[X] - E[Y]]^2 P(x,y)$$
$$= \Sigma_x \Sigma_y [(x - E[X]) + (y - E[Y])]^2 P(x,y)$$
$$= \Sigma_x \Sigma_y (x - E[X])^2 P(x,y) + \Sigma_x \Sigma_y (y - E[Y])^2 P(x,y) + 2\Sigma_x \Sigma_y (x - E[X])(y - E[Y]) P(x,y)$$
$$= Var[X] + Var[Y] + 2Cov[X,Y]$$
$$= \boxed{Var[X] + Var[Y]}$$

**Question 4.**

(a)

$$E[Ax + b] = E[Ax] + E[b]$$
$$= \boxed{AE[x] + b}$$

(b)

$$Cov[Ax + b] = E[(Ax + b - E(Ax + b))(Ax + b - E(Ax + b))^T]$$
$$= E[(Ax + b - AE[x] - b)(Ax + b - AE[x] - b)^T]$$
$$= E[A(x - E[x])A^T(x - E[x])^T]$$
$$= AA^T E[(x - E[x])(x - E[x])^T]$$
$$= \boxed{AA^T Cov[x]}$$

(c) $y$ will be a multivariate Gaussian with expectation $AE[x] + b$ and covariance matrix $AA^T Cov[x]$ as seen from previous parts.

**Question 5.**

(a) Done in Python. Red and green indicate the two possible outcomes for each applicant. The 2031 admissions data is linearly separable while the 2030 data is not.

2030 Admissions
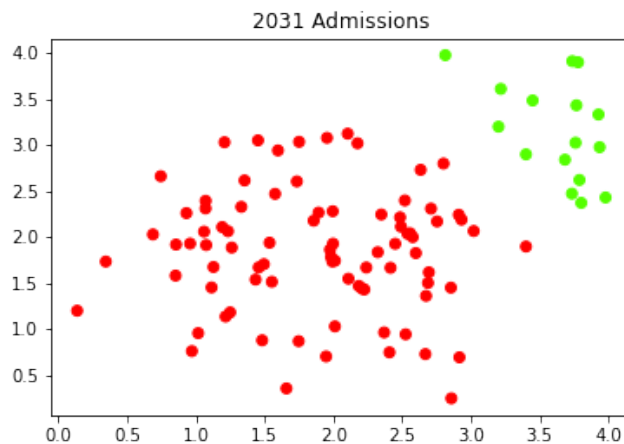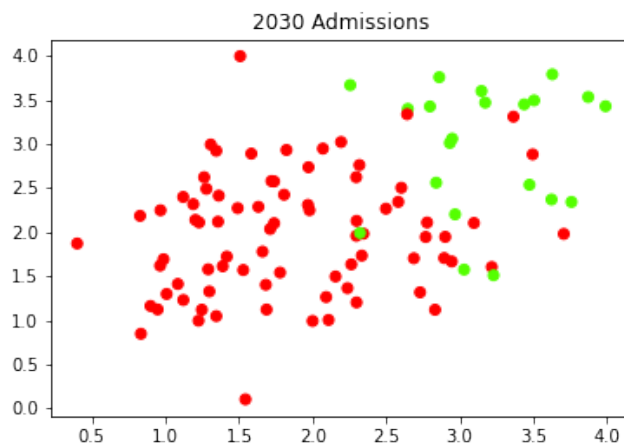
2031 Admissions

(b) For 2030 admissions. The algorithm did not converge and clearly had a wrong answer at the end since the data is not linearly separable.

- Weights: [6.17445 -1.46852]
- Bias: -31

For 2031 admissions. The algorithm did converge and from the plot we can see it did separate the data, however it did not optimise for the margin as expected from what we've seen before.

- Weights: [4.53609 2.42091]
- Bias: -22

2030 Admissions



2031 Admissions
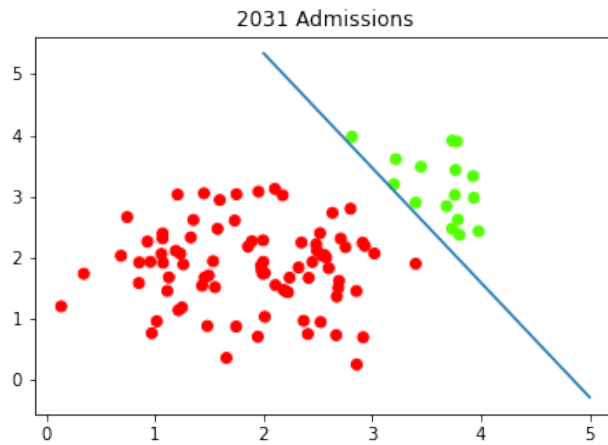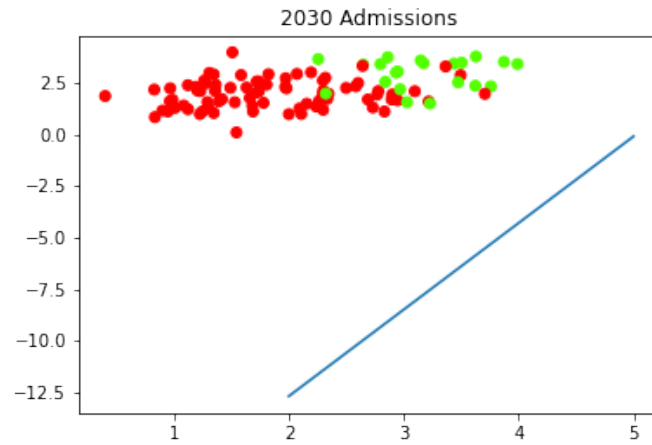
**(c)** The point closest to the hyperplane in the 2031 dataset is at (3.2, 3.2) at a distance of 0.051 from the plane.