hw 8

Problem 1

$$\left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 2 & 5 & 2 & 3 \\ 1 & 2 & 4 & 11 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 11 \\ 2 & 5 & 2 & 3 \\ 4 & -1 & 1 & 8 \end{array}\right] \begin{array}{l} \\ -2R_1 \\ -4R_1 \end{array}$$

$A \quad x$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 11 \\ 0 & 1 & -6 & -19 \\ 0 & -9 & -15 & -36 \end{array}\right] /3 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 11 \\ 0 & 1 & -6 & -19 \\ 0 & 3 & 5 & 12 \end{array}\right] -3R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 11 \\ 0 & 1 & 6 & -19 \\ 0 & 0 & 23 & 69 \end{array}\right] /23 \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 11 \\ 0 & 1 & -6 & -19 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$U \quad y$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & & \\ 0 & 1 & -6 & & 1 & \\ 0 & 0 & 1 & & & 1 \end{array}\right] +6R_3$$

$U \quad I$

$$-2R_2 - 4R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 6 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & & & 1 & -2 & -16 \\ & 1 & & 0 & 1 & 6 \\ & & 1 & 0 & 0 & 1 \end{array}\right]$$

$I \quad U^{-1}$

$$Ax = b \rightarrow Ux = y \rightarrow x = U^{-1}y$$

$$x = \begin{bmatrix} 1 & -2 & -16 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 11 \\ -19 \\ 3 \end{bmatrix} = \boxed{\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}}$$

Problem 2

a.

$$\begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 7 & 2 & -1 \end{bmatrix} \begin{array}{l} \\ +\frac{1}{2}R_1 \\ -\frac{7}{2}R_1 \end{array} = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 7 & 2 & -1 \end{bmatrix}$$

$$\boxed{\begin{array}{l} R_2 = R_2 + \frac{1}{2}R_1 \\ R_3 = R_3 - \frac{7}{2}R_1 \end{array}}$$

b. P_1 for $R_2 = R_2 + \frac{1}{2}R_1$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = P_1}$$

P_2 for $R_3 = R_3 - \frac{7}{2}R_1$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{7}{2} & 0 & 1 \end{bmatrix} = P_2}$$

They are lowertriangular.

c. $P_2 P_1 = P$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{7}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -\frac{7}{2} & 0 & 1 \end{bmatrix}}$$

P is lower triangular.

Problem 3

a. Find LI.

$$(LI)_{ij} = \sum_{n=1}^{i} L_{in} \gamma_{nj}$$

since $LI = I$, $(LI)_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$

$$0 = \sum_{n=1}^{i} L_{in}\gamma_{nj} = \left(\sum_{n=1}^{i-1} L_{in}\gamma_{nj}\right) + L_{ii}\gamma_{ij}$$

$$= (LI)_{i-1,j} + L_{ii}\gamma_{ij}$$

So we have

$$0 = (LI)_{i-1,j} + L_{ii}\gamma_{ij}$$
$$= (I)_{i-1,j} + L_{ii}\gamma_{ij}$$

In the I matrix we observe

$$I_{ij} = \gamma_{ij} = -\frac{(I)_{i-1,j}}{L_{ii}}$$

$I_{i-1,j}$ is only nonzero when $i-1=j$ or $i=j+1$

so I_{ij} is only non zero here for when the row is greater than the column - that is, lower ~~diagonal~~. tri.

$\rightarrow$ $\boxed{I \text{ lower triangular}}$

b Let $L = L_1L_2$ for $n \times n$ matrices.

Note $(L_1)_{ij}$ for $i<j$ is 0 same for L_2

$$L_{ij} = \sum_{k=1}^{n} (L_1)_{ik}(L_2)_{kj}$$
$$= \underbrace{\sum_{k=1}^{j-1} (L_1)_{ik}(L_2)_{kj}}_{} + \sum_{k=j}^{n} (L_1)_{ik}(L_2)_{kj}$$

Here $k<j$ so $(L_2)_{kj} = 0$

so term is 0

$$= \sum_{k=j}^{n} (L_1)_{ik}(L_2)_{kj}$$

We have $k \geq j$, so when $j>i$, $k>i$, and so $(L_1)_{ik}=0$.

$\rightarrow$ $L_{ij}=0$ for $i<j$ $\boxed{\text{so } L_1L_2 \text{ is lower triangular}}$

Problem 4

a I used the matrix equation in HW 5. Given the following boundary condition equations:

$$\begin{cases} a = -0.29004996 \\ a + 0.1b + 0.01c + 0.001d = -0.56079734 \\ e = -0.56079734 \\ e + 0.1f + 0.01g + 0.001h = -0.81401972 \\ b + 0.2c + 0.03d = f \\ 2c + 0.6d = 2g \\ 0 = 2c \\ 2g + 0.6h = 0 \end{cases}$$

We get the matrix eq:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.1 & 0.01 & 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.1 & 0.01 & 0.001 \\ 0 & 1 & 0.2 & 0.03 & 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0.6 & 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0.6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix}$$
$$= [-0.29004996, -0.56079734, -0.56079734, -0.81401972, 0, 0, 0, 0]^T$$

b solution is

$$s(x:[0.1,0.2]) = -0.29004996 - 2.7512863(x-0.1) + 4.38125(x-0.1)^2$$

$$s(x:[0.2,0.3]) = -0.56079734 - 2.6198488(x-0.2) + 2.6198488(x-0.2)^2 - 4.38125(x-0.2)^3$$

Problem 5

Eigenvalues λ of A satisfy

$Ax = \lambda x$

note $\langle x,y \rangle = x^T y$.

We have $A = A^T$ and

$$
\begin{aligned}
\langle x, Ax \rangle &= x^T A x \\
&= (A^T x)^T x \\
&= (Ax)^T x \\
&= \lambda x^T x \\
&= \lambda \langle x,x \rangle > 0
\end{aligned}
$$

Since $\langle x,x \rangle \geq 0$ by def of the inner product,

$\boxed{\lambda \text{ must be real and positive.}}$