ASTR0 81 - Assignment #6

due Sunday before 5 pm

1. What is the approximate eccentricity of a planet whose temperature is 5% higher at periapse than it is at apoapse?

2. We've talked a lot about our favorite star, S0-2, which is orbiting the black hole at the center of the Galaxy 8 kpc away. S0-2's orbital period is 16 years, and its apparent semi-major axis is 0.126 arcseconds. The mass of the black hole in the Galactic center is much greater than the masses of any of the individual stars.

   (a) Calculate the mass of the black hole under the assumption that S0-2's orbit is in the plane of the sky.

   (b) If S0-2's velocity is 20 times greater at periapse than it is at apoapse, what is the eccentricity of its orbit?

   (c) Describe one reason why the apparent eccentricity of the orbit does not *look* consistent with the value of eccentricity calculated in part (b).

3. Consider a binary star system of two stars in a circular orbit that are separated by a total distance of 10 AU. The mass of the two stars are $M_1 = 1M_\odot$ and $M_2 = 4M_\odot$. Assume that the inclination of the binary system is 90 degrees, and that the center-of-mass of the system is moving away from the Earth at a speed of 20 km/s.

   (a) Draw, to relative scale, the orbits of M_1 and M_2 about the common center-of-mass. Label all lengths in units of AU. Indicate the relative positions of M_1 and M_2 on your diagram at some arbitrary time.

   (b) If you can observe Hα absorption lines in the spectrum of M_1, calculate the maximum redshifted wavelength of the Hα line that you could observe. Recall that the rest wavelength of Hα is 6563 Angstroms.

   (c) Draw (on a single plot) the radial velocity curve of the two stars over one full period. Label which line corresponds to which star, and label the axes numerically (e.g. indicate the numerical values of the max/min observed radial velocity of each star, as well as the period of the orbit).

   (d) Explain (qualitatively) how your plot in part (c) would differ if the inclination were actually 45 degrees.

4. If you observe an eclipsing binary system, what can you assume about its inclination? Why?

5. Consider an eclipsing binary system. Star X has an effective temperature of 5,000 K. Star Y has an effective temperature of 15,000 K and a radius four times that of Star X.

   (a) What is the ratio of stellar luminosities?

   (b) Which star is eclipsed at primary minimum?

   (c) Is the primary minimum a total eclipse (e.g. the entire star is eclipsed) or an annular eclipse (e.g. only a portion of the star is eclipsed)?

   (d) Compare the depth of the primary minimum to that of the secondary minimum. Here, 'depth' refers to the difference between the maximum observed flux and the flux at the respective minimum.

   (e) Explain qualitatively how the primary and secondary eclipses may differ if the temperature of Star X were 15,000 K and the effective temperature of Star Y were 5,000 K, assuming their radii remained the same as defined above ($R_y = 4R_x$).

6. Consider an eclipsing, spectroscopic binary with an orbital period of 6 years and an eccentricity of zero. The maximum radial velocities of stars A and B are 5 km/sec and 20 km/sec, respectively. From the light curve, you observe that it takes 12 hours for the total light to be at a minimum once the eclipse begins, and that the duration of the primary minimum is also 12 hours. The apparent bolometric magnitudes of the maximum, primary minimum, and secondary minimum are 5 magnitudes, 9 magnitudes, and 5.5 magnitudes, respectively. From this information, determine the following quantities:

   (a) Ratio of stellar masses

   (b) Sum of the masses (assuming i $\sim 90^o$)

   (c) Individual masses

   (d) Individual radii

   (e) Ratio of the effective temperatures of the two stars.

7. Given the radius and mass of the Earth and no additional information, is it possible to estimate each of the following quantities? Briefly explain each of your answers (just qualitatively).

   (a) The pressure at the center of the Earth?

   (b) The temperature at the center of the Earth?

8. Consider three main sequence stars of mass 0.5, 10, and 50 $M_\odot$.

   (a) Estimate the central pressure for each of the stars in units of the central pressure of the sun.

(b) Estimate the central temperature for each of the stars in units of the central pressure of the sun.