Lecture A. Homework 1

Covered Contents: Calculus, Errors, Convergence Rates, MATLAB (Lec 1-2)
Deadline: 10/4/2021, 23:59 PST
Total points: Pen-and-Paper (70) + Coding (30) = 100
Please submit all files in a single "hw1.zip" through Canvas

Pen and Paper

What to submit: *Your solutions to the pen-and-paper exercises should be put in a single PDF file named 'hw1pen.pdf'. You can either Latex/Markdown it very nicely, or use any editor of your choice (as long as you convert it to PDF), or take clear pictures of your hand-written solutions (but please make sure it's clean: do it via a PDF scanner APP on your phone, or postprocess photos with things like https://onlinecamscanner.com/ to make them look like sharp black-and-white scans). If you need to merge PDF files, you can use things like https://www.pdf2go.com/merge-pdf.*

A.1 (4 points)**.** Use a calculator/software of your choice, compute the absolute error and the relative error in the following approximations of $p = 14$ by $p^* = 3.7$. Express your result with at least 5 digits after the decimal point.

A.2 (8 points)**.** Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$. Approximate $\sqrt{0.5}$, $\sqrt{0.75}$, $\sqrt{1.25}$, and $\sqrt{1.5}$ using $P_3(x)$ and compute the absolute errors.

A.3 (10 points)**.** Prove the existence of a solution to $x\cos(x) - 2x^2 = 1 - 3x$ on the interval $[0.2, 0.3]$.

A.4 (16 points)**.** Let $f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99$ be a function.

(a) Come up with a nested form for $f(x)$ using the polynomial nesting technique.

(b) Use three-digit rounding arithmetic, the assumption that $e^{1.53} = 4.62$, and the fact that $e^{nx} = (e^x)^n$ to evaluate $f(1.53)$. Use the original naive form of $f(x)$. Write down all intermediate steps.

(c) Redo the calculation described in (b), but now using the nested form you showed in (a). Write down all intermediate steps.

(d) Compare the two approximations to the true three-digit result $f(1.53) = -7.61$.

A.5 (12 points). (a) Show that the sequence $p_n = \left(\frac{1}{10}\right)^n$ converges linearly to $p = 0$. (b) Show that the sequence $p_n = 10^{-2n}$ converges quadratically to $p = 0$.

A.6 (12 points). A function $f : [a, b] \to \mathbb{R}$ is said to satisfy a Lipschitz condition on $[a, b]$ with Lipschitz constant L (where $L \in [0, \infty)$) if, for every $x.y \in [a, b]$, we have $|f(x) - f(y)| \leq L|x - y|$. Sometimes we also equivalently say, in this case, f is $L-$Lipschitz continuous on $[a.b]$.

(a) Show that if f is $L-$Lipschitz on $[a, b]$, then $f \in C[a, b]$. (Hint: continuity of a function $f(x)$ at a point x_0 means that for every $\epsilon > 0$, there exists a $\delta > 0$ such that for all x: $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \epsilon$.)

(b) For a differentiable function $f : [a, b] \to \mathbb{R}$, show that if f's derivative is bounded on $[a, b]$ by L (i.e., $|f'(x)| \leq L$, $\forall x \in [a, b]$), then f is $L-$Lipschitz continuous on $[a, b]$. (Hint: use the Mean Value Theorem or the Taylor's Theorem.)

(c) Give an example of a function that is continuous on a closed interval but does not satisfy a Lipschitz condition on the interval. Explain why it is not Lipschitz.

A.7 (8 points). Consider the big O notation in the limit of $x \to 0$. Determine true or false and explain:

(a) $3x^2 = O(x^4)$
(b) $x^{10} + 4x^3 = O(x^3)$
(c) $4x^4 + 3x^3 + 2x^2 = O(x)$
(d) $e^x = 1 + x + \frac{x^2}{2} + O(x^3)$

Coding

What to submit: *By reading the description below you will see that you should turn in 2 PDF files ('Sin.pdf' and 'Sin2.pdf') for the coding task, as well as your modified code (please name it 'Assignment1_modified.m').*

(30 points)

The purpose of this exercise is to have you become familiar with the software programs that you will be using to do the computational assignments for this class. It involves your creating, modifying, and then executing a script (an m-file) that plots $\sin(x^2)$ for $x \in [0, 2\pi]$.

(a) The first part of this assignment is to verify that by using MATLAB, you can create and save a plot of $\sin(x)$ for $x \in [0, 2\pi]$. Please take the following steps:

- Create a class directory for all of your 151A assignments, and then a sub-directory for Assignment 1. Download 'Assignment1.m' from CCLE to this subdirectory.
- Start up MATLAB/Octave and change the directory to your Assignment 1 sub-directory.
- Open the 'Assignment1.m' file in the editor and select 'Run'. Alternatively, at the command prompt you can type the name of the script (the m-file) without the .m extension. For example:

```
>> Assignment1
```

- Create a .pdf file of the resulting plot with the name ‘Sin.pdf’ by selecting File/Save As in the plot window. Alternatively, at the command prompt you can execute the command:

```
>> print('Sin.pdf','-dpdf')
```

(b) Modify, and then run ‘Assignment1.m’ so that it creates a plot of $sin(x^2)$ for $x \in [0, 2\pi]$. Save the plot in a PDF file called ‘Sin2.pdf’. Rename your file into ‘Assignment1_modified.m’ and submit.