Astronomy 82 – problem set #1 – due by midnight on Sunday, April 4 submit to Top Hat

1. Assume that a molecular cloud of hydrogen (H_2) number density of 10^4 cm^{-3} collapses to form a solar mass protostar with a radius equal to 3 times the solar radius, but without losing any angular momentum.
a) What is the angular velocity of the initial cloud, if the star that it forms ends up rotating at its marginal break-up speed (the speed at which the centrifugal force at the equator is equal to the gravitational force)?
b) How does that compare to the angular velocity of the Galaxy at the solar circle, where the rotational velocity of stars in the Galaxy is 240 km/s? The distance to the Galactic center (that is, the radius of the solar circle) is 8 kiloparsecs.

2. Referring to equation 7-50 in the text, estimate what the radius of a planetesimal would need to be in order for its cross-section for accumulating new particles to be twice as large as its geometric cross-section. Assume that the density of the planetesimal is 3 gm cm^{-3}, and that the mean relative velocity of the planetesimal and the particles that it encounters is 1% of the Earth's orbital velocity.

3. There are 30 magnitudes of visual extinction towards the Galactic center. What fraction of the visual light photons emitted at the Galactic center in the direction of the Earth actually reach the Earth? There are about 2.5 magnitudes of extinction at a near-infrared wavelength of 2.2 microns. What fraction of near-IR photons reaches the Earth? [context: much of the research on the Galactic center done by UCLA researchers is done in the near-infrared, at a wavelength of 2.2 microns.]

4. The typical abundance ratio of carbon-12 (^{12}C) to carbon-13 (^{13}C) in the nearby interstellar medium is about 65.
a) Consider a nearby molecular cloud in which the intensity ratio of rotational line emission (say, the J = 1→0 line) from the two species is measured to be $T(^{12}CO)/T(^{13}CO) = 10$. What is the optical depth of the ^{12}CO line? Assume that the two isotopologues have the same excitation temperature.
b) The abundance ratio $[^{12}C]/[^{13}C]$ in the solar system is 89. Offer a plausible hypothesis for why it might be different from the same ratio in the nearby interstellar medium.

5. Derive an expression for the radius of a Strömgren sphere around a massive, hot star that emits N_* *Lyman continuum*[1] photons per second. Assume that the gas surrounding the star consists entirely of atomic hydrogen and has a uniform number density, n_H (cm^{-3}). In an ionized region where the proton density is n_H, an electron will recombine with protons at a rate equal to αn_H, where α is the *recombination coefficient,* which has units of cm^3/s.

[1] Lyman continuum: the portion of the electromagnetic spectrum in which the photons have energies exceeding the threshold energy needed to ionize hydrogen from its ground state