HW9: PHYSICS 115A-2021F

Late Policy: 0 credits

P1: 15 points

$*$Problem 3.13

(a) Prove the following commutator identity:

$$[AB, C] = A[B, C] + [A, C]B. \qquad [3.64]$$

(b) Show that

$$[x^n, p] = i\hbar n x^{n-1}.$$

(c) Show more generally that

$$[f(x), p] = i\hbar \frac{df}{dx}, \qquad [3.65]$$

for any function $f(x)$.

P2: 15 points

$*$Problem 3.14 Prove the famous "(your name) uncertainty principle," relating the uncertainty in position ($A = x$) to the uncertainty in energy ($B = p^2/2m + V$):

$$\sigma_x \sigma_H \geq \frac{\hbar}{2m} |\langle p \rangle|.$$

For stationary states this doesn't tell you much—why not?

P3: 35 points

Problem 3.30 Suppose

$$\Psi(x, 0) = \frac{A}{x^2 + a^2}, \quad (-\infty < x < \infty)$$

for constants A and a.

(a) Determine A, by normalizing $\Psi(x, 0)$.

(b) Find $\langle x \rangle$, $\langle x^2 \rangle$, and σ_x (at time $t = 0$).

(c) Find the momentum space wave function $\Phi(p, 0)$, and check that it is normalized.

(d) Use $\Phi(p, 0)$ to calculate $\langle p \rangle$, $\langle p^2 \rangle$, and σ_p (at time $t = 0$).

(e) Check the Heisenberg uncertainty principle for this state.

P4: (35 points)

∗∗Problem 3.40

(a) Write down the time-dependent "Schrödinger equation" in momentum space, for a free particle, and solve it. *Answer:* $\exp(-ip^2t/2m\hbar)\,\Phi(p, 0)$.

(b) Find $\Phi(p, 0)$ for the traveling gaussian wave packet (Problem 2.43), and construct $\Phi(p, t)$ for this case. Also construct $|\Phi(p, t)|^2$, and note that it is independent of time.

(c) Calculate $\langle p \rangle$ and $\langle p^2 \rangle$ by evaluating the appropriate integrals involving Φ, and compare your answers to Problem 2.43.

(d) Show that $\langle H \rangle = \langle p \rangle^2/2m + \langle H \rangle_0$ (where the subscript 0 denotes the *stationary* gaussian), and comment on this result.