Physics 115A - Homework 5

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Question 1.

Expectation values of harmonic oscillator.

$$\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}\int_{-\infty}^{\infty}\Psi_n^*(a_+\Psi_n + a_-)\Psi_n\,\mathrm{d}x = \sqrt{\frac{\hbar}{2m\omega}}\int_{-\infty}^{\infty}\left(\sqrt{n+1}\Psi_n^*\Psi_{n+1} + \sqrt{n}\Psi_n^*\Psi_{n-1}\right)\mathrm{d}x = \boxed{0}$$

$$\langle p\rangle = m\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = \boxed{0}$$

$$\begin{aligned}\left\langle x^2\right\rangle &= \frac{\hbar}{2m\omega}\int_{-\infty}^{\infty}\Psi_n^*(a_+^2 + a_-^2 + a_+a_- + a_-a_+)\Psi_n\,\mathrm{d}x \\ &= \frac{\hbar}{2m\omega}\int_{-\infty}^{\infty}\left[\sqrt{(n+1)(n+2)}\Psi_n^*\Psi_{n+2} + n\Psi_n^*\Psi_n + (n+1)\Psi_n^*\Psi_n + \sqrt{(n-1)n}\Psi_n^*\Psi_{n-2}\right]\mathrm{d}x \\ &= \frac{\hbar}{2m\omega}\int_{-\infty}^{\infty}(2n+1)|\Psi_n|^2\,\mathrm{d}x = \frac{(2n+1)\hbar}{2m\omega} = \boxed{(n+\frac{1}{2})\frac{\hbar}{m\omega}}\end{aligned}$$

$$\left\langle p^2\right\rangle = -\frac{m\hbar\omega}{2}\int_{-\infty}^{\infty}\Psi_n^*(a_+^2 + a_-^2 + a_+a_- + a_-a_+)\Psi_n\,\mathrm{d}x = \boxed{(n+\frac{1}{2})m\hbar\omega}$$

Uncertainty principle check.

$$\sigma_x\sigma_p = \sqrt{\langle x^2\rangle - \langle x\rangle^2}\sqrt{\langle p^2\rangle - \langle p\rangle^2} = \sqrt{(n+\frac{1}{2})\frac{\hbar}{m\omega} - (0)^2}\sqrt{(n+\frac{1}{2})m\hbar\omega - (0)^2} = (n+\frac{1}{2})\hbar \geq \frac{\hbar}{2}$$

Question 2.

Since $E_n = (n+\frac{1}{2})\hbar\omega_n$, the energies are now $E_n' = (2n+1)\hbar\omega_n$ for $n = 0, 1, 2, \ldots$.

Probability of getting $E_n' = \frac{\hbar\omega}{2}$ is $\boxed{0}$ since there is no $n \in$ for which $2n+1 = \mathbb{Z}^+ = \frac{1}{2}$.

Probability of getting $E_n' = \hbar\omega$ is as follows.

$$\begin{aligned}C_0 &= \int_{-\infty}^{\infty}\Psi(x,0)\Psi'(x)\,\mathrm{d}x = \int_{-\infty}^{\infty}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{m\omega}{2\hbar}x}\left(\frac{2m\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{m\omega}{\hbar}x}\,\mathrm{d}x \\ &= 2^{1/4}\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}e^{-\frac{3m\omega}{2\hbar}x}\,\mathrm{d}x = 2^{1/4}\sqrt{\frac{2}{3}} = \boxed{0.9428}\end{aligned}$$

Question 3.

Probability of finding particle outside classical region for ground state in harmonic oscillator.

$$E_{max} = \frac{1}{2}m\omega^2 x_{max}^2 = \frac{1}{2}\hbar\omega \quad \text{then} \quad x_{max} = \sqrt{\frac{\hbar}{m\omega}}$$

$$P = 2\sqrt{\frac{m\omega}{\pi\hbar}}\int_{\sqrt{\frac{\hbar}{m\omega}}}^{\infty} e^{-\frac{m\omega}{\hbar}x^2}\,\mathrm{d}x = 2\sqrt{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{\hbar}{m\omega}}\int_1^{\infty} e^{-u^2}\,\mathrm{d}u = \frac{2}{\sqrt{\pi}}\frac{\sqrt{\pi}}{2}(1 - erf(1)) = \boxed{0.1573}$$

Question 4.

Question 5.

Solutions can no longer include even solutions because the wavefunction must go to 0 at $x = 0$. Since only odd solutions exist we recover the same energies as the regular harmonic oscillator but only for odd integers, $E_n = (n + \frac{1}{2})\hbar\omega, n = 1, 3, 5, \ldots$.