Math 151A - Homework Pen and Paper 3

Zooey Nguyen

zooeyn@ucla.edu

October 18, 2021

Question 1.

Use NM to get p_2.

$$
\begin{aligned}
p_{n+1} &= p_n - \frac{-p_n^3 - \cos p_n}{-3p_n^2 + \sin p_n} = p_n + \frac{\cos p_n + p_n^3}{\sin p_n - 3p_n^2} \\
p_1 &= (-1) + \frac{\cos(-1) + (-1)^3}{\sin(-1) - 3(-1)^2} = -1 + \frac{0.5403 - 1}{-0.8414 - 3} = (-0.8803) \\
p_2 &= (-0.8803) + \frac{\cos(-0.8803) + (-0.8803)^3}{\sin(-0.8803) - 3(-0.8803)^2} = -0.8803 + \frac{0.6369 - 0.6822}{-0.7709 - 2.3248} = \boxed{-0.8657}
\end{aligned}
$$

If we use $p_0 = 0$, the denominator $\sin(0) - 3(0)^2$ becomes 0, and we cannot iterate.

Question 2.

Use SM to get p_3.

$$p_{n+1} = p_n - \frac{(-p_n^3 - \cos p_n)(p_n - p_{n-1})}{(-p_n^3 - \cos p_n) - (-p_{n-1}^3 - \cos p_{n-1})}$$

$$p_2 = (0) - \frac{(-(0)^3 - \cos(0))((0) - (-1))}{(-(0)^3 - \cos(0)) - (-(-1)^3 - \cos(-1))} = -0.6850$$

$$p_3 = (-0.6850) - \frac{(-(-0.6850)^3 - \cos(-0.6850))((-0.6850) - (0))}{(-(-0.6850)^3 - \cos(-0.6850)) - (-(0)^3 - \cos(0))} = \boxed{-1.2523}$$

Question 3.

Showing linear convergence of Newton's method.

$$p_{n+1} - p = (p_n - p)\frac{f(p_n)}{f'(p_0)}$$
$$\lim \frac{|p_{n+1} - p|}{|p_n - p|} = \lim \frac{|f(p_n)|}{|f'(p_0)|} = \frac{|f(p)|}{|f'(p_0)|} \to \text{Positive finite constant}$$

Question 4.

Secant method for solving. Note solution to five digits from a computer is $1.82938\ldots$ and that the secant method took about five steps.

$$p_{n+1} = p_n - \frac{(e_n^p + 2^{-p_n} + 2\cos(p_n) - 6)(p_n - p_{n-1})}{e_n^p + 2^{-p_n} + 2\cos(p_n) - 6 - (e_{n-1}^p + 2^{-p_{n-1}} + 2\cos(p_{n-1}) - 6)}$$

$$p_{n+1} = p_n - \frac{(e_n^p + 2^{-p_n} + 2\cos(p_n) - 6)(p_n - p_{n-1})}{e_n^p - e_{n-1}^p + 2^{-p_n} - 2^{-p_{n-1}} + 2\cos(p_n) - 2\cos(p_{n-1})}$$

$$p_0 = 1$$

$$p_1 = 2$$

$$p_2 = 2 - \frac{(e^2 + 2^{-2} + 2\cos(2) - 6)(2-1)}{e^2 - e^1 + 2^{-2} - 2^{-1} + 2\cos(2) - 2\cos(1)} = 1.67830$$

$$p_3 = 1.67830 - \frac{(e^{1.67830} + 2^{-1.67830} + 2\cos(1.67830) - 6)(1.67830 - 2)}{e^{1.67830} - e^2 + 2^{-1.67830} - 2^{-2} + 2\cos(1.67830) - 2\cos(2)} = 1.80810$$

$$p_4 = 1.80810 - \frac{(e^{1.80810} + 2^{-1.80810} + 2\cos(1.80810) - 6)(1.80810 - 1.67830)}{e^{1.80810} - e^{1.67830} + 2^{-1.80810} - 2^{-1.67830} + 2\cos(1.80810) - 2\cos(1.67830)} = 1.83230$$

$$p_5 = 1.83230 - \frac{(e^{1.83230} + 2^{-1.83230} + 2\cos(1.83230) - 6)(1.83230 - 1.80810)}{e^{1.83230} - e^{1.80810} + 2^{-1.83230} - 2^{-1.80810} + 2\cos(1.83230) - 2\cos(1.80810)} = 1.82933$$

$$p_6 = 1.82933 - \frac{(e^{1.82933} + 2^{-1.82933} + 2\cos(1.82933) - 6)(1.82933 - 1.83230)}{e^{1.82933} - e^{1.83230} + 2^{-1.82933} - 2^{-1.83230} + 2\cos(1.82933) - 2\cos(1.83230)} = \boxed{1.82938}$$

Question 5.

Show order of convergence as function of e_n's.

$$\begin{aligned}
e_{n+1} &= \lambda e_n^\alpha \\
e_n &= \lambda e_{n-1}^\alpha \\
\frac{e_{n+1}}{e_n} &= \frac{\lambda e_n^\alpha}{\lambda e_{n-1}^\alpha} = \left(\frac{e_n}{e_{n-1}}\right)^\alpha \\
\log\left(\frac{e_{n+1}}{e_n}\right) &= \alpha \log\left(\frac{e_n}{e_{n-1}}\right) \\
\alpha &= \boxed{\frac{\log(e_{n+1}/e_n)}{\log(e_n/e_{n-1})}}
\end{aligned}$$