Astronomy 5 – Problem Set 5

Due on Top Hat before Sunday, May 9, at midnight

1) a) At what distance from a 2 $M_\odot$ neutron star would a planet like the Earth be tidally disrupted (that is, literally pulled apart)? That is, how close would the planet need to be to the NS for the difference between the NS's gravity at the center of the planet and at the surface of the planet to be greater than the gravity holding the planet together? This is the tidal disruption radius.

   b) Would the asteroid Pallas be able to get any closer? (Pallas has a radius of 256 km and a mass of 2.1×10^{23} gm.)

2) The innermost stable circular orbit around a black hole has a radius equal to 3 times the Schwarzschild radius. Consider a spaceship orbiting a supermassive black hole at that radius. Your adventurous colleague in the spaceship is tweeting to you with a transmitter tuned to a frequency of 1 GHz.

   a) To what frequency would you need to tune your receiver to get his tweets (assume that you are essentially an infinite distance away)?

   b) By how much might that frequency change, depending on where he is in his orbit?

3) The pulsar in the middle of the Crab Nebula spins at a rate of about 30 revolutions per second. As it slows down, its *period derivative*, dP/dt, is 4×10^{-13} seconds per second.

   a) what is its characteristic age, $P/\dot{P}$?

   b) assuming that the density of the neutron star is constant, that the mass is 1.4 $M_\odot$, and that the radius is 12 km, calculate the rate at which the neutron star loses rotational energy. This energy is emitted as powerful hydromagnetic waves that transfer the energy to the surrounding Crab Nebula supernova remnant, and the energy loss rate of the pulsar is the "luminosity" of the nebula.

4) a) If the density of matter perpendicular to the disk of the Galaxy has an exponential distribution, $\rho(z) = \rho_o \exp(-z/z_o)$, where z is the distance above the disk, z_o is the scale height of matter in the disk, and ρ_o is the density in the disk midplane, then what is the surface density of the disk?

   b) Evaluate the surface density (in solar masses per square parsec) for a midplane density of 0.1 $M_\odot$ pc^{-3}, and a scale height of 300 pc.

5) Assuming that the Galactic disk has an infinite extent in directions parallel to the disk, calculate the vertical acceleration experienced by a star located at a height z above the midplane in terms of ρ_o and z_o (use the information from problem 4).

6) In addition to orbiting around the Galactic center, a star in the Galactic disk will oscillate up and down across the Galactic disk because of the acceleration it experiences from the gravity of the disk. Calculate the period of oscillation, using the acceleration calculated in problem 2, and again using the information in problem 1. You may assume that the vertical excursions are small (although you don't have to), in which case the exponential is in the linear regime: $\exp(-z/z_o) = 1 - z/z_o$.