(Late policy: 0% credit)

P1. (25 points)

Problem 2.40 A particle of mass m is in the potential

$$V(x) = \begin{cases} \infty & (x < 0), \\ -32\hbar^2/ma^2 & (0 \le x \le a). \\ 0 & (x > a). \end{cases}$$

(a) How many bound states are there?

(b) In the highest-energy bound state, what is the probability that the particle would be found *outside* the well ($x > a$)? *Answer:* 0.542, so even though it is "bound" by the well, it is more likely to be found outside than inside!

P2. (25 points)

**Problem 2.33 Determine the transmission coefficient for a rectangular *barrier* (same as Equation 2.145, only with $V(x) = +V_0 > 0$ in the region $-a < x < a$). Treat separately the three cases $E < V_0$, $E = V_0$, and $E > V_0$ (note that the wave function inside the barrier is different in the three cases). *Partial answer:* For $E < V_0$,[42]

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right).$$

P3. (25 points)

Problem 2.52 The scattering matrix. The theory of scattering generalizes in a pretty obvious way to arbitrary localized potentials (Figure 2.22). To the left (Region I), $V(x) = 0$, so

$$\psi(x) = Ae^{ikx} + Be^{-ikx}, \quad \text{where } k \equiv \frac{\sqrt{2mE}}{\hbar}. \qquad [2.173]$$

To the right (Region III), $V(x)$ is again zero, so

$$\psi(x) = Fe^{ikx} + Ge^{-ikx}. \qquad [2.174]$$

In between (Region II), of course, I can't tell you what ψ is until you specify the potential, but because the Schrödinger equation is a linear, second-order differential equation, the general solution has got to be of the form

$$\psi(x) = Cf(x) + Dg(x),$$

where $f(x)$ and $g(x)$ are two linearly independent particular solutions.[48] There will be four boundary conditions (two joining Regions I and II, and two joining

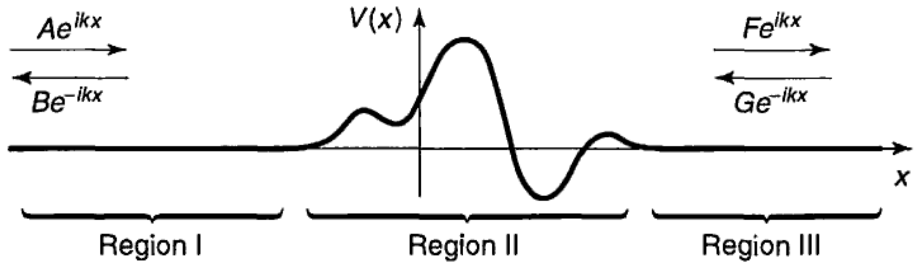

FIGURE 2.22: Scattering from an arbitrary localized potential ($V(x) = 0$ except in Region II); Problem 2.52.

Regions II and III). Two of these can be used to eliminate C and D, and the other two can be "solved" for B and F in terms of A and G:

$$B = S_{11}A + S_{12}G, \quad F = S_{21}A + S_{22}G.$$

The four coefficients S_{ij}, which depend on k (and hence on E), constitute a 2×2 matrix $\mathbf{S}$, called the **scattering matrix** (or ***S*-matrix**, for short). The S-matrix tells you the outgoing amplitudes (B and F) in terms of the incoming amplitudes (A and G):

$$\begin{pmatrix} B \\ F \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} A \\ G \end{pmatrix}. \qquad [2.175]$$

In the typical case of scattering from the left, $G = 0$, so the reflection and transmission coefficients are

$$R_l = \left.\frac{|B|^2}{|A|^2}\right|_{G=0} = |S_{11}|^2, \quad T_l = \left.\frac{|F|^2}{|A|^2}\right|_{G=0} = |S_{21}|^2. \qquad [2.176]$$

For scattering from the right, $A = 0$, and

$$R_r = \left.\frac{|F|^2}{|G|^2}\right|_{A=0} = |S_{22}|^2, \quad T_r = \left.\frac{|B|^2}{|G|^2}\right|_{A=0} = |S_{12}|^2. \qquad [2.177]$$

(a) Construct the S-matrix for scattering from a delta-function well (Equation 2.114).

(b) Construct the S-matrix for the finite square well (Equation 2.145). *Hint:* This requires no new work, if you carefully exploit the symmetry of the problem.

P4: (25 points)

Problem 2.31 The Dirac delta function can be thought of as the limiting case of a rectangle of area 1, as the height goes to infinity and the width goes to zero. Show that the delta-function well (Equation 2.114) is a "weak" potential (even though it is infinitely deep), in the sense that $z_0 \to 0$. Determine the bound state energy for the delta-function potential, by treating it as the limit of a finite square well. Check that your answer is consistent with Equation 2.129. Also show that Equation 2.169 reduces to Equation 2.141 in the appropriate limit.