Physics 112 - Homework 4

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May 11, 2021

Question 1.

Blackbody radiation of Earth given Sun's blackbody.

$$
\begin{aligned}
4\pi\sigma R_E^2 T_E^4 &= \frac{L_\odot}{4\pi D^2}\pi R_E^2 \\
4\pi\sigma R_E^2 T_E^4 &= \frac{4\pi\sigma R_\odot^2 T_\odot^4}{4\pi D^2}\pi R_E^2 \\
T_E^4 &= \frac{R_\odot^2 T_\odot^4}{4D^2} \\
T_E &= \boxed{396\,\mathrm{K}}
\end{aligned}
$$

Question 2.

Partition function of photon gas.

$$
\begin{aligned}
Z &= \Pi_n \sum_i e^{-E_i/\tau} \\
Z &= \Pi_n \sum_i e^{-i\hbar\omega/\tau} \\
Z &= \Pi_n \frac{1}{1-e^{\hbar\omega/\tau}} \\
Z &= \boxed{\Pi_n [1-e^{\hbar\omega/\tau}]^{-1}}
\end{aligned}
$$

Helmholtz free energy of photon gas.

$$
\begin{aligned}
F &= 2\int_0^\infty \frac{4\pi n^2 dn}{8}\tau \ln\left(1-e^{n\hbar\pi c/\tau L}\right) \\
F &= \pi\tau \int_0^\infty n^2 \ln\left(1-e^{n\hbar\pi c/\tau L}\right) \mathrm{d}n \\
F &= \pi\tau \left[n^3 \ln\left(1-e^{n\hbar\pi c/\tau L}\right)\Big|_0^\infty - \int_0^\infty \frac{n^3 e^{n\hbar\pi c/\tau L}(-\pi\hbar c)}{1-e^{n\hbar\pi c/\tau L}(\tau L)} \right] \\
F &= \boxed{-\frac{\pi V \tau^4}{45\hbar^3 c^3}}
\end{aligned}
$$

Question 3.

Heat shield absorbs and emits. Total thermal flux on each side is added to heat shield flux (halved due to two sides).

$$
\begin{aligned}
\sigma T_u^4 + \sigma T_l^4 &= \sigma T_m^4 \\
T_m &= (T_u^4 + T_l^4)^{1/4} \\
J_m &= \sigma(T_u^4 + T_l^4)/2 \\
J_{net,u} &= \sigma T_u^4 - \sigma(T_u^4 + T_l^4)/2 \\
J_{net,u} &= \sigma(T_u^4 - T_l^4)/2 \\
J_{net,u} &= \boxed{J_u/2} \\
J_{net,l} &= -\sigma T_l^4 + \sigma(T_u^4 + T_l^4)/2 \\
J_{net,l} &= \sigma(T_u^4 - T_l^4)/2 \\
J_{net,l} &= \boxed{J_u/2}
\end{aligned}
$$

Question 4.

Debye temperature.

$$
\begin{aligned}
\theta &= \frac{hv}{k_b}\left[\frac{\pi^2 N}{V}\right]^{1/3} \\
\theta &= \frac{(6.626\times10^{-34}\,\mathrm{J\,s^{-1}})(2.383\times10^{4}\,\mathrm{cm\,s^{-1}})}{1.38\times10^{-23}\,\mathrm{J\,K^{-1}}}\left[\pi^2\frac{0.145\,\mathrm{g/cm^3}}{4.002\,602\,\mathrm{g\,mol^{-1}}}(6.022\times10^{23}\,\mathrm{mol^{-1}})\right]^{1/3} \\
\theta &= 68.5\,\mathrm{K}
\end{aligned}
$$

Heat capacity per gram given Debye temperature. Take it in the limit $T \ll \theta$. Ends up being smaller somehow. Maybe I messed up. I don't know.

$$
\begin{aligned}
\frac{C_V}{\rho V} &= \frac{\frac{12 k_b N \pi^4 T^3}{5\theta^3}}{\rho V} \\
\frac{C_V}{\rho V} &= \frac{12T^3}{5(68.5\,\mathrm{K})^3}\frac{k_b\pi^4}{\rho}\frac{N}{V} \\
\frac{C_V}{\rho V} &= k_b\pi^4\frac{12T^3}{5(68.5\,\mathrm{K})^3}\frac{1}{0.145\,\mathrm{g/cm^3}}\frac{0.145\,\mathrm{g/cm^3}(6.022\times10^{23}\,\mathrm{mol^{-1}})}{4.002\,602\,\mathrm{g\,mol^{-1}}} \\
\frac{C_V}{\rho V} &= \boxed{.001\times T^3}
\end{aligned}
$$