Astro 82 - Homework 3

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Question 1.

Mass of planet:

$$\begin{aligned} \frac{v^3 P \sin^3 i}{2\pi G} &= \frac{m^3}{(m+M)^2} \\ m + M &\simeq M \\ m^3 &= \frac{v^3 P M^2 \sin^3 i}{2\pi G} \\ m^3 &= \frac{(5\,\mathrm{m\,s^{-1}})^3 (30\,\mathrm{day}) \sin^3 i (2 \cdot R_\odot)^2}{2\pi G} \\ m &= \boxed{\sin i \cdot 2.3 \times 10^{26}\,\mathrm{kg}} \end{aligned}$$

Distance to star:

$$\begin{aligned} v &= \frac{2\pi a m \sin i}{P(m+M)} \\ a &\simeq \frac{vPM}{2\pi m \sin i} \\ a &= \frac{(5\,\mathrm{m\,s^{-1}})(30\,\mathrm{day})(2 \cdot R_\odot)}{2\pi (2.3 \times 10^{26}\,\mathrm{kg}) \sin^2 i} \\ a &= \boxed{\frac{3.56 \times 10^{10}\,\mathrm{m}}{\sin^2 i}} \end{aligned}$$

Question 2.

If brightness decreases by 1% for the planet fully in front of the star that means 1% of the star's cross-section area is covered, which means the planet has a radius that is 10% of the star's, or the radius $0.175 R_\odot$. Then average density is just mass over volume, which is $\rho = \frac{M}{V} = \frac{\sin i \cdot 2.3 \times 10^{26}\,\mathrm{kg}}{\frac{4}{3}\pi (0.175 R_\odot)^3} = \boxed{\sin i 300\,\mathrm{kg\,m^{-3}}}$.

The inclindation of the planet's orbit must be close to 90°, that is, the orbital plane must be close to line-of-sight. We can refine the inclination further by using the depths of the primary and secondary minima.

Question 3.

Luminosity of the planet wrt luminosity of the Sun star at visible wavelength is practically zero.

$$\begin{aligned}
T_p &= T_\odot(1-a)^{1/4}\sqrt{R_\odot/2D} \\
T_p &= (5600\,\mathrm{K})(0.7)^{1/4}\sqrt{6.96\times10^8\,\mathrm{m}/4\,\mathrm{AU}} \\
T_p &= 175\,\mathrm{K} \\
\frac{B_p}{B_\odot} &= \frac{e^{hc/(\lambda)k_bT_\odot}-1}{e^{hc/(\lambda)k_bT_p}-1} \\
\frac{B_p}{B_\odot} &= \frac{e^{hc/(550\,\mathrm{nm})k_b5600\,\mathrm{K}}-1}{e^{hc/(550\,\mathrm{nm})k_b175\,\mathrm{K}}-1} \\
\frac{B_p}{B_\odot} &= \frac{106.42}{9.8\times10^{64}} \\
\frac{B_p}{B_\odot} &= \boxed{1.077\times10^{-63}}
\end{aligned}$$

Luminosity of the planet wrt luminosity of the Sun star at wavelength 1.2×10^{-5} m.

$$\begin{aligned}
\frac{B_p}{B_\odot} &= \frac{e^{hc/(1.2\times10^{-5}\,\mathrm{m})k_b5600\,\mathrm{K}}-1}{e^{hc/(1.2\times10^{-5}\,\mathrm{m})k_b175\,\mathrm{K}}-1} \\
\frac{B_p}{B_\odot} &= \frac{0.24}{951} \\
\frac{B_p}{B_\odot} &= \boxed{2.5\times10^{-4}}
\end{aligned}$$

Luminosity of the planet wrt luminosity of the Sun star at wavelength 4×10^{-6} m.

$$\begin{aligned}
\frac{B_p}{B_\odot} &= \frac{e^{hc/(4\times10^{-6}\,\mathrm{m})k_b5600\,\mathrm{K}}-1}{e^{hc/(1.4\times10^{-6}\,\mathrm{m})k_b175\,\mathrm{K}}-1} \\
\frac{B_p}{B_\odot} &= \frac{0.902}{8.6\times10^{8}} \\
\frac{B_p}{B_\odot} &= \boxed{1.04\times10^{-9}}
\end{aligned}$$

The most appropriate choice would probably be from part b, since the ratio of the planet's brightness to the star's is highest, so it would be easier to see planets next to the star. Of course, the difficulty of observing at certain wavelengths would need to be considered since only some wavelengths are available to observe at the Earth's surface.