DRAFT
Swift longitudinal modes approximations

Zouhair Mahboubi

November 11^{th}, 2009

Stanford University

1 Introduction

This is a quick summary of the two approaches used to obtain approximations to the longitudinal modes for the Swift plane, namely the phugoid and the short period mode. This is 'quick & dirty' and is not yet validated, but a comparison of the two methods indicate that we are at least within the right order of magnitude.

It's worth mentioning that the moments of inertia are NOT precise. They were computed assuming that the wing is the main contributor. The CAD model should be used for more accurate results.

2 Analysis

2.1 Methods

We use two approaches to obtain the phugoid and short period. In one case, we start by extracting 2D sections from the CAD drawings of the wing, and we use Xfoil to compute 2D section properties (Cl_α, Cm_0, etc.) We then feed these values to LinAir which we use to compute some of the stability derivatives as well as the eigen values of the dynamic system.

In the second approach we use 'back-of-the-envelope' computations from approximations given by 'Etkin' [1] for the phugoid and short period frequencies. The necessary aerodynamic coefficients were computed by John Melton using a NS code. A matlab script used to do the 'back-of-the-envelope' is included in appendix and lists all the reference values and flight conditions assumed.

2.1.1 Approximations of modes

We use the approximations of the modes as outlined in equations 6.3,12 and 6.3,15 of [1]. These are given in terms of aerodynamic coefficients and stability derivatives as well as other non-dimensional quantities summarized in tables 4.1, 4.2 and 4.4 of [1].

Phugoid $w_n = \frac{2\pi}{T_n}$ where $T_n = \pi\sqrt{2}\frac{u_0}{g}$, $\zeta = \frac{C_D}{C_L}$

Short Period

$$w_n{}^2 = -\frac{1}{t^{*2}\hat{I}_y}(C_{m_\alpha} - \frac{C_{m_q}C_{z_\alpha}}{2\mu})$$

$$\zeta = \frac{B}{2w_n} \text{ where } B = -\frac{1}{t^*}\left[\frac{C_{z_\alpha}}{2\mu} + \frac{C_{m_q} + C_{m_{\dot\alpha}}}{\hat{I}_y}\right]$$

2.2 Results

These are the results obtained from the two methods. The computed values are different mainly because LinAir is computing somewhat different stability derivatives.

For the moment being the approximations from 'Etkin' are probably more accurate since it's not certain that the wing is properly modeled in LinAir. However, once it is we should obtain more similar results.

2.2.1 Approximation from 'Etkin'

Phugoid : frequency = 0.99 rad/s, damping = 0.05
Short period: frequency = 6.25rad/s, damping = 0.25

2.2.2 LinAir

Phugoid : frequency = 0.51 rad/s, damping = 0.16
Short period: frequency = 12.55 rad/s, damping = 0.83

A Matlab script

```
%%
%Zouhair Mahboubi, 11-11-2009
%Stability derivatives approximations for Swift
%Reference Etkins, Dynamics of Flight: stability and control

%%
%Defining flight conditions
%All units are SI unless otherwise specified
%angles in radians and frequencies in radians/sec

g = 9.81; %m/s^2
rho = 1.22; %density at sea-level
mu_d = 2e-5; %dynamic viscocity at sea-level

M = 190; %Mass
W = M*g; %Weight ~418lbs
%Moments of Inertia use empty weight and assume that the major contribution
%comes from the wing. These are likely best computed from the CAD model...
Ix=1374.;Iy=16.6; Iz= 1374.;
Iyz = 0; Ixy = 0; Ixz = 0; %Ixz should not be zero, but not needed for now

Sref = 12.5; %reference area ~ 134.5 ft^2
bref = 12.8; %reference span ~ 42 ft
cavg = Sref/bref; %average chord ~ 3.2 ft
cbar = 1.07; %mean aerodynamic chord, 3.5 ft (given by John)
AR = bref^2/Sref; %Aspect ratio

U = 50*1e3/3600; %Cruising at 50km/h
L_D = 20; %L over D


%Non-dimensional quantities
Re = rho*U*cbar/mu_d; %Reynolds number
q = .5*rho*U^2;
CL = W / (q*Sref);
CD = CL/L_D;

mu = M/(rho*Sref*cbar/2); %non-dimensionalized mass
t_star = cbar/(2*U);
Iy_bar = Iy / (rho*Sref*(cbar/2)^3);

%Estimated stability derivatives
CL_alpha = 5.09; %Given by John Melton, Linair Computes 5.98
Cz_alpha = - (CL_alpha + CD); % Eq 5.2,4
```

```
Cm_alpha = -.406; %Given by John Melton, LinAir computes -1.347
Cm_alphadot = 0; %Given by John Melton
Cm_q = -0.0649 ; %Given by John Melton, LinAir computes -5.65


%%
%Lancaster approximation of phugoid period
%equations 6.3,5 and 6.3,12 of Etkins
T_phugoid = sqrt(2)*pi*U/g;
w_phugoid = 2*pi/T_phugoid;
zeta_phugoid = CD/CL;

%short period approximation
C = -(1/t_star^2/Iy_bar) * (Cm_alpha - Cm_q*Cz_alpha/2/mu );
B = -(1/t_star)*(Cz_alpha/2/mu+ (Cm_q + Cm_alphadot)/Iy_bar);
w_sp = sqrt(C);
zeta_sp = B/(2*w_sp);

fprintf('Phugoid frequency %f rad/s , damping %f \n', w_phugoid,zeta_phugoid);
fprintf('SP frequency %f rad/s , damping %f \n', w_sp,zeta_sp);

%results
%Phugoid frequency 0.998887 rad/s , damping 0.050000
%SP frequency 6.258862 rad/s , damping 0.248401

%Linair computes:
% Phugoid     : -0.0836692      +/- 0.5058501j => wn = 0.51 rad/s zeta = 0.16
% Short period: -10.46046 +/- 6.9383464j   => wn = 12.55 rad/s zeta = 0.83
%It seems like things are more damped according to LinAir. Not to be
%trusted for the moment being...
```

References

[1] Etkin B. Reid L., Dynamics of Flight: Stability and Control 3rd Edition. 1996.