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Interactive Optimization of Game Controls

anonymous

ABSTRACT

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Categories and Subject Descriptors

pick

H.4 [Information Systems Applications]: Miscellaneous; D.2.8 [Software Engineering]: Metrics—*complexity measures, performance measures*

General Terms

pick

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Game design and development often involves a final phase of substantial fine-tuning of game mechanics. Paradigmatic examples include varying the settings of player character movement parameters, altering opponent combat statistics, or varying low-level parameters around movement and collision of game objects. Tuning is often a time-consuming and expensive process due to the need for human testing (rather than analytic solutions or simulation results) and the large space of possible parameter settings for a game.

Our long term goal is to develop an intelligent system that can learn to model the design of computer games. To do so will enable the development of practical systems and tools that can:

- Help game developers visualize, understand, and select among game parameters with complex tradeoffs.
- Automatically tune the parameters of a game to meet design goals, taking into account a large population of diverse players.

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- Automatically tune the parameters of a game to tailor game play to specific individual players.

Our central insight is to formulate the iterative design process as a form of active learning in which game players respond to an AI-chosen set of tuning parameters. Specifically, an *Interactive Bayesian Optimization* model for active learning makes explicit the trade-offs between improving a design and testing alternatives, while developing a model of the design space of a given game for later **[(re)use]**.

Rather than replace game designers, a low-level game design AI system allows for a new kind of design process where an AI system automatically player-tests games and intelligently searches for designs that meet a designer-specified goal. That is, developing this design space formalization with an AI system enables game designers to specify abstract objectives for a game design—achieving a given level of player performance in a game or getting good user ratings for provided content—rather than remain tethered to manually making numerous minor low-level changes.

We employ interactive Bayesian optimization in an AI system to choose parameter settings that are most informative about the design space while achieving given design goals for player behavior in an online human-in-the-loop iterative process [1]. By explicitly modeling the trade-off between exploring very different designs and exploiting designs similar to existing ones, our approach **[[theoretically?]]** reduces the need for extensive amounts of play-testing while also automating the tuning process. Employing non-parametric models of the game design space (here Gaussian Processes) we demonstrate the application of interactive Bayesian optimization to two cases studies of game design tuning in a shoot-'em-up game: (1) adjusting enemy design parameters to achieve a desired level of player success and (2) optimizing controls to player preferences.

In this paper we make four primary contributions:

1. Formulation of game design tuning as an active learning process.
2. Demonstration that interactive Bayesian optimization improves performance over standard randomized (“A/B”) testing methods in terms of better modeling with the same quantity of data.
3. Application of this approach to optimize enemy designs for a design objective on player performance.
4. Application of this approach to optimizing and learning player preferences for control settings.

For enemy design optimization we show how a designer-specified objective function for player performance statistics can guide building a regression model from enemy parameter settings to desired player performance. To optimize controls we use preference learning to select control settings to test and evaluate against the previous set of controls. In both studies, Bayesian optimization affords automatic exploration-exploitation trade-offs that enable rapidly (globally) optimizing for the design objective (player performance or preference).

1. RELATED WORK

A game design AI system synthesizes elements from several existing paradigms for the use of AI in game design. Procedural content generation examines how an AI can create content within a space of parameters. Dynamic difficulty adjustment concerns itself with adapting game mechanics in real time to changes in users. By contrast, we focus on the problem of learning a model of how game mechanics impact player behavior. Unlike player modeling, our goal is an AI system that *actively* adjusts game mechanics to explore the design space for that game both to achieve a given design objective and understand how that design space works. That is, we model how a set of design parameters relates to desired outcomes such as performance in a game or preference for game settings.

Approaches to game tailoring and adaptation combine a player modeling technique with a content adaptation or generation method. Many early efforts employed a game-specific player model using vector of attributes (e.g. skills or use of content) and reactively selected new content to guide players toward a desired level of skill or intended level of performance. Hunnicke and Chapman [4] track the average and variance of player damage and inventory levels and employ a hand-crafted policy to adjust levels of enemies or powerups. Systems by Magerko et al. [8], El-Nasr [12], and Thue et al. [15] model players as vectors of skills, personality traits, or pre-defined “player types” and select content to fit players using rule-based approaches. In contrast, an interactive Bayesian optimization approach generalizes across games, automatically determines how complex the player model should be, and does not require designers to tune a set of rules (or other parameters) to generate content while retaining designer control over the algorithm objectives.

Evolutionary computing and machine learning provide frameworks for modeling players and optimizing game parameters to achieve adaptation goals.

Don’t just enumerate, pull out the principles, like in the preceding paragraph. This paragraph should be shorter.

Hastings et al. [3], Shaker et al. [10, 14], and Yannakakis et al. [17] model player preferences using neuro-evolutionary techniques and optimize the output of these models to select potential game parameters. Missura et al. [9] and Yu and Trawick [18] cluster players, train regression or classification models to predict player responses to game content, and choose sets of parameters that optimize the desired player response outcomes. Our approach formalizes these intuitions by making the design objective explicit and enabling the AI system to test variations of model parameters to create the best possible model. The approaches above focus on optimizing game parameters to players but risk ignoring alternative sets of parameters that have not yet

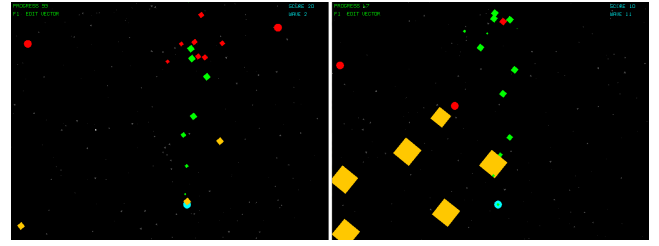


Figure 1: Experimental game interface illustrating player, enemies, and shots fired by both at two points along adaptation process. Preference learning experiments varied parameters related to player movement; enemy tuning involved the speed, size, and rate of fire of enemy bullets. [[Each image full-column-width, top and bottom]]

been tried but have promise to give a better outcome. We incorporate this trade-off of exploitation—optimizing to a given objective—and exploration—attempting new designs that may be better, but are currently unknown.

2. GAME DOMAIN

We test the interactive Bayesian Optimization model in a shoot-’em-up arcade game. Shoot-’em-up games emphasize reflexes and pattern recognition abilities as a player maneuvers a ship to dodge enemy shots and return fire. Arcade games serve as an ideal starting domain for low-level parameter tuning:

- There are a number of parameters that can potentially interfere with each other: size and speed of enemies and enemy bullets, rate of enemy fire, player speed, player rate of fire, etc.
- The game can be played in a series of waves, enabling our system to naturally test game parameter settings and gather player feedback.
- Action-oriented gameplay reduces the complexity of player long-term planning and strategizing.
- A scoring system makes gameplay goals and progress clear, unlike domains involving puzzle-solving or aesthetic enjoyment of a game world or setting.

In the case of shoot-’em-up games, we are interested in two different types of game design goals: abstract goals for player performance such as player success rates or score achieved, and getting good user ratings on the feel of the controls.

We implemented a shoot-’em-up arcade game based on space ship combat, as shown in Figure 1. During each wave a series of enemies appear that fire bullets at the player. We focused on varying three parameters governing enemy behavior: size of enemy bullets, speed of enemy bullets, and rate that enemies fire bullets. Increasing bullet size requires the player to move more carefully to avoid bullets. Faster bullets require quicker player reflexes to dodge incoming fire. More rapid firing rates increase the volume of incoming fire. Together these three parameters govern how much players must move to dodge enemy attacks, in turn challenging player reflexes. Getting approximate settings for these parameters is easy, but fine-tuning them for a desired level of difficulty can be challenging.

Player controls are limited to shooting at enemies and moving the ship. Ship movement is governed by two controllable parameters: drag and thrust. Drag is the “friction” applied to a ship that decelerates the moving ship at a constant rate when it is moving—larger values cause the ship to stop drifting in motion sooner. Thrust is the “force” a player movement press applies to accelerate the ship—larger values cause the ship to move more rapidly when the player presses a key to move. Combinations of thrust and drag are easy to tune to rough ranges of playability. However, the precise values needed to ensure the player has the appropriate controls are difficult to find as player movement depends on how enemies attack and individual player preferences for control sensitivity (much like mouse movement sensitivity).

Our experiments investigate how well an AI system can learn the right parameter settings to achieve a desired level of player performance (in terms of rate of being hit by enemies) or a most preferred set of controls (in terms of player subjective responses).

3. INTERACTIVE BAYESIAN OPTIMIZATION

We use Interactive Bayesian Optimization (IBO) to model an iterative game design process informed by playtesting, i.e., player feedback. IBO is itself a form of sequential Bayesian optimization where data points are user feedback gathered in interaction of the AI with the user [1]. Sequential Bayesian optimization optimizes a function through iteratively testing a sequence of points, each one selected by some algorithm based on previous points. Two functions are required: (1) the *objective function* that maps inputs to outputs; and (2) the *acquisition function* that maps potential inputs to their value for optimizing the objective function output. In our application of IBO to **[[game design space learning]]**, we employ Gaussian processes (GPs) for the objective function—the game design model from parameters to player behavior—and a modified expected improvement acquisition function—trading exploration of the design space against exploitation of designs known to achieve the design objective.

Gaussian processes are a non-parametric modeling technique able to capture complex non-linear relationships within a data set. Intuitively, non-parametric models allow an infinite number of variables to account for the data before selecting only the subset needed to explain a given set of observations. In practice, this leads to models that automatically become more complex to fit a data set as needed. Bayesian formulations of GP regression and classification automatically trade off between model complexity and fit to a data set, avoiding over-fitting and poor generalization problems that occur with other optimization approaches. Bayesian model specifications of GPs also require reduced or simplified parameter specification from users—in this case designers.

The IBO approach can use existing GP models to automate design tuning and learning a design space model. *Gaussian process regression* enables automatic difficulty adjustment by modeling player performance in a game as a non-linear function of game parameters. *Gaussian process preference learning* enables optimization of game parameters (here controls) to player preferences by modeling player preferences for a set of game parameters as a non-linear func-

tion of game parameters that is forced to pairwise choices between alternatives. We integrate these GP models with active learning techniques to automatically identify the next parameter setting to test, guided by a designer-specified expected improvement acquisition function such as achieving a given level of in-game performance or player preference over control settings.

3.1 Gaussian Process Regression

Gaussian processes are formally defined as “a collection of random variables, any finite number of which have a joint Gaussian distribution” [11]. While allowing an infinite number of variables to be used, any GP model can be computed through a multivariate Gaussian distribution based on the input and output values. Gaussian processes are specified by their mean function ($m(x)$) and covariance function ($k(x, x')$):

$$f(x) \sim GP(m(x), k(x, x'))$$

Intuitively, GP regression learns a model predicting that similar inputs—according to the covariance function—should yield similar outputs. Different covariance functions define different notions of similarity. In our work we employ the automatic relevancy detection (ARD) version of a squared exponential distance:

$$k(x, x') = \exp\left(-\frac{1}{2} \sum_{l=1}^d \kappa^l (x^l - x'^l)^2\right)$$

where $\kappa^l > 0$ is the ARD parameter for the l -th feature of a d -dimensional data set, serving to control the contribution of this feature to the model. Automatic relevancy detection allows us to optimize model parameters during the fitting process, automatically scaling input dimensions to minimize the impact of irrelevant aspects of the data. See Rasmussen and Williams [11] for additional details on GP regression.

For our performance regression model we predict player performance (number of times hit) from game parameters controlling enemy attacks (speed and size of bullets along with firing rate). We fit a GP regression model to player data and optimize the covariance function ARD parameters using stochastic gradient descent after each training point received. Since GP regression has a closed-form solution for learning and prediction the primary computational bottleneck is optimizing the ARD parameters (needing 13 seconds for 10 training points on a standard desktop machine).

3.2 Gaussian Process Preference Learning

[[To learn the design space mapping ship control parameters to preferences]], we employ a pairwise preference learning model. We use pairwise ratings (e.g., A is better than B) instead of a numeric rating scale because sequential numeric ratings are subject to a cognitive anchoring bias where earlier numeric ratings influence choices on subsequent ratings [16]. We thus employ a model that generalizes information gained from pairwise rankings to the underlying preference of users for different instances (here game parameter settings). Games can only be played sequentially during comparisons, motivating an approach of pairwise preference ratings comparing each new instance to the previous one.

Gaussian process preference learning models user choices in a two-part model: (1) a GP regression model specifying the underlying (unobserved) value of a single instance; (2) a

probit model of how a choice is generated based on two instances being compared [2]. The GP model allows a flexible specification of how users value a given instance specified in terms of its parameters. The probit model converts a pair of latent values into a comparison judgment according to the function:

$$P(x_i \succ x_j | f(x_i), f(x_j)) = \Phi\left(\frac{f(x_j) - f(x_i)}{\sqrt{2\sigma_{noise}}}\right)$$

where x_i, x_j are two instances, $f(x_i)$ is the GP latent value of an instance, Φ is the cumulative normal distribution, and σ_{noise} is the inherent noisiness of comparative judgments. Intuitively, the probit model encodes preference judgments as based on the difference in underlying value of two instances, allowing for noise in preference ratings.

Due to the non-linear probit model used GP preference learning has no analytic learning model. Instead, we follow work by [2] and use a Laplace approximation to learn the underlying GP model's parameters. We employ a GP with zero mean and the ARD covariance function and optimize its parameters along with the selection of σ_{noise} using a grid search over the space of possible parameters. These nested optimization processes are possible using off-the-shelf solvers, requiring 9 seconds for 10 training points. We note that optimization may be performed using any-time algorithms (such as DIRECT [5]), allowing optimization of parameters to occur while the player plays a new option. In our experiments we optimize parameter values and force players to wait in order to test the best-case performance of our approach.

3.3 Active Learning

[[need transition. Remind us what AL is for.]]

Active learning (AL) is an approach to machine learning problems with a large set of unlabeled instances where a computer asks a human to provide information about given instances to learn a model of the instances as a whole [13]. AL is well suited to our application where the space of game parameterizations is very large and information can only be gained through the expensive process of having a human play and provide feedback about a game instance. Acquisition functions specify how a given AL algorithm weights potential instances to test based on a goal of optimizing the objective function. In our case, the GP regression model seeks to minimize the difference between desired and actual player performance and the GP preference model seeks to find the highest latent value instance.

Many possible acquisition functions exist, varying in how the functions balance the exploration-exploitation trade-off guiding how locally-tethered the search for large objective function values is [13]. Expected improvement (EI) is an acquisition function that balances the value of unseen instances against the uncertainty regarding their values. EI integrates over all possible results to get an average-case estimate of the result, rather than seeking a best-case (or worst-case) scenario. We employ a modified EI function that incorporates a slack parameter ($\xi \geq 0$) to control the relative weighting of exploration and exploitation goals [7]:

$$EI(x) = \begin{cases} (f(x) - f(x^+) - \xi\Phi(Z) + \sigma(x)\phi(Z)) & \text{if } \sigma(x) > 0 \\ 0 & \text{if } \sigma(x) = 0 \end{cases}$$

where $f(x)$ is the function value at x , x^+ is the instance with the current greatest function value, σx is the uncertainty in the value of the instance, $\phi(Z)$ is the Gaussian distribution density at Z where Z is defined as:

$$Z = \begin{cases} \frac{f(x) - f(x^+) - \xi}{\sigma(x)} & \text{if } \sigma(x) > 0 \\ 0 & \text{if } \sigma(x) = 0 \end{cases}$$

Intuitively, Z is the noise-scaled difference between the test point x and the current best point x^+ , and the expected improvement takes a weighted combination of this gain against the uncertainty of the point. Points that are more uncertain and expected to have higher values are preferred to those with lower values or high values that are highly certain. ξ allows an explicit specification of how heavily to emphasize exploration

4. EXPERIMENT

In this section we describe two empirical human studies we conducted. The first study examines the use of GP regression to find and continually adjust game parameter settings to achieve a designer-specified level of performance. The second study examines the use of GP preference modeling to optimize controls to player preferences. We show our approach, compared to random sampling: improves a player model's predictive power with the same sampling budget, does equally well or better at achieving a design objective, and improves the ratio of positive to negative responses to sample points in preference learning. Random sampling is the common standard used to evaluate the efficacy of AL models for improving model fits for a given data budget and is similar to A/B testing approaches that capture large amounts of data before acting on the results [13].

4.1 Methods

We used 73 sessions of performance optimization (38 active learning and 35 random sampling) and 28 sessions (14 and 14) of control optimization data. Players logged into the game online and played the game against a series of waves of enemies. Players were assigned in a balanced factorial design to either a control case that sampled random possible game parameter values or a test case that selected parameter values using interactive Bayesian optimization. We recorded player performance in terms of number of times the player was hit and all parameter settings for each wave. Player preferences were recorded in a four option forced choice pane, selecting whether the control settings of the current wave compared to the previous wave were: better, worse, no different, or different but of equal quality. For analysis we examined only results from players who completed at least ten waves and only examine data from those ten waves. Preference responses only used better or worse choices.

In the performance optimization study a GP regression model was fit to predict the difference between the actual number of times the player was hit and a desired designer-specified value. We specified a designer target of players being hit 6 times over the course of a 20 second wave of combat. Between waves the model and covariance function parameters were all optimized and EI used to select the next test point. Learning had a one wave lag—players were allowed to play a wave while the model was fit on a server

machine to all data but the most recent wave. This enabled the game to play continuously without pausing for learning to occur at the cost of a model that was unaware of the most recent test point results.

In the control optimization study a GP preference model was fit to predict the underlying preference players had for different control settings. After each wave players were prompted to indicate whether the most recent wave had better or worse controls than the wave before that. Model fitting used the same timing as above. In this case EI selected an instance to test in comparison to the last tested instance—every comparison was the next point that would be most useful compared to the last level the player completed.

4.2 Results

wave	AL	random	p-value
1	13.18	8.89	0.19
2	8.68	12.57	0.02
3	3.89	10.86	0.01
4	7.71	5.37	0.25
5	7.34	5.63	0.96
6	13.84	6.80	0.01
7	9.79	4.60	0.07
8	10.92	6.57	0.12
9	12.18	6.00	0.00
10	10.37	7.17	0.46

Table 1: Error rates of active learning (AL) and random sampling (random) for achieving target performance rate of 6 hits per wave.

We evaluated our IBO model for two capabilities: (1) achieving a given design objective, and (2) modeling the design space. Achieving a design objective meant either enforcing the target player performance goal or optimizing player preferences. We evaluated this as error from the target level of performance and proportion of better or worse choices from player subjective feedback. Modeling the design space meant building a model with better generalization to new data. We evaluated this using 10-fold cross-validation—training a model on nine tenths of our data and testing it on the remaining one tenth.

For performance optimization we found our IBO approach performs equivalently to a random sampling strategy on a wave-by-wave basis (Table 1). We evaluated error as the difference between the number of times a player was hit and the target rate of 6 hits for each wave. We compared error rates between the two models using a Kruskal-Wallis rank sum test, a non-parametric alternative to the standard t-test used when data is not normally distributed (as is ours and most human subject data). These results showed the IBO approach had significantly ($p < 0.05$) smaller error on two waves, higher error on two waves, and no difference on the remaining waves. Both methods were unable to achieve the design objective, likely due to the difficulty of enforcing such a low rate of hitting players. Alternative acquisition functions may also allow for better performance on design objectives by putting heavier emphasis on exploitation over attempting new parameter values.

For preference optimization we found the IBO model had a statistically significantly greater proportion of “better” responses (and lower “worse” responses) on a wave-by-wave



Figure 2: Count of player subjective responses to control settings in different sampling techniques.

	better		worse	
wave	AL	random	AL	random
2	4	6	4	4
3	6	2	3	7
4	12	7	1	6
5	0	7	14	4
6	3	4	2	8
7	7	6	4	6
8	4	7	3	5
9	8	4	2	7
10	2	6	5	7

Table 2: Comparison of frequency of preference ratings by sampling method.

basis compared to the random sampling approach (χ^2 test, $p < 0.05$ for both). Players also showed significantly greater proportion of “no difference” responses (χ^2 test, $p < 0.05$) on a wave-by-wave basis. There were too few “equal quality” responses ($n=44$ over 10 waves) to achieve statistical significance. Together, these results show the active learning model is able to capture aspects of player preference and attempt to optimize for them.

Building a good design space model entails exploring the space in a way that generalizes to new parameter settings. We evaluated this by training an active learning model or random sampling strategy on a subset of nine tenths of the data before testing on the remaining one tenth of the data. We used the following standard method for evaluating active learning models: (1) set aside a random selection of 10% of our data for testing; (2) train either random sampling or active learning on 100 sample points of the 657 points in the remaining 90% of our data; (3) test the trained model on the test 10%; (4) repeat the above for 10 repetitions [13]. In random sampling step (2) randomly selected training points. In active learning step (2) was seeded with 10 random points, then allowed to iteratively select one new training point to sample from the remaining points in the 657 point pool and retrained with the new set of data until 100 points were being used in the AL model. To examine the efficiency of the two approaches we compared the error (or accuracy) of

the models as the number of samples increased.

For both performance and control optimization IBO more effectively finds the right parameter settings to test to build a useful model of the design space. We compared mean squared error of model predictions on the held-out test data using the Kruskal-Wallis rank sum test. For performance optimization we found the mean squared error of predictions against actual performance levels of IBO to be significantly less ($p < ???$)

fill

) than that of random sampling when comparing on a wave-by-wave basis. For control optimization we found the IBO model accuracy for predicting player choices was significantly higher ($p < ???$)

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) on a wave-by-wave basis. Thus, IBO uses human playtesting more effectively to automatically model a design space.

While difficult to quantify, IBO shows clear trends of exploring design alternatives (e.g., Figures 3, 4). During performance optimization IBO gradually shifts the values of enemy parameters tested, distinct from the uniform sampling approach from random sampling. During control optimization IBO shows a similar trend of trying alternatives for ship drag and thrust. These results suggest IBO is both intentionally exploring the design space and uncovering general patterns of player responses across the design space that may go unnoticed unless designers are careful to try the full range of options.

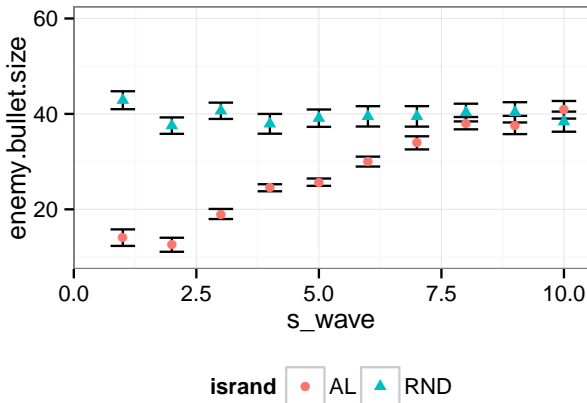


Figure 3:

5. DISCUSSION

6. ACKNOWLEDGMENTS

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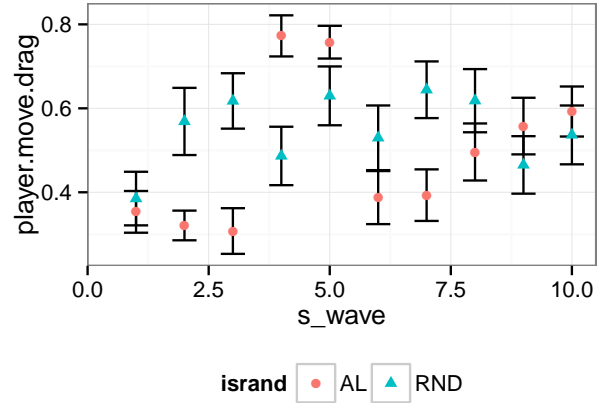


Figure 4:

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