Laboratory Session 03 : April 14, 2022

Exercises due on : May 1, 2022

Exercise 1

- The triangular distribution, in the interval (a, b), is given by the following:

$$f(X) = \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)} & a \le x < c \\[2ex] \dfrac{2(b-x)}{(b-a)(b-c)} & c \le x \le b \\[2ex] 0 & \text{otherwise} \end{cases}$$

  where $c \in [a, b]$.

a) plot the function, given the interval (a, b)

b) and write an algorithm to generate random numbers from the triangular distribution

c) generate 10^4 random number from the distribution, show them in an histogram and superimpose the analytical curve

Exercise 2 - Markov's inequality

- Markov's inequality represents an upper bound to probability distributions:

$$P(X \ge k) \le \frac{E[X]}{k} \text{ for } k > 0$$

- having defined a function

$$G(k) = 1 - F(k) \equiv P(X \ge k)$$

  plot $G(k)$ and the Markov's upper bound for

a) the exponential, $\text{Exp}(\lambda = 1)$, distribution function

b) the uniform, $\mathcal{U}(3, 5)$, distribution function

c) the binomial, $\text{Bin}(n = 1, p = 1/2)$, distribution function

d) a Poisson, $\text{Pois}(\lambda = 1/2)$, distribution function

Exercise 3 - Chebyshev's inequality

- Chebyshev's inequality tell us that

$$P(\left|X - \mu\right| \geq k\sigma) \leq \frac{1}{k^2}$$

- which can also be written as

$$P(\left|X - \mu\right| < k\sigma) \geq 1 - \frac{1}{k^2}$$

- use R to show, with a plot, that Chebyshev's inequality is is an upper bound to the following distributions:

a) a normal distribution, N($\mu = 3, \sigma = 5$)

a) an exponential distribution, Exp($\lambda = 1$)

b) a uniform distribution $\mathcal{U}(1 - \sqrt{2}, 1 + \sqrt{2})$

d) a Poisson, Pois($\lambda = 1/3$), distribution function

Exercise 4 - Six Boxes Toy Model : inference

- The six boxes toy model is described in reference [1].

- Labeling the boxes as follows:

H_0	H_1	H_2	H_3	H_4	H_5
●●●●●	○●●●●	○○●●●	○○○●●	○○○○●	○○○○○

- write a program in R that:

1) allows the user to insert the color of a randomly extracted box and

3) prints on the standard output the probability of selecting each box

4) plots the probability for each box as a function of the extraction step

Exercise 5 - Six Boxes Toy Model : simulation

- consider again the six boxes toy model of the previous exercise and write a simulation program that:

1) selects a random box

2) makes random sampling from the box

3) prints on the standard output the probability of selecting each box

4) plots the probability for each box as a function of the number of trial

Bibliography

[1] G. D'Agostini, *Probability, propensity and probabilities of propensities (and of probabilities)*, https://arxiv.org/pdf/1612.05292.pdf
G. D'Agostini, *More lessons form the six box toy experiment*, https://arxiv.org/pdf/1701.01143.pdf