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Implied Volatility and Greeks for Airbnb Inc.: An Empirical Study

1 Introduction

The aim of this report is to provide an empirical analysis of the implied volatility of the asset Airbnb Inc. and to derive the Greek letters for different maturity times from the data. The report is organized as follows: Section 2 provides a description of Airbnb Inc., Section 3 clarifies the concepts of implied volatility smile and skew, Section 4 presents the methodology used for the analysis and the results, and Section 5 discusses the conclusions.

2 Airbnb, Inc.

Airbnb, Inc. is a San Francisco-based American company that operates an online marketplace for short-term homestays and experiences. The company acts as a broker and charges a commission for each booking of private rooms, primary homes, or vacation homes. It was founded in 2008 by Brian Chesky (who is currently the CEO), Nathan Blecharczyk (currently the Chief Strategy Officer Director), and Joe Gebbia (currently the Chairman of Airbnb.org and Director). Other important figures in the company include David Stephenson, the CFO and Head of Employee Experience, Aristotle Balogh, the Chief Technology Officer, and Catherine Powell, the Global Head of Hosting. The company has a market capitalization of $75.326 billion and an enterprise value of $65.34 billion[1].

Trailing P/E	40.75
Forward P/E	31.55
PEG Ratio (5 yr expected)	1.58
Price/Sales (ttm)	9.20
Price/Book (mrq)	12.88
Enterprise Value/Revenue	7.78
Enterprise Value/EBITDA	31.21

Table 1: Valuation Measures for Airbnb Inc., data provided by Morningstar, Inc.

Airbnb, Inc. does not provide dividends. As of the latest available data, All Insider holds 1.80% of the shares, while institutions hold 66.49%. Among the 1,336 institutions that hold shares, the top institutional holder is Jennison Associates LLC, which owns 8,971,146 shares of the company with a total value of $1 billion.

[1]according to Yahoo Finance, accessed on April 28, 2023

3 Implied Volatility Smile and Skew

A volatility smile is a graphical representation of the implied volatility of an option with a certain life as a function of its strike price. For European options, the implied volatility for call and put options is always the same when they have the same strike price and maturity date. This result is also approximately true for American options, which is convenient for traders as they do not have to distinguish between call and put options when analyzing a volatility smile. The shape of the volatility smile depends on the current price of the underlying asset, with the lowest point typically close to the current exchange rate. As an option moves in or out of the money, the implied volatility tends to increase, resulting in a smile-like shape.
On the other hand, the volatility skew refers to the asymmetry of the implied volatility when plotted against the strike price for different expiration dates. It shows that for a given strike price, the implied volatility tends to be higher for options with shorter maturities than for options with longer maturities. This is because short-term options are more sensitive to changes in the underlying asset's price, and therefore, have a higher implied volatility.
Volatility smile and skew are both important in options pricing because they reflect the market's perception of the risk and uncertainty associated with the underlying asset. Traders and investors use these measures to manage their risks and adjust their trading strategies accordingly. For example, if the volatility smile is flat, it may indicate a lack of uncertainty in the market, which could be an opportunity for traders to sell options. Conversely, if the volatility skew is steep, it may indicate a higher level of uncertainty, and traders may adjust their strategies accordingly to manage the risk.

4 Methodology and Results

The data used for this analysis was collected from Yahoo Finance[2]. Specifically, the stock price S_0 on the date of data collection was 117.93.

4.1 Implied Volatility Analysis

The initial aim of the analysis was to research the volatility smile. To do this, data on the variation of volatility of call options according to maturity time T and strike prices K were collected and reported in Table 4.1.
Subsequently the implied volatility values have been plotted in a 3D surface in Microsoft Excel, presented in Figure 1.

[2]https://finance.yahoo.com/quote/ABNB/options?date=1685059200p=ABNB, accessed on April 28, 2023

K ($)	T (months) 1	2	3	4	6	9	14	21
95	0.6931	0.606	0.5743	0.5942	0.5687	0.571	0.4413	0.5608
100	0.6082	0.5752	0.5425	0.5695	0.5491	0.5549	0.5533	0.553
105	0.592	0.5453	0.5228	0.5394	0.5348	0.5437	0.5397	0.5467
110	0.5647	0.5265	0.5053	0.5249	0.5098	0.5275	0.5313	0.5357
115	0.5258	0.5039	0.4861	0.5087	0.5002	0.5176	0.5211	0.5297
120	0.5065	0.4769	0.4653	0.4972	0.4937	0.5028	0.512	0.522
125	0.4912	0.464	0.4509	0.4871	0.4792	0.4963	0.5007	0.5119
130	0.4736	0.448	0.4385	0.4698	0.4686	0.4892	0.497	0.5045
135	0.47	0.4399	0.4289	0.4612	0.4604	0.4778	0.4903	0.5045
140	0.468	0.4373	0.4243	0.4547	0.4486	0.4699	0.4831	0.4992
145	0.4644	0.436	0.417	0.4468	0.4451	0.4671	0.4755	0.4926
150	0.519	0.4373	0.4158	0.4385	0.439	0.4591	0.4696	0.4854
155	0.5127	0.4443	0.4128	0.4326	0.4323	0.4555	0.4636	0.4816
160	0.5225	0.4497	0.4111	0.4286	0.428	0.4483	0.4595	0.4757

Table 2: Implied volatilities in function of the strike price K and the maturity time T.

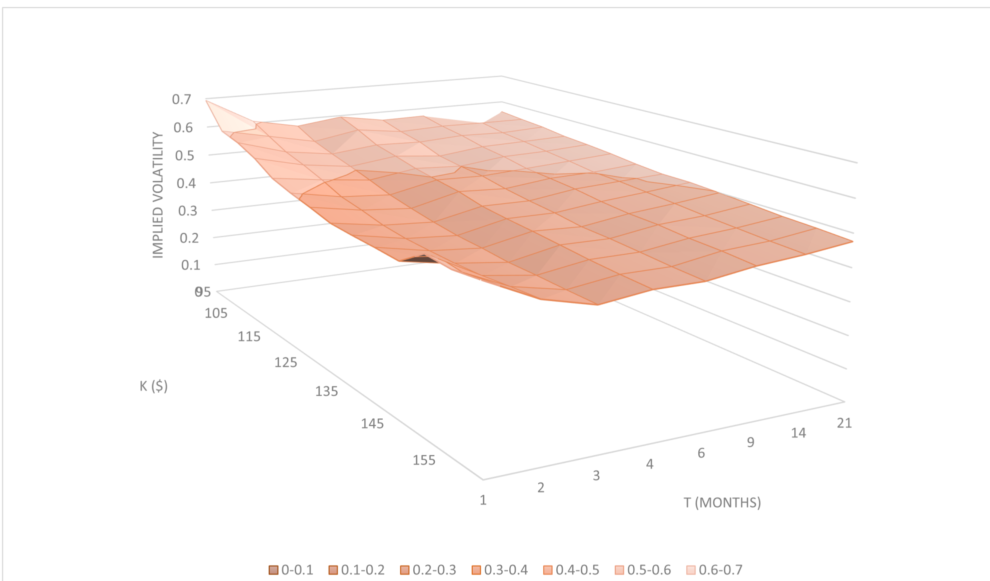

Figure 1: Implied Volatility Surface

Figure 1 and Table 4.1 show that the implied volatility is mostly flat, but it is still possible to observe that for small maturity times, the furthest in-the-money and out-of-the-money options have slightly higher volatility than at-the-money options, as expected. As the maturity time increases, the volatility presents a skew, because σ decreases as the strike price increases. Additionally, it can be observed that the volatility for shorter maturity times is higher compared to longer maturity times, as described in Section 3. The overall flatness of the surface suggests that there are good opportunities for traders to sell options.

S_0	K ($)	T (months)	q
117.93	[95, 100, ..., 160]	1, 2, 3, 4, 6, 9, 14, 21	0

Table 3: Parameter values for the Greek letters calculation.

4.2 Greek letters computation

In this section the Greek letters Δ, Θ, Γ, ν and ρ have been computated for the chosen asset at different strikes prices and maturity times using a VBA module provided in the Appendix. The variables used in the computation are reported in Table 4.2. In particular, the rate of dividend yiel has been set equal to zero because the asset does not provide dividends. Regarding the volatility values, for each couple (K, T) has been used the reference value reported in Table 4.1. In order to recover the r values, an analysis using the Box Spread Strategy has been performed and reported in Section 4.2.1.
The resulting surfaces of the Greeks as a function of the strike price K and the maturity time T are presented in the following figures. It should be noted that although the maturity time is expressed in months in the following plots, it has been expressed in years in the computation formulas, as required.

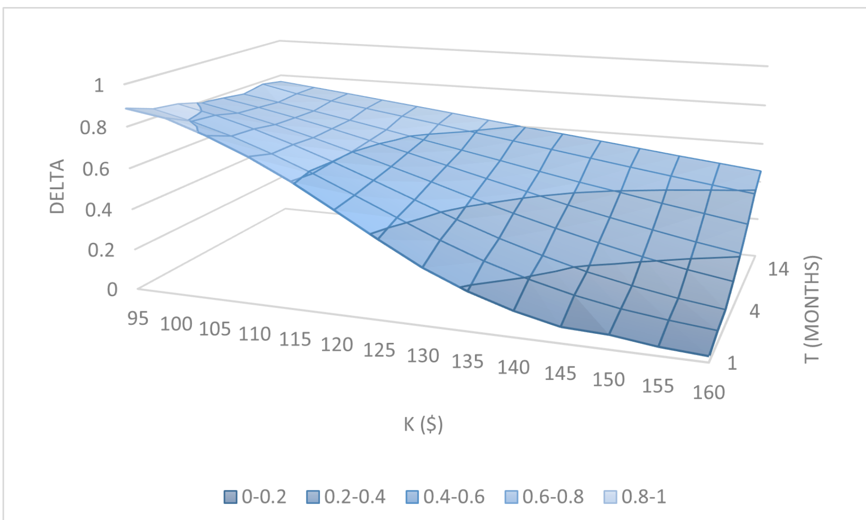

Figure 2: Δ surface

In Figure 2, it can be observed that as T increases, the value of Δ decreases more smoothly with K compared to smaller values of T.

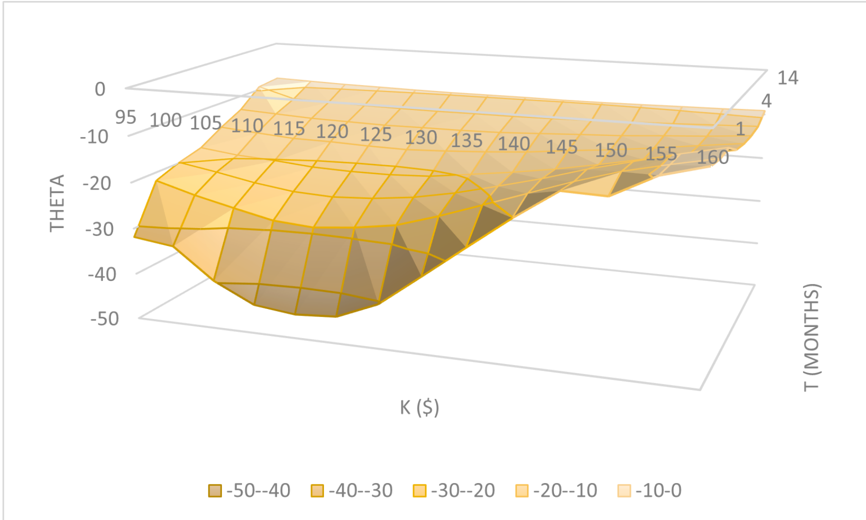

Figure 3: Θ surface

The values of Θ become less negative as maturity time increases, as can be appreciated in Figure 3.

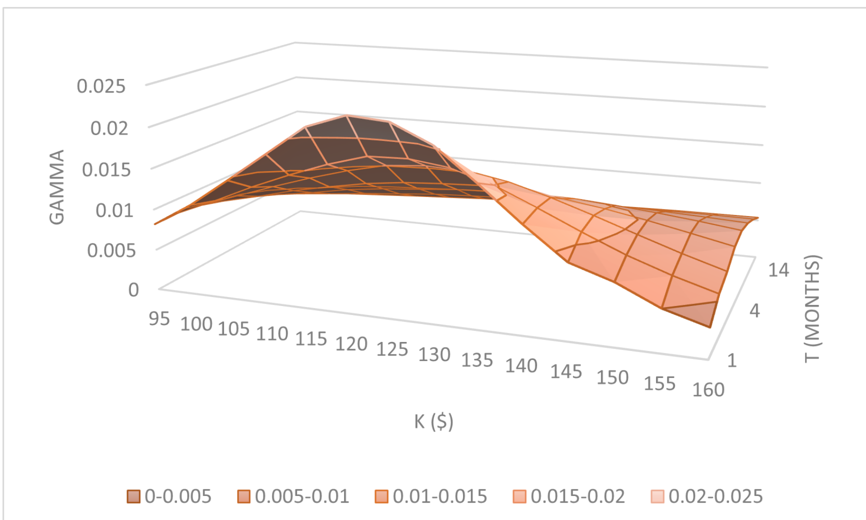

Figure 4: Γ surface

Similary to Θ, also Γ values become closer to zero while T increases, as shown in Figure 4

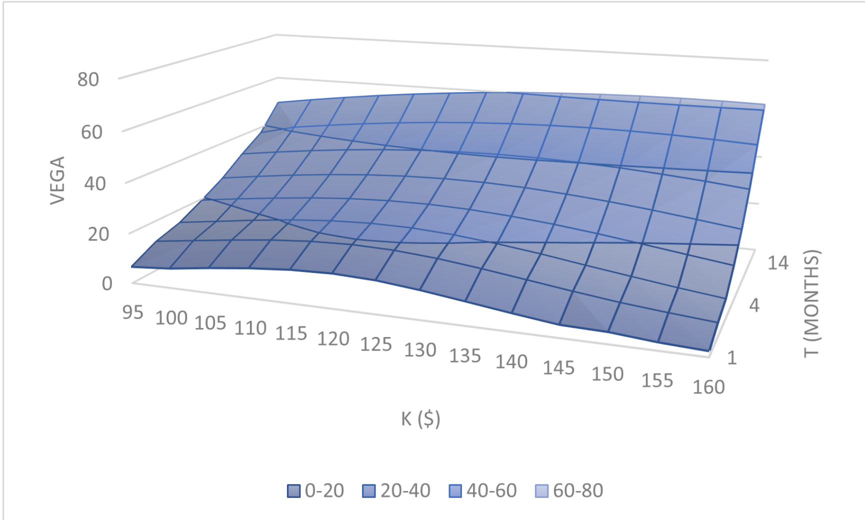

Figure 5: ν surface

Figure 5 shows that ν values increases with T.

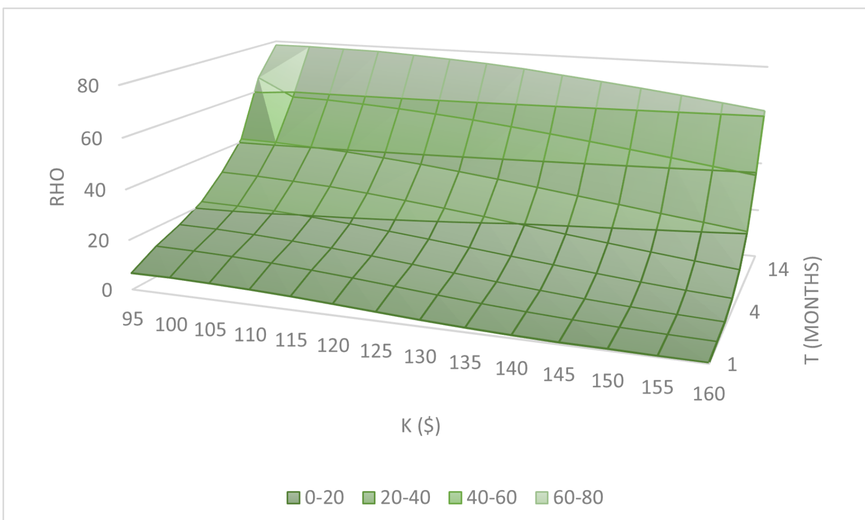

Figure 6: ρ surface

Analogously to ν, Figure 6 shows that ρ increases significantly while T grows.

4.2.1 Recovering r using the Box Spread Strategy

The risk-free rates available on global-rates.com[3] covered only maturity times of $T = 1, 3, 6$ months, reported in Table 4.2.1. The values were therefore calculated from prices of call and put options using the Box Spread Strategy. The discount rate D(0, T) can be obtained by selecting two strike prices K_+ and K_- (where $K_+ > K_-$) at a certain maturity time T based on the

[3] consulted on April 28, 2023

$T=1$	$T=3$	$T=6$
5.03414 %	5.29914 %	5.38686 %

Table 4: Risk-free rates provided by global-rates.com

$T=1$	$T=2$	$T=3$	$T=4$	$T=6$	$T=9$	$T=14$	$T=21$
5.613 %	1.001 %	2.676 %	2.007 %	2.685 %	2.241 %	3.648 %	2.135 %

Table 5: Recovered r using Box Spread Strategy.

current stock price S_0, and considering the prices of both call and put options for these strikes. Since the payoff for the strategy is constant, the discount rate can be calculated using the following formula:

$$D(0,T) = \frac{p_0^{call}(K_-) - p_0^{call}(K_+) + p_0^{put}(K_+) - p_0^{put}(K_-)}{K_+ - K_-}. \tag{1}$$

When using this strategy, it is important to choose appropriate values for K_+ and K_-; otherwise, they might provide a discount rate greater than 1, leading to paradoxical results such as negative dividends. The condition to be checked for each possible strike price pair K_+ and K_- is the following:

$$p_0^{call}(K_-) - p_0^{call}(K_+) + p_0^{put}(K_+) - p_0^{put}(K_-) < K_+ - K_-. \tag{2}$$

Assuming that the discount rate $D(0,T)$ has the expression $D(0,T) = e^{-rT}$, the value of r was recovered. The results of this analysis are reported in Table 4.2.1. Comparing Table 4.2.1 with Table 4.2.1, the r values do not match. It is thought that the Box Spread Strategy may not be suitable for this type of asset and, in general, may not be accurate enough for recovering discount rates due to its sensitivity to the strike values K_+ and K_-.

5 Conclusions

This report has presented an empirical analysis of the implied volatility and the Greeks for the asset Airbnb Inc. The implied volatility analysis revealed that the asset exhibits both a smile and a skew, but is mostly flat, indicating that it may be a good time for traders to sell options. The analysis of the Greeks surfaces highlighted how traders expect to hedge in further maturity times. Additionally, the report showed how the Box Spread Strategy can be used to recover risk-free rates when they are not available. Overall, these findings provide useful insights for traders and investors in making informed decisions about their trading strategies and risk management.

6 Appendix

```
'Greek Letters calculation
'd1
```

```
Function d1(S, K, T, r, q, sigma)
    d1 = (Log(S / K) + (r - q + sigma   2 / 2) * T) / (sigma * Sqr(T))
End Function
'd2
Function d2(S, K, T, r, q, sigma)
    d2 = d1(S, K, T, r, q, sigma) - sigma * Sqr(T)
End Function
'N'(x)
Function pdf(x)
    pdf = Exp(-WorksheetFunction.Power(x, 2) / 2) / Sqr(2 * Application
       .Pi())
End Function
'DELTA
'call
Function deltacall(S, K, T, r, q, sigma)
    deltacall = Exp(-q * T) * WorksheetFunction.NormSDist(d1(S, K,
       T, r, q, sigma), True)
End Function
'put
Function deltaput(S, K, T, r, q, sigma)
    deltaput = Exp(-q * T) * (deltacall(S, K, T, r, q, sigma) - 1)
End Function
'THETA
'call
Function thetacall(S, K, T, r, q, sigma)
    thetacall = -(Exp(-q * T) * S * pdf(d1(S, K, T, r, q, sigma)) *
       sigma) / (2 * Sqr(T)) - r * K * Exp(r * T) * WorksheetFunction.
       NormSDist(d2(S, K, T, r, q, sigma), True) + q * S * Exp(-q *
       T) * WorksheetFunction.NormSDist(d1(S, K, T, r, q, sigma),
       True)
End Function
'put
Function thetaput(S, K, T, r, q, sigma)
    thetaput = thetacall(S, K, T, r, q, sigma) + r * T * Exp(-r * T)
       - 2 * q * S * Exp(-q * T) * WorksheetFunction.NormSDist(d1(S,
        K, T, r, q, sigma), True)
End Function
'GAMMA
Function gammanew(S, K, T, r, q, sigma)
    gammanew = Exp(-q * T) * pdf(d1(S, K, T, r, q, sigma)) / (S *
       sigma * Sqr(T))
End Function
'VEGA
Function veganew(S, K, T, r, q, sigma)
    veganew = Exp(-q * T) * S * Sqr(T) * pdf(d1(S, K, T, r, q, sigma)
       )
End Function
```

```
'RHO
'call
Function rhocall(S0, K, T, r, q, sigma)
    rhocall = K * T * Exp(-r * T) * WorksheetFunction.NormSDist(d2(
       S0, K, T, r, q, sigma), True)
End Function
'put
Function rhoput(S0, K, T, r, q, sigma)
    rhoput = rhocall(S0, K, T, r, q, sigma) - K * T * Exp(-r * T)
End Function
```

Listing 1: VBA module for Greek letters computation using S, K, r, T, σ and q as variables.