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Recovering Implicit Dividends from Option Prices

1 Introduction

The aim of this report is to recover the implicit dividends for the International Business Machines Corporation (IBM) from the prices of options at different maturity times using the Put-Call Parity and using the Box Spread Strategy to recover the discount factor. The report is organized as follow: Section 2 provides a brief description of the IBM Corporation, while Section 3 presents the Call-Put Parity in presence of dividends. Section 4 explains the methodology applied and presents the results, and finally, Section 5 discusses the conclusions.

2 International Business Machines Corporation

IBM is a multinational technology company headquartered in Armonk, New York, USA. It was founded in 1911 as the Computing-Tabulating-Recording Company (CTR), which later became IBM in 1924. It is one of the oldest and largest information technology companies in the world, with operations in more than 170 countries. Additionally, IBM is the largest industrial research organization, with a long history of innovation and having been awarded more U.S. patents than any other company for 29 consecutive years (from 1993 to 2021). Nowadays, the corporation provides integrated solutions and services worldwide and operates through four business segments: Software, Consulting, Infrastructure, and Financing. The company also has a strong commitment to corporate social responsibility and sustainability and has been recognized for its efforts in these areas. In particular, the Environment, Social, and Governance (ESG) Risk Ratings gives IBM a Low Score (15) with a significant low Environment Risk Score (0.5).
The company has a market capitalization of \$116.262B and an enterprise value of \$158.93B[1]. Other valuable information is collected in Table 1.

Trailing P/E	63.27
Forward P/E	12.92
PEG Ratio (5 yr expected)	1.98
Price/Sales (ttm)	1.86
Price/Book (mrq)	5.10
Enterprise Value/Revenue	2.63
Enterprise Value/EBITDA	22.15

Table 1: Valuation Measures for IBM, data provided by Morningstar, Inc.

[1] according to Yahoo Finance, consulted on March 30, 2023

According to Morningstar, Inc., the Forward Annual Dividend Rate is 6.6, while the Forward Annual Dividend Yield is 5.10%. The Payout Ratio is estimated to be 337.95%. Since 2020, the CEO and Chairman is Arvind Krishna, who previously served as the senior vice president of IBM's cloud and cognitive software division. He supports the expansion of new markets for IBM in artificial intelligence, cloud computing, quantum computing, and blockchain technology.
IBM is known for its pioneering work in the field of computer technology, particularly in the areas of mainframe computers. Their System/360 was the dominant computing platform during the 1960s and 1970s. In the 1980s, they introduced the IBM Personal PC, which used an open architecture and led to a large market for third-party add-in boards and applications, setting the standard for personal computers.
IBM has faced some challenges in recent years, including declining revenues and profitability, as well as increased competition from other technology companies. In 2005, the company sold its personal computer division to Lenovo Group due to increasing competition in this field.
However, the company continues to invest heavily in research and development and remains committed to remaining at the forefront of technological innovation. This includes services related to supercomputers, artificial intelligence, cloud computing, data analytics, and cybersecurity.

3 The Call-Put Parity in case of Dividends

The Call-Put Parity is based on the simple formula $x^+ - x^- = x$: since at maturity time T the payoff of a call is $(S_T - K)^+$ and the payoff of a put is $(K - S_T)^+ = (S_T - K)^-$, then:

$$(S_T - K)^+ - (S_T - K)^- = S_T - K, \tag{1}$$

consequently, taking the expectation values and multiplying them for the discount rate $D(0,T)$:

$$D(0,T) \cdot \mathbb{E}^{\mathbb{Q}}[(S_T - K)^+] - D(0,T) \cdot \mathbb{E}^{\mathbb{Q}}[(S_T - K)^-] = D(0,T) \cdot \mathbb{E}^{\mathbb{Q}}[S_T] - D(0,T) \cdot K. \tag{2}$$

Considering a framework in which there the absence of arbitrage opportunities (*AAO*) condition is true, the pricing of a call and put option are respectively, $p_0^{call} = D(0,T) \cdot \mathbb{E}^{\mathbb{Q}}[(S_T - K)^+]$ and $p_0^{put} = D(0,T) \cdot \mathbb{E}^{\mathbb{Q}}[(S_T - K)^-]$ and since $\mathbb{Q}$ is a martingale measure because of *AAO*: $D(0,T) \cdot \mathbb{E}^{\mathbb{Q}}[S_T] = S_0$. The call-put parity is therefore obtained:

$$p_0^{call} - p_0^{put} = S_0 - K \cdot D(0,T). \tag{3}$$

The previous equation allows to obtain the price of a call option from the price of a put one and vice-versa. In a framework where dividends are allowed, the dividend yield δ expected by the market is incorporated into forward prices $F(0,T)$, which is not quoted directly. In order to recover both the values, a portfolio is built as reported in Table 2, using the present value of dividends during the life of the options:

$$DIV = \frac{S_0 - S_0 \cdot e^{-\delta T}}{D(0,T)}, \tag{4}$$

where $1/D(0,T)$ corresponds to the capitalization factor.

	$t=0$	$t=T$
long position	0	$S_T - F(0,T)$
short position	S_0	$-S_T - DIV$
long position in riskless market	$-D(0,T) \cdot (F(0,T) + DIV)$	$+F(0,T) + DIV$
TOT	$S_0 - D(0,T) \cdot (F(0,T) + DIV)$	0

Table 2: Portfolio for a forward contract with dividends

The call-put parity equation defined in 3 therefore results:

$$p_0^{call} - p_0^{put} = S_0 - K \cdot D(0,T) - DIV \cdot D(0,T) = D(0,T) \cdot (F(0,T) - K), \tag{5}$$

or expliciting DIV:

$$p_0^{call} - p_0^{put} = S_0 \cdot e^{-\delta T} - K \cdot D(0,T). \tag{6}$$

From equation 5 the price $F(0,T)$ can be derived as:

$$F(0,T) = \frac{p_0^{call} - p_0^{put}}{D(0,T)} + K. \tag{7}$$

Knowing $F(0,T)$, is then possible to calculate the Dividend Yield δ for T:

$$\delta = -\frac{1}{T} \ln \frac{F(0,T) \cdot D(0,T)}{S_0} \tag{8}$$

4 Methodology and Results

4.1 Recover the Discount term using the Box Spread Strategy

The discount rate $D(0,T)$ is obtained using the Box Spread Strategy, which involves selecting two strike prices K_+ and K_- (where $K_+ > K_-$) at a certain maturity time T based on the current stock price S_0, and considering the prices of both call and put options for these strikes. The sum of the long call K_- and short call K_+ is known as a bull spread, while the sum of the long put K_+ and short put K_- is called a bear spread.
Since the payoff for the strategy is constant, the discount rate can be calculated using the following formula:

$$D(0,T) = \frac{p_0^{call}(K_-) - p_0^{call}(K_+) + p_0^{put}(K_+) - p_0^{put}(K_-)}{(K_+ - K_-)}. \tag{9}$$

When using this strategy, it is important to choose appropriate values for K_+ and K_-. If these strike prices are too far out of the money, the market will expect a significant increase in the underlying asset's price, leading to a higher implied forward price and a discount rate greater than 1. This could bring paradoxal results as negative dividends. The same result could be obtained if the options market is pricing in an excessive amount of volatility or risk for the underlying asset, in this case it indicates that the Box Spread Strategy may not be suitable for that asset.

For each selected maturity time T_i, where i is the index corresponding to the number of months considered, we checked the condition for each possible strike price pair K_+ and K_- using the following equation:

$$p_0^{call}(K_-) - p_0^{call}(K_+) + p_0^{put}(K_+) - p_0^{put}(K_-) < (K_+ - K_-), \tag{10}$$

otherwise, the result would be greater than 1 and we would fall in the situation previously descripted.
The chosen strike prices K_+ and K_- that satisfy this condition, along with the corresponding prices of the calls and puts (calculated using the average value of the bid and the ask), are reported in Tables 3. In these tables, the first term of equation 10 is referred to as "diff". All data were collected the 29th of March 2023 on Yahoo Finance.[2]

	K_+	$p_0^{call}(K_+)$	$p_0^{put}(K_+)$	K_-	$p_0^{call}(K_-)$	$p_0^{put}(K_-)$	diff	$K_+ - K_-$
T_1	132	2,835	4,85	122	9,375	1,43	9,96	10
T_3	175	0,02	49,8	75	48,6	0,06	98,32	100
T_6	150	1,665	22,1	120	14,375	5,075	29,735	30
T_{12}	155	1,94	27,575	70	55,575	0,425	80,785	85

Table 3: Recovered discount terms for different maturity times

The discount rates found are reported in Table 4 together with the corresponding R, assuming that $D(0,T) \sim e^{-RT}$, and the comparison with the true discount rate R_{true}[3].

	K_+	K_-	$D(0,T)$	R	R_{true}	% err
T_1	132	122	0,996	0,048	0,049	0,004
T_3	175	75	0,983	0,068	0,052	0,403
T_6	150	120	0,991	0,018	0,052	1,732
T_{12}	155	70	0,950	0,051	0,052	0,074

Table 4: Recovered discount terms for different maturity times

4.2 Recover the Dividend Yield δ

The formulas derived from the call-put parity relation with dividends, as described in Section 3, have been applied to calculate the Dividend Yield δ for each maturity time T_i. As of March 29, 2023, the stock price was $S_0 = 129.75\$$ and the strike prices at the money K_i were derived by taking the average of the bid and ask prices collected from Yahoo Finance. These strike prices are reported in Table 5.
The price of the forward contract can be obtained by using equation 7, while DIV can be calculated using equation 5 and the Dividend Yield δ can be derived from equation 8. Table 4 presents the results along with the expected stock prices at the end of the dividend period, which are given by $S_0 e^{-\delta T}$ in the case of ordinary dividends and $S_0 e^{\delta T}$ in the case of extra dividends.

[2]https://finance.yahoo.com/quote/IBM?p=IBM
[3]found consulting https://www.global-rates.com/en/interest-rates/libor/american-dollar/american-dollar.aspx on March 29, 2023.

	K	p_0^{call}	p_0^{put}
T_1	130	3,825	3,85
T_3	130	5,2	5,7
T_6	130	8,975	8,275
T_{12}	130	9,975	10,65

Table 5: Strike prices

	$F(0,T)$	DIV	$\delta(\%)$	$S_0 \cdot e^{-\delta T}$	$S_0 \cdot e^{\delta T}$
T_1	129,97	0,296	2,73	129,45	130,05
T_3	129,49	2,476	7,57	127,32	132,23
T_6	130,71	0,200	0,31	129,55	129,95
T_{12}	129,29	7,23	5,44	122,88	137,01

Table 6: Recovered terms for different maturity times

As reported in Section 2, the estimated value for the Forward Annual Dividend Yield is 5.10%, providing an error of 6,69 % on the value presented in Table 6. his suggests that the procedure is not perfect, perhaps due to the fact that the options analyzed are not European, as the strategy assumes. Nonetheless, the results provide a general sense of the possible outcomes. However, the results for 6 months of maturity time seem unrealistic, as the δ value is significantly smaller than the other values. This may be a consequence of the inaccurate discount rate obtained for this time period, which had a percentage error higher than 1%, as shown in Table 4.

5 Conclusions

In this report, we have demonstrated that the Box Spread Strategy, in conjunction with the Call-Put Parity, can be used to recover plausible discount rate and dividend yield terms. However, it is crucial to note that the accuracy of this method relies heavily on the precise selection of the strike prices K_+ and K_-. Failure to choose suitable values can lead to erroneous results. Furthermore, it should be noted that the strategy presented in this report is intended for use with European options, whereas American options were used in our analysis. Therefore, it is essential to exercise caution when applying the Box Spread Strategy to American options, as the early exercise feature of American options may introduce additional complexities and risks.