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Pricing a Call Option using the One-Period Binomial Model

1 Introduction

This report describes the procedure to price a call option for a financial asset using a simple, discrete time model based on the assumption that arbitrage opportunities do not exist: the binomial model. The chosen asset is The Walt Disney Company, which is one of the biggest entertainment companies worldwide, with a current market capitalization of 175.063B $ (according to Yahoo Finance as of March 24, 2023). The model is applied first for a three-month maturity time and then for a six-month one. The structure of the report is as follows: Section 2 summarizes the principal information about The Walt Disney Company, Section 3 presents the one-period Binomial Model, Section 4 describes the methodology applied and presents the results, and Section 5 provides the conclusions.

2 The Walt Disney Company

The Walt Disney Company is a global entertainment company that operates through two segments: Disney Media and Entertainment Distribution, and Disney Parks, Experiences and Products. The company produces and distributes film and television content under various brands and also offers direct-to-consumer streaming services. Additionally, Disney operates theme parks and resorts worldwide and provides various travel experiences. The company licenses its intellectual property to third parties, offers consumer products, and provides post-production services. The company was founded in 1923 by Walt and Roy O. Disney and is based in Burbank, California.
As mentioned in the introduction, the market capitalization of The Walt Disney Company (DIS in the Nasdaq Stock market) amounts to $175.063B, while the estimated enterprise value is $210.17B. The average volume in the previous 3 months has been $11.56M. Other valuable information are collected in Table 1.

Trailing P/E	51.21
Forward P/E	22.37
PEG Ratio (5 yr expected)	0.97
Price/Sales (ttm)	2.02
Price/Book (mrq)	1.77
Enterprise Value/Revenue	2.49
Enterprise Value/EBITDA	17.19

Table 1: Valuation Measures of The Walt Disney Company, provided by Morningstar, Inc.

The company does not provide dividends or splits. 65.08% of the shares of the company are held by institutions (with the number of institutions holding shares being 3895), and the largest shares belong to The Vanguard Group, Inc. (8.01%), Blackrock Inc. (6.60%), State Street Corporation (3.87%), and Morgan Stanley (2.53%). Only 0.06% of shares are held by insiders. The top mutual fund holders are Vanguard Total Stock Market Index Fund (2.98%) and Vanguard 500 Index Fund (2.26%).
From 2005 to 2020 and since November 2022, the CEO and Director of the company has been Robert "Bob" Iger. It was he who decided to buy Pixar in 2006 for $7.4 billion in an all-stock transaction and Marvel Entertainment and its associated assets for $4 billion in 2009. It is interesting to note that just with the box office earnings of Marvel movies, The Walt Disney Company has grossed more than the expense. Another significant purchase ($4 billion) was made in 2012 when Disney acquired Lucasfilm, which made them the owner of the Star Wars multimedia franchise. In 2018, under his direction, 21st Century Fox shareholders approved a deal that allowed Disney to purchase Fox assets. From 2020 to late 2022, the CEO of the company was Bob Chapek until the role was once again assigned to Iger. During the next two years, Iger will try to identify a successor.

3 The One Period Binomial Model [1]

The Binomial Model is a discrete-time model whose simplest configuration (one-period binomial model) describes the behavior of a financial market between time $t = 0$ and $t = T$, where T is called the *maturity time*. The model considers two assets, the stock S and the bond B. The bond is described by a deterministic process where $B_0 = 1$ evolves into $B_T = 1 + R \cdot T$, with R being the spot rate for the period. The initial stock value S_0 can either increase with a probability q to $S_T = S_0 \cdot u$, where $u > 1$, or decrease with a probability $1 - q$ to $S_T = S_0 \cdot d$, where $d < 1$. These two scenarios lead to different payoffs, denoted as f^{up} and f^{down}, respectively.
To derive the price of an option, a replicating portfolio investment strategy is built in the (S, B) market. This strategy involves a long position in Δ shares and a short position in one option. The value of Δ that makes the portfolio riskless can be obtained by matching the two possible portfolio values at $t = T$:

$$\Delta = \frac{f^{up} - f^{down}}{S_0 \cdot u - S_0 \cdot d}. \tag{1}$$

In order to ensure the absence of arbitrage opportunities (*AAO*), this portfolio must earn the risk-free interest rate $R \cdot T$. In particular, it can be demonstrated that the portfolio satisfies the *AAO* if the following condition holds:

$$d < 1 + R \cdot T < u. \tag{2}$$

The above condition is equivalent to stating that $1 + R \cdot T$ can be represented as a convex combination of u and d, weighted by probabilities $p^u = q$ and $p^d = 1 - q$, respectively. These probabilities define a probability measure $\mathbb{Q}$.

[1] the procedure described is a summary of the more detailed theoretical explanation contained in the book from Thomas Bjork *Arbitrage Theory in Continuous Time*, 3rd ed. Oxford University Press, 2009.

Consequently, if we consider:

$$\frac{\mathbb{E}^{\mathbb{Q}}(S_T)}{1+R\cdot T} = \frac{q\cdot u\cdot S_0 + (1-q)\cdot d\cdot S_0}{1+R\cdot T} = S_0\frac{1+R\cdot T}{1+R\cdot T} = S_0, \tag{3}$$

we discover that $\mathbb{Q}$ is a martingale measure. The martingale probabilities are therefore:

$$\begin{cases} p^{up} = q = \frac{1+R\cdot T-d}{u-d} \\ p^{down} = 1 - p^{up} = 1-q \end{cases} \tag{4}$$

Defining $1/B_T$ as the discounting rate, the pricing formula is:

$$p_0 = \frac{\mathbb{E}^{\mathbb{Q}}[f_T]}{B_T} = \frac{f^{up}q + (1-q)f^{down}}{1+R\cdot T}, \tag{5}$$

where f_T indicates the payoff at maturity time. In particular, for a call option:

$$f_T = (S_T - K)^+ \tag{6}$$

where K is the strike price.

4 Methodology and Results

The one-period Binomial Model has been applied to The Walt Disney Company for three-month and six-month maturity times using the following approach. All data were collected from Yahoo Finance[2] on March 22, 2023, and elaborated using Microsoft Excel.
On March 22, 2023, the stock price was $S_0 = 96,23$, and in order to compare the model with real prices, two call options with strike values at the money ($K_j \sim S_0$, $j = 3,6$) and maturity times of June 16, 2023, and October 20, 2023, respectively, were collected:

	June 16, 2023	October 20, 2023
Strike ($)	95	95
Mid Price ($)	7.60	11.72

Table 2: Strike values at the money

In Table 2 also the Mid Prices of the two strikes are reported: they are calculated as the average between the Bid and Ask prices and they will be used as targets for the model.
In order to calculate the two calibrating parameters u and d, the historical daily data of, respectively, the last three and six months have been imported and the daily returns have been calculated as:

$$R_i = \frac{S_i - S_{i-1}}{S_{i-1}} \tag{7}$$

where S_i is the Close value of the i-th day. At this point the daily volatilities of the stock can be calulated considering the standard deviations σ_{daily} of the returns. From these values the yearly

	σ_y
T_3	0.353
T_6	0.340

Table 3: year volatility σ_y

sigmas have been found considering the conventional number of work days in a year (252): $\sigma_y = \sigma_{daily} \cdot \sqrt{252}$.
The calibrating parameters u and d have been calucated according to:

$$u, d = \exp\left(\pm\sigma_y \cdot \sqrt{T}\right) \tag{8}$$

where T indicates the maturity time expressed in relation to a unit of 1 year (therefore $T_3 = 3/12$ and $T_6 = 6/12$). At this point, the Libor rates for the two maturity times have been collected from global-rates.com[3]:

	R (%)	$1/(1+RT)$
T_3	5.018	0.988
T_6	5.007	0.976

Table 4: Libor Rates and discounting factors

The rates in Table 4 allowed us to demonstrate that $d < 1 + R \cdot T < u$, then that we are in a condition of *AAO*:

	d	$1 + R \cdot T$	u
T_3	0.838	1.012	1.193
T_6	0.786	1.025	1.272

Table 5: AAO proof

The binomial model therefore was applied using:

$$q = \frac{1 + R \cdot T - d}{u - d}, \tag{9}$$

in particular the estimated price for the call has been obtained using equation 5 with equation 6, considering that since $u > 1$ and $d < 1$: $f^{up} = (S_T^{up} - K_j)$ and $f^{down} = 0$. The results are summarized in tables 6 and 7, in particular in Table 6 the Δ value has been calculated using equation 1.
In Table 7 the relative errors have been calculated considering as true values the Mid Prices reported in Table 2.

[2]https://finance.yahoo.com/quote/DIS?p=DIS.tsrc=fin-srch

[3]https://www.global-rates.com/en/interest-rates/libor/american-dollar/american-dollar.aspx, consultated the 22nd March 2023

	$S_T^{up}(\$)$	$S_T^{down}(\$)$	$f^{up}(\$)$	$f^{down}(\$)$	q	Δ
T_3	113.36	79.61	18.36	0	0.491	0.544
T_6	120.81	74.70	25.81	0	0.492	0.560

Table 6: Asset values, payoffs and Δ at T_3 and T_6

	p_0^{call} (\$)	err (%)
T_3	8.91	17.20
T_6	12.38	5.62

Table 7: Estimated call pricing and % error for T_3 and T_6

5 Conclusions

The one-period binomial model is a relatively simple approach to pricing call options, yet this report demonstrates that it can still offer valuable insights and reasonably accurate results. The six-month maturity time window produced the most reliable results, implying that the volatility factor plays a significant role in enhancing the model's accuracy. However, it is important to note that the model has its limitations and may not be appropriate for all financial instruments or circumstances. More complex scenarios may require more advanced models, such as the Black-Scholes model, which takes into account continuous time and other factors that the one-period binomial model does not consider. Moreover, in real-world scenarios, the volatility parameter is not constant, and it is often necessary to incorporate additional variables to accurately price options. Despite its limitations, the one-period binomial model can serve as a useful tool for basic option pricing and for understanding the fundamental principles underlying option valuation.