**VECM**

**A vector correction model of the US stock market.**

**Introduction**

We examine the long term-equilibrium relationships between the US stock index and selected macroeconomic variables. Upon testing appropriate vector error-correction model we detected that change of money supply is not integrated of the same order as change in U.S stock market. However, changes in U.S stock market level do form a cointegrating relationship with changes in price levels, long-term interest rates and the U.S dollar index.

**1. US stock market and macroeconomic variables.**

We hypothesize a relationship between the US stock market and macroeconomic variables: long term interest rates, consumer price index and the U.S dollar index. We hypothesize that stock prices are positively related to increasing money supply and negatively related to interest rates. An increase in the rate of interest raises the opportunity cost of holding cash and is likely to lead to a substitution effect between stocks and interest securities. Since the rate of inflation is positively related to money growth (Fama, 1981), an increase in the money supply may lead to an increase in the discount rate and lower stock prices. An increase in expected inflation rate, under general circumstances, is likely to lead to economic tightening policies that would have a negative effect upon stock prices. The rise in the rate of inflation, additionally, increase the nominal risk-free rate. Hence, the hypothesized relationship between stock prices and inflation is negative or insignificant, because the public is aware of the credibility of the MAS in maintaining price stability. And finally, the U.S index is likely to a negative relation with stock prices through its connection with interest rates. As interest rates rise, they tend to strengthen the US dollar.

**2. Methodology and data selection**

Using Johansen’s vector error-correction model, this empirical analysis examines the dynamic relations between macroeconomics variables and the U.S stock market. Although Engle and Granger’s (1987) may also be used in a multivariate context, the VECM yields more efficient estimators of cointegrating vectors. This is because the VECM is a full information maximum likelihood estimation model, which allows for testing for cointegration in a whole system of equation in one step and without requiring a specific variable to be normalized. This allows us to avoid carrying over the errors from the first step into the second, as would be the case if Engle-Granger’s methodology is used. It also has advantage of no requiring a priori assumption of endogenity or exogenity of the variables.

**The VECM formula**

Where is the vector autoregressive (VAR) component in first difference, and the error-correction components in level. is variables and is integrated of order 1. is a vector of constants. is a lag structure, while is a vector of white noise terms. is a matrix that represent short-term adjustment among variables across equation at the th lag. is a matrix of speed of adjustment parameters representing the speed of error correction mechanism. A larger suggest a faster convergence toward long-run equilibrium in case of short-run equilibrium. In estimating the VECM, we first check for stationarity and unit roots through performing Dickey-Fuller (ADF) test on the variables in level and first differences. Only variables integrated of the same order may be cointegrated, and the unit test root test will help us determine which variables are integrated of order one, or .

The choice of lag lengths we will be decided using Akaike information criterion (AIC). The general formula for AIC is where k is the number of parameters estimated in the model, and L is the maximum likelihood of the model. The higher the likelihood, the better the model explains the observed data. The model is estimated by regressing the matrix against the lagged differences of and and determine the rank of . The eigenvectors in are estimated from the canonical correlation of the set of residuals form the regression equations. To determine the rank of , which will give the order of cointegration, r, we calculate the characteristic root or eigenvalues of , . Furthermore, we test for r using the and test statistics, where and . The choice of maximum cointegrating relationship will be based on the tests. The test is used to test specific alternative hypotheses. We will reject models where has a full rank since in such situation is stationary and has no unit root, and so there would be no error correction.

Having determined the order of cointegration, we select and analyse the relevant cointegrated vector and speed adjustment coefficient. Since we consider the natural logarithm of the price index, LSP500, to be the dependent variable, we will normalize with respect to the coefficient for LSP500.

The macroeconomic variables selected for this study are prensented in Table 1. The monthly time-series were taken from yahoo finance and the Federal Reserve Bank of ST Louis: LSP500t, Rates10Ratesyrt, LCpit, Usdt.

Table 1 Definitions of variables and time-series transformations

|  |  |
| --- | --- |
| Variables | Definitions of Variables |
| LSP500t  Rates10yrt  LCpit  Usdt | Natural logarithm of month-start of S&P 500.  Monthly CBOE Interest Rate 10 Year T No.  Natural logarithm of month-start of the Consumer Price Index.  Monthly US Dollar Index of month-start. |
| Transformation | Definitions of Transformations |
| ∆SP500t = LSP500t - LSP500t-1  ∆Rates10yrt = Rates10yrt - Rates10yrt-1  ∆SCpit = LCpi­t - LCpi­t-1  ∆Usdt = LUsdt – Lusdt-1­ | Monthly return on the S&P 500.  Monthly change in exchange rate.  Monthly change on Consumer Price Index U.S.  Monthly change on the Us Dollar Index. |

To arrive at the stationary variables needed in the ECM, S&P 500 prices and Consumer Price Index are converted into natural logarithms, and their first difference are taken. The data are seasonally adjusted, month-start data for the period January 1990 to October 2024.

**3. Results**

3.1 Descriptive

3.1 Unit root test

Cointegration requires the variables to be integrated of the same order. We test the variables for unit roots to verify their stationarity of order zero. We do this through the augmented Dickey-Fuller test. The results are reported in Table 2. The hypothesis for this test are, H0, the series has a unit root, H1, we reject H0.

We found that all variables are integrated of order one and have no deterministic trend. All variables in first differences are stationary at 5% significance level. ADF statistics are smaller than critical values at 5% (-2.868) and p-value are smaller than 0.05, meaning we can reject the null hypothesis of non-stationarity.

Table 2 Unit root tests (Augmented Dickey Fuller)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Variables | Order of Integration | ADF test statistic | Critical value 5% | p-value | Hypothesis |
| LSP500t  ∆SP500t  Rates10yrt  ∆Rates10yrt  LCpit  ∆SCpit  Usdt  ∆Usdt | I(0)  I(1)  I(0)  I(1)  I(0)  I(1)  I(0)  I(1) | -0.529  -20.22  -2.505  -15.036  0.276  -3.906  -2.131  -18.478 | -2.868  -2.868  -2.868  -2.868  -2.868  -2.868  -2.868  -2.868 | 0.886  0.000  0.114  0.000  0.976  0.002  0.232  0.000 | Non-stationary  Stationary  Non-stationary  Stationary  Non-Stationary  Stationary  Non-Stationary  Stationary |

3.2 Selected lags based on Akaike information criterion

The model with the lowest AIC was the one for k = 1, follow by the one for k = 2. We report results for our model selection. We report the results for k=1 as it best meets our model-selection criteria. Table 4 reports the results for AIC, BIC, FPE and HQIC information criterion.

Table 4 Results for information criterion

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| k | AIC | BIC | FPE | HQIC |
| 0  1  2  3  4  5  6  7  8  9  10 | -37.92  -38.22\*  -38.21  -38.19  -38.17  -38.13  -38.12  -38.11  -38.06  -38.03  -38.02 | -37.88  38.02\*  -37.85  -37.68  -37.49  -37.29  -37.12  -36.95  -36.75  -36.56  -36.39 | 3.409e-17  2.520e-17\*  2.551e-17  2.588e-17  2.646e-17  2.771e-17  2.796e-17  2.821e-17  2.950e-17  3.053e-17  3.075e-17 | -37.90  -38.14\*  -38.07  -37.99  -37.90  -37.79  -37.72  -37.65  -37.54  -37.45  -37.38 |

3.3 Johansen’s test

The hypothesis for this test are, H0, there are no cointegrating relationships among the variables, H1, Reject H0.

By performing the Johansen’s test, we reported the results of the test for k = 1 in table 5. Tests indicate that the first trace statistic is 573.27, which is larger than critical value at the 5% significance level. We reject the null hypothesis of no cointegrating relationship r = 0. This suggest that there is at least one cointegrated vector between variables S&P 500, Rates10yr, CPI and USD. As far as 369.29 larger than critical value 24.28 (r ≤ 1), 197.27 larger than 12.32 (r ≤ 2), and 57.14 larger than 4.14 (r ≤ 3). We reject the null hypothesis of at most three cointegrating vectors.

Table 5 Result and critical values for the test.

|  |  |  |
| --- | --- | --- |
| H0 | λtrace | Critical value (5%) |
| r = 0  r ≤ 1  r ≤ 2  r ≤ 3 | 573.269  369.294  197.270  57.1450 | 40.174  24.276  12.321  4.129 |

We conclude that there are three cointegrating vector r = 3.

**4. VECM model**

4.1 Long run

Table 6 Vector error-correction model (VECM) long run.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variables | coef | std err | z | P>|z| |
| beta.1  beta.2  beta.3  beta.4 | 1.0000  -16.7883  -2.7579  -0.7625 | 0  1.573  0.933  0.199 | 0.000  -10.671  -2.955  -3.827 | 0.000  0.000  0.003  0.000 |

The table 6 represent the long-term equilibrium relationship.

Normalizing with respect to the coefficient for LSP500, the cointegrating vector for k=1 is given by = (1, -16.788, -2.757, -0.762). This yields the following cointegration relationship:

Through exanimating the , we note that when k=3, coefficients for, Rates10yr, LCpi, Usdt are negative. We see that coefficients are significant thanks to the p-value.

Rates10yr: p-value = 0.000 < 0.05%

Cpi: p-value = 0.003 < 0.05%

Usd: p-value = 0.000 < 0.05%

The coefficient for S&P500 beta.1 is 1.00. This mean that S&P 500 is the anchor variable in this cointegrating relation, and other variables are interpreted relative to S&P 500.

The S&P 500 relationship with first lag of long-term interest rate is negative. For every 1% increase in 10-year interest rates CBOE, there is an expected 16.79% decrease in S&P 500 in the long run, all else being equal. The long-term interest rate may serve as a proxy for the nominal risk-free component of the discount rate in stock valuation model.

The association between US Dollar index has also a negative relationship S&P 500 prices is also negative. For every 1% increase in USD, there is an expected 0.76% decrease in S&P 500 in the long run. An appreciation of the currency would make exports less competitive, thus lowering earnings. Given this, it is likely that the appreciation of the US Dollar is receive as unfavourable news by the US stock market and hence generate negative returns.

The negative relationship between price level and stock market levels is consistent. For every 1% increase in CPI, there is an expected 2.76% decrease in S&P 500 in the long run. The relationship between stock market and inflation has been the focus of many studies. A negative relationship between inflation and real activity because the nominal quantity of money did not vary sufficiently with real activity. This is a plausible explanation in S&P 500’s case.

The important factor affecting S&P 500 are interest rates and consumer price index. The impact of exchange rate seems to be less substantial.

4.2 Short-run & speed of adjustment

Table 7 show how the stock market adjusts in response to its own past behavior and macroeconomic variables.

Table 7 Vector error-correction model (VECM) short-run.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variables | coef | std err | z | P>|z| |
| L1.SP500  L1.Rates10yr  L1.Cpi  L1.Usd | -0.2841  -3.5803  -2.2224  -0.2454 | 0.052  0.821  0.891  0.095 | -5.431  -4.363  -2.493  -2.593 | 0.000  0.000  0.013  0.010 |

L1.Rates10yr: p-value = 0.000 < 0.05%

L1.Cpi: p-value = 0.013 < 0.05%

L1.Usd: p-value = 0.010 < 0.05%

A 1% increase in S&P 500 in the previous period leads to a 0.284% decrease in the current S&P 500. This coefficient suggest that the previous period’s value of S&P 500 have a negative short-term impact on the current S&P 500 price.

The S&P 500 relationship with first lag of long-term interest rate is negative. A 1% increase in 10-year interest rates CBOE leads to a 3.58% decrease in SP500 in the current period.

The association between US Dollar index has also a negative relationship S&P 500 prices is also negative. A 1% increase in the USD index in the previous period leads to a 0.245% decrease in S&P 500 in the current period.

The negative relationship between price level and stock market levels is consistent. A 1% increase in CPI in the previous period results in a 2.22% decrease in S&P 500 in the current period.

The important factor affecting S&P 500 are interest rates and consumer price index. The impact of exchange rate seems to be less substantial.

The short-run adjustment is captured by the coefficients alpha and the coefficient of the lagged difference of the variable.

The speed adjustment is significant for all variables, showing that the market is responsive to these. The speed of adjustment reflects how quickly variables adjust back toward their long run equilibrium relationship after a short-run deviation. The speed of adjustment coefficients are crucial for understanding how each variable react to imbalances in the cointegration relationship.

The hypothesis for this test are, H0, the speed of adjustment coefficient (α) is zero, meaning the system does not adjust towards the long-run equilibrium, H1, Reject H0.

In the table, these coefficients are present in each equation as L1.ec1. These coefficients show the speed of convergence towards long-run equilibrium. In the short run, the variables can deviate from long-run equilibrium or cointegration relationship. The adjustment coefficient depicts how the deviation will be corrected.

In our equation, the adjustment coefficient must be negative and significant. This will ensure convergence toward long-run equilibrium. Table 8 represent the full set of adjustment coefficient in the VECM for the same value of k (k = 1). The adjustment coefficients are shown in the vector “gamma”.

Table 8 Adjustment speed

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| variables | γ (ec1) | std err | z | P>|z| |
| ∆SP500  ∆Rates10yr  ∆Cpi  ∆Usd | -0.3821  0.0331  0.0093  0.1756 | 0.052  0.003  0.003  0.04 | -7.357  11.410  3.316  7.272 | 0.000  0.000  0.001  0.010 |

The deviation from the estimated equilibrium (), lagged by one month, can then be used to estimate a short run model in difference over rolling samples.

Here is the error correction term, which is the extend of the deviation form long-run equilibrium at time t-1. The adjustment term -0.382 indicates how much of this deviation will be corrected in the next period. 38.21% if the deviation from equilibrium is corrected each period.

**5. Impulse response function**

The study uses impulse response function as an additional check of the cointegration test’s finding. Followed by Cholesktype of contemporaneous identifying restrictions are employed to draw a meaningful interpretation. Impulse response Function (IRFs) are used to study the effect of shocks or impulse in VECM system. It traces out one unit or one standard deviation shock to an endogenous variable and its effects on all endogenous variables in a VECM, keeping all other variables and shocks constant. They are useful in understanding the dynamic behavior of variables in a system, and predicting the effect of impulse and policy analysis.

The IRF shows the dynamic impact of a shock to one variable on itself and the other variables on over multiple periods, while VECM represent the impact of past value of each variable on the current value of the dependent variable in the next time. These coefficients show the immediate adjustment in one period due to past values, not cumulative impact over many period like the IRF.

The IRF can show cumulative effect that evolves overtime and might reverse the initial direction of the short-term response seen in the VECM. The S&P 500 might adjust negatively in the next month following an interest rate hike, but after several month, feedback from other variables might mitigate or reverse this effect, leading to the overall trend seen in the IRF.

Over the period from January 2010 to October 2024 we found results of figure 1.

Figure 1. Impulse responses (10 periods).

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Regarding interest rate the initial impact is negative and significant meaning that a shock in has a negative effect on S&P500. Over the next few periods, the negative impact decrease and stabilizes but remains negative. A shock in inflation lead to an initial large positive response in the S&P 500. After the initial spike, the positive effect declines but stays positive. The US Dollar shock initially has a significant negative impact on the S&P 500, indicating that a stroger USD initially depresses stock prices. Over time, the impact of the USD shock remains negative, stabilizing around a slightly negative value. This indicating that after the initial drop, the S&P 500 stay negatively affected by a stronger dollar for an extend period.

In VAR, the effects of a shocks or impulse always converge to zero after some time period. But this is not the case for VECM models. The IRFs or effects of the shocks may or may not converge to zero in a VECM model.

6. Forcast

Figure 2.

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