MATH 353-753 ORD & PRTL DIFF EQUATIONS-SPRING 2023 EXAM 1

Instructor: _____ *Date* _____

*Student Name:*_____

Closed book, notes, internet, cell phones.
Allowed: Class formula sheet

Please sign "no assistance" pledge_____

ALL ANSWERS SHOULD BE CIRCLED

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
TOTAL	

1. READ CAREFULLY: Check the boxes in the table, in which the ODE has the corresponding property. Points will be taken out for wrong checks. Leave the other boxes blank.

   READ CAREFULLY: It is possible that more than one boxes should be checked for the same ODE. It is also possible that no boxes at all should be checked. You may have to do simple algebra to bring an ODE to one of the standard solavable forms. DO NOT SOLVE THE ODEs.

   REMEMBER: A separable equation is necessarily also exact.

   Notation: y' is the derivative of y with respect to x.

ODE	lin. [H]	lin. [NH]	separable	exact	autonomous
$y' = xy + x$		✓		✓	
$y' + y\cos y = 0$					
$(x^2 + y^2)y' = x$					
$xy' = 5x^2 - y$					
$y' + 2x^2y = x^2$					
$(2xy + y^2)dx + (2xy + x^2)dy = 0$					

2. Find the solution of the ODE

$$y' = -2xy + 2x,$$

in two ways.

(a) Using an integrating factor. Circle your answer.

(b) Using the general solution of the [H] equation and an easy to find particular solution of the [NH] equation. Circle your answer.

3. (a) Find the equilibrium solutions of the ODE

$$y' = y^2 - 4y + 3,$$

and determine whether they are stable or unstable.

Circle your answer.

(b) Determine the limit of the solution $y(x)$ as $x \to +\infty$, given the initial value $y(0) = 2$.

Circle your answer.

(c) Consider the solution of the ODE with initial value $y(0) = 4$. Name two different scenarios for this solution as x grows from the value zero.

1)

2)

4. You are given the ODE

$$y'' + 2py' + y = 0, \quad y = y(x).$$

a)Assuming that $0 < p < 1$, calculate the distance between two successive points of the x-axis, at which a solution $y(x)$ equals zero. Circle your answer

b) Assuming that $0 < p < 1$, make a rough graph of a nonzero solution of the ODE. Circle your answer

c) Calculate the smallest value of $p \in \mathbb{R}$ for which the solutions of the ODE exhibit no oscillations.

5. Use the Laplace transform to solve the initial value problem

$$y' + y = 1 - u_1(t); \qquad y(0) = 0.$$

Circle your answer.

6. Use the method of undetermined coefficients to find the general solution of the ODE

$$y'' + y = \sin 2t + e^{-t}$$

Circle your answer.

Hint: Linearity allows you to solve the problem separately for each of the terms on the right.