ORD & PRTL DIFF EQUATIONS-MATH 353-753 FALL 2024-EXAM 1

Name _____

Section 02
Wednesday, October 9, 2024, 3:05-4:20 PM, Physics 235.

Calculators, cellphones, computers not allowed. Closed book and notes.
Allowed: Formula sheet given to you plus one standard page sheet written by you on both sides
PLACE ALL ANSWERS IN BOXES

Student "no assistance" pledge. Please sign:

MATH 353, ORD & PRTL DIFF EQUATIONS-EXAM 1

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	

1. Identify the type(s) of each ODE by checking the appropriate boxes. Leave the other boxes blank (points taken off for incorrect checks). Remember: "separable" implies "exact". y' stands for dy/dt.

	linear homog.	linear nonhom.	separable	exact
$ty' = (y + \sin y)(t^2 + 1)$				
$y' = -\frac{2t+y\cos t+\sin y}{\sin t+t\cos y}$				
$y' = \frac{f(t)}{g(y)+2y}$				
$y' + 5t^2 + 5y\cos t = \sin^2 t$				
$y'^2 = y^2 - t^2$				

2. (a) What does it mean to solve the eigenvalue problem of a given square matrix A?

(b) Write down a third order linear nonhomogeneous ODE with coefficients that are not all constants. Do not attempt to solve it.

(c) Write down a third order linear homogeneous ODE with constant coefficients. Do not solve it, but explain the solution process in a few words.

(d) Write down the first order ODE that has general solution $t^2+3t^3y+y^3 = \text{constant}$.

(e) Plot the solutions of the ODE $xdx + ydy = 0$ in the x, y plane.

3. a) Find a basis for the vector space of all real-valued solutions of the ODE

$$y'' - 2y' + 5y = 0$$

b) Solve the the ODE

$$y' + 4ty - 5t = 0$$

4. Consider the ODE

$$y'' + 4y = \cos at \quad a > 0$$

Provide formulae for the solution that would cover ALL the values of a

5. Consider the initial value problem

$$\frac{dy}{dt} = y(y^2 - 3y + 2), \qquad y(0) = 1.5$$

a) Find all equilibrium solutions and classify them as stable or unstable.

b) The solution $y(t)$ approaches what value as $t \to +\infty$? Justify your answer.

6. Use the Laplace transform to solve the initial value problem,

$$y'' + 4y = \delta(t-1), \qquad y(0) = 0, \quad y'(0) = 1.$$