MATH 353-ORD & PRTL DIFF EQUATIONS FALL 2019-EXAM 1

Name ______________________

Section ______________________

Tuesday, October 1, 2016.

Closed books and notes. Use of calculators and cellphones is not allowed.
Formula sheet in sakai general site allowed.
You may use the back of the pages.

Please sign "no assistance" pledge______________________

ALL ANSWERS SHOULD BE CIRCLED

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
TOTAL	

1. For each ODE, place a check in one or more boxes, where the ODE has the indicated property. Leave the other boxes blank (points taken off for incorrect checks). Remember that a separable ODE is necessarily exact. Also a homogeneous ODE (not the [NH] type) can be made separable. DO NOT SOLVE THE ODEs.

   Notation: y' is the derivative of y with respect to x.

ODE	linear homog.	linear nonhom.	separable	exact
$(x+y)y' = x - y$				
$y' = xy^2 + 1$				
$xy' - (\sin x)y - x\cos x = 0$				
$(x^2 + \ln x)y' = x^2(y^2+1)$				
$y' + \frac{y\cos x - xy\sin x - 2x\cos y}{x\cos x + x^2\sin y} = 0$				
$(\sin x)y' = y\cos x$				

2. True or false? In each of the following statements, circle the appropriate word (points taken off for wrong guesses):

   (a) Consider the ODE
   $$y' = (y-1)(y-2)(y-3)(y-5)$$

      i. The ODE has exactly two stable equilibrium solutions. **True** **False**

      ii. If $y(0) = 6$, then $\lim_{x\to+\infty} y(x) = +\infty$. **True** **False**

      iii. The limit $\lim_{x\to+\infty} y(x) = -\infty$ does not occur for any initial data $y(0)$. **True** **False**

   (b) The following operation is linear.

      i. The Lapalce transform. **True** **False**

      ii. Taking the square root of a positive function. **True** **False**

      iii. Taking the second derivative of a function **True** **False**

   (c) The initial value problem $y' = y^{\frac{1}{3}}, \quad y(0) = 0$ has a unnique solution. **True** **False**

   (d) If one solution of a second order linear homogeneous ODE is known, a formula for a second,linearly independent solution of the ODE can be derived. **True** **False**

   (e) If the complex power series $\sum_{n=0}^{\infty} a_n z^n$ converges when $z = 3 + 4i$, where $i = \sqrt{-1}$, it necessarily converges when $z = 5i$. **True** **False**

   (f) The set of solutions of a linear homogeneous third order ODE is a three dimensional vector space **True** **False**

   (g) If the coefficient functions of a linear ODE and the forcing term are bounded, then all its solutions are also bounded. **True** **False**

   (h) The general solution of a linear nonhomogeneous ODE equals the sum of the general solution of the homogeneous ODE plus a particular solution. **True** **False**

3. (a) Use the methodof undetermined coefficients to derive the solution of the ODE

$$y'' + 4y = \sin(2t)$$

with zero initial conditions, $y(0) = 0, \quad y'(0) = 0$. Circle your answer.

(b) Without using series, find a basis of the space of solutions of the ODE

$$x^2 y'' - 2y = 0.$$

HINT. Look for solutions of the form $y = x^n$, where n is an integer.
Circle your answer.

4. Find the general solution of the ODE

$$(2x + 3x^2 + 6xy)dx + (3x^2 + 1)dy = 0$$

Circle your answer.

5. (a) Use the Laplace transform to solve the following initial value problem.

$$\begin{cases} y'' + 4y = \delta(t-1) \\ y(0) = 0, \quad y'(0) = 0. \end{cases}$$

where $\delta(t)$ is the Dirac delta function. Graph your result. Circle your answer.

(b) Solve the same problem without using the Laplace transform.
HINT. y'' displays a delta function at $t = 1$. What does this mean for y'?

6. Consider the ODE

$$y'' + y' + xy = 0.$$

We are seeking a power series solution centered at $x = 0$,

$$y = \sum_{-\infty}^{\infty} a_n x^n, \qquad a_n = 0 \text{ when } n < 0.$$

(a) Identify the free variables and find the recurrence relation. Circle your answer.

(b) Determine the first four nonzero coefficients of the solution that satisfies $y(0) = 1$ and $y'(0) = 2$. Circle your answer.