ORD & PRTL DIFF EQUATIONS-MATH 753-FALL 2020-EXAM 1

Name _____

Section _____

Thursday, September 24, 12:00-1:15 pm.

Please sign the "no assistance" pledge:

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	

NOTICE: y' stands for dy/dx.

1. Check all appropriate boxes. Leave the other boxes blank (points taken off for incorrect checks). DO NOT SOLVE THE ODEs.

   Hint: An ODE that is separable is automatically exact. An ODE that is homogeneous is automatically separable.

ODE	linear [H]	linear [NH]	separable	exact	homogeneous
$y' \sin x + x \cos x = -y \cos x$					
$y' = \frac{3x^2y^2+y^2\sin x}{-2x^3y+2y\cos x}$					
$y' = \frac{x^2-y^2}{x^2+y^2}$					
$y' = \frac{(x^2+1)\cos y}{y^2+1}$					
$y' + y^2 \cos x = \sin x$					

2. Find the general solution of the following differential equations. If you cannot solve for x or y it is OK to leave your answer in implicit form. CIRCLE YOUR ANSWERS.

(a) $xy' = y + 2x^3$, $\qquad x > 0$.

(b) $y''' + y'' - 2y' = 1$

3. (a) What does it mean for an operator $\mathbb{L}$ to be linear?

(b) Prove that the second order differential operator $\mathbb{L} = \frac{d^2}{dx^2} + q(x)$, where $q(x)$ is a function, is linear.

4. (a) Consider the initial value problem

$$\frac{dy}{dx} = y(y^2 - 3y + 2), \quad y(0) = 1.5$$

The solution $y(x)$ approaches what value as $x \to +\infty$? Justify your answer.

For what initial values of y does the solution of the ODE (not necessarily the IVP) display a vertical asymptote? CIRCLE YOUR ANSWER

(b) The solution of a first order ODE is given by

$$x^3 y + y^2 x^2 - 4x^5 = c$$

where c is an arbitrary constant. What is the ODE? CIRCLE YOUR ANSWER.

5. (a) Find the value of b for which the ODE

$$\frac{dy}{dx} = -\frac{xy^2 + bx^2y}{x^3 + x^2y + y^3}$$

is exact. CIRCLE YOUR ANSWER.

(b) Calculate the general solution of the ODE for this value of b. CIRCLE YOUR ANSWER.

6. Find the general solution of the ODE

$$y'' - 4y = e^{-x}$$

CIRCLE YOUR ANSWER.