MATH 353.01-ORD & PRTL DIFF EQUATIONS-Fall 2021-EXAM 1

Name _____

Section _____

Thursday, September 23, 2021.

Open book and notes. Please sign "no assistance" pledge_____

ALL ANSWERS SHOULD BE CIRCLED

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
TOTAL	

1. Check the boxes in the table, in which the ODE has the corresponding property. It is possible that more than one boxes have to be checked, or no boxes at all have to be checked for one ODE. Points will be taken out for wrong checks. You may have to do simple algebra to bring an ODE to one of the standard solavable forms. DO NOT SOLVE THE ODEs.

   Notation: y' is the derivative of y with respect to x.

ODE	lin. [H]	lin. [NH]	separable	exact	homogeneous	autonomous
$y' = xy + y + x + 1$						
$(x^2 + 1)y' = \cos x - y$						
$(x^2 + 1)y' + xy^2 + xy = 0$						
$(x + y)y' = x - y$						
$y' + \frac{2xy^2+y+1}{2x^2y+x+1} = 0$						
$y' - 2y^2 - 3 = 0$						

2. (a) Find the general solution of the ODE

$$xy' + y = x^2.$$

Circle your answer.

(b) Find the general solution of the ODE

$$(2x + y + 1)dx + (2y + x)dy = 0,$$

Circle your answer.

3. Use the method of undetermined coefficients to find a particular solution of the ODE

$$2y'' - y' - y = 6e^x, \quad y(0) = 1.$$

Circle your answer.

4. Given the ODE

$$\frac{dy}{dt} = -(y-1)(y-3)(y-5),$$

(a) Determine the equilibrium solutions and characterize their stability. Beware of the minus sine up front.

Circle your answer.

(b) Consider the initial value problem of the ODE with initial condition $y(0) = 4$. What does the solution $y(t)$ do, as $t \to +\infty$. What does it do, as $t \to -\infty$.

Circle your answer.

5. (a) Solve the initial value problem

$$y'' + a^2 y = \cos x, \qquad a > 0; \qquad y(0) = 0, \quad y'(0) = 0.$$

Circle your answer.

(b) Draw a rough graph of the maximum of the solution $|y(x)|$ over x, versus the positive values of a.

6. In each of the following true-false statements, circle true or false (points taken off for wrong guess). On the series questions, "explain" basically means stating an appropriate convergence test (points taken off for wrong guess).

   (a) A first order ODE that is exact is necessarily also separable. **True** **False**

   (b) The set of solutions of the ODE $y''' + 5xy'' - x^2y' + y = 0$ is a vector space of dimension 3. **True** **False**

   (c) All solutions $y(x)$ of the ODE $y'' + 4y = 3\sin 2x + 4$ are bounded, *i.e.*, there is a constant M, such that $|y(x)| < M$ for all x. **True** **False**

   (d) The general solution of the ODE $y' = ay, \quad a \neq 0$ is:

   (e) The general solution of the ODE $y'' + a^2y = 0, \quad a \neq 0$ is:

   (f) The series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$ converges. **True** **False**
   Explain:

   (g) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$ converges. **True** **False**
   Explain:

   (h) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$ converges. **True** **False**
   Explain:

   (i) Express the decimal number 0.77777777..... as a series:
   Does the series converge?
   Explain