MATH 353.02-ORD & PRTL DIFF EQUATIONS-Fall 2022-EXAM 1

Name ______________________

Section ______________________

Monday, October 3, 2022.

Closed book, notes, internet, cell phones.
Allowed: Class formula sheet
Please sign "no assistance" pledge______________________

<u>ALL ANSWERS SHOULD BE CIRCLED</u>

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
TOTAL	

1. READ CAREFULLY: Check the boxes in the table, in which the ODE has the corresponding property. Points will be taken out for wrong checks.

   READ CAREFULLY: It is possible that more than one boxes should be checked for the same ODE. It is also possible that no boxes at all should be checked. You may have to do simple algebra to bring an ODE to one of the standard solavable forms. DO NOT SOLVE THE ODEs.

   Notation: y' is the derivative of y with respect to x.

ODE	lin. [H]	lin. [NH]	separable	exact	homogeneous	autonomous
$y' = xy + y + x$						
$y' + 3y^2 - 5 = 0$						
$(x^2+2)y' + xy^3 + xy = 0$						
$(x+2y)y' = 5x - y$						
$y' + \frac{2xy^2+y}{2x^2y+x} = 0$						
$(x^2+1)y' = 3x + \sin y$						

2. (a) Solve the initial value problem of the ODE

$$y' = 2y^2 + xy^2, \quad y(0) = -1$$

and determine the subset of the x axis over which the solution exists as a continuous function. Circle your answers.

(b) Find the general solution of the ODE

$$(3y + 1)dx + (3x + 6y)dy = 0,$$

Circle your answer.

3. Use the method of undetermined coefficients to find the solution of the ODE initial value problem (only one initial condition is given).

$$y'' + y = \cos x, \quad y(0) = 0.$$

Circle your answer. Make a rough graph of a solution. The x positions at which $y(x) = 0$ should be placed correctly in the graph.

4. Given the ODE

$$\frac{dy}{dt} = \frac{(y-1)(y-3)}{y^2+1},$$

(a) Determine the equilibrium solutions and characterize their stability. Circle your answer.

(b) Consider the initial value problem of the above ODE with initial condition $y(0) = 2$. What does the solution $y(t)$ do, as $t \to +\infty$. What does it do, as $t \to -\infty$.

Circle your answer.

(c) Given $y(0) = 4$, what are the limits of y and $\frac{dy}{dt}$ as t tends to $+\infty$?

Circle your answers.

5. a) Given the ODE (known as the Airy equation)

$$y'' - xy = 0$$

calculate the recurrence relation for the coefficients of power series solutions of the ODE centered at $x = 0$.

b) Calculate the radius of convergence of the series $\sum_{n=0}^{\infty} \dfrac{x^n}{n!}$.

6. Use the Laplace transform to solve the initial value problem

$$y' + y = 1 - u_1(x); \qquad y(0) = 0.$$

7. EXTRA CREDIT PROBLEM

The second order differential operator $-\frac{d^2}{dx^2}$ is applied to twice differentiable functions $y(x)$ that are defined on the interval $0 \leq x \leq \pi$ and are equal to zero when $x = 0$ and $x = \pi$.

The eigenvalue problem for the operator is

$$-\frac{d^2}{dx^2}y = \lambda y \quad 0 \leq x \leq \pi; \qquad y(0) = 0, \quad y(\pi) = 0.$$

There are infinitely many eigenvalues λ_1, λ_2, λ_3, $\cdots$ and corresponding eigenfunctions $y_1(x)$, $y_2(x)$, $y_3(x)$, $\cdots$ Calculate the eigenfunctions and eigenvalues. Alternatively, it is acceptable to look at the eigenvalue problem carefully and figure out the answers with very little calculation.

Answer: (fill in)

$y_n(x) =$ $\qquad\qquad$ $\lambda_n =$ $\qquad\qquad$ $n = 1, 2, 3, \cdots$