Lab 1. Speed of Light

The speed of light, denoted as c, is a fundamental quantity in electrodynamics following from Maxwell's Equations. From Einstein's relativistic considerations it has been found to be the "speed limit" of the universe: Only electromagnetic waves (light included) and gravitational waves move at this speed. In this lab, you will utilize basic optical principles to measure c. Through this lab you will gain experience with different optical components, a laser, mirrors, and lenses; the thin lens equation; image analysis tools; and error/uncertainty analysis.

Background

The speed of light in a vacuum was first measured on earth by Hippolyte Fizeau in the mid Eighteenth Century, though it was measured astronomically by Romer almost two-hundred years prior. Previous terrestrial experiments, for example by Galileo, were unable to measure c because of its magnitude – the time delay measured even from large distances was not resolvable. Fizeau was able to circumvent the problem of requiring linear distances to resolve a time delay by using a system of mirrors and a rotating gear that only allowed light to pass either way periodically. Shortly after, Foucault created an experiment that used a rotating mirror to measure the speed of light with greater precision than Fizeau. He also showed that the speed of light decreases in water compared with air!

Experimental setup

For this experiment we borrow the main idea and technique of Foucault's experiment. Figure 1 shows the simplified diagram of the experiment. A beam from a laser is directed to the center of a rotating mirror that spins with a constant frequency f. For now, we will consider the laser beam to be an ideal ray. The divergent nature of the laser beam will be addressed later. Also, we will call the laser beam/ray that propagates from laser the forward beam/ray. While the rotating mirror is spinning, it reflects the forward ray all around, and at some orientation of the rotating mirror the ray is directed toward the center of the return mirror. If the return mirror is aligned perpendicularly to the forward ray, then it reflects the ray back to the center of the rotating mirror. This reflected ray/beam we will call the return ray/beam. While the ray is traveling from the rotating mirror to the return mirror and back, the rotating mirror changes its orientation by a small angle θ. As result, the return beam is reflected by the rotating mirror under some angle α with respect to the forward ray path, and it misses the laser aperture by some small distance x. This distance x depends on the geometry of the experiment, frequency of the rotating mirror f and the speed of light c.

Derive the dependence of angle α between the return and forward rays on angle θ that the rotating mirror rotates while the ray travels from the rotating mirror to the return mirror and back. Create a diagram to illustrate your derivation.

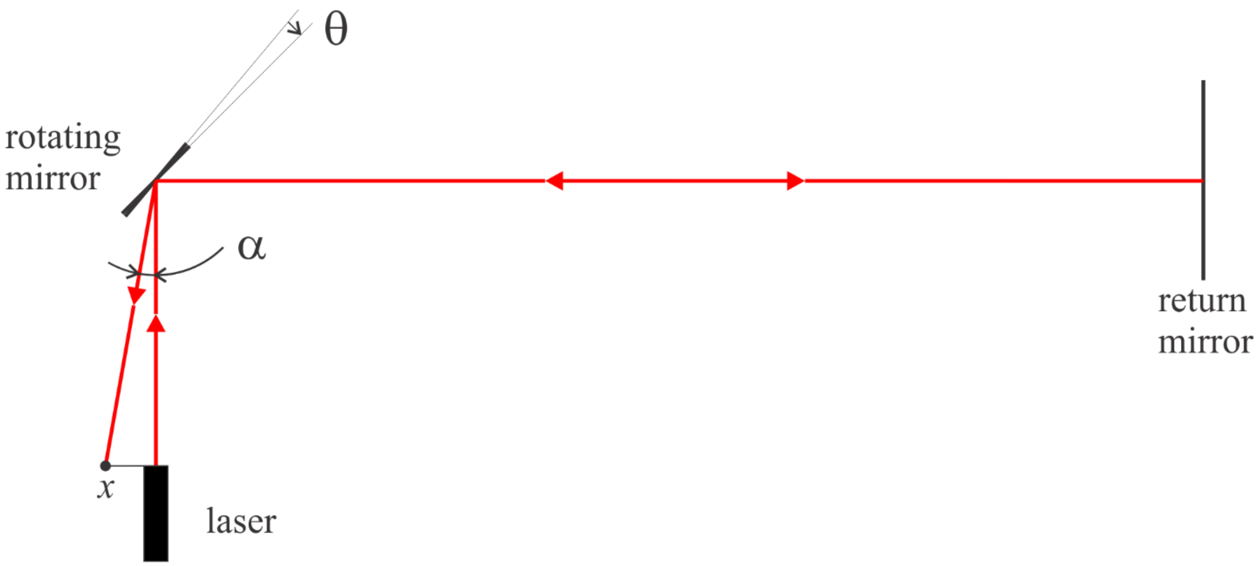


Figure 1. Simplified diagram of the experiment.

Derive the dependence of distance x on the distance from the laser to the rotating mirror, the distance from the rotating mirror to the return mirror, the frequency f of the rotating mirror, and the speed of light c.

This simplified version of the experiment works only for the single orientation of the rotating mirror, the orientation when the forward ray is reflected exactly to the center of the return mirror. For all other orientations the return ray will not come back to the rotating mirror through the same path, and as result it will not come back to the laser aperture plane with the same displacement x, see Figure 2.

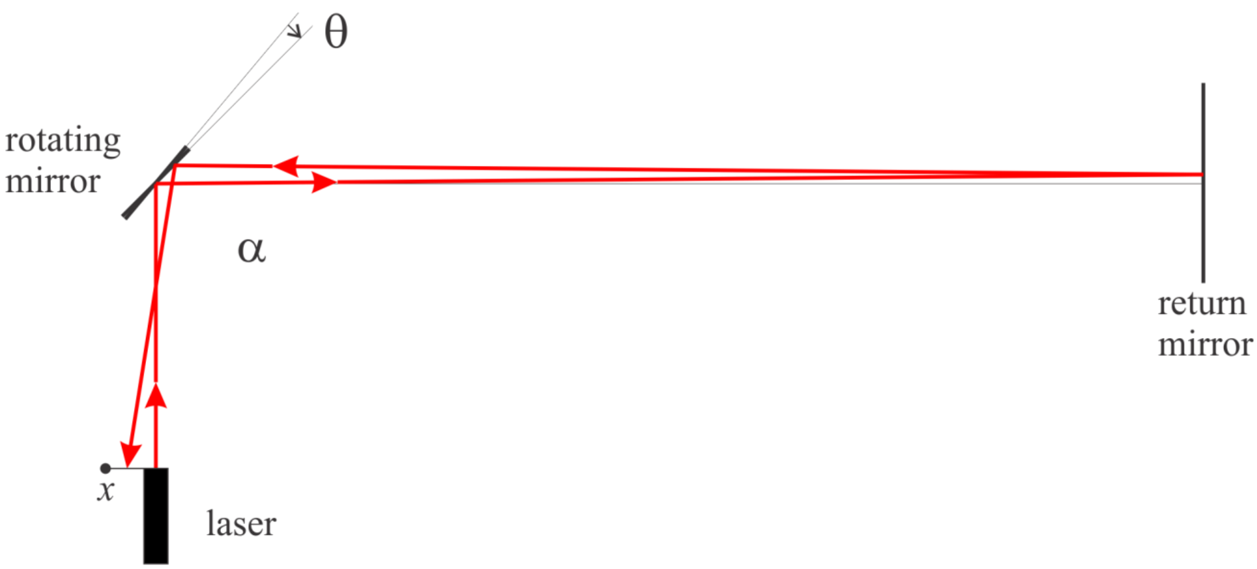


Figure 2. Path of the return ray when the forward ray is not centered at the return mirror.

This problem could be solved by introducing a convex lens between the rotating mirror and the return mirror. This lens should be placed at the focal distance F from the rotating mirror, such that the center of the rotating mirror is located at the focal point of the lens, see Figure 3.

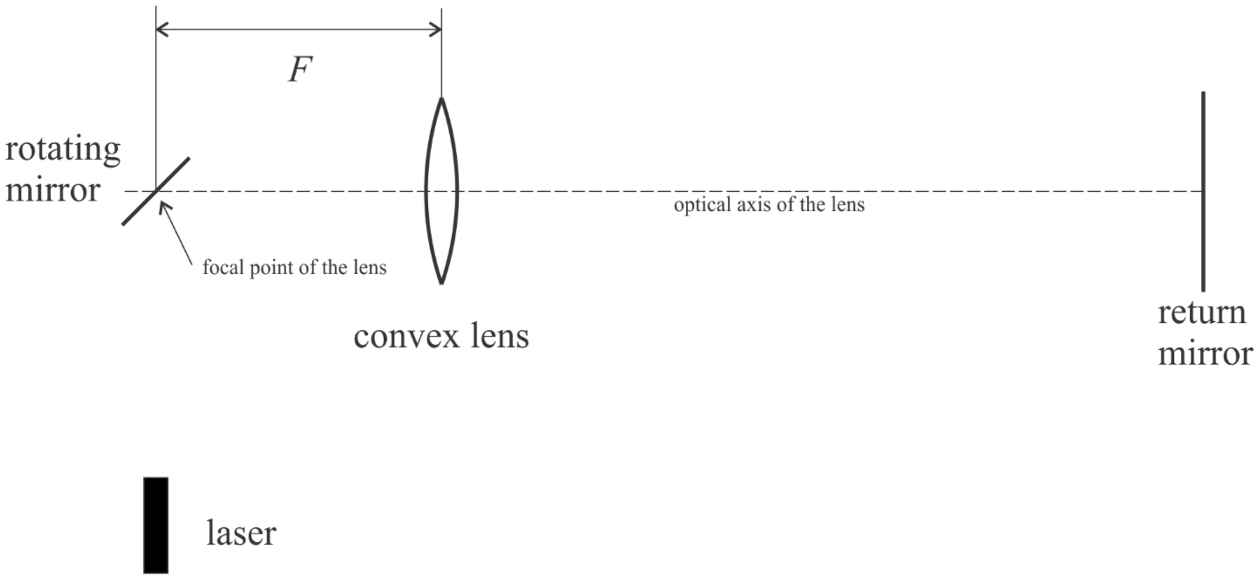
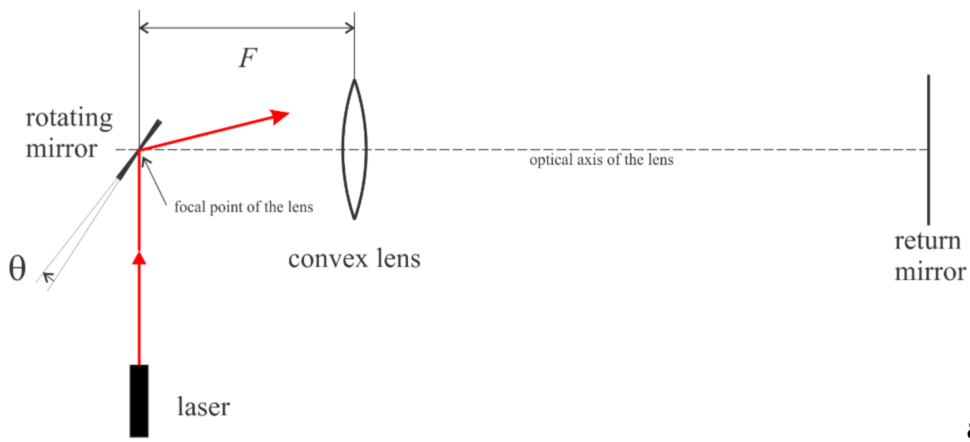


Figure 3. Diagram of the experimental setup with the convex lens.

Figure 4 shows three different orientations of the rotating mirror and corresponding reflections of the forward ray. Assuming that it takes the same time for the ray to travel from the rotating mirror to the return mirror and back (it results in the angle θ being the same in all three cases), complete all three diagrams by tracing the ray all the way to the return mirror and back to the plane of the laser aperture. Based on competed diagrams, can you make a conclusion whether the ray comes back with the same displacement x in all three cases or not? Explain your conclusion.



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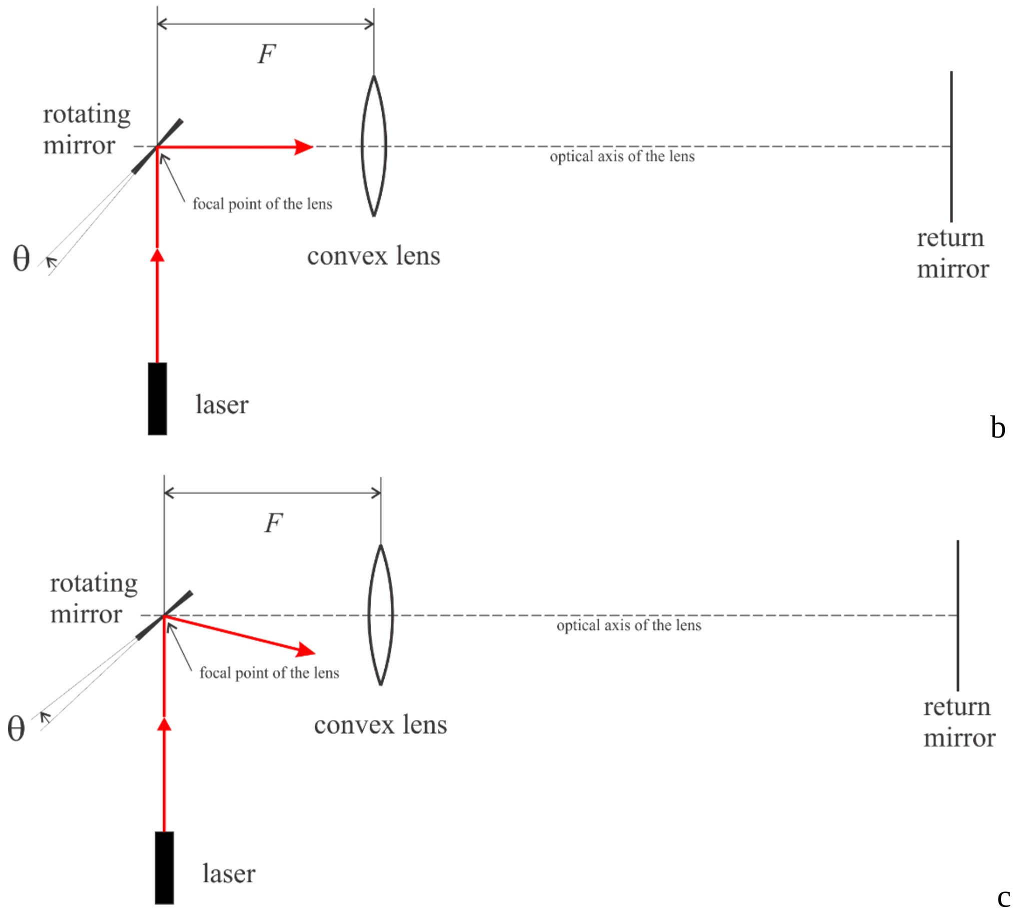

Figure 4. Trace the laser ray from rotating mirror to the return mirror and back to the plane of the laser aperture for three different orientations of the rotating mirror.

We assumed that laser ray spends the same time traveling between the rotating mirror and the return mirror and back for all initial orientations of the rotating mirror that are shown in Figure 4. As result the angle θ that the rotating mirror rotates during that time is the same for all three cases. Examine this assumption.

Does the laser ray spend the same time traveling from the rotating mirror to the return mirror and back for 3 positions of the rotating mirror shown in Figure 4? Explain why.

So far, we have treated the laser beam as an ideal ray. The real laser beam is a divergent beam. It could be represented as a cone that spreads around the ray that we used as an idealization in our previous discussion. The laser we use in this experiment has a divergence angle of 1°. It is beneficial to have the laser beam focused when it comes back to the laser plane in order to measure its position with higher accuracy and precision. To focus the laser beam we can repurpose the lens that already is a part of the experimental setup. The position of the lens is fixed, it should be at the focal distance from the rotating mirror. So, instead of adjusting the position of the lens, we can adjust the position of the return mirror and place it at the distance from the lens where the image of the laser will be formed. Use thin lens equation and calculate the distance from the lens where the image of the laser is formed. The focal length F of the lens used in this experiment is $2\ m$. The distance from the laser to the rotating mirror you have measured in the lab.

Assuming that the return mirror in Figure 5 is placed at the distance where the image of the laser is formed, complete the diagram and show how the divergent laser beam goes through the lens and riches the return mirror. Also show how beam propagates back to the plane of the laser aperture. When the beam comes back to the plane of the laser aperture, is it focused or not? Explain why.

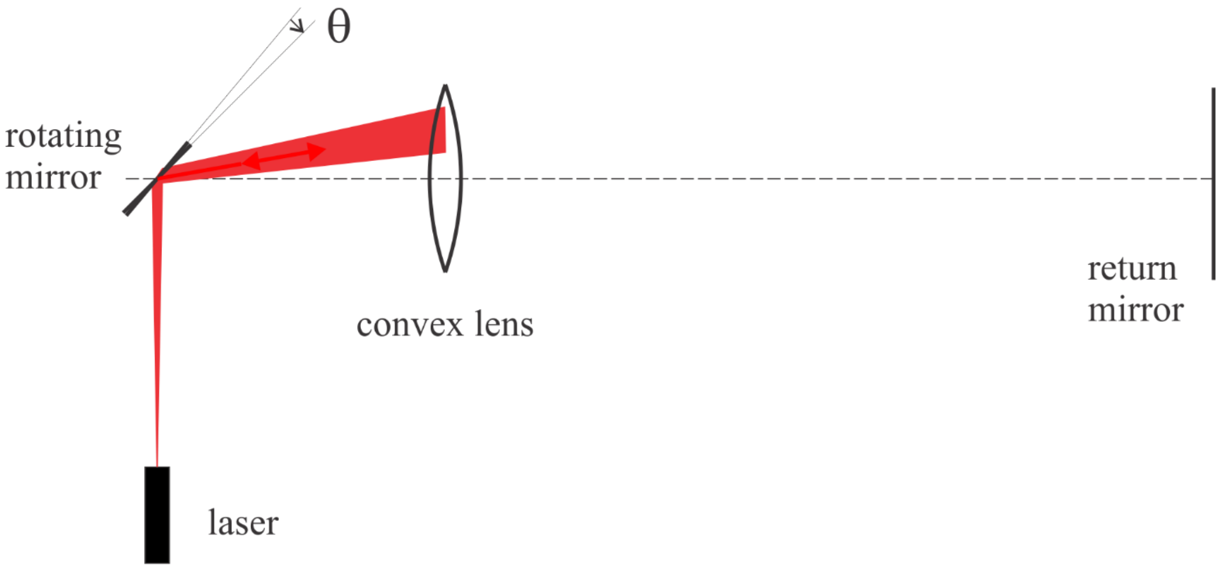

Figure 5. Complete the diagram and show how the laser beam travels from the laser to the return mirror and back.

Additional equipment

The displacement x of the return beam is so small that any device that we can try to use to measure it will naturally block the laser aperture. To solve this problem, we will use a beam splitter that will redirect the return beam to the CCD camera sensor, see Figure 6. The lens is not shown in the figure. We will also use a 1000x neutral density filter (reduces the intensity of the laser beam 1000 times) attached to the camera mount to reduce the intensity of the laser beam and prevent the camera sensor from being overexposed. Keeping the camera sensor from being overexposed is important for more precise measurement of the laser spot position

In addition, we will use a photodiode connected to an oscilloscope to measure the frequency of the rotating mirror. The photodiode should be placed on the way of the laser beam reflected from the rotating mirror (do not block the lens). Every time the laser beam crosses the aperture of the photodiode it causes voltage pulse. The frequency of the voltage pulses could be measured by the oscilloscope.

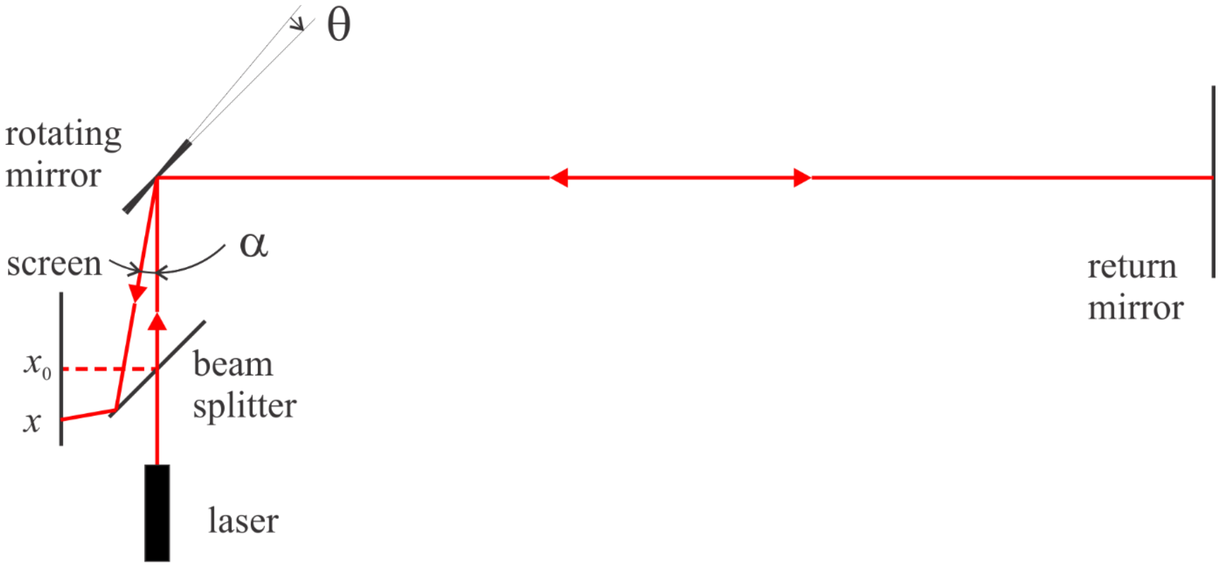

Figure 6. Using beam splitter to redirect the return beam to the CCD camera that is used to measure the position of the return beam spot.

Data collection and analysis

Before collecting data practice to operate the rotating mirror, measuring mirror frequency, and adjust the exposure time of the camera to have a steady and not overexposed image of the laser. Figure 7 shows a typical image of the laser spot.

Figure 7. Typical image of the laser spot.

Take about 10 images of the laser spot at different frequencies of the rotating mirror. The minimal stable frequency of the rotating mirror is around 100 Hz, and the maximal frequency is about 300 Hz.

A typical image of the laser spot is shown in Figure 7. Use a programming language of your choice and extract the position of the laser spot as a function of frequency. The size of the camera sensor pixel is $4.8\ \mu m$. Use it to convert the position of the laser spot from pixels to meters.

Plot the vertical and horizontal position of the laser spot as a function of frequency and discuss whether they behave according to the model that you developed earlier. Fit your data with a proper function and extract the speed of light.