Final Year Project

Semileptonic Decays of the Triply Heavy Omega Baryon

Final Defence

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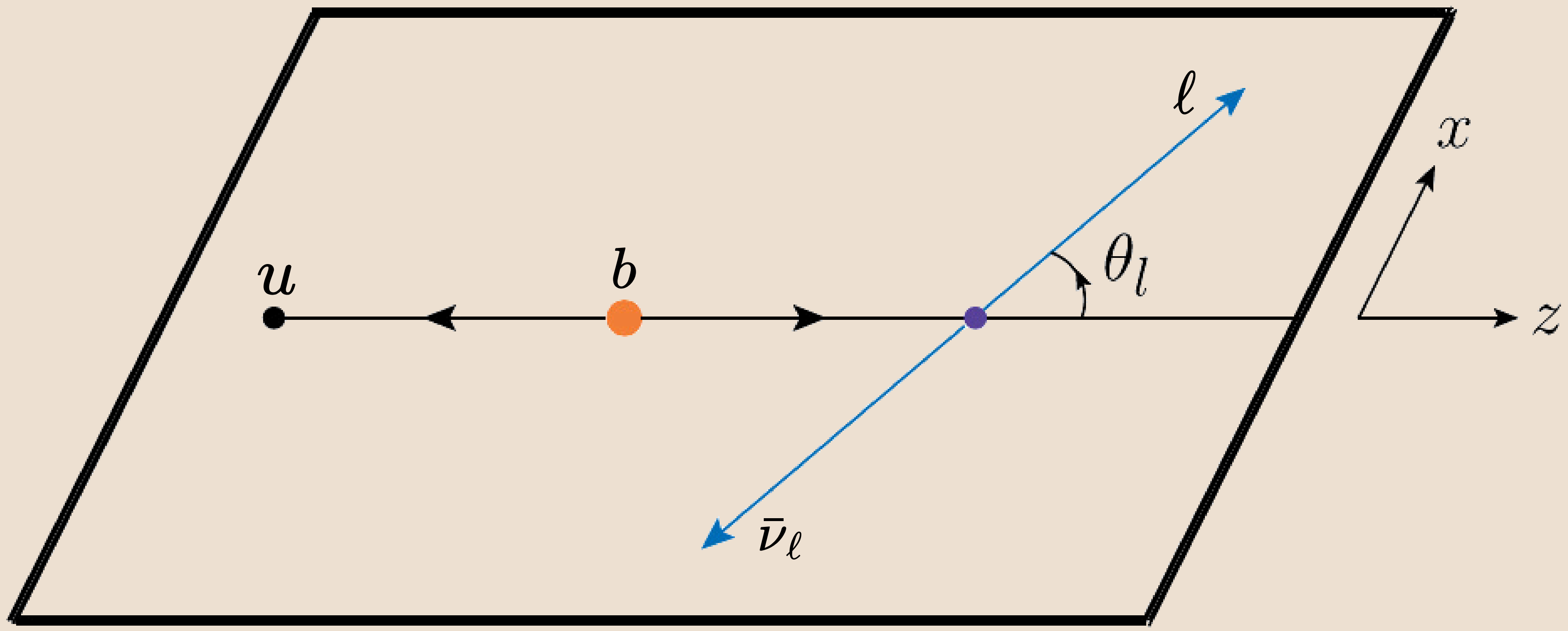
Introduction

$$\Omega_{ccb}^{+} \to \Xi_{cc}^{++} \ \ell \bar{\nu}_\ell$$



Motivation

- In 2017 LHCb, reported the discovery of the doubly charmed Xi baryon.
- The detection of triply heavy baryons now seems increasingly likely and represent the last missing category of standard baryons.
- Allows us to test the SM
- Mass spectra are well studied but decay processes with spin 1/2 to spin 1/2 are not
- Extension to other triply heavy baryons
- If there is a difference in SM vs our result, we can explain in new physics

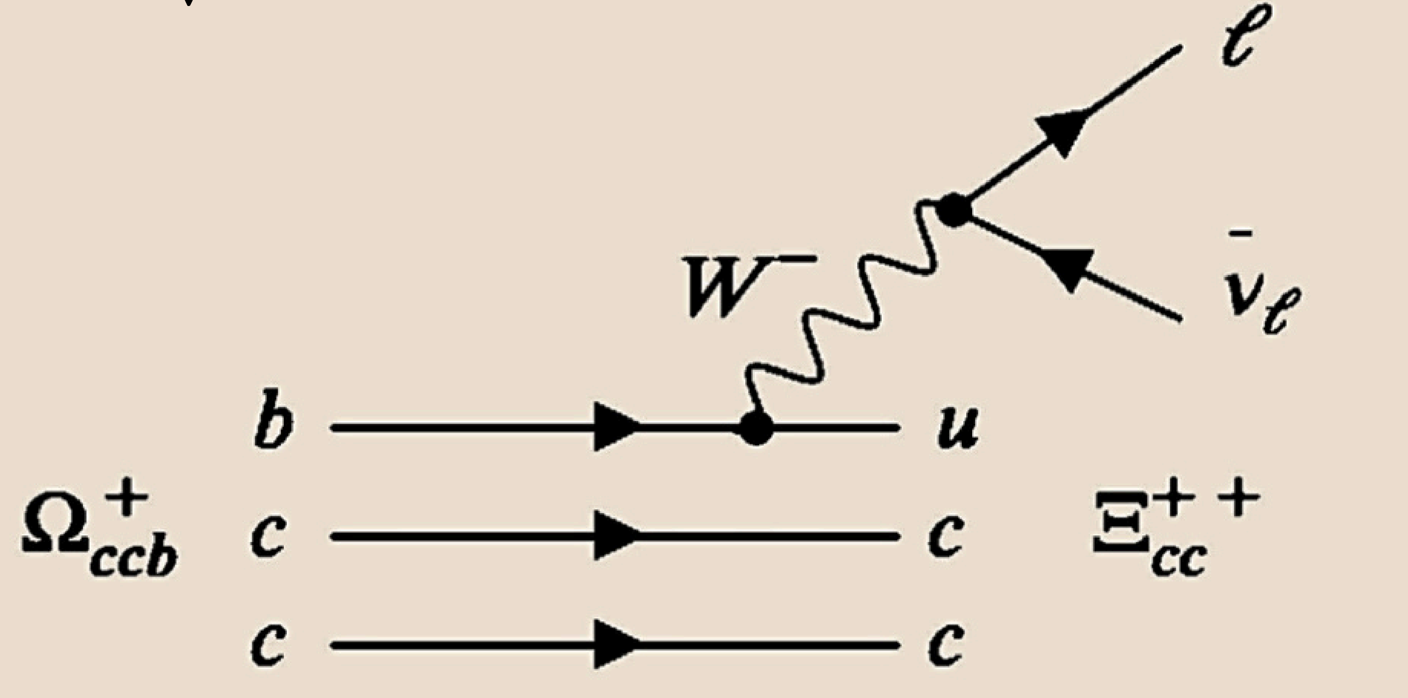
Amplitude

The full amplitude using electroweak theory Feynman rules mediated by the W boson propagator is given by

$$-iM = V_{ub}\left[\frac{-ig}{\sqrt{2}}\bar{u}\frac{1}{2}\gamma_\mu\,(1-\gamma_5)b\right]\frac{1}{i}\,\frac{g^{\mu v}-\frac{k^{\mu v}}{m_w^2}}{k^2-m_w^2}\left[\frac{-ig}{\sqrt{2}}\bar{l}\frac{1}{2}\gamma_v\,(1-\gamma_5)v_l\right]$$

Using effective field theory to separate the high energy physics and simplify the problem in the limit where $k^2 >> m_w^2$

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}V_{ub}\bar{u}\gamma_\mu\,(1-\gamma_5)b\bar{\ell}\gamma^\mu\,(1-\gamma_5)\nu_\ell$$



Amplitude

$$\langle \ell \bar{v}_l \Xi | H_{eff} | \Omega \rangle = \frac{G_F}{\sqrt{2}} V_{ub} \langle \Xi | \bar{u} \gamma_\mu \left(1 - \gamma^5\right) b | \Omega \rangle \, \langle \ell \bar{v}_l | \bar{\ell} \gamma^\mu \left(1 - \gamma_5\right) v_l | 0 \rangle$$

The polarization of the virtual boson satisfies the completeness relation:

$$g^{\mu\mu'} = \sum_{m=0,\pm,t} \epsilon^{\mu'}(m) \epsilon^{*\mu}(m) \delta_m$$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ub} \sum_{m=0,\pm,t} \epsilon^{\mu'}(m) \epsilon^{*\mu}(m) \delta_m \langle \Xi | \bar{u} \gamma_\mu \left(1 - \gamma^5\right) b | \Omega \rangle \, \langle \ell \bar{v}_l | \bar{\ell} \gamma_{\mu'} \left(1 - \gamma_5\right) v_l | 0 \rangle$$

$$H_m^{\lambda_b, \lambda_c} = \epsilon^{*\mu}(m) \langle \Xi \left(\lambda_c\right) | \bar{u} \gamma_\mu \left(1 - \gamma_5\right) b | \Omega \left(\lambda_b\right) \rangle$$

$$L_m^{\lambda_l, \lambda_{v_l}} = \epsilon^{\mu'}(m) \, \langle \ell(\lambda_l) \bar{\nu}_l \left(\lambda_v\right) | \bar{l} \gamma_{\mu'} \left(1 - \gamma_5\right) | 0 \rangle$$

Helicity Basis

It is implied that H and L have two components i.e. Vector and Axial which will be split and computed separately later

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ub} \sum_{m=0,\pm,t} H_m^{\lambda_b,\lambda_c} L_m^{\lambda_l,\lambda v_l} \delta_m$$

$$\mathcal{M}^\dagger = \frac{G_F}{\sqrt{2}} V_{ub} \sum_{n=0,\pm,t} L_n^{\dagger\lambda_l,\lambda v_l} H_n^{\dagger\lambda_b,\lambda_c} \delta_n$$

Next we compute $\mathcal{M}\mathcal{M}^\dagger$ by summing over all the final spins and polarizations.

This results in a very large number of components in the sum which can be simplified thanks to the presence of the Kronecker Delta leaving only those terms that have the same polarization state

$$MM^\dagger = |M|^2 = N \sum_{\lambda_b,\lambda_c,\lambda_l,\lambda_v} \sum_{m=0,\pm,t} H_m^{\lambda_b\lambda_c} H_m^{\dagger\lambda_b\lambda_c} L_m^{\lambda_l\lambda_v} L_m^{\dagger\lambda_l\lambda_v} \delta_{mn}$$

the following vector and axial low energy matrix elements are defined in terms of six form factors which are derived using QCD sum rules that we are borrowing

$$\langle \Xi_{cc}^{++}(p',s')|V^\mu|\Omega_{ccb}^{+}(p,s)\rangle = \bar{u}_{\Xi_{cc}^{++}}(p',s')\left[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{p^\mu}{m_{\Omega_{ccb}^+}} + F_3(q^2)\frac{p'^\mu}{m_{\Xi_{cc}^{++}}}\right]u_{\Omega_{ccb}^+}(p,s),$$

$$\langle \Xi_{cc}^{++}(p',s')|A^\mu|\Omega_{ccb}^{+}(p,s)\rangle = \bar{u}_{\Xi_{cc}^{++}}(p',s')\left[G_1(q^2)\gamma^\mu + G_2(q^2)\frac{p^\mu}{m_{\Omega_{ccb}^+}} + G_3(q^2)\frac{p'^\mu}{m_{\Xi_{cc}^{++}}}\right]\gamma_5 u_{\Omega_{ccb}^+}(p,s),$$

Spinors

We will calculate $|\mathcal{M}|^2$ in helicity basis by introducing spinors that are kinematic dependent. The ones we are using are given by:

$$\eta^\mu(\pm) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \mp 1 \\ i \\ 0 \end{pmatrix}, \quad \eta^\mu(0) = \frac{1}{\sqrt{q^2}} \begin{pmatrix} |\boldsymbol{q}| \\ 0 \\ 0 \\ -q_0 \end{pmatrix}, \quad \eta^\mu(t) = \frac{1}{\sqrt{q^2}} \begin{pmatrix} q_0 \\ 0 \\ 0 \\ -|\boldsymbol{q}| \end{pmatrix}$$

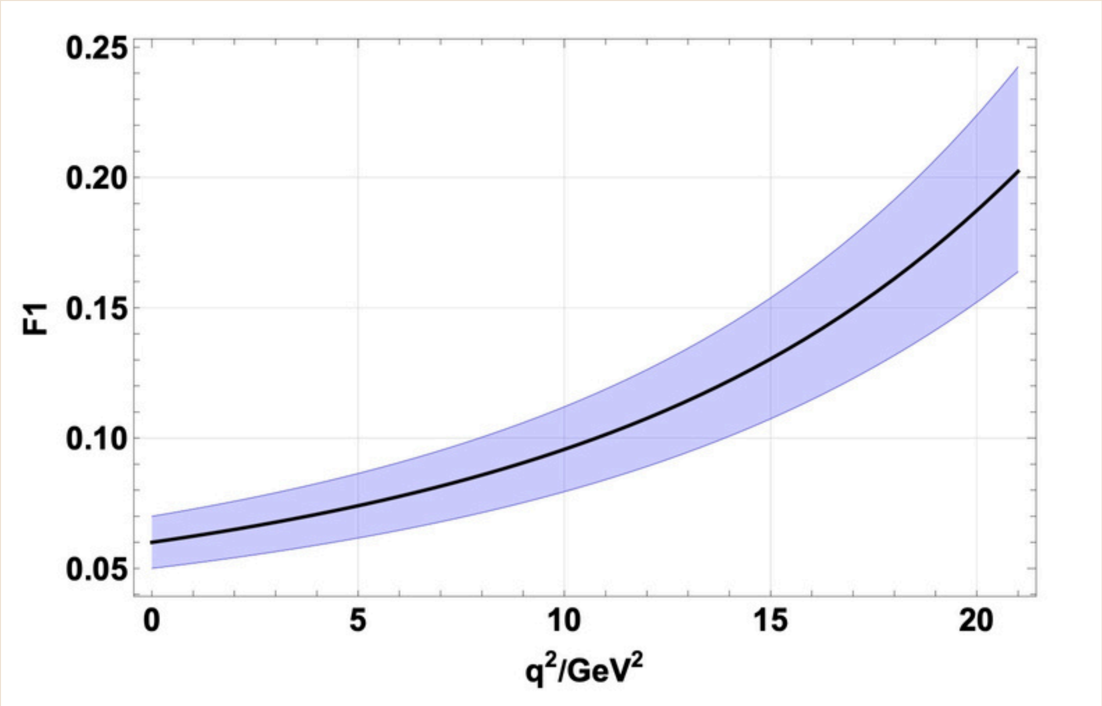
The dirac spinors to calculate Leptonic part are given by

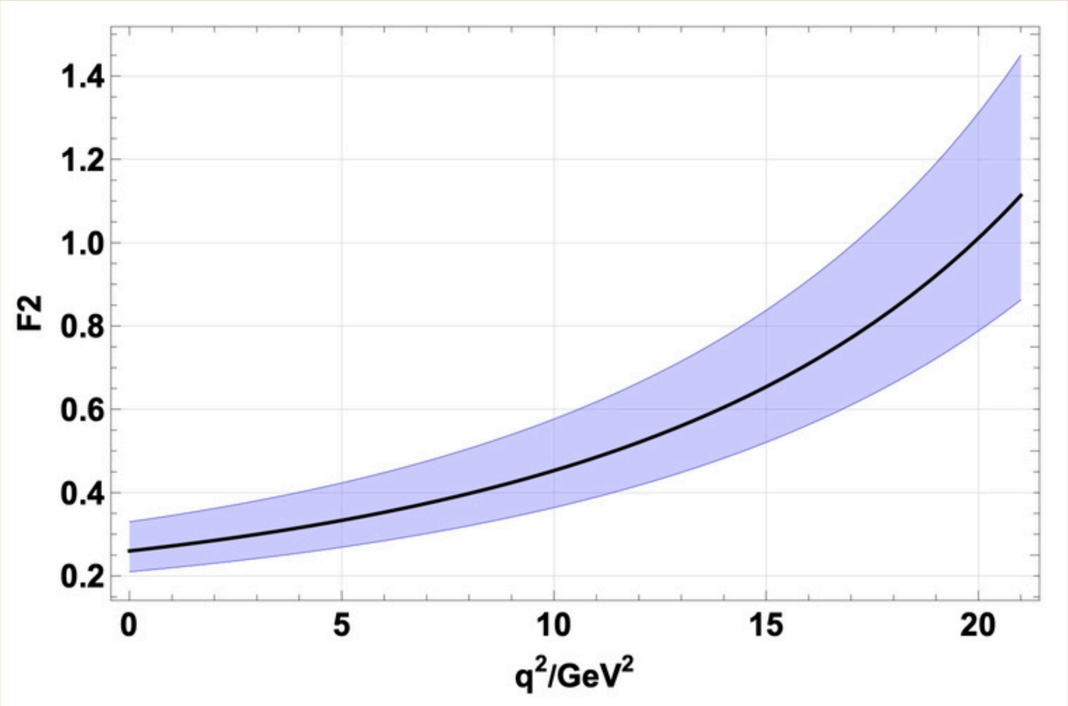
$$u(\lambda = \pm 1/2) = \begin{pmatrix} \sqrt{E \mp |\boldsymbol{p}|}\, \xi_\pm \\ \sqrt{E \pm |\boldsymbol{p}|}\, \xi_\pm \end{pmatrix}, \quad v(\lambda = \pm 1/2) = \begin{pmatrix} -\sqrt{E \pm |\boldsymbol{p}|}\, \xi_\mp \\ \sqrt{E \mp |\boldsymbol{p}|}\, \xi_\mp \end{pmatrix}$$

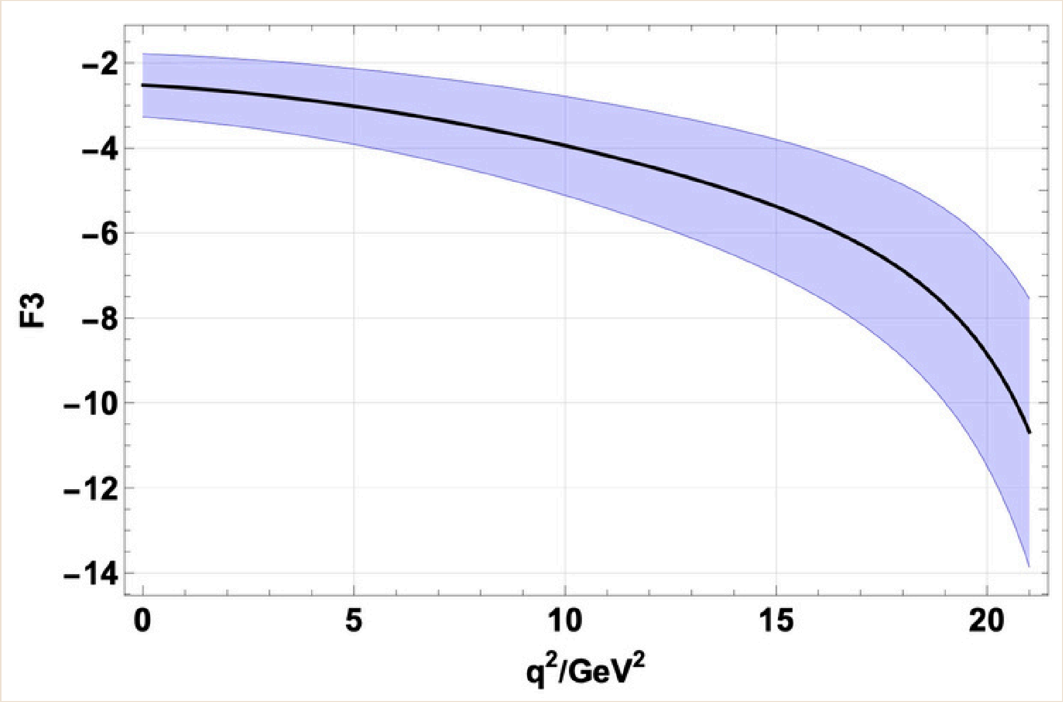
$$\xi_+ = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\phi} \end{pmatrix}, \qquad \xi_- = \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix},$$

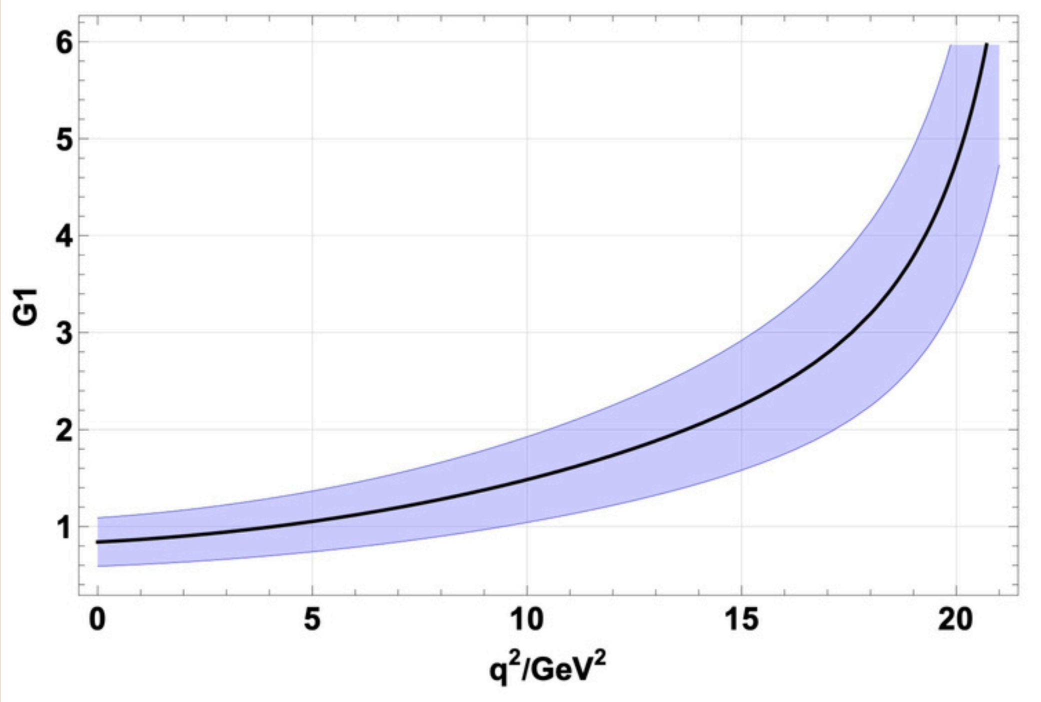
Form Factors

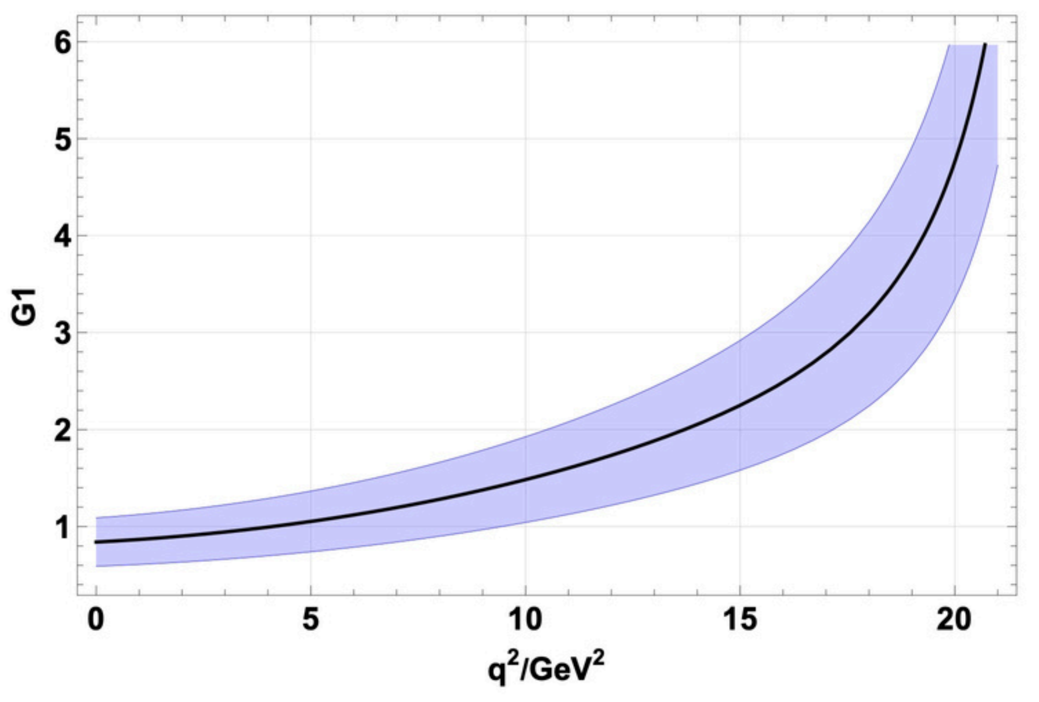
- The leptonic part is simple: it's made of pointlike particles, so we can compute it analytically using standard QFT rules.
- The hadronic part, however, is complicated: it's a matrix element between QCD-bound states (baryons), involving strong interaction effects - the quarks aren’t free.
- We borrow QCD sum rules to paramatrize the hadronic part using form factors to obtain:

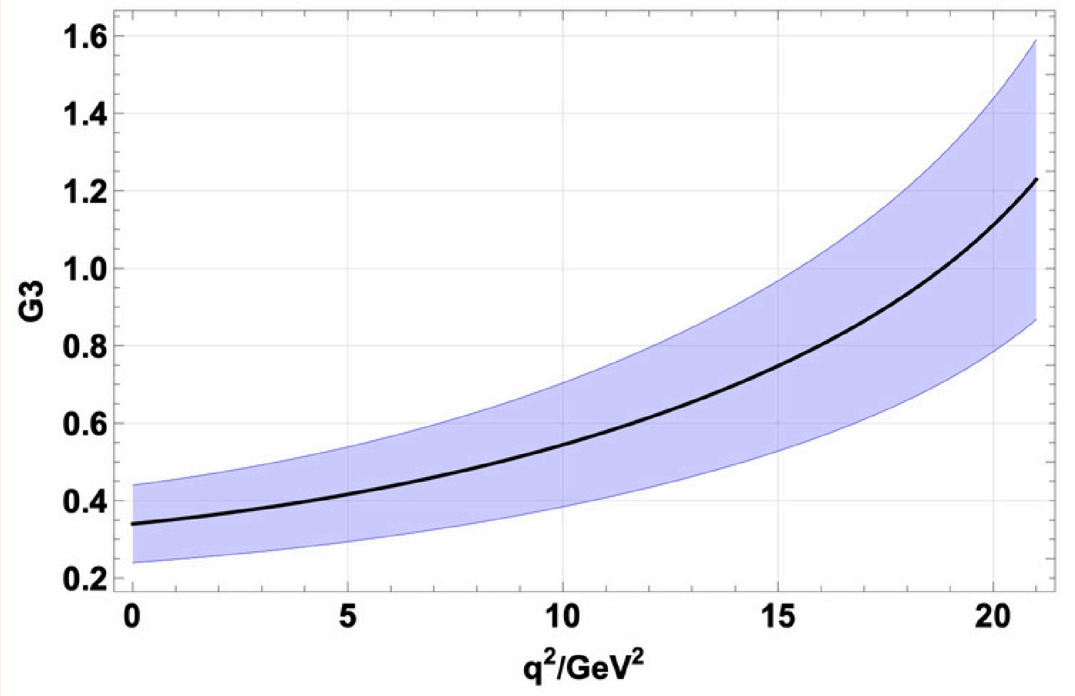












- **The fitting of the form factors is achieved using the following expression:**

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{\left(1 - a_1 \frac{q^2}{m^2_{\Omega^+_{ccb}}} + a_2(\frac{q^2}{m^2_{\Omega^+_{ccb}}})^2 + a_3(\frac{q^2}{m^2_{\Omega^+_{ccb}}})^3 + a_4(\frac{q^2}{m^2_{\Omega^+_{ccb}}})^4\right)}.$$

Decay Width

The differential decay width which we know is given through Fermi's Golden Rule by:

$$\frac{d\Gamma(\Omega_{ccb}^{+} \to \Xi_{cc}^{++}\ \ell\bar{\nu}_\ell)}{dq^2} = \frac{G_F^2}{(2\pi)^3}|V_{ub}|^2 \frac{\lambda^{1/2}(q^2 - m_\ell^2)^2}{48 m_{\Omega_{ccb}^{+}}^3 q^2} \mathcal{H}_{tot}(q^2),$$

Where the total helicity is given after utilizing the fitted form factors and the analytically computed leptonic parts:

$$\mathcal{H}_{\text{tot}}(q^2) = \left[\left|H_{+\frac{1}{2},+1}\right|^2 + \left|H_{-\frac{1}{2},-1}\right|^2 + \left|H_{+\frac{1}{2},0}\right|^2 + \left|H_{-\frac{1}{2},0}\right|^2\right]\left(1 + \frac{m_\ell^2}{2q^2}\right) + \frac{3m_\ell^2}{2q^2}\left[\left|H_{+\frac{1}{2},t}\right|^2 + \left|H_{-\frac{1}{2},t}\right|^2\right]$$

The calculated decay widths for all lepton channels are given by:

$\Gamma\ [\Omega_{ccb}^{+} \to \Xi_{cc}^{++} e\ \overline{\nu}_e] \times 10^{14}$	$\Gamma\ [\Omega_{ccb}^{+} \to \Xi_{cc}^{++} \mu\ \overline{\nu}_\mu] \times 10^{14}$	$\Gamma\ [\Omega_{ccb}^{+} \to \Xi_{cc}^{++} \tau\ \overline{\nu}_\tau] \times 10^{15}$
$1.23^{+0.28}_{-0.56}$	$1.22^{+0.28}_{-0.55}$	$7.32^{+1.48}_{-3.16}$

Decay widths (in GeV) for the semileptonic $\Omega_{ccb}^{+} \to \Xi_{cc}^{++} \ell \overline{\nu}_\ell$ transition at different channels.

In conclusion, our results can be used to confirm the SM or open up avenues for new physics. It can also be used to assist in the experimental search for Triply Heavy Baryons.

Possible Extensions

- Plot the graph of decay width against different observable
- Apply the same procedure to calculate branching ratios of other baryons decays
- Check for polarization and helicity asymmetry of our model

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Thank You

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