Final Year Project Report

Title:

Semileptonic Decays of the Triply Heavy Omega Baryon

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Contents

Abstract

This report investigates the weak semileptonic decay of $\Omega_{ccb}^{+} \to \Xi_{cc}^{++}\ell\bar{\nu}_{\ell}$, where a theoretically predicted triply heavy omega baryon of spin 1/2 decays into an observed doubly heavy Xi baryon with spin 1/2. We develop the theoretical formalism required to explain this phenomenon and briefly outline the methodology of QCD sum rules and relevant form factors. We plot the form factors and calculate the exclusive widths across all three lepton channels. Finally, we discuss how our predicitons may help the present and future experiments in the search for doubly heavy and triply baryons.

1 Introduction

1.1 Motivation

Baryons are a type of subatomic particle made up of three quarks, belonging to the hadron family. They are strongly interacting fermions; that is, they are acted on by the strong nuclear force and are described by Fermi–Dirac statistics. Since LHCb reported the discovery of the doubly charmed baryon Ξ_{cc}^{+} in 2017, great progress has been made in the field of doubly heavy baryons (DHBs). The discovery of theoretically predicted triply heavy baryons (THBs) now seems increasingly likely. Triply heavy baryons which consist of three heavy c or b quarks are of great theoretical interest since they refrain from light-quark contaminations. They are the last missing category of standard hadrons. While theoretical studies of triply heavy baryons have largely focused on their mass spectra, relatively little attention has been given to their decay properties. Decay processes predominantly discuss transitions from spin 3/2 to spin 1/2 or from spin 1/2 to spin 3/2. Thus, spin 1/2 to spin 1/2 transitions have not been explored and present an avenue to explore further.

Most importantly,the decay widths computed here allow us to test the standard model in the future if triply heavy baryons are found. These theoretical results can be compared with experimental ones to either confirm the SM or open avenues for new physics.

1.2 Standard Model

The Standard Model of particle physics provides the foundational framework for understanding the fundamental constituents of matter and their interactions. Crucially, the Standard Model's SU(3) × SU(2) × U(1) gauge group structure governs these interactions, with SU(3) describing quantum chromodynamics. For baryonic systems at energy scales below the electroweak unification (100 GeV), the model achieves remarkable predictive accuracy. In the context of heavy baryon decays, the electroweak sector becomes particularly relevant. The weak interaction, mediated by massive W± and Z bosons, facilitates quark flavor transitions through charged current processes. This force operates at femtometer scales due to the large boson masses ($m_W \approx 80GeV/c$), making it inherently short-ranged compared to electromagnetic effects. The Standard Model's mathematical formulation encodes these interactions through covariant derivatives in the Lagrangian,

1.3 Structure of Triply Heavy Baryons

The quark model revolutionized hadron physics by classifying baryons as bound states of three valence quarks. Since top quark does not materialize into hadrons, only the c and b quarks participate in the formation of these baryons. Therefore, the possible triply heavy baryons would be Ω_{ccc}, Ω_{ccb}, Ω_{cbb}, Ω_{bbb} baryons in a c or a b quark fragmentation.

The triply heavy baryon Ω_{ccb} contains two charm quarks and one bottom quark (quark content ccb), giving it overall charge +1 and baryon number 1. It lies in the family of heavy baryons consisting of three heavy quarks, which are the heaviest members of the baryon spectrum. Unlike light-flavor baryons, Ω_{ccb} has no light (u, d, s) quarks, so it does not fit into the usual SU(3) flavor octets or decuplets. Instead, one may loosely categorize it by heavy-flavor symmetry or extended flavor groups. In fact, triply-heavy baryons are at the "top layer" of any flavor symmetry (involving c, b). Its isospin is $I = 0$ (there is no light-quark doublet), charm number $C = 2$, and bottomness $B' = -1$.

The ground-state Ω_{ccb} baryon is expected to have spin-parity $J^P = 1/2^+$ or $3/2^+$, corresponding to the two possible couplings of the quark spins. Because two of its quarks are identical charm quarks, they obey Fermi statistics. In the color-singlet baryon, the two charm quarks must couple to an overall symmetric flavor wavefunction (both are c) and an overall color-antisymmetric state. This forces their spin wavefunction to be symmetric (spin-1) in the ground (s-wave) state. The total spin of Ω_{ccb} then comes from coupling this cc-diquark (with spin $s = 1$) with the spin-1/2 of the b quark. One obtains a spin-1/2 ground state and a spin-3/2 excited state, both with positive parity. In heavy-quark symmetry language, the cc pair can be treated as a tightly bound diquark in a color-$\bar{\mathbf{3}}$, forming a heavy object with $s = 1$, to which the b quark is bound in a relative s-wave. Hence the two lowest states are $J^P = (1/2)^+$ and $(3/2)^+$.

1.4 Decay Channel

Semileptonic decays of heavy baryons involve a weak transition where one heavy quark emits a virtual W boson, subsequently decaying into a lepton-neutrino pair. The semileptonic decay $\Omega_{ccb} \to \Xi_{cc}^{++}\,\ell^-\bar{\nu}_\ell$ is governed by the charged weak interaction. At the quark level the dominant process is:

$$b \to u\,\ell^-\bar{\nu}_\ell,$$

mediated by a virtual W^- boson. In this decay, the leptons $(\ell^-, \bar{\nu}_\ell)$ carry away charge -1, and the hadronic transition $b \to u$ increases the charge of the hadronic system by $+1$, so that the initial $+1$ charge of Ω_{ccb} matches the final $+2$ of Ξ_{cc}^{++} plus the lepton charges.

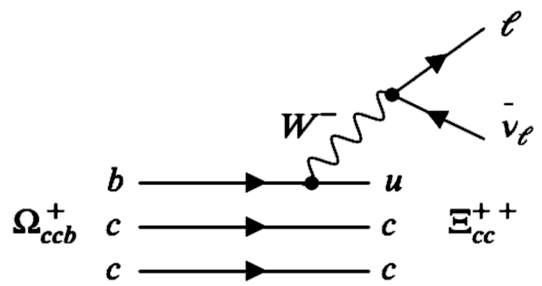

Figure 1: Feynmann diagram for the $\Omega_{ccb}^+ \to \Xi_{cc}^{++}\ell\bar{\nu}_\ell$ semileptonic decay

1.4.1 Details on Kinematics

The decay $b \to u\,\ell^-\bar{\nu}_\ell$ can be conveniently regarded as a quasi-two body decay with $b \to u\,W^-$ followed by $W^- \to \ell^-\bar{\nu}_\ell$.

In the rest frame of the parent particle b we choose the daughter particle u to move along the negative z direction, and the virtual boson in the positive z direction. Considering W decays in the x-z plane, the lepton ℓ makes an angle θ_l with the z-axis. (See Fig. 2)

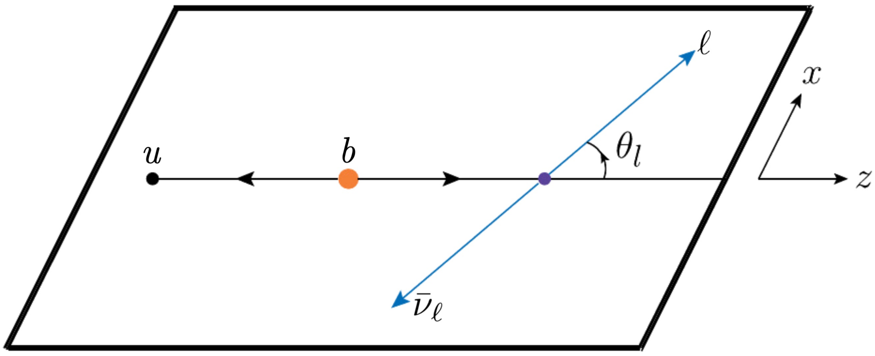

Figure 2: Kinematics of the $\Omega_{ccb}^{+} \to \Xi_{cc}^{++}\ell\bar{\nu}_\ell$ decay.

2 Theoretical Framework

2.1 Effective Field Theory

In order to describe physical phenomena spread out over different energy and length scales we make use of effective field theory. This is done through low energy parameters and the low energy interaction with higher energies not taken into account. EFT is applied at the energy scales for Fermi theory of weak interaction.

2.1.1 Feynman Rules for W boson

The W boson's (W^+ and W^-), unlike the massless photon and gluons, are extremely heavy experimentally,

$$M_W = 80.40 \pm 0.03\,\mathrm{GeV}/c^2.$$

A massive particle of spin 1 has three allowed polarization states ($m_s = 1, 0, -1$). To get the completeness relation for a W boson we only impose the lorentz condition:

$$\epsilon_\mu q^\mu = 0. \tag{2.1}$$

Hence, we have three independent polarization states. If we take the W boson to be in the z-direction, two of the polarization states can be chosen to be circularly polarized:

$$\epsilon_\mu^- = \frac{1}{\sqrt{2}}(0, 1, -i, 0) \quad \text{and} \quad \epsilon_\mu^+ = -\frac{1}{\sqrt{2}}(0, 1, i, 0).$$

The third polarization, which will be orthogonal to the circularly polarized states can be written as:

$$\epsilon_\mu^L = (a, 0, 0, b).$$

Imposing the lorentz condition gives us the value for a and b, so the longitudinal polarization state becomes:

$$\epsilon_\mu^L = \frac{1}{mc}(q_z, 0, 0, q_0).$$

Summing over all these polarization states gives us the completeness relation for a W boson:

$$\sum_\lambda \epsilon_\mu^{\lambda *}\epsilon_\nu^\lambda = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}. \tag{2.2}$$

Hence, the feynman rule associated with the exchange of the W boson is:

$$\frac{-i}{q^2 - m_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right). \tag{2.3}$$

2.1.2 Fermi Theory

In 1933 Fermi proposed his theory of beta decay. This was an effective theory of weak interactions between four point like particles, without an intermediate gauge boson.
Hence, the matrix element according to fermi theory was given by:

$$\mathcal{M}_{fi} = G_F g_{\mu\nu} \left[\bar{\psi}_3 \gamma^\mu \psi_1\right] \left[\bar{\psi}_4 \gamma^\nu \psi_2\right], \tag{2.4}$$

Where, G_F is the strength of the weak interaction.
However, in his theory, he did not account for parity violation, which is exactly what happens in the weak interaction. This corresponds to the V-A structure of the parity-violating part. Thus, the vertex factor for an exchange of W bosons is given by:

$$\frac{-ig_W}{2\sqrt{2}} \gamma^\mu (1-\gamma^5), \tag{2.5}$$

Where γ^μ represents the vector coupling and $\gamma^\mu\gamma^5$ is the axial vector.
After the discovery of the parity violation by Wu in 1957, Fermi's theory was modified to:

$$\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}} g_{\mu\nu} \left[\bar{\psi}_3 \gamma^\mu (1-\gamma^5) \psi_1\right] \left[\bar{\psi}_4 \gamma^\nu (1-\gamma^5) \psi_2\right]. \tag{2.6}$$

Using Fermi Theory we approximate (2.3) in the limit $q^2 \ll m_W^2$, hence, the propagator takes the form:

$$i \frac{g_{\mu\nu}}{m_W^2}. \tag{2.7}$$

Thus, the amplitude for transition in accordance with the Feynman diagram Fig.1 is given by:

$$A = \left(\frac{-ig}{\sqrt{2}}\right)^2 V_{ub} (\bar{u}\gamma_\mu L b) \frac{1}{i} \frac{g^{\mu\nu} - \frac{k^{\mu\nu}}{m_W^2}}{k^2 - m_W^2} (\bar{v}_l \gamma_\nu L l), \tag{2.8}$$

Where $L = \frac{1}{2}(1-\gamma_5)$ is the left handed current. Expanding this in a taylor series we obtain,

$$\frac{g^2}{2i} V_{ub} \left[\frac{1}{m_W^2} (\bar{u}\gamma_\mu L b)(\bar{v}\gamma_\nu L e) + \frac{1}{m_W^4} (higher\, dim\, operators) + ...\right]. \tag{2.9}$$

Taking only the lowest order term we obtain our effective Hamiltonian describing our transition as :

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_\mu (1-\gamma_5) b \bar{l} \gamma^\mu (1-\gamma_5) v_l. \tag{2.10}$$

Where G_F is the fermi coupling constant and V_{ub} is the Cabbibo-Kobayashi-Maskawa (CKM) matrix element.

3 Semileptonic Decay

3.1 Formalism

Using the formalism developed in our first section, we write the effective Hamiltonian for the semileptonic decay $\Omega_{ccb}^+ \to \Xi_{cc}^{++} \ell \bar{\nu}_\ell$:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_\mu (1-\gamma_5) b \bar{\ell} \gamma^\mu (1-\gamma_5) v_\ell. \tag{3.1}$$

Sandwiching the effective Hamiltonian between initial triply and final doubly heavy baryon states, we obtain the amplitude:

$$\langle \Xi_{cc}^{++} | \mathcal{H}_{eff} | \Omega_{ccb}^+ \rangle = \frac{G_F}{\sqrt{2}} V_{ub} \ell \gamma^\mu (1-\gamma_5) v_\ell \langle \Xi_{cc}^{++} | \bar{u} \gamma_\mu (1-\gamma_5) b | \Omega_{ccb}^+ \rangle. \tag{3.2}$$

Where the first part of the equation indicates the leptonic transition and the second part the hadronic from b to u channel.

3.1.1 Helicity Amplitude

To write the matrix element in terms of the helcity amplitude we adopt the approach shown in [1, 2].
The polarization of the virtual boson satisfies the completeness relation:

$$g^{\mu\mu'} = \sum_{m=0,\pm,t} \epsilon^{\mu'}(m)\epsilon^{*\mu}(m)\delta_m. \tag{3.3}$$

Putting this in (3.2) :

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ub} \ell g^{\mu\mu'} \gamma_{\mu'}(1-\gamma_5)v_\ell \langle \Xi_{cc}^{++}|\bar{u}\gamma_\mu(1-\gamma_5)b|\Omega_{ccb}^{+}\rangle, \tag{3.4}$$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{ub} \sum_{m=0,\pm,t} \epsilon^{\mu'}(m)\epsilon^{*\mu}(m)\delta_m \gamma_{\mu'}(1-\gamma_5)v_\ell \ell \langle \Xi_{cc}^{++}|\bar{u}\gamma_\mu(1-\gamma_5)b|\Omega_{ccb}^{+}\rangle. \tag{3.5}$$

The leptonic matrix element can be written as:

$$\langle \bar{\ell} v_\ell | \bar{\ell}\gamma_{\mu'}(1-\gamma_5)v_\ell|0\rangle. \tag{3.6}$$

The amplitude can now be separated into two helicity amplitudes. One denoting the leptonic helicity and the other the hadronic.

$$H_m^{\lambda_b,\lambda_c} = \epsilon^{*\mu}(m)\langle \Xi_{cc}^{++}(\lambda_c)|\bar{u}\gamma_\mu(1-\gamma_5)b|\Omega_{ccb}^{+}(\lambda_b)\rangle, \tag{3.7}$$

$$L_m^{\lambda_{\bar{\ell}},\lambda_{v_\ell}} = \epsilon^{\mu'}(m)\langle \bar{\ell}(\lambda_{\bar{l}})v_\ell(\lambda_{v_\ell})|\bar{\ell}\gamma_{\mu'}(1-\gamma_5)v_\ell|0\rangle, \tag{3.8}$$

where $\lambda_b, \lambda_c, \lambda_{\bar{\ell}}, \lambda_{v_\ell}$ are the individual polarizations of the four particles of the interaction.
Summing over initial and final spins and polarizations, we obtain the amplitude squared of the the semileptonic decay $\Omega_{ccb}^{+} \to \Xi_{cc}^{++}\ell\bar{\nu}_\ell$:

$$|\mathcal{M}|^2 = \mathcal{N} \sum_{\lambda_b,\lambda_c,\lambda_{\bar{\ell}},\lambda_{v_\ell}} \sum_{m=0,\pm,t} H_m^{\lambda_b,\lambda_c} H_m^{\dagger\lambda_b,\lambda_c} L_m^{\lambda_{\bar{\ell}},\lambda_{v_\ell}} L_m^{\dagger\lambda_{\bar{\ell}},\lambda_{v_\ell}}. \tag{3.9}$$

3.1.2 Hadronic Form Factors

The hadronic matrix element in (3.2) consists of two currents, the vector ($V^\mu = \bar{u}\gamma_\mu b$), and the axial vector ($A^\mu = \bar{u}\gamma_\mu(1-\gamma_5)b$). These vector and axial matrix elements can be defined in terms of six form factors given by:

$$\begin{aligned}\langle \Xi_{cc}^{++}(p',s')|V^\mu|\Omega_{ccb}^{+}(p,s)\rangle = \bar{u}_{\Xi_{cc}^{++}}(p',s')\bigg[&F_1(q^2)\gamma^\mu + F_2(q^2)\frac{p^\mu}{m_{\Omega_{ccb}^{+}}} \\ &+ F_3(q^2)\frac{p'^\mu}{m_{\Xi_{cc}^{++}}}\bigg] u_{\Omega_{ccb}^{+}}(p,s),\end{aligned} \tag{3.10}$$

$$\begin{aligned}\langle \Xi_{cc}^{++}(p',s')|A^\mu|\Omega_{ccb}^{+}(p,s)\rangle = \bar{u}_{\Xi_{cc}^{++}}(p',s')\bigg[&G_1(q^2)\gamma^\mu + G_2(q^2)\frac{p^\mu}{m_{\Omega_{ccb}^{+}}} \\ &+ G_3(q^2)\frac{p'^\mu}{m_{\Xi_{cc}^{++}}}\bigg] \gamma_5 u_{\Omega_{ccb}^{+}}(p,s).\end{aligned} \tag{3.11}$$

The form factors F_i and G_i ($i = 1,2,3$), are functions of of the four momentum transferred to the lepton pair ($q = p - p'$), and p and p' denote the four momentas of the initial and final baryons. $u_{\Xi_{cc}^{++}}(p',s')$ and $u_{\Omega_{ccb}^{+}}(p,s)$ are the dirac spinors of the initial and final states. These six form factor are derived using the QCD sum rules.

3.1.3 Spinors

We use the dirac spinors in helicity basis for the calculation of both our leptonic and hadronic amplitudes. The dirac spinor and its anti spinor will be of the form:

$$u(\lambda = \pm 1/2) = \begin{pmatrix} \sqrt{E \mp |\mathbf{p}|}\,\xi_{\pm} \\ \sqrt{E \pm |\mathbf{p}|}\,\xi_{\pm} \end{pmatrix}, \quad v(\lambda = \pm 1/2) = \begin{pmatrix} -\sqrt{E \pm |\mathbf{p}|}\,\xi_{\mp} \\ \sqrt{E \mp |\mathbf{p}|}\,\xi_{\mp} \end{pmatrix}. \tag{3.12}$$

where the helicity eigenspinors are:

$$\xi_+ = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix}, \quad \xi_- = \begin{pmatrix} -\sin\frac{\theta}{2}e^{-i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix}. \tag{3.13}$$

3.1.4 Decay Width

The Hadronic and Leptonic amplitudes in helicity basis can be split into two parts: Vector and Axial

$$H_m^{\lambda_b,\lambda_c} = H_m^{V\lambda_b,\lambda_c} - H_m^{A\lambda_b,\lambda_c}, \tag{3.14}$$

$$L_m^{\lambda_{\bar{\ell}},\lambda_{v_\ell}} = L_m^{V\lambda_{\bar{\ell}},\lambda_{v_\ell}} - L_m^{A\lambda_{\bar{\ell}},\lambda_{v_\ell}}. \tag{3.15}$$

Following an analogous procedure as shown in [3], [4], [5], we define the spinors and the polarizations vectors for J=1/2. We work in the rest frame of the parent baryon Ω_{ccb}^+ and the daughter baryon Ξ_{cc}^{++} moving in the negative z direction. The helcity of the parent baryon (λ_a) will be fixed by conservation of angular momentum, such that $\lambda_a = -\lambda_b + \lambda_c$. Thus we can take the four momentum and the polarization vectors to be:

$$\begin{aligned} q^\mu &= (q_0, 0, 0, |\mathbf{p}'|), \\ p^\mu &= (m_{\Omega_{ccb}^+}, 0, 0, 0), \\ p'^\mu &= (E_2, 0, 0, -|\mathbf{p}'|), \end{aligned} \tag{3.16}$$

$$\begin{aligned} \epsilon^\mu(t) &= \frac{1}{\sqrt{q^2}}(q_0, 0, 0, |\mathbf{p}'|), \\ \epsilon^\mu(0) &= \frac{1}{\sqrt{q^2}}(|\mathbf{p}'|, 0, 0, q_0), \\ \epsilon^\mu(\pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0). \end{aligned} \tag{3.17}$$

We now introduce the definitions for our kinematic variables:

$$\begin{aligned} m_\pm &= m_{\Omega_{ccb}^+} + m_{\Xi_{cc}^{++}}, \\ q_0 &= \frac{(m_+ m_- + q^2)}{2m_{\Omega_{ccb}^+}}, \\ q_\pm &= m_\pm^2 - q^2, \end{aligned} \tag{3.18}$$

so we can write the magnitude of the momentum in the z dirction in terms of these variables as:

$$|\boldsymbol{p_2}| = \frac{\sqrt{q_+ q_-}}{2m_{\Omega_{ccb}^+}}. \tag{3.19}$$

The full helicity amplitudes in terms of the baryonic form factors can then be calulated using the spinors defined in the previous section along with our kinematic notation:

$$\begin{aligned}
H^{V,A}_{+1/2,0} &= \frac{1}{\sqrt{q^2}}\sqrt{2m_{\Omega^+_{ccb}} m_{\Xi^{++}_{cc}}(\sigma \mp 1)}\times \\
&\quad \left[(m_{\Omega^+_{ccb}} \pm m_{\Xi^{++}_{cc}})\mathcal{F}^{V,A}_1(\sigma) \pm m_{\Xi^{++}_{cc}}(\sigma \pm 1)\mathcal{F}^{V,A}_2(\sigma) \pm m_{\Omega^+_{ccb}}(\sigma \pm 1)\mathcal{F}^{V,A}_3(\sigma)\right], \quad (3.20)\\
H^{V,A}_{+1/2,1} &= -2\sqrt{\frac{m_{\Omega^+_{ccb}}}{m_{\Xi^{++}_{cc}}}}\sqrt{m_{\Omega^+_{ccb}} m_{\Xi^{++}_{cc}}(\sigma \mp 1)}\mathcal{F}^{V,A}_1(\sigma), \quad (3.21)\\
H^{V,A}_{+1/2,t} &= \frac{1}{\sqrt{q^2}}\sqrt{2m_{\Omega^+_{ccb}} m_{\Xi^{++}_{cc}}(\sigma \pm 1)}\times \\
&\quad \left[(m_{\Omega^+_{ccb}} \mp m_{\Xi^{++}_{cc}})\mathcal{F}^{V,A}_1(\sigma) \pm (m_{\Omega^+_{ccb}} - m_{\Xi^{++}_{cc}}\sigma)\mathcal{F}^{V,A}_2(\sigma) \pm (m_{\Omega^+_{ccb}}\sigma - m_{\Xi^{++}_{cc}})\mathcal{F}^{V,A}_3(\sigma)\right], \quad (3.22)
\end{aligned}$$

where,

$$\sigma = \frac{m^2_{\Omega^+_{ccb}} + m^2_{\Xi^{++}_{cc}} - q^2}{2m_{\Omega^+_{ccb}} m_{\Xi^{++}_{cc}}}. \quad (3.23)$$

Using our kinematic notation and the amplitudes we follow the prescription given in [3] to write the two fold angular distribution for $\Omega^+_{ccb} \to \Xi^{++}_{cc}\ell\bar{\nu}_\ell$:

$$\frac{d\Gamma(\Omega^+_{ccb} \to \Xi^{++}_{cc}\ell\bar{\nu}_\ell)}{dq^2 cos\theta} = \frac{G^2_F}{(2\pi)^3}|V_{ub}|^2\frac{\lambda^{1/2}(q^2-m^2_\ell)^2}{48m^3_{\Omega^+_{ccb}} q^2}V(\theta, q^2), \quad (3.24)$$

where $V(\theta, q^2)$ contains terms in terms of the helcity structures of the hadronic part and can be found in [6]. Using that and integrating over $cos\theta$ we obtain the differntial decay width for our semileptonic decay:

$$\frac{d\Gamma(\Omega^+_{ccb} \to \Xi^{++}_{cc}\ell\bar{\nu}_\ell)}{dq^2} = \frac{G^2_F}{(2\pi)^3}|V_{ub}|^2\frac{\lambda^{1/2}(q^2-m^2_\ell)^2}{48m^3_{\Omega^+_{ccb}} q^2}\mathcal{H}_{tot}(q^2), \quad (3.25)$$

where,

$$\begin{aligned}
\mathcal{H}_{\rm tot}(q^2) &= \left[|H_{+\frac{1}{2},+1}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{+\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2\right]\left(1+\frac{m^2_\ell}{2q^2}\right) \\
&\quad + \frac{3m^2_\ell}{2q^2}\left[|H_{+\frac{1}{2},t}|^2 + |H_{-\frac{1}{2},t}|^2\right]. \quad (3.26)
\end{aligned}$$

The following combination of helcity amplitudes represent:

$$\begin{aligned}
&|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 && \text{transverse unpolarized,}\\
&|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 && \text{longitudinal unpolarized,}\\
&|H_{\frac{1}{2},t}|^2 + |H_{-\frac{1}{2},t}|^2 && \text{scalar unpolarized.}
\end{aligned}$$

4 Form Factors Computation

Using the QCD sum rules we find that the weak decay baryon form factors can be approximated by the following expression:

$$\mathcal{F}(q^2) = \frac{\mathcal{F}(0)}{\left(1 - a_1 \frac{q^2}{m^2_{\Omega^+_{ccb}}} + a_2 \left(\frac{q^2}{m^2_{\Omega^+_{ccb}}}\right)^2 + a_3 \left(\frac{q^2}{m^2_{\Omega^+_{ccb}}}\right)^3 + a_4 \left(\frac{q^2}{m^2_{\Omega^+_{ccb}}}\right)^4\right)}. \tag{4.1}$$

The values of the fitting parameters $\mathcal{F}(0), a_1, a_2, a_3$, are taken from Table III in [7]. From this we plot the form factors with their errors at the average values of the auxillary parameters. The computation for the six form factors $F_1, F_2, F_3, G_1, G_2, G_3$ was done in Mathematica in the following way:

```
x=F0/(1-a1*(p/m^2)+a2*(p/m^2)^2+a3*(p/m^2)^3+a4*(p/m^2)^4);
dF0s=D[x,F0];
dFm=D[x,m];
params={a1->2.52,a2->-0.25,a3->3.43,a4->0.99,F0->0.06,dF0->0.01,m
    ->8.15, dmDn-> 0.23, dmUp -> 0.27};
F[p_?NumericQ]:=(x/.params)/.p->p;
dFplus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2 + (dFm * dmUp)^2]/.params/.p->p;
dFminus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2 + (dFm * dmDn)^2]/.params/.p->p
    ;

Plot[Evaluate[{F[p],F[p]+dFplus[p],F[p]-dFminus[p]}],{p,0,21},Frame ->
    True,PlotStyle->{{Thick,Black},Directive[Blue,Thin],Directive[Blue,
    Thin]},FrameLabel->{Style[Row[{Superscript["q",2],"/",Superscript["
    GeV",2]}],Bold,18],Style["F1",Bold,18]},Filling->{2->{3}},
    FillingStyle->{{Opacity[0.0]}},FrameStyle->Directive[Black,FontSize
    ->18,Bold],
AxesLabel->{"p (GeV\.b2)","F1(p)"},GridLines->Automatic,ImageSize->
    Large]
```

Listing 1: Mathematica code to plot F_1

```
x=F0/(1-a1*(p/m^2)+a2*(p/m^2)^2+a3*(p/m^2)^3+a4*(p/m^2)^4);
dF0s=D[x,F0];dFm=D[x,m];params={a1->2.92,a2->-0.82,a3->11.13,a4
    ->-11.32,F0->0.26,dFu->0.07,dFd->- 0.05, m->8.15, dmDn-> 0.23, dmUp
    -> 0.27 };
F[p_?NumericQ]:=(x/.params)/.p->p;
dFplus[p_?NumericQ]:=Sqrt[(dF0s*dFu)^2 + (dFm * dmUp)^2]/.params/.p->p;
dFminus[p_?NumericQ]:=Sqrt[(dF0s*dFd)^2  + (dFm * dmDn)^2]/.params/.p->
    p;
Plot[Evaluate[{F[p],F[p]+dFplus[p],F[p]-dFminus[p]}],{p,0,21},Frame ->
    True,PlotStyle->{{Thick,Black},Directive[Blue,Thin],Directive[Blue,
    Thin]},FrameLabel->{Style[Row[{Superscript["q",2],"/",Superscript["
    GeV",2]}],Bold,18],Style["F2",Bold,18]},Filling->{2->{3}},
    FillingStyle->{{Opacity[0.0]}},FrameStyle->Directive[Black,FontSize
    ->18,Bold],
AxesLabel->{"p (GeV\.b2)","F1(p)"},GridLines->Automatic,ImageSize->
    Large]
```

Listing 2: Mathematica code to plot F_2

```
x=F0/(1-a1*(p/m^2)+a2*(p/m^2)^2+a3*(p/m^2)^3+a4*(p/m^2)^4);
dF0s=D[x,F0];dFm=D[x,m];params={a1->1.41,a2->-15.54,a3->78.45,a4
    ->-124.53,F0->-2.52,dFu->0.74,dFd->  0.75, m->8.15};
F[p_?NumericQ]:=(x/.params)/.p->p;
dFplus[p_?NumericQ]:=Sqrt[(dF0s*dFu)^2]/.params/.p->p;
dFminus[p_?NumericQ]:=Sqrt[(dF0s*dFd)^2]/.params/.p->p;
Plot[Evaluate[{F[p],F[p]+dFplus[p],F[p]-dFminus[p]}],{p,0,21},Frame ->
    True,PlotStyle->{{Thick,Black},Directive[Blue,Thin],Directive[Blue,
    Thin]},FrameLabel->{Style[Row[{Superscript["q",2],"/",Superscript["
    GeV",2]}],Bold,18],Style["F3",Bold,18]},Filling->{2->{3}},
    FillingStyle->{{Opacity[0.0]}},FrameStyle->Directive[Black,FontSize
    ->18,Bold],
AxesLabel->{"p (GeV\.b2)","F1(p)"},GridLines->Automatic,ImageSize->
    Large]
```

Listing 3: Mathematica code to plot F_3

```
x=F0/(1-a1*(p/m^2)+a2*(p/m^2)^2+a3*(p/m^2)^3+a4*(p/m^2)^4);
dF0s=D[x,F0];
params={a1->1.80,a2->-17.87,a3->92.95,a4->-145.86,F0->0.84,dF0->0.25, m
    ->8.15};
F[p_?NumericQ]:=(x/.params)/.p->p;
dFplus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2]/.params/.p->p;
dFminus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2]/.params/.p->p;
Plot[Evaluate[{F[p],F[p]+dFplus[p],F[p]-dFminus[p]}],{p,0,21},Frame ->
    True,PlotStyle->{{Thick,Black},Directive[Blue,Thin],Directive[Blue,
    Thin]},FrameLabel->{Style[Row[{Superscript["q",2],"/",Superscript["
    GeV",2]}],Bold,18],Style["G1",Bold,18]},Filling->{2->{3}},
    FillingStyle->{{Opacity[0.0]}},FrameStyle->Directive[Black,FontSize
    ->18,Bold],
AxesLabel->{"p (GeV\.b2)","F1(p)"},GridLines->Automatic,ImageSize->
    Large]
```

Listing 4: Mathematica code to plot G_1

```
x=F0/(1-a1*(p/m^2)+a2*(p/m^2)^2+a3*(p/m^2)^3+a4*(p/m^2)^4);
dF0s=D[x,F0];
params={a1->3.12,a2->-10.14,a3->67.61,a4->-106.61,F0->0.45,dF0->0.13,m
    ->8.15};
F[p_?NumericQ]:=(x/.params)/.p->p;
dFplus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2]/.params/.p->p;
dFminus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2]/.params/.p->p;
Plot[Evaluate[{F[p],F[p]+dFplus[p],F[p]-dFminus[p]}],{p,0,21},Frame ->
    True,PlotStyle->{{Thick,Black},Directive[Blue,Thin],Directive[Blue,
    Thin]},FrameLabel->{Style[Row[{Superscript["q",2],"/",Superscript["
    GeV",2]}],Bold,18],Style["G2",Bold,18]},Filling->{2->{3}},
    FillingStyle->{{Opacity[0.0]}},FrameStyle->Directive[Black,FontSize
    ->18,Bold],
AxesLabel->{"p (GeV\.b2)","F1(p)"},GridLines->Automatic,ImageSize->
    Large]
```

Listing 5: Mathematica code to plot G_2

```
x=F0/(1-a1*(p/m^2)+a2*(p/m^2)^2+a3*(p/m^2)^3+a4*(p/m^2)^4);
dF0s=D[x,F0];
params={a1->2.14,a2->-5.94,a3->29.93,a4->-39.92,F0->0.34,dF0->0.10,m
    ->8.15};
F[p_?NumericQ]:=(x/.params)/.p->p;
dFplus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2]/.params/.p->p;
dFminus[p_?NumericQ]:=Sqrt[(dF0s*dF0)^2]/.params/.p->p;
Plot[Evaluate[{F[p],F[p]+dFplus[p],F[p]-dFminus[p]}],{p,0,21},Frame ->
    True,PlotStyle->{{Thick,Black},Directive[Blue,Thin],Directive[Blue,
    Thin]},FrameLabel->{Style[Row[{Superscript["q",2],"/",Superscript["
    GeV",2]}],Bold,18],Style["G3",Bold,18]},Filling->{2->{3}},
    FillingStyle->{{Opacity[0.0]}},FrameStyle->Directive[Black,FontSize
    ->18,Bold],
AxesLabel->{"p (GeV\.b2)","F1(p)"},GridLines->Automatic,ImageSize->
    Large]
```

Listing 6: Mathematica code to plot G_3

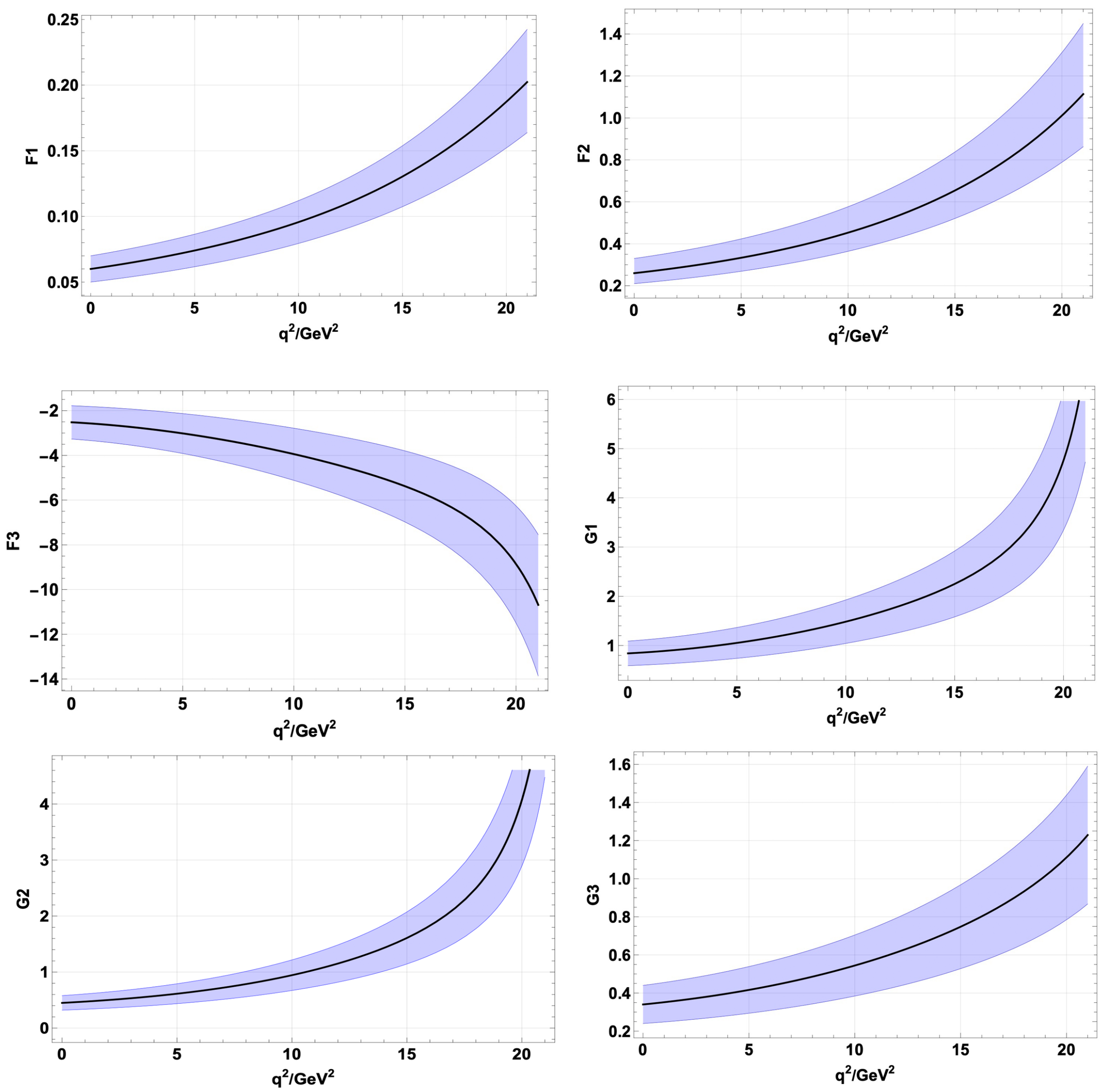

Figure 3: Form factors with their errors at average values of auxillary parameters.

5 Results

The calculated decay widths for all lepton channels along with their associated errors are provided in Table 1. The decay width itself serves as an important indicator for guiding experimental searches and theoretical analysis.

Table 1: Exclusive semileptonic decay widths of $\Omega_{ccb} \to \Xi_{cc}^{++} \ell \bar{\nu}_\ell$ for $\ell = e, \mu, \tau$. All widths are given in units of GeV.

Decay Channel	**Decay Width Γ (GeV)**
$\Omega_{ccb} \to \Xi_{cc}^{++} e\, \bar{\nu}_e$	$(1.23^{+0.28}_{-0.56}) \times 10^{-14}$
$\Omega_{ccb} \to \Xi_{cc}^{++} \mu\, \bar{\nu}_\mu$	$(1.22^{+0.28}_{-0.55}) \times 10^{-14}$
$\Omega_{ccb} \to \Xi_{cc}^{++} \tau\, \bar{\nu}_\tau$	$(7.32^{+1.48}_{-3.16}) \times 10^{-15}$

As seen from the table, the e and μ channels have essentially identical widths, while the τ channel is roughly half as large. This trend is a direct consequence of the larger τ mass: heavier leptons have reduced phase space, leading to kinematic suppression of the decay rate. These numbers will serve as input for predicting branching fractions once the lifetime of Ω_{ccb} is measured.

Assessing the ratio of decay width in τ and e/μ channels is advantageous due to its uncertainty. We find:

$$R = \frac{\Gamma\left[\Omega_{ccb}^{+} \to \Xi_{cc}^{++} \tau \bar{\nu}_\tau\right]}{\Gamma\left[\Omega_{ccb}^{+} \to \Xi_{cc}^{++} e(\mu) \bar{\nu}_{e(\mu)}\right]} = 0.60^{+0.07}_{-0.02}. \tag{5.1}$$

6 Conclusion

To date, no experimental evidence has been found for triply heavy baryons, which represent the final predicted category of heavy baryons in the quark model. Theoretical studies on their weak decays are still limited, with most focusing on transitions between spin-3/2 triply heavy baryons or on decays from spin-1/2 to spin-3/2 states. This work marks one of the first investigations into weak decays where both the initial and final baryons are triply heavy spin-1/2 states, covering all three lepton channels.

The analysis shows that the semileptonic decay of the Ω_{ccb} baryon is dominated by the light-lepton channels, with the electron and muon decays having nearly equal rates and the tau channel reduced by about a factor of two. This is fully consistent with standard model expectations: lepton universality implies $\Gamma_e \simeq \Gamma_\mu$, while the larger τ mass incurs a phase-space suppression.

The overall scale of the widths ($\sim 10^{-14}$ GeV) suggests that, once the Ω_{ccb} lifetime is known, the branching fractions for these modes will be small but potentially measurable - thus all the channels are accessible.

In practice, this means experimental searches (for example at LHCb or future colliders) should expect comparable signals in the e and μ channels and roughly half as many events in the τ channel. In terms of theoretical consistency, our findings agree with analogous analyses of heavy-baryon decays (where heavier leptons yield smaller rates) and with general heavy-quark effective theory expectations. No previous experimental data exist for the Ω_{ccb} (it remains undiscovered), so these predictions are novel; however, they fit into the established pattern seen in other semileptonic heavy-flavor decays.

Looking ahead, further work could improve the precision (for example, by including higher-order corrections or lattice QCD inputs for the form factors) and extend to related channels. Moreover, the polarization and helicity asymmetries can be checked. The methodology developed here can also be extended to other triply heavy baryons and their corresponding decay widths calculated.

Overall, our computed decay widths not only provide a consistency check of the theory, but also serve as practical guidance for the search and study of the Ω_{ccb} baryon in upcoming experiments.

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