复杂系统建模论文报告

Defectors in bad circumstances possessing higher reputation can promote cooperation

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Content

Chaos

Defectors in bad circumstances possessing higher reputation can promote cooperation

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Research background

In nature and human society, social relationships and behavior patterns are usually unpredictable. The concept of "reputation" can provide some information to mitigate(means weaken) such uncertainty.

In previous studies, researchers have considered that only **cooperators** are able to maintain a high reputation.
In reality, however, some individuals will be forced to defect to protect themselves against exploitation. Therefore, **defectors in bad circumstances** could also obtain higher reputations, and cooperators can maintain higher reputations in comfortable circumstances.

Research Method

In this work, the reputations of individuals are calculated using the **fraction of their neighbors who have the same strategy**. Therefore, some defectors in a population may obtain higher reputations than some cooperators.

Research process and method

- The reputation rule using **heterogeneous investments in public goods games**.
- Dynamical evolution is observed in *Monte Carlo simulations*.
- The effects of the noise intensity of the irrational population and the original proportion of cooperation in the population.
- Use numerical simulation to indicate conclusions.

Research significance

Main results and research significance

1. The reputation rule and heterogeneous investments can better stimulate cooperation.
2. Stronger investment heterogeneity can further increase the level of cooperation.
3. The conclusion is the same when we consider network structure and total investment.

Content

Preliminary

- Theory of game.
- Spatial PGG(Public Goods Game) model.
- Barabási–Albert (BA) scale-free network.
- Monte Carlo simulations.
- Fermi rule.
- Numerical simulation by **Fortran**.

PGG model

PGG model

The public goods game(PGG)[2] is a standard of experimental economics. In the basic game, subjects secretly choose how many of their private tokens to put into a public pot. The tokens in this pot are multiplied by a factor (greater than one and less than the number of players, N) and this "public good" payoff is evenly divided among players. Each subject also keeps the tokens they do not contribute.

Barabási–Albert (BA) model

BA model

The Barabási–Albert (BA)[1] model is an algorithm for generating random scale-free networks using a preferential attachment mechanism.
Several natural and human-made systems, including the Internet, the World Wide Web, citation networks, and some social networks are thought to be approximately scale-free and certainly contain few nodes (called hubs) with unusually high degree as compared to the other nodes of the network.

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Paper Structure

1. the reputation rule and the *spatial PGG model*
2. the results of numerical simulations
3. summarizes the paper and puts forward directions for future research

Research hypothesis

1. An individual's reputation may reflect the willingness of their neighbors to cooperate with them.
2. Most previous studies of reputation have assumed that social interactions are public, which means that everyone knows about the details of these interactions.
3. ★ Some individuals in bad circumstances will be forced to defect to protect themselves against exploitation.
4. A cooperator (defector) with more cooperative (defective) neighbors can obtain a higher reputation.

The reputation rule

The reputation $R_i(t) \in [0,1]$ of individual i at step t:

$$R_i(t) = \begin{cases} \dfrac{N_i(t-1)}{k_i(t-1)} & \text{if } s_i(t-1) = 1, \\ 1 - \dfrac{N_i(t-1)}{k_i(t-1)} & \text{if } s_i(t-1) = 0. \end{cases} \tag{3.1}$$

where

- $N_i(t-1)$: the number of cooperators among the neighbors of individual i at step $t-1$;
- k_i: the degree of individual i;
- s_i: the strategy of individual i($s_i = 1$ if individual i is a cooperator,otherwise $s_i = 0$).

Spatial PGGs in structured populations

Consider a $L \times L$ square lattice with periodic boundary conditions. Each individual is located on a single node surrounded by four nearest neighbors.
In a square lattice, there are five different reputation levels.
The possible reputation values for each individual are $[0, 1/4, 1/2, 3/4, 1]$.
Also consider a lattice with eight nearest neighbors and the Barabási–Albert (BA) scale-free network.

Monte Carlo simulations

Three elementary procedures

1. investment allocation;
2. payoff accumulation;
3. strategy updating.

investment allocation

The total investments c of each cooperator are equal and $c = 1$.
In contrast to the traditional PGG, define **heterogeneous investments induced by reputation**.
At step t, $I_{i \to j}(t)$ is the investment in the group organized by individual j from cooperator i.

investment allocation

$$I_{i\to j}(t) = c \cdot \frac{e^{\alpha \cdot R_j(t-1)}}{\sum_{k\in\Omega_i} e^{\alpha \cdot R_k(t-1)}} \tag{3.2}$$

where

- Ω_i: the collection of individual i and its neighbors;
- $\alpha(\alpha \geq 0)$: a tunable parameter controlling the strength of investment heterogeneity.

When $\alpha = 0$, the model collapses to the traditional PGG, in which cooperators invest in all common pools with no differences.

investment allocation

The detailed calculation of the investment using Eq 3.2 starts at the second step.
All investments $I_{i \to j}$ take a value of 0.2 at the first step. In real life, individuals tend to invest more in other individuals with higher reputations, this corresponds to the case of $\alpha > 0$.
A defector with only one cooperative neighbor is also considered to have a better reputation; the cooperative neighbor will be willing to provide certain help to the benefit of this defector.

investment allocation

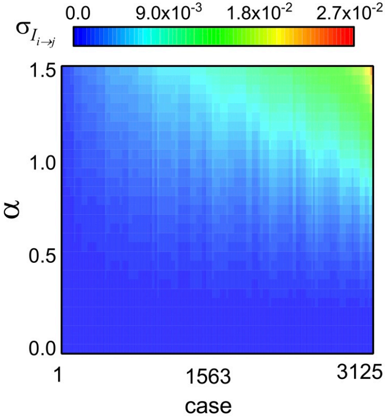
In Eq 3.2, α is a key parameter to distinguish from the tradition PGG. To better demonstrate the impact of α on the investment, we measure the investment heterogeneity of an individual by calculating the variance of the investment $\sigma_{I_{i\to j}}$.

$$\sigma_{I_{i\to j}} = \frac{1}{G}\sum_{k\in\Omega_i}\left(I_{i\to k} - \frac{c}{G}\right)^2, \tag{3.3}$$

where

- G: the number of individuals in one group;
- $\alpha(\alpha \geq 0)$: a tunable parameter controlling the strength of investment heterogeneity.

investment allocation



In the left several columns, all $\sigma_{I_{i\to j}} = 0$ whatever α is because of the same reputation of all group members and $I_{i\to j} = 0.2$. Except for the several columns, $\sigma_{I_{i\to j}}$ grows with increasing α in the other cases. The investment heterogeneity of an individual becomes stronger as α increases as long as the reputations are not identical.

Fig 1a: Color map of the investment heterogeneity $\sigma_{I_{i\to j}}$ in the α and case space

investment allocation

Calculate the mean of the investment variances for all individuals in a square lattice with the random initial condition:

$$\bar{\sigma}_{I_{i\to j}} = \frac{1}{N}\sum_{i=1}^{N}\sigma_{I_{i\to j}}, \tag{3.4}$$

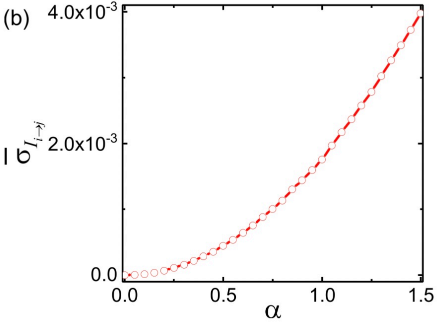


Fig 1b: The relationship between the investment heterogeneity $\sigma_{I_{i\to j}}$ calculated at step $t = 2$ and α. Simulations are carried out for the square lattice.

payoff accumulation

The payoff of individual i at step t can be calculated from

$$U_i(t) = \frac{r}{G} \sum_{j \in \Omega_i} \sum_{k \in \Omega_j} I_{k \to j}(t) - s_i(t) \cdot c. \tag{3.5}$$

strategy updating

Use the **Fermi rule** as the strategy-update rule. In each step, each individual i randomly selects a neighbor j. Then, i learns the strategy of j with a probability P calculated by the Fermi function:

$$P\left(s_j \rightarrow s_i\right)(t)=\frac{1}{1+\exp\left[\left(U_i(t)-U_j(t)\right)/\kappa\right]}, \tag{3.6}$$

κ: the noise intensity of the irrational population, κ is set as 0.1 in this work.

- $\kappa \rightarrow 0$: individual i will adopt the strategy of individual j as long as j has a higher payoff.
- $\kappa \rightarrow +\infty$: random section.

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Numerical simulation

All simulation results are obtained on lattice comprising $N = 100 \times 100$ individuals or a BA scale-free network comprising $N = 3000$ individuals. At the beginning of each simulation, individuals choose one of the strategies of cooperation and defection randomly.

Monitor key quality f_C measures the cooperation frequency in the population.

Calculate f_C within the last 3000 steps of a total of 2×10^4 steps.

Cooperative behavior in a square lattice

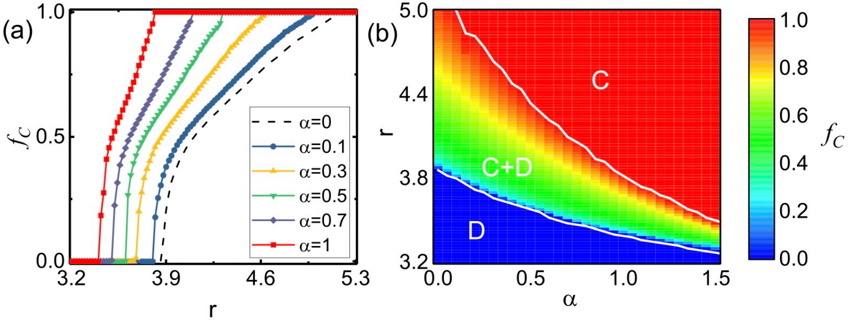

Fig 2:(a) Relationship between the cooperation frequency f_C and the synergy factor r at several heterogeneity strengths α. (b) Color map of the cooperation frequency f_C in the r–α parameter space. The white curves denote the critical values r_{c1} (lower) and r_{c2} (upper).

Dynamical evolution of cooperation

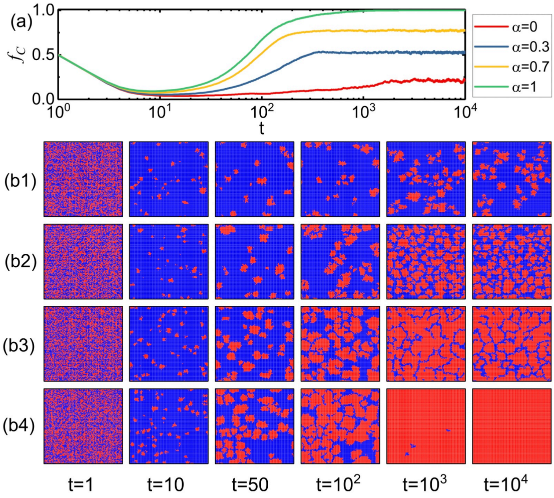

Fig 3 :(a) Time courses of the cooperation frequency f_C with several α values. (b1) – (b4) Snapshots of the distributions of cooperators (red) and defectors (blue) at several typical time nodes: (b1) $\alpha = 0$, (b2) $\alpha = 0.3$, (b3) $\alpha = 0.7$, and (b4) $\alpha = 1.0$. In all cases, $r = 3.9$.

Dynamical evolution of cooperation

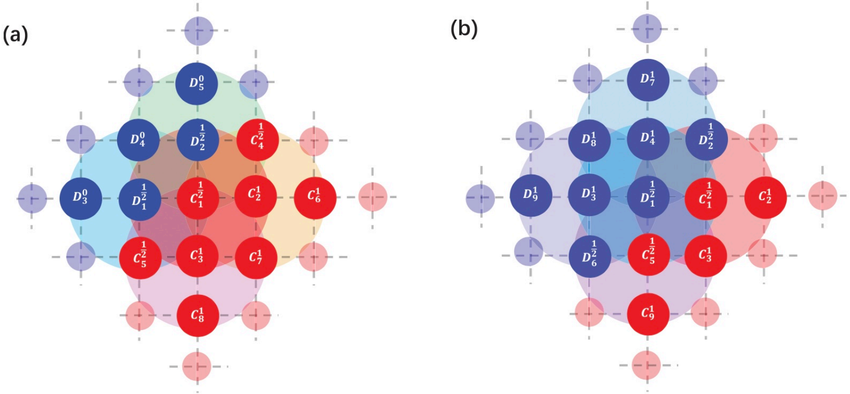

Fig 4: PGGs staged on the edges of C-clusters. (a) PGG focused on a cooperator. (b) PGG focused on a defector. Red and blue points represent cooperators and defectors, respectively. The subscripts denote the serial numbers of individuals, and their corresponding reputations are shown by superscripts.

Dynamical evolution of cooperation

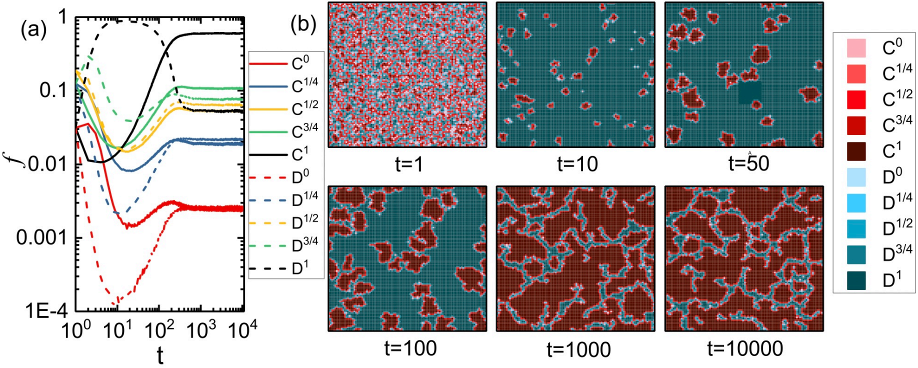

Fig 5 :(a) Time courses of the fractions of ten kinds of individuals. (b) Snapshots of the distributions of these ten kinds of individuals at several representative times. Other parameters: $r = 3.7$, $\alpha = 1.0$.

Influence of other parameters

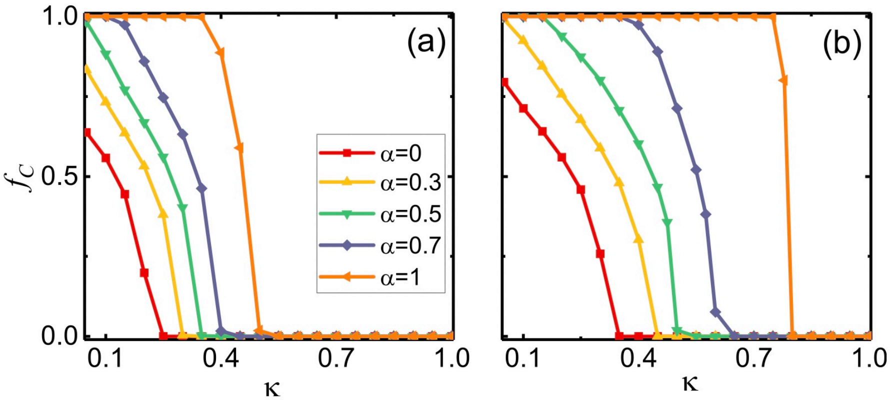

Fig 6 :Relationship between the cooperation frequency f_C and the noise intensity κ at several heterogeneity strengths α: (a) $r = 4.2$ and (b) $r = 4.5$.

Influence of other parameters

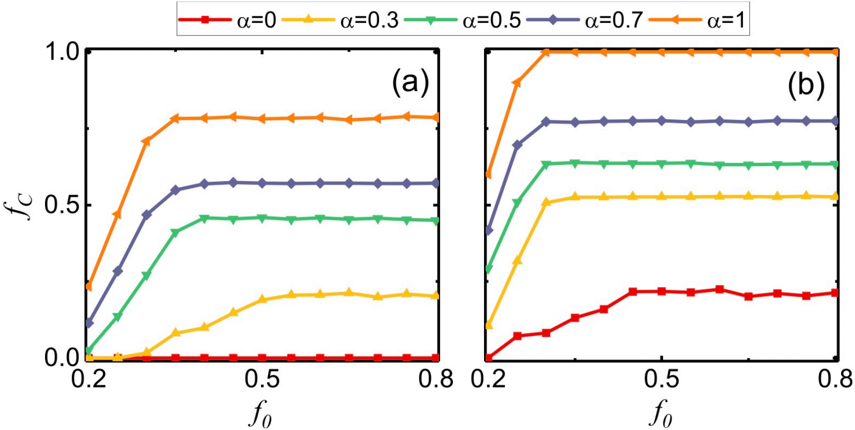

Fig 7 :Relationship between the cooperation frequency f_C and the original proportion of cooperation f_0: (a) $r = 3.7$ and (b) $r = 3.9$.

Robustness of the results

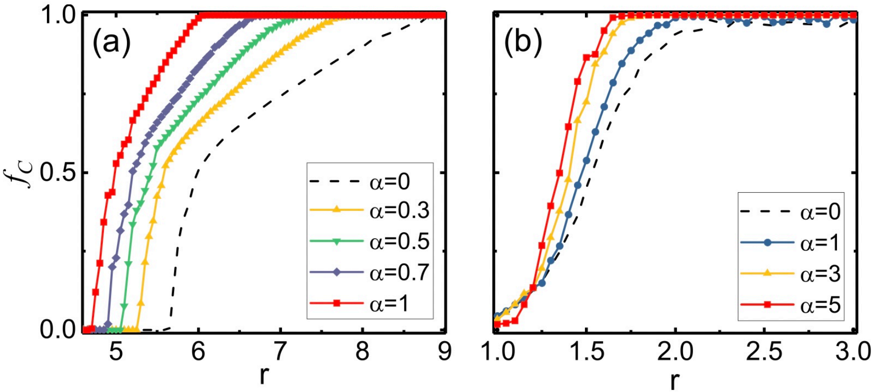

Fig 8 :Relationship between the cooperation frequency f_C and the synergy factor r at several heterogeneity strengths: (a) lattice with a Moore neighborhood and (b) a BA scale-free network with the average degree $\langle k \rangle = 4$.

Robustness of the results

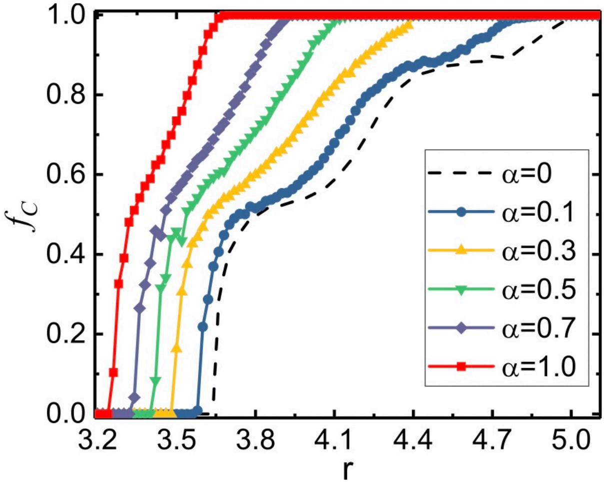

Fig 9 :Relationship between the cooperation frequency f_C and the synergy factor r at several heterogeneity strengths α with $c = 5$.

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Summary and prospect

A reputation mechanism is commonly introduced as being necessary to favor cooperation. In previous studies, only cooperators have been given the chance to maintain a higher reputation.
However, this ignores the fact that some individuals in bad circumstances will be forced to defect to protect themselves against exploitation.
In this work, we worked on the principle that individuals who hold the same strategies as their neighbors can gain higher reputations. This reputation rule is introduced through heterogeneous investments to **spatial PGGs**. The results show that this model achieves better performance in terms of cooperative behavior, and cooperation is more favorable with stronger heterogeneity of investments.

Summary and prospect

In this work, we found that the reputation calculation method can provide defectors in bad circumstances a higher reputation. It should be noted that, in this model, the reputation value depends only on the strategies chosen by each individual. In real-world situations, however, the reputations of individuals may be influenced by other social factors. In future research, a combination of strategies and other social factors could be considered. We hope this work contributes to the understanding of social reputation in structured populations from an evolutionary perspective.

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Thank you!