Let $g = GCD(n,k)$ so $n = n_1 \times g$; $k = k_1 \times g$; (n_1, k_1) coprime
$X_0 = x(0) \wedge x(g) \wedge x(2g) \wedge ... \wedge x((n_1-1)g)$
$X_1 = x(1) \wedge x(g+1) \wedge x(2g+1) \wedge ... \wedge x((n_1-1)g+1)$
...
$X_{g-1} = x(g-1) \wedge x(2g-1) \wedge x(3g-1) \wedge ... \wedge x(n_1 g - 1)$

As: $k = k_1 \times g$ turning k consecutive $x(j)$ changes exactly $k_1$bits in each X_i

$$\begin{aligned}
S(N) &= \sum_{n_1=1}^{n_1=N} \sum_{g=1}^{g\times n_1 \le N} \sum_{\substack{k_1=1,\\ gcd(k_1,n_1)=1}}^{k_1 \le n_1} 2^{n_1 g - g + 1 + IsOdd(k_1)} \\
&= \sum_{n_1=1}^{n_1=N} \sum_{g=1}^{g\times n_1 \le N} 2^{n_1 g - g} \sum_{\substack{k_1=1,\\ gcd(k_1,n_1)=1}}^{k_1 \le n_1} 2^{1 + IsOdd(k_1)} \\
&= \sum_{n_1=1}^{n_1=N} \sum_{g=1}^{g\times n_1 \le N} 2^{(n_1-1)^g} \sum_{\substack{k_1=1,\\ gcd(k_1,n_1)=1}}^{k_1 \le n_1} 2^{1 + IsOdd(k_1)} \\
&= \sum_{n_1=1}^{n_1=N} \sum_{g=1}^{g\times n_1 \le N} 2^{(n_1-1)^g} \times \begin{cases} 2\varphi(n_1) & n_1 \text{ even} \\ \varphi(n_1) + \varphi(n_1)/2 & n_1 \text{ odd} \end{cases} \\
&= \sum_{n_1=1}^{n_1=N} 2^{n_1-1}[1 + 2^{n_1-1} + (2^{n_1-1})^2 + ... + (2^{n_1-1})^{gmax}] \times \begin{cases} 2\varphi(n_1) & n_1 \text{ even} \\ \varphi(n_1) + \varphi(n_1)/2 & n_1 \text{ odd} \end{cases}
\end{aligned}$$

as if n_1 even then k_1 odd, so $\varphi(n_1) =$ number of k_1 $gcd(k_1, n_1) = 1$
and if n_1 odd then $gcd(k_1, n_1) = 1 \Leftrightarrow gcd(n_1 - k_1, n_1) = 1$ and $k_1, n_1 - k_1$have different parity