

For a string  $s$ ,  $L(k)$  is the length of the longest substring of  $s$  which appears at least  $k$  times in  $s$ .  
Let us define  $K(l)$  as the maximum occurrences of any substring of  $s$  with length  $l$ .  
It is easy to see that  $L(k)$  and  $K(l)$  are decreasing functions and inverse each other.

So  $\sum_{k \geq 1} L(k) = \sum_{l \geq 1} K(l)$  as illustrated by figures above for  $S_{20}$