$$S(n) = \sum_{\substack{i,j,k,l \\ 0<i<j<k<l\leq n}} (a_i + a_j + a_k + a_l) \times 1_{a_i<a_j<a_k<a_l} = \sum_{\substack{i,j,k,l \\ 0<i<j<k<l\leq n}} ((a_i + a_j) + (a_k + a_l)) \times 1_{a_i<a_j<a_k} \times 1$$

$$= \sum_{\substack{i,j,k,l \\ 0<i<j<k<l\leq n}} ((a_i + a_j) \times 1_{a_i<a_j<a_k} \times 1_{a_k<a_l} + (a_k + a_l) \times 1_{a_i<a_j<a_k} \times 1_{a_k<a_l})$$

$$= \sum_{0<k\leq n} \{ \sum_{\substack{i,j \\ 0<i<j<k}} ((a_i + a_j) \times 1_{a_i<a_j<a_k}) \times (\sum_{\substack{l \\ k<l\leq n}} 1_{a_k<a_l})\} + \sum_{0<j\leq n} \{ \sum_{\substack{i \\ 0<i<j}} 1_{a_i<a_j} \times (\sum_{\substack{k,l \\ j<k<l\leq n}} ((a_k + a_l) \times 1_{a_j}$$

$$= \sum_{0<k\leq n} S_{L2}(k) \times W_H(h) + \sum_{0<j\leq n} S_{H2}(j) \times W_L(j)$$

Where:

$W_L(j) = \sum_{\substack{i \\ 0<i<j}} 1_{a_i<a_j}$ *(resp.)* $W_H(k) = \sum_{\substack{l \\ k<l\leq n}} 1_{a_k<a_l}$

$S_{L2}(k) = \sum_{\substack{i,j \\ 0<i<j<k}} ((a_i + a_j) \times 1_{a_i<a_j<a_k})$ *(resp.)* $S_{H2}(j) = \sum_{\substack{k,l \\ j<k<l\leq n}} ((a_k + a_l) \times 1_{a_j<a_k<a_l})$

$S_{L2}(k)$ can be computed by :

$S_{L2}(k) = \sum_{\substack{j \\ 0<j<k}} 1_{a_j<a_k}(S_{L1}(j) + a_j \times W_L(j))$ with: $S_{L1}(j) = \sum_{\substack{i \\ 0<i<j}} a_i \times 1_{a_i<a_j}$

We can compute by Fenwick tree for any function f(i) $F(i \to f(i), j) = \sum_{\substack{i \\ 0<i<j}} f(i) \times 1_{a_i<a_j}$

(resp) $F^T(i \to f(i), j) = \sum_{\substack{i \\ j<i\leq n}} f(i) \times 1_{a_j<a_i}$

So the algorithm is:

$W_L(j) = F(i \to 1, j)$

$S_{L1}(j) = F(i \to a_i, j)$; $S_{L1*}(j) = S_{L1}(j) + a_j W_L(j)$; $S_{L2}(j) = F(i \to S_{L1*}(j), j)$

$W_H(j) = F^T(i \to 1, j)$

$S_{H1}(j) = F^T(i \to a_i, j)$; $S_{H1*}(j) = S_{H1}(j) + a_j W_H(j)$; $S_{H2}(j) = FT(i \to S_{H1*}(j), j)$

$S(n) = \sum_{\substack{j \\ 0<j\leq n}} S_{L2}(j)W_H(j) + S_{H2}(j)W_L(j)$

$$S_{L2}(k) = \sum_{\substack{i,j \\ 0<i<j<k}} ((a_i + a_j) \times 1_{a_i<a_j} \times 1_{a_j<a_k}) = \sum_{\substack{j \\ 0<j<k}} 1_{a_j<a_k} \times \sum_{\substack{i \\ 0<i<j}} ((a_i + a_j) \times 1_{a_i<a_j})$$

$$= \sum_{\substack{j \\ 0<j<k}} 1_{a_j<a_k} \times S_{L1*}(j)$$

with: $S_{L1*}(j) = \sum_{\substack{i \\ 0<i<j}} (a_i + a_j) \times 1_{a_i<a_j} = \sum_{\substack{i \\ 0<i<j}} ((a_i \times 1_{a_i<a_j}) + a_j \times \sum_{\substack{i \\ 0<i<j}} 1_{a_i<a_j}$

$= S_{L1}(j) + a_j \times W_L(j)$ with: $S_{L1}(j) = \sum_{\substack{i \\ 0<i<j}} a_i \times 1_{a_i<a_j}$

Remark : to implement the division by 2, we compute $R_{\Delta-1}$