Let the generative function for n drones $f_n(x) = \sum_{k=0}^{\propto} p_{n,t=k}\ x^k$

Where $p_{n,t=k}$ is the probability that the last drone starts at time $t = k$

by decomposing an even for n+1 drones with the time the n^{th} start :

$f_{n+1}(x) = f_n(\frac{n}{n+1}x) \times g_{n+1}(x)$

where $g_{n+1}(x) = \sum_{k=0}^{\propto} \frac{1}{n+1}(\frac{n}{n+1})^k x^{k+1} = \frac{x}{n+1} \times \frac{1}{1-\frac{n}{n+1}x} = \frac{x}{n+1-nx}$

so we have $f_{n+1}(x) = f_n(\frac{n}{n+1}x) \times \frac{x}{n+1-nx}$ and $f_1(x) = x$

$$\text{it is easy to show that :} f_n(x) = \frac{1}{n}x^n \prod_{k=1}^{n-1} \frac{1}{n-kx}$$

$$\begin{aligned}
f_{n+1}(x) &= f_n(\frac{n}{n+1}x) \times \frac{x}{n+1-nx} \\
&= \frac{1}{n}(\frac{n}{n+1}x)^n \ (\prod_{k=1}^{n-1} \frac{1}{n-k(\frac{n}{n+1}x)}) \times \frac{x}{n+1-nx} \\
&= \frac{1}{n+1}x^{n+1} \ (\prod_{k=1}^{n-1} \frac{n}{n+1} \times \frac{1}{n-k(\frac{n}{n+1}x)}) \times \frac{1}{n+1-nx} \\
&= \frac{1}{n+1}x^{n+1} \ (\prod_{k=1}^{n-1} \frac{1}{n+1-kx}) \times \frac{1}{n+1-nx} = \frac{1}{n+1}x^{n+1} \prod_{k=1}^{n} \frac{1}{n+1-kx}
\end{aligned}$$

By observing that the sum of drone speeds increases by one at each second, we have Sum(v)=t and so the traveled distance for end t=k is $S_d = 1 + 2 + ... + k = \dfrac{k(k+1)}{2}$

so the expected distance is $E(n) = \dfrac{1}{n} \displaystyle\sum_{k=0}^{\propto} p_{n,t=k} \ \dfrac{k(k+1)}{2}$

as $f_n^{'}(x) = \displaystyle\sum_{k=1}^{\propto} p_{n,t=k} \ k \ x^{k-1}$ and $f_n^{''}(x) = \displaystyle\sum_{k=1}^{\propto} p_{n,t=k} \ k(k-1) \ x^{k-2}$

$f_n^{'}(1) = \displaystyle\sum_{k=1}^{\propto} p_{n,t=k} \ k$ and $f_n^{''}(1) = \displaystyle\sum_{k=1}^{\propto} p_{n,t=k} \ k(k-1)$

and $\displaystyle\sum_{k=0}^{\propto} p_{n,t=k} \ k(k+1) = f_n^{''}(1) + 2f_n^{'}(1)$

so $E(n) = \dfrac{1}{n}(\dfrac{f_n^{''}(1)}{2} + f_n^{'}(1))$

We can show :$f_n^{'}(1) = nS_1(n)$ and $f_n^{''}(1) = n^2 S_2(n) + n^2 S_1(n)^2 - 2nS_1(n)$

so $E(n) = \dfrac{1}{n}(\dfrac{f_n^{''}(1)}{2} + f_n^{'}(1)) = \dfrac{n}{2}[S_2(n) + (S_1(n))^2]$

where $S_1(n) = \sum_{k=1}^{n} \frac{1}{k}$ and $S_2(n) = \sum_{k=1}^{n} \frac{1}{k^2}$

From $f_n(x) = \frac{1}{n} x^n \prod_{k=1}^{n-1} \frac{1}{n-kx}$ and $f_n(1) = 1$

$$f_n^{'}(x) = f_n(x) \times [\frac{n}{x} + \sum_{k=1}^{n-1} \frac{k}{n-kx}]$$

so $f_n^{'}(1) = f_n(1) \times [n + \sum_{k=1}^{n-1} \frac{k}{n-k}] = 1 + \sum_{k=1}^{n-1} (1 + \frac{k}{n-k})$

$$= 1 + \sum_{k=1}^{n-1} \frac{n}{n-k} = n \sum_{k'=1}^{n} \frac{1}{k'} = nS_1(n)$$

From $f_n^{'}(x) = f_n(x) \times [\frac{n}{x} + \sum_{k=1}^{n-1} \frac{k}{n-kx}]$

$$f_n^{''}(x) = f_n^{'}(x) \times [\frac{n}{x} + \sum_{k=1}^{n-1} \frac{k}{n-kx}] + f_n(x) \times [\frac{n}{x} + \sum_{k=1}^{n-1} \frac{k}{n-kx}]^{'}$$

$$= f_n^{'}(x) \times [\frac{n}{x} + \sum_{k=1}^{n-1} \frac{k}{n-kx}] + f_n(x) \times [\frac{-n}{x^2} + \sum_{k=1}^{n-1} (\frac{k}{n-kx})^2]$$

so $f_n^{''}(1) = n^2 S_1(n)^2 + [-n + \sum_{k=1}^{n-1} (\frac{k}{n-k})^2]$

$$= n^2 S_1(n)^2 - 1 + \sum_{k=1}^{n-1} ((\frac{k}{n-k})^2 - 1)$$

$$= n^2 S_1(n)^2 - 1 + \sum_{k=1}^{n-1} (\frac{2kn - 2n^2 + n^2}{(n-k)^2})$$

$$= n^2 S_1(n)^2 - 2 - 2 \sum_{k=1}^{n-1} (\frac{n}{(n-k)}) + 1 + \sum_{k=1}^{n-1} (\frac{n^2}{(n-k)^2})$$

$$= n^2 S_1(n)^2 - 2nS_1 + n^2 S_2(n)$$