First we show that for $k \geq 2$ the decimal representation of $\dfrac{1}{3^k}$ has a periodicity divisor or equal to 3^{k-2}

As by recurrence 3^k divide $10^{3^{k-2}} - 1$ (True for k=2) and :

$10^{3^{k-1}} - 1 = 10^{3^{k-2}\times 3} - 1 = (10^{3^{k-2}})^3 - 1 = (10^{3^{k-2}} - 1)(10^{2\times 3^{k-2}} + 10^{3^{k-2}} + 1)$

by recurrence $(10^{3^{k-2}} - 1) \equiv 0 \ mod(3^k)$ and $(10^{2\times 3^{k-2}} + 10^{3^{k-2}} + 1) \equiv 0 \ mod(3)$

For $\dfrac{1}{3^k} < (N - nb)$ the nb decimal digits beginning at the Nth are identical

to digits beginning at $N \ mod(3^{k-2})$ and also beginning at $N \ mod(3^{k-2+i})$, $i \geq 0$

As 3^k is a multiple of the period 3^{k-2} the nb decimal digits are the same for $\dfrac{1}{3^k 10^{3^k}}$

For $K = 33$: $\dfrac{1}{3^K} < (10^{16} - nb) < 10^{16} < \dfrac{1}{3^{K+1}}$

we need to compute nb digits begining at 10^{16} or $\Delta = (10^{16} \ mod \ 3^{K-2})$ of

$$\sum_{k=1}^{K} \frac{1}{3^k} = \frac{1 - (\frac{1}{3})^{K+1}}{1 - \frac{1}{3}} - 1 = \frac{1 - \frac{1}{3^K}}{2}$$

as we can compute digits for $\dfrac{1}{3^K}$ by recursion : $R_0 = 1$; $d_{n+1} = \lfloor \dfrac{R_n}{3^K} \rfloor$; $R_{n+1} = R_n - d_{n+1} \times 3^K$

to skip Δ digits we compute $R_\Delta = (10^\Delta \ mod(3^K))$ by fast exponentation.

Remark : to implement the division by 2, we compute $R_{\Delta-1}$