$S(n) = 2^{i-1} + S(n - F_i) - S(F_{i+1} - n - 3)$ with : $F_i \leq n < F_{i+1}$
Let $L(n) = n - F_i$ and $R(n) = F_{i+1} - n - 3$
The recursion is : (1) $S(n) = 2^{i-1} + S(L(n)) - S(R(n))$
If we represent n by its binary Zeckendorf decomposition the function L is equivalent to remove the hightest bit
Ex: $74 = F_9 + F_6 + F_4 + F_1 = 100101001b$
$L(100101001b) = 1010001b = F_6 + F_4 + F_1 = 19 = 74 - 55$
$R(100101001b) = 89 - 74 - 3 = 12 = F_5 + F_3 + F_1 = 10101b$

Let $E_L = \{i \geq 0; L^{(i)}(n)\}$ and $E_R = \{i \geq 0; L^{(i)}(R(n))\}$
$E_L = \{100101001b, 101001b, 1001b, 1b\}$; $E_R = \{10101b, 101b, 1b\}$
It can be proved that $R(E_L) \subset E_R$ and $R(E_R) \subset E_L$

if $F_i \leq n < F_{i+1}$ and $F_j \leq n - F_i < F_{j+1}$
we have $j \leq i - 2$ *(no consecutive Fibonacci numbers)*
$R(n) = F_{i+1} - n - 3$ and $R(L(n)) = F_{j+1} - (n - F_i) - 3$
so $R(n) = R(L(n)) + F_{i+1} - F_i - F_{j+1} = R(L(n)) + F_{i-1} - F_{j+1}$
the Zeckendorf decomposition of $F_{i-1} - F_{j+1}$ uses Fibonacci with index $\geq j$
the Zeckendorf decomposition of $R(L(n))$ uses Fibonacci with index $\leq j - 2$
$R(L(n))$ is a low part of Zeckendorf decomposition of $R(n) \Rightarrow R(L(n)) \in E_R$

the method is simple : we compute $E_{LR} = E_L \bigcup E_R$
as foreach element $k \in E_{LR}$ $L(k), R(k) \in E_{LR}$
we can compute $S(k)$ using (1) for k in ascending order