

let k the number of persons not passed. In this group we design by :

- n_2 the number with 2 slips in the hat.
- n_1 the number with 1 slip in the hat.
- n_0 the number with 0 slip in the hat.

Let n_p the number of slips in the hat from persons already passed

We have the relations $k = n_2 + n_1 + n_0$; $n_p = 2 \times (k - n_2) - n_1$.

By recursion on k from N to 2 we compute the probability of $P_k(n_2, n_1)$

Remark : as $k = n_2 + n_1 + n_0$ the number of states (n_2, n_1) is equal to $\frac{k(k+1)}{2}$

FIRST we choose the next people to pass among the k

- from n_2 with proba $p = \frac{n_2}{k}$; $\Rightarrow n_2 = n_2 - 1$
- from n_1 with proba $p = \frac{n_1}{k}$; $\Rightarrow n_1 = n_1 - 1$
- from n_0 with proba $p = \frac{n_0}{k}$ No action

The number of different possible couples of slip is equal to $N2 = \binom{2 \times n_2 + n_1 + n_p}{2}$

SECOND, we treats the different couples:

- $n_2 \otimes n_2$ IF($n_2 \geq 2$) with proba $p = \frac{4 \times \binom{n_2}{2}}{N2}$; $\Rightarrow P_{k-1}(n_2 - 2, n_1 + 2)$
- $[n_2]$ IF($n_2 > 0$) with proba $p = \frac{n_2}{N2}$; $\Rightarrow P_{k-1}(n_2 - 1, n_1)$
- $n_2 \otimes n_1$ IF($n_2 > 0$ and $n_1 > 0$) with proba $p = \frac{2 \times n_2 \times n_1}{N2}$; $\Rightarrow P_{k-1}(n_2 - 1, n_1)$
- $n_2 \otimes n_p$ IF($n_2 > 0$ and $n_p > 0$) with proba $p = \frac{2 \times n_2 \times n_p}{N2}$; $\Rightarrow P_{k-1}(n_2 - 1, n_1 + 1)$
- $n_1 \otimes n_1$ IF($n_1 \geq 2$) with proba $p = \frac{\binom{n_1}{2}}{N2}$; $\Rightarrow P_{k-1}(n_2, n_1 - 2)$
- $n_1 \otimes n_p$ IF($n_1 > 0$ and $n_p > 0$) with proba $p = \frac{n_1 \times n_p}{N2}$; $\Rightarrow P_{k-1}(n_2, n_1 - 1)$
- $n_p \otimes n_p$ IF($n_p \geq 2$) with proba $p = \frac{\binom{n_p}{2}}{N2}$; $\Rightarrow P_{k-1}(n_2, n_1)$

END: the result is equal to $P = P_1(1, 0) + P_1(0, 1)$