For a palindrom N, equal to a sum of n consecutive squares from k+1 to k+n

$N = \sum_{i=1}^{i=n+k} i^2 - \sum_{i=1}^{i=k} i^2 = \dfrac{n \times [6k^2 + 6k(n+1) + (n+1)(2n+1)]}{6}$

$\Leftrightarrow\ 6N = n \times [6k^2 + 6k(n+1) + (n+1)(2n+1)]$ (1)

$\Leftrightarrow\ 6k^2 + 6k(n+1) + (n+1)(2n+1) - \dfrac{6N}{n} =\ 0$

$\Leftrightarrow\ P[k] = 6k^2 + k \times b + c - \dfrac{6N}{n} = 0$ with $b = 6(n+1)$, $c = (n+1)(2n+1)$

for $k \geqslant 0$, (1) $\ \Rightarrow\ 6N \geqslant n \times c$

so the only positive solution is $k = \dfrac{-b + \sqrt{\Delta}}{12}$ with $\Delta = b^2 - 24(c - \dfrac{6N}{n})$

and k integer $\Longleftrightarrow \Delta$ is a perfect square