We must compute $S(n) = \sum_{i=0}^{i \leq n} b(i)^2 \times i^2$ where b(i) is the number of bits set to 1 for i

for $p = 2^k > n$ we have:

$$S(n+2^k) = \sum_{i=0}^{i \leq n+2^k} b(i)^2 \times i^2 = S(2^k-1) + \sum_{i=2^k}^{i \leq n+2^k} b(i)^2 \times i^2$$

$$= S(2^k-1) + \sum_{i=0}^{i \leq n} b(i+2^k)^2 (i+2^k)^2 = S(2^k-1) + \sum_{i=0}^{i \leq n} (b(i)+1)^2 (i+p)^2$$

$$= S(2^k-1) + \sum_{i=0}^{i \leq n} (b(i)^2 + 2b(i) + 1)^2 (i^2 + 2\ i\ p + p^2)$$

if we note by $S_{\alpha\beta}(n) = \sum_{i=0}^{i \leq n} b(i)^{\alpha} \times i^{\beta}$ for $\alpha = \{1,2\}, \beta = \{0,1,2\}$

$$S_{22}(n+2^k) = S_{22}(2^k-1) + S_{22}(n) + 2pS_{21}(n) + p^2 S_{20}(n) + 2S_{12}(n) + 4pS_{11}(n) + 2p^2 S_{10}(n) + \sum_{i=0}^{i \leq n} (i+p)^2$$

$$S_{22}(n+2^k) = S_{22}(2^k-1) + S_{22}(n) + 2pS_{21}(n) + p^2 S_{20}(n) + 2S_{12}(n) + 4pS_{11}(n) + 2p^2 S_{10}(n) + \sum_{i=p}^{i \leq n+p} i^2 \ (1)$$

we can apply (1) to compute in ascending order for k $S_{\alpha\beta}(2^k-1)$
and next compute $S_{22}(N)$ by adding in ascending order the bits set to 1 in base 2 of N

**Remark: we compute $\mathbf{S_{\alpha\beta}}$ only for $\mathbf{\log_2(N) + b(N)}$ different values**