

The 360-chandelier is equivalent to 12 independant 30-chandeliers.

So we need only to compute $f(30, i)$ for $i = 1, 2, \dots, 20$ and
distribute the 20 candles between the 12 x 30-chandeliers

The number of distributions is reduced as a balanced chandelier must contain 2 or more candles.

to compute $f(30, i)$ for $i = 1, 2, \dots, 20$ we use:

a) any balanced group is a combination of 2-group 3-group and 5-group

b) first we compute $g(30, i)$ for $i = 1, 2, \dots, 20$ without any anti-symetric candels ($k, k+15$)

c) - any 3-group or 5-group is even or odd

- the anti-symetric of any 3-group (resp. 5-group) is a 3-group (resp. 5-group) of opposite parity

- any (3-group, 5-group) of same parity contains a commun candle, so in a combination

3-group and 5-group must have different parity

- any (3-group, 5-group) of different parity, contains an anti-symetric pair(2-group) which must be removed

- so $n_3 \times 3\text{-g} + n_5 \times 5\text{-g}$ has $(3n_3 + 5n_5 - 2n_3 \times n_5)$ candles and $\binom{5}{n_3} \times \binom{3}{n_5} \times 2^{\delta(n_3, n_5)}$ occurrences

where $\delta(n_3, n_5) = 1$ if $n_3 > 0$ and $n_5 > 0$ $\delta(n_3, n_5) = n_3 + n_5$ otherway.

d) exeption: $3 + 3 + 3 + 3 + 3$ and $5 + 5 + 5$ lead to the same candles in $g(30, 15)$

e) we add the 2-groups by: $f(30, i) = \sum_{j=0}^{2j \leq i} g(30, i - 2j) \times \binom{15 - (i - 2j)}{j}$

f) we need only to compute $f(30, i)$ for $i \leq 15$ as by complementarity $f(30, 30 - i) = f(30, i)$

A little faster solution is:

We only to compute $g(30, i)$ for $i = 1, 2, \dots, 20$ and

distribute the 20 candles using $g(30, i)$ between the 12 x 30-chandeliers

We add the 2-groups globally for the 12 chandeliers

This version is faster as :

- $g(30, i)$ is different from 0 only for $i=3, 5, 6, 7, 8, 9, 10, 12, 15$ so the number of distribution is reduced

- as we add 2-groups globally we need to consider only distributions for even sums ≤ 20