It is easy to show that each solution is of the form:

$$\begin{aligned} r &= k \cdot b^2 \\ d &= k \cdot a \cdot b \\ q &= k \cdot a^2 \end{aligned}$$

$$m^2 = n = d \cdot q + r = b \cdot k \cdot \left(k \cdot a^3 + b\right) \tag{1}$$

where : a,b,k integers and $a > b$; $a \wedge b = 1$*(d and q can be exchanged)*
For each element m, we will design its square part by m_s and its square free part by m_f
so: $m = m_f \cdot m_s^2$
if $b = b_f \cdot b_s^2$; $k = k_f \cdot k_s^2$
as n is a perfect square and by (1) : b_f is a divisor of k and k_f is a divisor of b
So if $\delta_f = \gcd(b_f, k_f)$, $b_{\bar{f}} = \frac{b_f}{\delta_f}$, $b_{\bar{s}} = \frac{b_s}{k_{\bar{f}}}$ we have $b = \delta_f \cdot b_{\bar{f}} \cdot k_{\bar{f}}^2 \cdot b_{\bar{s}}^2$
and respectively for k: $k_{\bar{f}} = \frac{k_f}{\delta_f}$, $k_{\bar{s}} = \frac{k_s}{b_{\bar{f}}}$, $k = \delta_f \cdot k_{\bar{f}} \cdot b_{\bar{f}}^2 \cdot k_{\bar{s}}^2$
by replacing in (1) :

$$\begin{aligned} m^2 &= \delta_f^2 \cdot b_{\bar{f}} \cdot k_{\bar{f}} \cdot k_{\bar{f}}^2 \cdot b_{\bar{s}}^2 \cdot b_{\bar{f}}^2 \cdot k_{\bar{s}}^2 \left(\delta_f \cdot k_{\bar{f}} \cdot b_{\bar{f}} \left(a^3 \cdot b_{\bar{f}} \cdot k_{\bar{s}}^2 + k_{\bar{f}} \cdot b_{\bar{s}}^2\right)\right) \\ &= \delta_f^2 \cdot b_{\bar{f}}^2 \cdot k_{\bar{f}}^2 \cdot k_{\bar{f}}^2 \cdot b_{\bar{s}}^2 \cdot b_{\bar{f}}^2 \cdot k_{\bar{s}}^2 \left(\delta_f \left(a^3 \cdot b_{\bar{f}} \cdot k_{\bar{s}}^2 + k_{\bar{f}} \cdot b_{\bar{s}}^2\right)\right) \\ &= \delta_f^2 \cdot b_{\bar{f}}^4 \cdot k_{\bar{f}}^4 \cdot b_{\bar{s}}^2 \cdot k_{\bar{s}}^2 \left(a^3 \cdot b_f \cdot k_{\bar{s}}^2 + k_f \cdot b_{\bar{s}}^2\right) \\ &= b \cdot k \cdot b_{\bar{f}} \cdot k_{\bar{f}} \left(a^3 \cdot b_f \cdot k_{\bar{s}}^2 + k_f \cdot b_{\bar{s}}^2\right) \end{aligned}$$

so: n is perfect square $\Leftrightarrow \left(a^3 \cdot b_f \cdot k_{\bar{s}}^2 + k_f \cdot b_{\bar{s}}^2\right) = \bar{m}^2$ is perfect square.
Remark : we can only search the primary cases where $k_{\bar{s}} \wedge b_{\bar{s}} = 1$ (prime) and then find other solutions by adding a common square factor.
Next, we show that: $\bar{m} = \lfloor k_{\bar{s}} \cdot \sqrt{a^3 \cdot b_f} \rfloor + 1$
- obviously $\bar{m} > k_{\bar{s}} \cdot \sqrt{a^3 \cdot b_f}$
- and $\bar{m} < k_{\bar{s}} \cdot \sqrt{a^3 \cdot b_f} + 1 \Leftrightarrow \bar{m}^2 < \left(k_{\bar{s}} \cdot \sqrt{a^3 \cdot b_f} + 1\right)^2$
$\Leftrightarrow m^2 < b \cdot k \cdot b_{\bar{f}} \cdot k_{\bar{f}} \left(k_{\bar{s}} \cdot \sqrt{a^3 \cdot b_f} + 1\right)^2$
$\Leftrightarrow m^2 < b \cdot k \cdot b_{\bar{f}} \cdot k_{\bar{f}} \left(a^3 \cdot b_f \cdot k_{\bar{s}}^2 + 2 \cdot k_{\bar{s}} \cdot \sqrt{a^3 \cdot b_f} + 1\right)$
$\Leftrightarrow m^2 < b \cdot k^2 \cdot a^3 + b \cdot k \cdot a \cdot 2 \cdot b_{\bar{f}} \cdot k_{\bar{f}} \cdot \sqrt{a \cdot b_f} + ...$
always true from (1) and $b < a$
It conducts to the algorithm:
- loop on couples (b_f, k_f) init: $b_0 = \left(\frac{b_f}{\delta_f}\right)^2$, $k_0 = \left(\frac{k_f}{\delta_f}\right)^2$
  - sub-loop on $a > b_0$ *and* $b_0 \cdot k_0^2 \cdot a^3 < N$
    - sub-loop on $k_{\bar{s}}$
      - compute $\bar{m} = \lfloor k_{\bar{s}} \cdot \sqrt{a^3 \cdot b_f} \rfloor + 1$
      - check if $\left(\bar{m}^2 - a^3 \cdot b_f \cdot k_{\bar{s}}^2\right)$ if equal to $k_f \cdot b_{\bar{s}}^2$ for some $b_{\bar{s}}$
        - new primitive solution $b = b_0 \cdot k_f \cdot b_{\bar{s}}^2$, $k = k_0 \cdot k_{\bar{s}}^2$ if $b < a$, $n < N$
          - add a common square factor to b and k while $b < a$, $n < N$

-Remark, for the main loop: lcm $(b_f, k_f) < N^{1/6}$ as $a < b$, $n > b \cdot k^2 \cdot a^3$