As $m \vee n = m \wedge n + m \oplus n$

We must only compute $f(n) = \sum_{k=0}^{k \leq n} (n-k) \vee k$ and $G(N) = 2 \times \sum_{n=0}^{n \leq N} f(n)$

We have the recursive formulas :

$(1a)$ $f(2n+1) = 4 \times f(n) + 2 \times (n+1)$

$(1b)$ $f(2n) = 2 \times f(n-1) + 2 \times f(n) + n$

$(2a)$ $g(2n+1) = 8 \times g(n) - 2 \times f(n) + \frac{(3n+4)(n+1)}{2}$

$(2b)$ $g(2n) = 8 \times g(n) - 6 \times f(n) + 3 \times n \times (n+1)$

Example of demo for (1b):

$$f(2n) = \sum_{k=0}^{k \leq 2n} (2n-k) \vee k = \sum_{k=0}^{k \leq n} (2n-2k) \vee 2k + \sum_{k=0}^{k \leq n-1} (2n-2k-1) \vee (2k+1)$$

$$= \sum_{k=0}^{k \leq n} 2((n-k) \vee k) + \sum_{k=0}^{k \leq n-1} ((2n-2-2k+1) \vee (2k+1))$$

$$= 2\sum_{k=0}^{k \leq n} ((n-k) \vee k) + \sum_{k=0}^{k \leq n-1} ((2n-2-2k) \vee (2k)) + 1$$

$$= 2 \times f(n) + 2 \sum_{k=0}^{k \leq n-1} ((n-1-k) \vee k) + \sum_{k=0}^{k \leq n-1} 1$$

$$= 2 \times f(n) + 2 \times f(n-1) + n$$

we can construct N bit by bit from the most significant bit by:

$n_0 = 0$, $n_1 = 2 \times n_0 + b_1$, $n_2 = 2 \times n_1 + b_2$

... $n_k = 2 \times n_{k-1} + b_k$ ... $N = 2 \times n_{nb-1} + b_{nb}$

where $b_1 b_2 ... b_k ... b_{nb}$ are the bits in base 2 for N, b_1 *most significant*

Now we can compute by recursion $\{g(n_k), f(n_k), f(n_k - 1)\}$

using (2a) and (2b) for $g(n_k)$

and noting for $(f(n_k), f(n_k - 1))$ that if:

$b_k = 1$ *then* $(n_k, n_k - 1) = (2 \times n_{k-1} + 1, 2 \times n_{k-1})$

$b_k = 0$ *then* $(n_k, n_k - 1) = (2 \times n_{k-1}, 2 \times n_{k-1} - 1) = (2 \times n_{k-1}, 2 \times (n_{k-1} - 1) + 1)$