Let $N = N_L = d_0 d_1 ... d_L$
where d_l are the digits of N, d_0 is the most significant digit
and $N_l = d_0 d_1 .. d_l$ so $N_{l+1} = 10 \times N_l + d_{l+1}$

Define by $\sigma_l = \sum_{k=0}^{k \leq l} d_k$, the sum functions $S_l^*(k) = \sum_{\substack{n=1 \\ d(n)=k}}^{n<N_l} n$

and the count functions $C_l^*(k) = \sum_{\substack{n=1 \\ d(n)=k}}^{n<N_l} 1$

We have the recursive formulas :

(1) $S_{l+1}^*(k) = \sum_{d=0}^{d \leq 9} \{10 \times S_l^*(k-d) + d \times C_l^*(k-d)\} + \sum_{d=0}^{d<d_{l+1}} \{(10 \times N_l + d)\delta(k == \sigma_l + d)\}$

(2) $C_{l+1}^*(k) = \sum_{d=0}^{d \leq 9} C_l^*(k-d) + \sum_{d=0}^{d<d_{l+1}} \delta(k == \sigma_l + d)$

The final result is computed by: $F(N_L) = \sum_{k=1}^{k<kmax} \frac{S_L^*(k)}{k} + \frac{N_L}{\sigma_L}$

Remarks :

a) the functions C_l^*, S_L^* exclude the limit values N_l

b) range for k for functions C_l^*, S_L^* is $0 < k \leq 9 \times l$

c) Recursion (1) and (2) on C_l^*, S_l^* can be done inplace by reverse order on k, so we need only $9 \times L$ values