It is easy to see that:
$$N_i = \sum_{\frac{r^2}{(i+1)^2} \leqslant x^2 + y^2 < \frac{r^2}{i^2}} \delta_{gcd(x,y)=1}$$
 (1)

So, if
$$D_i = \sum_{x^2 + y^2 < \frac{r^2}{i^2}} \delta_{gcd(x,y)=1}$$
 then $N_i = D_i - D_{i+1}$ (2)

To compute D_i same methods as for problem 153 can be used :

So, if
$$C_i = \sum_{x^2 + y^2 < \frac{r^2}{i^2}} 1$$
, then $D_i = C_i - \sum_{2 \le j \ ; \ i \times j < r} D_{i \times j}$ (3)

 $\big\lfloor \frac{r^2}{i^2} \big\rfloor$