

Survey on the Recent Design and Tuning Approaches for model-free Fuzzy PID/PI Controllers

Kamel Sabahi (✉ k.sabahi@uma.ac.ir)

University of Mohaghegh Ardabili <https://orcid.org/0000-0002-9571-2208>

Ardashir Mohammadzadeh

University of Bonab

Mehdi Tavan

Islamic Azad University

Saleh Mobayen

National Yunlin Institute of Technology: National Yunlin University of Science and Technology

Wudhichai Assawinchaichote

King Mongkut's University of Technology Thonburi

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Survey on the Recent Design and Tuning Approaches for model-free Fuzzy PID/PI Controllers

Kamel Sabahi¹, Ardashir Mohammadzadeh², Mehdi Tavan³, Saleh Mobayen⁴, Wudhichai Assawinchaichote⁵,

¹Department of Engineering Sciences, Faculty of Advanced Technologies, University of Mohaghegh Ardabili, Namin, Iran.

²Department of Electrical Engineering, University of Bonab, Bonab, Iran.

³Department of Electrical Engineering, Nour branch, Islamic Azad University, Nour, Iran.

⁴Future Technology Research Center, National Yunlin University of Science and Technology, Douliu 64002, Taiwan.

⁵Department of Electronic and Telecommunication Engineering, Faculty of Engineering, King Mongkut's University of Technology Thonburi, Bangkok 10140, Thailand.

Abstract:

There is good experience in applying classical linear proportional, integral and derivative (PID) controllers to industrial plants due to its simple structure and easy tuning property. However, considering the fact that these linear controllers have fixed parameters, the uncertainties and nonlinearities in the controlled system can degrade the control performance. To tackle the mentioned problems in the application of PID controllers, one of the ways is to incorporate fuzzy logic systems (FLSs) into the PIDs and fine-adjust them. Like the PID controllers, one of the advantages of the FLS-based PID controllers is that they do not require a system mathematical model for control problems, thus these controllers have shown to be a viable control solution for various complicated nonlinear systems. This study presents a survey of development and analysis of model-free PIDs incorporated with FLSs. We focus especially on the widely used fuzzy gain scheduling PID (FGPID) and fuzzy PID (FPID) controllers which are two important combinations of the FLSs and PID controllers. In this study, different structures of FGPID and FPID controllers in which type-1 FLS (T1FLS), interval type-2 FLS (IT2FLS), and general type-2 FLS (GT2FLS) are used as the inference realization part, have also been reviewed. Moreover, the tuning methods of FGPID and FPID controllers' parameters and their closed-loop stability problems are discussed.

Keywords: Fuzzy systems; Machine learning; Learning algorithms; PID; Survey; Stability

1 Introduction:

Regarding the simplicity of proportional, integral and derivative (PID) controllers, they are employed in wide practical systems (Ziegler and Nichols 1993). Since they are linear and some trial-and-error methods are used to tune their gains, uncertainties and variation in the controlled systems' parameters can degrade

the closed-loop performance. Because of the capability of the fuzzy logic systems (FLS)s and artificial neural networks (ANN)s, they are extensively used to ease the problem of PID controllers dealing with the problem of tuning procedures, nonlinearities, and variation in the parameters of the controlled systems. The classical PID/PD controllers are used in combination with intelligent ANN and FLS controllers in which the classical controller handles the transient stability and the intelligent controllers are designed to tackle the nonlinearities, i.e., feedback error learning (FEL) approaches (Sabahi and Teshnehlab 2009, Kayacan, Cigdem et al. 2011, Sabahi 2011, Sabahi, Ghaemi et al. 2014, Sabahi, Ghaemi et al. 2016, Sabahi, Ghaemi et al. 2019). Alternatively, as discussed in this paper, the FLSs are exploited to improve the PID/PI controller performance. Two groups of these controllers are categorized as follows: (i) the PIDs are tuned via FLS. In this structure, which is famous for fuzzy gain scheduling PID (FGPID) controllers, the PID controller generates the actual signal (Talaq and Al-Basri 1999, Çam and Kocaarslan 2005, Cherrat, Boubertakh et al. 2018). (ii) The rules are used to create the PID, and the controller is extracted directly from the FLS. This scheme is named fuzzy PID (FPID) controller (Yeşil, Güzelkaya et al. 2004, Sabahi and Tavan 2020). In the FGPID approaches, the FLS is used to tackle the tuning problems of classical PID controllers, which in most industrial systems are based on trial and error methods (Ang, Chong et al. 2005). Like the classical linear PID controllers, the structure of the FPID is defined in such a way that this controller's input-output relation is like the classical PID controller which has proportional, integral and derivative terms (Sakalli, Kumbasar et al. 2014). Both the FPID and FGPID controllers can be realized using three types of FLSs: type-1 FLSs (T1FLS)s, interval type-2 FLSs (IT2FLS)s (Ghaemi, Sabahi et al. 2019) and general T2FLSs (GT2FLS)s (Sakalli, Kumbasar et al. 2020). T1FLSs are the most basic type of linguistic scheme, and then, they can only describe a limited level of imprecision or ambiguity. The idea of uncertainty and imprecision was introduced as intervals with IT2FLSs. In a perfect world, these intervals would represent an infinite number of embedded T1FLSs. Although more computationally demanding than T1FLSs, they enhance the overall fuzzy model by being more resistant to measurement noise when inferred. The second important kind of FLS representation is GT2FLSs, which, like IT2FLSs, handle uncertainty inherently. In essence, this kind of uncertainty representation is substantially more noise-resistant than the IT2FLSs. In both FPID and FGPID control design approaches, the important matter is how to determine the FLS parameters such as the consequent/antecedent MFs and the input/output scaling factors. In this way, some optimization algorithms such as gradient descent (GD)-based algorithms and evolutionary optimizations methods have been exploited. The evolutionary optimization-based design methods are offline tuning methods and try to minimize a specific objective function to adjust the controller parameters, while the gradient descent-based algorithms work in an online manner. Another important issue that must be considered in constructing both FPID and FGPID is the closed-loop stability and the boundedness of the adaptive parameters. In these controllers, because the model of the controlled plant is often not involved in

the design procedures, proving the closed-loop stability and boundedness adaptive parameters are challenging problems and interesting topics for future researches. In this paper, we have tried to review the different structures that have been introduced to realize the both FPID and FGPIID controllers. Moreover, the ways of the adjusting parameters of these controllers and their closed-loop stability problems are discussed carefully. The remainder of this paper is as follows: In section 2, different types of FLSs are explained. The FGPIID and FPID controllers are provided in section 3 and 4, respectively. Finally, the conclusion is in section 5.

2 Different types of FLSs

A FLS is distinctive in that it is capable of handling numerical-linguistic knowledge simultaneously. There are two techniques to designing FLSs, according to Lotfi Zadeh: T1FLSs and T2FLSs. The second is presented as a continuation of the first. Expertise and experience are required while developing T1FLSs to choose the MFs and fuzzy rules. For various experts, the language concepts employed in MFs and rules have varied meanings. For the same rule foundation, specialists often arrive at various results. T1FLSs, are unable to deal with rule uncertainty directly. To address this issue, Zadeh proposed the notion of type-2 FLSs as a generalization of T1FLSs, with the goal of being able to better describe the fuzziness that unavoidably exists in the system's rules. T2FLSs can manage the ambiguity inherent in linguistic terms better than T1FLSs. The uncertainties are represented by a fuzzy MF. As a result, T2FLSs are better suited to situations where determining the precise MF for a fuzzy set (FS) is challenging, which is particularly beneficial for incorporating uncertainties (Mendel 2017, Tavoosi, Mohammadzadeh et al. 2021). Type-1 FS (T1FS)s have completely certain MFs, whereas type-2 FS (T2FS)s have fuzzy MFs. As a result, the rules are ambiguous at T2FSs. A type-1 membership grade is a crisp integer in $[0, 1]$, but a type-2 membership grade, known as primary membership, maybe any subset in $[0, 1]$. Furthermore, each main membership has a secondary membership which specifies the possibilities of basic memberships (Wagner and Hagra 2010, Almohammadi, Hagra et al. 2017).

2.1 T1FLS structure

A T1FS, A , in terms $x \in X$, is represented as:

$$A = \int_{x \in X} \mu_A(x)/x \quad (1)$$

where, \int denotes union. Using a type-1 triangular MF, $\mu_A(x)$ is represented in Fig 1.

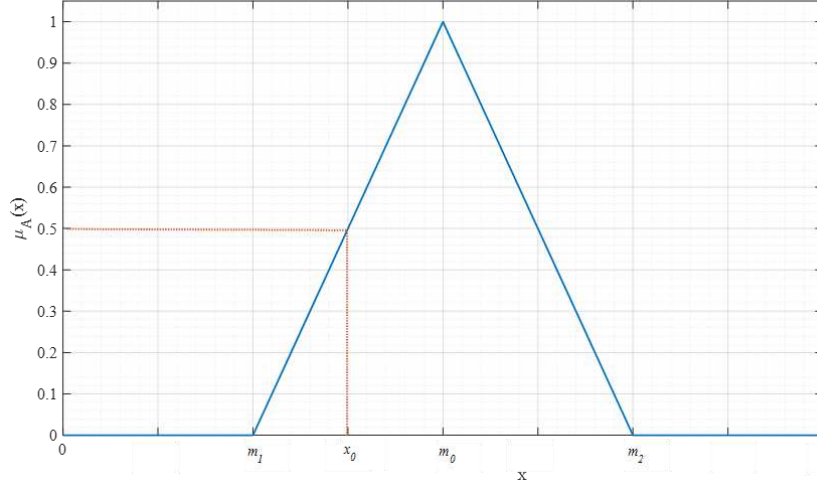


Fig 1: type-1 triangular MF

From this figure, it can be seen $\mu_A(x)$ is constrained to be between 0 and 1 for all $x \in X$, and is a two-dimensional function and, therefore, there exists an obvious membership for $x = x_0$ the value of $\mu_A(x = x_0)$ is 0.5. As can be seen, the triangle MF itself has three tunable parameters m_1 , m_0 and m_2 . In this way, the block diagram of T1FLS, which can be constructed using T1FSs, is shown in Fig 2.

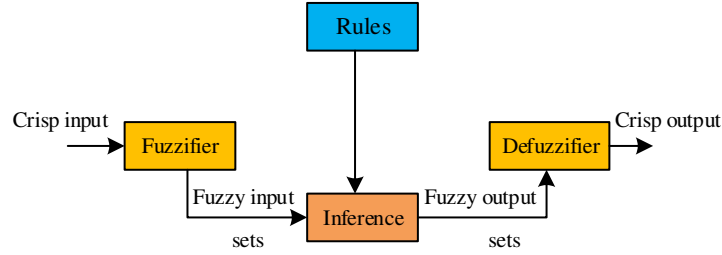


Fig 2: the block diagram of T1FLS

2.2 T2FLS structure

T2FSs are derived from the expansion of T1FSs, which have been shown to have a better ability to deal with uncertainties. A T2FS, \tilde{A} , is written as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad (2)$$

Where \int denotes union. Also, J_x is MF of x and $\mu_{\tilde{A}}(x, u)$ is secondary MF. As given in Fig 4, in type- 2 FSs all $\mu_{\tilde{A}}(x, u)$ are equal 1 and in the general one, shown in Fig 3, $\mu_{\tilde{A}}(x, u)$ is in the interval of $[0, 1]$. Like the T1FSs and for a tringle MF, both $\bar{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$ have three tunable parameters.

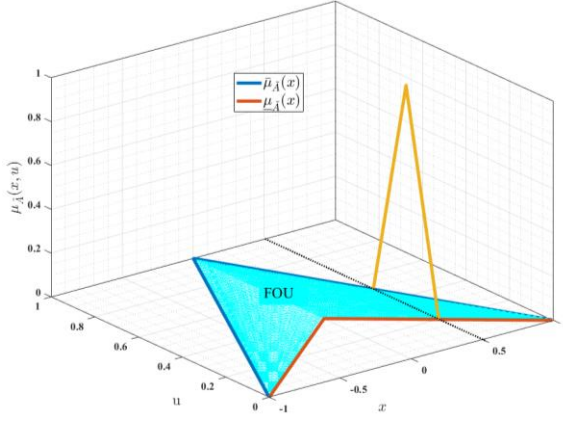


Fig 3: General type-2 fuzzy set

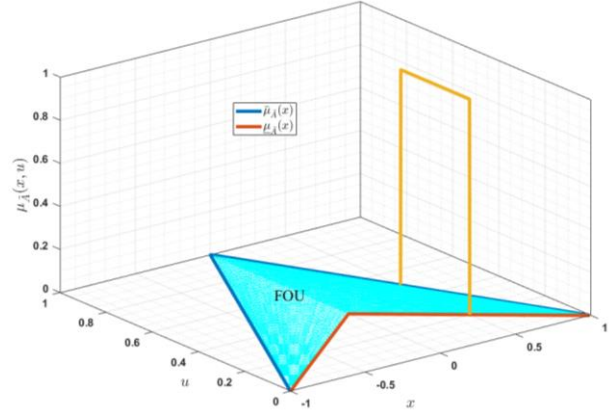


Fig 4: Interval type-2 fuzzy set

The general and interval T2FLSs can be constructed using T2FSs in which the block diagram of that is similar to T1FLS, except that in the T2FLSs there is an extra unit called type reducer to convert the output into a T1FLS. A block diagram of a T2FLS is depicted in Fig 5.

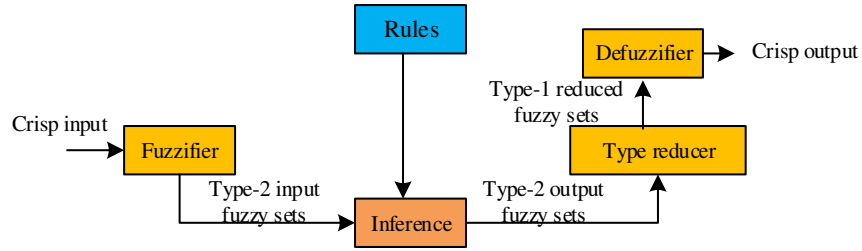


Fig 5: the block diagram of T2FLS

The T2FLSs rules are similar to the T1FLSs, except T2FSs are used to express the antecedents and/or consequents. As a result, the rules sections of T2FLSs are identical to those of T1FLSs, but the inference outputs are T2FS. Some techniques known as type-reduction approaches may be used to reduce type-2 fuzzy consequences to type-1, as detailed in greater detail in (Torshizi, Zarandi et al. 2015). Using the center-of-set (COS) type-reduction method and after defuzzification, the final output (crisp output) of an IT2FLS can be represented as follow:

$$y = \frac{1}{2}(y_l + y_r) \quad (3)$$

where y_l and y_r are the end points and:

$$y_l = \frac{\sum_{i=1}^p \bar{f}^i y_l^i + \sum_{p+1}^M \underline{f}^i y_l^i}{\sum_{i=1}^p \bar{f}^i + \sum_{p+1}^M \underline{f}^i} \quad (4)$$

$$y_r = \frac{\sum_{i=1}^q \bar{f}^i y_r^i + \sum_{q+1}^M \underline{f}^i y_r^i}{\sum_{i=1}^q \bar{f}^i + \sum_{q+1}^M \underline{f}^i}$$

where \bar{f}^i and \underline{f}^i are the upper/lower firing levels (for the i – th rule). Also, y_l^i and y_r^i , which are tunable parameters, represent the centroid of the consequent set (for the i – th rule) (Mendel 2017) and M is the number of the fuzzy rules.

3 FGPID controllers

The basic structure of the FGPID controller is depicted in Fig 6. The linear PID with tunable parameters of K_p , K_i and K_D generates the final control signal ($u(t)$), and we have:

$$G(s) = K_p + \frac{K_i}{s} + K_D s \quad (5)$$

In this structure, the fuzzy logic controller (FLC) which can be implemented using T1FLS, IT2FLS and GT2FLS is utilized to tune the PID controller parameters. In this structure, the FLC inputs are the error and its derivative, and parameters of PID are given by the FLC outputs instantly.

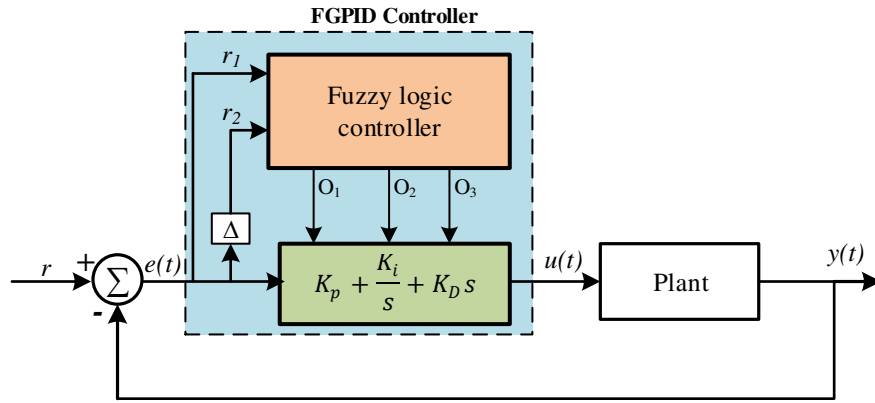


Fig 6: The basic structure of the FGPID controller

In the FGPID controller, after the MFs are defined for both inputs and outputs, the next step is to define the fuzzy control rule. The FLC rules can be prepared by human experts in the form of linguistic descriptions. These principles are generally derived through trials of the step response. An example of the defined inputs and outputs MFs for FLC for the structure shown in Fig 6 is illustrated in Fig 7 (Zhao, Tomizuka et al. 1993).

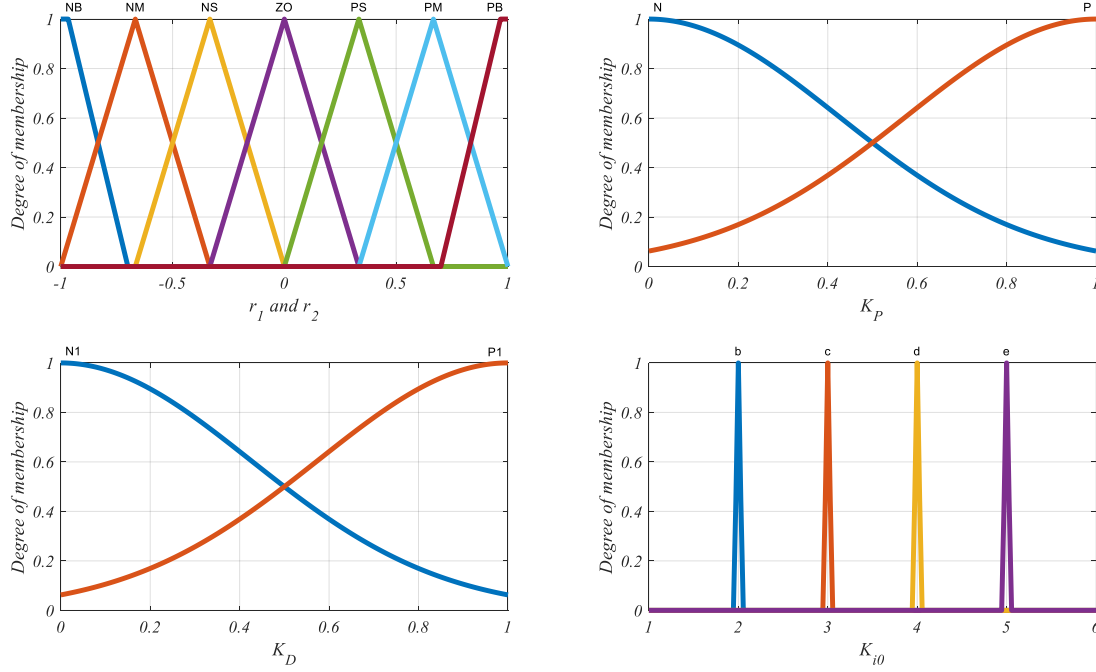


Fig 7: MFs for r_1 , r_2 , K_p , K_D and K_{i0}

Based on the defined MFs, the general form of the fuzzy rules was given as follow:

$$IF \ r_1 \text{ is } A_1 \text{ and } r_2 \text{ is } A_2 \text{ THEN } K_p \text{ is } B_1 \text{ and } K_{i0} \text{ is } B_2 \text{ and } K_D \text{ is } B_3 \quad (6)$$

where A_1 and A_2 are the antecedent MFs and B_1 , B_2 and B_3 represent the consequent MFs. In this FGPID controller, the proportional and derivative gains are directly deduced from the related fuzzy outputs i.e., $K_p = O_1$ and $K_D = O_3$, and the integral gain is indirectly determined by the fuzzy-logic controller (FLC) output, i.e., $K_i = \frac{K_p^2}{(K_{i0} \times K_D)}$ where $K_{i0} = O_2$. According to Table 1 which demonstrates the effects of changes in the closed loop performance with changes in the PID gains, the FLC rules can be constructed to better adjust the PID.

Table 1

Effects of gains in the amount of rise time, overshoot, and settling time

Gains	Effects		
	Rise time	Overshoot	Settling Time
K_p	reduce	amplify	Small change
K_i	reduce	amplify	amplify
K_D	reduce	reduce	reduce

According to the general rule of thumb between changes in the PID controller parameters and closed loop

system performance, the complete rule bases for the three outputs FLC, K_p , K_D , and K_{i0} are given in Table 2, Table 3, and Table 4, respectively.

Table 2: Rules for K_p

$r_1 \downarrow r_2 \rightarrow$	NR	NM	NL	ZO	PL	PM	PR
NR	P	P	P	P	P	P	P
NM	N	P	P	P	P	P	S
NL	N	N	P	P	P	N	N
ZO	N	N	N	P	N	N	N
PL	N	N	P	P	P	N	N
PM	N	P	P	P	P	P	N
PR	P	P	P	P	P	P	P

Table 3: Rules for K_D

$r_1 \downarrow r_2 \rightarrow$	NR	NM	NL	ZO	PL	PM	PR
NR	N1	N1	N1	N1	N1	N1	N1
NM	P1	P1	N1	N1	N1	P1	P1
NL	P1	P1	P1	N1	P1	P1	P1
ZO	P1	P1	P1	P1	P1	P1	P1
PL	P1	P1	P1	P1	P1	P1	P1
PM	P1	P1	N1	N1	N1	P1	P1
PR	N1	N1	N1	N1	N1	N1	N1

Table 4: Rules for K_{i0}

$r_1 \downarrow r_2 \rightarrow$	NR	NM	NL	ZO	PL	PM	PR
NR	b	b	b	b	b	b	b
NM	c	c	b	b	b	c	c
NL	d	c	c	b	c	c	d
ZO	e	d	c	c	c	d	e
PL	d	c	c	b	c	c	d
PM	c	c	b	b	b	c	c
PR	b	b	b	b	b	b	b

The MF shapes are important in some problems when designing a FLC because they affect the fuzzy inference system. They may be triangular, trapezoidal, Gaussian, and other forms. Each of these MFs has a number of parameters, as mentioned earlier, in the triangular MFs the tunable parameters are m_1 , m_0 and m_2 , whose proper adjustment of them can defiantly improve the performance of FLC. In this way, in (Çam and Kocaarslan 2005, Çam and Kocaarslan 2005, Kocaarslan and Çam 2005) FGPI controllers were designed for frequency regulation in which it was shown that the intervals and shape of fuzzy MFs (MF)s have a severe effect on the performance. For example, the FLC in (Çam and Kocaarslan 2005) consists of seven MFs for inputs (error and its derivative) and outputs (K_p and K_i) defined in the range of [-1,1]. While the number and shape of MFs in (Çam and Kocaarslan 2005) are similar to defined one in (Çam and Kocaarslan 2005), the first input (error) range for the FLC is [-0.0171, +0.0171], the second input range is [-0.0283,+0.0283], and the outputs (K_p and K_i) range are [-1,1].

For this reason, and to improve the efficiency of the FGPIDs, in (Ahn and Truong 2009) an online tuning

algorithm based on the extended Kalman filter (EKF) was designed to adjust the centre of triangle MFs and the weight of the control output. With this online tuning method, the FGPID controller is able to deal with variation in the operating points and the external disturbances. The EKF optimized FGPID controller was applied for a force control problem and the simulation evaluation and experimental results illustrated that the designed controller could achieve good tracking in the case of variation of disturbances. One of the major disadvantages of the proposed method is the large number of parameters that must be adjusted. To avoid this high computational time the online tuning methods, the offline tuning methods such as genetic algorithm (GA) optimization and particle swarm optimization (PSO) are the solution. For this, the PSO was employed to determine the fuzzy MFs parameters in (Bevrani, Habibi et al. 2012). For the PSO algorithm, the system closed loop error can be considered as the cost function which should be minimized by finding the optimal values for the MF parameters. This tuning method of FLS's parameters is an offline method and its efficiency has been shown in some applications. In (Li, Mao et al. 2017), the gravitational search algorithm (GSA) based on Cauchy mutation and mass weighing (GSA-CW) was exploited to optimize the FLS's parameters of the FGPID controller that applied to regulate the frequency/power of generating unit. In this work, since Gaussian MF is a smooth and non-zero at all points, it was utilized for the fuzziness of the output which claimed it reduced the complexity of the calculations. To decrease the impact of uncertainties and time-delay in the performance of the FPID controller, a predictive functional control (PFC) was designed in the feedback path to predict of the future process behavior (p -step ahead of the output) (Wang, Jin et al. 2017). This structure is shown in Fig 8. Using the coke furnace as a case study system, it was shown that the combination of PFC and FPID controller has considerably improved the performance regarding the impact of uncertainty and disturbance.

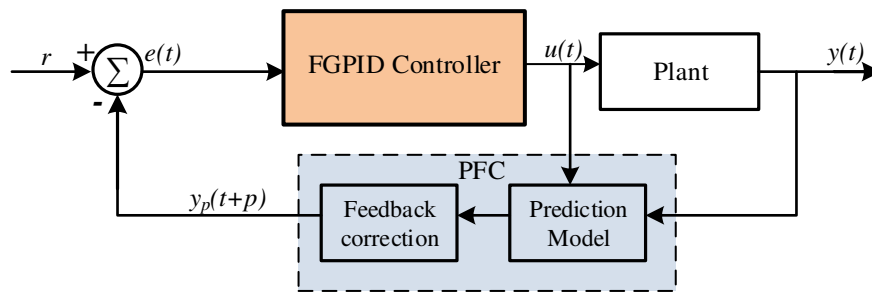


Fig 8: The predictive functional control FPID controller

To tackle the effects of structure uncertainties and measurement noise in a pendulum on a cart, an IT2FPID controller was designed in (El-Bardini and El-Nagar 2014). The inputs and outputs MFs for the interval type-2 FLC are defined as follows:

Five triangular MFs in the range of $[-3,3]$ and $[-1.5,1.5]$ for (error) and (derivative of error), respectively,

four triangular MFs in the range of [0,100] and [0,50] for the first (to generate K_p) and second (to generate K_i) outputs, respectively, and three triangular MFs in the range of [0,10] for the third (to generate K_D) output. Also, In (El-Nagar and El-Bardini 2014) and using the hardware-in-the-loop (HIL) simulation, the IT2FGPID was applied to control of a nonlinear pendulum. It was shown that the proposed type-2 FLC outperforms the type-1 one in some closed loop criteria.

In (Sabahi, Ghaemi et al. 2017) an interval type-2 FGPID (IT2FGPID) was presented for frequency tuning of an interconnected system in which just the consequent MFs was optimized using PSO algorithm. This controller was trained in the off-line manner and it was shown that it is capable to infer the PID gains appropriately. The modified harmony search algorithm (MHSA) and PSO algorithm were exploited to optimize the structures of the general type-2 FPI (GT2FGPI) controllers to frequency regulation of micro-grid power system and power-line inspection (PLI) robot in (Khooban, Niknam et al. 2016) and (Zhao, Chen et al. 2020), respectively. In (Zhao, Chen et al. 2020), as the controlled system owns four states, to reduce the input dimension for the general type-2 FLC, using a fusion information and quadratic optimal control theory, fuzzy inputs were reduced to 2. In that work, using three triangular MFs for each output, the first output (to generate K_p) is in the range of [40,120], the second one (to generate K_i) is in the range of [5,20], and the third one (to generate K_D) is in the range of [10,25].

Based on the fact that in the most of real-world control design problems, the PID controller design matters can be considered as a multi-objective (MO) problem, a multi-objective GA (MOGA) was utilized to optimize two conflicting objectives (overshoot and settling time) for the FGPID controller applied to control of inverted pendulum fourth order nonlinear systems (Mahmoodabadi and Jahanshahi 2016). Unlike the single objective optimization method, in the MO design approaches and according to the importance of the objectives, different PID gains can be obtained. In (Fathy, Kassem et al. 2020) and (Feng, Wu et al. 2020), the structures of FGPID controllers were optimized using mine blast (MB) and PSO algorithms to regulate the frequency in power system and to control of mold level, respectively. The FGPID controller tuning methods based on evolutionary optimization algorithms are offline methods and the problem of closed loop stability has not been investigated. In (Cherrat, Boubertakh et al. 2018) a gradient descent based learning algorithm combined with the Lyapunov theory was used to update the FLS consequent parameters in the FGPID controller. In this online tuning method, PID controller gains according to the FLS output can be given as follow:

$$K_{PID} = [K_p, K_i, K_D]^T = W^T c \quad (7)$$

where W is the normalized firing strength and c is the consequent tunable parameters. Using a Lyapunov function, the adaptation algorithm for c was derived as

$$\dot{c} = \gamma W^T E (\dot{s} + ks) \quad (8)$$

where γ is learning rate, $E = [e, \int e, \dot{e}]^T$, and s is error surface. The proposed adaptive FGPID controller was applied to control of a two-link manipulator robot and it was shown that the tracking error converge to zero and all signals in the closed-loop system remain bounded. In Table 5 a summary of contributions to design of the FGPID controllers has been illustrate. The following criteria were used to make the comparison in this table: authors (publication, year), FLS's parameters tuning approach(s) and if the closed loop stability is studied.

Table 5

Some presented contributions to design of the FGPID controllers

Author(s)(pub.year)	FLS's parameters tuning approach(s)	Closed-loop stability analysis?
(Zhao, Tomizuka et al. 1993)	Analytical design	NO
(Çam and Kocaarslan 2005)	Analytical design	NO
(Ahn and Truong 2009)	Extended Kalman filter	NO
(Bevrani, Habibi et al. 2012)	PSO	NO
(Li, Mao et al. 2017)	gravitational search algorithm	NO
(Wang, Jin et al. 2017)	Analytical design	NO
(El-Bardini and El-Nagar 2014)	Analytical design	NO
(Sabahi, Ghaemi et al. 2017)	PSO	NO
(Zhao, Chen et al. 2020)	PSO	NO
(Mahmoodabadi and Jahanshahi 2016)	MOGA	NO
(Fathy, Kassem et al. 2020)	mine blast algorithm	NO
(Feng, Wu et al. 2020)	PSO	NO
(Cherrat, Boubertakh et al. 2018)	Lyapunov based gradient descent	YES

4 FPID Controller

As previously stated, the traditional linear PIDs produce the final signal in the FGPID controller, which is susceptible to uncertainties and nonlinearities. Using a FLS as a direct controller is one solution to these issues. The FPID controller is the name for this approach. The FPID controller is comparable to standard PID controller in terms of input-output relationship, except that the FPID controller has a nonlinear structure and is an effective solution for regulating nonlinear and uncertain systems. The well-known construction of an FPID controller type is seen in Fig 9 (Yeşil, Güzelkaya et al. 2004). Unlike the FGPID, the fuzzy inference generates the main control signal $u(t)$ to the plant.

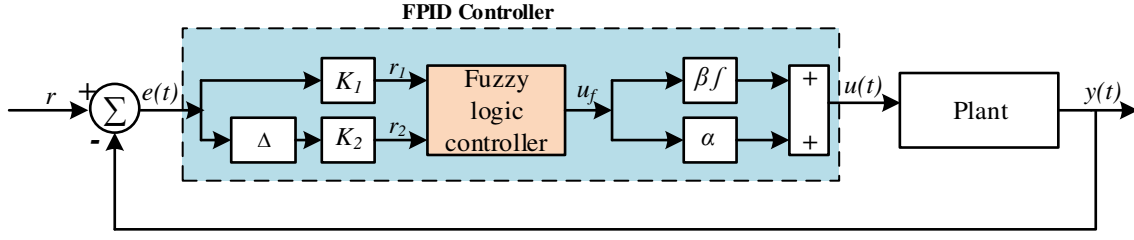


Fig 9: the structure of FPID

In this controller, u is the final signal (output of FPID controller), $e(t)$ represents the error (the input of the FPID controller), u_f denotes the output of FLS, α and β are scaling factors, and K_1 and K_2 are input scaling gains. In this structure, the real-world inputs are normalized using the input scaling factors to a range in which MFs are considered. The scaling factor has a crucial role in the efficiency of the FPID (Kumbasar and Hagrass 2015). For a FPID we have:

$$u(t) = \alpha u_f(t) + \beta \int_0^t u_f(t) dt \quad (9)$$

Then the controller is:

$$u_f(t) = A + Pr_1 + Dr_2 \quad (10)$$

where A , P , and D are fixed values and $r_1 = K_1 e$ and $r_2 = K_2 \dot{e}$ are inputs of FLS. From (9) and (10), the output of FPID is as:

$$u(t) = \alpha A + \beta A t + \alpha K_1 P e + \beta K_2 D e + \beta K_1 P \int e(t) dt + \alpha K_2 D \dot{e} \quad (11)$$

It is worth pointing out that FPID is similar to a conventional PID by variable parameters (for example α , β , K_1 , K_2 can be tuned in both online and offline manners) (Yeşil, Güzelkaya et al. 2004). To control of nonlinear time-delay system, a fuzzy PI type controller was proposed in (Genc, Yesil et al. 2009). This structure which is illustrated in Fig 10, has three output-input scaling factor parameters ke , kde and ko . According to the defined MFs, the symmetrical rule base, which is in the general and frequently used form, for the delay-free case is given in Table 6. In that work, to deal with the time-delay, the idea of “shifting the rules to the appropriate reign” is used to deal with time-delay; and; it is shown that if this shift is done correctly, it would result in better controller performance.

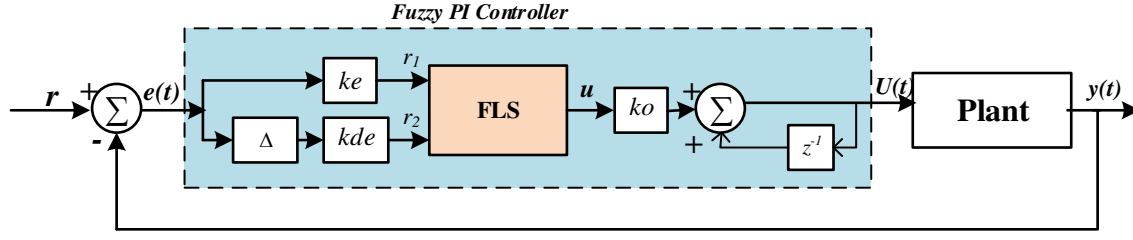


Fig 10: the structure of fuzzy PI type controller proposed in (Genc, Yesil et al. 2009)

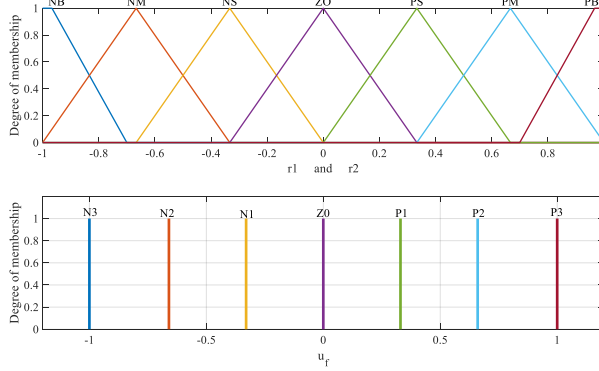


Fig 11: inputs and output MFs for fuzzy PI type controller

Table 6: Fuzzy tuning rules for u

$r_1 \downarrow r_2 \rightarrow$	NB	NM	NS	ZO	PS	PM	PB
NB	Z0	P1	P2	P3	P3	P3	P3
NM	N1	Z0	P1	P2	P3	P3	P3
NS	N2	N1	Z0	P1	P2	P3	P3
ZO	N3	N2	N1	Z0	P1	P2	P3
PS	N3	N3	N2	N1	Z0	P1	P2
PM	N3	N3	N3	N2	N1	Z0	P2
PB	N3	N3	N3	N3	N2	N1	Z0

An adaptive mechanism based on the peak observer idea was designed for online tuning of the output scaling factor β and input coefficient K_2 of the FPID for frequency regulation in (Yeşil, Güzelkaya et al. 2004). The online tuning algorithm (for the strategy given in Fig 9) is as follows:

$$\begin{aligned} \beta &= \delta \beta_0 \\ K_2 &= \frac{K_0}{\delta} \end{aligned} \quad (12)$$

where δ is the peak value for step response, and β_0 and K_0 are the initial values for β and K_2 , respectively. In that observer-based FPID controller, as given in Fig 12, the FLS rule base was constructed using two MFs and four rules. The related rule bases are given in Table 7.

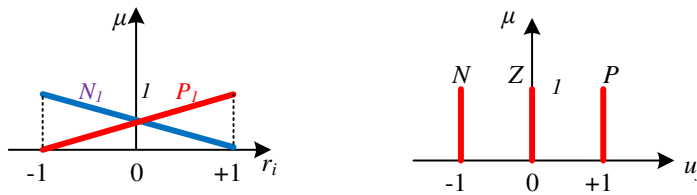


Fig 12: The antecedent and consequent MFs

Table 7: Fuzzy tuning rules for u

$r_1 \downarrow r_2 \rightarrow$	N_1	P_1
N_1	N	Z
P_1	Z	P

In (Karasakal, Guzelkaya et al. 2011), the k th rule of the FPID controller in Fig 9 was defined as follow:

if r_1 is N_k and r_2 is M_k Then u_f is C_k with θ_k (13)

where N_k and M_k are the input MFs, C_k is the output MF, and θ_k is the weighting vector. In the case of singleton MFs for output, the output of FLC can be written as follow:

$$u_f = \frac{\sum_{i=1}^m f_k c_k \theta_k}{\sum_{i=1}^m f_k \theta_k} \quad (14)$$

where f_k , c_k , and w_k are the firing degree, parameter and the output for k th rule. In this work, the weights are used to apply the high or low importance of a rule. Considering the regionalized step response of a controlled system shown in Fig 13(a), the defined MFs in Fig 13(b), and the complete rule base in table, at the region 1, as the error decreases, the significance of the rules R_8 / R_7 decreases, while the effect of rules R_5/R_4 increases. Thus, the weights are adjusted as $\theta_5 = 1 - Abs(e)$, $\theta_4 = 1 - Abs(e)$, $\theta_7 = Abs(e)$, and $\theta_8 = Abs(e)$.

Table 8: FPID rules

$r_1 \downarrow r_2 \rightarrow$	N	Z	P
N	N(θ_1)	N(θ_2)	Z(θ_3)
Z	N(θ_4)	Z(θ_5)	P(θ_6)
P	Z(θ_7)	P(θ_8)	P(θ_9)

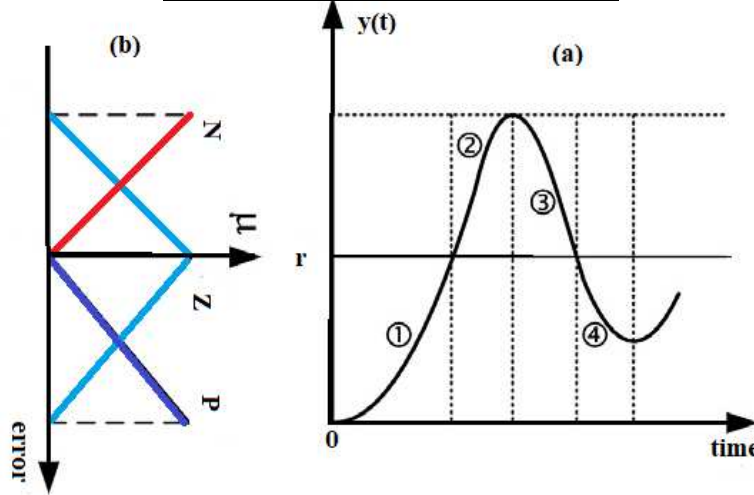


Fig 13: the regionalized step response (a) and the error input MFs (b) (Karasakal, Guzelkaya et al. 2011)

Using two MFs for inputs and four for outputs (as shown in Fig 14(a) and (b)), a bounded-input/bounded-output (BIBO) stable T1FPID controller was designed for some linear and nonlinear control system in (Carvajal, Chen et al. 2000). This controller has three inputs that in the discrete time $Ep = Kp(e(nT) - e(nT - T))$, $Ei = Ki(e(nT))$, and $Ed = Kd(\dot{e}(nT) - \dot{e}(nT - T))$, in which T is sampling time and n

represents the sample.

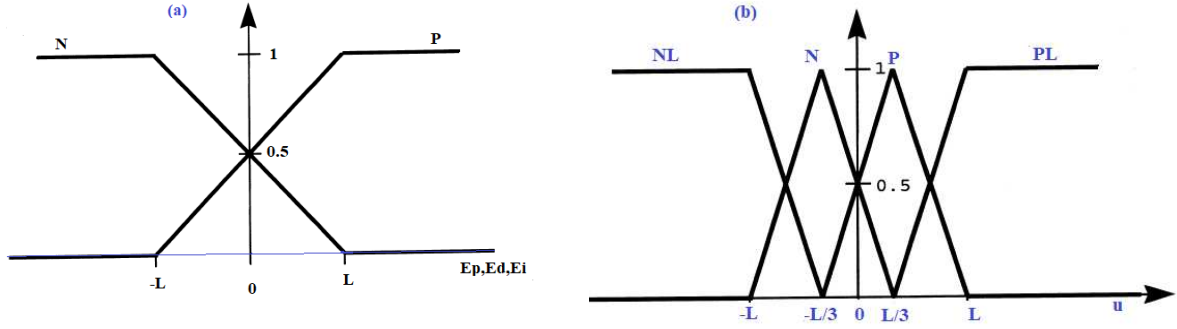


Fig 14: input MFs (a) and output MFs (b)

According to the input/output MFs, the rules of FPID controller were written as follows:

If E_p is N and E_i is N E_d is N then u is NL

If E_p is N and E_i is N E_d is P then u is N

If E_p is P and E_i is N E_d is N then u is N

If E_p is N and E_i is P E_d is N then u is NL

If E_p is P and E_i is N E_d is P then u is P

If E_p is N and E_i is P E_d is N then u is N

If E_p is N and E_i is P E_d is P then u is P

If E_p is P and E_i is P E_d is N then u is P

If E_p is P and E_i is P E_d is P then u is PL

Using the small gain theorem and according to the response of the control system, the BIBO stability of the designed FPID controller has been illustrated.

In (Yesil, Kumbasar et al. 2014), to handle the uncertainty and measurement noise, the peak observer based self-tuning FPID controller in (Yeşil, Güzelkaya et al. 2004) was developed using IT2FLS with three triangle fuzzy MFs and five rules. In this case, the number of rules of the T2FLS is nine. It was shown that interval T2FPID (IT2FPID) outperforms the T1FPID, because of its MFs and the more degree of freedom. The big bang–big crunch (BB–BC) approach was employed to adjust MFs of IT2FPID, to stabilize a power system and a path tracking problem in (Yesil 2014), (Kumbasar and Hagrass 2014) and (Shokouhandeh and Jazaeri 2018), respectively. Fractional order GT2FPID and IT2FPID controllers were designed to control

of the inverted pendulum and to speed control of a nonlinear DC motor in (Shi 2020) and (Khooban, ShaSadeghi et al. 2017), respectively. In (Khooban, ShaSadeghi et al. 2017), a multi-objective stochastic tuning algorithm was developed for the online tuning of IT2FPID. An bee colony-GA (ABC-GA) has been exploited to tune a fractional ordered IT2FPID controller in (Kumar and Kumar 2017). The designed controller was implemented on MIMO two-link robot under uncertainties, and it was illustrated that the performance of the fractional ordered IT2FPID is better. PSO and GA approaches were utilized to tune the parameters of T1FPID and IT2FPID controllers applied to frequency regulation in power system and 5-DOF redundant robot manipulators in (Haroun and Li 2017) and (Kumar and Kumar 2017), respectively. In (Kumbasar 2014) a single input IT2FPID controller was proposed to enhance the efficiency. This control structure is depicted in Fig 15. The single input IT2FPID includes all three parts of classical PID. It was shown that, in this scheme, the closed form can be obtained in a simpler way, through which its parameters can be designed in such that the accuracy can be improved. Using an input for the FLC, the i th rule can be written as follow:

$$\text{if } e \text{ is } \tilde{A}_i \text{ Then } u_f \text{ is } B_i \quad (15)$$

And the final output is:

$$u_f = \frac{u_f^l + u_f^r}{2} \quad (16)$$

Using fully overlapping triangular FSs, u_f^l and u_f^r can be simply written as:

$$u_f^l = \frac{\bar{\mu}_{\tilde{A}_i(e)} \cdot B_i + \underline{\mu}_{\tilde{A}_{i+1}(e)} \cdot B_{i+1}}{\bar{\mu}_{\tilde{A}_i(e)} + \underline{\mu}_{\tilde{A}_{i+1}(e)}} \quad (17)$$

$$u_f^r = \frac{\underline{\mu}_{\tilde{A}_i(e)} \cdot B_i + \bar{\mu}_{\tilde{A}_{i+1}(e)} \cdot B_{i+1}}{\underline{\mu}_{\tilde{A}_i(e)} + \bar{\mu}_{\tilde{A}_{i+1}(e)}}$$

where $\bar{\mu}_{\tilde{A}_i(e)}$ and $\underline{\mu}_{\tilde{A}_i(e)}$ are an upper/lower MFs.

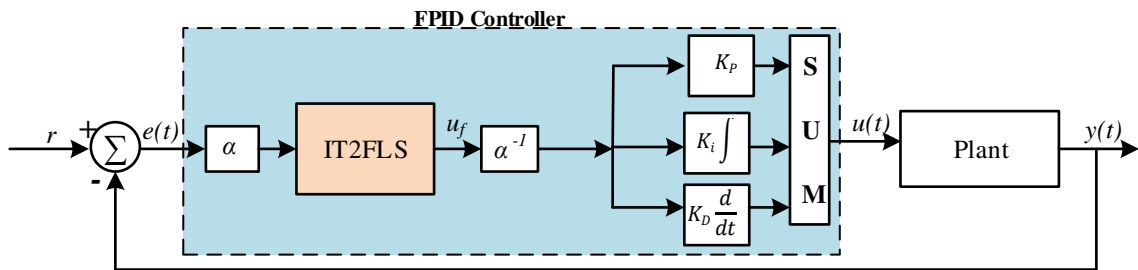


Fig 15: The structure of single input IT2FPID controller

Although the mentioned FPID controllers' design procedure doesn't require the mathematical model of the plant which can be an impotent advantage, they suffer from stability guarantee. Therefore, it is desirable to derive a FPID controller's design method do not ensure stability. By Lyapunov-Krasovskii (L-K) approach a stable IT2FPID controller was schemed to steer the frequency variation to zero under time delay and uncertainties in (Sabahi, Hajizadeh et al. 2021). In that model free method, the adaptation laws were extracted using the following L-K functional:

$$V(t) = \frac{1}{2}e(t)^2 + \frac{1}{2}\int_{t-\tau_{max}}^t e(s)^2 ds + \int_{-\tau_{max}}^0 \int_{t+\theta}^t e(\theta)^2 d\theta ds \quad (18)$$

where $e(t)$ is closed loop error and τ_{max} is the maximum amount of the delay. As can be seen from the L-K functional, despite the convergence of error to zero, the boundedness of α and β cannot be concluded.

The adaptation laws for (α and β) were derived as:

$$\begin{aligned} \dot{\beta} &= -0.5(1 + \tau_{max})e(J \int u_f)^{-1} \\ \dot{\alpha} &= -0.5(1 + \tau_{max})e(J u_f)^{-1} \end{aligned} \quad (19)$$

In (Sabahi, Tavan et al. 2021) a Lyapunov based adaptive IT2FPID was suggested for the frequency adjustment of a micro-grid. In this approach, it was shown that adaptive gains remain bonded and the error approaches to zero properly. To evaluate the close-loop stability and to derive the adaptive scheme for IT2FPID, the following Lyapunov function was used:

$$V(t) = \frac{1}{2}e(t)^2 + \frac{1}{2\gamma_1}\tilde{\alpha}^2 + \frac{1}{2\gamma_2}\tilde{\beta}^2 \quad (20)$$

where

$$e(t) = y_d(t) - y(t) \quad (21)$$

and y is the power system frequency, y_d is the reference signal, $\tilde{\beta} = \beta^* - \beta$, $\tilde{\alpha} = \alpha^* - \alpha$. β^* and α^* are the optimal parameters. The adaptation of (α and β) were written as:

$$\begin{aligned} \dot{\alpha}(t) &= \gamma_1(eJ)\dot{u}_f \\ \dot{\beta}(t) &= \gamma_1(eJ)u_f \end{aligned} \quad (22)$$

In (Sabahi, Tavan et al. 2020), the suggested controller in (Sabahi, Tavan et al. 2021) was developed for a frequency control that suffers from nonlinearities and measurement noise. Using center of gravity defuzzification (COG) method, FPID controller was experimentally used to control of a magnetic system which suffers from nonlinearities, uncertainties and time-delay in (Sain and Mohan 2021). In Table 9 a

summary of contributions to design of the FPID controllers has been illustrated. The comparison is on basis of following scheme: author name, year, reference, Input-output scaling factors tuning method and if the closed loop stability is studied.

Table 9
presented contributions to design and application of the FPID controllers

Author(s)(pub.year)	Input-output scaling factors tuning method	Closed-loop stability analysis?
(Yeşil, Güzelkaya et al. 2004)	Peak observer idea	NO
(Genc, Yesil et al. 2009)	Analytical design	NO
(Karasakal, Guzelkaya et al. 2011)	Analytical design	NO
(Carvajal, Chen et al. 2000)	Analytical design	Yes
(Yesil, Kumbasar et al. 2014)	Peak observer idea	NO
(Yesil 2014)	big bang–big crunch (BB–BC) algorithm	NO
(Kumbasar and Hagrass 2014)	BB–BC	NO
(Shokouhandeh and Jazaeri 2018)	BB–BC	NO
(Kumar and Kumar 2017)	ABC-GA	NO
(Haroun and Li 2017)	PSO	NO
(Kumar and Kumar 2017)	GA	NO
(Kumbasar 2014)	Analytical design	NO
(Sabahi, Hajizadeh et al. 2021).	Lyapunov-Krasovskii based GD	Yes
(Sabahi, Tavan et al. 2021)	Lyapunov based GD	Yes
(Sain and Mohan 2021)	Analytical design	NO

5 Conclusion

In this study, the fuzzy PID controllers, which own the ability of FLSs and simplicity of PID controllers, were reviewed. it was investigated that these controllers can be realized in two cases: (i) adjusting the PID using FLS is known as FGPID controllers and (ii) realizing the classical PID controller using FLS rules which is famous as the FPID controllers. Based on the survey, it was found that the use of evolutionary optimization algorithms and analytical design methods are the common method of adjusting the parameters of the FLSs in the FGPID controllers. Analyzing the stability and tuning the FLS parameters are one of the

important challenges of these controllers. In the FPID controllers, the analytical design methods were often used to adjust the FLS parameters, while some evolutionary optimization algorithms and peak observer idea have often been used to adjust the input-output factors. As discussed, in a number of works the Lyapunov theory was exploited to analyze the closed-loop stability and to obtain the tuning rules of the output scaling factors of the FPID controllers.

Ethical Declarations

- **Funding:** This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.
- **Conflicts of interest/Competing interests:** The authors declare that they have no conflict of interest regarding the publication of this paper.
- **Informed Consent:** informed consent was not required as no humans or animals were involved.

Authors' contribution

- Kamel Sabahi has supervised the project and took the lead in writing the manuscript. Mehdi Tavan, Ardashir Mohammadzadeh, Saleh Mobayen and Wudhichai Assawinchaichote contributed to the final version of the manuscript and provided critical feedbacks.

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