Math 3020, Probability and Statistics for CSC

Test 1 Formulas

- Probability of the Sample Space:
$$P(\Omega) = 1$$
- Probability of the Empty Set:
$$P(\varnothing) = 0$$
- Complement Rule for any event A:
$$P(A^c) = 1 - P(A)$$
- Addition Rule for Two Arbitrary Events:
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
- Addition Rule for Three Arbitrary Events:
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$
- Multiplication Rule for Two Independent Events:
$$P(A \cap B) = P(A)P(B)$$
- Formula for "Permutations with Replacement":
$$P_r(n,k) = n \cdot n \cdot n ... \cdot n = n^k$$
- Formula for "Permutations without Replacement":
$$P(n,k) = \frac{n!}{(n-k)!} = n(n-1)(n-2)(n-3)...(n-k+1)$$
- Formula for "Combinations without Replacement":
$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$
- Formula for "Combinations with Replacement":
$$C_r(n,k) = \binom{k+n-1}{k} = \frac{(n+k-1)!}{k!(n-1)!}$$
- Multiplication Rule for general events:
$$P(A \cap B) = P(A|B) \cdot P(B)$$
- Bayes Rule:
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

- The Expectation is defined as:

$$\mu = \mathbf{E}(X) = \sum_x xP(x)$$

- The Variance is defined as:

$$\sigma^2 = \mathrm{Var}(X) = \mathbf{E}(X - \mathbf{E}(X))^2 = \sum_x (x-\mu)^2 P(x),$$

- Binomial Distribution:

  - The pmf is:

  $$P(x) = \mathbf{P}\{X = x\} = \binom{n}{x} p^x q^{n-x}, \quad \text{for } x = 0, 1, 2, ..., n.$$

  - The expectation is:

  $$\mu = \mathbf{E}(X) = np.$$

  - The variance is:

  $$\sigma^2 = \mathrm{Var}(X) = npq.$$

- Geometric Distribution:

  - The pmf is:

  $$P(x) = \mathbf{P}\{X = x\} = (1-p)^{x-1}p, \quad \text{for } x = 1, 2, ...$$

  - The expectation is:

  $$\mu = \mathbf{E}(X) = \frac{1}{p}$$

  - The variance is:

  $$\sigma^2 = \mathrm{Var}(X) = \frac{1-p}{p^2}$$

- Poisson Distribution:

  - The pmf is:

  $$P(x) = \mathbf{P}\{X = x\} = e^{-\lambda} \cdot \frac{\lambda^x}{x!}, \quad \text{for} \quad x = 0, 1, 2, ...$$

  - The expectation is:

  $$\mu = \mathbf{E}(X) = \lambda$$

  - The variance is:

  $$\sigma^2 = \mathrm{Var}(X) = \lambda$$

* To solve the probability of Binomial Distributions using R:

  - $\mathbf{P}\{X = a\} = \text{dbinom}(a, n, p)$
  - $\mathbf{P}\{X \leq a\} = \text{pbinom}(a, n, p)$
  - $\mathbf{P}\{X > a\} = 1 - \text{pbinom}(a, n, p)$
  - $\mathbf{P}\{X \geq a\} = 1 - \text{pbinom}(a - 1, n, p)$

* To solve the probability of Poisson Distributions using R:

  - $\mathbf{P}\{X = a\} = \text{dpois}(a, \lambda)$
  - $\mathbf{P}\{X \leq a\} = \text{ppois}(a, \lambda)$
  - $\mathbf{P}\{X > a\} = 1 - \text{ppois}(a, \lambda)$
  - $\mathbf{P}\{X \geq a\} = 1 - \text{ppois}(a - 1, \lambda)$