Test 2 Formulas

- **Bernoulli Distribution** with the probability of success p:
  - The pmf is
  $$P(x) = \begin{cases} p, & x = 1 \\ 1-p, & x = 0 \end{cases}$$
  - The expectation is
  $$\mu = \mathbf{E}(X) = p.$$
  - The variance is:
  $$\sigma^2 = \mathrm{Var}(X) = p(1-p) = pq.$$

- **Binomial Distribution** with n trials and probability of success p:
  - The pmf is:
  $$P(x) = \mathbf{P}\{X = x\} = \binom{n}{x} p^x q^{n-x}, \quad \text{for } x = 0, 1, 2, ..., n.$$
  - The expectation is:
  $$\mu = \mathbf{E}(X) = np.$$
  - The variance is:
  $$\sigma^2 = \mathrm{Var}(X) = npq.$$

- **Geometric Distribution** with the probability of success p:
  - The pmf is:
  $$P(x) = \mathbf{P}\{X = x\} = (1-p)^{x-1} p, \quad \text{for } x = 1, 2, ...$$
  - The expectation is:
  $$\mu = \mathbf{E}(X) = \frac{1}{p}$$
  - The variance is:
  $$\sigma^2 = \mathrm{Var}(X) = \frac{1-p}{p^2}$$

- **Poisson Distribution** with the frequency λ:
  - The pmf is:
  $$P(x) = \mathbf{P}\{X = x\} = e^{-\lambda} \cdot \frac{\lambda^x}{x!}, \quad \text{for} \quad x = 0, 1, 2, ...$$
  - The expectation is:
  $$\mu = \mathbf{E}(X) = \lambda$$
  - The variance is:
  $$\sigma^2 = \mathrm{Var}(X) = \lambda$$

- **Uniform Distribution** on an interval (a, b):
  - The pdf is
  $$f(x) = \frac{1}{b-a} \quad \text{for} \quad a < x < b$$
  (and $f(x) = 0$ otherwise).
  - The expectation is:
  $$\mu = \mathbf{E}(X) = \frac{a+b}{2}$$
  - The variance is:
  $$\sigma^2 = \mathrm{Var}(X) = \frac{(b-a)^2}{12}$$

- **Exponential Distribution** with frequency λ:
  - The pdf is
  $$f(x) = \lambda e^{-\lambda x} \quad \text{for} \quad x > 0$$
  (and $f(x) = 0$ otherwise).
  - The cdf is
  $$F(x) = 1 - e^{-\lambda x} \quad \text{for} \quad x > 0$$
  (and $F(x) = 0$ otherwise).
  - The expectation is:
  $$\mu = \mathbf{E}(X) = \frac{1}{\lambda}$$
  - The variance is:
  $$\sigma^2 = \mathrm{Var}(X) = \frac{1}{\lambda^2}$$

- **Normal Distribution** with the mean μ and standard deviation σ:
  - The pdf is
  $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$
  - The cdf is
  $$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right] dt$$
  - The expectation is:
  $$\mathbf{E}(X) = \mu$$
  - The variance is:
  $$\mathrm{Var}(X) = \sigma^2$$

* To solve the probability of Standard Normal Distribution (i.e., $Z \sim N(0,1)$) using R :

  * $\mathbf{P}\{Z \leq z\} = \mathbf{P}\{Z < z\} = \text{pnorm}(z)$
  * $\mathbf{P}\{Z \geq z\} = \mathbf{P}\{Z > z\} = 1 - \text{pnorm}(z)$
  * $\mathbf{P}\{z_1 \leq Z \leq z_2\} = \text{pnorm}(z_2) - \text{pnorm}(z_1)$
  * if $\mathbf{P}\{Z \leq z\} = a$, then $z = \text{qnorm}(a)$

- **Central Limit Theorem (informally)**

Let $X_1, X_2, X_3, ...$ be independent random variables with the same expectation $\mu = \mathbf{E}(X_i)$ for all i and with the same standard deviation $\sigma = \text{Std}(X_i)$ for all i, and let

$$S_n = \sum_{i=1}^{n} X_i = X_1 + X_2 + X_3 + ... + X_n$$

As $n \to \infty$, the sum S_n follows a Normal Distribution:

$$S_n \sim \text{Normal}(n\mu, \sigma\sqrt{n})$$

- Summary of Discrete distributions and Continuous distributions:

Distribution:	**Discrete**	**Continuous**
Defined by:	$P(x) = \mathbf{P}\{X = x\}$ (pmf)	$f(x) = F'(x)$ (pdf)
Calculating probabilities:	$\mathbf{P}\{a < X \leqslant b\} = \sum_{a < x_j \leqslant b} P(x_j)$	$\mathbf{P}(a < X \leqslant b\} = \int_a^b f(x)\,\mathrm{d}x$
Cumulative distribution function (cdf):	$F(x) = \sum_{x_j \leqslant x} P(x_j)$	$F(x) = \int_{-\infty}^{x} f(t)\,\mathrm{d}t$
Total probability:	$\sum_{x_j} P(x_j) = 1$	$\int_{-\infty}^{+\infty} f(x)\,\mathrm{d}x = 1$
Expectation:	$\mu = \mathbf{E}(X) = \sum_{x_j} x_j P(x_j)$	$\mu = \mathbf{E}(X) = \int_{-\infty}^{+\infty} x f(x)\,\mathrm{d}x$
Variance:	$\sigma^2 = \text{Var}(X) = \sum_{x_j} (x_j - \mu)^2 P(x_j)$	$\sigma^2 = \text{Var}(X) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,\mathrm{d}x = \mathbf{E}(X^2) - \mu^2$