

Quotient Exercises

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1. Let \sim be the equivalence relation on $[0, 1]$ with $0 \sim 1$ and $x \sim x$ for all $x \in [0, 1]$. Prove that $[0, 1]/\sim$ is homeomorphic to the circle S^1 .
2. Below we list several equivalence relations \sim for various spaces X . State (without proof) what familiar topological space X/\sim is homeomorphic to.
 - (a) X is a Euclidean disk $D^2 = \{(x, y) \in \mathbb{R}^2 : |x|^2 + |y|^2 \leq 1\}$ with $(x, y) \sim (x', y')$ iff $|x|^2 + |y|^2 = |x'|^2 + |y'|^2$ (i.e. two points are identified if they have the same radius).
 - (a) $X = \mathbb{R}^2$ with $(x, y) \sim (x', y')$ iff $|x|^2 + |y|^2 = |x'|^2 + |y'|^2$.
 - (b) $X = D^2$ with $(x, y) \sim (x, y)$ for all $(x, y) \in D^2$ and $(x, y) \sim (x', y')$ whenever $|x|^2 + |y|^2 = |x'|^2 + |y'|^2 = 1$ (i.e. all points on the boundary of D^2 are identified).
 - (c) $X = \mathbb{R}^n$ with $x \sim y$ iff $x - y \in \mathbb{Z}^n$ (Hint: we saw $n = 1$ in class; solve $n = 2$ and guess the pattern).