Super (Saiyan) Permutations

A Bridge Between Combinatorics and Anime

A lower bound on the length of the shortest superpattern

Anonymous 4chan Poster, Robin Houston, Jay Pantone, and Vince Vatter

October 25, 2018

The Haruhi Problem

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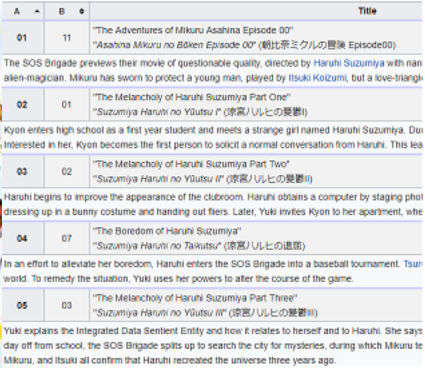

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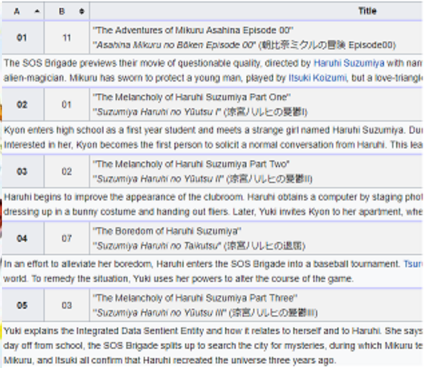

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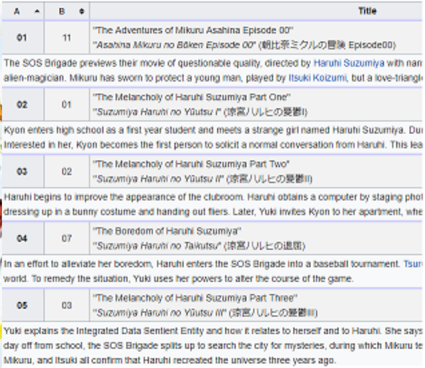

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```
123                1234123
 231                    2314231
  312                       3124312
    213                         2134213
     132                            1324132
      321                               3214321
---------          ---------------------------------
123121321   =>     123412314231243121342132413214321
```

Picture from Jeffrey A. Barnett.

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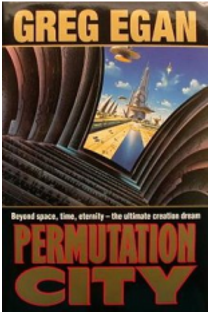
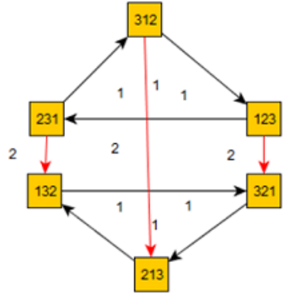


Superpermutations

What about lower bounds?

Superpermutations

What about lower bounds? Construct a weighted digraph as follows. Let your vertex set consist of all permutations on n. Draw an edge between every two permutations where the weight of the edge from π to σ is the minimal number of symbols we need to add to π to get σ. Delete all edges for which the associated transformation produces an intermediate permutation.



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Proposition

$$wt(\pi_1 \ldots, \pi_m) \geq p(\pi_1, \ldots, \pi_m) - 1.$$

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Corollary

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Let π be a superpermutation whose corresponding walk in the digraph is $\pi_1, \ldots, \pi_m$. We can build π by first placing down the n symbols of π_1 and then add symbols according to the walk.

Superpermutations

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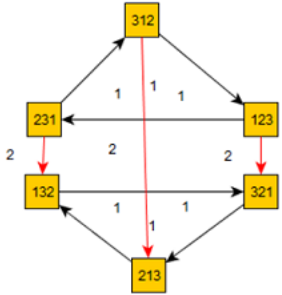
Let π be a superpermutation whose corresponding walk in the digraph is $\pi_1, \ldots, \pi_m$. We can build π by first placing down the n symbols of π_1 and then add symbols according to the walk. Thus the number of additional symbols we must add is exactly

$$wt(\pi_1 \ldots, \pi_m) \geq p(\pi_1, \ldots, \pi_m) - 1 = n! - 1,$$

since we assumed the walk of π visits every permutation.

Superpermutations

Define the 1-loop of a permutation π to be the set of permutations that π can reach by only using edges of weight 1. Observe that the number of 1-loops is $(n-1)!$.



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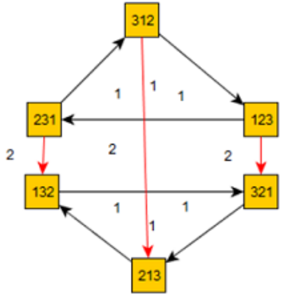

Define $c(\pi_1, \ldots, \pi_m)$ to be the number of 1-loops that the walk $\pi_1, \ldots, \pi_{m-1}$ has completely gone through.

Superpermutations

Proposition

$$wt(\pi_1, \ldots, \pi_m) \geq p(\pi_1, \ldots, \pi_m) + c(\pi_1, \ldots, \pi_m) - 1.$$

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If $wt(\pi_{m-1}, \pi_m) = 1$ then the walk didn't leave its 1-loop, so either (1) it didn't visit a new permutation or (2) it didn't finish a 1-loop. In either case the righthand side increases by at most 1. We conclude the result.

Superpermutations

Corollary (Ashlock and Tillotson, 1993)

$$s(n) \geq n! + (n-1)! + n - 2.$$

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This was all that was known by the combinatorics community. However, while working on the Haruhi problem, someone on 4chan managed to improve this bound!

Superpermutations

Observe that there is a unique edge from π of weight 2 going to $\pi(3)\cdots\pi(n)\pi(2)\pi(1)$. E.g. 51234 goes to 23415.

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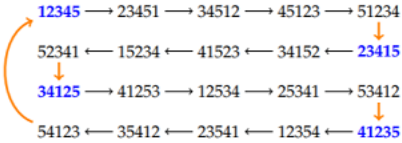
The 2-loop generated by π is defined as the set of vertices visited by the walk that starts at π, follows $n-1$ consecutive edges of weight 1, then follows the (unique) edge of weight 2, and then repeats these steps $n-2$ more times.



Picture from "A Lower Bound on the Length of the Shortest Superpattern."

Superpermutations

$$
\begin{tikzcd}
\mathbf{12345} \arrow[r] & 23451 \arrow[r] & 34512 \arrow[r] & 45123 \arrow[r] & 51234 \arrow[d] \\
52341 \arrow[d] & 15234 \arrow[l] & 41523 \arrow[l] & 34152 \arrow[l] & \mathbf{23415} \arrow[l] \\
\mathbf{34125} \arrow[r] & 41253 \arrow[r] & 12534 \arrow[r] & 25341 \arrow[r] & 53412 \arrow[d] \\
54123 \arrow[uuu, bend left] & 35412 \arrow[l] & 23541 \arrow[l] & 12354 \arrow[l] & \mathbf{41235} \arrow[l]
\end{tikzcd}
$$

Observe that this 2-loop is generated precisely by all of the bold permutations in the above picture (i.e. by fixing the last entry of 12345 and then cyclically generating the elements).

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Observe that this 2-loop is generated precisely by all of the bold permutations in the above picture (i.e. by fixing the last entry of 12345 and then cyclically generating the elements). Also observe that each 2-loop contains exactly $n(n-1)$ elements.

Superpermutations

We say that a walk visits the 2-loop generated by π if it follows an edge of weight 2 or more to arrive at π. Note that this means that the 2-loop we are at depends not only on the vertex we are currently at, but also how we got there.

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We say that a walk visits the 2-loop generated by π if it follows an edge of weight 2 or more to arrive at π. Note that this means that the 2-loop we are at depends not only on the vertex we are currently at, but also how we got there.
Let $t(\pi_1, \ldots, \pi_m)$ denote the number of 2-loops visited by the walk where we let $t(\pi_1) = 1$. Note that since each 2-loop contains only $n(n-1)$ permutations, a walk visiting every permutation must enter at least $(n-2)!$ different 2-loops.

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Theorem

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Superpermutations

Corollary (4chan 2018)

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$$\begin{aligned} wt(\pi_1, \ldots, \pi_m) &\geq p(\pi_1, \ldots, \pi_m) + c(\pi_1, \ldots, \pi_m) + t(\pi_1, \ldots \pi_m) - 2 \\ &\geq n! + (n-1)! - 1 + (n-2)! - 2, \end{aligned}$$

so we conclude the result.

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Do such strings exist for all n? If so, how many are there?

De Bruijn Sequences

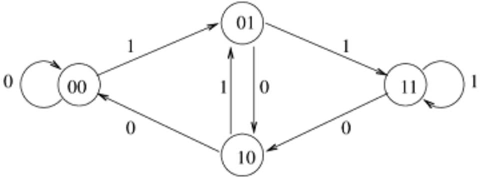
THIS DE BRUIJN STUFF IS COOL BUT CAN IT GET YOU A JOB?

Combinatorics sometimes seems to be about solving puzzles and asking riddles. It can be tricky but sometimes it doesn't seem like "real math." De Bruijn sequences are a perfect case in point. Can you really get paid to think about such stuff and, if you can, what kinds of things do you think about?

A snapshot of this kind of thinking occurred recently in an exotic location—a Banff resort in the Canadian Rockies. The Banff International Research Station is a mathematics institute that runs week-long conferences on focused topics.

De Bruijn Sequences

D_3 (picture from Richard Stanley):



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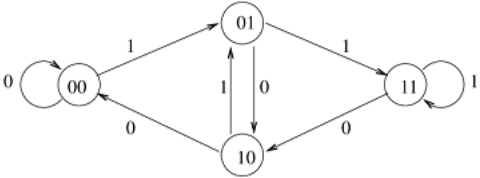

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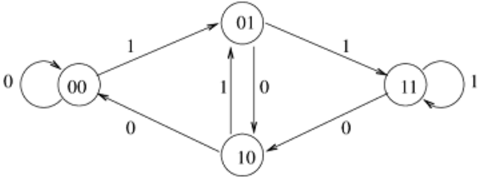

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$$00 \to 00 \to 01 \to 10 \to 01 \to 11 \to 11 \to 01 \to 00$$

$$\implies 00010111$$

De Bruijn Sequences

Theorem (Euler)

A digraph has an Eulerian tour iff it is connected and balanced (every vertex has indegree equal to its outdegree).

De Bruijn Sequences

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Corollary

There exists a De Bruijn sequence of degree n for all n.

De Bruijn Sequences

For a digraph D, let m_{ij} denote the number of edges starting at v_i and ending at v_j, and let ℓ_i denote the number of loops at v_i. Let $L(D)$ be the matrix defined by

$$L_{ij}(D) = \begin{cases} -m_{ij} & i \neq j \\ \mathrm{outdeg}(v_i) - \ell_i & i = j. \end{cases}$$

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Theorem

Let D be a connected, balanced digraph on p vertices. Let $\mu_1, \ldots, \mu_{p-1}$ denote the non-zero eigenvalues of $L(D)$. Then the number of Eulerian tours of D is precisely

$$\frac{1}{p}\mu_1 \cdots \mu_{p-1} \prod_{u \in V} (outdeg(u) - 1)!.$$

De Bruijn Sequences

Corollary

The number of De Bruijn sequences of degree n is equal to

$$2^{2^{n-1}} = \sqrt{2^{2^n}}.$$

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*For n sufficiently large.

Coloring the Plane

Say you're an alchemist trying to perform human transmutation. To do this, you need to color the plane in such a way that no two points at distance 1 from each other have the same color.

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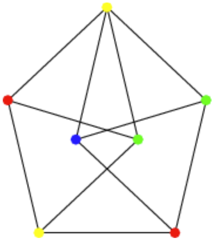
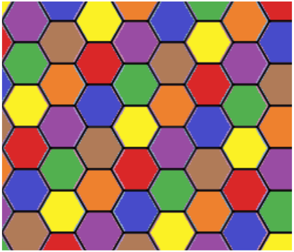
Equivalently, define an infinite graph whose vertex set is $\mathbb{R}^2$ where two points are adjacent if they're at unit distance. The Hadwiger-Nelson problem asks for the chromatic number of this graph, and some people would give an arm and a leg to solve this problem!

Coloring the Plane

It's been known since 1950 that

$$4 \leq \chi(\mathbb{R}^2) \leq 7.$$





Pictures from Wolfram Alpha and math stackexchange

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Theorem (de Grey 2018)

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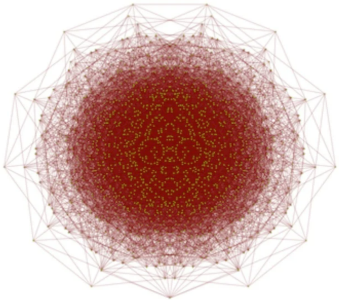
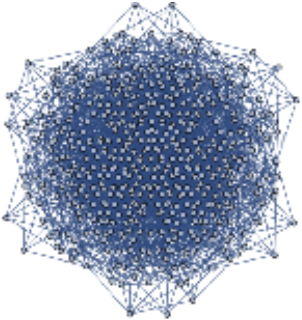
One again, an amateur mathematician made this breakthrough! de Grey proved this by constructing a unit distance graph on 1581 vertices with chromatic number 5.



De Grey's 1,581-point graph (De Grey)

Coloring the Plane

Since then Heule has found constructions with 529 vertices.



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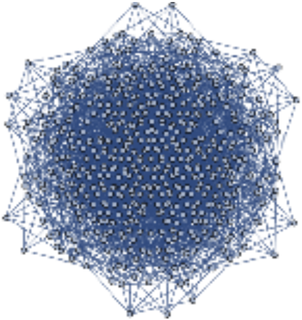

Picture from Wolfram Alpha.

It is a major problem to find smaller graphs with this property, and in particular to find a unit distance graph showing $\chi(\mathbb{R}^2) \geq 6$.

あなたは多分日本語が読めません！