SYSTEM OF LINEAR EQUATIONS AND INEQUALITIES

1. SYSTEM OF LINEAR EQUATIONS IN TWO VARIABLES

Two or more linear equations form a system of linear equations.

Example: Solve the following system of equations by elimination.

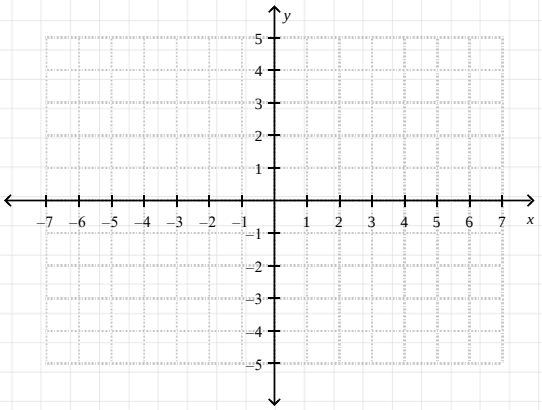
$$\begin{cases} 2x - y = 8 \\ x + y = 7 \end{cases}$$

Example: Solve the following system of equations by substitution.

$$\begin{cases} 6a_1 - 7a_2 = 4 \\ 2a_1 = 3a_2 - 4 \end{cases}$$

Example: Solve the following system of equations by graph.

$$\begin{cases} x + y = 3 \\ 4x + 2y = 8 \end{cases}$$



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2. SYSTEM OF LINEAR EQUATIONS IN THREE VARIABLES

Example: Solve the following system of equations by elimination.

$$\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ 3x - z = 0 \end{cases}$$

Example: Solve the following system of equations by substitution.

$$\begin{cases} 2x + y - z = 2 \\ x - y + z = 7 \\ 2x + 2y + z = 4 \end{cases}$$

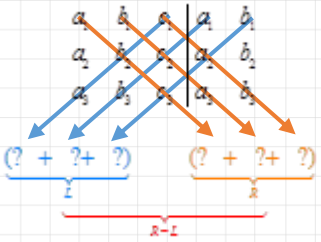
Note: Graph method cannot be applied to linear equations in 3 variables.

Sarrus Method

Given the system of $\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$

Let's form the following representation and define multiplications as shown.

$$\begin{array}{ccc|cc} a_1 & b_1 & c_1 & a_1 & b_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 \\ a_3 & b_3 & c_3 & a_3 & b_3 \end{array}$$



Then,

$$x=\frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}\begin{matrix} d_1 & b_1 \\ d_2 & b_2 \\ d_3 & b_3 \end{matrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}} \qquad y=\frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}\begin{matrix} a_1 & d_1 \\ a_2 & d_2 \\ a_3 & d_3 \end{matrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}}$$

$$z=\frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix}}$$

Example: Solve the following system of equations by Sarrus.

$$\begin{cases} 3x-y+z=9 \\ -2x+y-2z=-8 \\ x+2y+5z=3 \end{cases}$$

Example: Solve the following system of equations by Sarrus.

$$\begin{cases} x-y+z-8=0 \\ x-y-3z+4=0 \\ 2x-2y-3z-1=0 \end{cases}$$

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3. SYSTEM OF LINEAR INEQUALITIES IN TWO VARIABLES

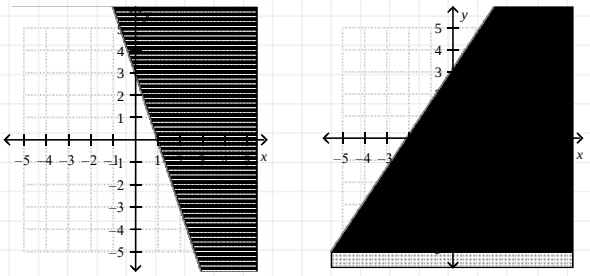
Two or more linear inequalities form a system of linear inequalities.

How to represent the solution set of linear inequalities?

- Represent each of the inequalities on the same coordinate plane.
- Choose the intersection part of each solution sets.

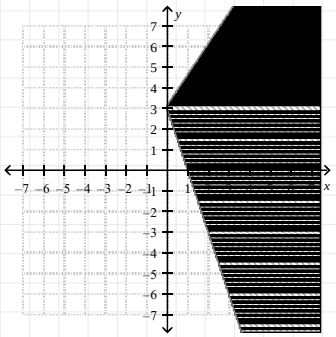
Example: $\begin{cases} 3x+y\geq 3 \\ 2y-3x\leq 6 \end{cases}$

$3x+y\geq 3$ $\qquad$ $2y-3x\leq 6$



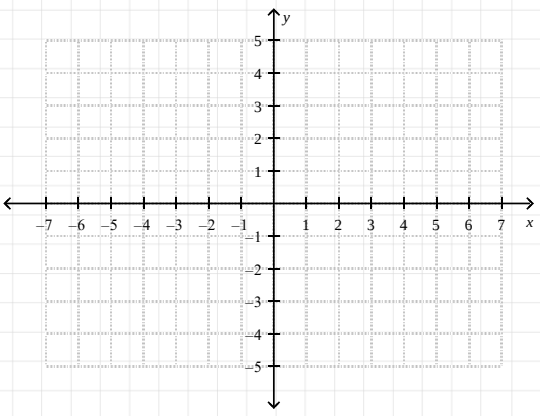
$\Downarrow$

$$\begin{cases} 3x+y\geq 3 \\ 2y-3x\leq 6 \end{cases}$$



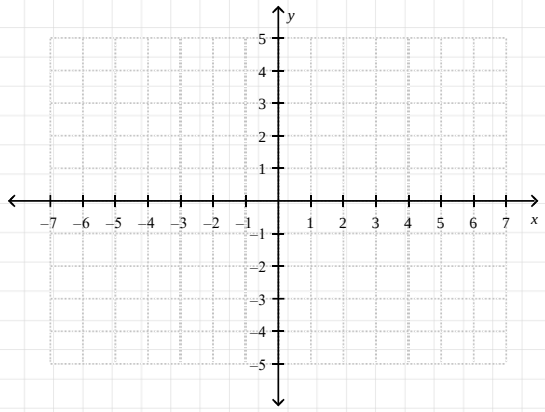
Example: Show the solution set of

$$\begin{cases} 2x+y-4\leq 0 \\ 3x-2y-6>0 \end{cases}$$



Example: Show the solution set of

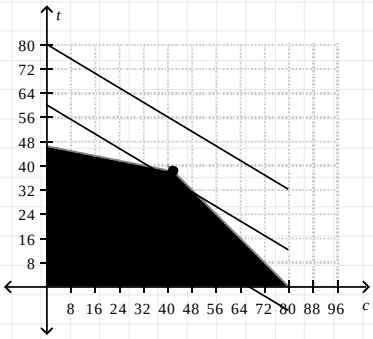
$$\begin{cases} x+2y \geq 0 \\ x-y+3 \geq 0 \\ 3x-2y \leq 0 \end{cases}$$



Maximize / Minimize the function on a specified region

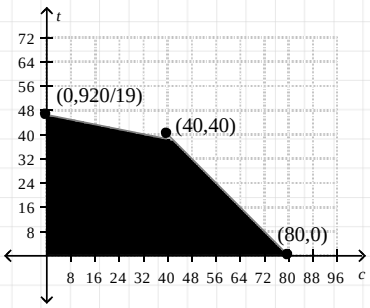
1st Way: Sliding Line:

- Find the solution set of all inequalities.
- Sketch the objective function.
- Slide up to maximize.
- Slide down to minimize.



2nd Way: Corner Point:

- Determine all corner points.
- Substitute in objective function.
- Choose the maximum/minimum value.



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Example: Find the maximum value of the function $f(x,y)=x+y$ on the set given by

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x+3y \leq 6 \\ 3x+y \leq 10 \end{cases}$$

Example: Find the maximum and minimum value of the function $f(x,y)=8x-5y+6$ on the set given by

$$\begin{cases} x \geq -6 \\ y \leq 7 \\ -5x+6y \geq -30 \end{cases}$$

Review Test

1. Hanna has $11.20 in a jar that contains only nickels and dimes. There are 140 coins in the jar. How many dimes are in Hanna's jar?

A) 28 dimes B) 42 dimes C) 56 dimes
D) 84 dimes E) 92 dimes

2. Matt starts with $15 and saves $10 a week. At the same time, Julie starts with $45 and saves $5 a week. In how many weeks will they have the same amount of money?

A) 2 weeks B) 4 weeks C) 6 weeks
D) 8 weeks E) 10 weeks

3. Raquel starts with $75 and saves $5 a week. At the same time, Pedro starts with $120 and spends $10 a week. In how many weeks will they have the same amount of money?

A) 2 weeks B) 3 weeks C) 4 weeks
D) 5 weeks E) 6 weeks

4. What is the *x*-value of the solution to the system below?

$$\begin{cases} x+y=4 \\ 2x+3y=-2 \end{cases}$$

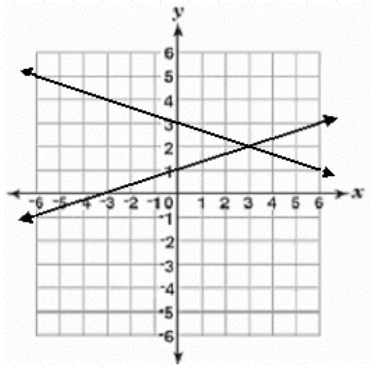
A) 2 B) 6 C) 10 D) 14 E) 16

5. What is the *y*-value of the solution to the system below?

$$\begin{cases} \frac{1}{2}x+y=10 \\ \frac{1}{2}x-y=0 \end{cases}$$

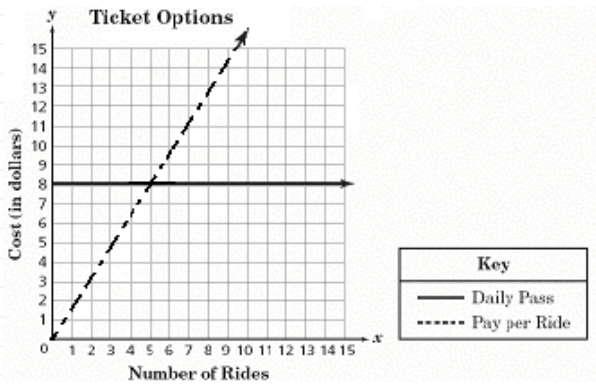
A) 3 B) 5 C) 10 D) 15 E) 18

6. What is the solution to the graphed system of equations?



A) (-3, 0) B) (2, 3) C) (3, 2) D) (1, 3) E) (1, -3)

7. The graph shows two options to buy tickets for amusement park rides. Julie can pay $8 for a daily pass or she can pay $1.60 per ride. For what number of rides is the cost of both options the same?



A) 5 B) 6 C) 7 D) 8 E) 10

8. $$\left.\begin{array}{l} 3x-y+z=9 \\ -2x+y-2z=-8 \\ x+2y+5z=3 \end{array}\right\} \Rightarrow x+y+z=?$$

A) 0 B) 1 C) 2 D) 3 E) None of them

9. $\left.\begin{array}{l} x+z=6 \\ z-3y=7 \\ 2x+y+3z=15 \end{array}\right\} \Rightarrow (x,y,z)=?$

A) $(0,0,0)$ B) $(1,2,-1)$ C) $(-2,4,-1)$
D) $(-2,1,4)$ E) $(2,-1,4)$

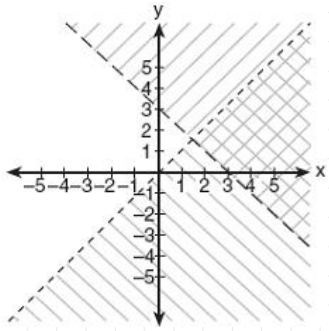
10. $\left.\begin{array}{l} x+y+z=6 \\ 2y+5z=-4 \\ 2x+5y-z=27 \end{array}\right\} \Rightarrow (x,y,z)=?$

A) $(3,-2,3)$ B) $(5,3,-2)$ C) $(5,-3,2)$
D) $(3,5,-2)$ E) $(3,5,2)$

11. $\left.\begin{array}{l} x-2y+3z=7 \\ 2x+y+z=4 \\ -3x+2y-2z=-10 \end{array}\right\} \Rightarrow (x,y,z)=?$

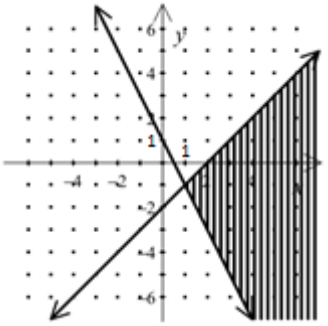
A) $(-1,2,-1)$ B) $(-1,2,1)$ C) $(2,-1,1)$
D) $(2,1,-1)$ E) $(-2,-1,1)$

12. Which ordered pair is in the solution set of the system of inequalities shown in the accompanying graph?



A) $(3,2)$ B) $(0,0)$ C) $(1,5)$ D) $(0,1)$ E) $(-5,0)$

13. Which system of inequalities describes the graph?



A) $\begin{array}{l} y \geq -2x+1 \\ y \leq x-2 \end{array}$ B) $\begin{array}{l} y \leq -2x+1 \\ y > x-2 \end{array}$ C) $\begin{array}{l} y \leq -2x+1 \\ y \geq x-2 \end{array}$
D) $\begin{array}{l} y < -2x+1 \\ y \geq x-2 \end{array}$ E) $\begin{array}{l} y > -2x+1 \\ y \leq x-2 \end{array}$

14. Given the inequalities: $x \geq 0, y \geq 0, 8x + 9y \geq 72,$

Find the minimum value of the function $Z = 2x + 3y + 1$.

A. 0 B. 18 C. 19 D. 24 E. 25

15. Given the inequalities: $x \geq 0, y \geq 0, 3x + y \geq 6, 5x + 4y \geq 20,$

Find the minimum value of the function $z = 3x + 4y$.

A. 0 B. $10\frac{2}{7}$ C. 12 D. $18\frac{6}{7}$ E. 24