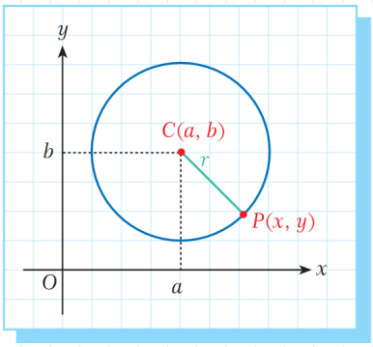
ANALYTIC ANALYSIS OF CIRCLES

1. EQUATION OF A CIRCLE

Standard Equation of a Circle

The equation of circle with center $C(a,b)$ and radius *r* is

$$(x-a)^2+(y-b)^2=r^2 .$$

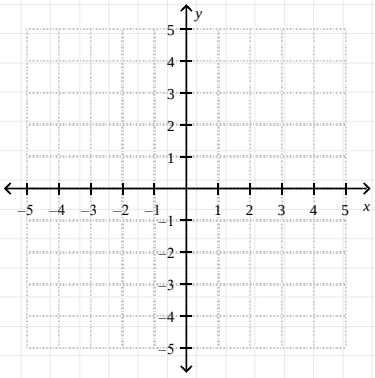


Example: Write the equation of circle with center $C(1,-3)$ and radius 4.

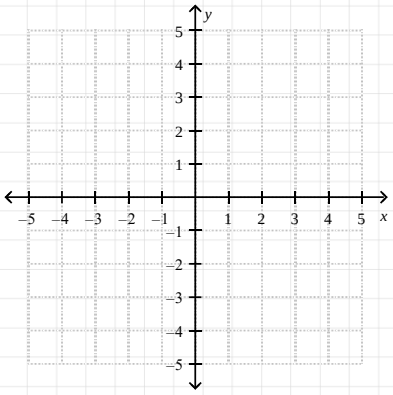
Example: Find the center and radius of the circle

$$(x+\frac{3}{4})^2+(y-2)^2=\frac{1}{9} .$$

Example: Find the equation of the circle with center at the origin and radius 4. Show it on the coordinate plane.



Example: Determine the radius and center of the circle $x^2+(y-2)^2=9$ and sketch its graph.



Example: Determine the equation of the circle with center $C(-3,5)$ passes through the point $P(1,2)$.

Example: Write the equation of the circle centered at $C(2,-3)$ which is tangent to the line $-3x+4y-7=0$.

Example: What is the equation of the circle which passes through the points $O(0,0)$, $A(0,8)$ and $B(6,0)$?

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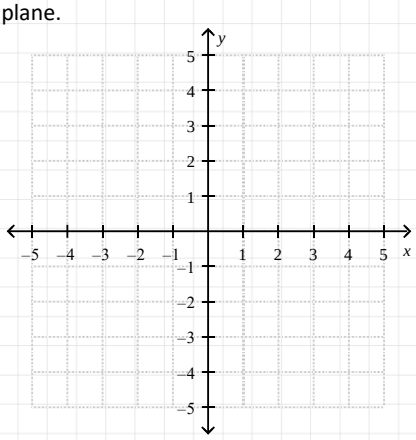
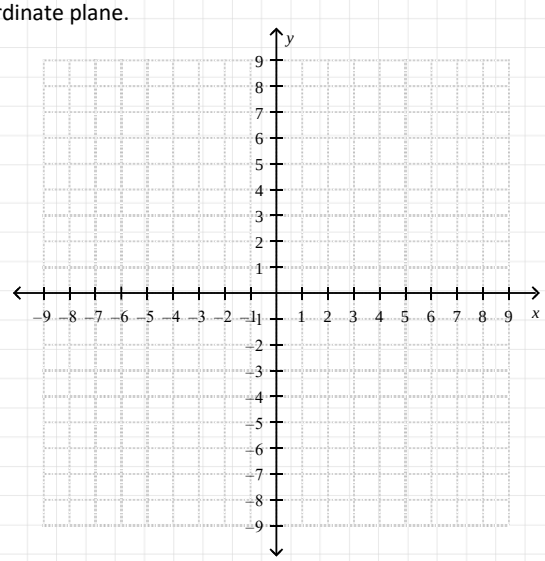
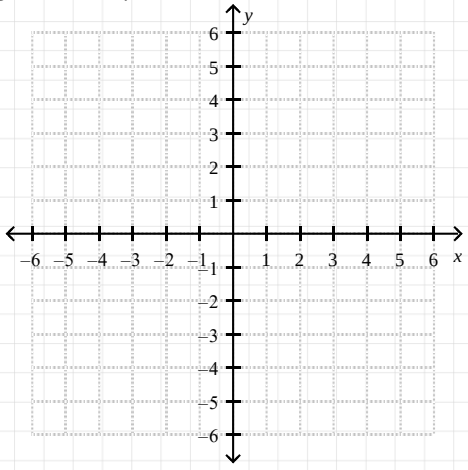
Example: Find the equation of the circle which is tangent to the x-axis and whose center is $C(3,-2)$. Show the circle on following coordinate plane.



Example: Find the equation of the circle which is tangent to the y-axis and whose center is $C(-5,3)$. Show the circle on following coordinate plane.



Example: Find the equation of the circle which is tangent to the x-axis and *y*-axis with center $C(-3,3)$. Show the circle on following coordinate plane.



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General Equation of a Circle

The equation of circle with center $C(a,b)$ and radius *r* is $(x-a)^2+(y-b)^2=r^2$.

by expanding $(x-a)^2+(y-b)^2=r^2$

we get $x^2+y^2-2ax-2by+a^2+b^2-r^2=0$

let us take $D=-2a, E=-2b, F=a^2+b^2-r^2$

so,

the general equation of a circle with center $C(a,b)$ and radius *r* is

$$x^2+y^2+Dx+Ey+F=0$$

where

$$D=-2a, E=-2b, F=a^2+b^2-r^2$$

Example: Find the general equation of the circle with center $C(2,-5)$ and radius $r=4$.

Example: Find the center and radius of the circle whose equation is $x^2+y^2-2x+6y-6=0$.

Example: The circle $x^2+y^2+5x+(2-m)y-m+2=0$ passes through the point $A(1,-2)$.

a) Find the value of m .

b) Find the center of circle.

Example: (UN 2006)

Persamaan lingkaran yang berpusat di $(1,-10)$ dan menyinggung garis $3x-y\sqrt{3}-3=0$ adalah ...

The general equation of circle which is centered at $(1,-10)$ and tangent to $3x-y\sqrt{3}-3=0$ line is ...

Definition: Given the equation $x^2+y^2+Dx+Ey+F=0$,

$\boxed{D^2+E^2-4F}$ is called the ***discriminant*** of the circle.

Observation: $D^2+E^2-4F=4r^2$

Therefore;

- $D^2+E^2-4F<0 \Rightarrow r \notin \mathbb{R}$
  $\Rightarrow$ equation does not represent a circle.
- $D^2+E^2-4F=0 \Rightarrow r=0$
  $\Rightarrow$ equation represents only point.
- $D^2+E^2-4F>0 \Rightarrow r \in \mathbb{R}$
  $\Rightarrow$ equation represents a circle with
  center $(-\frac{D}{2},-\frac{E}{2})$ and radius $\frac{1}{2}\sqrt{D^2+E^2-4F}$.

Note that;
If $F<0$, then $D^2+E^2-4F>0$.
So, it always represents a circle.

Example: Find the possible values of *m* such that
$x^2+y^2+5x-7y+m=0$ represents a circle.

Example: If $x^2+y^2+3x-y-2n+4=0$ represents a point, find *n*.

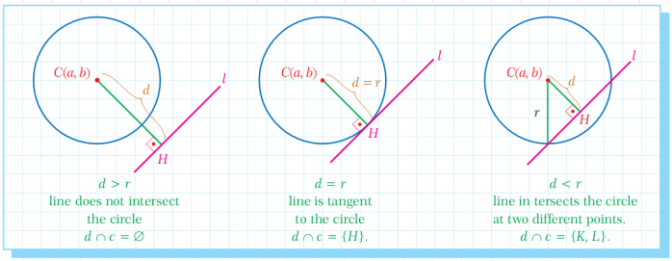
Example: The radius of the circle
$4x^2+4y^2-32x+24y+16m=0$ is 3 units. Find m.

Example: The equation
$(k+2)x^2+(5-2k)y^2+12x+15y-6=0$ represents a circle.
Find *k*, radius and center of the circle.

Exercises 2.1 – Page 113 in Zambak
From 1 to 30

2. POSITIONS OF LINES AND CIRCLES

Relative Position of a Line and a Circle



Example: Describe the position of the circle $(x-4)^2+(y+5)^2=16$ relative to each line.

- $5x-12y-15=0$

- $-5x+12y+28=0$

- $5x+12y+1=0$

Example: The circle $(x-3)^2+(y+1)^2=37$ is tangent to the line $x+6y+m=0$. Find the value(s) of *m*.

Example: Find the possible values of k such that the circle $(x+2)^2+(y+3)^2=4$ and the line $8x-15y+k=0$ intersect at two points.

Example: Find the minimum and maximum distance between the circle $(x-2)^2+(y+1)^2=9$ and the line $3x-4y+15=0$.

Observation:

If $y=mx+n$ is replaced into the equation of the circle $x^2+y^2+Dx+Ey+F=0$, second degree equation with one unknown is obtained.

Based on the discriminant (Δ) of this equation;

- If $\Delta<0$, then the line does not intersect the circle.
- If $\Delta=0$, then the line is tangent to the circle. The point of tangency is the solution of this equation.
- If $\Delta>0$, then the line intersects the circle at two different points. The intersection points are the solution of this equation.

Example: Describe the relative position of $y=x+1$ and $x^2+y^2-5x+7y-6=0$. If the line and circle are tangent or intersecting, find the solution.

Example: Describe the relative position of $-2x+y-2=0$ and $x^2+y^2+2x-y+1=0$. If the line and circle are tangent or intersecting, find the solution.

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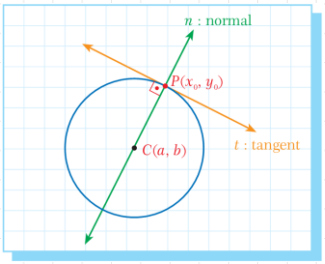
Equations of Tangent and Normal Lines



Tangent Line: a line which meets a circle at only one point.
Normal Line: a line which is perpendicular to tangent of a circle at point of tangency.

Equation of the Tangent and Equation of the Normal through a Point on a Circle

Tangent Line:

$$m_t=-\frac{x_0-a}{y_0-b}\Rightarrow\begin{cases}y-y_0=-\dfrac{x_0-a}{y_0-b}(x-x_0)\\ or\\ (y-y_0)(y_0-b)+(x-x_0)(x_0-a)=0\end{cases}$$

Normal Line:

$$m_n=\frac{y_0-b}{x_0-a}\Rightarrow\begin{cases}y-y_0=\dfrac{y_0-b}{x_0-a}(x-x_0)\\ or\\ (y-y_0)(x_0-a)-(x-x_0)(y_0-b)=0\end{cases}$$

Example: Write the equation of the tangent and normal to the circle $(x-2)^2+(y+3)^2=25$ at the point $P(5,0)$.

Example: Find the equation of the tangent and normal to the circle $(x+2)^2+(y-4)^2=34$ at the point $P(1,-1)$.

Example: Find the equation of the tangent and normal to the circle $x^2+y^2-2x+8y+9=0$ at the point $P(3,-2)$.

Practical ways to find the equation of the tangent through
$P(x_0, y_0)$

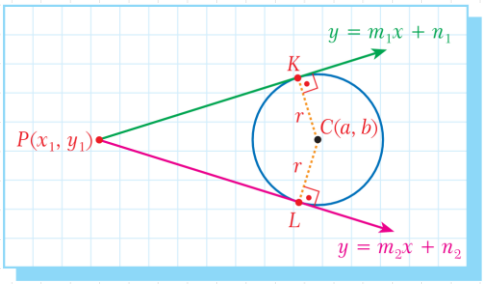
- $(x-a)^2+(y-b)^2=r^2$
  can be written as $(x-a)(x-a)+(y-b)(y-b)=r^2$
  then
  Equation of tangent through $P(x_0, y_0)$
  $(x-a)(x_0-a)+(y-b)(y_0-b)=r^2$

- $x^2+y^2+Dx+Ey+F=0$
  Can be written as
  $x\cdot x+y\cdot y+\frac{D}{2}(x+x)+\frac{E}{2}(y+y)+F=0$
  then
  Equation of tangent through $P(x_0, y_0)$
  $x\cdot x_0+y\cdot y_0+\frac{D}{2}(x+x_0)+\frac{E}{2}(y+y_0)+F=0$

Example: Find the equation of the tangent to the circle $x^2+y^2+4x-8y+3=0$ at the point $P(2,3)$.

Example: (UN 2011 PAKET 12)
Persamaan garis singgung lingkaran $x^2+y^2-6x+4y-12=0$ di titik $(7,1)$ adalah...
The equation of tangent line to the circle $x^2+y^2-6x+4y-12=0$ at point $(7,1)$ is ...

Equations of Tangents Drawn from an External Point



Let $y=mx+n$ be a tangent line.
Distance of a point $C(a,b)$ to $y=mx+n$ can be calculated by
$y=mx+n \Leftrightarrow -mx+y-n=0$.

$$r=\frac{|-ma+b-n|}{\sqrt{m^2+1}} \quad (1)$$

On the other hand, $P(x_1, y_1)$ must satisfy $y=mx+n$, that is,

$$y_1=mx_1+n \quad (2)$$

By using (1) and (2), equations of lines can be obtained.

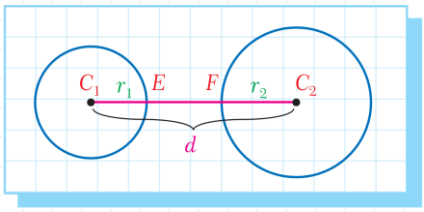
Example: Find the equation of the tangent lines to the circle $x^2+y^2=10$ drawn from the external point $P(2,4)$.

Example: (UN 2007 PAKET B)
Persamaan garis singgung lingkaran $x^2+y^2-2x+2y-2=0$ yang bergradien 10 adalah...
The equation of tangent line to the circle $x^2+y^2-6x+4y-12=0$ with a slope of 10 is ...
Hint: (use $y=10x+n$ (or $10x-y-n=0$) as a tangent line equation, then think about the distance from the center of circle to tangent line)

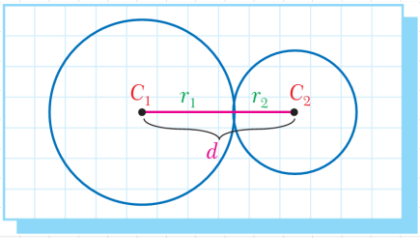
Example: (UN 2010 PAKET A)
Persamaan garis singgung lingkaran $(x-3)^2+(y+5)^2=80$ yang sejajar dengan garis $y-2x+5=0$ adalah...
The tangent line to the circle $x^2+y^2-6x+4y-12=0$ is parallel to the line $y-2x+5=0$. The equation of this tangent line is ...

Relative Position of Two Circles

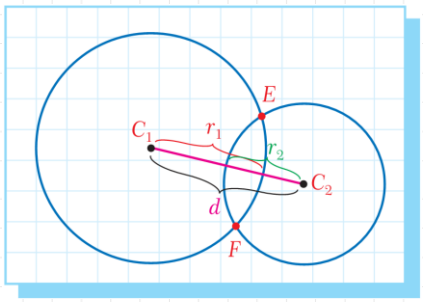
- $d \succ r_1 + r_2 \Rightarrow$ circles do not intersect.



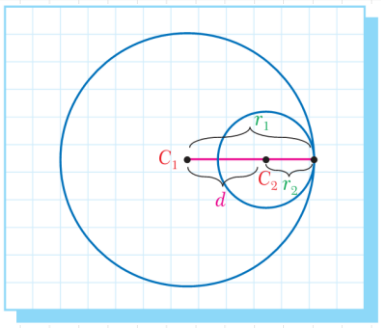
- $d = r_1 + r_2 \Rightarrow$ circles are tangent to each other externally.



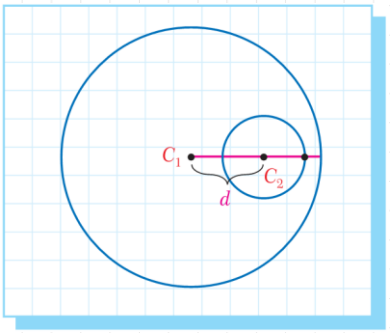
- $|r_1 - r_2| \prec d \prec r_1 + r_2 \Rightarrow$ circles intersect at two points



- $d = |r_1 - r_2| \Rightarrow$ circles are tangent to each other internally.



- $d \prec |r_1 - r_2| \Rightarrow$ circles do not intersect and one of them is completely inside of the other.



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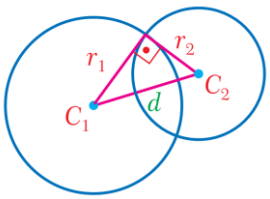
Example: $(x-1)^2 + y^2 = r^2$ and $(x-9)^2 + (y-15)^2 = 64$ are externally tangent. Find *r*.

Example: Find the distance between the nearest and farthest points of the circles $(x+7)^2 + (y+3)^2 = 16$ and $(x-8)^2 + (y-5)^2 = 49$.

Example: If the circles $x^2 + (y-6)^2 = r^2$ and $(x-8)^2 + y^2 = 25$ are internally tangent, then find *r*.

Definition: If two intersecting circles satisfy the equality $d^2 = r_1^2 + r_2^2$, they are called **orthogonal circles**.



Example: The circles $(x+2)^2 + (y-4)^2 = r^2$ and $(x-6)^2 + (y+2)^2 = 36$ are orthogonal circles. Find r.

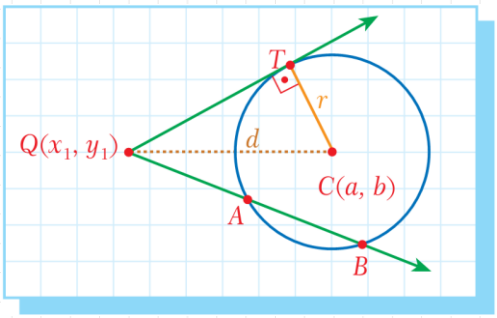
Example: A chord of the circle $x^2+y^2+6x-10y-135=0$ is tangent to the circle $x^2+y^2+6x-10y+9=0$. Find the length of this chord.

Exercises 2.2 – Page 130 in Zambak

From 1 to 24

3. POWER OF A POINT, RADICAL AXIS, AND RADICAL CENTER

Power of a Point



Definition: Power of the point $Q(x_1,y_1)$ with respect to the circle is

$$P = d^2 - r^2$$

Since $d^2 = (x_1-a)^2 + (y_1-b)^2$

- $P = (x_1-a)^2 + (y_1-b)^2 - r^2$

Since $d^2 - r^2 = |QT|^2$ from ΔQTC

- $P = |QT|^2$

Since $|QT|^2 = |QA| \cdot |QB|$

- $P = |QA| \cdot |QB|$

Observation:

$Q(x_1,y_1)$ is at the exterior of circle

$$\Rightarrow d > r \;\Rightarrow\; P > 0$$

$Q(x_1,y_1)$ is on the circle

$$\Rightarrow d = r \;\Rightarrow\; P = 0$$

$Q(x_1,y_1)$ is at the interior of circle

$$\Rightarrow d < r \;\Rightarrow\; P < 0$$

Conclusion:

Power of $Q(x_1,y_1)$ wrt $(x-a)^2 + (y-b)^2 = r^2$ is

$$(x_1-a)^2 + (y_1-b)^2 - r^2$$

Power of $Q(x_1,y_1)$ wrt $x^2 + y^2 + Dx + Ey + F = 0$ is

$$x_1^2 + y_1^2 + Dx_1 + Ey_1 + F$$

Example: Find the power of $Q(-2,1)$ with respect to $(x-2)^2 + (y+3)^2 = 4$.

Example: Find the length of tangent segment drawn from $A(8,-3)$ to $x^2 + y^2 + 8x - 4y - 5 = 0$.

Example: The length of tangent segment drawn from $M(4,1)$ to $x^2 + y^2 - 2x + 6y + p = 0$ is 4. Find p.

Example: If $M(4,-5)$ is in the interior region of $(x+2)^2 + (y-5)^2 = r^2$. Find the minimum integer value of r.

Exercises 2.3 – Page 141 in Zambak

1, 2, 3, 4, 5, 8, 9, 10

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4. FURTHER APPLICATIONS ON CIRCLES

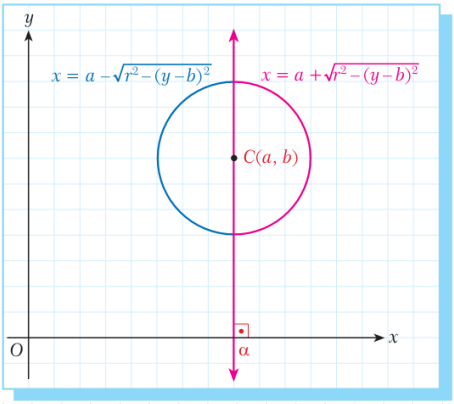
Equation of a Semi-Circle

$(x-a)^2+(y-b)^2=r^2$ is given.

solve for x.

$$x=a\pm\sqrt{r^2-(y-b)^2}$$

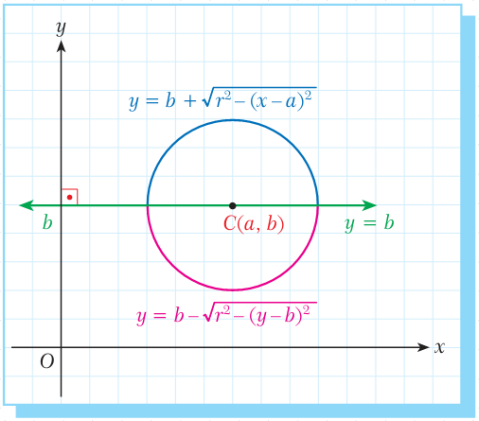
are the semi-circles separated by the line $x=a$.



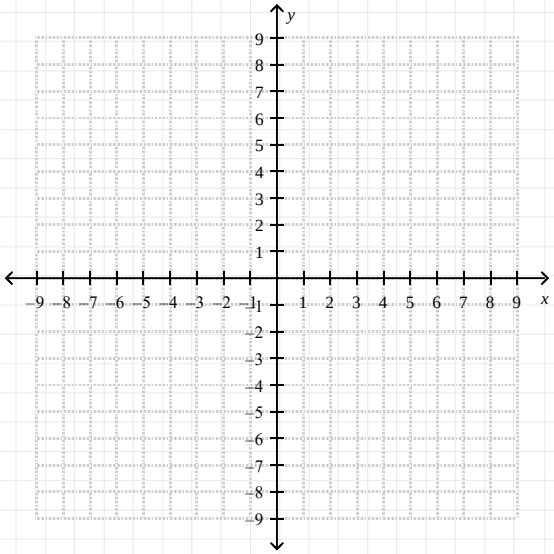
solve for y.

$$y=b\pm\sqrt{r^2-(x-a)^2}$$

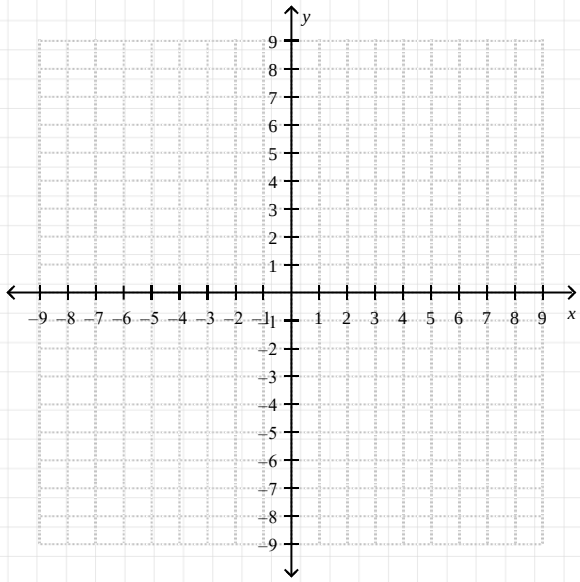
are the semi-circles separated by the line $y=b$.



Example: Draw the graph of semi-circle given by the equation $y=\sqrt{49-x^2}$.



Example: Draw the graph of semi-circle given by the equation $x = \sqrt{16-(y-3)^2} - 1$.

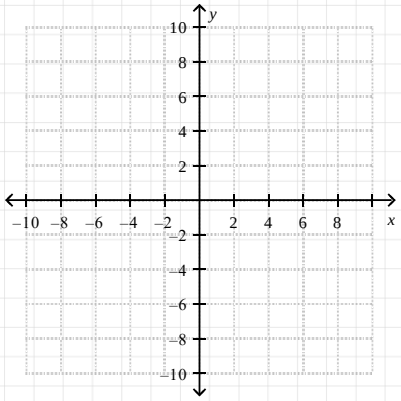


Inequalities Involving Circles

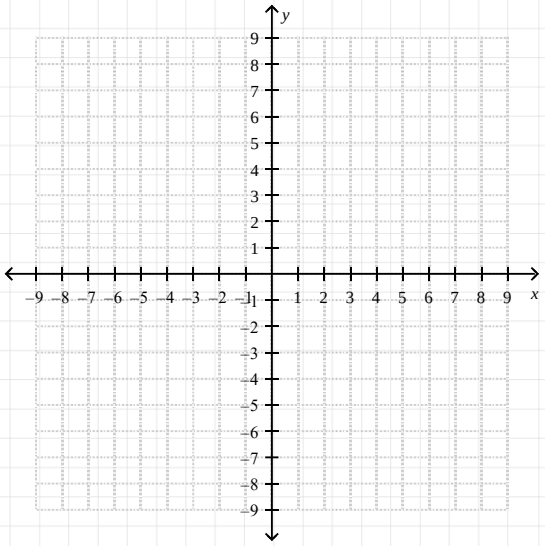
Let $(x-a)^2+(y-b)^2 = r^2$ given,

- $(x-a)^2+(y-b)^2 < r^2 \Rightarrow$ interior of circle.
- $(x-a)^2+(y-b)^2 = r^2 \Rightarrow$ circle itself.
- $(x-a)^2+(y-b)^2 > r^2 \Rightarrow$ exterior of circle.

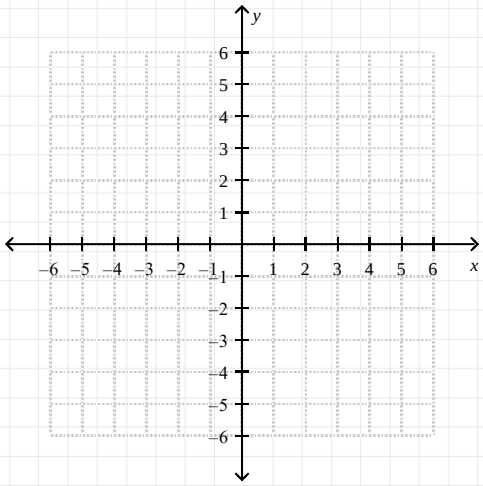
Example: Show the points satisfying the inequality $x^2+y^2 \le 36$.



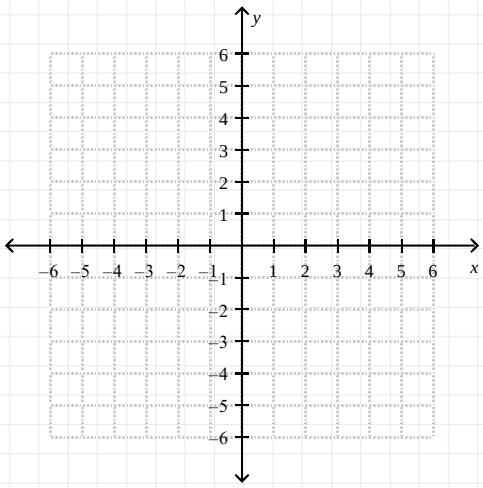
Example: Draw the graph of inequality $(x-2)^2+(y-3)^2 > 9$.



Example: Draw the graph of $4 \le x^2+y^2 < 16$



Example: Show the points on the coordinate plane which satisfy the system of inequalities $\left.\begin{matrix} x^2+(y-2)^2 \le 9 \\ y \ge 3 \end{matrix}\right\}$.



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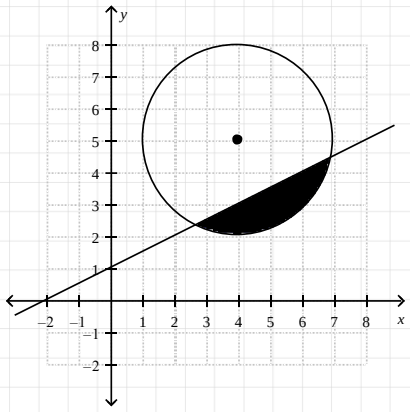
Example: Write the appropriate inequalities for the shaded region in the figure.



Exercises 2.4 – Page 153 in Zambak

From 4 to 13

Review Test

1) Which of the following is the equation of the circle centered at $M(-1,2)$ with radius 4 cm?

A) $(x+1)^2+(y-2)^2=4$
B) $(x+1)^2+(y-2)^2=16$
C) $(x-1)^2+(y+2)^2=16$
D) $(x-2)^2+(y+1)^2=16$
E) $(x+1)^2-(y-2)^2=16$

2) What is the radius of the circle $(x-2)^2+(y+3)^2=8$?

A) 4 B) 3 C) $2\sqrt{2}$ D) $\sqrt{6}$ E) 2

3) Which of the following is the equation of the circle centered at $M(3,4)$ and passes through the origin?

A) $(x+3)^2+(y+4)^2=25$
B) $(x-3)^2+(y-4)^2=5$
C) $(x-3)^2+(y-4)^2=25$
D) $(x-4)^2+(y-4)^2=25$
E) $(x-1)^2+(y+3)^2=25$

4) Which one is the equation of the circle having center at $C(-3,1)$ and passing through the point $P(1,-2)$?

A) $(x-3)^2+(y+1)^2=4$
B) $(x+3)^2+(y-1)^2=25$
C) $(x+1)^2+(y-3)^2=25$
D) $x^2+y^2=5$
E) $(x-2)^2+(y+1)^2=25$

5) What is the radius of the following circle $x^2+y^2-2x-4y+1=0$?

A) 1 B) 2 C) 3 D) 4 E) 5

6) What is the radius of the following circle $x^2+y^2-4x-8y+11=0$?

A) 1 B) 2 C) 3 D) 4 E) 5

7) The points $A(-2,1)$ and $B(2,5)$ are given. Which one of the following is the equation of the circle with diameter $[AB]$?

A) $x^2+(y+3)^2=2\sqrt{2}$
B) $(x-3)^2+y^2=8$
C) $(x+3)^2+y^2=2\sqrt{2}$
D) $x^2+(y-3)^2=4\sqrt{2}$
E) $x^2+y^2=8$

8) If the point $(3,\sqrt{3})$ is on the circle $x^2+y^2-4a=0$, then find the radius of the circle.

A) $\sqrt{2}$ B) $\sqrt{3}$ C) $\sqrt{6}$ D) $2\sqrt{2}$ E) $2\sqrt{3}$

9) Which one of the followings is the center C and the radius r of the circle $4x^2+4y^2-8x+24y=81$?

A) $C(4,-12), r=9$
B) $C(1,-3), r=9$
C) $C(1,-3), r=11$
D) $C(1,-3); r=\frac{11}{2}$
E) $C(1,-3), r=\frac{9}{2}$

10) If a circle is tangent to the lines $y=2$ and $y=-4$, and centered on the line $x+y=-1$, then find the equation of the circle.

A) $(x-1)^2+(y-2)^2=9$
B) $(x-2)^2+y^2=6$
C) $(x+1)^2+y^2=6$
D) $x^2+y^2=36$
E) $x^2+(y+1)^2=9$

11) Find one of the intersection point of the line $x+y=1$ and the circle $x^2+y^2=1$?

A) $(1,1)$ B) $(1,-1)$ C) $(1,0)$ D) $(-1,0)$ E) $(0,-1)$

12) If the line $y=3x-a$ is tangent to the circle $x^2+y^2=10$, then find the sum of the values of a .

A) 0 B) 1 C) 2 D) -2 E) 10

13) What is the equation of a circle which is centered at $M(3,4)$ and tangent to the line $3x+4y=-5$?

A) $(x-3)^2+(y-4)^2=16$

B) $(x-3)^2+(y-4)^2=36$

C) $(x-4)^2+(y-3)^2=16$

D) $x^2+y^2=36$

E) $x^2+y^2+3x+4y=5$

14) If the circles $x^2+y^2=9$ and $(x-3)^2+(y-a)^2=4$ are externally tangent to each other, then find the value of a . ($a>0$)

A) 3 B) 4 C) 5 D) 6 E) 8

15) Which one of the following is the equation of the line that is tangent to the circle $x^2+y^2-8x+2y-8=0$ at the point $A(1,3)$?

A) $3x+4y-15=0$

B) $4x+3y-13=0$

C) $3x-5y+12=0$

D) $3x-4y+9=0$

E) $4x+5y-10=0$

16) What is the equation of the tangent line at point $(3,2)$ to the circle $(x-2)^2+(y-3)^2=2$?

A) $x+y=1$ B) $y-x=-2$ C) $x-y=1$

D) $x=y$ E) $x+y=2$

17) (UN 2012/E25)

Lingkaran $L\equiv(x+1)^2+(y-3)^2=9$ memotong garis $y=3$.
Garis singgung lingkaran yang melalui titik potong antara lingkaran dan garis tersebut adalah ...

The circle $L\equiv(x+1)^2+(y-3)^2=9$ intersects the line $y=3$.
The lines passing through the intersection points of line and circle are ...

A) $x=2$ dan $x=-4$
B) $x=2$ dan $x=-2$
C) $x=-2$ dan $x=4$
D) $x=-2$ dan $x=-4$
E) $x=8$ dan $x=-10$

18) (UN 2009 PAKET A/B)

Lingkaran $(x-4)^2+(y-4)^2=16$ memotong garis $y=4$.
Garis singgung lingkaran yang melalui titik potong antara lingkaran dan garis tersebut adalah ...

The circle $(x-4)^2+(y-4)^2=16$ intersects the line $y=4$.
The lines passing through the intersection points of line and circle are ...

A) $y=8-x$
B) $y=0$ dan $y=8$
C) $x=0$ dan $x=8$
D) $y=x+8$ dan $y=x-8$
E) $y=x-8$ dan $y=8-x$

19) (UN 2008 PAKET A/B)

Persamaan garis singgung melalui titik $(2,3)$ pada lingkaran $x^2+y^2=13$ adalah ...

The equation of tangent line to the circle $x^2+y^2=13$ at point $(2,3)$ is ...

A) $2x-3y=13$
B) $2x+3y=-13$
C) $2x+3y=13$
D) $3x-2y=-13$
E) $3x+2y=13$

20) (UN 2011 PAKET 46)

Persamaan garis singgung lingkaran $x^2+y^2-6x+4y+11=0$ di titik $(2,-1)$ adalah ...

The equation of tangent line to the circle $x^2+y^2-6x+4y+11=0$ at point $(2,-1)$ is ...

A) $x-y-12=0$
B) $x-y-4=0$
C) $x-y-3=0$
D) $x+y-3=0$
E) $x+y+3=0$

21) (UN 2007 PAKET A)
Persamaan garis singgung lingkaran $x^2+y^2-6x+4y-12=0$ di titik $P(7,-5)$ adalah ...
The equation of tangent line to the circle $x^2+y^2-6x+4y-12=0$ at point $P(7,-5)$ is ...

A) $4x-3y=43$
B) $4x+3y=23$
C) $3x-4y=41$
D) $10x+3y=55$
E) $4x-5y=53$

22) (UN 2010 PAKET B)
Salah satu persamaan garis singgung lingkaran $(x-4)^2+(y-5)^2=8$ yang sejajar dengan garis $y-7x+5=0$ adalah ...
The equation of one of the tangent line to the circle $(x-4)^2+(y-5)^2=8$ which is parallel to the line $y-7x+5=0$ is...

A) $y-7x-13=0$
B) $y+7x+3=0$
C) $-y-7x+3=0$
D) $-y+7x+3=0$
E) $y-7x+3=0$

23) (UN 2004)
Salah satu persamaan garis singgung lingkaran $x^2+y^2-4x-8y+15=0$ yang tegak lurus garis $x+2y=6$ adalah ...
The equation of one of the tangent line to the circle $x^2+y^2-4x-8y+15=0$ which is perpendicular to the line $x+2y=6$ is...

A) $2x-y+3=0$
B) $2x-y+5=0$
C) $2x-y+7=0$
D) $2x-y+13=0$
E) $2x-y+25=0$

24) Find the intersection points of the circle $x^2+y^2-8x+11=0$ and the line $y-x+1=0$.

A) (3,0) (2,0) B) (0,-2) (2,0) C) (3,2) (2,1) D) (3,-1) (2,2) E) (1,2) (3,1)

25) Find the power of the point $Q(3,5)$ with respect to the circle $(x+1)^2+(y-2)^2=16$.

A) 6 B) 7 C) 8 D) 9 E) 10

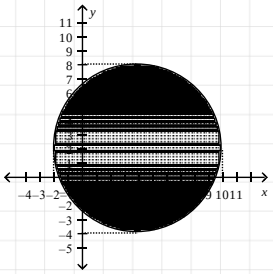
26) The equation $(5-2m)x^2+(m-1)y^2-4x+6y+2m=0$ represents a circle. What is the radius of this circle?

A) 2 B) 3 C) 4 D) 5 E) 6

27) What is the relative position of the two circles $x^2+y^2=9$ and $(x-3)^2+(y-3)^2=81$?

A) the circles do not intersect.
B) the circles are tangent to each other externally.
C) the circles intersect at two points.
D) the circles are tangent to each other internally.
E) the circles do not intersect and one of them is completely inside of the other.

28) Find the expression representing the graph below?



A) $(x-5)^2+(y-5)^2\le 25$
B) $(x-4)^2+(y-6)^2\le 36$
C) $(x-4)^2+(y-6)^2\ge 36$
D) $(x-4)^2+(y-2)^2\le 36$
E) $(x-6)^2+(y-2)^2=36$

29) The slope of the tangent lines drawn from the point $P(3,2)$ to the circle $(x+1)^2+(y-2)^2=9$ are m_1 and m_2. Find m_1+m_2.

A) 0 B) $-\sqrt{3}$ C) $-\sqrt{7}$ D) $\frac{\sqrt{7}}{7}$ E) $-\frac{3\sqrt{7}}{7}$

30) Find the distance from the nearest point on the circle $(x-2)^2+(y+1)^2=9$ to the point P(-1 , 3).

A)2 B)4 C)6 D)8 E)10