CHAPTER 1: INDEFINITE INTEGRALS

1. ANTIDERIVATIVE AND INDEFINITE INTEGRAL

A. Definition of the Indefinite Integral

If $F'(x) = f(x)$, then

$F(x)$ is called **primitive** or **antiderivative** of $f(x)$, and

$F(x) + c$ is called **indefinite integral** of $f(x)$.

$\int f(x)\ dx$ means **the integral of $f(x)$ with respect to x.**

Example: $\int 2x\ dx =$ integral of $f(x) = 2x$ with respect to x.

We think that derivative of which function is $f(x) = 2x$.

It is $F(x) = x^2 \rightarrow$ antiderivative

So, $\int 2x\ dx = x^2 + c$

Example: $\int \cos x\ dx =$

Example: $\int dx =$

C. Basic Integration Formulas

Properties

$\int a \cdot f(x)\ dx = a \cdot \int f(x)\ dx \qquad for\ a \in \mathbb{R}$

$\int [f(x) \pm g(x)]\ dx = \int f(x)\ dx \pm \int g(x)\ dx$

Basic Integration Formulas – 1

$\int x^n\ dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq 1$

Example: Find the following integrals.

- $\int 2\ dx =$
- $\int -\sqrt{3}\ dx =$
- $\int x^3\ dx =$
- $\int x^{-4}\ dx =$
- $\int \frac{5}{x^7}\ dx =$
- $\int (3x^2 + 2x - 1)\ dx =$
- $\int \sqrt[5]{x^4}\ dx =$
- $\int \left(\frac{x^4 + 2x^3 + 1}{x^2} \right) dx =$

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Basic Integration Formulas – 2

$\int \frac{1}{x}\ dx = \ln|x| + c$

$\int \frac{u'(x)}{u(x)}\ dx = \ln|u(x)| + c$

Example: Find the following integrals.

- $\int \frac{3}{x}\ dx =$
- $\int \frac{3x^2}{x^3}\ dx =$
- $\int \frac{5}{x+4}\ dx =$
- $\int \frac{\cos x}{\sin x}\ dx =$

Check Yourself 3 – Page 8 in Zambak

Basic Integration Formulas – 3

$$\int e^x \; dx = e^x + c$$

$$\int a^x \; dx = \frac{a^x}{\ln a} + c$$

Example: Find the following integrals

- $\int 2^x \; dx =$
- $\int 7 \cdot 4^x \; dx =$
- $\int 5 \cdot e^x \; dx =$
- $\int 6 \cdot e^{x+3} \; dx =$
- $\int 4 \cdot 5^{4x-1} \; dx =$

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Basic Integration Formulas – 4

$$\int \sin x \; dx = -\cos x + c$$

$$\int \sin(ax+b) \; dx = -\frac{1}{a}\cos(ax+b) + c$$

$$\int \cos x \; dx = \sin x + c$$

$$\int \cos(ax+b) \; dx = \frac{1}{a}\sin(ax+b) + c$$

Example: Find the following integrals

- $\int 3\sin x \; dx =$
- $\int \cos(2x-5) \; dx =$
- $\int 5\sin(4x-1) \; dx =$
- $\int \sin^2 x \; dx =$

$$\int \frac{1}{\cos^2 x} \; dx = \int \sec^2 x \; dx = \int (1+\tan^2 x) \; dx = \tan x + c$$

$$\int \frac{1}{\cos^2(ax+b)} \; dx =$$

$$\int \sec^2(ax+b) \; dx =$$

$$\int (1+\tan^2(ax+b)) \; dx = \frac{1}{a}\tan(ax+b) + c$$

$$\int \frac{1}{\sin^2 x} \; dx = \int \csc^2 x \; dx = \int (1+\cot^2 x) \; dx = -\cot x + c$$

$$\int \frac{1}{\sin^2(ax+b)} \; dx =$$

$$\int \csc^2(ax+b) \; dx =$$

$$\int (1+\cot^2(ax+b)) \; dx = -\frac{1}{a}\cot(ax+b) + c$$

Example: Find the following integrals

- $\int \frac{1}{\sin^2 3x} \; dx =$
- $\int \sec^2(6x+4) \; dx =$
- $\int \tan^2 x \; dx =$
- (UN 2010 PAKET A)
  $\int \left(\sin^2 x - \cos^2 x\right) \; dx =$

- $\int \cos^2 5x \; dx - \int \sin^2 5x \; dx =$

- (UN 2010 PAKET B)
  $\int 3 - 6\sin^2 x \; dx =$

- $\int 3\cot^2(7x-5) \; dx =$

$$\int \frac{1}{\sqrt{1-x^2}} \; dx = \arcsin x + c_1 = -\arccos x + c_2$$
$$\int \frac{1}{1+x^2} \; dx = \arctan x + c_1 = -\operatorname{arccot} x + c_2$$

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Exercises 1.1 – Page 16 in Zambak

Part A,B-5,6, C

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2. INTEGRATION METHODS

A. Integration by Substitution

Substitution Method

$\int f(u(x)) \cdot u'(x)\ dx = F(u(x)) + c$ where $F(x)$ is antiderivative of $f(x)$

Example: Find the following integrals

- $\int (2x+5)^5\ dx =$
- $\int 2x(x^2+4)^3\ dx =$
- (UN 2011 PAKET 46)
  $\int 6x\sqrt{3x^2+5}\ dx =$
- $\int \frac{5}{(3x-2)^6}\ dx =$
- $\int \sin(2x-3)\ dx =$
- $\int (2x+3) \cdot \cos(x^2+3x-1)\ dx =$
- (UN 2012/E52)
  $\int \sin^2 x \cdot \cos x\ dx =$
- $\int \cos^3 x\ dx =$
- $\int \frac{\sin(\ln x)}{x}\ dx =$
- $\int e^{3x-1}\ dx =$

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- $\int 2x \cdot e^{x^2+2}\ dx =$
- $\int 3^{2x+3}\ dx =$
- $\int \frac{e^x}{\sqrt{1-e^{2x}}}\ dx =$
- $\int \frac{1}{3x+5}\ dx =$
- $\int \frac{e^x}{e^x+2}\ dx =$
- $\int \frac{1}{\cos^2(3x+2)}\ dx =$
- $\int \frac{2}{\sqrt{1-4x^2}}\ dx =$
- $\int \frac{3}{1+9x^2}\ dx =$
- $\int (x+3)(x-1)^3\ dx =$
- (UN 2003)
  $\int x\sqrt{x+1}\ dx =$

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B. Integration by Parts

Integration by Parts

$\int u \; dv = u \cdot v - \int v \; du$ where

u is easy to differentiate
dv is easy to integrate

Example: Find the following integrals

- $\int x \cdot e^x \; dx =$
- $\int \ln x \; dx =$
- $\int x^3 \cdot \ln x \; dx =$
- $\int \arcsin x \; dx =$
- $\int x \cdot \sin x \; dx =$
- (UN 2004)
  $\int x^2 \cdot \sin 2x \; dx =$
- (UN 2005)
  $\int \left(x^2+1\right) \cdot \cos x \; dx =$
- (UN 2006)
  $\int \left(x^2-3x+1\right) \cdot \sin x \; dx =$
- $\int e^{2x} \cdot \sin e^x \; dx =$
- $\int \sin(\ln x) \; dx =$
- $\int e^x \cdot \cos x \; dx =$

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C. Integrating Partial Fractions

1. $\int \frac{P(x)}{Q(x)}$ with $deg[P(x)] = deg[Q(x)] - 1$

Example: Find the following integrals

- $\int \frac{4}{3x-2}\,dx =$
- $\int \frac{2x+5}{x^2+5x-5}\,dx =$
- $\int \frac{1}{x^2+2x+1}\,dx =$

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2. $\int \frac{P(x)}{Q(x)}$ with $deg[P(x)] < deg[Q(x)]$ and $Q(x)$ reducible

Reducing $Q(x)$

1. $\frac{P(x)}{(ax+b)\cdot(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$
2. $\frac{P(x)}{(ax+b)^n} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \dots + \frac{D}{(ax+b)^n}$
3. $\frac{P(x)}{(ax+b)\cdot(cx^2+dx+e)} = \frac{A}{ax+b} + \frac{Bx+C}{cx^2+dx+e}$

Example: Find the following integrals

- $\int \frac{1}{x^2-5x}\,dx =$
- $\int \frac{3}{x^2-x-2}\,dx =$

- $\int \frac{11x+4}{2x^2+x-3}\,dx =$
- $\int \frac{x+1}{x^3-1}\,dx =$
- $\int \frac{5x-1}{(2x+3)^2}\,dx =$

3. $\int \frac{P(x)}{Q(x)}$ with $deg[P(x)] < deg[Q(x)]$ and $Q(x)$ not reducible

Example: Find the following integrals

- $\int \frac{1}{x^2-2x+2}\,dx =$
- $\int \frac{1}{x^2+6x+13}\,dx =$

4. $\int \frac{P(x)}{Q(x)}$ with $deg[P(x)] \geq deg[Q(x)]$

Example: Find the following integrals

- $\int \frac{4x^2+14x+3}{x^2+3x}\,dx =$

- $\int \frac{x^2}{x+1}\,dx =$

- $\int \frac{x+5}{x-1}\,dx =$

Check Yourself 9 – Page 37 in Zambak

D. Integrating Radical Functions

1. Integrating Simple Radical Functions

Try to eliminate radical by substituting as u^2, u^3 etc.

Example: Find the following integrals

- $\int \sqrt{5x-1}\,dx =$

- $\int \sqrt[3]{2x+1}\,dx =$

- $\int \frac{x}{\sqrt{x^2+5}}\,dx =$

- $\int \frac{4x}{\sqrt{x-1}}\,dx =$

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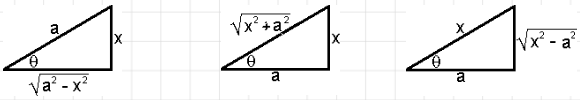
2. Integrals of the Form $\int \sqrt{a^2 \pm x^2}\ dx$ or $\int \sqrt{u^2 \pm x^2}\ dx$

Use the following right triangles depending on question.



Example: Find the following integrals

- $\int \sqrt{1-x^2}\ dx =$

- $\int \frac{\sqrt{9x^2-1}}{x}\ dx =$

- $\int \frac{1}{x^2 \cdot \sqrt{9+4x^2}}\ dx =$

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E. Integrating Trigonometric Functions

1. Integrals of the Form $\int \sin^m x \cdot \cos^n x\ dx \quad (m, n \in N)$

Example: Find the following integrals

- $\int \sin^3 x \cdot \cos^5 x\ dx =$

- $\int \sin^2 x \cdot \cos^3 x\ dx =$

- $\int \sin^2 x \cdot \cos^2 x\ dx =$

Note: If both powers are even, use following identities.

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

2. Integrals of the Form $\int \sin mx \cdot \cos nx\ dx$, $\int \sin mx \cdot \sin nx\ dx$ or $\int \cos mx \cdot \cos nx\ dx$

Note: For this case, we can use following inverse conversion identities.

$$\sin a \cdot \sin b = -\frac{1}{2}\left[\cos(a+b) - \cos(a-b)\right]$$

$$\sin a \cdot \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

$$\cos a \cdot \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

Example: Find the following integrals

- (UN 2009 PAKET A/B)
  $\int 4\sin 5x \cdot \cos 3x\ dx =$

- $\int \cos 8x \cdot \cos 4x\ dx =$

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3. Substituting $t = tan\ \frac{x}{2}$

For integrands containing only the first power of $\sin x$ and/or $\cos x$.

The $t = tan\ \frac{x}{2}$ Substitution		
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$dx = \dfrac{2}{1+t^2}\,dt$

Example: Find the following integrals

- $\displaystyle\int \frac{1}{\cos x + 1}\,dx =$

- $\displaystyle\int \frac{1}{1+\cos x - \sin x}\,dx =$

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Exercises 1.2 – Page 47 in Zambak

1, 2, 3, 4-a,d,e,i,j,m,n,o,r,s 5-b,c,e,f,h 6-a,b,c,d,e,h 7-a,b,h,i,l,n,p 8-a,b,d

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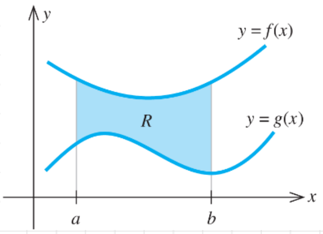
CHAPTER 2: DEFINITE INTEGRALS

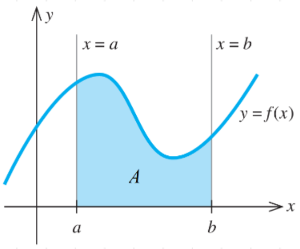
1. EVALUATING DEFINITE INTEGRALS

A. Definition of the Definite Integral

The area between $f(x)$ and $g(x)$ on the interval $[a,b]$ where $f(x) \geq g(x)$ for all x values is called **definite integral** of $f(x)-g(x)$ on the interval $[a,b]$

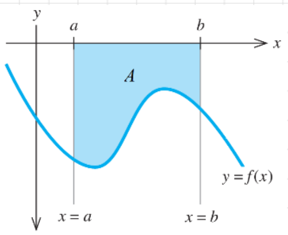


$$R=\int_a^b [f(x)-g(x)]\,dx$$



$$A=\int_a^b [f(x)-0]\,dx$$

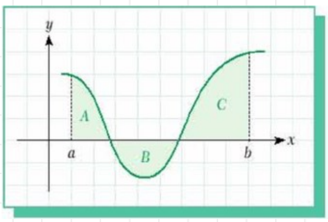
$$\boxed{A=\int_a^b f(x)\,dx}$$



$$A=\int_a^b [0-f(x)]\,dx=\int_a^b -f(x)\,dx$$

$$A=-\int_a^b f(x)\,dx \text{ or } \boxed{-A=\int_a^b f(x)\,dx}$$

Example: In the figure, A=9, B=5, and C=13.
Find the total area and calculate the integral on the interval $[a,b]$



B. Fundamental Theorem of Calculus

If $f(x):[a,b]\to\mathbb{R}$ and $\int f(x)\,dx=F(x)+c$ then

The Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx=F(b)-F(a)$$

Example: Calculate the following integrals

- $\int_2^4 2x\,dx=$
- $\int_0^2 x^3\,dx=$
- $\int_0^{\frac{\pi}{2}} \cos x\,dx=$
- $\int_0^1 e^{4x}\,dx=$

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C. Properties of the Definite Integral

Properties

Let $f:[a,b]\to\mathbb{R}$ and $g:[a,b]\to\mathbb{R}$ be two integrable functions. Then,

1. $\int_a^a f(x)\,dx=0$
2. $\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx$
3. $\int_a^b c\cdot f(x)\,dx=c\cdot\int_a^b f(x)\,dx$
4. $\int_a^b |f(x)\pm g(x)|\,dx=\int_a^b f(x)\,dx\pm\int_a^b g(x)\,dx$
5. $\int_a^c f(x)\,dx=\int_a^b f(x)\,dx+\int_b^c f(x)\,dx$ where $a\leq b\leq c$
6. $\int_a^b f(x)\,dx\geq\int_a^b g(x)\,dx$ where $f(x)\geq g(x)$ for all x
7. $\left|\int_a^b f(x)\,dx\right|\leq\int_a^b |f(x)|\,dx$

Example: Calculate the following integrals

- $\int_2^2 x^2\,dx=$

- (UN 2010 PAKET A)
  $\int_1^2 \left(x^2 - \frac{1}{x^2}\right) dx =$

- (UN 2012/B25)
  $\int_1^3 \left(2x^2 + 4x - 3\right) dx =$

- $\int_0^1 e^{-2x-1}\, dx =$

- (UN 2012/E52)
  $\int_0^{\frac{\pi}{2}} \sin(2x - \pi)\, dx =$

- $\int_0^{\frac{\pi}{2}} \left(\sin x + \cos x\right) dx =$

- (UN 2003)
  $\int_0^{\pi} x \cos x\, dx =$

- $\int_0^1 \sqrt{5x-3}\, dx =$

- $\int_1^e x \cdot \ln x\, dx =$

- (UN 2007 PAKET B)
  $\int_1^p \left(3t^2 + 6t - 2\right) dx = 14 \Rightarrow (-4p) =$

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D. Leibniz's Rule

Leibniz's Rule

Let $u(x)$ and $v(x)$ be two differentiable functions.

$$F(x) = \int_{u(x)}^{v(x)} f(t)\, dt \Rightarrow F'(x) = f\left(v(x)\right) \cdot v'(x) - f\left(u(x)\right) \cdot u'(x)$$

Example: Find the following derivatives

- $F(x) = \int_1^x \cos(2t)\, dt \;\Rightarrow\; F'(x) =$

- $F(x) = \int_x^{x^2} \left(t^2 - 3t + 2\right) dt \;\Rightarrow\; F'(x) =$

- $F(x) = \int_{2x}^{x^3} \left(-2t^2 + t\right) dt \;\Rightarrow\; F'(1) =$

E. The Mean Value Theorem

Mean Value Theorem

Let $f : [a,b] \to \mathbb{R}$ be a continuous function. Then there exists at least one real number $c \in [a,b]$ such that

$$f(c) = \frac{\int_a^b f(x)\, dx}{b-a}$$

$f(c)$ is called the mean value of $f(x)$ on the interval $[a,b]$

Example: Find the mean value of $f(x)=x^2-3x+1$ on the interval $[0,2]$

Example: Find the mean value of $f(x)=\cos x$ on the interval $\left[0,\frac{\pi}{2}\right]$

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Exercises 2.1 – Page 68 in Zambak

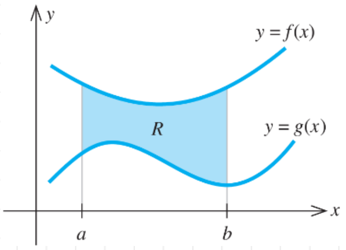
CHAPTER 3: APPLICATIONS OF DEFINITE INTEGRALS

1. FINDING THE AREA BETWEEN CURVES

Finding Area

Let $f:[a,b]\to\mathbb{R}$ and $g:[a,b]\to\mathbb{R}$ be continuous functions such that for every x, $f(x)\geq g(x)$. Then the area of the region between $f(x)$ and $g(x)$ on the interval $[a,b]$ is

$$R=\int_a^b\left[f(x)-g(x)\right]dx$$



Example: Find the area of the region bounded by the line $y=3+x$, x-axis and y-axis.

Example: Find the area of the region bounded by the graph of $y=4-x^2$ and x-axis.

Example: Find the area of the region bounded by the graph of $y=1-3x$ and the x-axis on the interval $[2,5]$.

Example: Find the area of the region bounded by the graph of $y=x^2-3x-4$ and the x-axis on the interval $[-1,7]$.

Example: (UN 2008 PAKET A/B)

Luas daerah yang dibatasi oleh kurva $y=\sqrt{x+1}$, sumbu X dan $0\leq x\leq 8$ adalah ...

Find the area of the region bounded by the graph of $y=\sqrt{x+1}$, the x-axis, and $0\leq x\leq 8$.

Example: Find the area of the region bounded by the graph of $y=2x+1$, $x=0$, $y=1$ and $y=3$.

Example: Find the area of the region bounded by the graph of $x=y^2-4$ and the y-axis.

Example: (UN 2012/A13)
Luas daerah yang dibatasi oleh kurva $y = x^2 - 4x + 3$ dan $y = 3 - x$ adalah ...
Find the area of the region bounded by the graph of $y = x^2 - 4x + 3$ and $y = 3 - x$.

Example: (UN 2003)
Luas daerah yang dibatasi oleh kurva $y = x^2 - 9x + 15$ dan $y = -x^2 + 7x - 15$ adalah ...
Find the area of the region bounded by the curves $y = x^2 - 9x + 15$ and $y = -x^2 + 7x - 15$.

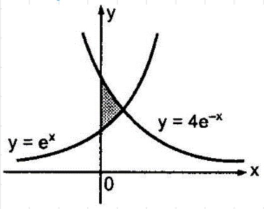
Example: Find the area of the region bounded by the graphs of $y = 2x^2 - 3x + 1$ and $y = -8x + 4$.

Example: (UN 2010 PAKET A)
Luas daerah yang dibatasi parabola $y = x^2 - x - 2$ dengan garis $y = x + 1$ pada interval $0 \le x \le 3$ adalah ...
Find the area of the region bounded by the curves $y = x^2 - x - 2$ and $y = x + 1$ on the interval $0 \le x \le 3$.

Example: Find the area of the region bounded by the graphs of $y = x^3$, $x = 0$, $y = -1$, $y = 3$.

Example: Find the area of the region bounded by the graphs of $y = \cos x$ and $y = \sin x$ on the interval $[0, \pi]$.

Example: Find the area of the shaded region given in the figure.



Example: (UN 2010 PAKET A)
Luas daerah tertutup yang dibatasi oleh kurva $x = y^2$ dan garis $y = x - 2$ adalah ...
Find the area of the region bounded by the curve $x = y^2$ and the line $y = x - 2$.

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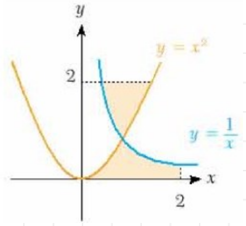
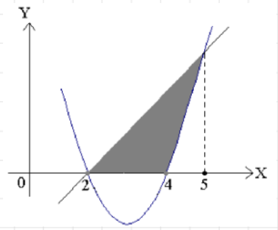
Example: Find the area of the shaded region in the figure.



Example: (UN 2009 PAKET A/B)

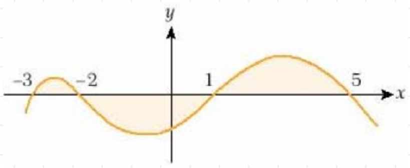
Luas daerah yang dibatasi oleh parabola $y = x^2 - 6x + 8$, garis $y = x - 2$ dan sumbu X dapat dinyatakan dengan …

What is the formula to find the area of the region bounded by the parabola $y = x^2 - 6x + 8$, line $y = x - 2$, and x-axis?



Example: The figure shows the graph of the function $f(x)$.

$\int_{-2}^{1} f(x)\, dx = -5$ and $\int_{-3}^{5} f(x)\, dx = 5$ are given. Find the total area of the shaded region.



Exercises 3.1 – Page 69 in Zambak

Exercises 3.1 – Page 89 in Zambak

3,5,7,8,9,11,12,14,15,16,17,18,21,22,23,24,25,26,28,29,30,31,32,33,34,35,36,37,38,40,41,42

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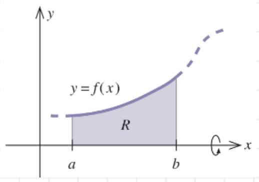
2. OTHER APPLICATIONS

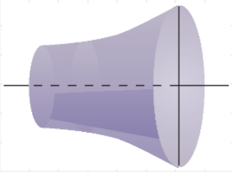
A. Calculating the Volume of a Solid of Revolution

Finding Volume

Let $f:[a,b]\to\mathbb{R}$ be a continuous function defined on $[a,b]$.
Then the volume V of the solid generated by rotating the area between the graph of $f(x)$ and the $x-axis$ on $[a,b]$ about $x-axis$ is

$$V=\pi\cdot\int_a^b\left[f(x)\right]^2\,dx$$





Example: Find the volume of the solid figure generated by rotating the area of the region bounded by $y=2x+5$, $x=2$ and $x=3$ around the x-axis.

Example: (UN 2008 PAKET A/B)
Daerah yang dibatasi oleh kurva $y=4-x$, $x=1$, $x=3$, dan sumbu X diputar mengelilingi sumbu X sejauh 360^o, maka volume benda putar yang terjadi adalah …
Find the volume of the solid figure generated by rotating the area of the region bounded by $y=4-x$, $x=1$, $x=3$ and the x-axis.

Example: Find the volume of the solid figure generated by rotating the area of the region bounded by $y=x^2+1$ and the x-axis on $[0,1]$ about the x-axis.

Note:
If we rotate a figure around the y-axis then the volume is created by $x=f(y)$ and we integrate it with respect to dy :

$$V=\pi\cdot\int_c^d\left[f(y)\right]^2\,dy$$

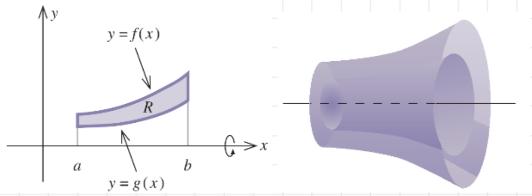
Example: Find the volume of the solid figure generated by rotating the area of the region bounded by $y=4x-1$, $y=0$, $y=3$ and the y-axis around the y-axis.

Example: Find the volume of the solid figure generated by rotating the area of the region bounded by $y=2x^2-1$, the y-axis, and the lines $y=1$ and $y=3$ about the y-axis.

Example: (UN 2007 PAKET A)
Volume benda putar yang terjadi jika daerah yang dibatasi oleh kurva $y = x^2 + 1$ dan $y = 3$ diputar mengelilingi sumbu Y sejauh 360^o adalah ...
Find the volume of the solid figure generated by rotating the area of the region bounded by $y = x^2 + 1$ and $y = 3$ around the y-axis.

Note:
If we rotate the area between two curves $f(x)$ and $g(x)$ on the interval $[a,b]$ then the volume of the solid figure generated is

$$V = \pi \cdot \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx$$



Example: Find the volume of the solid figure generated by rotating the area of the region bounded by $y = x^2 + 4$ and $y = 2$ on the interval $[1,3]$ about the x-axis.

Example: (UN 2012/B25) (UN 2011 PAKET 12) (UN 2007 PAKET A)
Volume benda putar yang terjadi untuk daerah yang dibatasi oleh kurva $y = x^2$ dengan $y = 2x$ diputar mengelilingi sumbu X sejauh 360^o adalah ...
Find the volume of the solid figure generated by rotating the area of the region bounded by $y = x^2$ and $y = 2x$ about the x-axis.

Example: (UN 2012/A13)
Volume benda putar yang terjadi bila daerah yang dibatasi oleh kurva $y = x^2$ dan $y = 4x - 3$ diputar mengelilingi sumbu X adalah ...
Find the volume of the solid figure generated by rotating the area of the region bounded by $y = x^2$ and $y = 4x - 3$ about the x-axis.

Example: Find the volume of the solid figure generated by rotating the area of the region bounded by $y = 5 - x^2$ and $y = x^2 + 3$ about the x-axis.

Example: (UN 2010 PAKET B)
Volume benda putar yang terjadi bila daerah yang dibatasi oleh kurva $y = x^2$ dan $y = \sqrt{x}$ diputar mengelilingi sumbu X sejauh 360^o adalah ...
Find the volume of the solid figure generated by rotating the area of the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the x-axis.

Example: Find the volume of the solid figure generated by rotating the area of the region bounded by $y = 1 - x^2$, the x-axis, $x = 1$ and $x = 3$ about the y-axis.

Example: (UN 2005)
Volume benda putar yang terjadi karena daerah yang dibatasi oleh kurva $y = x^2$ dan $y^2 = 8x$ diputar 360^o mengelilingi sumbu Y adalah ...
Find the volume of the solid figure generated by rotating the area of the region bounded by $y = x^2$ and $y^2 = 8x$ about the y-axis.

Example: (UN 2003)
Volume benda putar yang terjadi karena daerah yang dibatasi oleh sumbu X, sumbu Y, dan kurva $y = \sqrt{4-x}$ diputar terhadap sumbu Y sejauh 360^o, dapat dinyatakan dengan ...
What is the formula for the volume of the solid figure generated by rotating the area of the region bounded by x-axis, y-axis and $y = \sqrt{4-x}$ curve around the y-axis?

Check Yourself 3 – Page 95 in Zambak

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Exercises 3.2 – Page 99 in Zambak

3,4,5,6,7,10,12,13,14,16,17,18,19,20,21,23,26,27,30

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