1. CARTESIAN PRODUCT

Ordered Pair

Given that a,b are any two elements.
(a,b) is called **ordered pair** where a is the first and b is the second component.

Note: $(a,b) \neq (b,a)$

Example: Give two examples of ordered pairs (x,y) satisfying $x+2y=3$.

Equality of Ordered Pairs

$(a,b)=(c,d) \Leftrightarrow a=c \;\; \& \;\; b=d$

Example: Given that $(2x+3,6)=(11,5y-4)$, find x and y.

Notation:

$A=[a,b]=\{x \in \mathbb{R} \text{ such that } a \leq x \leq b\}$
all real numbers between a and b where a and b are included.

$A=(a,b)=\{x \in \mathbb{R} \text{ such that } a < x < b\}$
all real numbers between a and b where a and b are excluded.

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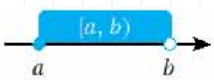

$A=(a,b]=\{x \in \mathbb{R} \text{ such that } a < x \leq b\}$
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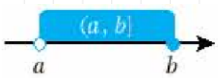


Example: If $A=[-1,4)$ and $B=(-1,5)$, find $A \cup B$ and $A \cap B$.

Cartesian Product

Let A and B be two non-empty sets.
Set of all ordered pairs whose first component is from A and whose second component is from B is called **Cartesian product of A and B**, and denoted by $A \times B$.

Example: Given $A=\{a,b,c\}$ and $B=\{1,2\}$. Find

$A \times B=$

$B \times A=$

Number of Elements of Cartesian Product

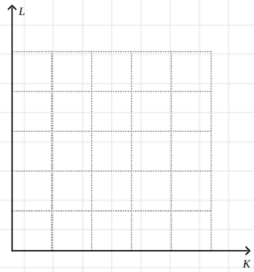
$n(A \times B)=n(B \times A)=n(A) \cdot n(B)$

Example: Given $M=\{1,2,3,4\}$ and $N=\{x,y,z\}$. $n(M \times N)=?$

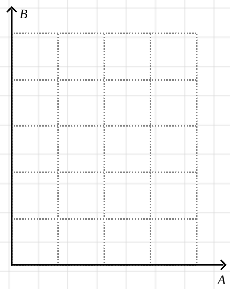
Example: Given that $K=\{\text{black, white, red}\}$ and $L=\{1,2,3,4\}$. Represent the $K \times L$.

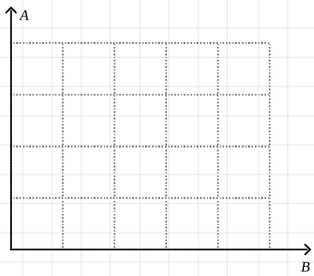
List: $K \times L=$

Coordinate Method:



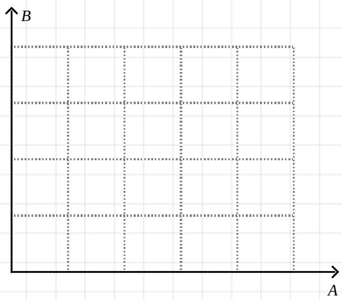
Example: Represent $A \times B$ and $B \times A$ if $A=\{1,2,3\}$ and $B=\{x \in \mathbb{R} \text{ such that } 3 \leq x \leq 5\}$.





$n(A \times B)=\ldots$ $\qquad$ $n(B \times A)=\ldots$

Example: Represent $A \times B$ if $A=(5,7)$ and $B=[2,3]$.



2. RELATIONS

Relation

Let A and B be two non-empty sets.
Each non-empty subset of $A \times B$ is called **relation from A to B**.

Example: Let $A = \{1,2,3,4,5\}$ and $B = \{a,b,c\}$.
Check whether the following sets are relation from A to B.

- $R_1 = \{(1,a),(3,c),(2,a)\}$
- $R_2 = \{(1,b),(2,c),(4,a),(6,b)\}$
- $R_3 = \{(5,b),(3,a),(1,b),(b,2),(5,c)\}$

Domain & Range

Let R be any relation from A to B .
A is called **domain.**
B is called **codomain.**
The set of all second components of R is called **range.**

Example: Let $A = \{1,2,3,4,5\}$ and $B = \{a,b,c,d,e,f\}$.
Find the domain and range of following relations from A to B.

- $R_1 = \{(1,a),(3,c),(2,a)\} \Rightarrow \begin{cases} \text{domain} = \\ \text{codomain} = \\ \text{range} = \end{cases}$
- $R_2 = \{(1,a),(3,c),(2,e),(5,f)\} \Rightarrow \begin{cases} \text{domain} = \\ \text{codomain} = \\ \text{range} = \end{cases}$
- $R_3 = \{(2,d),(3,b),(1,e),(2,b),(1,f)\} \Rightarrow \begin{cases} \text{domain} = \\ \text{codomain} = \\ \text{range} = \end{cases}$

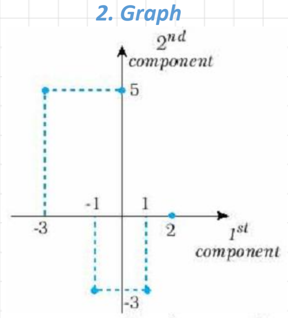
How to Represent a Relation

Let's represent the relation of
$R = \{(0,5),(1,-3),(-1,-3),(2,0),(-3,5)\}$ where
domain $= \{-3,-1,0,1,2\}$ and range $= \{-3,0,5\}$.

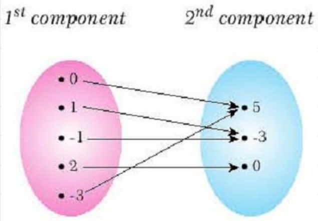
1. Table

1^{st} component	2^{nd} component
0	5
1	−3
−1	−3
2	0
−3	5

2. Graph



3. Map



Properties of Relations

Let R be a relation from A to A .

Reflexive Relations

R is reflexive if $(a,a) \in R$ for each $a \in A$.

Example: Check whether following relations are reflexive.

- $A = \{1,2,3\}$ $R = \{(1,3),(2,2),(3,1),(3,3),(1,2),(1,1)\}$
- $A = \{a,b,c\}$ $R = \{(a,a),(a,b),(a,c),(b,b),(b,c)\}$

Symmetric Relations

R is symmetric if $(b,a) \in R$ whenever $(a,b) \in R$.

Example: Check whether following relations are symmetric.

- $A = \{1,2,3\}$ $R = \{(1,1),(1,2),(1,3),(2,2),(2,1),(3,1),(3,3)\}$
- $A = \{a,b,c\}$ $R = \{(a,a),(b,b),(c,c),(b,a)\}$

Transitive Relations

R is transitive if $(a,c) \in R$ whenever $(a,b) \& (b,c) \in R$.

Example: Check whether following relations are transitive.

- $A = \{1,2,3\}$ $R = \{(1,1),(1,2),(2,2),(2,1),(3,3)\}$
- $A = \{1,2,3\}$ $R = \{(1,1),(1,2),(2,2),(2,3),(3,3),(3,2)\}$

Anti-symmetric Relations

R is anti-symmetric if $a = b$ whenever $(a,b) \& (b,a) \in R$.

Example: Check whether following relations are anti-symmetric.

- $A = \{2,4,5\}$ $R = \{(2,2),(4,4),(5,5),(4,2)\}$
- $A = \{1,2,3\}$ $R = \{(1,1),(1,2),(2,2),(2,1),(3,3)\}$

Equivalent Relations

R is equivalent if and only if R is reflexive, symmetric and transitive.

Example: Check whether following relation is equivalent.

- $A = \{1,2,3\}$ $R = \{(1,1),(1,2),(2,2),(2,1),(3,3)\}$

3. FUNCTIONS

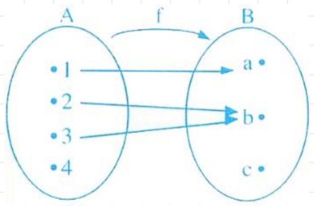
Function

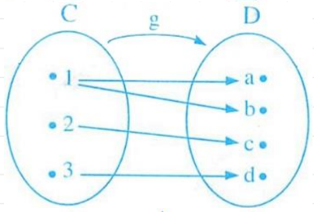
Let A and B be two non-empty sets.
The mapping of each elements of A to exactly one element of B is called a **function from A to B**.

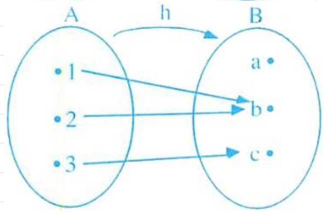
Note: Any function is a relation in which all elements of domain set are paired with only one element in range set.

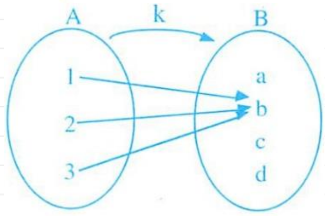
That is, function is a relation whose first components of ordered pairs occur once!

Example: Check the following relations, determine whether they are functions and state your reason.

- 

- 

- 

- 

Example: Given the sets $A=\{1,2,3\}$ and $B=\{a,b,c,d\}$. Check the following relations, determine whether they are functions and state your reason.

- $f_1=\{(1,a),(2,b)\}$
- $f_2=\{(1,b),(2,a),(2,c),(3,d)\}$
- $f_3=\{(1,a),(2,b),(3,d)\}$
- $f_4=\{(1,b),(2,c),(3,c)\}$

Example: Examine whether each of the following relations is a function.

- $R_1=\{(r,A): r \text{ is the radius and } A \text{ is the area of the circle}\}$
- $R_2=\{(a,b): a \text{ is the set of people and b is the places they visit}\}$

Notation:
If there is a certain pattern between the components of ordered pairs, it can be represented by formula.
Observe the following relation from natural numbers to real numbers:
$R=\{(0,0),(1,1),(2,8),(3,27),(4,64),\dots\}$
As it is seen easily, there is a relationship between first and second components of each ordered pairs.
That is, the second component is equal to cube of the first component.
If the first one is represented by x, then second one becomes x^3
That is;
$R=\{(x,y): y=x^3, x\in\mathbb{N}\}$ or
$y=x^3$ where $x\in\mathbb{N}$ or
$f:\mathbb{N}\to\mathbb{R}$ such that $f(x)=x^3$

Example: Examine whether each of the following relations is a function.

- $R_3=\{(x,y): -2x+y=1\}$
- $R_4=\{(x,y): y=x^2\}$

Example: Given $g:\mathbb{Z}\to\mathbb{Z}$ such that $g(x)=x+5$. Find the value of

- $g(0)=$
- $g(-2)=$
- $g(0.5)=$

Example: Given $g:\mathbb{R}\to\mathbb{R}$ such that $g(x)=x+5$. Find the value of

- $g(0)=$
- $g(-2)=$
- $g(0.5)=$

Example: Given $f(x)=\begin{cases}3-x & if \quad x>1\\ x^2+1 & if \quad -1\le x\le 1\\ 2x & if \quad x<1\end{cases}$

$f(0)+f(-4)+f(10)=$

Example: If $h(2x-1)=x+6$, then $h(3)=$

Example: If $k(x)=x^2-3x$, then $k(m-2)=$

Example: If $f(\frac{1}{x})=x-1$, then

$f(2)=$

$f(x)=$

Example: If $f(x+1)=2f(x)$ and $f(1)=4$, then

$f(4)=$

$f(2013)=$

How to find the largest possible domain for functions:

Example: Find the domain of $f(x)=\sqrt{3x+6}$

Example: Find the domain of $g(x)=\sqrt[3]{x+9}$

Example: Find the domain of $h(x)=\dfrac{2x+5}{x+3}$

How to find the functions from range to domain:

Example: If a function f maps x to y where $y=\dfrac{x+2}{2x-6}$, then find the function g which maps y to x.

Example: If a function f maps x to y where $y=\dfrac{3x-2}{x+5}$, then find the function g which maps y to x

Review Test

1. Given that $(4, x-y)=(2x, 3x+1)$, find y.

A) -5 B) -9 C) -4 D) 5 E) 9

2. $A=\{1,2,3\}$ and $B=\{m,n\}$ are given. Which one of the following is $A\times B$?

A) $\{(1,m),(2,m),(3,m)\}$

B) $\{(1,m),(2,m),(3,m),(1,n),(2,n),(3,n)\}$

C) $\{(1,n),(2,n),(3,n)\}$

D) $\{(m,1),(m,2),(m,3),(n,1),(n,2),(n,3)\}$

E) $\{(m,1),(m,2),(m,3)\}$

3. $A=\{1,2,3\}$ and $B=\{a,b,c\}$ are given. Which one of the following is not a subset of $A\times B$?

A) $\{(1,a),(2,a),(3,a)\}$

B) $\{(1,a),(2,b),(3,a),(1,b),(2,a),(3,b)\}$

C) $\{(1,c),(2,c),(3,c)\}$

D) $\{(1,a),(1,b),(1,c),(2,d),(2,a),(2,b),(2,c),(3,a)\}$

E) $\{(1,a)\}$

4. Given that $n(A)=3$, and $n(B)=4$, find $n(A\times B)$.

A) 81 B) 3 C) 4 D) 7 E) 12

5. $A=\{1,2,3\}$ and $B=\{a,b,c\}$ are given. Which one of the following is **not** a relation from A to B ?

A) $\beta_1=\{(1,a),(2,a),(3,a)\}$

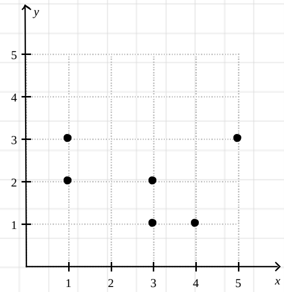
B) $\beta_2=\{(1,a),(1,a),(1,c)\}$

C) $\beta_3=\{(1,c),(2,c),(3,c)\}$

D) $\beta_4=\{(1,a),(2,a),(3,a),(4,a)\}$

E) $\beta_5=\{\ \}$

6. What is the range of the following relation?



A) $\{1,2,3,4,5\}$

B) $\{1,2,3,4\}$

C) $\{1,2,3\}$

D) $\{4,5\}$

E) $\{3,5\}$

7. Which of the following is a function?

A) $f:\mathbb{Z}\to\mathbb{Z},\ f(x)=\dfrac{3x-5}{4}$

B) $f:\mathbb{N}\to\mathbb{N},\ f(x)=3x-1$

C) $f:\mathbb{R}\to\mathbb{R},\ f(x)=\dfrac{x^2-5}{2^x-1}$

D) $f:\mathbb{N}\to\mathbb{N},\ f(x)=2^x-3$

E) $f:\mathbb{Z}\to\mathbb{Z},\ f(x)=3^x+1$

8. $A=\{1,2,3\}$, $f:A\to B$, and $f(x)=x+2$ are given. Which one of the following is $f(A)$ (range of A under f)?

A) $\{-1,0,1\}$ B) $\{1,2,3\}$ C) $\{3,4,5\}$ D) $\{2,4,6\}$ E) $\{1,4,9\}$

9. Which one of the following is domain of $f(x)=x^2+3x-4$?

A) $\{-4,1\}$ B) $R-\{-4,1\}$ C) $Z-\{-4,1\}$ D) $\mathbb{R}$ E) $\mathbb{Z}$

10. Which one of the following is domain of $f(x)=\dfrac{3}{x-2}$?

A) $\{2\}$ B) $R-\{2\}$ C) $\mathbb{Z}-\{2\}$ D) $\mathbb{R}$ E) $\mathbb{Z}$

11. Which one of the following is domain of $f(x)=\sqrt{x+3}$?

A) $[-3,\infty)$ B) $(-3,\infty)$ C) $(-\infty,-3)$ D) $\mathbb{R}$ E) $(-\infty,-3]$

12. If $f(x)=3x-1$, then find $f(4)$.

A) 13 B) 12 C) 11 D) 10 E) 9

13. $f:\mathbb{R}\to\mathbb{R}$, $f(x)+f(x+1)=2x-1$ and $f(5)=4$ are given. Find $f(3)$.

A) 6 B) 5 C) 4 D) 3 E) 2

14. $f(2x-3)=3x^2+2x+4$ is given. Find $f(1)$.

A) 9 B)12 C)15 D)18 E)20

15. $f(2x-3)=5x+1$ and $f(a)=1$ are given. Find a.

A) -3 B) -1 C) 3 D) 5 E) 7

Active Note Book

16. $f\left(\frac{x}{2}\right)=3x-2.f\left(\frac{2}{x}\right)$ is given. Find $f(2)$.

A) $-\frac{5}{2}$ B) -2 C)-3 D)3 E)6

17. If $f\left(\frac{x+3}{x+1}\right)=\left(\frac{x+1}{x+3}\right)^2$, then find $f(x)$.

A) $2x^2$ B) $\frac{1}{x^2}$ C) $\frac{1}{x}$ D) x E) x^3

18. If $f(x)=5x+1$ and $\frac{f(x+1)}{f(x-1)}=2$, then find x.

A) -1 B) 0 C) $\frac{2}{5}$ D) $\frac{14}{5}$ E) 3

19. If a function f maps x to y where $y=2x-1$, then find the function g which maps y to x.

A) $\frac{y+1}{2}$ B) $2y-1$ C) $\frac{-y-1}{2}$ D) $\frac{1}{y+1}$ E) $\frac{2}{y+1}$

20. If a function f maps x to y where $y=\frac{x-1}{x+2}$, then find the function g which maps y to x.

A) $\frac{y-1}{y+2}$ B) $\frac{y+1}{y-2}$ C) $\frac{-2y+1}{-y-1}$ D) $\frac{-2y+1}{y+1}$ E) $\frac{-2y-1}{y-1}$