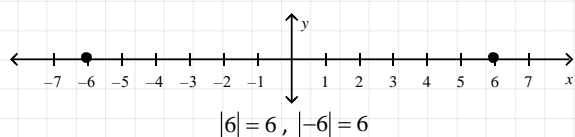


1. ABSOLUTE VALUE

Definition: The absolute value of a number x is the distance from the number to the origin, and denoted by $|x|$.



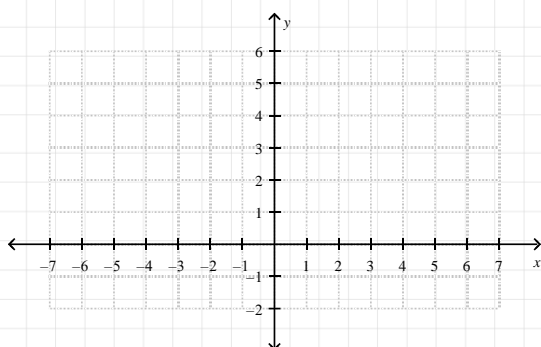
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Example: $|0| =$ $|-21| =$ $|15| =$

Let's draw the graph of $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

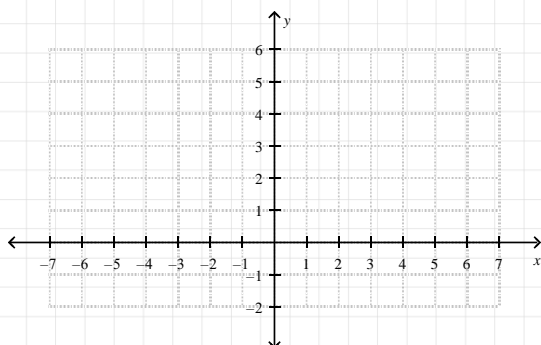
Fill the following table and show the points on the coordinate plane.

x	-4	-2	-1	0	1	2	4
$f(x) = x $							



Example: Draw the graph of $f(x) = |x - 2|$ by filling the following table.

x	-3	-2	-1	0	1	2	3
$f(x)$							



What can be inferred from last two graphs? Can you guess the graph of $f(x) = |x - p|$ without calculating coordinate values?

Observation:

Properties of Absolute Value

- $|x| \geq 0$
- $|x| = a \Rightarrow x = \pm a$
- $|x \cdot y| = |x| \cdot |y|$
- $\left| \frac{x}{y} \right| = \frac{|x|}{|y|}$
- $|x^n| = |x|^n$
- $|x| = |y| \Rightarrow x = \pm y$
- $|x + y| \leq |x| + |y|$

Example: Solve the following equations.

- $|x - 5| = 12$
- $|x - 1| \cdot |x + 3| = 5$
- $|2x + 7| = |x - 4|$

Let's solve the following equation.

$$|2 - x| = 2x + 6$$

Find the roots of each terms in absolute value.

$$2 - x = 0 \Rightarrow x = 2$$

Form the following table and find the result by using definition of absolute value.

$x < 2 \Rightarrow 2 - x > 0$	$x > 2 \Rightarrow 2 - x < 0$
$ 2 - x = 2x + 6$	$ 2 - x = 2x + 6$
\Downarrow	\Downarrow
$2 - x = 2x + 6$	$-(2 - x) = 2x + 6$
\Downarrow	\Downarrow
$x = -\frac{4}{3}$ is valid since $-\frac{4}{3} < 2$	$x = -8$ is not valid since $-8 < 2$

Example: Solve the following equations.

- $|x - 2| = 3x + 2$

- $\sqrt{(3-x)^2} + 2x = 10$

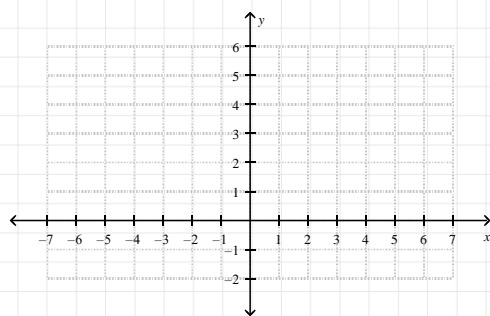
- $|4x - 2| + 3|1 - 2x| = 2x$

- $|2x - 4| - |x - 2| = x + 1$

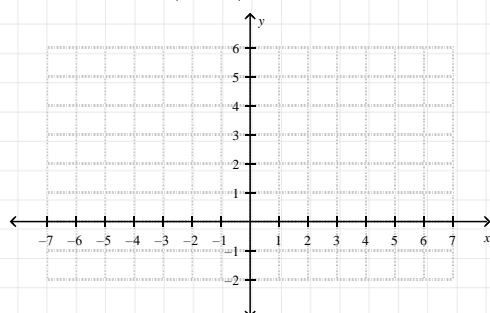
Assignment:

1. Draw the graphs of the functions below.

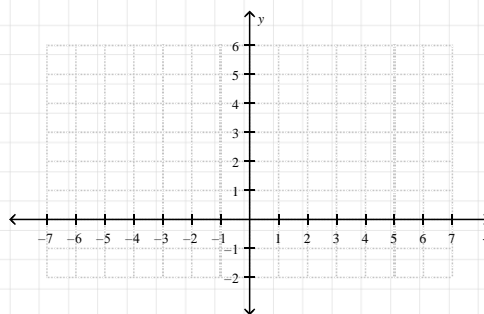
- $f(x) = |x - 4|$ for $-7 \leq x \leq 7$



- $g(x) = |3x - 4|$ for $-7 \leq x \leq 7$



- $h(x) = |2x - 1|$ for $-7 \leq x \leq 7$



2. $|ax + b| = \begin{cases} \dots & \text{if } \dots \geq \dots \\ \dots & \text{if } \dots < \dots \end{cases}$

3. Solve the following equations.

- $|3x + 5| = 7$

- $|x - 1| + |x - 2| = 1$

- $|x| + |x + 2| = 5$

- $|2x - 4| - |x - 2| = x + 1$

2. LINEAR EQUATIONS

Example: Formulate the following problem mathematically and solve.

Andi had a certain amount of money. First day he spent $\frac{1}{3}$ of the total amount. Second day he spent Rp 10.000 more than the amount of previous day. Third day he spent $\frac{1}{2}$ of the amount spent in second day. He still had Rp 20.000. What is the amount of money at the beginning?

Example: Formulate the following problem mathematically and solve.

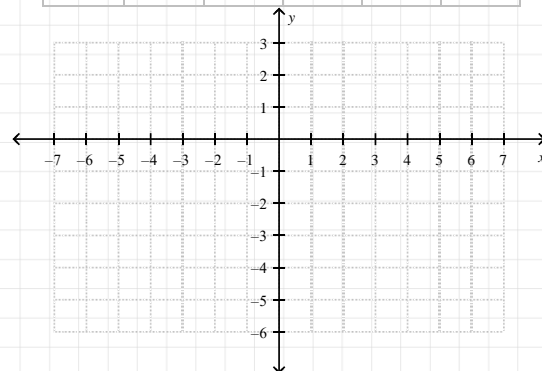
A father's age is 10 more than the sum of the ages of his two sons. Eight years ago his age was three times the sum of the ages of his sons. How old is the father now?

Definition: The equation of the form $ax + b = 0$ where $a, b \in \mathbb{R}$ and $a \neq 0$ is called a **linear equation in one variable**.

Definition: The equation of the form $ax + by + c = 0$ where $a, b, c \in \mathbb{R}$ and $a \neq 0, b \neq 0$ is called a **linear equation in two variables**.

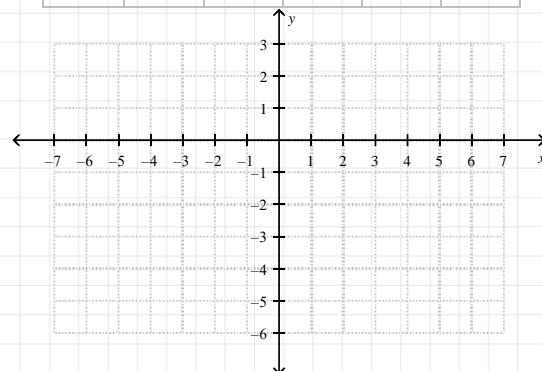
Example: Show the solutions of $x - y - 2 = 0$ by graph.

x	-4	-2	0	2	4
y					

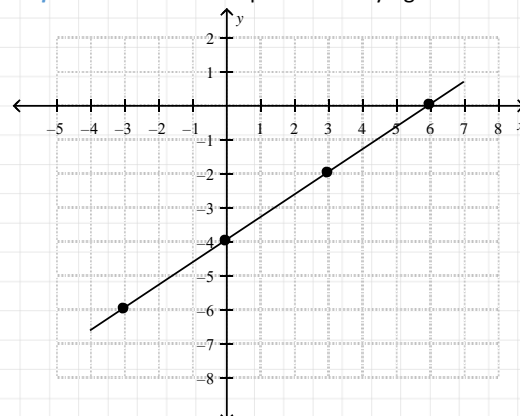


Example: Show the solutions of $5x + 4y + 20 = 0$ by graph.

x					
y					



Example: Find the linear equation satisfying the following graph.



3. LINEAR INEQUALITIES

Example: Formulate the following problem mathematically and solve.

Budi's father is younger than his uncle but older than his mother. His aunt is just one year older than his mother but one year younger than his father. Budi plans to sort the age of father, mother, uncle, and aunt.

Example: Formulate the following problem mathematically and solve.

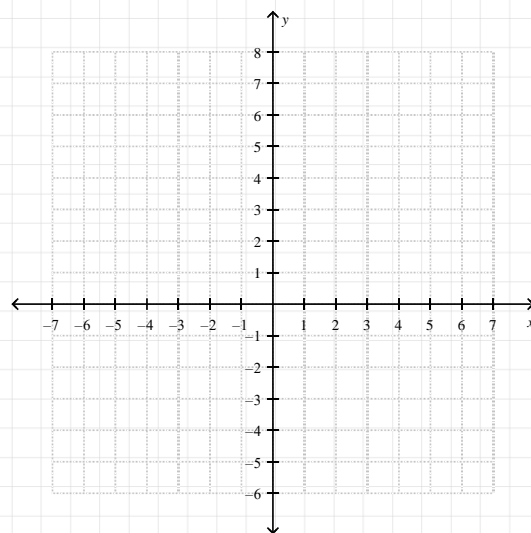
Taxi charges a \$1.75 flat rate in addition to \$0.65 per kilometer. Katie has no more than \$10 to spend on a ride. How many kilometers can Katie travel without exceeding her limit?

Definition: The inequality of the forms: $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$ or $ax + by + c \leq 0$ where $a, b, c \in \mathbb{R}$ and $a \neq 0, b \neq 0$ is called a **linear inequality in two variables**.

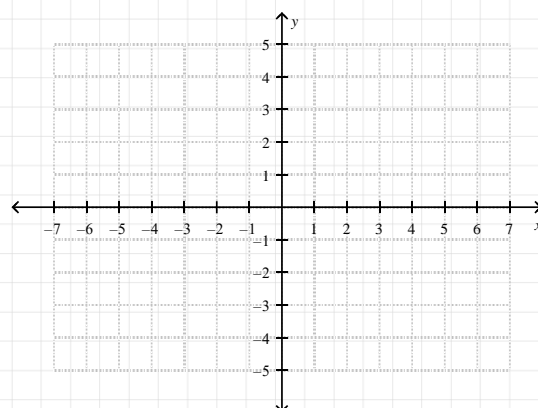
How to represent the solution set of linear inequalities?

- Draw the line of $ax + by + c = 0$.
Dashed line for $ax + by + c < 0$ and $ax + by + c > 0$.
Solid line for $ax + by + c \geq 0$ and $ax + by + c \leq 0$
- Choose the correct region of the coordinate plane separated by line by testing a point.

Example: Show the solution set of $3x + 2y - 12 > 0$.



Example: Show the solution set of $4x - y - 4 \geq 0$.



Properties of Absolute Value in Inequalities

1. $|x| < a \Rightarrow -a < x < a$

2. $|x| > a \Rightarrow \begin{cases} x > a & \text{if } x \geq 0 \\ -x > a & \text{if } x < 0 \end{cases}$

Example: Solve the following inequalities.

• $|x - 3| < 5$

• $|x + 2| > 4$

Let's solve the following inequality.

$$|2x - 6| - |x - 4| < 6$$

Find the roots of each terms in absolute value.

$$2x - 6 = 0 \Rightarrow x = 3 \quad \text{and} \quad x - 4 = 0 \Rightarrow x = 4$$

Form the following table and find the result by using definition of absolute value.

$x = 3$		$x = 4$	
$x < 3 \Rightarrow \begin{cases} 2x - 6 < 0 \\ x - 4 < 0 \end{cases}$		$3 < x < 4 \Rightarrow \begin{cases} 2x - 6 > 0 \\ x - 4 < 0 \end{cases}$	
$ 2x - 6 - x - 4 < 6$		$ 2x - 6 - x - 4 < 6$	
\Downarrow		\Downarrow	
$-(2x - 6) - (-(x - 4)) < 6$		$(2x - 6) - (-(x - 4)) < 6$	
\Downarrow		\Downarrow	
$-4 < x$		$x < 8$	
\Downarrow		\Downarrow	
$-4 < x < 3$		$4 < x < 8$	
		\Downarrow	
		$3 < x < 4$	

$x = 3$ and $x = 4$ satisfy the inequality.

$x = -4$ and $x = 8$ do not satisfy the inequality.

So, the solution set is $S = (-4, 8)$.

Example: Solve the following inequalities.

• $|x| + 3 > |x + 3|$

• $|3x - 1| < |x + 2|$

• $|x - 2| + |x - 1| > 3$

Assignment:

1. Brad graduated from college and started a new job. The table below shows his anticipated yearly salary for each of the next four years.

Year	Salary
1	\$36,500
2	\$38,600
3	\$40,700
4	\$42,800

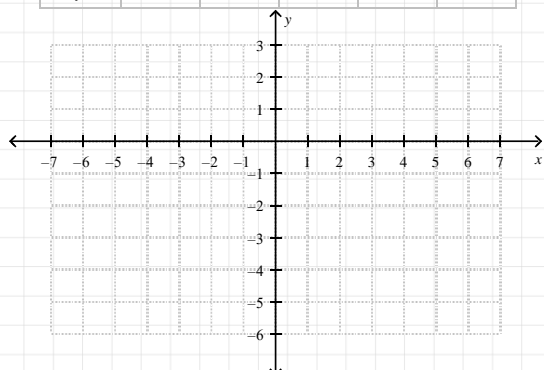
If his salary continues to increase in the same manner, what will be his salary for the 7th year at this job?

2. Tina will set up chairs for a concert. She will set up 20% more chairs than the number of tickets sold. There were 130 tickets sold. How many chairs will Tina set up?

3. Represent the solution set of the following equations.

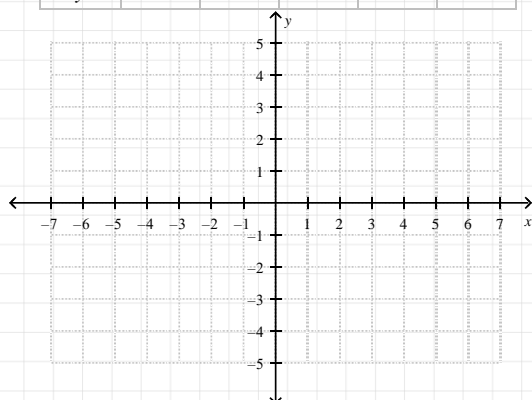
- $5x - 3y + 15 = 0$

x					
y					

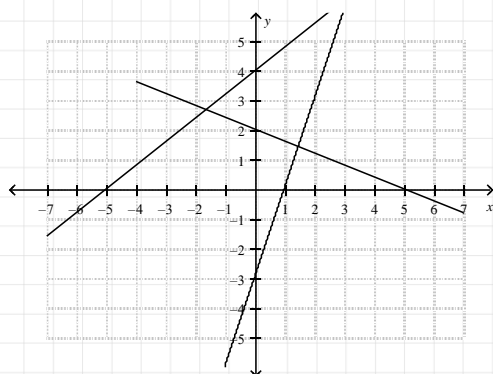


- $4x + 3y - 6 = 0$

x					
y					

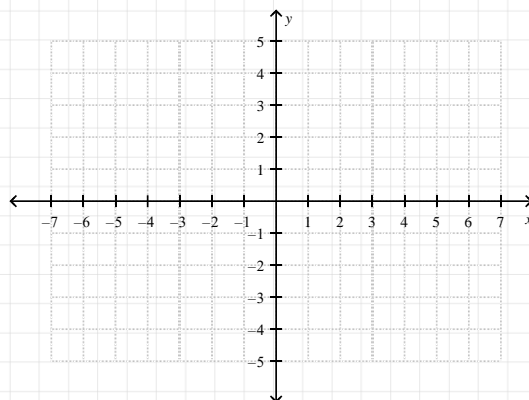


4. Find the linear equations satisfying the following graphs.

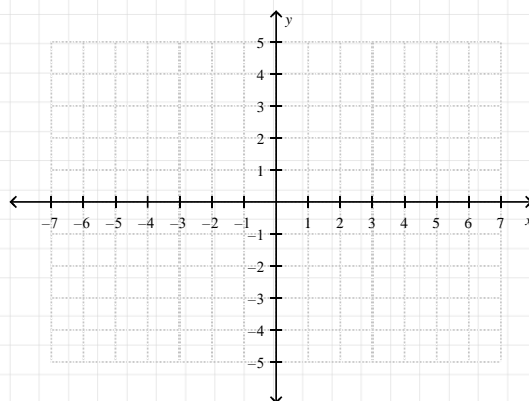


5. Show the solution set of following inequalities.

- $x - 2y < -4$



- $x \leq 2y + 5$



6. Solve the following inequalities.

- $|2x + 7| \leq 5$

- $|x - 10| > 4$

- $|x + 3| > |x - 8|$

- $|x + 4| - |2x + 6| > 5$

Review Test

1. $\frac{|-5|-|-6|}{|-11|-|-12|}=?$

- A) $\frac{2}{3}$ B) -2 C) 1 D) -1 E) -2

2. $\frac{|-2002|-|-2003|}{|-2001|-|-2000|}=?$

- A) 1 B) -1 C) 2 D) $\frac{1}{2}$ E) 12

3. $\frac{|-5|^2-|-5^2|+|(-5)^2|}{|-5^2|+|5|^2-|(-5)^2|}=?$

- A) 1 B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) -1 E) 3

4. Evaluate the following $|1-\sqrt{2}|+|1+\sqrt{2}|=?$.

- A) -1 B) 2 C) $-2\sqrt{2}$ D) 0 E) $2\sqrt{2}$

5. $|2x+1|=7 \Rightarrow x=?$

- A) {-4,4} B) {-4,7} C) {7,3} D) {-4,3} E) R

6. How many solution does the following expression have?
 $|x^2-3|+9=0$

- A) 0 B) 1 C) 2 D) 3 E) 4

7. Solve $|x-2|+|x+3|=7$ for $x < -3$.

- A) 4 B) -4 C) 5 D) -5 E) 7

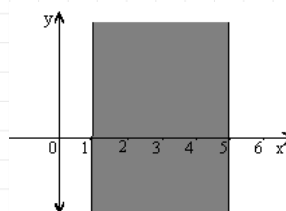
8. $\frac{|x|-|-x|}{|x|+|-x|}=?$ for $x \neq 0$.

- A) 0 B) 1 C) $2x+1$ D) x E) undefined

9. If $y = x+2$, then evaluate $|x-y|+|y-x|$.

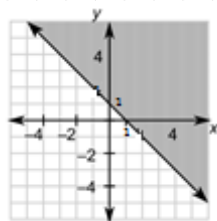
- A) -4 B) -2 C) 0 D) 2 E) 4

10. Which one of the following inequalities has the solution as the shaded part in the graph below?



- A) $|x-1| \leq 5$
B) $|x-3| \leq 2$
C) $1 < x < 5$
D) $|x-2| \leq 3$
E) $|x-4| \leq 1$

11. Which statement describes the graph below?



- A) $y > -x + 1$ B) $y \geq -x + 1$ C) $y < -x + 1$
D) $y = -x + 1$ E) $y \leq -x + 1$

12. Find the solution set of $|x - 3| < 4$.

- A) $(-1, 7)$ B) $(-7, 1)$ C) $(0, 4)$ D) $(4, -3)$ E) $(-3, 7)$

13. Which one is the solution of $|x - 2| > 2 - x$?

- A) $(-\infty, 2]$ B) $(2, \infty)$ C) $(-\infty, 2)$ D) $[5, \infty)$ E) $[-2, 2]$

14. What is the solution set of $|2x - 7| + |2x + 9| = 2x + 18$?

- A) $\{-1, 8\}$ B) $\{-1, 1\}$ C) $\{-2, -1\}$ D) $\{0, -1\}$ E) $\{1\}$

15. Kylan graduated from college and started a new job. The table below shows his anticipated yearly salary for each of the next four years.

Year	Salary
1	\$40,300
2	\$42,500
3	\$44,700
4	\$46,900

If his salary continues to increase in the same manner, what will be his salary for the 8th year at this job?

- A) \$49,700 B) \$51,200 C) \$53,100
D) \$55,700 E) \$58,500

16. How many integers satisfy $|x - 2| - |x + 3| \geq 1$ for $-3 \leq x < 2$?

- A) 5 B) 4 C) 3 D) 2 E) 1

17. The profit (P) an ice cream store makes in one day when producing x gallons of ice cream is given by the equation $P = 30x - 240$. For what value of x is the store's profit equal to \$90?

- A) 10 B) 11 C) 15 D) 18 E) 20

18. Which equation below could represent the data in the table?

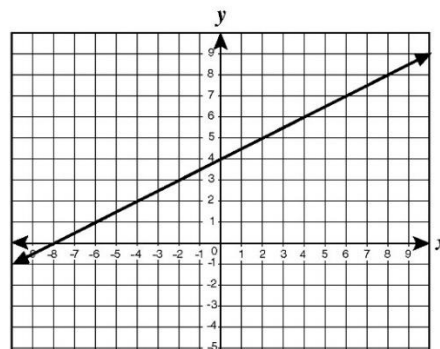
Table of Values	
x	y
-2	-1
-1	1
0	3
1	5
2	7

- A) $y = x + 2$ B) $y = x + 4$ C) $y = 2x + 3$
D) $y = 3x + 1$ E) $y = -3x + 2$

19. The bill (parts and labor) for the repair of a car was \$458. The cost of the part was \$339. The cost of the labor was \$34 per hour. What equation can be used to find x , the number of hours of the labor?

- A) $339 - 34x = 458$ B) $x + 339 = 458$ C) $34 + 339x = 458$
D) $34x + 339 = 458$ E) $34x - 339 = 458$

20. Which equation represents the line shown on the graph?



- A) $y = -\frac{1}{2}x - 8$ B) $y = \frac{1}{2}x - 8$ C) $y = \frac{1}{2}x + 4$
D) $y = \frac{1}{2}x - 4$ E) $y = -\frac{1}{2}x - 4$