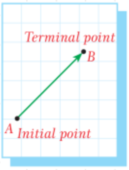
CHAPTER 1: VECTORS IN THE PLANE

1. ANALYSIS OF VECTORS GEOMETRICALLY

A. Basic Vector Concepts

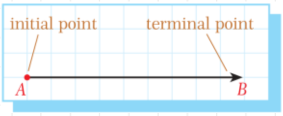
Directed line segment is a line segment with initial and terminal (end) point.



Directed line segment AB is denoted by $\overrightarrow{AB}$

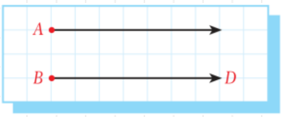
Example: Write all directed line segments whose end points are M, N, P, K

Vector in the plane is a directed line segment.



Equal Vectors:

Two vectors that have the same direction and length are called equal vectors.



$\overrightarrow{AC} = \overrightarrow{BD}$

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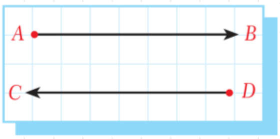
Opposite Vectors:

Two vectors are opposite iff their magnitudes (lengths) are same but directions are different.



$\overrightarrow{AB} = -\overrightarrow{CD}$

Zero Vector:

A vector whose initial and terminal points are same is called zero vector, and denoted by $\vec{0}$.

B. Vector Operations

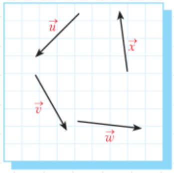
Addition of Vectors

Let $\overrightarrow{AC}$ and $\overrightarrow{BD}$ be two vectors.

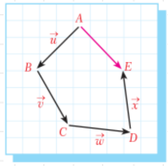
$\overrightarrow{AC} + \overrightarrow{BD} \Rightarrow$ sum of the vectors $\overrightarrow{AC}$ and $\overrightarrow{BD}$

Method 1: The Polygon Method

Place each vectors' initial point to the end of the previous vector

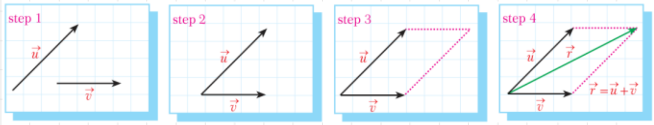


$\vec{u} + \vec{v} + \vec{w} + \vec{x} \Rightarrow$



Method 2: The Parallelogram Method

Place initial points of two vectors shown as following figure

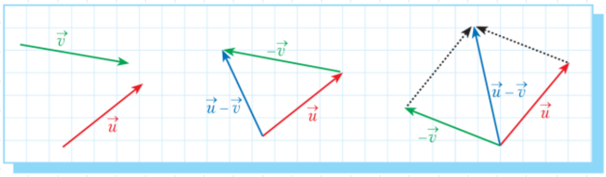


Properties of Vector Addition

1. Sum of any two vectors is a vector
2. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
3. $\vec{u} + \left(\vec{v} + \vec{w}\right) = \left(\vec{u} + \vec{v}\right) + \vec{w}$
4. $\vec{u} + \left(-\vec{u}\right) = \vec{0}$ ($-\vec{u}$ is an additive inverse of $\vec{u}$)

Subtraction of Vectors

$\vec{u} - \vec{v}$ means $\vec{u} + \left(-\vec{v}\right)$



Example: Draw the resultant vector by using the followings

$\vec{u} + \vec{v}$

$\vec{u} - \vec{v}$

$\vec{v} + \vec{w}$

$\vec{w} + \vec{v} + \vec{u}$

Check Yourself 2 – Page 8 in Zambak

Multiplication of a Vector by a Scalar

Scalar multiplication changes the length and direction of vector.

Properties of the Multiplication of a Vector by a Scalar

1. $a \cdot \vec{u}$ is a vector
2. $(a \cdot b) \cdot \vec{u} = a \cdot (b \cdot \vec{u})$
3. $(a+b) \cdot \vec{u} = a \cdot \vec{u} + b \cdot \vec{u}$
4. $a \cdot (\vec{u}+\vec{v}) = a \cdot \vec{u} + a \cdot \vec{v}$
5. $1 \cdot \vec{u} = \vec{u}$
6. $a \cdot \vec{0} = \vec{0}$

Example: Draw the resultant vector by using the followings

$\frac{1}{2} \cdot \vec{v}$ $\qquad$ $2 \cdot \vec{w}$

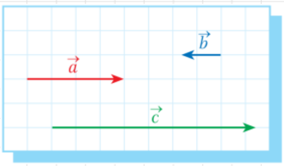
$-\frac{1}{2} \cdot \vec{u}$ $\qquad$ $-2 \cdot \vec{v}$

Check Yourself 3 – Page 10 in Zambak

C. Parallel Vectors

Parallel Vectors

$\vec{a}$ and $\vec{b}$ are parallel vectors iff $\vec{a} = k \cdot \vec{b}$ ($k \neq 0,\ k \in \mathbb{R}$)



$\vec{a} \parallel \vec{b} \parallel \vec{c}$

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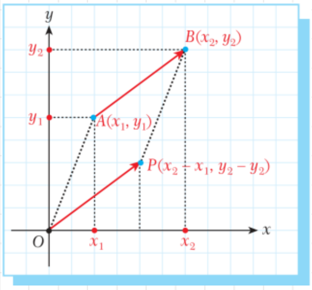
2. ANALYSIS OF VECTORS ANALYTICALLY

A. Basic Concepts of Vectors in the Analytic Plane

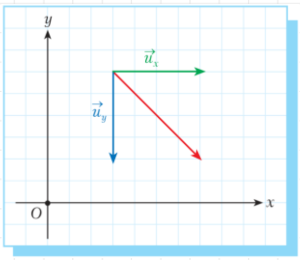
Position vector:

A vector $\overrightarrow{OP}$ whose initial point is at the origin of the rectangular coordinate plane and which is parallel to a vector $\overrightarrow{AB}$ is called the position vector of $\overrightarrow{AB}$.

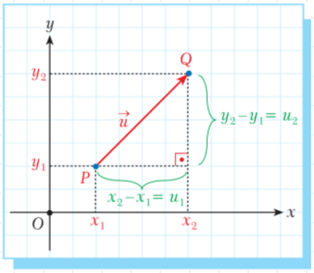


Example: Find the position vector of $\overrightarrow{MN}$ with end points $M(3,1)$ and $N(6,3)$

Components of a Vector



Any vector can be represented by the sum of a horizontal ($\vec{u_x}$) and vertical ($\vec{u_y}$) vectors.



$\vec{u}$ can be represented as an ordered pair of real numbers.

$\vec{u} = (u_1, u_2)$ or $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

From above construction, the length of vector $\vec{u} = (u_1, u_2)$ is

$$|\vec{u}| = \sqrt{u_1^{\,2} + u_2^{\,2}}$$

Example: Find the length of each vector.

- $\vec{u} = (3,-2)$
- $\vec{v} = (-4,0)$
- $\vec{w} = \left(\frac{5}{13}, -\frac{12}{13}\right)$
- $\vec{u}$ with initial point $(1,4)$ and terminal point $(-2,2)$

Check Yourself 6 – Page 18 in Zambak

Equal Vectors

$\vec{u}=(u_1,u_2)$ and $\vec{v}=(v_1,v_2)$ are equal iff $u_1=v_1$ and $u_2=v_2$

Example: If $\vec{u}=(m-n,5)$ and $\vec{v}=(5,2m-1)$ are equal, then find m and n.

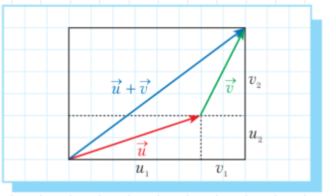
Example: The vector $\vec{w}=(1,-5)$ has end point $(-4,5)$. What is the initial point?

Check Yourself 7 – Page 19 in Zambak

B. Vector Operations

Addition of Vectors

If $\vec{u}=(u_1,u_2)$ and $\vec{v}=(v_1,v_2)$, then $\vec{u}+\vec{v}=(u_1+v_1,u_2+v_2)$



Properties of Vector Addition

1. Sum of any two vectors is a vector
2. $\vec{u}+\vec{v}=\vec{v}+\vec{u}$
3. $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$
4. $\vec{u}+(-\vec{u})=\vec{0}$ ($-\vec{u}$ is an additive inverse of $\vec{u}$)

Example: $\vec{u}=(-2,5)$ and $\vec{v}=(4,-6)$. Find $\vec{u}+\vec{v}$

Subtraction of Vectors

If $\vec{u}=(u_1,u_2)$ and $\vec{v}=(v_1,v_2)$, then $\vec{u}-\vec{v}=(u_1-v_1,u_2-v_2)$

Properties of Vector Subtraction

1. Difference of any two vectors is a vector
2. $\vec{u}-\vec{v}\neq\vec{v}-\vec{u}$
3. $\vec{u}-(\vec{v}-\vec{w})\neq(\vec{u}-\vec{v})-\vec{w}$
4. $\vec{u}-\vec{0}\neq\vec{0}-\vec{u}$ ($-\vec{u}$ is an additive inverse of $\vec{u}$)

Example: $\vec{u}=(3,8)$ and $\vec{v}=(-3,10)$. Find $\vec{u}-\vec{v}$

Multiplication of a Vector by a Scalar

If $\vec{v}=(v_1,v_2)$ and $c\in\mathbb{R}$, then $c\cdot\vec{v}=(c\cdot v_1,c\cdot v_2)$

Properties of the Multiplication of a Vector by a Scalar

If $\vec{u}=(u_1,u_2)$, $\vec{v}=(v_1,v_2)$ and $c,d\in\mathbb{R}$, then

1. $c\cdot(\vec{u}+\vec{v})=c\cdot\vec{u}+c\cdot\vec{v}$
2. $(c+d)\cdot\vec{u}=c\cdot\vec{u}+d\cdot\vec{u}$
3. $(c\cdot d)\cdot\vec{u}=c\cdot(d\cdot\vec{u})=d\cdot(c\cdot\vec{u})$
4. $1\cdot\vec{u}=\vec{u}$
5. $0\cdot\vec{u}=\vec{0}$
6. $c\cdot\vec{0}=\vec{0}$
7. $|c\cdot\vec{u}|=|c|\cdot|\vec{u}|$

Example: If $\vec{u}=(2,-5)$, $\vec{v}=(-1,7)$ then find

$-2\cdot\vec{u}$

$4\cdot\vec{v}$

$4\cdot\vec{u}-3\cdot\vec{v}$

Check Yourself 8 – Page 22 in Zambak

Unit Vector is a vector of length 1. (Example; $\vec{u}=\left(\frac{3}{5},\frac{4}{5}\right)$)

For any non-zero vector $\vec{u}=(u_1,u_2)$, $\frac{\vec{u}}{|\vec{u}|}$ is a unit vector.

$\frac{\vec{u}}{|\vec{u}|}$ is also used for direction of $\vec{u}=(u_1,u_2)$.

Standard Base Vectors are $\vec{i}=(1,0)$ and $\vec{j}=(0,1)$

Any vector $\vec{u}=(u_1,u_2)$ can be expressed in terms of $\vec{i}$ and $\vec{j}$ as

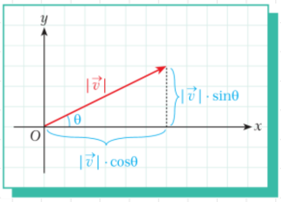
$\vec{u}=(u_1,u_2)=u_1\cdot\vec{i}+u_2\cdot\vec{j}$

Example: Write the vector $\vec{v}=(-7,4)$ in terms of $\vec{i}$ and $\vec{j}$

Let $\vec{v}$ be a vector in the plane with its initial point at the origin.

As observe in the following figure,

$$\vec{v}=(v_1,v_2)=v_1\cdot\vec{i}+v_2\cdot\vec{j}=|\vec{v}|\cdot\cos\theta\cdot\vec{i}+|\vec{v}|\cdot\sin\theta\cdot\vec{j}$$



Example:

Check Yourself 9 – Page 24 in Zambak

C. Parallel Vectors

Let $\vec{u}=(u_1,u_2)$ and $\vec{v}=(v_1,v_2)$ be vectors

$\vec{u}\parallel\vec{v}$ iff $\dfrac{u_1}{v_1}=\dfrac{u_2}{v_2}=c$

Example: Show that $\vec{u}=\left(-3,\frac{1}{2}\right)$ and $\vec{v}=\left(2,-\frac{1}{3}\right)$ are parallel.

Check Yourself 11 – Page 27 in Zambak

D. Linear Combination of Vectors

Let $\vec{u_1},\vec{u_2},\ldots,\vec{u_k}$ be vectors in the plane and let $c_1,c_2,\ldots,c_k$ be scalars. Then,
$c_1\cdot\vec{u_1}+c_2\cdot\vec{u_2},\ldots,c_k\cdot\vec{u_k}$ is called a **linear combination** of vectors.

Example: Find the vector $\vec{m}$ if $\vec{m}=3\cdot\vec{a}-2\cdot\vec{b}$ if $\vec{a}=(3,-5)$ and $\vec{b}=(-4,10)$

Example: Express $\vec{w}=(16,3)$ as a linear combination of the vectors $\vec{u}=(-2,1)$ and $\vec{v}=(3,4)$

3. THE DOT PRODUCT OF TWO VECTORS

A. Dot Product

Let $\vec{u}=(u_1,u_2)$ and $\vec{v}=(v_1,v_2)$ be two vectors.

The **dot product** of $\vec{u}$ and $\vec{v}$, denoted by $\vec{u}\cdot\vec{v}$, is defined by

$$\vec{u}\cdot\vec{v}=u_1\cdot v_1+u_2\cdot v_2$$

Example:

$\vec{u}=(1,-2)$, $\vec{v}=(3,5)\Rightarrow\vec{u}\cdot\vec{v}=$

$\vec{u}=\vec{i}-4\cdot\vec{j}$, $\vec{u}=2\cdot\vec{i}+7\cdot\vec{j}\Rightarrow\vec{u}\cdot\vec{v}=$

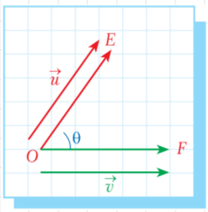
Properties of Dot Product

1. $\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$
2. $\vec{u}\cdot(\vec{v}+\vec{w})=\vec{u}\cdot\vec{v}\pm\vec{u}\cdot\vec{w}$
3. $c\cdot(\vec{u}\cdot\vec{v})=(c\cdot\vec{u})\cdot\vec{v}$
4. $\vec{u}\cdot\vec{u}=|\vec{u}|^2$
5. $\vec{u}\cdot\vec{u}=0$ iff $\vec{u}=\vec{0}$

Example: Find the length of $\vec{u}=(-5,12)$ by using dot product.

Check Yourself 13 – Page 34 in Zambak

Angle between $\vec{u}$ and $\vec{v}$ is the smaller angle represented by θ



Dot Product Theorem $\vec{u}\cdot\vec{v}=|\vec{u}|\cdot|\vec{v}|\cdot\cos\theta$

Example: Given $\vec{u}=(3,-1)$, $\vec{v}=(4,7)$, and $\vec{w}=(-2,5)$, find

- $\vec{u}\cdot\vec{v}=$
- $(\vec{u}\cdot\vec{v})\cdot\vec{w}=$
- $\vec{u}\cdot(2\cdot\vec{w})=$
- $|\vec{w}|^2$

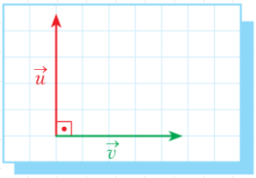
To find the angle between two non-zero vectors $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$

Example: Find the cosine of the angle between the vectors $\vec{u}=(3,4)$ and $\vec{v}=(-2,6)$

Example: Find the angle between the vectors $\vec{u}=(1,\sqrt{2})$ and $\vec{v}=(2,2\sqrt{2})$

Check Yourself 14 – Page 36 in Zambak

Perpendicular (Orthogonal) Vectors



$\vec{u}$ and $\vec{v}$ are perpendicular iff $\vec{u}\cdot\vec{v}=0$

Example: Are the vectors $\vec{u}=(4,2)$ and $\vec{v}=(-3,6)$ perpendicular?

Example: Are the vectors $\vec{u}=(7,-1)$ and $\vec{v}=(-3,3)$ perpendicular?

Example: Find the equation of line passing through $A(-2,1)$ which is perpendicular to $\vec{m}=(2,3)$.

Parallel Vectors

$\vec{u}$ and $\vec{v}$ are parallel iff $\vec{u}\cdot\vec{v}=|\vec{u}|\cdot|\vec{v}|$ or $\vec{u}\cdot\vec{v}=-|\vec{u}|\cdot|\vec{v}|$

Theorem:

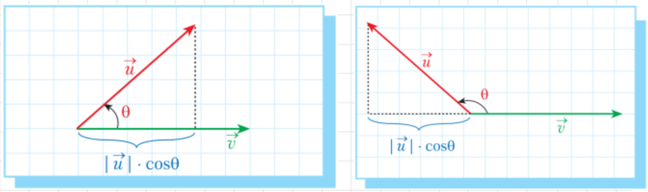
$$|\vec{u}+\vec{v}|^2=|\vec{u}|^2+2\cdot\vec{u}\cdot\vec{v}+|\vec{v}|^2=|\vec{u}|^2+2\cdot|\vec{u}|\cdot|\vec{v}|\cdot\cos\theta+|\vec{v}|^2$$

$$|\vec{u}-\vec{v}|^2=|\vec{u}|^2-2\cdot\vec{u}\cdot\vec{v}+|\vec{v}|^2=|\vec{u}|^2-2\cdot|\vec{u}|\cdot|\vec{v}|\cdot\cos\theta+|\vec{v}|^2$$

Example: $|\vec{u}|=2$, $|\vec{v}|=5$, and the angle between $\vec{u}$ and $\vec{v}$ is 120^o . Find $|3\vec{u}-4\vec{v}|$

Check Yourself 15 – Page 40 in Zambak

B. Component of $\vec{u}$ along $\vec{v}$



Length of Projection of $\vec{u}$ along $\vec{v}$ $=|\vec{u}|\cos\theta=\dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$

Projection vector of $\vec{u}$ along $\vec{v}$ $=\dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\cdot\vec{v}$

Example: Find the length of projection of $\vec{u}=(4,3)$ along $\vec{v}=(-2,6)$

Example: Find the projection vector of $\vec{u}=(4,3)$ along $\vec{v}=(-2,6)$

Check Yourself 16 – Page 42 in Zambak

Exercises 3 – Page 45 in Zambak

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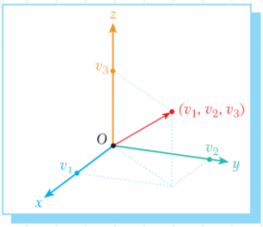
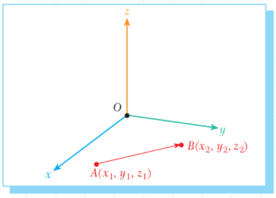
CHAPTER 2: VECTORS IN THE SPACE

1. ANALYSIS OF VECTORS GEOMETRICALLY

A. Basic Vector Concepts



$\vec{v}=(v_1,v_2,v_3)$



$\overrightarrow{AB}=(x_2-x_1,y_2-y_1,z_2-z_1)$

Example: Find the vector with initial point $A(3,-1,5)$ and terminal point $B(-1,0,7)$

Example: The points $A(-5,4,-2)$, $B(0,6,-2)$ and $C(1,7,4)$ are given. Write the vectors $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{AC}$.

Example: The point $A(4,-1,5)$ and the vector $\overrightarrow{AB}=(10,5,-6)$ are given. Find the coordinates of point B

Length (Norm) of Vector

The length (norm) of the vector $\vec{v}=(v_1,v_2,v_3)$ is $|\vec{v}|=\sqrt{v_1^2+v_2^2+v_3^2}$

Example: Find the length of each vector.

- $\vec{u}=(-3,-4,5)$
- $\vec{v}=(-4,0,3)$
- $\vec{w}$ with initial point $(1,4,-6)$ and terminal point $(-5,-3,2)$

Example: (UN 2005)
Diketahui segitiga ABC dengan koordiat $A(2,-3,4)$, $B(5,0,1)$, dan $C(4,2,5)$. Titik P membagi AB sehingga $AP:AB=2:3$. Panjang vektor PC adalah ...
ABC triangle with $A(2,-3,4)$, $B(5,0,1)$, and $C(4,2,5)$ is given. P point divides AB in the ratio of $AP:AB=2:3$. Length of PC vector is ...

Zero Vector $\vec{0}=(0,0,0)$

Equal Vectors

$\vec{u}=(u_1,u_2,u_3)$ and $\vec{v}=(v_1,v_2,v_3)$ are equal iff $u_1=v_1$, $u_2=v_2$, $u_3=v_3$

Example: If $\vec{u}=(a-1,2,b)$ and $\vec{v}=(b+4,c,3-a)$ are equal, then find a,b and c .

Example: Find the length of

- $\vec{u}=(1,2,-2)$
- $\vec{v}=(-5,\sqrt{2},-3)$
- $\vec{w}=(\sqrt{6},-\sqrt{3},4)$

Example: Given $M(2,3,-1)$ and $N(1,3,-1)$. Find the length of $\overrightarrow{MN}$

B. Vector Operations

Let $\vec{u}=(u_1,u_2,u_3)$ and $\vec{v}=(v_1,v_2,v_3)$ be vectors and $c\in\mathbb{R}$. Then

$\vec{u}+\vec{v}=(u_1+v_1,u_2+v_2,u_3+v_3)$

$\vec{u}-\vec{v}=(u_1-v_1,u_2-v_2,u_3-v_3)$

$c\cdot\vec{u}=c\cdot(u_1,u_2,u_3)=(c\cdot u_1,c\cdot u_2,c\cdot u_3)$

Example: Given $\vec{u}=(-4,0,3)$ and $\vec{v}=(2,-1,-5)$. Find

- $|\vec{u}|=$
- $\vec{u}+\vec{v}=$
- $\vec{v}-3\cdot\vec{u}=$
- $4\cdot\vec{u}+5\cdot\vec{v}=$

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Example: (UN 2004)

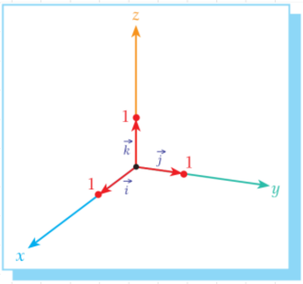
If $a = i + 2j + 3k$, $b = -3i - 2j - k$ and $b = i - 2j + 3k$, then $2a + b - c = \ldots$

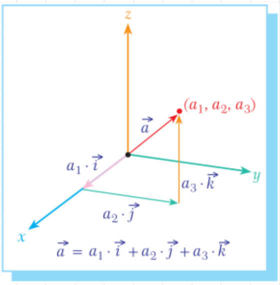
Unit Vector is a vector with length 1.

Example: Find a if $\vec{u} = \left(a, 0, \frac{\sqrt{3}}{2}\right)$ is a unit vector.

Direction of Non-zero Vector $\vec{v}$ is $\frac{\vec{v}}{|\vec{v}|}$

Standard Basis Vectors are $\vec{i} = (1,0,0)$, $\vec{j} = (0,1,0)$ and $\vec{k} = (0,0,1)$





Example: Write $\vec{v} = (-4, 7, 3)$ in terms of standard basis vectors

Example: Find the unit vector in the direction of $3\vec{i} + \vec{j} - 4\vec{k}$

C. Parallel Vectors

Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ be vectors

$\vec{u} \parallel \vec{v}$ iff $\frac{u_1}{v_1} = \frac{u_2}{v_2} = \frac{u_3}{v_3} = c$

D. Dot Product

Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$ be two vectors.

The **dot product** of $\vec{u}$ and $\vec{v}$, denoted by $\vec{u} \cdot \vec{v}$, is defined by

$$\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + u_3 \cdot v_3$$

Example:

$\vec{u} = (-3, 0, 2)$, $\vec{v} = (2, -3, 5) \Rightarrow \vec{u} \cdot \vec{v} =$

$\vec{u} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{v} = \vec{i} - 7 \cdot \vec{j} - 2\vec{k} \Rightarrow \vec{u} \cdot \vec{v} =$

Properties of Dot Product

1. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} \pm \vec{u} \cdot \vec{w}$
3. $c \cdot (\vec{u} \cdot \vec{v}) = (c \cdot \vec{u}) \cdot \vec{v}$
4. $\vec{u} \cdot \vec{u} = |\vec{u}|^2$
5. $\vec{u} \cdot \vec{u} = 0$ iff $\vec{u} = \vec{0}$

Example: Find the length of $\vec{v} = (5, -3, 1)$ by using dot product.

Angle between $\vec{u}$ and $\vec{v}$ is the smaller angle represented by θ

Dot Product Theorem $\quad \vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$

To find the angle between two non-zero vectors $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|}$

Example: Find the cosine of the angle between the vectors $\vec{u} = (3, -2, 1)$ and $\vec{v} = (-1, 2, -6)$

Example: (UN 2012/A13)

Diketahui vektor $\vec{a} = 4\vec{i} + 2\vec{j} + 2\vec{k}$ dan $\vec{b} = 3\vec{i} + 3\vec{j}$. Besar sudut antara vektor $\vec{a}$ dan $\vec{b}$ adalah...

$\vec{a} = 4\vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{b} = 3\vec{i} + 3\vec{j}$ vectors are given. The angle value between $\vec{a}$ and $\vec{b}$ vectors is ...

Example: (UN 2011 PAKET 46)

Diketahui segitiga ABC dengan $A(2,1,2)$, $B(6,1,2)$, dan $C(6,5,2)$. Jika u mewakili $\overrightarrow{AB}$ dan v mewakili $\overrightarrow{AC}$, maka sudut yang dibentuk oleh vektor u dan v adalah ...

ABC triangle with $A(2,1,2)$, $B(6,1,2)$ and $C(6,5,2)$ is given. The measure of angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is ...

Example: (UN 2009 PAKET A/B)

Diketahui balok $ABCD.EFGH$ dengan $AB = 2$ cm, $BC = 3$ cm, dan $AE = 4$ cm. Jika $\overrightarrow{AC}$ wakil vektor u dan v wakil $\overrightarrow{DH}$ adalah vektor v, maka sudut antara vektor u dan v adalah ...

$ABCD.EFGH$ is rectangular prism with $AB = 2$ cm, $BC = 3$ cm, and $AE = 4$ cm. The measure of angle between $\overrightarrow{AC}$ and $\overrightarrow{DH}$ is ...

Perpendicular (Orthogonal) Vectors

$\vec{u}$ and $\vec{v}$ are perpendicular iff $\vec{u}\cdot\vec{v} = 0$

Example: For what value of a are the vectors $\vec{u} = (4, -2, a)$ and $\vec{v} = (-1, a, 6)$ orthogonal?

Example: (UN 2008 PAKET A/B)

Jika vektor $a = xi - 4j + 8k$ tegak lurus vektor $b = 2xi + 2xj - 3k$, maka nilai x yang memenuhi adalah...

If $a = xi - 4j + 8k$ vector is perpendicular to $b = 2xi + 2xj - 3k$, then x value is ...

Example: (UN 2012/A13)

Diketahui vektor $\vec{a} = \begin{pmatrix} p \\ 2 \\ -1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix}$, dan $\vec{c} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$. Jika $\vec{a}$ tegak lurus $\vec{b}$, maka hasil dari $(\vec{a} - 2\vec{b})\cdot 3\vec{c}$ adalah...

$\vec{a} = \begin{pmatrix} p \\ 2 \\ -1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix}$, and $\vec{c} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ are given. If $\vec{a}$ is perpendicular to $\vec{b}$, then $(\vec{a} - 2\vec{b})\cdot 3\vec{c} = \ldots$

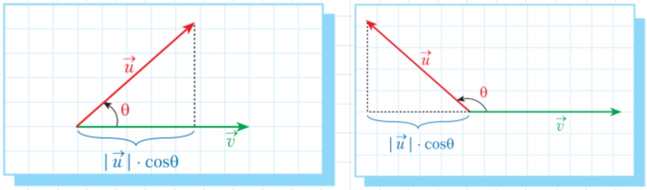
Theorem:

$$\left|\vec{u}+\vec{v}\right|^2 = \left|\vec{u}\right|^2 + 2\cdot\vec{u}\cdot\vec{v} + \left|\vec{v}\right|^2 = \left|\vec{u}\right|^2 + 2\cdot\left|\vec{u}\right|\cdot\left|\vec{v}\right|\cdot\cos\theta + \left|\vec{v}\right|^2$$

$$\left|\vec{u}-\vec{v}\right|^2 = \left|\vec{u}\right|^2 - 2\cdot\vec{u}\cdot\vec{v} + \left|\vec{v}\right|^2 = \left|\vec{u}\right|^2 - 2\cdot\left|\vec{u}\right|\cdot\left|\vec{v}\right|\cdot\cos\theta + \left|\vec{v}\right|^2$$

Example: $\vec{a}$ and $\vec{b}$ are the vectors where $\left|\vec{a}+\vec{b}\right| = \sqrt{5}$. If $\left|\vec{a}-\vec{b}\right| = \sqrt{3}$ then, $\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 = \ldots\ldots.$

C. Component of $\vec{u}$ along $\vec{v}$



Length of Projection of $\vec{u}$ along $\vec{v}$ $= |\vec{u}|\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\left|\vec{v}\right|}$

Projection vector of $\vec{u}$ along $\vec{v}$ $= \dfrac{\vec{u}\cdot\vec{v}}{\left|\vec{v}\right|^2}\cdot\vec{v}$

Example: Find the projection of $\vec{u} = (3, 4, 5)$ along $\vec{v} = (2, 1, -1)$

Example: (UN 2008 PAKET A/B)

Jika vektor $a = -3i - j + xk$ dan vektor $b = 3i - 2j + 6k$. Jika panjang proyeksi vektor a pada b adalah 5, maka nilai $x = \ldots$

Given $a = -3i - j + xk$ and $b = 3i - 2j + 6k$ vectors. If the length of projection of a on b is 5, then $x = \ldots$

Example: (UN 2012/A13)

Diketahui $\vec{a} = 5\vec{i} + 6\vec{j} + \vec{k}$ dan $\vec{b} = \vec{i} - 2\vec{j} - 2\vec{k}$. Proyeksi orthogonal vektor $\vec{a}$ pada $\vec{b}$ adalah...

$\vec{a} = 5\vec{i} + 6\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} - 2\vec{j} - 2\vec{k}$ are given. Orthogonal projection of $\vec{a}$ on $\vec{b}$ is ...

Example: (UN 2011 PAKET 12)

Diketahui vektor $a = 4i - 2j + 2k$ dan vektor $b = 2i - 6j + 4k$.
Proyeksi vektor orthogonal vektor a pada vektor b adalah ...
Given $a = 4i - 2j + 2k$ and $b = 2i - 6j + 4k$ vectors. Orthogonal projection vector of a on b is ...

Example: (UN 2010 PAKET B)

Diketahui segitiga ABC dengan koordinat $A(2,-1,-1)$, $B(-1,4,-2)$, dan $C(5,0,-3)$. Proyeksi vektor $\overrightarrow{AB}$ pada $\overrightarrow{AC}$ adalah ...
ABC triangle with $A(2,-1,-1)$, $B(-1,4,-2)$ and $C(5,0,-3)$ is given. The projection vector of $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is ...

Review Test

1. The vectors $\vec{v} = (-2,1,3)$ and $\vec{u} = (0,-1,2)$ are given. Find $3\vec{u} + 4\vec{v}$.

   A) (-2,0,5) B) (-6,1,5) C) (-6,-0,17) D) (-2,1,17) E) (-6,-1,17)

2. The vectors $\vec{v} = (-2, a-1, 3)$ and $\vec{u} = (b+1, 2, 3)$ are equal vectors. Find $a + b$.

   A) -2 B)-1 C) 0 D) 1 E) 2

3. Modulus of vector $(1,-2,\ 3)$ is......

   A) $\sqrt{10}$ B) $\sqrt{11}$ C) $\sqrt{12}$ D) $\sqrt{13}$ E) $\sqrt{14}$

4. If $\vec{v} = \left(a, 0, \frac{\sqrt{3}}{2}\right)$ is a unit vector, find a.

   A) -1 B) $-\frac{1}{2}$ C) -2 D) $\frac{3}{2}$ E) 1

5. Find a unit vector with the same direction as $\vec{a} = 8\vec{i} - \vec{j} + 4\vec{k}$

   A) $\left(\frac{8}{9}, -\frac{1}{9}, \frac{4}{9}\right)$ B) 9.(8,-1,4) C) 81.(8,-1,4) D) $\left(\frac{8}{9}, \frac{1}{9}, -\frac{4}{9}\right)$ E) $\frac{1}{81}(8,-1,4)$

6. If $Q(1,2,3), R(0,3,a)$, and $\left|\overrightarrow{QR}\right| = \sqrt{3}$`. The value of a is....

   A) 0 or 1 B) 1 or 2 C) 2 or 4 D) 3 E) 4

7. ABCDEF is a regular hexagon of center 0. If each of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ is expressed by vectors $\vec{u}$ and $\vec{v}$ then $\overline{DC}$ is equal to....

   A) $\vec{u} + \vec{v}$ B) $\vec{u} - \vec{v}$ C) $2\vec{v} - 3\vec{u}$ D) $2\vec{u} - 3\vec{v}$ E) $\vec{v} - \vec{u}$

8. Given that the point M is the center of the square ABCD. If $\overrightarrow{AC} = \vec{U}$ and $\overrightarrow{MD} = \vec{V}$ then $\overrightarrow{CD} =$......

   A) $\vec{V} = \frac{1}{2}\vec{U}$ B) $\vec{U} - \frac{1}{2}\vec{V}$ C) $\vec{V} - \frac{1}{2}\vec{U}$ D) $\vec{V} + \frac{1}{2}\vec{U}$ E) $\frac{1}{2}(\vec{U} + \vec{V})$

9. Given that $\left|\vec{a}\right| = 3$ and $\left|\vec{b}\right| = 2$. Cosine of angle between $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}$. The value of $\left|\vec{a} + \vec{b}\right| =$......

   A) $\sqrt{15}$ B) $\sqrt{16}$ C) $\sqrt{17}$ D) $\sqrt{18}$ E) $\sqrt{19}$

10. Given that $p = i - + 2k$ and $q = 2i + 2j - k$. Cosine of angle between *p* and *q* is.........

    A) $-\frac{1}{2}$ B) $-\frac{1}{2}\sqrt{5}$ C) $\frac{1}{2}\sqrt{5}$ D) $-\frac{1}{9}\sqrt{6}$ E) $\frac{1}{9}\sqrt{6}$

11. Given that the point K is on the line segment $\overrightarrow{BC}$ and K is between B and C where BK: KC = 3: 2. If B (1, 3, 2) and C (6, 8, 2), then coordinate of the point K is........

    A) (2, 1, 3) B) (1, 3, 2) C) (2, 6, 4) D) (4, 6, 2) E) (5, 10, 5)

12. The vectors $\vec{v} = (p, -2, 5)$ and $\vec{u} = (1, p, -4)$ are orthogonal. Find *p*.

    A) –22 B) –20 C) -18 D)-16 E) -14

13. The angle between $\vec{v} = \left(1, -\sqrt{3}, 2\right)$ and $\vec{u} = \left(-1, \sqrt{3}, t\right)$ is 60^0. Find t.

    A) 1 B) 4 C) -4 D) 2 E) 3

14. The vectors $\vec{a} = (4, t, -6)$ and $\vec{b} = (k, -1, 3)$ are parallel. Find $t \cdot k$.

    A) -9 B)-4 C) -1 D) 1 E) 4

15. $\vec{u} = (a, 4, -1)$, $\vec{v} = (-3, -4, -1)$ and $\vec{u} \cdot \vec{v} = -9$ are given. Find a.

    A) -2 B) -1 C) 0 D) 1 E) 2

16. $\vec{u} = (-1, 3, 0)$ and $\vec{v} = (2, -1, 0)$ are given. Find $|\vec{u} - 2\vec{v}|$.

    A) $\sqrt{2}$ B) $5\sqrt{2}$ C) $3\sqrt{2}$ D) $4\sqrt{2}$ E) $\sqrt{9}$