Proposition is an assertion (claim) which is definitely either true or false. (Generally shown by $p,q,r,s,...$)

Example:

- Dogs have six legs → proposition
- $2+3=5$ → proposition
- $3\cdot 5=21$ → proposition
- The area of a rectangle is the product of its length and width → proposition
- The rectangle is negative → not proposition because it is not making sense
- Come here → not proposition because it is an order
- Who is that? → not proposition because it is a question
- Great man! → not proposition because it is an exclamation
- He is very great → not proposition because it is subjective truth

Truth Values of Propositions

Any proposition can be True or False

It is shown by $p\equiv 1$ if p is True
$p\equiv 0$ if p is False

For one proposition;

$$p \begin{cases} \nearrow 1 \rightarrow True \\ \searrow 0 \rightarrow False \end{cases} \rightarrow$$

p
1
0

$2^1=2$ possible conditions

For two propositions;

$$p \begin{cases} \nearrow 1\ (q) \begin{cases} \nearrow 1 \\ \searrow 0 \end{cases} \\ \searrow 0\ (q) \begin{cases} \nearrow 1 \\ \searrow 0 \end{cases} \end{cases} \rightarrow$$

p	q
1	1
1	0
0	1
0	0

$2^2=4$ possible conditions

For three propositions; (draw the chart and fill in the blanks)

p	q	r

$\ldots=\ldots$ possible conditions

Generalization: For n propositions, there are 2^n possible conditions.

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Example: Decide whether following are propositions or not. Determine the truth values.

- p: the area of a right triangle is the half of the product of its legs
- t: $2+3-1=7$
- v: Trees are speaking English

Equivalent Propositions

If the truth values of p and q are equal, then p and q are called equivalent propositions and denoted by $p\equiv q$.

Example:

p: Jakarta is the capital city. $\rightarrow p\equiv 1$
q: $5\cdot 3+4=19$ $\rightarrow q\equiv 1$
$\rightarrow p\equiv q$

Negation of a Proposition (not *p*)

Negation is an operation which reverses the truth value of proposition. It is denoted by p' or $\sim p$ or $\overline{p}$.

p	$\sim p$
1	0
0	1

Some symbols	negation
$=$	$\neq$
$>$	$\leq$
$\geq$	$<$
$\in$	$\notin$

Observation: It is obvious that $\sim(\sim p)\equiv p$

Example:

p: Ali passed the class last year.

$\sim p$: Ali did not pass the class last year. (Ali failed the class)

$\sim(\sim p)$: Ali passed the class last year.

Example: Find the negation of following propositions

- p: Tomorrow is Friday.
  $\sim p$:
- q: Dwi does not play football.
  $\sim q$:
- r: $4+5=9$
  $\sim r$:
- s: $2+3<4$
  $\sim s$:

Example: (UN 2008 PAKET A/B)
Ingkaran dari pernyataan "Semua anak-anak suka bermain air." Adalah ...
The negation of the proposition "All children like playing with water" is ...

Compound Propositions

Compound Proposition is a proposition which is formed by connecting two or more propositions with connective words such as ***and, or, if … then, if and only if***.

Example:

- Dwi passed the class ***and*** Dewi did not pass the class.
- Agus works 3 days in a week ***or*** Surya goes to the university.
- ***If*** it is raining, ***then*** it is cloudy.

words	symbol	operation
and	$\wedge$	Conjunction
or	$\vee$	Disjunction
if … then	$\Rightarrow$	Implication
if and only if	$\Leftrightarrow$	Biimplication

Conjunction (∧)

$p \wedge q$ is true whenever both p and q are true; otherwise it is false.

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

Example:

- $\underbrace{\text{Jakarta is the capital city}}_{p}$ $\underset{\wedge}{\text{and}}$ $\underbrace{\text{Istanbul is in Turkey}}_{q}$.

  $p \equiv 1 \wedge q \equiv 1 \rightarrow p \wedge q \equiv 1$

- $2+3=4$ and $6>5 \rightarrow \ldots$

Observation: $p \wedge 0 \equiv 0$, $p \wedge 1 \equiv p$, $p \wedge \sim p \equiv 0$

Disjunction (∨)

$p \vee q$ is true whenever one of them is true; it is false if both are false.

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

Example:

- $\underbrace{\text{All animals have two legs}}_{p}$ $\underset{\vee}{\text{or}}$ $\underbrace{\text{chicken has two legs}}_{q}$.

  $p \equiv 0 \vee q \equiv 1 \rightarrow p \vee q \equiv 1$

Observation: $p \vee 0 \equiv p$, $p \vee 1 \equiv 1$, $p \vee \sim p \equiv 1$

Example: Find the truth values of p and q if $(p \vee \sim q) \wedge q \equiv 1$.

Common Properties of "∧" and "∨"

1. Commutative $\rightarrow$ $p \vee q \equiv q \vee p$; $p \wedge q \equiv q \wedge p$
2. Idempotent $\rightarrow$ $p \vee p \equiv p$; $p \wedge p \equiv p$
3. Associative $\rightarrow$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$; $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
4. De Morgan's Law $\rightarrow$ $\sim (p \vee q) \equiv \sim p \wedge \sim q$; $\sim (p \wedge q) \equiv \sim p \vee \sim q$
5. Distributive $\rightarrow$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$; $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Example: Prove the property number 3 and 4 by constructing truth table.

Example: Negate the following propositions.

- Ali can swim or Fendy plays football.
- $(p \wedge q) \vee r$
- $(\sim p \vee q) \wedge r$

Example: Prove that $\left[(p \vee q) \wedge (p \vee \sim q)\right] \vee \sim p \equiv 1$ is true.

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Implication (Conditional) ($\Rightarrow$)

It is the form of "if p then q" or "$p \Rightarrow q$" or "p implies q"

Example:

- If $\underbrace{\text{the team wins}}_{p}$, then $\underbrace{\text{they will get trophy}}_{q}$. $\rightarrow p \Rightarrow q$
- If $x=2$, then $2x+5=9$

"$p \Rightarrow q$" is false whenever p is true and q is false; otherwise it is true.

p	q	$p \Rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

Definition:

Inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$

Converse of $p \Rightarrow q$ is $q \Rightarrow p$

Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

Example: Write the inverse, converse and contrapositive of following propositions.

- If an object is rectangular, then it is parallelogram.
  Inverse:

  Converse:

  Contrapositive:

- $p \Rightarrow (\sim q \vee r)$
  Inverse:

  Converse:

  Contrapositive:

Properties

1. $p \Rightarrow q \equiv \sim p \vee q$
   Proof:

2. $p \Rightarrow q \equiv \sim q \Rightarrow \sim p$
   Proof:

3. $\sim (p \Rightarrow q) \equiv p \wedge \sim q$
   Proof:

Example: $p \Rightarrow q$: If $\underbrace{\text{it is shiny}}_{p}$, then $\underbrace{\text{the weather is not cloudy}}_{q}$.

$\sim p \vee q$:

$p \wedge \sim q$:

Example: Given $(p \wedge q) \Rightarrow r \equiv 0$. Find the truth value of

$\sim[(p \wedge r) \vee (q \vee r)] \Rightarrow (r \Rightarrow \sim p)$

Example: (UN 2005)
Invers dari pernyataan p $\Rightarrow$ (p ∧ q) adalah ...
The inverse of p $\Rightarrow$ (p ∧ q) is ...

Biimplication (Biconditional) ($\Leftrightarrow$)

"$p \Leftrightarrow q$" is true whenever p and q have the same truth values; otherwise it is false.

p	q	$p \Leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

Example: A triangle is equilateral if and only if it is equiangular.

Example: Identify the truth value of following propositions

- World is a planet if and only if coal is black.
- $x+5=6 \Leftrightarrow x=1$
- Flower is a plant if and only if snow is black.

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Properties

1. $p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

Proof:

2. $p \Leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Proof:

3. $\sim (p \Leftrightarrow q) \equiv \sim p \Leftrightarrow q \equiv p \Leftrightarrow \sim q$
$\equiv (p \wedge \sim q) \vee (\sim p \wedge q)$

Proof:

Tautology and Contradiction

Tautology is a proposition which is true for all possible cases.
Contradiction is a proposition which is false for all possible cases.

Example:
$p \vee \sim p \equiv 1 \rightarrow$ tautology
$p \wedge \sim p \equiv 0 \rightarrow$ contradiction

Example: Show that following proposition is a tautology.
$\sim \left[p \wedge (\sim p \vee q) \right] \vee q$

Example: Show that following proposition is a contradiction.
$q \wedge \left\{ (p \Rightarrow q) \Rightarrow \left[\sim q \wedge (p \Rightarrow q) \right] \right\}$

Quantified Propositions

Universal Quantifier ($\forall$) : for all, for every, for each.

Existential Quantifier ($\exists$) : there is, for some, there is at least one.

Example: Rewrite the following propositions as quantified propositions.

- If a real number is an integer, then it is a rational number.
  $\forall x \in \mathbb{R}$, if x is an integer then x is a rational number.
  $\forall x \in \mathbb{R},\ x \in \mathbb{Z} \Rightarrow x \in \mathbb{Q}$

- For any real number x ,there is another real number greater than x

  $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R}$ such that $y > x$

Negation of Quantified Propositions

$\sim (\forall x, p(x)) \equiv (\exists x, \sim p(x))$

$\sim (\exists x, p(x)) \equiv (\forall x, \sim p(x))$

Example: Express the negation of the following propositions.

- All pencils are black. $\Rightarrow (\forall$ pencils are black$)$
  $\sim (\forall$ pencils are black$) \equiv \exists$ pencils which are not black
- There exists a student with red pencil.
  $\Rightarrow (\exists$ student with red pencil$)$
  $\sim (\exists$ student with red pencil$) \equiv \forall$ students without red pencil

Example: (UN 2012/A13)
Negasi dari pernyataan : “Jika semua siswa SMA mematuhi disiplin sekolah maka Roy siswa teladan.”,adalah…

A. Semua siswa SMA mematuhi disiplin sekolah dan Roy bukan siswa teladan
B. Semua siswa SMA mematuhi disiplin sekolah dan Roy siswa teladan
C. Ada siswa SMA mematuhi disiplin sekolah dan Roy bukan siswa teladan
D. Ada siswa SMA mematuhi disiplin sekolah dan Roy siswa teladan
E. Jika Siswa SMA disiplin maka Roy siswa teladan

Negation of the proposition “If all SMA students obey the school discipline, then Roy is honorable student” is …

A. All SMA students obey the school discipline and Roy is not honorable student
B. All SMA students obey the school discipline and Roy is honorable student
C. There is SMA student who obeys the school discipline and Roy is not honorable student
D. There is SMA student who obeys the school discipline and Roy is honorable student
E. If SMA students obey the school discipline, then Roy is honorable student

Drawing Conclusions

Premise is the sequence of propositions serving as evidence.
Conclusion is the proposition inferred from premises.

Notation: $\therefore \rightarrow$ therefore

Modus Ponens: $((p \Rightarrow q) \wedge p) \Rightarrow q$

Premise 1 : $p \Rightarrow q$
Premise 2 : $\underline{p}$
Conclusion : $\therefore q$

Modus Tollens: $((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$

Premise 1 : $p \Rightarrow q$
Premise 2 : $\underline{\sim q}$
Conclusion : $\therefore \sim p$

Syllogisms:

1) $((p \vee q) \wedge \sim p) \Rightarrow q$ & $((p \vee q) \wedge \sim q) \Rightarrow p$

Premise 1 : $p \vee q$
Premise 2 : $\underline{\sim p}$
Conclusion : $\therefore q$

Premise 1 : $p \vee q$
Premise 2 : $\underline{\sim q}$
Conclusion : $\therefore p$

2) $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow p \Rightarrow r$

Premise 1 : $p \Rightarrow q$
Premise 2 : $\underline{q \Rightarrow r}$
Conclusion : $\therefore p \Rightarrow r$

Example: (UN 2012/C37)

Diketahui premis-premis sebagai berikut:
Premis 1 : Jika hari ini hujan deras, maka Bona tidak ke luar rumah.
Premis 2 : Bona keluar rumah.
Kesimpulan yang sah dari premis tersebut adalah…
A. Hari ini hujan deras.
B. Hari ini hujan tidak deras.
C. Hari ini hujan tidak deras atau Bona tidak keluar rumah.
D. Hari ini tidak hujan dan Bona tidak keluar rumah.
E. Hari ini hujan deras atau Bona tidak keluar rumah.

Premise 1 : If it is downpour today, then Bona will not go out.
Premise 2 : Bona goes out.
The conclusion from above premises is …
A. It is downpour today.
B. It is not downpour today.
C. It is not downpour today or Bona does not go out.
D. It is not rainy today and Bona does not go out.
E. It is downpour today or Bona does not go out.

Example: (UN 2012/A13)

Diketahui premis-premis sebagai berikut :
Premis I : "Jika Cecep lulus ujian maka saya diajak kebandung."
Premis II : "Jika saya diajak ke Bandung maka saya pergi ke Lembang."
Kesimpulan yang sah dari premis-premis tersebut adalah…..
A. Jika saya tidak pergi ke Lembang maka Cecep lulus ujian.
B. Jika saya pergi ke Lembang maka Cecep lulus ujian
C. Jika Cecep Lulus Ujian maka saya pergi ke Lembang.
D. Cecep lulus ujian dan saya pergi ke Lembang
E. Saya jadi pergi ke Lembang atau Cecep tidak lulus ujian

Premise I : "If Cecep pass the exam, then I will be invited to Bandung."
Premise II : "If I am invited to Bandung, then I will go to Lembang."
The conclusion from above premises is …
A. If do not go to Lembang, then Cecep will pass the exam
B. If go to Lembang, then Cecep will pass the exam
C. If Cecep passes the exam, then I will go to Lembang.
D. Cecep passes the exam and I go to Lembang
E. I go to Lembang or Cecep does not pass the exam

Example: (UN 2011 PAKET 12)

Diketahui premis-premis:
1) Jika hari hujan, maka ibu memakai payung.
2) Ibu tidak memakai payung.
Penarikan kesimpulan yang sah adalah …
a. Hari tidak hujan.
b. Hari hujan.
c. Ibu memakai payung.
d. Hari hujan dan Ibu memakai payung.
e. Hari tidak hujan dan Ibu memakai payung.

1) If it is rainy, then Mama uses umbrella.
2) Mama does not use umbrella.
The conclusion from above premises is …
a. It is not rainy.
b. It is rainy.
c. Mama uses umbrella.
d. It is rainy and Mama uses umbrella.
e. It is not rainy and Mama uses umbrella.

Example: (UN 2003)

Kesimpulan dari 3 premis berikut adalah…
The conclusion from following premises is …
P_1 : $p \Rightarrow q$
P_2 : $q \Rightarrow r$
P_3 : $\underline{\sim r}$
$\therefore$: ………….
a. $\sim q \Rightarrow p$
b. $q \Rightarrow p$
c. $\sim (q \Rightarrow p)$
d. $\sim p$
e. $\sim q$

Review Test

1. Which one of the following is not a proposition?

A) Prime numbers are even.
B) Sum of interior angles of a triangle is 180° .
C) Bandung is in Indonesia.
D) Mathematics is the best subject.
E) There are 12 hours in a day.

2. Which one of the following is the negation of the proposition “Agus got gold medal in national Mathematics Olympiad”?

A) Agus will get gold medal in national Mathematics Olympiad.
B) Agus got silver medal in national Mathematics Olympiad.
C) Agus got bronze medal in national Mathematics Olympiad.
D) Agus got silver or bronze medals in national Mathematics Olympiad.
E) Agus didn’t get gold medal in national Mathematics olympiad.

3. How many of the following sentences are propositions?

I. Ankara is the capital city of Germany.
II. What time is it?
III. 6+3=14.
IV. Answer this question.
V. $x + y = y + x$ for every pair of real numbers x and y.

A) 5 B) 4 C) 3 D) 2 E) 1

4. Which one of the following is not a tautology?

A) $[p' \wedge (p \vee q)] \Rightarrow q$
B) $(p \wedge q) \Rightarrow (p \Rightarrow q)$
C) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$
D) $(p' \Rightarrow q) \Rightarrow (p' \Rightarrow q')'$
E) $[p \wedge (p \Rightarrow q)] \Rightarrow q$

5. The converse of the statement "I come to class whenever there is a quiz" is

A) If I come to class, then there will be quiz.
B) If I do not come to class, then there will be quiz.
C) If I do not come to class, then there will not be quiz.
D) If there is a quiz, then I will not come to the class.
E) If there is no quiz, then I will come to class.

6. The negation of statement "All men are honest" is:

A) No men are honest.
B) All men are dishonest.
C) Some men are dishonest.
D) No men are dishonest.
E) Some men are honest.

7. Propositions p and q are given.

p: Dwi is hard working.
q: Surya doesn’t play basketball.

Which of the following represents the proposition “It is not true that Dwi is hardworking and Surya doesn’t play basketball“?

A) $p \Rightarrow q'$ B) $p' \vee q$ C) $p \wedge q'$ D) $p' \Rightarrow q'$ E) $p' \wedge q'$

8. Which one is the contrapositive of $(p' \wedge q) \Rightarrow r'$?

A) $r \Rightarrow (q \Rightarrow p)$ B) $r \Rightarrow (p \Rightarrow q)$
C) $r' \Rightarrow (p' \wedge q)$ D) $r' \Rightarrow (p \vee q')$
E) $r' \Rightarrow (p' \wedge q)$

9. Which one of the following is equivalent to the negation of the proposition $p \Rightarrow (p \vee q)$?

A) $p \vee q'$ B) $p' \wedge q$ C) $p \wedge q$ D) 0 E) 1

10. If $p' \Rightarrow (q \vee r) \equiv 0$, then which one of the following is always true?

A) $p \vee r$ B) $p' \wedge q'$ C) $q \vee r$ D) $r \vee p$ E) $q \wedge r'$

11. If $q' \wedge (p' \Rightarrow r)' \equiv 1$, what is the truth values of p, q and r respectively?

A)0,1,0 B)0,1,1 C)1,0,0 D)1,1,0 E)0,0,0

12. (UN 2005) According to the following, which ones are true?

i. $\begin{array}{l} p \vee q \\ \underline{p'} \\ \therefore q' \end{array}$ ii. $\begin{array}{l} p' \vee q \\ \underline{q'} \\ \therefore p' \end{array}$ iii. $\begin{array}{l} p \Rightarrow q \\ \underline{q' \vee r} \\ \therefore r' \Rightarrow p' \end{array}$ iv. $\begin{array}{l} q' \Rightarrow p' \\ \underline{r' \Rightarrow q'} \\ \therefore p \Rightarrow r \end{array}$

A) i and ii
B) ii and iii
C) iii and iv
D) i, ii and iii
E) ii, iii and iv

13. The inverse of the statement "If it snows today, I will ski tomorrow" is ...

A) I will ski tomorrow, only if it snows today.
B) If I won't ski tomorrow, then it doesn't snow today.
C) I will ski tomorrow if it doesn't snow today
D) If I won't ski tomorrow, then it snows today.
E) I will not ski tomorrow, if it doesn't snow today.

14. (UN 2004) What is the negation of the proposition "Today is not raining and I don't bring an umbrella".

A) Today is not raining but I don't bring an umbrella.
B) Today is not raining but I bring an umbrella.
C) Today is not raining or I don't bring an umbrella.
D) Today is raining and I bring an umbrella.
E) Today is raining or I bring an umbrella.

15. Which one of the following is the negation of the proposition of "If all students in the school go to outing program then all classrooms will be locked"?

A) If there is a student in the school who doesn't go to outing program then there is a classroom which is not locked.
B) If there is a classroom which is not locked then there is a student in the school who doesn't go to outing program.
C) If all classrooms are locked then all students in the school go to outing program.
D) All students in the school go to outing program and there is a classroom which is not locked.
E) All classrooms are not locked and there is a student in the school who doesn't go to outing program.

16.

Premise 1: If tomorrow is raining then I will not go to school.

Premise 2: If I will not go to school then I will watch football match on TV.

Which one of the following **conclusion** is true for the given premises?

A) If tomorrow is raining then I will not watch football match on TV.
B) If tomorrow is raining then I will watch football match on TV.
C) Tomorrow is raining and I will watch football match on TV.
D) I will not watch football match on TV or tomorrow is not raining.
E) Tomorrow is not raining, I don't go to school but I will watch football match on TV.