CHAPTER I: DIFFERENTIATION

1. INTRODUCTION TO DERIVATIVES

Definition:

The derivative of the function $f(x)$ with respect to x is the function $f'(x)$ (read as " f prime of x ") defined by

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Example: Find the derivative of $f(x)=x^2$.

Example: Find the derivative of $f(x)=x^2+4x-6$ at $x=-1$.

Example: If $f(x)=\frac{1}{x}$, then find $f'(x)$ at $x=1$.

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2. TECHNIQUES OF DIFFERENTIATION

A. Basic Differentiation Rules

CONSTANT FUNCTION

$$f(x)=c\Rightarrow f'(x)=c'=0$$

Example: Find the derivative of functions below.

- $f(x)=15\Rightarrow f'(x)=$
- $g(x)=\sqrt{3}\Rightarrow g'(x)=$
- $h(x)=a^2+a\Rightarrow h'(x)=$

POWER FUNCTION

$$f(x)=x^n\Rightarrow f'(x)=(x^n)'=n\cdot x^{n-1}$$

Example: Find the derivative of functions below.

- $g(x)=x^6\Rightarrow g'(x)=$
- $f(x)=x\Rightarrow f'(x)=$
- $h(x)=\sqrt[4]{x^5}\Rightarrow h'(x)=$
- $u(x)=\frac{1}{x^2}\Rightarrow u'(x)=$

CONSTANT MULTIPLE RULE

$$[c\cdot f(x)]'=c\cdot f'(x)$$

Example: Find the derivative of functions below.

- $f(x)=5x\Rightarrow f'(x)=$
- $h(x)=-\frac{2}{3}x^7\Rightarrow h'(x)=$
- $h(x)=8\sqrt[3]{x^5}\Rightarrow h'(x)=$
- $u(x)=3\sqrt[3]{x^4}\Rightarrow u'(27)=$

SUM RULE

$$[f(x)\mp \qquad \mp$$

Example: Find the derivative of functions below.

- $f(x)=3x^2-4x+5\Rightarrow f'(x)=$
- (UN 2008 PAKET A/B)
  $f(x)=3x^3+4x+8\Rightarrow f'(3)=$
- $\left(x^{-3}+5\right)'=$
- $h(x)=\frac{x^3}{4}+\frac{4}{x^3}\Rightarrow h'(x)=$

B. The Product and The Quotient Rules

PRODUCT RULE

$$[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

Example: Find the derivative of functions below.

- $f(x)=\left(x^2+1\right)\cdot\left(x^3-1\right)\Rightarrow \dfrac{dy}{dx}=$
- $g(x)=\left(x+\sqrt{x}\right)\cdot(1-x)\Rightarrow g'(4)=$

Observation:

$$\left[f(x)\cdot g(x)\cdot h(x)\right]' = f'(x)\cdot g(x)\cdot h(x) + f(x)\cdot g'(x)\cdot h(x) + f(x)\cdot g(x)\cdot h'(x)$$

Example: Derive the formula above.

Example: Find the derivative of function below.

- $h(x)=\left(1-x^2\right)\cdot\left(3x^3-x^2\right)\cdot\left(x^4+4x\right)\Rightarrow h'(x)=$

QUOTIENT RULE

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2} \quad , g(x)\neq 0$$

Example: Find the derivative of functions below.

- $f(x)=\dfrac{x^2+1}{2x+1}\Rightarrow f'(x)=$
- (EBTANAS 2002)

  $y=\dfrac{x}{1-x}\Rightarrow y'=$
- $g(x)=\dfrac{x+3}{x-3}\Rightarrow g'(0)=$

Note: Sometimes performing division gives easier opportunity.

- $h(x)=\dfrac{2x^3+4x-2}{2x}\Rightarrow h'(x)=$

Example: $f(x)=\dfrac{g(x)}{x^2}$ where $g(-1)=-2$ and $g'(-1)=-3$. Find $f'(-1)$.

C. The Chain Rule

GENERAL POWER RULE

$$\left[f(x)^n\right]' = n\left(f(x)\right)^{n-1}\cdot f'(x)$$

Example: Find the derivative of functions below.

- $f(x)=(3x+1)^3\Rightarrow f'(x)=$
- $g(x)=(2x-1)^{-6}\Rightarrow g'(1)=$
- $h(x)=\sqrt{x^2+2x+5}\Rightarrow \dfrac{d}{dx}h(x)=$
- $u(x)=\dfrac{1}{x^3-4x^2}\Rightarrow u'(x)=$

CHAIN RULE

$$\left[f\left(g(x)\right)\right]' = f'\left(g(x)\right)\cdot g'(x)$$

Example: Find the derivative of functions below.

- $f(3x+2)=x^2-2x+1\Rightarrow f'(5)=$
- $f(x^3-1)=x^3+x+2\Rightarrow f'(0)=$

Example: If $f(x)=x^2+x$ and $g(x)=3-x^2$, then

- $(gof)'(2)=$

- $(fog)'(1)=$

D. Higher Order Derivatives

Notation:

$\frac{d}{dx}f(x)=f'(x):$ 1st derivative of $f(x)$

$\frac{d^2}{dx^2}f(x)=f''(x):$ 2nd derivative of $f(x)$

$\frac{d^3}{dx^3}f(x)=f'''(x):$ 3rd derivative of $f(x)$

$\frac{d^4}{dx^4}f(x)=f^{(4)}(x):$ 4th derivative of $f(x)$

$\vdots$ $\vdots$

$\frac{d^n}{dx^n}f(x)=f^{(n)}(x):$ nth derivative of $f(x)$

Note that $\underbrace{f^{(n)}(x)}_{n^{th}\ derivative\ of\ f(x)} \qquad \underset{n^{th}\ power\ of\ f(x)}{f^{n}(x)}$

Example: Find the derivative of functions below.

- $f(x)=x^4+3x^3-5x^2-7 \Rightarrow \frac{d^2}{dx^2}f(x)=$

- $\frac{d^3\left(5x^4-2x^3+4\right)}{dx^3}=$

- $f(x)=\frac{3x}{1-x} \Rightarrow f''(x)=$

- $f(x)=\frac{1}{x} \Rightarrow f^{(4)}(x)=$

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3. DERIVATIVES OF ELEMENTARY FUNCTIONS

A. Derivatives of Exponential and Logarithmic Functions

1. Exponential Functions

NATURAL EXPONENTIAL FUNCTION

$$(e^x)' = e^x$$

Example: Find the derivative of functions below.

- $f(x) = x^3 \cdot e^x \Rightarrow f'(x) =$
- $\frac{d}{dx}\left(2e^x + x \cdot e^x\right) =$

NATURAL EXPONENTIAL FUNCTION CHAIN RULE

$$\left(e^{f(x)}\right)' = e^{f(x)}.f'(x)$$

Example: Find the derivative of functions below.

- $\left(e^{2x-1}\right)' =$
- $f(x) = e^{x^2+2x+3} \Rightarrow f'(x) =$

EXPONENTIAL FUNCTION

$$(a^x)' = a^x \cdot \ln a \quad , \quad a \in \mathbb{R}$$

Example: Find the derivative of functions below.

- $f(x) = 5^x \Rightarrow f'(x) =$
- $h(x) = e^{2x} \cdot 3^x \Rightarrow h'(x) =$

EXPONENTIAL FUNCTION CHAIN RULE

$$\left(a^{f(x)}\right)' = a^{f(x)} \cdot \ln a \cdot f'(x) \quad , \quad a \in \mathbb{R}$$

Example: Find the derivative of functions below.

- $\left(7^{x^2+1}\right)' =$
- $\frac{d}{dx}\left(\frac{7^{3x-1}}{x^3-1}\right) =$

2. Logarithmic Functions

NATURAL LOGARITHMIC FUNCTION

$$(\ln x)' = \frac{1}{x} \quad , \quad x > 0$$

Example: Find the derivative of functions below.

- $(x \cdot \ln x)' =$
- $\frac{d}{dx}\left(\ln \sqrt[7]{x^4}\right) =$
- $f(x) = \ln x^5 + x^2 + 2x \Rightarrow f'(x) =$

NATURAL LOGARITHMIC FUNCTION CHAIN RULE

$$(\ln f(x))' = \frac{1}{f(x)} \cdot f'(x)$$

Example: Find the derivative of function below.

- $\frac{d}{dx}\left(\ln(x^3 - 2x + 5)\right) =$

LOGARITHMIC FUNCTION

$$(\log_a x)' = \frac{1}{x} \cdot \log_a e \quad , \quad x > 0, a > 0, a \neq 1$$

Example: Find the derivative of function below.

- $f(x) = \log_6 x \Rightarrow \frac{dy}{dx} =$

LOGARITHMIC FUNCTION CHAIN RULE

$$(\log_a f(x))' = \frac{f'(x)}{f(x)} \cdot \log_a e$$

Example: Find the derivative of functions below.

- $f(x) = \log_5(x+3) \Rightarrow f'(x) =$
- $g(x) = e^{x^2-1} \cdot \log_3(x^2 + 3x) \Rightarrow g'(x) =$

3. Logarithmic Differentiation

Logarithmic properties sometimes helps to find derivatives easier for some products, quotients, or power of functions.

Example: Find the first derivative of $f(x)=x(2x-3)(x^2-1)$ by using logarithmic differentiation.

Example: Find the derivative of $f(x)=\dfrac{x^{\frac{4}{5}}\cdot\sqrt[3]{x^2+1}}{(x+2)^5}$

Example: $f(x)=x^x,\ x>0\Rightarrow f'(x)=$

Check yourself 12 – Page 51 in Zambak

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B. Derivatives of Trigonometric Functions

SINE and COSINE
$(\sin x)'=\cos x$
$(\cos x)'=-\sin x$

Example: Find the derivative of functions below.

- $\dfrac{d}{dx}(\sin x-\cos x)^2=$
- $\left(\dfrac{\sin x}{x}\right)'=$

SINE and COSINE CHAIN RULE
$(\sin f(x))'=\cos x\cdot f'(x)$
$(\cos f(x))'=-\sin f(x)\cdot f'(x)$

Example: Calculate the following derivatives.

- $f(x)=\sin(3x+1)\Rightarrow f'(x)=$
- $g(x)=\cos(x^2+2x)\Rightarrow g'(x)=$

- (UN 2008 PAKET A/B)
  $y=\frac{1}{4}\sin 4x \Rightarrow y' =$
- $h(x)=\sin(\cos x) \Rightarrow h'(\frac{\pi}{2}) =$
- (UN 2007 PAKET B)
  $y=\sin^3(2x-4) \Rightarrow y' =$
- $m(x)=\cos^3(x^2) \Rightarrow m'(x) =$
- (UN 2007 PAKET A)
  $f(x)=\sqrt[3]{\sin^2 3x} \Rightarrow f'(x) =$

OTHER TRIGONOMETRIC FUNCTIONS

$$(\tan x)' = \sec^2 x = 1+\tan^2 x$$
$$(\cot x)' = -\csc^2 x = -(1+\cot^2 x)$$
$$(\sec x)' = \sec x \cdot \tan x = \frac{\sin x}{\cos^2 x}$$
$$(\csc x)' = -\csc x \cdot \cot x = -\frac{\cos x}{\sin^2 x}$$

OTHER TRIGONOMETRIC FUNCTIONS CHAIN RULE

$$(\tan f(x))' = \sec^2 f(x) \cdot f'(x)$$
$$(\cot f(x))' = -\csc^2 f(x) \cdot f'(x)$$
$$(\sec f(x))' = \sec f(x) \cdot \tan f(x) \cdot f'(x)$$
$$(\csc f(x))' = -\csc f(x) \cdot \cot f(x) \cdot f'(x)$$

Example: Find the derivative of functions below.

- $f(x)=\tan(x^2-2x+1) \Rightarrow f'(x) =$
- $(\sec(\ln x))' =$
- $\frac{d}{dx}\left(e^{-\tan^2 x}\right) =$

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D. Implicit Differentiation

IMPLICIT DIFFERENTIATION

1. Differentiate both sides of the equation with respect to x . Remember that y is really a function of x and use the Chain Rule when differentiating terms containing y .
2. Solve the resulting equation y' or $\frac{dy}{dx}$ in terms of x and y .

Example: Given $y=f(x)$ and $y^2+2y-x^2=0$, $\frac{dy}{dx}$=?

Example: Find $\frac{dy}{dx}$ given $y^3-y^2x+x^4-4=0$.

Example: The equation $x^2+y^2=25$ is given.

- Find $\frac{dy}{dx}$.
- Find the slope of the tangent line to the curve at the point $(3,4)$.
- Find the equation of the tangent line at this point.

E. Derivatives of Inverse Trigonometric Functions

Following formulas can be obtained by using **inverse trigonometric function definition** and **implicit differentiation.**

Example: $y=\arcsin x$, find y' .

INVERSE TRIGONOMETRIC FUNCTIONS

$$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)'=-\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)'=\frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)'=-\frac{1}{1+x^2}$$

INVERSE TRIGONOMETRIC FUNCTIONS CHAIN RULE

$$(\arcsin f(x))'=\frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

$$(\arccos f(x))'=-\frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

$$(\arctan f(x))'=\frac{f'(x)}{1+(f(x))^2}$$

$$(\operatorname{arccot} f(x))'=-\frac{f'(x)}{1+(f(x))^2}$$

Example: $f(x)=\arcsin(x^2)\Rightarrow f'(x)=$

Example: $y=\arctan(x^3)\Rightarrow y'=$

Example: $\left(\operatorname{arccot}(2x+1)\right)'=$

Example: $y=x^3-\operatorname{arccot}(e^x)\Rightarrow y'=$

Example: $y=\arcsin x+\arctan 3x\Rightarrow \frac{dy}{dx}=$

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Part A,B,D,F

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CHAPTER II: APPLICATIONS OF THE DERIVATIVE

1. L'HOSPITAL'S RULE

The Indeterminate Form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

L'HOSPITAL'S RULE

Let the function f and g be differentiable on an open interval that contains the point a . Suppose that

$\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0 (or\ \infty)$ and $\lim\limits_{x\to a} \frac{f'(x)}{g'(x)} = L$

Then,

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}\left(or \frac{\infty}{\infty}\right) \Rightarrow \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} = L$$

Example: Find the following limits.

- $\lim\limits_{x\to 1} \frac{x^2-5x+4}{3x^2-x-2} =$
- (UN 2007 PAKET A)
  $\lim\limits_{x\to 1} \frac{x^2-5x+4}{x^3-1} =$
- (UN 2011 PAKET 46)
  $\lim\limits_{x\to \sqrt{2}} \frac{x^2-2}{x-\sqrt{2}} =$
- $\lim\limits_{x\to 2} \frac{x^4-16}{x^2-4} =$
- $\lim\limits_{x\to 2} \frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2} =$
- (UN 2012/C37)
  $\lim\limits_{x\to 0} \frac{5x}{3-\sqrt{9+x}} =$
- $\lim\limits_{x\to 0} \frac{\sin x}{x} =$
- (UN 2011 PAKET 46)
  $\lim\limits_{x\to 0} \left(\frac{1-\cos 2x}{1-\cos 4x}\right) =$
- (UN 2006)
  $\lim\limits_{x\to \frac{\pi}{2}} \frac{\cos x - \sin\frac{\pi}{6}}{\frac{\pi}{6}-\frac{x}{2}} =$
- (UN 2009 PAKET A/B)
  $\lim\limits_{x\to -3} \frac{x^2+6x+9}{2-2\cos(2x+6)} =$
- $\lim\limits_{x\to 0} \frac{\cos x - 2\sin x - 1}{\cos 2x + \sin 2x - 1} =$
- $\lim\limits_{x\to 3} \frac{x-3}{\ln(x-2)} =$
- $\lim\limits_{x\to 1} \frac{e^x - ex}{(x-1)^2} =$

Example: Find the following limits.

- $\lim\limits_{x\to \infty} \frac{x^3-6x+1}{-3x^3+x+4} =$
- $\lim\limits_{x\to \infty} \frac{e^{2x}+3x}{e^{3x}-1} =$
- $\lim\limits_{x\to 0^+} \frac{\frac{1}{x}}{\ln x} =$

Note: L'Hospital's Rule cannot be applied directly to the indeterminate forms $\infty \cdot 0$ and $\infty - \infty$. We may convert them into the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ first and then apply L'Hospital's Rule.

Example: Find the following limits.

- $\lim_{x \to \infty}\left(x \sin\frac{1}{x}\right) =$

- $\lim_{x \to 0}(2x \cdot \cot 3x) =$

- $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right) =$

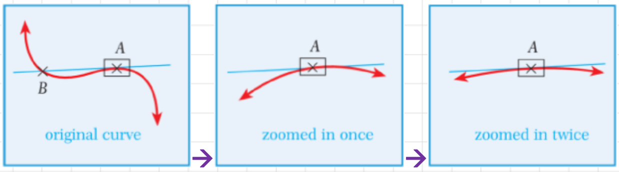
Exercises 2.1 – Page 87 in Zambak

2. TANGENT LINE

tangent → touching

tangent line to the curve → a line touching the curve at a point

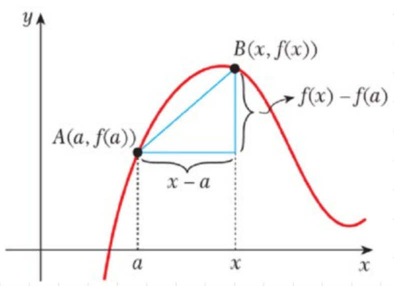
Look at the figures below.



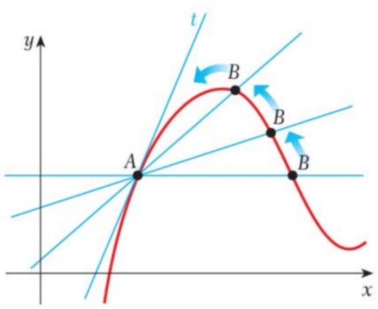
The tangent line is almost parallel to the curve at a touching point.

How to find equation of tangent line?

Let's find the slope of tangent line to the following curve at $x = a$.



$$m_{AB} = \frac{f(x) - f(a)}{x - a}$$



As a result,

Slope of tangent line to the curve $f(x)$ at $x = a$

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

In other words;

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Observation:
As we know the slope above is exactly the definition of derivative.
Thus, $f'(x)$ is the slope of tangent line to the graph of $f(x)$ at point x.

Example: Find the equation of tangent line to the curve $y = x^2 - 2$ at $x = 2$.

Example: Find the equation of tangent line to the curve $y = x^3 + 1$ at $x = -1$.

Example: Find the equation of normal line to the curve $y = \frac{1}{x}$ at $x = 2$.

Example: (UN 2010 PAKET B)

Garis singgung kurva $y = (x^2 + 2)^2$ yang melalui titik $(1,9)$ memotong sumbu y di titik ...

The tangent line to curve $y = (x^2 + 2)^2$ which passes through $(1,9)$ cuts the y axis at ...

Example: (UN 2009 PAKET A/B)

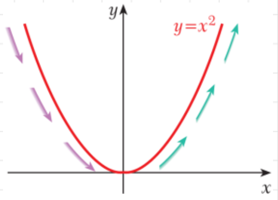
Garis l menyinggung kurva $y = 3\sqrt{x}$ di titik yang berabsis 4. Titik potong garis l dengan sumbu x adalah ...

Line l is tangent to curve $y = 3\sqrt{x}$ at the point whose abscissa is 4. The x intercept of the tangent line is ...

3. APPLICATIONS OF THE FIRST DERIVATIVE

A. Intervals of Increase and Decrease

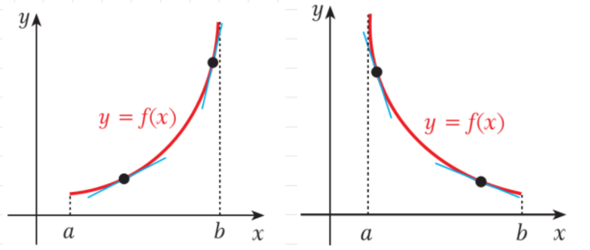
Let's observe $f(x)=x^2$ function.



the graph of $f(x)=x^2$

As we move from left to right along its graph, we see that the graph of f falls for $x<0$ and rises for $x>0$. The function f is said to be **decreasing** on $(-\infty,0)$ and **increasing** on $(0,\infty)$.

Observation:



f is increasing
positive slopes ($f'(x)>0$)

f is decreasing
negative slopes ($f'(x)<0$)

Thus,

> If $f'(x)>0$ for all $x\in I$, then $f(x)$ is increasing on I.
> If $f'(x)<0$ for all $x\in I$, then $f(x)$ is decreasing on I.

Example: Find the intervals where the function $f(x)=-x^2+4x+3$ is increasing and where it is decreasing.

Example: Find the increasing and decreasing intervals of $f(x)=x^3-12x+1$.

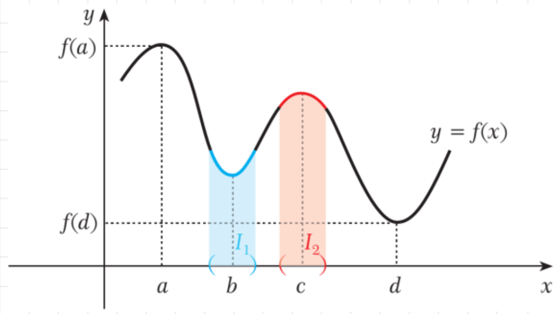
Example: For what values of x is the function $f(x)=\sqrt[3]{x-1}$ either increasing or decreasing?

Example: Determine where $f(x)=e^{x^2-4x+3}$ is increasing.

Example: For what values of m is the function

$f(x)=-\dfrac{1}{3}x^3+mx^2-4x+1$ decreasing for all real numbers?

B. Maximum and Minimum Values



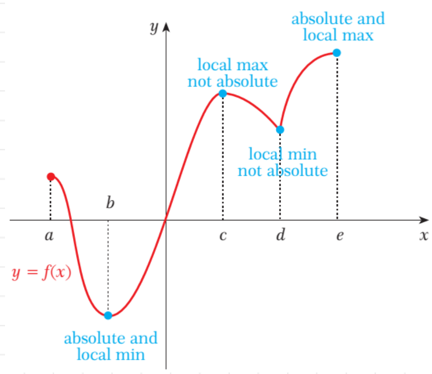
Definition:

Local Maximum: the greatest value of a function in a small interval.

Local Minimum: the smallest value of a function in a small interval.

Absolute Maximum: the greatest value of a function in a given domain.

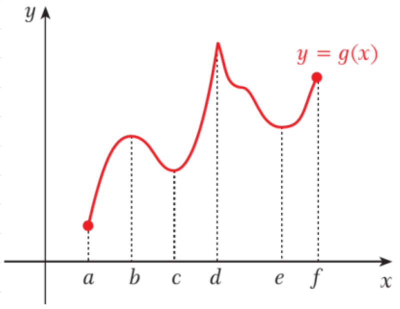
Absolute Minimum: the smallest value of a function in a given domain.



Note: (**extremum** is either maximum or minimum)

- A function has at most one absolute maximum and one absolute minimum. But it may have more than one local maximum or minimum.
- An absolute extremum of a function is either a local extremum or an endpoint.

Example: State whether the function has a local maximum or minimum, or an absolute maximum or minimum.



How to find Local Extrema

Definition: c is called a **critical point** if either

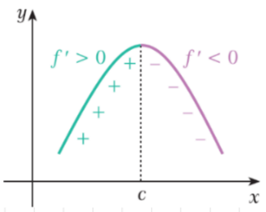
1) $f'(c)=0$, or
2) $f'(c)$ does not exist.

Example: Find the critical points of $f(x)=-x^3-3x^2+4$.

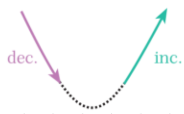
Example: Find the critical points of $f(x)=\dfrac{x^2}{x-1}$.

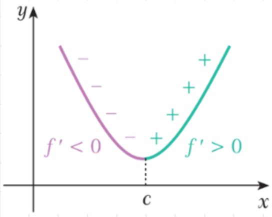
We will test the critical points by first derivative for classification.

If f is increasing on the left of c and decreasing on the right, then f should have a local maximum at $x = c$.

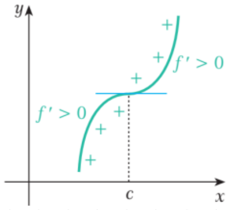


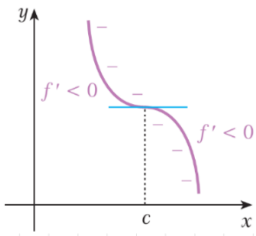
If f is decreasing on the left of c and increasing on the right, then f should have a local minimum at $x = c$.





If f is increasing on both sides or decreasing on both sides, then f should have neither a local maximum nor a local minimum at $x = c$.





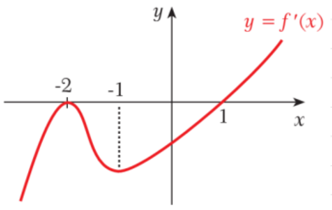
Example: Find the critical points of the function $f(x)=x^3-9x^2+24x-15$ and classify each critical point as a local maximum, a local minimum, or neither.

Example: Find the local extrema of the function $f(x)=x^{\frac{4}{5}}-1$.

Example: Find the local extrema of the function $f(x)=4x^3+5x$.

Example: If the function $f(x)=x^3+mx^2+nx+3$ has local extrema at $x=-2$ and $x=1$, then find m.

Example: The graph of the derivative of the function $f(x)$ is shown in the figure. Find the intervals where $f(x)$ is increasing or decreasing and find the local extrema of f.



Example: For what values of m does the function $f(x)=x^3-(m-1)x^2+3x-2$ have no local extrema?

How to find Absolute Extrema

1. Find the critical points of f on the interval $[a,b]$.
2. Evaluate $f(x)$ at each critical point.
3. Evaluate $f(a)$ and $f(b)$.
4. The largest of the values of f found in Steps 2 and 3 is the absolute maximum, the smallest of these values is the absolute minimum.

Example: Find the absolute extrema of the function $f(x)=-x^2+2x-1$ on $[-2,2]$.

Example: Find the maximum and minimum values of the function $f(x)=x^3-3x^2-9x+5$ on $[0,2]$.

Example: (EBTANAS 2002)

Nilai maksimum dari fungsi $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2+2x+9$ pada interval $0\le x\le 3$ adalah ...

The maximum value of function $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2+2x+9$ on the interval of $0\le x\le 3$ is ...

Example: (UN 2012 C37)

Suatu perusahaan memproduksi x unit barang dengan biaya $\left(4x^2-8x+24\right)$ dalam ribu rupiah untuk tiap unit. Jika barang tersebut terjual habis dengan harga Rp40.000,00 tiap unit, maka keuntungan maksimum yang diperoleh perusahaan tersebut adalah ...

A company produces x unit goods with costs in $\left(4x^2-8x+24\right)$ thousands of rupiah for each unit. If the goods are sold out at a price of Rp 40.000,00 per unit, the maximum profit obtained by the company is...

Example: (UN 2010 PAKET B)

Jarak yang ditempuh sebuah mobil dalam waktu diberikan oleh fungsi $s(t)=\frac{1}{4}t^4-\frac{3}{2}t^3-6t^2+5t$. Kecepatan maksimum mobil tersebut akan tercapai pada saat $t=$ ... detik .

The distance taken by a car in time is given by the function of $s(t)=\frac{1}{4}t^4-\frac{3}{2}t^3-6t^2+5t$. The maximum speed of the car will be achieved when $t=$ ... seconds.

Example: (UN 2008 PAKET A/B)

Suatu peluru ditembakan ke atas. Jika tinggi h meter setelah t detik dirumuskan dengan $h(t)=120t-5t^2$ maka tinggi maksimum yang dicapai peluru tersebut adalah ... meter.

A bullet fired upwards. If the height of bullet h meters after t seconds is formulated with $h(t)=120t-5t^2$, then the maximum height reached by the bullet is ... meters.

Exercises 2.2 – Page 105 in Zambak

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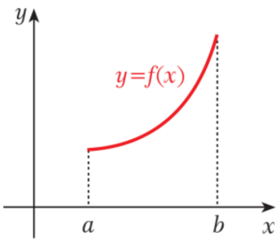
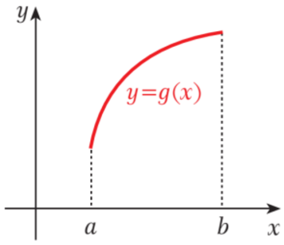
4. APPLICATIONS OF THE SECOND DERIVATIVE

A. Concavity

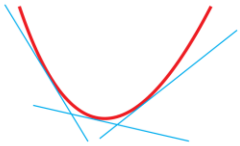


increasing, concave up



increasing, concave down

A function f is **concave up** on an interval I if the graph of f lies above all of its tangent lines on the interval I.



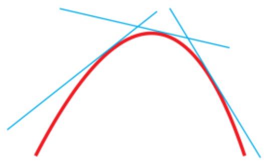
concave up (slopes increasing)

Thus, derivative of $f'(x)$ is increasing

That is,

$$f''(x)>0$$

A function f is **concave down** on an interval I if the graph of f lies below all of its tangent lines on the interval I.



concave down (slopes decreasing)

Thus, derivative of $f'(x)$ is decreasing

That is,

$$f''(x)<0$$

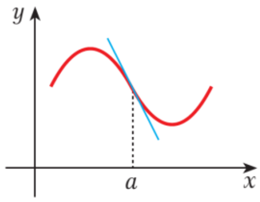
Example: Determine where the following functions are concave up and where they are concave down.

- $f(x)=x^3+x^2-2x+5$

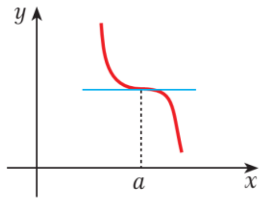
- $f(x)=16-x^4$

- $f(x)=e^x$

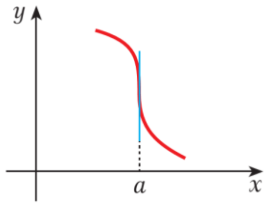
Definition: An **inflection point** is a point where a graph changes its direction of concavity.



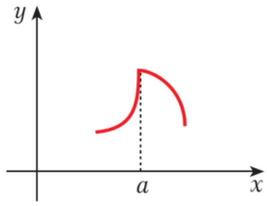
$f''(a)=0$



$f''(a)=0$



$f''(a)$ does not exist



$f''(a)$ does not exist

Definition: At each inflection point, either

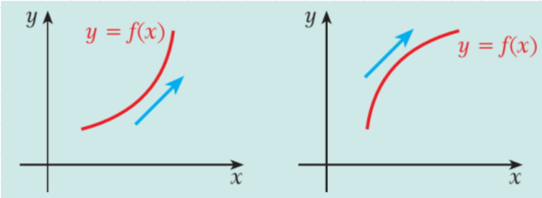
1) $f''(c)=0$, or
2) $f''(c)$ does not exist.

Example: Investigate $f(x)=x^3-6x^2+9x-14$ for concavity and find the inflection points.

Example: Find the intervals of concavity and the inflection points for $f(x)=x^4+x^3-2x+3$.

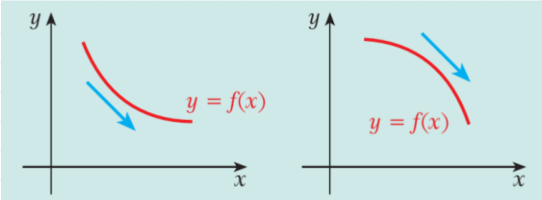
Example: Find a and b, if $f(x)=x^4-4x^3+ax^2+b$ has an inflection point at $(1,3)$.

Note: the increase and decrease of a function is independent of the concavity of the function.



$f' > 0$, f increasing
$f'' > 0$, f concave up

$f' > 0$, f increasing
$f'' < 0$, f concave down

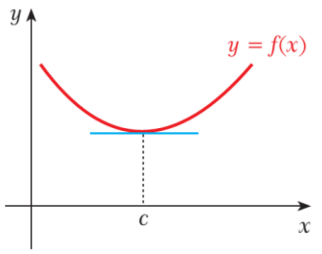


$f' < 0$, f decreasing
$f'' > 0$, f concave up

$f' < 0$, f decreasing
$f'' < 0$, f concave down

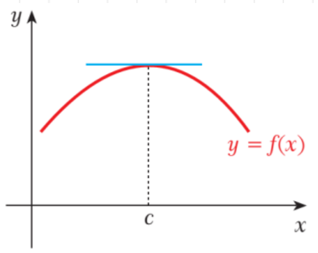
B. The Second Derivative Test

If $f''(c) > 0$, then f has a local **minimum** at $x = c$.



$f'(c) = 0, f''(c) > 0$
local minimum at c.

If $f''(c) < 0$, then f has a local **maximum** at $x = c$.



$f'(c) = 0, f''(c) < 0$
local maximum at c.

Example: Find the local extrema of the function $f(x) = x^3 - 3x^2 - 9x + 6$.

Note: If $f''(c) = 0$, then second derivative test fails. We must use the first derivative to find extrema.

Example: Find the local extrema of the function $f(x) = 3x^5 - 5x^3 + 3$.

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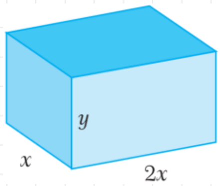
5. OPTIMIZATION PROBLEMS

Example: Find two positive numbers such that their sum is 30 and their product is as large as possible.

Example: A farmer has 100 m of fencing and wants to build a rectangular pen for his horse. Find the dimensions of the largest area he can enclose.

Example: A closed rectangular box is to be made with 192 cm² of material. The length of its base is twice its width. What is the largest possible volume of such a box?



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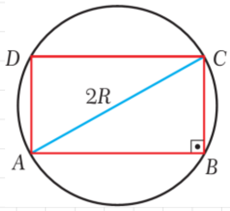
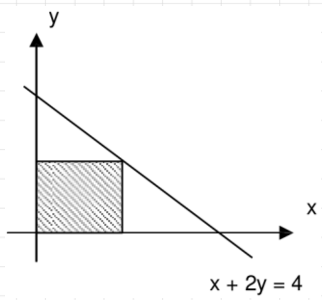
Example: Find the area of the largest rectangle that is inscribed in a circle with the radius R. ([AC] is the diameter of the circle because an angle inscribed in a semicircle is a right angle).



Example: (UN 2012/B25)

Sebuah segitiga dibatasi oleh garis $x + 2y = 4$, sumbu x dan sumbu y. Dari sebuah titik pada garis itu dibuat garis–garis tegak lurus pada sumbu x dan sumbu y sehingga membentuk sebuah persegi panjang seperti pada gambar berikut. Luas maksimum daerah persegi panjang yang diarsir adalah ... satuan luas.

Find the maximum area of the rectangle given in the figure below.



Example: (UN 2012/B25)

Selembar karton berbentuk persegi panjang dengan lebar 5 dm dan panjang 8 dm akan dibuat kotak tanpa tutup. Pada keempat pojok karton dipotong persegi yang sisinya x dm. Ukuran kotak tersebut (panjang, lebar, tinggi) agar volum maksimum berturut–turut adalah ...
A rectangular piece of cardboard with a width of 5 dm and a length of 8 dm will be used to make a box without lid. At the four corners of cardboard, square regions whose sides are x dm are cut. The box size (length, width, height respectively) in order to get the maximum volume is...

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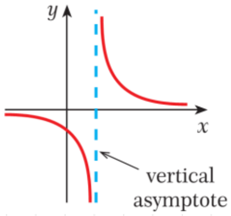
6. PLOTTING GRAPHS

A. Asymptotes

Vertical Asymptote:

If $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$, then the line $x = a$ is a vertical asymptote.



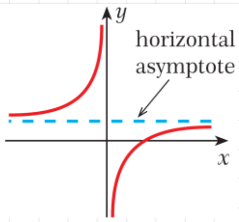
A rational function has a vertical asymptote at the zeros of denominator.

Example: Find the vertical asymptotes of the function $f(x) = \dfrac{x+2}{x^2 - x - 2}$.

Example: Find the vertical asymptotes of the function $f(x) = \dfrac{x^2 - x}{x^2 + 2x - 3}$.

Horizontal Asymptote:

If $\lim_{x\to -\infty} f(x) = b$ or $\lim_{x\to \infty} f(x) = b$, then the line $y = b$ is a horizontal asymptote.



Example: Find the horizontal asymptote of the function $f(x) = \dfrac{x+2}{x^3 - 2x}$.

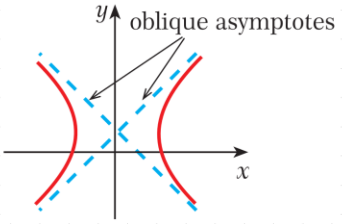
Example: Find the vertical and horizontal asymptotes of the function $f(x) = \dfrac{x^2 - 2x}{3x^2 + 7x + 2}$.

Example: Find the vertical and horizontal asymptotes of the function $f(x)=\dfrac{5x^2+4}{x^2+3}$.

Example: Find all asymptotes of $f(x)=2x^4-5x^3+4x-1$.

Oblique Asymptote:

If $\lim\limits_{x\to-\infty}\left[f(x)-(mx+n)\right]=0$ or $\lim\limits_{x\to\infty}\left[f(x)-(mx+n)\right]=0$, then the line $y=mx+n$ is an oblique asymptote.



For a rational function $\dfrac{P(x)}{Q(x)}$ where $\deg\left[P(x)\right]=\deg\left[Q(x)\right]+1$ use long division to find an oblique asymptote.

Example: Find all asymptotes of the function $f(x)=\dfrac{x^2+x}{x-2}$.

B. Curve Plotting

Steps of Curve Plotting

1. Domain
2. Intervals of Increase and Decrease
3. Local Extrema
4. Concavity and Inflection Points
5. Intercepts
6. Behavior at Infinity
7. Asymptotes
8. Graph

Example: Plot the graph of $f(x)=-x^4+8x^2-7$.

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Example: Plot the graph of the function $f(x)=\dfrac{x^2-x+4}{x-1}$.

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Review Test

1. If $y=-\frac{1}{333}$ is given, what is y'?

A) 0   B) -1   C) 1   D) 333   E)-333

2. If $y=x^4$ is given, what is y'?

A) $4x$ B) x^3 C) $4x^3$ D) 0   E) 4

3. If $f(x)=4x^2-2x-40$ is given, find $f'(3)$.

A) 49   B) 53   C) 22   D) 18   E) 23

4. If $y=\sqrt[4]{x^7}$ is given, what is y'?

A) $\frac{7}{4}\sqrt[4]{x^7}$ B) $\frac{7}{4}\sqrt[4]{x^3}$ C) $-\frac{7}{4}\sqrt[4]{x^3}$ D) $-7\sqrt[4]{x^3}$ E) $7\sqrt[4]{x^3}$

5. If $y=-\frac{2}{\sqrt{x^5}}$ is given, what is y'?

A) $\frac{1}{\sqrt{x^5}}$ B) $\frac{2}{5\sqrt{x^5}}$ C) $\frac{5}{\sqrt{x^5}}$ D) $-\frac{5}{\sqrt{x^7}}$ E) $\frac{5}{\sqrt{x^7}}$

6. If $y=\frac{4}{t^4}+t^3-t$ is given, what is y'?

A) $-\frac{4}{t^5}+3t^2-1$ B) $\frac{4}{t^5}+3t^2-1$ C) $\frac{16}{t^5}+3t^2-1$ D) $\frac{16}{t^5}+3t^2-t$ E) $-\frac{16}{t^5}+3t^2-1$

7. If $f(x)=\frac{4}{x^2}+0.02x$ is given then $f'(-2)=?$

A) 1   B) 1.002   C) 2   D) 2.002   E) 1.02

8. If $y=x^2(x+2)$ is given, what is y'?

A) $3x^2+4x$ B) $3(x^2+4x)$ C) $2x^2+2$ D) $2x^2+x$ E) $2x^2+4$

9. If $y=(x^3+x^2-2)(2x^2+1)$ is given, what is y'?

A) $10x^4+8x^3+3x^2-6x$ B) $10x^4+8x^3+3x^2$ C) $10x^3+8x^2+3x$ D) $10x^3+8x^2+3$ E) $10x^4+8x^2+3$

10. $y=(x^4-x^2)(x^2+2)$ is given. Calculate y'?

A) x^7+x^4-4x B) $6x^5+4x^3-4x$ C) $4x^3$ D) $6x^5+4x^3$ E) $6x^5+12x^3-4x$

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11. $y=\dfrac{3}{3x+5}$ is given. Calculate y'?

A) $\dfrac{-9}{(3x+5)}$ B) $\dfrac{9}{(3x+5)}$ C) $\dfrac{9}{(3x+5)^2}$ D) $\dfrac{-9}{(3x+5)^2}$ E) $\dfrac{-3}{(3x+5)^2}$

12. If $y=\dfrac{x+1}{3x+2}$ is given, what is y'?

A) $\dfrac{-1}{(3x+2)^2}$ B) $\dfrac{1}{(3x+2)^2}$ C) $\dfrac{5}{(3x+2)^2}$ D) $\dfrac{-5}{(3x+2)^2}$ E) $\dfrac{6x+5}{(3x+2)^2}$

13. If $y=\dfrac{x^2-1}{x^2+2x+1}$ is given, what is y'?

A) $\dfrac{2}{(x+1)^2}$ B) $\dfrac{x}{x+2}$ C) 0 D) $\dfrac{2x}{(x+1)^2}$ E) -1

14. (EBTANAS 2002)

$f(x)=\dfrac{x^2-3x}{x^2+2x+1} \Rightarrow f'(2)=$

A) $-\dfrac{2}{9}$ B) $\dfrac{1}{9}$ C) $\dfrac{1}{6}$ D) $\dfrac{7}{27}$ E) $\dfrac{7}{4}$

15. $y=(x^3+2x)^5$ is given, what is y'?

A) $15x^2+2$ B) $(3x^2+2)^4$ C) $(3x^2+2)^5$ D) $5(x^3+2x)^4(3x^2+2)$ E) $5(x^3+2x)^4$

16. If $y=(x^2-2)^{-4}$ is given, what is y'?

A) $-8x(x^2-2)^{-3}$ B) $-8x(x^2-2)^{-5}$ C) $(x^2-2)^{-3}$ D) $-4(x^2-2)^{-3}$ E) $-4(x^2-2)^{-5}$

17. If $y=(3x-2)^3(3x-2)^2$ is given, what is y'?

A) $9(3x-2)^2(3x-2)$ B) $6(3x-2)^2(3x-2)$ C) $6(3x-2)^4$ D) $5(3x-2)^4$ E) $15(3x-2)^4$

18. If $y=u^3-u$ and $u=2x+1$ are given, what is y'?

A) $6(2x+1)^2-2$ B) $3(2x+1)^2$ C) $6(2x+1)^2$ D) $3(2x+1)^2-2$ E) $3(2x+1)^2-1$

19. If $f(-2x+3)=x^3+7x^2+6x+1$ then $f\,'(11)=$?

A) -2 B) -1 C) 1 D) 2 E) 4

20. If $f(x)=\sqrt{x^2+3}$ and $\mathrm{g(x)}=\mathrm{x}^3+2x$ then $(gof)'(1)=$?

A) $8\sqrt{7}$ B) 7 C) $\dfrac{10}{\sqrt{7}}$ D) $\dfrac{5\sqrt{7}}{2}$ E) $4\sqrt{7}$

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21. If $y=3x^3-8x+2$ is given. Evaluate y''.

A) $9x^2$ B) $9x^2-8$ C) $18x$ D) $18x^2-8$ E) 18

22. If $y=(2x+1)^6$ is given. Evaluate y''.

A) $30(2x+1)^4$ B) $60(2x+1)^4$ C) $120(2x+1)^4$ D) $(2x+1)^4$ E) $60(2x+1)^5$

23. If $y=\dfrac{(3x-2)^4}{54}$ is given. Find y'''.

A) $36x$ B) $36x-2$ C) $36x-24$ D) 36 E) $24x$

24. If $y=\sqrt{3x-1}$ is given. Find y''.

A) $\dfrac{9}{\sqrt{(3x-1)^3}}$ B) $\dfrac{-1}{4\sqrt{(3x-1)^3}}$ C) $\dfrac{1}{4\sqrt{(3x-1)^3}}$ D) $\dfrac{9}{4\sqrt{(3x-1)^3}}$ E) $\dfrac{-9}{4\sqrt{(3x-1)^3}}$

25. If $y=\dfrac{-3}{x}$ is given. Find y'''.

A) $\dfrac{6}{x^3}$ B) $\dfrac{-18}{x^3}$ C) $\dfrac{-18}{x^4}$ D) $\dfrac{18}{x^4}$ E) $\dfrac{-6}{x^3}$

26. If $y=e^x$ is given. Find y'''.

A) 0 B) x C) 1 D) e E) e^x

27. If $y=x^3.e^x$ is given, calculate y'.

A) $x^2e^x(3+x)$ B) $e^x(3+x)$ C) x^2e^x D) $(3+x)$ E) $x^2(3+x)$

28. If $y=3^x$ is given, calculate y'.

A) 3^x B) $3^x\ln 3$ C) $3^x\ln 3^x$ D) $\ln 3^x$ E) $3x$

29. If $y=e^x3^x$ is given, calculate y'.

A) $3e^x+3xe^x$ B) $e^x(1+3^x\ln 3)$ C) $e^x3^x(1+\ln 3)$ D) $e^x3^x\ln 3$ E) e^x3

30. If $y=e^{x^2+2x}$ is given, then $y'=?$

A) $e(2x+2)$ B) $e(x^2+2x)$ C) e^{x^2+2x} D) $e^{x^2+2x}(2x+2)$ E) $2x+2$

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31. If $y=(e^x+3)^5$ is given, then $y'=?$

A) $(e^x+3)^5$ B) $5e^x(e^x+3)^4$ C) $5(e^x+3)^4$ D) $e^x(e^x+3)^4$ E) $(e^x+3)^4$

32. If $y=\ln x$ is given, then $y'=?$

A) $\ln x$ B) x C) 1 D) $\frac{1}{x}$ E) 0

33. If $y=2\ln(2x-1)$ is given $y'=?$

A) $\ln(2x-1)$ B) $2\ln(2x-1)$ C) $\frac{4}{2x-1}$ D) $\frac{2}{2x-1}$ E) $\frac{1}{2x-1}$

34. If $y=\ln(x^3)$ is given, then $y'=?$

A) $\ln(x^3)$ B) $\frac{3}{x}$ C) $3\ln x$ D) $3\ln(x^2)$ E) $\ln(3x)$

35. If $y=x\ln x$ is given, then $y'=?$

A) $x\ln x$ B) $\ln x$ C) x D) 1 E) $\ln x+1$

36. If $y=(e^x+\ln x)^3$ is given, then $y'=?$

A) $3(e^x+\frac{1}{x})^2$ B) $(e^x+\frac{1}{x})$ C) $3(e^x+\ln x)^2$ D) $3(e^x+\ln x)^2(e^x+\frac{1}{x})$ E) $\frac{3e^x(e^x+\ln x)^3}{x}$

37. Find derivative of $f(x)=3\log x$.

A) $\frac{3}{x\log x}$ B) $\frac{1}{x}$ C) $\frac{3}{x}$ D) $\frac{3}{x\ln 10}$ E) $\frac{1}{3x\log x}$

38. If $y=\log_2 x$ is given, then $y'=?$

A) $x\ln 2$ B) $\log_2 x$ C) $\frac{1}{x\ln 2}$ D) $2\log x$ E) 2

39. If $y=\log_3(x^2+1)$ is given, then find y'.

A) $\frac{2x}{(x^2+1)\ln 3}$ B) $\frac{2}{(x^2+1)\ln 3}$ C) $\log_3(x^2+1)$ D) $\frac{2x}{(x^2+1)}$ E) $\frac{3}{(x^2+1)}$

40. If $y=\sin(2x)$ is given, then find y'.

A) $2\cos x$ B) $-2\cos x$ C) $-2\sin(2x)$ D) $-2\cos(2x)$ E) $2\cos(2x)$

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41. What is derivative of $f(x)=\sin 3x+\cos 5x$?

A) $\cos 3x - \sin 5x$ B) $3\cos 3x - 5\sin 5x$
C) $3\cos 3x+5\sin 5x$ D) $\sin 3x - \cos 5x$
E) $3\sin 3x+5\cos 5x$

42. If $f(x)=\frac{1}{2}\cos 2x$ then what is $f'(\pi)=?$

A) 1 B) –1 C) 0 D) 0.05 E) –0.5

43. $y=\sin^3(x^2)\Rightarrow y'=?$

A) $6x\sin^2(x^2)$
B) $6x\sin^2(x^2)\cos(x^2)$
C) $3\sin^2(x^2)$
D) $3\sin^2(x^2)\cos(x^2)$
E) $3x\sin^2(x^2)\cos(x^2)$

44. $y=x^2\cos x$ is given, $y'=?$

A) $2x\cos x$
B) $-x^2\sin x$
C) $x^2\sin x$
D) $2x\cos x-x^2\sin x$
E) $2x\cos x+x^2\sin x$

45. If $y=\frac{\sin x}{1+\cos x}$ is given, $y'=?$

A) $\frac{\sin x}{1+\cos x}$ B) $\sin x$ C) $1+\cos x$
D) $\frac{1}{1+\cos x}$ E) $\tan x$

46. (UN 2006)
$f(x)=\sin^2(8x-2\pi)\Rightarrow f'(x)=$

A) $2\sin(8x-2\pi)$
B) $8\sin(8x-2\pi)$
C) $2\sin(16x-4\pi)$
D) $8\sin(16x-4\pi)$
E) $16\sin(16x-4\pi)$

47. (UN 2005)
$f(x)=\cos^3 x\Rightarrow f'(x)=$

A) $f'(x)=-\frac{3}{2}\cos x\cdot\sin 2x$
B) $f'(x)=\frac{3}{2}\cos x\cdot\sin 2x$
C) $f'(x)=-3\cos x\cdot\sin x$
D) $f'(x)=3\cos x\cdot\sin x$
E) $f'(x)=-3\cos^2 x$

48. (UAN 2003)
$f(x)=(3x^2-5)\cos x\Rightarrow f'(x)=$

A) $3x\cdot\sin x+(3x^2-5)\cos x$
B) $3x\cdot\cos x+(3x^2-5)\sin x$
C) $-6x\cdot\sin x-(3x^2-5)\cos x$
D) $6x\cdot\cos x+(3x^2-5)\sin x$
E) $6x\cdot\cos x-(3x^2-5)\sin x$

49. $y=\tan(3x^2)\Rightarrow y'=?$

A) $-6x\tan(3x^2)$ B) $\tan^2(3x^2)$ C) $6x\tan^2(3x^2)$
D) $\sec^2(3x^2)$ E) $6x\sec^2(3x^2)$

50. If $f(x)=x\cdot\cot x$ is given then $f'(x)=?$

A) $-\frac{x}{\sin^2 x}$ B) $\cot x$ C) $\cot x-\frac{x}{\sin^2 x}$
D) $1+\frac{x}{\sin^2 x}$ E) $1-\frac{x}{\sin^2 x}$

51. If $f(x)=\tan 4x+\cot 2x$ is given, then $f'\left(\frac{2\pi}{3}\right)=?$

A) $\frac{32}{3}$ B) 11 C) $\frac{35}{3}$ D) 13 E) $\frac{40}{3}$

52. If $f(x)=\cos(\sin x)$ then $f'\left(\frac{\pi}{2}\right)=?$

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{1}{2}$ C) 0 D) $\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

53. If $f(x)=3^{\sin x}$ then $f'(0)=?$

A) $-e^2$ B) $-e$ C) 1 D) e E) $\ln 3$

54. If $f(x)=3^{\cos x}$ is given then $f'\left(\frac{\pi}{2}\right)=?$

A) $-\ln 6$ B) $-\ln 3$ C) ln2 D) ln3 E) ln6

55. If $5x-y=3$ is given, find the first derivative of $5x-y=3$.

A)-5 B) $3x$ C) $5x-3$ D) $5x$ E)5

56. $x^2+x-xy=1 \Rightarrow \frac{dy}{dx}=$

A) $\frac{-y}{x}$ B) $2x+1$ C) $\frac{2x+1-y}{x}$ D) $\frac{2x-y}{x}$ E) $\frac{2x-1}{x}$

57. $y^4+y^2+2x=0 \Rightarrow \frac{dy}{dx}=$

A) $\frac{1}{4y^3+2y}$ B) $\frac{2}{4y^3+2y}$ C) $\frac{-1}{4y^3+2y}$ D) $\frac{-2}{4y^3+2y}$ E) $\frac{2x}{y^4+y^2}$

58. If $x^2-xy-y^2+5=0$ is given then $\frac{dy}{dx}=?$

A) $\frac{y-x}{x+2y}$ B) $\frac{y-2x}{x+2y}$ C) $\frac{y+2x}{x-2y}$ D) $\frac{2x-y}{x+2y}$ E) $\frac{y+x}{x-2y}$

59. $f(x)=\arctan(x^2-1) \Rightarrow f'(0)=?$

A)0 B) $\frac{3}{5}$ C) $\frac{4}{5}$ D) 1 E) $\frac{6}{5}$

60. $f(x)=\arctan x+(\arccos x)^2 \Rightarrow f'(0)=?$

A) $1+\pi$ B) $1-\pi$ C) $2+\pi$ D) $2-\pi$ E) $3+\pi$

61. (UN 2007 PAKET B)

$$\lim_{x\to 3}\frac{9-x^2}{4-\sqrt{x^2+7}}=$$

A) 8 B) 4 C) $\frac{9}{4}$ D) 1 E) 0

62. (UN 2012/B25)

$$\lim_{x\to 3}\frac{2-\sqrt{x+1}}{x-3}=$$

A) $-\frac{1}{4}$ B) $-\frac{1}{2}$ C) 1 D) 2 E) 4

63. (UN 2004)

$$\lim_{x\to 0}\frac{1-\cos 4x}{x^2}=$$

A) -8 B) -4 C) 2 D) 4 E) 8

64. (UAN 2003)

$$\lim_{x\to \frac{\pi}{4}}\frac{\cos 2x}{\cos x-\sin x}=$$

A) $-\sqrt{2}$ B) $-\frac{1}{2}\sqrt{2}$ C) $\frac{1}{2}\sqrt{2}$ D) $\sqrt{2}$ E) $2\sqrt{2}$

65. Find the equation of the line which is tangent to $f(x)=2x^2-1$ at $x=0$.

A) $f(x)=3x$ B) $f(x)=-1$ C) $f(x)=2$ D) $f(x)=x+1$ E) $f(x)=1-x$

66. Find the equation of line which is tangent to $f(x)=2\ln(x-1)$ at $x=3$.

A) $y=x-3$ B) $y=2\ln\left(\frac{2}{x-1}\right)$ C) $y=-x-3$ D) $y=x+2\ln 2$ E) $y=x-3+2\ln 2$

67. Find the point on curve of $y=x^3-3x+2$ so that the tangent line passing through the point is parallel to x-axis.

A) (1, -1) B) (1, 0) C) (-1, 1) D) (0, -1) E) (-1, 0)

68. Find the equation of tangent line of the function $y=2\cos 3x+1$ at $(\frac{\pi}{3},-1)$.

A) $y=1$ B) $y=-1$ C) $y=2x-\pi/3$ D) $y=2x+\pi/3$ E) $y=3x+\pi/3$

69. Find the equation of line that is parallel to the line $y=4x-3$ and tangent to the curve $y=x^2-1$.

A) $y=4x-5$ B) $y=2x-5$ C) $y=x-5$ D) $y=-4x-5$ E) $y=-2x+5$

70. The tangent line of curve $y=x^3$ at the point $A(2,8)$ intersects with the given curve at the point B. Find the abscissa of B.

A) -5 B) -4 C) -3 D) $-\frac{5}{2}$ E) $-\frac{3}{2}$

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71. (UN 2010 PAKET A)

Diketahui h adalah garis singgung kurva $y = x^3 - 4x^2 + 2x - 3$ pada titik $(1, -4)$. Titik potong garis h dengan sumbu x adalah ...

If h is a tangent line to curve $y = x^3 - 4x^2 + 2x - 3$ at point $(1, -4)$. The $x-$intercept of h is ...

A) $(-3, 0)$ B) $(-2, 0)$ C) $(-1, 0)$

D) $\left(-\frac{1}{2}, 0\right)$ E) $\left(-\frac{1}{3}, 0\right)$

72. (UAN 2003)

Diketahui kurva dengan persamaan $y = x^3 + 2ax^2 + b$. Garis $y = -9x - 2$ menyinggung kurva di titik dengan absis 1. Nilai $a = ...$

Given a curve with the equation of $y = x^3 + 2ax^2 + b$, the line $y = -9x - 2$ is tangent to the curve at a point whose abscissa is 1. The value of a is ...

A) -3 B) $-\frac{1}{3}$ C) $\frac{1}{3}$ D) 3 E) 8

73. Find the equation of tangent line of curve $x^3y^2 - 5xy^3 + 8x^2 - 4y + 24 = 0$ at point (2, 2).

A) $y - 2 = \frac{10}{23}(x - 2)$ B) $y = \frac{10}{23}(x + 2)$ C) $23y = 10(x - 2)$

D) $x + y = 4$ E) $x - y = 4$

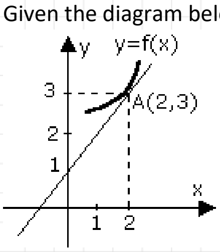
74. The function $y = f(x)$ is defined as $x^2 + 2xy - 3y^2 - 4x + 11 = 0$. Which one of the following is the equation of the line that is tangent to the graph of the function at the point (2,-1)?

A) $2x - y - 3 = 0$ B) $x - 5y - 7 = 0$

C) $x + 3y - 1 = 0$ D) $2x - 3y - 7 = 0$

E) $3x - y + 4 = 0$

75. Given the diagram below.

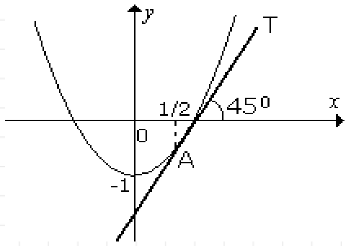


The equation of tangent line of curve $y = f(x)$ on $A(2,3)$ is $y = x + 1$.

Find the value of $g'(x)$ at $x = 2$ if $g(x) = f(x)(x^2 - 5)$.

A) 7 B) 8 C) 9 D) 10 E) 11

76. The equation of the parabola below is $y=ax^2+bx+c$. If the line AT is tangent of parabola at the tangency point A, then calculate the value of $a + b + c$.



A) -2 B) $-\frac{1}{2}$ C) 0 D) $\frac{2}{3}$ E) 1

77. What is the minimum value of the function $f(x) = 2x^3 - 3x^2 - 12x + 1$ in the interval [0, 3]?

A) -19 B) -17 C) -13 D) 8 E) 9

78. In which interval is the function $f(x) = x^3 - 3x$ increasing?

A) $[0,\infty)$ B) $[1,\infty)$ C) [-1, 1]

D) $(-\infty,-1]\cup[1,\infty)$ E) [0,1)

79. In which interval is the function $f(x) = 2x - 3$ increasing?

A) $(-\infty, +\infty)$ B) $[0,\infty)$ C) $[3/2,\infty)$

D) $[-3/2,\infty)$ E) (-3/2, 3/2)

80. What is the increasing interval of $f(x) = -4x^2 - 4x - 1$?

A) $(-\infty,+\infty)$ B) $(-\infty,-\frac{1}{2})$ C) $(-\infty, \frac{1}{2})$

D) $[-\frac{1}{2},\infty)$ E) $(-\frac{1}{2}, \frac{1}{2})$

81. Which one of the following is valid for the function $f(x) = 4x^2 - 6x - 7$?

A) x=3/4 – max point
B) x=-3/4 – min point
C) x=3/4 – min point
D) x=-3/4 – max point
E) x=6/7 – max point

82. Find the extreme point of the $f(x) = 3 + 4x - x^2$.

A) x=2 – max point
B) x=2 – min point
C) x= -2 – max point
D) x= -2 – min point
E) x=3/4 – min point

83. Which one of the following is true for the function $f(x) = 2x^3 + 3x^2 + 12x + 1$?

A) Its value is maximum at $x = 2$.
B) Its value is minimum at $x = -1$.
C) It has an inflection point at $x = \frac{1}{2}$.
D) It is an increasing function.
E) It is a decreasing function.

84. Which one of the following is the slope of the line that is tangent to the function $f(x) = 5x - x^2$ at point A (2, 6)?

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$

85. (UN 2012/E52)
Suatu perusahaan memproduksi x unit barang dengan biaya $(5x^2 - 10x + 30)$ dalam ribuan rupiah untuk tiap unit. Jika barang tersebut terjual habis dengan harga Rp.50.000,00 tiap unit, maka keuntungan maksimum yang diperoleh perusahaan tersebut adalah....
A company produces x unit goods with costs in $(5x^2 - 10x + 30)$ thousands of rupiah for each unit. If the goods are sold out at a price of Rp 50.000,00 per unit, the maximum profit obtained by the company is...

A) Rp10.000,00 B) Rp20.000,00 C) Rp30.000,00
D) Rp40.000,00 E) Rp50.000,00

86. (UN 2011 PAKET 12/46)
Suatu perusahaan menghsilkan x produk dengan biaya sebesar $(9000 + 1000x + 10x^2)$ rupiah. Jika semua hasil produk perusahaan tersebut habis dijual dengan harga Rp5.000,00 untuk satu produknya, maka laba maksimum yang dapat diperoleh perusahaan tersebut adalah ...
A company produces x unit goods with costs in $(9000 + 1000x + 10x^2)$ rupiah. If the goods are sold out at a price of Rp 5.000,00 per unit, the maximum profit obtained by the company is...

A) Rp149.000,00 B) Rp249.000,00 C) Rp391.000,00
D) Rp609.000,00 E) Rp757.000,00

87. (EBTANAS 2002)
Koordinat titik maksimum dan minimum dari grafik $y = x^3 + 3x^2 + 4$ berturut–turut adalah...
The coordinates of maximum and minimum of the function $y = x^3 + 3x^2 + 4$ is ... respectively.

A) (–2,4) dan (0,3) B) (0,3) dan (–2,4) C) (–2,6) dan (0,5)
D) (0,4) dan (–2,8) E) (–2,8) dan (0,4)

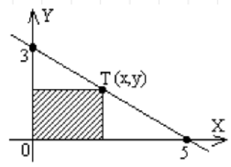
88. If the function $f(x) = 2x^3 + ax^2 + bx + c$ has an inflection point at $x = -\frac{1}{2}$, find the value of *a*.

A) 1 B) 2 C) 3 D) 4 E) 6

89. In which of the following intervals is the function $f: \Re \to \Re,\ f(x) = x^2 \cdot (x+3)$ concave down?

A) $(-\infty, -2)$ B) $(-2, 0)$ C) $(0, +\infty)$
D) $(-\infty, -1)$ E) $(-1, +\infty)$

90. (UN 2007 PAKET A)
Perhatikan gambar! Luas daerah yang diarsir pada gambar akan mencapai maksimum, jika koordinat T adalah ...
Look at the graph! The shaded area will be maximum if the coordinates of T is ...



A) $\left(3, \frac{5}{6}\right)$ B) $\left(\frac{5}{2}, \frac{3}{2}\right)$ C) $\left(2, \frac{9}{5}\right)$ D) $\left(\frac{3}{2}, \frac{21}{10}\right)$ E) $\left(1, \frac{12}{5}\right)$

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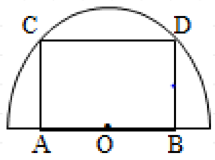
91. Find the maximum value of the area of the rectangle ABCD that can be inscribed in the semicircle whose radius is 1.



A) $\sqrt{5}$ B) $2\sqrt{5}$ C) $3\sqrt{5}$ D) $4\sqrt{5}$ E) $5\sqrt{5}$

92. (UN 2009 PAKET A/B)
Sebuah bak air tanpa tutup berbentuk tabung. Jumlah luas selimut dan alas bak air adalah $28m^2$. Volum akan maksimum, jika jari–jari alas sama dengan ...

A water tube without cover is in a cylinder shape. The surface area of the tube is $28m^2$. The volume will be maximum, if the radius of the base is equal to...

A) $\frac{1}{3\pi}\sqrt{7\pi}$ B) $\frac{2}{3\pi}\sqrt{7\pi}$ C) $\frac{4}{3\pi}\sqrt{7\pi}$

D) $\frac{2}{3\pi}\sqrt{21\pi}$ E) $\frac{4}{3\pi}\sqrt{21\pi}$

93. (UN 2006)
Santo ingin membuat sebuah tabung tertutup dari selembar karton dengan volum $16dm^3$. Agar luas permukaan tabung minimal, maka jari–jari lingkaran alasnya adalah ...

Santo wants to make a closed cylinder tube with a volume of $16dm^3$ by using a piece of cardboard. In order to get the minimum surface area, the radius of the base circle is...

A) $\sqrt[3]{\frac{4}{\pi}}dm$ B) $\frac{2}{\sqrt[3]{\pi}}dm$ C) $\frac{4}{\sqrt[3]{\pi}}dm$

D) $2\sqrt[3]{\pi}dm$ E) $4\sqrt[3]{\pi}dm$

94. A 500m wire will be constructed to a rectangular prism whose height is 25 m. Find the area of the rectangular base so that the volume maximum...

A) $500m^2$ B) $2500m^2$ C) $1500m^2$

D) $5000m^2$ E) $625m^2$

95. What is the vertical asymptote of the given function $f(x)=\frac{3x-1}{2x+5}$?

A) $\frac{3}{2}$ B) $-\frac{3}{2}$ C) 1 D) $-\frac{5}{2}$ E) $\frac{5}{2}$

96. What is the horizontal asymptote of the given function $f(x)=\frac{1-3x}{4x+2}$?

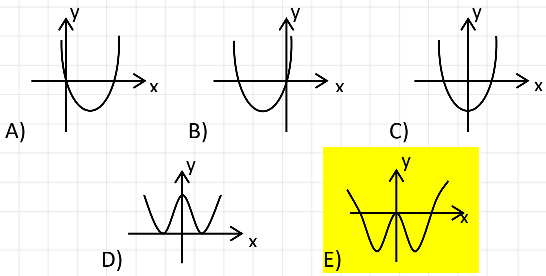
A) $\frac{3}{4}$ B) $-\frac{3}{4}$ C) -1 D) $-\frac{1}{2}$ E) $\frac{1}{2}$

97. What is the oblique asymptote of the given function $f(x)=\frac{x^2-3x+5}{x-2}$?

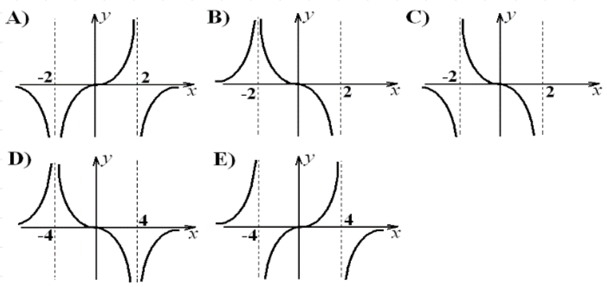
A) $y=x+1$ B) $y=2x-1$ C) $y=x-1$

D) $y=-x-1$ E) $y=-x+1$

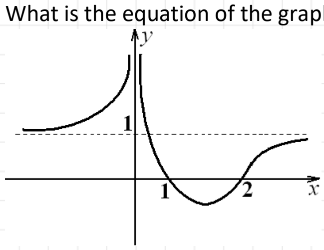
98. Which one of the followings may be the graph of the function $y=x^4-2x^2$?



99. Which one of the followings may be the graph of the function $y=\frac{2x}{x^2-4}$?



100. What is the equation of the graph below?



A) $y=\frac{x^2-3x+2}{x^2}$ B) $y=\frac{x^2-3}{x^2+1}$ C) $y=\frac{x^2-2x+1}{x^2}$

D) $y=\frac{x-2}{x-1}$ E) $y=\frac{x^2-2x+3}{x^2}$