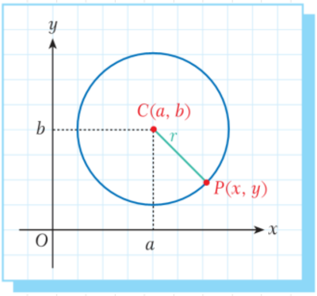
1. FUNDAMENTALS of TRIGONOMETRY

Unit Circle

Equation of Circle

The equation of circle which is centered at (a,b) with radius r is

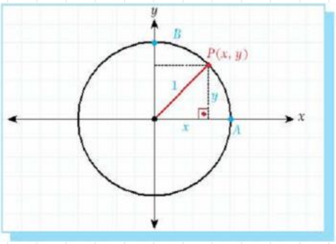
$$(x-a)^2+(y-b)^2=r^2$$



Unit Circle

Unit circle is a circle which is centered at $(0,0)$ with radius 1.

$$x^2+y^2=1$$



Example: $(a-3)x^2+(b+1)y^2=1$ is the equation of unit circle. Find a & b.

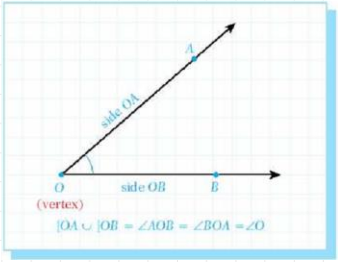
Example: Show that $P(\frac{1}{\sqrt{3}},\frac{\sqrt{6}}{3})$ is on the unit circle.

Example: The point $P(x,\frac{\sqrt{3}}{2})$ is on the unit circle. Find the possible values for x.

Angles and Directions

Angle

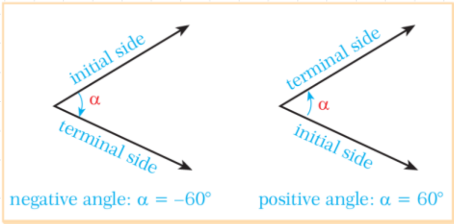
Angle is the union of two rays which have a common endpoint.



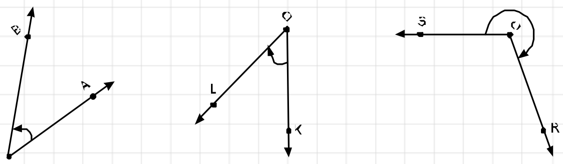
Directed Angle

The angle whose one side is initial side and the other is terminal side is called directed angle.

Counterclockwise = Positive
Clockwise = Negative



Example: Determine initial and terminal side of each angle and their directions.



Central Angle

The angle whose vertex is the center of circle is called **central angle**.

Arc

The segment of a circle between the two sides of a central angle is called an arc.
Longer is called **major arc**, shorter is **minor arc.**

Chord

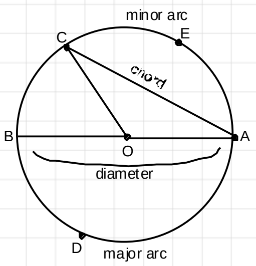
The line segment AB which joins two different points of circle is called chord.

Diameter

The chord which passes through center of circle is called **diameter**.

Semicircle

The equal arcs of a circle which are divided by a diameter are called **semicircles**.

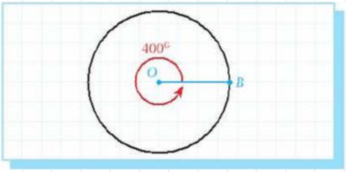


Units of Angle Measures

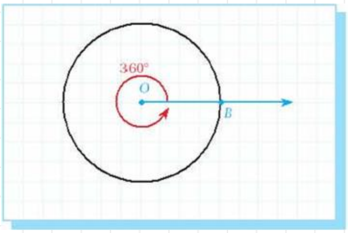
Complete Angle

The central angle which corresponds to one complete revolution around a circle is called **complete angle.**

1) ***Grad:*** The complete angle of circle measures 400^G .



2) ***Degree:*** The complete angle of circle measures 360° .



$\dfrac{1}{60}$ of degree is called **minute**. That is, $1^\circ = 60'$.

$\dfrac{1}{60}$ of minute is called **second**. That is, $1' = 60''$.

Let's consider the angle of 37 degrees, 45 minutes, 30 seconds.

It can be written in two ways:

In **degree-minute-second form**: $37^\circ\ 45'\ 30''$

In **decimal degree form:** $37^\circ + (45\cdot\dfrac{1}{60}) + (30\cdot\dfrac{1}{3600}) = 37.758\overline{3}^\circ$

Example: Write $56^\circ\ 20'\ 15''$ in decimal degree form.

Example: Write 17.86° in degree-minute-second form.

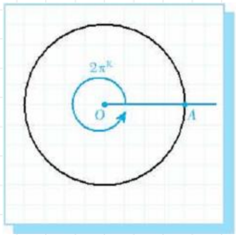
Example:

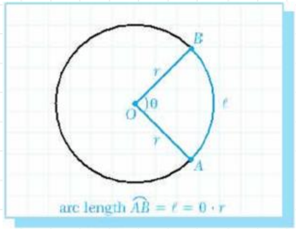
$x = 202^\circ\ 15'\ 36''$ $\qquad$ $x = 202^\circ\ 15'\ 36''$

$y = 114^\circ\ 57'\ 58''$ $\qquad$ $y = 114^\circ\ 57'\ 58''$

$x + y =$ $\qquad$ $x - y =$

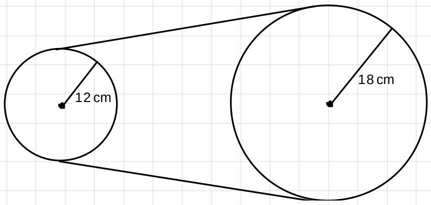
3) ***Radian :*** The complete angle of circle measures 2π radians.





length $AB = l = x \cdot r$

Example: When the small circle makes one full rotation, how many radians will the big circle rotate?



Converting Units of Angle Measures

Conversion Formula

$$\frac{D}{360} = \frac{R}{2\pi} = \frac{G}{400}$$

Example: Convert the angles below.

- 100° to radian :
- $\dfrac{5\pi}{12}$ to degree :

Primary Directed Angles

Standard Position of an Angle

The angle whose vertex is at the origin and whose initial side lies along positive x-axis is called **standard position of an angle**.

Coterminal Angles

Two or more angles whose terminal sides coincide with each other when they are in standard position are called **coterminal angles**.

Example: For each of the angles below, write set of coterminal angles.

- 175°
- $\dfrac{5\pi}{4}$

Example: Find the arc length which corresponds to central angle 40° on unit circle. ($\pi = 3$)

Primary Directed Angle

Let $\beta \geq 360^\circ$ be an angle.
The positive angle $\alpha \in [0,360)$ which is coterminal with β is called **primary directed angle of β .**

Example: Find primary directed angle of each of the angles below.

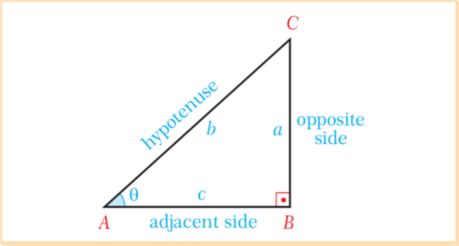
- 7320°
- -7320°
- $\dfrac{75\pi}{8}$
- $\dfrac{75\pi}{8}$
- $-\dfrac{75\pi}{8}$
- $-30^\circ\ 42'\ 15''$

Exercises 1 – Page 27 in Zambak

Active Note Book

2. RIGHT ANGLE TRIGOMETRY

Trigonometric Ratios



$\sin\theta = \dfrac{a}{b}$

$\cos\theta = \dfrac{c}{b}$

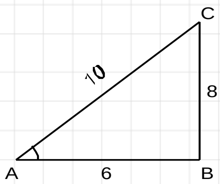
$\tan\theta = \dfrac{a}{c}$

$\cot\theta = \dfrac{c}{a} = \dfrac{1}{\tan\theta}$

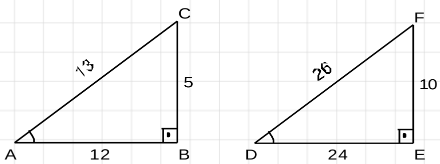
$\sec\theta = \dfrac{b}{c} = \dfrac{1}{\cos\theta}$

$\csc\theta = \dfrac{b}{a} = \dfrac{1}{\sin\theta}$

Example: Write six ratios above by using the following triangle.



Note: If triangles are similar then trigonometric ratios of same angles are same.



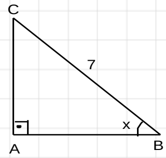
Calculating the Other Ratios From Given Ratio

Show the given ratio on right triangle and find the others.

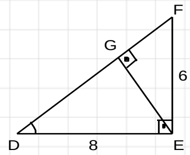
Example: Given that $\cos\theta = \dfrac{2}{3}$, find the other trigonometric ratios.

Example: If $\tan x = \dfrac{2}{7}\sqrt{6}$, then find $\cos x$ and $\sec x$.

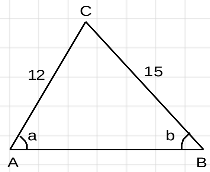
Example: If $\sec x = 0.6$, then $|BC| = ?$



Example: Find $|GF|$ in the triangle below.



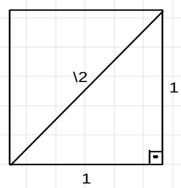
Example: Find $|AC|$ in the triangle below if $\cos a = \dfrac{9}{16}$ & $\cos b = \dfrac{3}{4}$.



Special Triangles and Ratios

45 Degree

Consider a square whose length is 1.

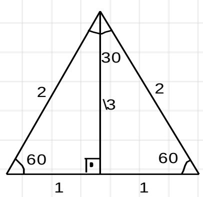


$\sin 45 = \dfrac{1}{\sqrt{2}}$ $\quad \cos 45 = \dfrac{1}{\sqrt{2}}$

$\tan 45 = 1$ $\quad \cot 45 = 1$

30 & 60 Degrees

Consider an equilateral triangle whose length is 2.



$\sin 30 = \dfrac{1}{2}$ $\quad \sin 60 = \dfrac{\sqrt{3}}{2}$

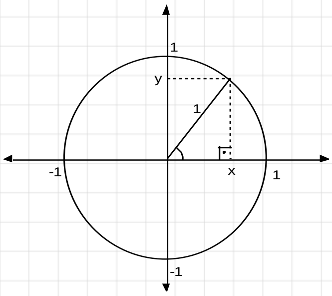
$\cos 30 = \dfrac{\sqrt{3}}{2}$ $\quad \cos 60 = \dfrac{1}{2}$

$\tan 30 = \dfrac{1}{\sqrt{3}}$ $\quad \tan 60 = \sqrt{3}$

$\cot 30 = \sqrt{3}$ $\quad \cot 60 = \dfrac{1}{\sqrt{3}}$

0 & 90 Degrees

Consider unit circle.



$\sin 0 = 0$ $\quad\sin 90 = 1$

$\cos 0 = 1$ $\quad\cos 90 = 0$

$\tan 0 = 0$ $\quad\tan 0 = \infty$

$\cot 0 = \infty$ $\quad\cot 0 = 0$

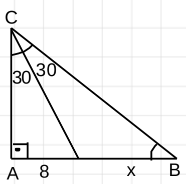
$\cos\alpha = x$ $\qquad \sin\alpha = y$

Easy way to remember these ratios

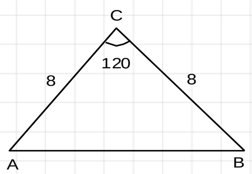
- Form the first row by given pattern.
- Write the first row results in reverse order to form second row.
- Use the identity of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ for third row.
- Use the identity of $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ for fourth row.

	0	30	45	60	90
sin	$\frac{1}{2}\sqrt{0}=0$	$\frac{1}{2}\sqrt{1}=\frac{1}{2}$	$\frac{1}{2}\sqrt{2}=\frac{\sqrt{2}}{2}$	$\frac{1}{2}\sqrt{3}=\frac{\sqrt{3}}{2}$	$\frac{1}{2}\sqrt{4}=1$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

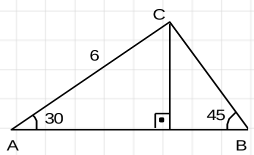
Example: Find x in the triangle below.



Example: Find $|AB|$ and $A(ABC)$ in the triangle below.

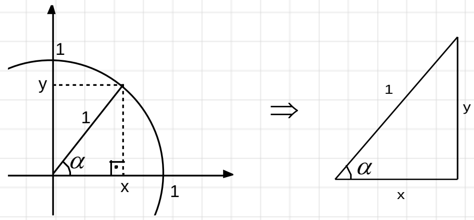


Example: Find $|AB|$ in the triangle below.



Trigonometric Identities

Consider unit circle & a right triangle formed by α .



$x = \cos\alpha$

$y = \sin\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$

Proof:

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

Proof:

$\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

Proof:

$1 + \tan^2\alpha = \sec^2\alpha$

Proof:

$1 + \cot^2\alpha = \csc^2\alpha$

Proof:

Example: $\tan x \cdot \cot x =$

Example: $(\cos x + 2\sin x)^2 + (2\cos x - \sin x)^2 =$

Example: $\tan x \cdot \cos x \cdot \csc x =$

Example: $\cos^3 x + \sin^2 x \cdot \cos x =$

Example: $\dfrac{\sec x - \cos x}{\tan x} =$

Example: $\dfrac{2 + \tan^2 x}{\sec^2 x} - 1 =$

Note: For verification problems, it is better to start with the complicated side to get the other side.

Example: Verify the following equalities.

- $\sin x \cdot \cot x = \cos x$

- $\csc x = \cos x \cdot (\tan x + \cot x)$

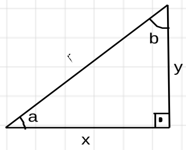
- $\dfrac{(\sin x + \cos x)^2}{\sin x \cdot \cos x} = 2 + \sec x \cdot \csc x$

- $\dfrac{\tan x}{\csc x} = \sec x - \cos x$

- $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$

Cofunctions

Two angles whose sum is 90 degree are called **complementary angles.**



a and b are complementary angles.

Observation:

$\underline{\sin a} = \dfrac{y}{r} = \underline{\cos b}$ $\quad$ $\underline{\tan a} = \dfrac{y}{x} = \underline{\cot b}$ $\quad$ $\underline{\sec a} = \dfrac{r}{x} = \underline{\csc b}$

In other words, $a = 90 - b$

$\sin a = \cos b \Leftrightarrow \boxed{\sin(90 - b) = \cos b}$

$\tan a = \cot b \Leftrightarrow \boxed{\tan(90 - b) = \cot b}$

$\sec a = \csc b \Leftrightarrow \boxed{\sec(90 - b) = \csc b}$

Example: $\cot 1 \cdot \cot 89 =$

Example: $\sin^2 27 + \sin^2 63 =$

Example: $\tan 1 \cdot \tan 2 \cdot \ldots\ldots \cdot \tan 88 \cdot \tan 89 =$

Example: $\sin^2 \dfrac{\pi}{7} + \left(\tan \dfrac{7\pi}{18} \cdot \tan \dfrac{\pi}{9}\right) + \sin^2 \dfrac{5\pi}{14} =$

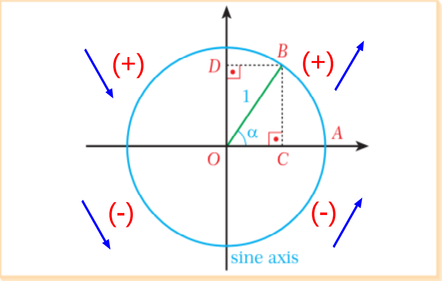
Exercises 2 – Page 47 in Zambak

Active Note Book

3. TRIGONOMETRIC FUNCTIONS

Trigonometric Functions

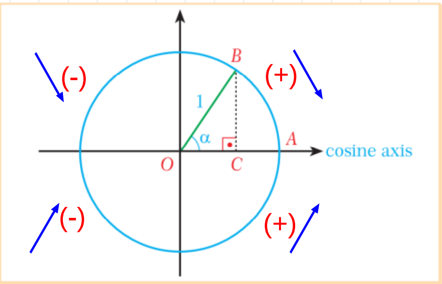
Sine Function



$y = f(\alpha) = \sin(\alpha)$

- $\alpha \in \mathbb{R}$
- $-1 \le \sin\alpha \le 1$
- Increasing in 1st and 4th quadrant.
  Decreasing in 2nd and 3rd quadrant.
- Positive in 1st and 2nd quadrant.
  Negative 3rd and 4th quadrant.

Cosine Function

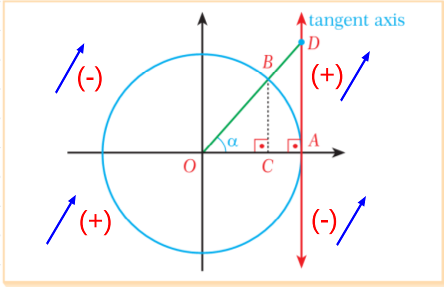


$y = f(\alpha) = \cos(\alpha)$

- $\alpha \in \mathbb{R}$
- $-1 \le \cos\alpha \le 1$
- Increasing in 3rd and 4th quadrant.
  Decreasing in 1st and 2nd quadrant.
- Positive in 1st and 4th quadrant.
  Negative 3rd and 2nd quadrant.

Example: Find the maximum and minimum values of $A = 3\cos x - 1$ and $B = 2 - 4\sin x$.

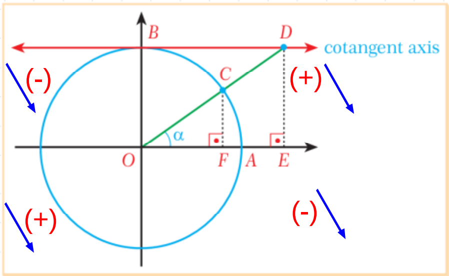
Tangent Function



$y = f(\alpha) = \tan(\alpha)$

- $a \in \mathbb{R} - \{\frac{\pi}{2} + k\pi;\ k \in \mathbb{Z}\}$
- $-\infty \le \tan\alpha \le \infty$
- Increasing in all quadrants.
- Positive in 1st and 3rd quadrant.
  Negative 2nd and 4th quadrant.

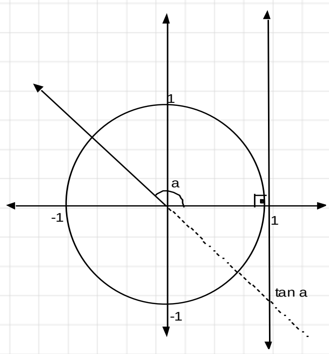
Cotangent Function



$y = f(\alpha) = \cot(\alpha)$

- $a \in \mathbb{R} - \{k\pi;\ k \in \mathbb{Z}\}$
- $-\infty \le \cot\alpha \le \infty$
- Decreasing in all quadrants.
- Positive in 1st and 3rd quadrant.
  Negative 2nd and 4th quadrant.

Note: If the angle cannot cut tangent axis or cotangent axis, we consider extension of the angle.



Example: Order following expressions.

$x = \tan 37$
$y = \tan 36$
$z = \tan 35$

Example: Order following expressions.

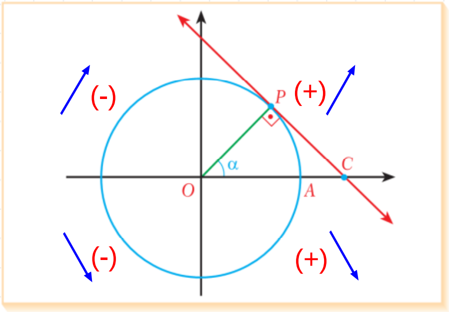
$a = \cos\frac{5\pi}{9}$
$b = \cot\frac{2\pi}{9}$
$c = \cot\frac{7\pi}{9}$

Example: Show that $\tan 37 < \sin 37$.

Example: Show that $\cos 57 < \cot 57$.

Example: Show that $\cos 72 = \sin 18$.

Secant Function

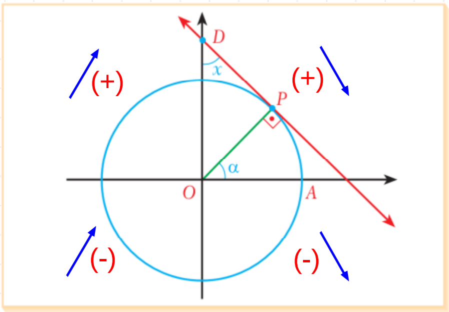


In OPC triangle, $\underline{\sec\alpha} = \frac{|OC|}{\underset{1}{|OP|}} = \underline{|OC|}$

$y = f(\alpha) = \sec(\alpha)$

- $a \in \mathbb{R} - \{\frac{\pi}{2} + k\pi;\ k \in \mathbb{Z}\}$
- $\sec a \in \mathbb{R} - (-1,1)$
- Increasing in 1st and 2nd quadrant.
  Decreasing in 3rd and 4th quadrant.
- Positive in 1st and 4th quadrant.
  Negative 3rd and 2nd quadrant.

Cosecant Function

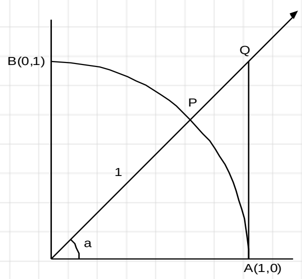


In OPD triangle, $x = \alpha \Rightarrow \underline{\csc\alpha} = \frac{|OD|}{\underset{1}{|OP|}} = \underline{|OD|}$

$y = f(\alpha) = \csc(\alpha)$

- $\alpha \in \mathbb{R} - \{k\pi;\ k \in \mathbb{Z}\}$
- $\csc\alpha \in \mathbb{R} - (-1,1)$
- Increasing in 2nd and 3rd quadrant.
  Decreasing in 1st and 4th quadrant.
- Positive in 1st and 2nd quadrant.
  Negative 3rd and 4th quadrant.

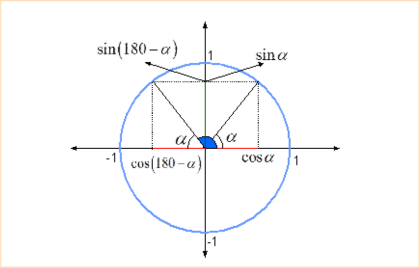
Example: Represent |PQ| in terms of trigonometric functions.



Calculating Trigonometric Values with Reference Angle

(Let $0 < a < 90$)

Angles of the form (180 – α)



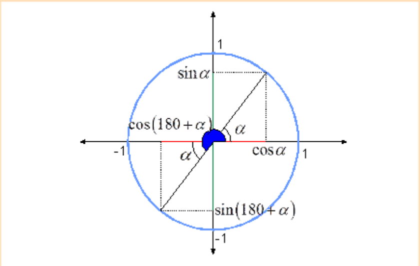
$\sin(180-\alpha)=\sin\alpha$

$\cos(180-\alpha)=-\cos\alpha$

Example:

- $\sin(120)=$
- $\cos(135)=$
- $\tan(150)=$

Angles of the form (180 + α)



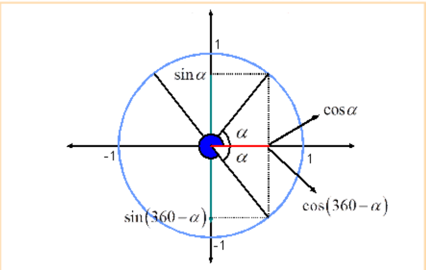
$\sin(180+\alpha)=-\sin\alpha$

$\cos(180+\alpha)=-\cos\alpha$

Example:

- $\sin(210)=$
- $\cos(240)=$
- $\tan(225)=$

Angles of the form (360 – α)



$\sin(360-\alpha)=-\sin\alpha$

$\cos(360-\alpha)=\cos\alpha$

Example:

- $\sin(315)=$
- $\cos(330)=$
- $\tan(300)=$

Example: $\cot 240+\tan 150+\cos 315+\sin 750=$

Example:

$$\cos(49\pi-\alpha)+\cot(-100\pi+\alpha)+\sin\left(\frac{71\pi}{2}+\alpha\right)+\tan\left(\frac{-37\pi}{2}-\alpha\right)=$$

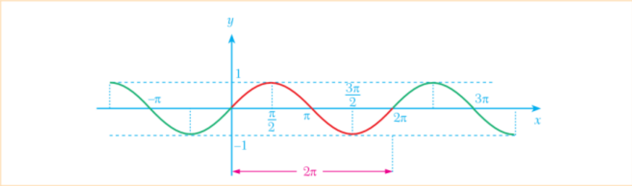
Exercises 3 – Page 73 in Zambak

Active Note Book

4. GRAPHS of TRIGONOMETRIC FUNCTIONS

$f(x)=\sin x$

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
$\sin x$	0	↗	1	↘	0	↘	-1	↗	0



Example: Sketch the graph of $f(x)=2\sin x$.

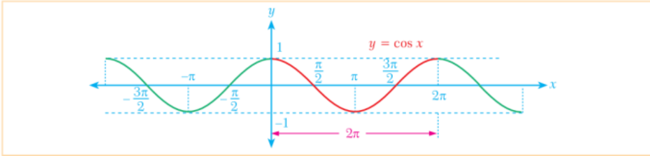
Observation:
$y=f(x)\to y=a\cdot f(x)$
Means multiply y values of your function with a .

So, if a is negative, it means that take the symmetry of function graph with respect to x -axis.

Example: Sketch the graph of $f(x)=-3\sin x$.

$f(x)=\cos x$

x	0		$\frac{\pi}{2}$		π		$\frac{3\pi}{2}$		2π
$\cos x$	1	↘	0	↘	-1	↗	0	↗	1



Example: Sketch the graph of $f(x)=\cos 2x$ where $-\pi\le x\le 0$.

Observation:
$y=f(x)\to y=f(a\cdot x)$

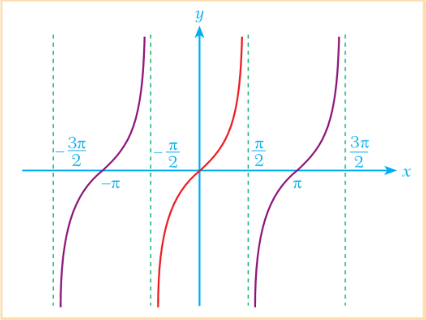
Means multiply x values of your function with $\frac{1}{a}$.

So, if a is negative, it means that take the symmetry of function graph with respect to y -axis.

Example: Sketch the graph of $f(x)=\cos(-4x)$ where $-2\pi\le x\le 2\pi$.

$f(x)=\tan x$

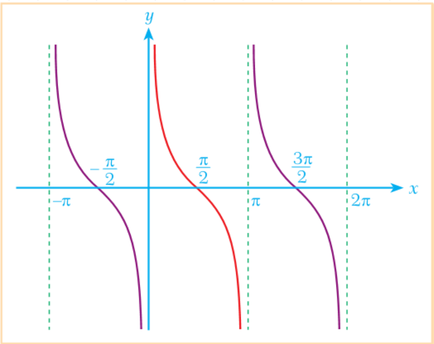
x	$-\frac{\pi}{2}$		$-\frac{\pi}{4}$		0		$\frac{\pi}{4}$		$\frac{\pi}{2}$
$\tan x$	$-\infty$	$\nearrow$	-1	$\nearrow$	0	$\nearrow$	1	$\nearrow$	∞



Example: Sketch the graph of $f(x)=2\tan 2x$ where $0\le x\le \pi$.

$f(x)=\cot x$

x	0		$\frac{\pi}{4}$		$\frac{\pi}{2}$		$\frac{3\pi}{4}$		π
$\cot x$	∞	$\searrow$	1	$\searrow$	0	$\searrow$	-1	$\searrow$	$-\infty$



Example: Sketch the graph of $f(x)=\cot 3x$ where $0\le x\le \pi$.

5. TRIGONOMETRIC EQUATIONS

Equations of the Form $\sin x=a$ **,** $\cos x=a$ **,** $\tan x=a$ **,** $\cot x=a$

Let's solve the equation of $\sin x=\frac{1}{2}$.

- We know that $\sin x=\frac{1}{2}$ when $x=\frac{\pi}{6}$.

  Since sine is periodic with 2π .

  $x=\frac{\pi}{6}+2\pi k,\ k\in\mathbb{Z}$ is a solution set for $\sin x=\frac{1}{2}$.

- We also know that $\sin\alpha=\sin(\pi-\alpha)$.

  So $\sin x=\underline{\sin(\pi-x)=\frac{1}{2}}$.

  That is, $\pi-x=\frac{\pi}{6}$.

  Since sine is periodic with 2π .

  $\pi-x=\frac{\pi}{6}+2\pi k,\ k\in\mathbb{Z}$.

  $x=\pi-\frac{\pi}{6}+2\pi k,\ k\in\mathbb{Z}$ is also solution set for $\sin x=\frac{1}{2}$.

$\sin x=a$

If $\alpha\in[0,2\pi)$ satisfies the equation of $\sin x=a$, then

$x=\alpha+2\pi k,\ k\in\mathbb{Z}$ or $x=(\pi-\alpha)+2\pi k,\ k\in\mathbb{Z}$

Example: Solve the following equations.

- $\sin x=1$

- $\sin x=-\frac{\sqrt{3}}{2}$

- $\sin 2x=0$

- $\sin\left(2x-\frac{\pi}{6}\right)=-1$

$\cos x = a$

If $\alpha\in[0,2\pi)$ satisfies the equation of $\cos x=a$, then

$$x=\alpha+2\pi k,\ k\in\mathbb{Z} \quad \text{or} \quad x=-\alpha+2\pi k,\ k\in\mathbb{Z}$$

Example: Solve the following equations.

- $\cos x=\frac{1}{2}$

- $\cos x=0$

- $\cos 3x=\frac{1}{2}$

Observation:

What about the solutions of $\cos x=\frac{3}{2}$ *or* $\sin x=-2$?

$\tan x = a$ *or* $\cot x = a$

If $\alpha\in[0,2\pi)$ satisfies the equation of $\tan x=a$ or $\cot x=a$, then

$$x=\alpha+\pi k,\ k\in\mathbb{Z}$$

Example: Solve the following equations.

- $\tan x=\sqrt{3}$

- $\tan x=1$

- $\cot x=-1$

Generalization

1. If $\sin(f(x))=\sin(g(x))$, then
   $f(x)=g(x)+2\pi k$ or $f(x)=\pi-g(x)+2\pi k$
2. If $\cos(f(x))=\cos(g(x))$, then
   $f(x)=g(x)+2\pi k$ or $f(x)=-g(x)+2\pi k$
3. If $\tan(f(x))=\tan(g(x))$ or $\cot(f(x))=\cot(g(x))$, then
   $f(x)=g(x)+\pi k$

Example: Solve the following equations.

- $\cos x=\cos\frac{\pi}{3}$

- $\tan x = \tan(2x - 10)$

- (UN 2010 PAKET B)
  $\cos 2x - \sin x = 0$ where $0 \le x \le 2\pi$

- $\sin\left(2x - \frac{\pi}{3}\right) = \cos x$

- (UN 2011 PAKET 12)
  $\cos 2x + \cos x = 0$ where $0^\circ \le x \le 180^\circ$

- $\tan(2x - \pi) = \cot\left(x + \frac{\pi}{2}\right)$

- $\cos\left(3x - \frac{\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right) = 0$ where $x \in [0, 2\pi)$

- $\sin x - 2 \cdot \cos x \cdot \sin x = 0$

- (UN 2012/C37)
  $\cos 2x - 2 \cdot \cos x = -1$ where $0 \le x \le 2\pi$

- (UN 2012/D49)
  $\cos 4x + 3\sin 2x = -1$ where $0^\circ \le x \le 180^\circ$

- $\cos 2x + 2\cdot\cot 2x = 0$

- (UN 2004)
  $2\cos x + 2\sin x = \sqrt{2}$ where $0^\circ \le x \le 360^\circ$

- $\sin 5x + \sin x - 2\sin 3x = 0$ where $x \in [0, 2\pi)$

- $\tan x - \cot x = 2\sqrt{3}$

Active Note Book

Equations of the Form $a\cdot\cos x + b\cdot\sin x = c$

1st Way:

$\frac{1}{a}(a\cdot\cos x + b\cdot\sin x = c)$

$\Leftrightarrow \cos x + \frac{b}{a}\cdot\sin x = \frac{c}{a}$ (Let $\frac{b}{a} = \tan\alpha$)

$\Leftrightarrow \cos x + \tan\alpha\cdot\sin x = \frac{c}{a}$

$\Leftrightarrow \cos x + \frac{\sin\alpha}{\cos\alpha}\cdot\sin x = \frac{c}{a}$

$\Leftrightarrow \frac{\cos\alpha\cdot\cos x + \sin\alpha\cdot\sin x}{\cos\alpha} = \frac{c}{a}$

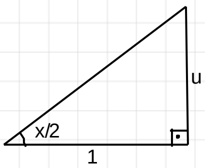
$\Leftrightarrow \frac{\cos(x-\alpha)}{\cos\alpha} = \frac{c}{a}$

$\Leftrightarrow \boxed{\cos(x-\alpha) = \frac{c}{a}\cdot\cos\alpha}$

And above equation can be solved as in previous case.

Example: Solve the equation of $3\sin x + \sqrt{3}\cos x = \sqrt{3}$.

2nd Way:



Let $u = \tan\frac{x}{2}$ then

$\sin x = 2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}$

$= 2\cdot\frac{u}{\sqrt{1+u^2}}\cdot\frac{1}{\sqrt{1+u^2}}$

$= \frac{2u}{1+u^2}$

$\cos x = 1 - 2\cdot\sin^2\frac{x}{2}$

$= 1 - 2\cdot\left(\frac{u}{\sqrt{1+u^2}}\right)^2$

$= \frac{1-u^2}{1+u^2}$

So,

$a\cdot\cos x + b\cdot\sin x = c$

$\Leftrightarrow a\cdot\frac{1-u^2}{1+u^2} + b\cdot\frac{2u}{1+u^2} = c$

$\Leftrightarrow (a+c)u^2 - 2bu - (a-c) = 0$

Example: Solve the following equations.

- $\sqrt{3}\cos x + \sin x = \sqrt{2}$ where $0 \le x \le 2\pi$ (UN 2004)

- $\sin x - 3\cos x = 1$

- $2\cos^2 x + \sqrt{3}\sin 2x = 1 + \sqrt{3}$ where $0 < x < \frac{\pi}{2}$ (UN 2006)

Active Note Book

Note: If $c = 0$, then $a \cdot \cos x + b \cdot \sin x = 0$

So, we can use 1st way or 2nd way or $a \cdot \cos x + b \cdot \sin x = 0$

$$\Leftrightarrow \frac{a \cdot \cos x + b \cdot \sin x = 0}{\cos x} \Leftrightarrow \tan x = -\frac{b}{a}$$

Example: Solve the equation of $\sqrt{3}\cos x - \sin x = 0$.

Equations of the Form $a \cdot \cos^2 x + b \cdot \cos x \cdot \sin x + c \cdot \sin^2 x = 0$

Divide either by $\cos^2 x$ or $\sin^2 x$.

Example: Solve the equation of $\sin^2 x + 2 \cdot \sin x \cdot \cos x - 3\cos^2 x = 0$.

Exercises 7

Solve for $0 \le x \le 2\pi$.

1. $\sin 3x = 0$

2. $\cot\left(\frac{\pi}{3} - 2x\right) = \sqrt{3}$

3. $\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{2}$

4. $-\tan\left(x - \frac{\pi}{5}\right) = 1$

5. $\tan 2x = \tan\frac{5\pi}{6}$

Solve each over real numbers.

1. $\cos\left(x - \frac{\pi}{3}\right) = \cos\left(2x - \frac{\pi}{2}\right)$

2. $\sin\left(\pi - x\right) = \sin(\frac{2\pi}{3} - 3x)$

3. $\sin\left(2x - \frac{\pi}{4}\right) = \cos\left(x + \frac{\pi}{2}\right)$

4. $\tan 2x = \cot(x - \pi)$

Active Note Book

Solve for $0 \le x \le 2\pi$

1. $2\cos x \sin x + \cos x = 0$

2. $2\cos^2 x - \sin x - 1 = 0$

3. $\sin(20 - x) + \cos(2x - 10) = 0$

Review Test

1) What is the principle measure of $\frac{35\pi}{6}$?

A) $\frac{5\pi}{6}$ B) $\frac{11\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$ E) $\frac{\pi}{6}$

2) What is the value of the angle $(-\frac{57\pi}{6})$ in $(0, 2\pi)$?

A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$ C) $\frac{3\pi}{4}$ D) $\frac{3\pi}{2}$ E) $\frac{\pi}{6}$

3) If $\tan\alpha = 1$ then find $\sin\alpha$.

A) 1 B) $\sqrt{2}$ C) $\frac{1}{2}$ D) $\frac{\sqrt{3}}{2}$ E) $\frac{\sqrt{2}}{2}$

4) If $\tan 60^\circ = \sqrt{3}$, find $\cot 30^\circ$.

A) $\sqrt{2}$ B) 1 C) $\sqrt{3}$ D) $\frac{1}{\sqrt{3}}$ E) 2

5) If $\sin x = \frac{7}{25}$ evaluate $\tan x . \cos x$.

A) $\frac{7}{24}$ B) $\frac{49}{576}$ C) $\frac{24}{25}$ D) $\frac{16}{25}$ E) $\frac{7}{25}$

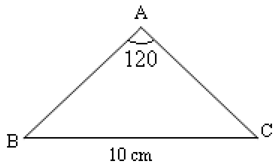
6) Evaluate $\sin 45 \cdot \cos 30 \cdot \sin 60 \cdot \cot 60 \cdot \tan 30 \cdot \tan 60$.

A) $\frac{2\sqrt{3}}{4}$ B) $\frac{4\sqrt{3}}{7}$ C) 16 D) -16 E) $\frac{\sqrt{6}}{8}$

7) Evaluate $\frac{\sin 30^\circ . \cos 60^\circ}{2\tan 45^\circ}$.

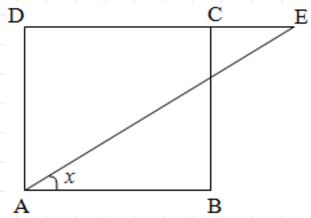
A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{8}$ D) $\frac{1}{16}$ E) $\frac{1}{32}$

8) ABC is an isosceles triangle. If $m(\angle A) = 120^\circ$ and $|BC| = 10$ cm then find A(ABC) .



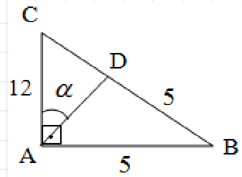
A) $\frac{50}{\sqrt{3}}$ B) $\frac{25}{\sqrt{3}}$ C) $25\sqrt{3}$ D) $50\sqrt{3}$ E) 60

9) In the given figure; ABCD is a square. $|DC| = 3|CE|$. What is $\cot x$?



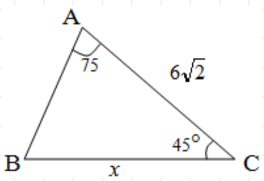
A) $\sqrt{3}$ B) $\frac{4}{3}$ C) 1 D) $\frac{3}{4}$ E) $\frac{2}{3}$

10) In the figure,
$m(B\hat{A}C) = 90^\circ$,
$|AB| = |BD| = 5$ cm,
$|AC| = 12$ cm
Find $\tan\alpha$.



A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ E) $\frac{4}{3}$

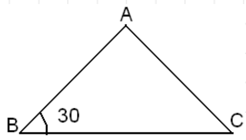
11) Find the value of x according to the figure.



A) 9 B) $2\sqrt{3}+6$ C) $2\sqrt{2}+6$

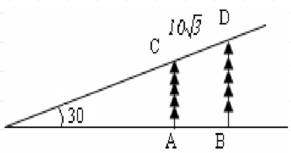
D) $6\sqrt{3}+6$ E) $6\sqrt{2}+6$

12) In a triangle ABC, $m(\angle B)=30^\circ$, $|BC|=9$ cm and $|AB|=8$ cm then find Area of triangle (ABC).



A) 45 B) 36 C) 27 D) 18 E) 9

13) In the given figure If $|CD|=10\sqrt{3}$ cm then Find the distance between tree A and tree B.



A) $10\sqrt{3}$ B) $15\sqrt{3}$ C) 15 D) 30 E) 45

14) Evaluate $\left(10.\sin 30^\circ + 20.\cos 60^\circ\right)-\left(10.\cos 60^\circ + 20.\sin 30^\circ\right)$.

A) 1 B) $\frac{1}{2}$ C) 0 D) -1 E) $\frac{3}{2}$

15) Which of the following is false?

A) $\sin 45^\circ.\cos 45^\circ = 1$

B) $\sin^2 30^\circ + \cos^2 30^\circ = 1$

C) $\cos x.\tan x = \sin x$

D) $(\sin x + \cos x)^2 = 1 + 2\sin x.\cos x$

E) $\tan x.\cot x = 1$

16) Which one of the followings is equal to $\sin(35°)$?

A) $\sin 215°$ B) $\cos 145°$ C) $\sin 65°$

D) $\sin(-35°)$ D) $\cos(-55°)$

17) Simplify $\dfrac{\sin 35°\cdot\cos 35°}{\sin 55°\cdot\cos 55°}$.

A) 0 B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 1 E) -1

18) $\cot x\cdot(\sec x - \dfrac{\cos x}{1+\sin x}) = ?$

A) $\tan x$ B) $\sin x$ C) $\cos x$ D) $\cot x$ E) 1

19) If $\dfrac{4\sin 47^\circ - \cos 43^\circ}{3\cos 43^\circ - 2\sin 47^\circ} - 1 = a$ then find a .

A) -1 B) 0 C) 1 D) 1.5 E) 2

20) Calculate $\dfrac{\cos 42^\circ + \sin 48^\circ}{2\sin 48^\circ}$.

A) $\frac{1}{2}$ B) $\sin 42^\circ$ C) $\sin 48^\circ$ D) 1 E) -1

21) Evaluate $\dfrac{\sin^2 x.\cot^2 x-1}{\sin^2 x}$.

A) 1 B) $\dfrac{1}{2}$ C) 0 D) -1 E) -2

22) Calculate $\dfrac{3\cos 70^\circ.\cos 50^\circ}{\sin 20^\circ.\sin 40^\circ}-4\cos 60^\circ.\sin 30^\circ$.

A) -1 B) 0 C) 1 D) 2 E) 3

23) What is the simplest form of $\dfrac{\sin^4 x-\cos^4 x}{\sin x-\cos x}$?

A) $\sin x$ B) $1+\sin x$ C) $\cos x$ D) $1+\cos x$ E) $\sin x+\cos x$

24) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}-\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}=?$

A) $\sec\theta-1$ B) $2\csc\theta$ C) $2\tan\theta$ D) $\sec\theta\cdot\cot\theta$ E) $\sec\theta+1$

25) If $\sin\alpha+\cos\alpha=\dfrac{1}{3}$, then $\sin\alpha\cdot\cos\alpha=?$

A) $\dfrac{11}{18}$ B) $-\dfrac{4}{9}$ C) $-\dfrac{3}{4}$ D) $\dfrac{5}{2}$ E) $\dfrac{5}{8}$

26) What is the greatest value of $\dfrac{\sin x-1}{2}$?

A) -2 B) -1 C) 0 D) 1 E) 2

27) What is the greatest value of $\dfrac{\cos x-2}{2}$?

A) $\dfrac{11}{18}$ B) $-\dfrac{4}{9}$ C) $-\dfrac{1}{2}$ D) $\dfrac{5}{2}$ E) $\dfrac{5}{8}$

28) Find the order of the signs of the given functions.

$\sin 179^o$, $\cos 269^o$, $\tan 271^o$, $\cot 359^o$

A) + - -- B) + - - + C) - - - - D) + - + - E) - + - +

29) What is the sign of the angles

$\sin\dfrac{2\pi}{5},\cos\dfrac{\pi}{7},\tan\dfrac{3\pi}{5},\cot\dfrac{18\pi}{5}$ respectively?

A) + - + - B) + + - + C) + + - - D) - + + - E) - + + -

30) Calculate $\tan 3630^o$.

A) 1 B) $\sqrt{3}$ C) -$\sqrt{3}$ D) -1 E) $\dfrac{1}{\sqrt{3}}$

Active Note Book

31) $\sin(-\frac{\pi}{2}).\tan(225°)+\cos(-\frac{\pi}{3}).\cot(315°)=?$

A) 2 B) 1 C) −3/2 D) −1 E) −2

32) $\cos(\frac{\pi}{5})+\cos(\frac{2\pi}{5})+\cos(\frac{3\pi}{5})+\cos(\frac{4\pi}{5})=?$

A) −1 B) −1/2 C) 0 D) 2 E) 3

33) $\frac{\sin(410°)\cdot\tan(-15°)\cdot\sin(210°)}{\cot(105°)\cdot\cos(220°)}=?$

A) 1 B) 2 C) $\frac{1}{2}$ D) $-\frac{1}{2}$ E) $\frac{\sqrt{3}}{2}$

34) $\frac{\sin(\frac{17\pi}{2}+\alpha)\cdot\cos(\alpha-5\pi)}{\sin(-\frac{5\pi}{2}+\alpha)}=?$

A) $\cos\alpha$ B) $-\cos\alpha$ C) $\sin\alpha$ D) $-\sin\alpha$ E) 1

35) $\frac{\sin(-33\pi-x)\cdot\cos(-\frac{13\pi}{2}+x)}{\sin(\frac{39\pi}{2}+x)\cdot\cos(45\pi+x)}=?$

A) $\sin^2 x$ B) $\sin x\cdot\tan x$ C) $\cot^2 x$ D) $\tan^2 x$ E) $-\tan^2 x$

36) In $\triangle ABC$, if $a=\sqrt{5}$, $b=\sqrt{2}+1$ and $c=\sqrt{2}-1$, what is $m(\angle A)$?

A) 15^0 B) 30^0 C) 45^0 D) 60^0 E) 75^0

37) $\frac{\tan\frac{7\pi}{4}+\sin\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)}{\cot\left(-\pi+\frac{\pi}{4}\right)+\cos\left(\frac{\pi}{3}\right)}=?$

A) $\frac{-2-\sqrt{3}}{3}$ B) $\frac{-1-\sqrt{3}}{2}$ C) $\frac{1+\sqrt{3}}{3}$ D) −1 E) 1

38) Which one of the following is the simplest form of

$$\frac{\sin(-\alpha)\cdot\cos(\pi+\alpha)}{2\cdot\sin(\pi-\alpha)}+\frac{\sin(\frac{3\pi}{2}-\alpha)\cdot\cos(\frac{\pi}{2}-\alpha)}{2\cdot\cos(\frac{\pi}{2}+\alpha)}?$$

A) $\sin(2\alpha)$ B) $\cos(2\alpha)$ C) $\sin(\alpha)$ D) $\tan(\alpha)$ E) $\cos(\alpha)$

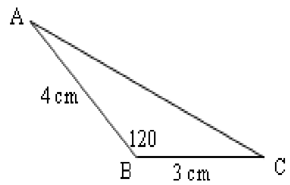
39) In the given triangle

$m(ABC)=120°$,

$|BC|=3$ cm

$|AB|=4$ cm

find $|AC|$.



A) $\sqrt{5}$ B) 5 C) $\sqrt{37}$ D) $\sqrt{47}$ E) $\sqrt{52}$

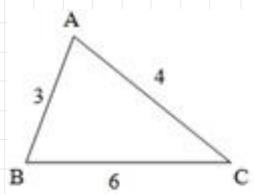
40) In the figure,

$|AB|=3\ cm$

$|BC|=6\ cm$

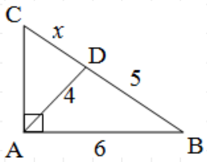
$|AC|=4\ cm$

What is $\cos A$?



A) $-\frac{11}{24}$ B) $\frac{7}{6}$ C) $\frac{15}{26}$ D) $\frac{21}{32}$ E) $\frac{43}{48}$

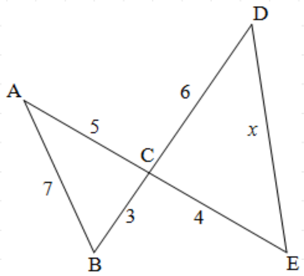
41) In the given figure.
$|AD| = 4$ cm
$|AB| = 6$ cm
$|BD| = 5$ cm
$[AB] \perp [AC]$
Find $|CD| = x$.



A) 2 B) 3 C) 4 D) 5 E) 6

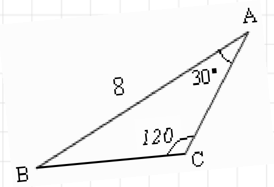
42) In the given figure;
$|AB| = 7$ $|BC| = 3$
$|CD| = 6$ $|CE| = 4$
$|AC| = 5$ $|DE| = x$

Find the value of x.



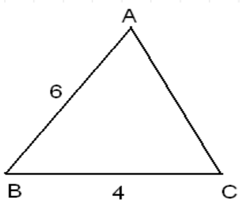
A) $2\sqrt{19}$ B) $\sqrt{74}$ C) $\sqrt{37}$ D) 4 E) 3

43) In the given figure,
$m(\angle BCA) = 120^0$,
$m(\angle BAC) = 30^0$,
$|AB| = 8$ cm then
find $|AC|$.



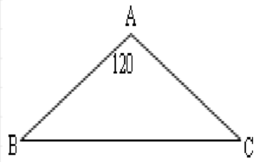
A) $\frac{4}{\sqrt{3}}$ B) $4\sqrt{3}$ C) $\frac{3}{\sqrt{3}}$ D) $8\sqrt{3}$ E) $\frac{8}{\sqrt{3}}$

44) In a triangle ABC, If
$|AB| = 6$ cm, $|BC| = 4$
cm, $A(ABC) = 6\sqrt{3}$
cm² then what is the
value of angle B?



A) 30 B) 45 C) 60 D) 75 E) 90

45) ABC is an isosceles
triangle.
$m(\angle A) = 120°$
$|AB| = |AC| = 5$ cm
Find $A(\triangle ABC)$.



A) 12 B) $9\sqrt{3}$ C) 15 D) $\frac{25\sqrt{3}}{4}$ E) $12\sqrt{3}$

46) In a triangle ABC, $|AC| = 20\sqrt{2}$ cm, $|BC| = 6$ cm and
$m(\angle C) = 45^o$ find A(ABC).

A) 45 B) 60 C) 75 D) 85 E) 90

47) Find the area of triangle ΔABC if a = 2, *b* = 3, *c* = 4.

A) $\frac{7}{8}$ B) $\frac{4}{3}$ C) $\frac{3}{4}\sqrt{15}$ D) $\frac{3}{4}\sqrt{5}$ E) 12

48) Triangle ABC is given with a = 13 cm, b = 14 cm, dan c = 15 cm. Find the height of AC side.

A) 6 B) 8 C) 10 D) 11 E) 12

49) $\frac{1}{\sin 75°} - \frac{1}{\cos 75°} = ?$

A) $\sqrt{2}$ B) $2\sqrt{2}$ C) $-2\sqrt{2}$
D) $2\sqrt{6}$ E) $-2\sqrt{6}$

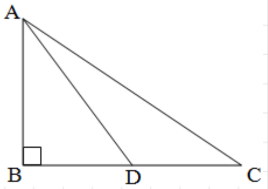
50) $\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = ?$

A) tanx B) cot2x C) 2cos2x
D) 2cot2x E) 2sin2x

51) If $\cos 22^0 = k$ then $\sin 56^0 + \sin 146^0 = ?$

A) $2\sqrt{k}$ B) $\sqrt{2+k}$ C) $\sqrt{k}$ D) $\sqrt{k+1}$ E) $1+\sqrt{k}$

52) Given the diagram;
$[AD]$ is the median
$m(\angle C) = 60°$ then
what is $\tan(\angle DAC)$?



A) $\frac{\sqrt{3}}{7}$ B) $\frac{1}{3}$ C) $\frac{\sqrt{3}}{3}$ D) $\frac{\sqrt{3}}{2}$ E) $\sqrt{3}$

53) $\sin^2 20 - \cos^2 20 + 2\sin 25 \cos 25 = ?$

A) -1 B) $-\frac{\sqrt{3}}{2}$ C) 0 D) $\frac{\sqrt{3}-\sqrt{2}}{6}$ E) 1

54) Simplify $\frac{1+\cos 2x}{\sin 2x}$.

A) $\tan x$ B) $\cot x$ C) $-\tan x$ D) 1 E) $\sin 2x$

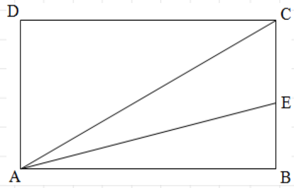
55) If $\tan 65° = a$, then find $\tan 40°$ in terms of a.

A) $\frac{1-a^2}{a}$ B) $\frac{1-a^2}{2a}$ C) $\frac{a^2-1}{2a}$ D) $\frac{a^2+1}{a}$ E) $\frac{a^2+1}{2a}$

56) $2\cos(140) \cdot \sin(-40) = ?$

A) $-\sin 10°$ B) $-\cos 10°$ C) $\cos 10°$ D) $\sin 40°$ E) $\cos 20°$

57) Given that;
ABCD is a rectangle.
$|AB| = 2.|BC|$
$|BE| = |EC|$
What is $\tan(\angle CAE)$?



A) $\frac{2}{9}$ B) $\frac{1}{2}$ C) $\frac{1}{4}$ D) $\frac{3}{5}$ E) $\frac{2}{3}$

58) If $\cos^2 \frac{\pi}{8} = \frac{a+1}{4}$, what is a ?

A) $\frac{1}{\sqrt{2}}$ B) 1 C) $\sqrt{2}+1$ D) 2 E) $2+\sqrt{2}$

59) If $10x = \pi$, then $\frac{\cos 7x + \cos 5x}{\cos 5x + \cos 3x} = ?$

A) 1 B) -1 C) 2 D) -2 E) $\frac{1}{2}$

60) In a right triangle $\tan x = \frac{4}{3}$ then find $\cot \frac{x}{2}$.

A) $\frac{1}{3}$ B) $\frac{3}{5}$ C) $\frac{4}{3}$ D) 2 E) $\frac{5}{2}$

61) $\tan\frac{A}{2}+\cot\frac{A}{2}=?$

A) 2sinA B) 2cosA C) 2secA D) 2cosecA E) 1

62) Evaluate $\left(\sin 25^\circ+\cos 25^\circ\right)^2+\left(\sin 25^\circ-\cos 25^\circ\right)^2$.

A) 1 B) 2 C) $2\cos 50^\circ$ D) $\sin 50^\circ$ E) $\sin^2 25^\circ$

63) If $\tan 2x=0,75$ then find $\tan x$.

A) 1 B) $\frac{3}{5}$ C) $\frac{1}{3}$ D) $\frac{\sqrt{10}}{3}$ E) $\frac{1}{\sqrt{3}}$

64) Calculate $\sin 45^\circ.\cos 45^\circ-\tan 45^\circ.\cot 45^\circ$.

A) -2 B) -1 C) 1 D) $-\frac{1}{2}$ E) 2

65) If $\cos x+\sin x=\frac{1}{4}$, what is $\cos 4x$?

A) $-\frac{97}{128}$ B) $-\frac{3}{16}$ C) $\frac{4}{25}$ D) $\frac{1}{2}$ E) 1

66) $\dfrac{\sin 10^\circ+\sqrt{3}\cdot\cos 10^\circ}{\cos 20^\circ}=?$

A) 1 B) 2 C) 3 D) 4 E) 5

67) If $\dfrac{\cos 2x-3\sin x+1}{2+\sin x}=-\frac{1}{5}$, then find $\cot 2x$.

A) $\frac{24}{7}$ B) $\frac{7}{24}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$ E) 1

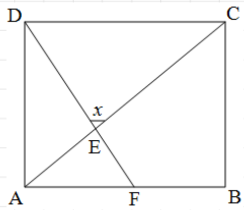
68) If $\cos 6^\circ=a$ then $\sin 78^\circ=?$

A) $2a+1$ B) $2a^2-1$ C) $2a$ D) $\frac{a}{2}$ E) $\frac{a+1}{2}$

69) If $\sin x=\frac{2}{3}$ then $\sin(45^\circ+x)\times\sin(45^\circ-x)=?$

A) $\frac{-5}{9}$ B) $\frac{-4}{9}$ C) $\frac{1}{18}$ D) $\frac{4}{9}$ E) $\frac{5}{9}$

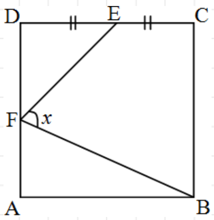
70) Given that;
ABCD is a square. If $|CE|=4|AE|$ then what is $\cot x$?



A) $-\frac{5}{3}$ B) $\frac{3}{5}$ C) 1 D) $\frac{5}{3}$ E) $\sqrt{3}$

Active Note Book

71) ABCD is a square.
If $|DE| = |EC|$ and
$\dfrac{|AF|}{|FD|} = 5$
what is $\tan x$?



A) $\dfrac{21}{13}$ B) $\dfrac{-1}{\sqrt{2}}$ C) $\dfrac{1}{\sqrt{3}}$ D) $\dfrac{1}{2}$ E) $\dfrac{3}{5}$

72) Find the period of the function $f(x) = 2\cos^2 6x$.

A) $\dfrac{\pi}{6}$ B) $\dfrac{\pi}{3}$ C) $\dfrac{\pi}{4}$ D) π E) $\dfrac{\pi}{2}$

73) Find the fundamental period of $f(x) = 2\,\mathrm{Cot}^6\,3x$.

A) $\dfrac{\pi}{3}$ B) $\dfrac{\pi}{2}$ C) $\dfrac{2\pi}{3}$ D) π E) 2π

74) What is the fundamental period of the function
$f(x) = 3 + 2\sin^3(4x-3)$?

A) $\dfrac{\pi}{4}$ B) $\dfrac{\pi}{3}$ C) $\dfrac{\pi}{2}$ D) π E) 2π

75) What is the fundamental period of
$y = 3\mathrm{Sin}^3\,2x + \cos^2(3x-1)$?

A) $\dfrac{\pi}{2}$ B) $\dfrac{\pi}{3}$ C) $\dfrac{\pi}{6}$ D) π E) 2π

76) What is the fundemental period of the function
$y = \sin^4(4x-5) - \cos^3(2x+3) + \cot^6(3x+3)$?

A) 45 B) 60 C) 90 D) 180 E) 360

77) $\arcsin\left(-\dfrac{1}{2}\right)$ =?

A) –60 B) 120 C) –30 D) 150 E) 180

78) $\dfrac{12}{\pi}\left[\arccos\left(-\dfrac{\sqrt{3}}{2}\right) + \arcsin\left(-\dfrac{\sqrt{3}}{2}\right) + \arctan\left(-\sqrt{3}\right)\right]$ =?

A) 1 B) 2 C) 4 D) 3 E) 5

79) $\cos\left(2\arcsin\dfrac{5}{13}\right) = ?$

A) 1 B) $\dfrac{12}{13}$ C) $\dfrac{10}{13}$ D) $\dfrac{69}{65}$ E) $\dfrac{119}{169}$

80) $\arccos\left(\sin\dfrac{27\pi}{7}\right) = ?$

A) $\dfrac{8\pi}{7}$ B) $\dfrac{9\pi}{14}$ C) $\dfrac{6\pi}{7}$ D) $-\dfrac{47\pi}{14}$ E) $\dfrac{4\pi}{7}$

81) $\sin(2\text{arccot}\frac{2}{3}) = ?$

A) $\frac{12}{13}$ B) $\frac{4}{13}$ C) $\frac{6}{13}$ D) $\frac{12}{\sqrt{13}}$ E) $\frac{9}{\sqrt{13}}$

82) $\cos(\arcsin(-\frac{\sqrt{3}}{2}))=?$

A) $\frac{\sqrt{3}}{2}$ B) $-\frac{\pi}{6}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) $-\frac{\sqrt{3}}{2}$

83) $\sin(\arctan\frac{3}{4}+\text{arccot}\frac{4}{3})=?$

A) $\frac{3}{5}$ B) $\frac{6}{25}$ C) $\frac{9}{25}$ D) $\frac{16}{25}$ E) $\frac{24}{25}$

84) $\sin\left[\arcsin\frac{12}{13}+\arccos\left(\frac{4}{5}\right)\right]=?$

A) 1 B) $\frac{63}{65}$ C) $\frac{9}{13}$ D) $\frac{5}{13}$ E) $\frac{-33}{65}$

85) Find the smallest acute angle *x* that satisfies the equality $\sin(5x)=\cos(35^\circ)$.

A) 7° B) 11° C) 18° D) 24° E) 35°

86) The solution set of the equation $2+\cos 2x = 3\cos x$ in the interval $[0,2\pi]$ is ...

A) $\left\{0,\frac{\pi}{3},\frac{5\pi}{3},2\pi\right\}$ B) $\left\{\frac{\pi}{6},\frac{5\pi}{6},\frac{5\pi}{3}\right\}$ C) $\left\{0,\frac{2\pi}{3},\frac{4\pi}{3},2\pi\right\}$ D) $\left\{\frac{\pi}{2},\frac{5\pi}{4},\frac{7\pi}{6}\right\}$ E) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$

87) If $0 < x < 90°$, then the number of the roots of the equation $\sin 6x - \cos 3x = 0$ is ...

A) 1 B) 2 C) 3 D) 4 E) 5

88) Which one of the following is the number of the roots of the equation $\cos 2x + \sin x = 0$ in the interval $[0,3\pi]$?

A) 1 B) 2 C) 3 D) 4 E) 5

89) Find the solution set of the equation $3\tan(3x)=3$ in the interval of $[0,\pi]$.

A) $\{60^\circ,90^\circ,120^\circ,150^\circ,180^\circ\}$
B) $\{30^\circ,150^\circ\}$
C) $\{45^\circ,105^\circ,165^\circ\}$
D) $\{15^\circ,75^\circ,135^\circ\}$
E) $\{0^\circ,180^\circ\}$

90) Get the solution set of the equation $2\cos^2 x-5\cos x=-3$ in the interval $[0,2\pi]$.

A) $\{\pi\}$ B) $\{0,2\pi\}$ C) $\{0,\pi,2\pi\}$ D) $\left\{\frac{\pi}{2},\pi,\frac{3\pi}{2}\right\}$ E) $\left\{\frac{\pi}{3},\frac{2\pi}{3},\pi\right\}$

Active Note Book

91) Get the solution set of the equation cos(30+2x)+1=0.

A) $x=\frac{\pi}{2}-\frac{\pi}{12}+\pi k$ B) $x=-\frac{\pi}{12}+\pi k$ C) $x=75+\pi k$

D) $x=-\frac{\pi}{12}+2\pi k$ E) $x=-\frac{\pi}{6}+\pi k$

92) Solve $\frac{\sin 2x}{2}-\cos^2 x=0$

A) $\frac{\pi}{4}+2k\pi;\frac{\pi}{2}+k\pi$ B) $\frac{\pi}{6}+k\pi;\frac{\pi}{4}+k\pi$ C) $\frac{\pi}{2}+2k\pi$

D) $\frac{\pi}{3}+k\pi;\frac{\pi}{2}+k\pi$ E) $\frac{\pi}{2}+k\pi;\frac{\pi}{4}+\frac{k\pi}{2}$

93) Get the solution set of the equation (Cosx+2)(tg2x+1)=0 in $[0,2\pi]$

A) $\left\{\frac{\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}\right\}$

B) $\left\{\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

C) $\left\{\frac{5\pi}{6},\frac{11\pi}{6}\right\}$

D) $\left\{\frac{3\pi}{8},\frac{11\pi}{8},\frac{15\pi}{8}\right\}$

E) $\left\{\frac{7\pi}{8},\frac{15\pi}{8}\right\}$

94) Solve (tanx – 1).(2.sin²x + 3) = 0

A) $\frac{\pi}{4}+k\pi$ B) $\frac{\pi}{4}+2k\pi$ C) $\frac{\pi}{6}+2k\pi$

D) $\pm\frac{\pi}{3}+k\pi$ E) $\pm\frac{\pi}{6}+k\pi$

95) If $\sin\left(\frac{x}{2}-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$, then find $x\in\left[0^\circ,360^\circ\right]$.

A) $\{0^\circ,180^\circ,270^\circ,360^\circ\}$ B) $\{0^\circ,120^\circ,210^\circ\}$

C) $\{180^\circ,360^\circ\}$ D) $\{135^\circ,270^\circ\}$

E) $\{135^\circ,270^\circ,315^\circ\}$

96) Solve $2\cos\left(x-\frac{\pi}{6}\right)=\sqrt{3}$ for x.

A) $\pm\frac{\pi}{6}+\frac{\pi}{6}+\pi k$ B) $\pm\frac{\pi}{3}+2\pi k$

C) $\pm\frac{5\pi}{6}+\frac{\pi}{6}+2\pi k$ D) $\pm\frac{\pi}{6}+\frac{\pi}{6}+2\pi k$

E) $\pm\frac{\pi}{3}+\frac{\pi}{6}+\pi k$

97) Solve $2\sin^2x+2\sin x=\sqrt{3}+\sqrt{3}\sin x$ where $0\le x\le 2\pi$.

A) $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{3\pi}{2}\right\}$

B) $\left\{\frac{\pi}{3},\frac{3\pi}{2},\frac{5\pi}{3}\right\}$

C) $\left\{\frac{\pi}{3},\frac{2\pi}{3},\frac{7\pi}{3}\right\}$

D) $\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\}$

E) No Solution

98) Solve $\tan\left(\frac{x}{2}-\pi\right)=1$

A) $\frac{\pi}{2}+k\pi$ B) $\frac{\pi}{2}+2k\pi$ C) $\frac{5\pi}{2}+2k\pi$

D) $\frac{\pi}{4}+k\frac{\pi}{2}$ E) $4\pi+\frac{\pi}{2}k$

99) Find the solution set of $\cos 2x+3\sin x=2$ for $0^\circ\le x\le 360^\circ$.

A) $\{30,90\}$ B) $\{30,150\}$ C) $\{0,30,90\}$

D) $\{30,90,150\}$ E) $\{30,90,150,180\}$

100) Find the solution set of $\cos 2x+7\sin x+3=0$ for $0^\circ<x<360^\circ$.

A) $\{0,90\}$ B) $\{90,270\}$ C) $\{30,130\}$

D) $\{210,330\}$ E) $\{180,360\}$