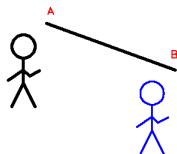


1. TRANSLATION

Movement of each point on a plane in a certain distance and direction (which is represented by a vector).

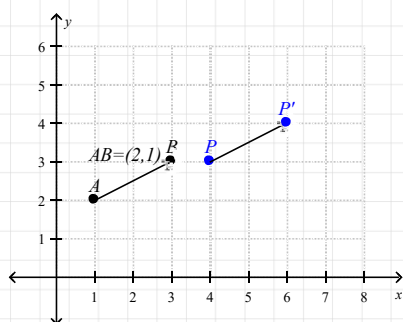
Following image is translated under the vector \overrightarrow{AB} .



Let $A(1,2)$ and $B(3,3)$ be two points.

Let's observe the translation of point $P(4,3)$ under the vector

$$\overrightarrow{AB} = (2, 1) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$



$P'(6,4)$ is the image of $P(4,3)$ under translation represented by

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ vector.}$$

In Matrix notation, translation can be formulized as following:

Point + Translation Vector = Image

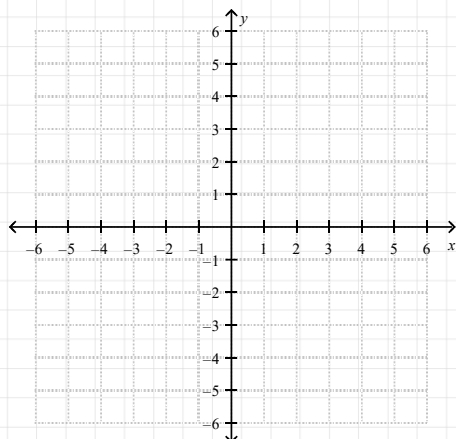
That is,

$$\begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

Example: Find the image of points $A(-4,3)$ $B(3,2)$ $C(0,5)$

under the translation $T = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and show them on the

following coordinate plane.



Example: Find the image of $y = 2x + 1$ under translation

$$T = \begin{pmatrix} 2 \\ -3 \end{pmatrix}.$$

Solution 1:

$$\begin{pmatrix} x \\ 2x+1 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} x+2 \\ 2x-2 \end{pmatrix}$$

Let $x+2 = u$ then $2x-2 = 2u-6$

That is, $\begin{pmatrix} x+2 \\ 2x-2 \end{pmatrix} = \begin{pmatrix} u \\ 2u-6 \end{pmatrix}$, here convert u into x , so $\begin{pmatrix} x \\ 2x-6 \end{pmatrix}$

That is, $y = 2x - 6$

Solution 2:

If $y = 2x + 1$ is translated under $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$, then the image is

$$y - (-3) = 2(x - 2) + 1$$

$$\text{So, } y = 2x - 6$$

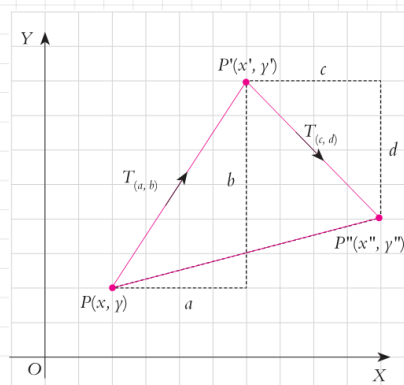
Example: Find the image of $y = x^2 - 2x + 1$ under translation

$$T = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

Composition of Translation

Let $P(x, y)$ is translated to $P'(x', y')$ under $T_1 = \begin{pmatrix} a \\ b \end{pmatrix}$,

then $P'(x', y')$ is translated to $P''(x'', y'')$ under $T_2 = \begin{pmatrix} c \\ d \end{pmatrix}$.



That is, $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} x'' \\ y'' \end{pmatrix}$ or

$$T_1 \circ T_2 = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a+c \\ b+d \end{pmatrix} = \begin{pmatrix} x'' \\ y'' \end{pmatrix}$$

Example: Find the image of $M(4, -6)$ when translated through

$$T_1 = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \text{ and then } T_2 = \begin{pmatrix} 4 \\ -1 \end{pmatrix}.$$

Example: If point $M(4, -6)$ is translated through $T_1 = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ and then through T_2 , then image is $M''(-5, 11)$. Find T_2 .

Example: (UAN 2003)

The $2x + 3y = 6$ line is translated with $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ matrix and then

translated again with $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ matrix. The image is ...

Assignment:

1. Find the image of $A(-4, 1)$, $B(-5, 6)$ and $C(-1, 7)$ under translation $T = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$.

2. The image of $(4, 9)$ under translation T is $(-1, 12)$. Find translation T .

3. Given the points $A(1, 4)$, $B(-3, 2)$ and $C(1, 4)$. Find
a. the image of A under translation of vector BC .

b. the image of C under translation of vector BA .

4. Find the image of $3x - 4y + 2 = 0$ under translation $T = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

5. Find the image of parabola $y^2 + 2x + 3 = 0$ under translation $T = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$.

6. Find the image of circle $x^2 + y^2 - 3x + 4y + 9 = 0$ under translation $T = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

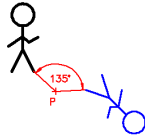
7. What is the image of $A(-1, 4)$ when translated through $T_1 = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$ and then $T_2 = \begin{pmatrix} -2 \\ 9 \end{pmatrix}$.

8. What is the image of $B(2, -1)$ when translated through $T_1 \circ T_2 \circ T_3$ if $T_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, $T_2 = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$, and $T_3 = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$.

2. ROTATION

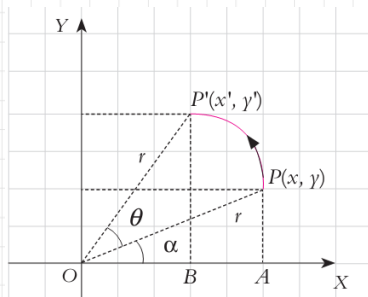
Revolving around a fixed point. Rotation is determined by three important items:

1. $P \rightarrow$ center of the rotation
2. clockwise $(-)$ \rightarrow direction of the rotation
3. $135^\circ \rightarrow$ measure of the rotation



1. Rotation of θ (degree or radian) around the origin $O(0,0)$

If $P(x, y)$ is rotated by θ around the origin $O(0,0)$ in a counterclockwise $(+)$ direction, then the image is $P'(x', y')$.



$$x = R \cdot \cos \alpha \quad y = R \cdot \sin \alpha$$

$$\begin{aligned} x' &= R \cdot \cos(\alpha + \theta) = R \cdot (\cos \alpha \cdot \cos \theta - \sin \alpha \cdot \sin \theta) \\ &= \underbrace{x \cdot \cos \theta}_x - \underbrace{y \cdot \sin \theta}_y \end{aligned}$$

$$\begin{aligned} y' &= R \cdot \sin(\alpha + \theta) = R \cdot (\sin \alpha \cdot \cos \theta + \cos \alpha \cdot \sin \theta) \\ &= \underbrace{y \cdot \cos \theta}_y + \underbrace{x \cdot \sin \theta}_x \\ &= x \cdot \sin \theta + y \cdot \cos \theta \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Example: Find the image of $P(2,5)$ under the rotation about origin of

- $\frac{\pi}{2}$

- $-\frac{\pi}{3}$

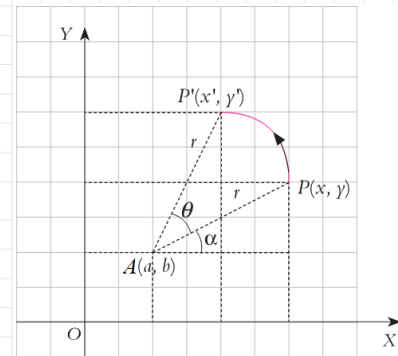
Example: Find the rotation of $y = 3x - 5$ under the rotation about origin of $-\pi$.

Example: (UN 2008 PAKET A/B)

Persamaan bayangan garis $y = 5x - 3$ karena rotasi dengan pusat $O(0,0)$ bersudut -90° adalah ...

The rotation of $y = 5x - 3$ under the rotation about origin of -90° is ...

2. Rotation of θ (degree or radian) around the center $A(a, b)$



$$x - a = R \cdot \cos \alpha \quad y - b = R \cdot \sin \alpha$$

$$\begin{aligned} x' - a &= R \cdot \cos(\alpha + \theta) = R \cdot (\cos \alpha \cdot \cos \theta - \sin \alpha \cdot \sin \theta) \\ &= \underbrace{(x - a) \cdot \cos \theta}_x - \underbrace{(y - b) \cdot \sin \theta}_y \end{aligned}$$

$$\begin{aligned} y' - b &= R \cdot \sin(\alpha + \theta) = R \cdot (\sin \alpha \cdot \cos \theta + \cos \alpha \cdot \sin \theta) \\ &= \underbrace{(y - b) \cdot \cos \theta}_y + \underbrace{(x - a) \cdot \sin \theta}_x \\ &= (x - a) \cdot \sin \theta + (y - b) \cdot \cos \theta \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} x - a \\ y - b \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$

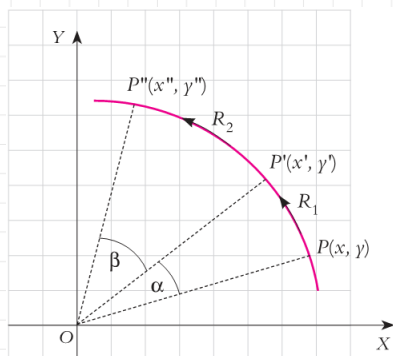
Example: Find the image of $P(2,5)$ under the rotation around the point $A(1,2)$ of

- $-\frac{\pi}{2}$

- $\frac{\pi}{3}$

Composition of Rotation

Let $P(x, y)$ is rotated about O through α to $P'(x', y')$,
then $P'(x', y')$ is rotated about O through β to $P''(x'', y'')$.



As observed, $P''(x'', y'')$ is the rotation of $P(x, y)$ about O through $\alpha + \beta$. So,

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} \cos(\alpha + \beta) & -\sin(\alpha + \beta) \\ \sin(\alpha + \beta) & \cos(\alpha + \beta) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Example: Find the image of $P(4, 2)$ rotated through 15° , then rotated through 75° about origin.

Assignment:

1. Find the images of $A(-3, 2)$, $B(5, -4)$ and $C(2, -6)$ under following rotations

Rotation Center & Rotation Angle	$A(-3, 2)$	$B(5, -4)$	$C(2, -6)$
$O(0, 0)$ & $\frac{\pi}{2}$			
$O(0, 0)$ & $\frac{\pi}{3}$			
$O(0, 0)$ & $-\frac{\pi}{4}$			
$Q(2, -1)$ & $-\frac{\pi}{2}$			
$Q(-3, 4)$ & π			

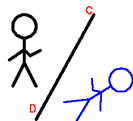
2. Find the image of $x - 2y - 1 = 0$ under the rotation of $\frac{\pi}{2}$ around the origin.

3. Find the image of $3x + y + 1 = 0$ under the rotation of $-\frac{\pi}{2}$ around $Q(1, 3)$.

4. Find the image of $P(-2, 5)$ rotated through 75° , then through 45° about origin.

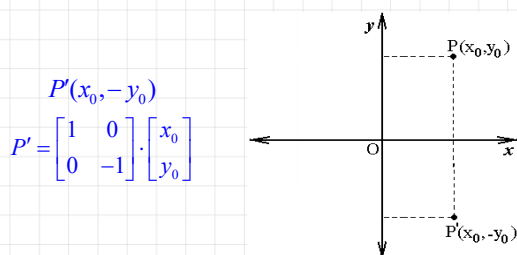
3. REFLECTION

Mirroring with respect to axis of reflection (or axis of symmetry)

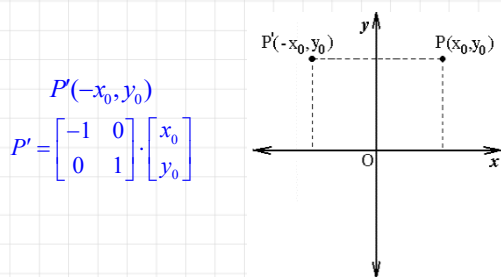


1. Reflection of a Point with respect to the Coordinate axes

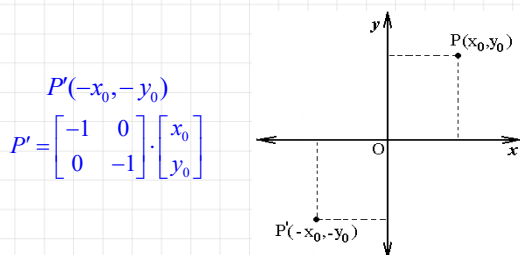
The reflection of $P(x_0, y_0)$ wrt x-axis is



The reflection of $P(x_0, y_0)$ wrt y-axis is



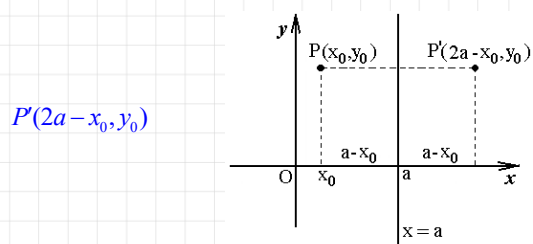
The reflection of $P(x_0, y_0)$ wrt origin is



Example: Find the reflection of $B(-4, -5)$, $D(-4, 5)$ & $E(a - b, -b)$ with respect to the x-axis, y-axis, and origin.

2. Reflection of a Point with respect to Lines Parallel to axes

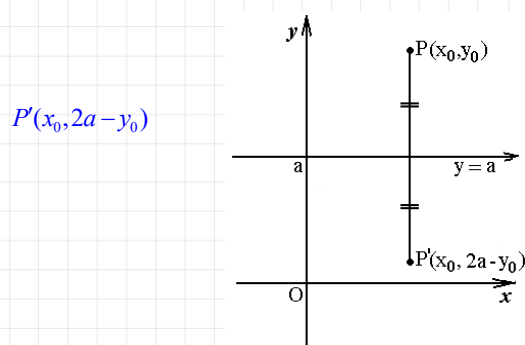
The reflection of $P(x_0, y_0)$ wrt $x = a$ is



$$P' = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} a \\ 0 \end{bmatrix} \right) + \begin{bmatrix} a \\ 0 \end{bmatrix} \Rightarrow P' = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 2a \\ 0 \end{bmatrix}$$

Example: Find the reflection of $M(-12, 7)$ with respect to the line $x = -3$.

The reflection of $P(x_0, y_0)$ wrt $y = a$ is

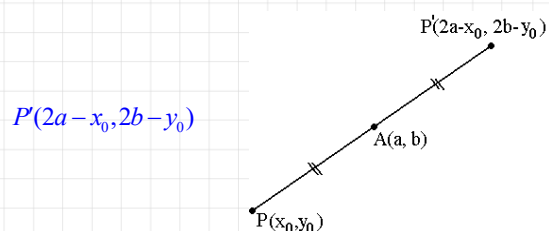


$$P' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \left(\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} 0 \\ a \end{bmatrix} \right) + \begin{bmatrix} 0 \\ a \end{bmatrix} \Rightarrow P' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2a \end{bmatrix}$$

Example: Find the reflection of $M(10, -8)$ with respect to the line $y = -10$

3. Reflection of a Point with respect to Another Point

The reflection of $P(x_0, y_0)$ wrt $A(a, b)$ is



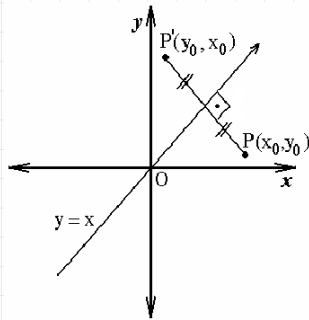
$$P' = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \left(\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} a \\ b \end{bmatrix} \right) + \begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow P' = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$

Example: Find the reflection of $A(5, -3)$ with respect to $(-4, 2)$

4. Reflection of a Point wrt the Lines $y = x$ and $y = -x$

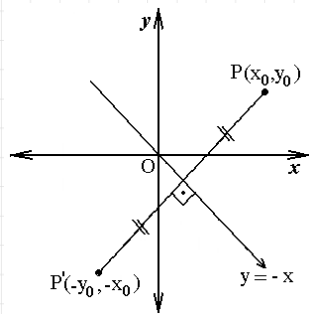
The reflection of $P(x_0, y_0)$ wrt $y = x$ is

$$P' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$



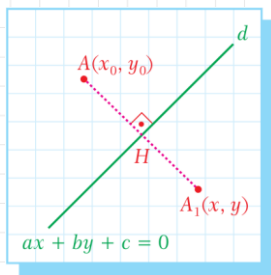
The reflection of $P(x_0, y_0)$ wrt $y = -x$ is

$$P' = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$



Example: Find the reflection of $L(-7, 4)$ with respect to the line $y = x$ and $y = -x$.

5. Reflection of a Point with respect to a Line



Find the equation of line AA_1

Find intersection point H

Since H will be midpoint of AA_1 ,

We can find A_1

Example: Find the reflection of $(-2, 4)$ wrt $x - y - 6 = 0$.

Example: Find the reflection of $y = 2x - 1$ wrt $y = x$ axis.

Hint: Choose two points of line $y = 2x - 1$

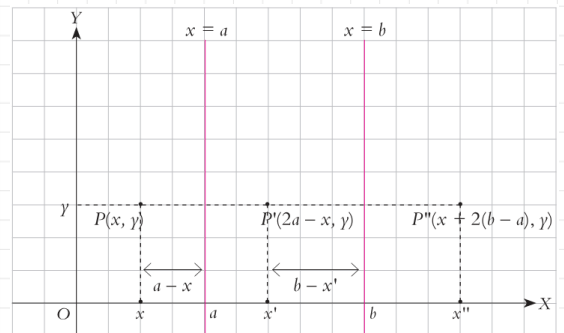
Reflect these points.

Find the line passing through these image points

Composition of Reflection

1. Reflection of a Point with respect to Parallel Lines

$P(x, y)$ is reflected wrt $x = a$ and then P' is reflected wrt $x = b$.

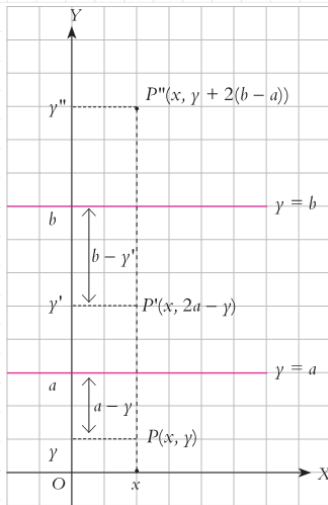


$$P'' = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2a \\ 0 \end{bmatrix} \right) + \begin{bmatrix} 2b \\ 0 \end{bmatrix} \Rightarrow$$

$$P'' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2(b - a) \\ 0 \end{bmatrix}$$

Example: Find the image of $A(-2, 4)$ when reflected wrt the line $x = 2$ and then reflected wrt the line $x = -5$

$P(x, y)$ is reflected wrt $y = a$, then P' is reflected wrt $y = b$.



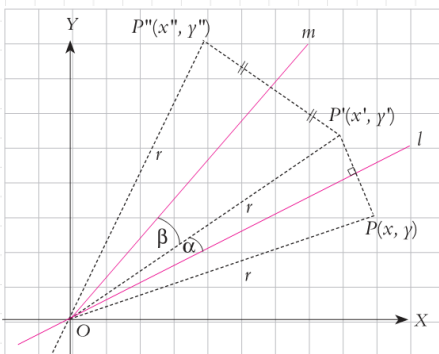
$$P'' = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2a \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 2b \end{bmatrix} \Rightarrow$$

$$P' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2(b-a) \end{bmatrix}$$

Example: Find the image of $B(3, -5)$ when reflected wrt the line $y = -3$ and then reflected wrt the line $y = 7$.

2. Reflection of a Point with respect to Intersecting Lines

$P(x, y)$ is reflected wrt l line and then reflected wrt m line.



As observed, $m(\angle POP'') = 2(\alpha + \beta)$. So, we can find the image by rotation of $2(\alpha + \beta)$ around O , where O is the intersection point.

Example: (UN 2011 PAKET 12)

Persamaan bayangan garis $y = 2x - 3$ karena refleksi terhadap garis $y = -x$, dilanjutkan refleksi terhadap $y = x$ adalah ...

The image of $y = 2x - 3$ line when it is reflected with respect to $y = -x$ line and then reflected with respect to $y = x$ line is ...

Assignment:

- Find the image of $A(-3, 2)$, $B(5, -4)$ and $C(2, -6)$ with respect to following reflection axes.

with respect to	$A(-3, 2)$	$B(5, -4)$	$C(2, -6)$
x -axis			
y -axis			
origin			
$x = 4$			
$y = -3$			
$R(4, 12)$			
$y = x$			
$y = -x$			
$y = 2x - 4$			

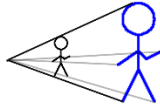
- Find the reflection of $y = 3x + 5$ wrt $y = -x$ axis.

- Find the reflection of $y = x^2 - 2x - 3$ wrt $y = 3$ line.

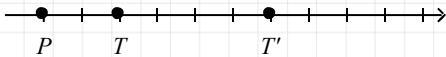
4. DILATION

Enlarging or reducing a plane without changing the shape.
Dilation is determined by two important items:

1. $P \rightarrow$ center of the dilation
2. $k \rightarrow$ dilation factor

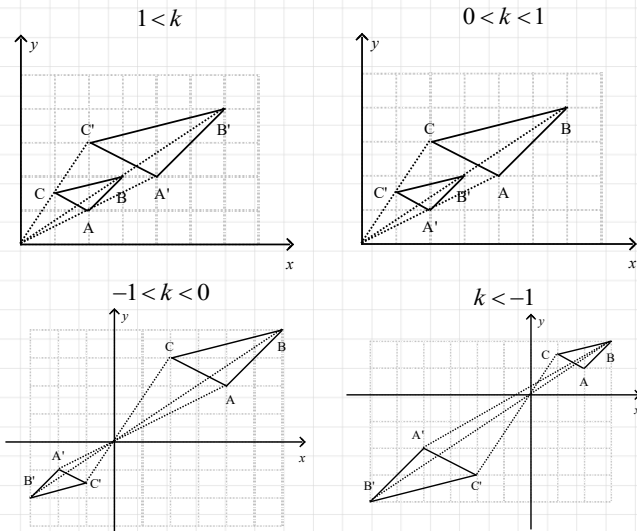


If T is dilated with center P by scale factor k , then T' is the image. So, $|PT'| = k \cdot |PT|$



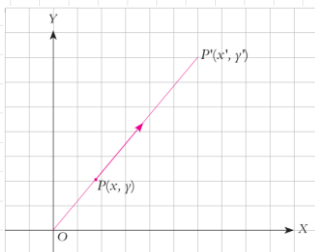
Dilation with center at P by scale factor k is denoted by $[P, k]$

Based on scale factor k , there are following possibilities:



1. Dilation with center at origin $O(0, 0)$

Image of $P(x, y)$ under dilation $[O, k]$ is $P'(x', y')$.



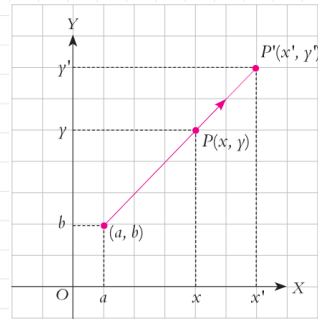
$$\begin{aligned} x' &= k \cdot x \\ y' &= k \cdot y \\ \begin{bmatrix} x' \\ y' \end{bmatrix} &= \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \end{aligned}$$

Example: Find the following images under given dilations

Point	Dilation	Image
(4, 12)	$[O, 2]$	
	$[O, -3]$	
	$[O, -1/4]$	
	$[O, 1/2]$	

2. Dilation with center at point $A(a, b)$

Image of $P(x, y)$ under dilation $[A(a, b), k]$ is $P'(x', y')$.



$$\begin{aligned} x' - a &= k \cdot (x - a) \\ y' - b &= k \cdot (y - b) \end{aligned}$$

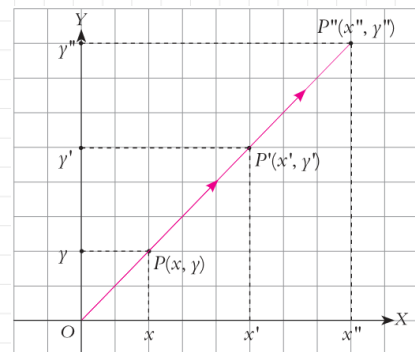
$$\begin{bmatrix} x' - a \\ y' - b \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \cdot \begin{bmatrix} x - a \\ y - b \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \cdot \begin{bmatrix} x - a \\ y - b \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix}$$

Example: Find the following images under given dilations

Point	Dilation	Image
(4, 8)	$[A(2, 3), 2]$	
	$[A(2, 3), 1/2]$	
	$[A(2, 3), -1/3]$	

Composition of Dilation

Let $P(x, y)$ is dilated by scale factor k to $P'(x', y')$,
then $P'(x', y')$ is dilated by scale factor l to $P''(x'', y'')$



$$\begin{aligned} \begin{bmatrix} x' \\ y' \end{bmatrix} &= \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} l & 0 \\ 0 & l \end{bmatrix} \cdot \left(\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \right) \Rightarrow \\ \begin{bmatrix} x'' \\ y'' \end{bmatrix} &= \begin{bmatrix} l \cdot k & 0 \\ 0 & l \cdot k \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \end{aligned}$$

Example: Find the image of $M(-2, 4)$ if it is dilated under $[O, 3]$,
and then dilated under $[O, -2]$.

Assignment:

1. Find the following images under given dilations

Point	Dilation	Image
(10,4)	$[O, 1/2]$	
(3,5)	$[O, 3]$	
(-6,9)	$[O, -1/3]$	
(0,2)	$[M(2,1), 4]$	
(7,-6)	$[N(-2,4), -1/5]$	
(3,6)	$[P(-1,2), 1/4]$	

2. Find the image of line $x - 2y + 3 = 0$ under the dilation of

- $[O, 2]$
- $\left[O, -\frac{1}{2}\right]$
- $[A(2,3), -2]$

5. COMBINATION OF TRANSFORMATIONS**Example: (UN 2007 PAKET B)**

Bayangan garis $3x - y + 2 = 0$ apabila direfleksikan terhadap garis $y = x$, dilanjutkan rotasi sebesar 90° dengan pusat $O(0,0)$ adalah ...

The image of $3x - y + 2 = 0$ when it is reflected with respect to $y = x$, then rotated around $O(0,0)$ by 90° is ...

Example: (UN 2007 PAKET B)

Persamaan bayangan lingkaran $x^2 + y^2 = 4$ bila dicerminkan terhadap garis $x = 2$ dilanjutkan dengan translasi $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ adalah ...

The image of circle $x^2 + y^2 = 4$ when it is reflected with respect to $x = 2$, then translated through $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ is...

Example: (UN 2010 PAKET A)

Sebuah garis $3x + 2y = 6$ ditranslasikan dengan matriks $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, dilanjutkan dilatasi dengan pusat di O dan faktor 2. Hasil transformasinya adalah ...

Line $3x + 2y = 6$ is translated through $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and then dilated with center O by scale factor 2. The result of the transformation is...

Example: (UN 2012/D49)

Bayangan kurva $y = 3x - 9x^2$ jika dirotasi dengan pusat $O(0,0)$ sejauh 90° dilanjutkan dengan dilatasi dengan pusat $O(0,0)$ dan faktor skala 3 adalah ...

The image of curve $y = 3x - 9x^2$ when it is rotated around $O(0,0)$ by 90° and then dilated with center $O(0,0)$ by scale factor 3 is ...

Review Test

1. The point B is translated by $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ gives the point B^1 . Then the point B^1 is translated by $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ gives the point B^{11} (6, -1). Coordinate of the point B is ...

A.(3, 2) B.(2, 3) C.(0, -7) D.(-7, 0) E.(1, 8)

2. Given the line K : $2x + 3y = 5$. If the line K is translated by $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ gives the line k^1 , then the equation of the line k^1 is ...

A. $2x = 3y = 12$ B. $2 = 3y = 16$ C. $2x = 3y = 24$

D. $2x = 3y = 25$ E. $2x + 3y = 28$

3. The point P (2, -4) is reflected to the line $x = -3$ gives the point p^1 . Coordinate of the point p^1 is ...

A.(-6, -4) B.(-8, -4) C.(-10, -4) D.(-12, -4) E.(-15, -4)

4. The point Q (1, -6) is reflected to the line $y = -5$ gives the point Q^1 . Coordinate of the point Q^1 is ...

A.(1, -10) B.(1, -16) C.(1, 16) D.(1, -4) E.(1, 4)

5. The point $(-2, 4)$ is reflected to the line $y = -x$ gives the point ...

A. $(-4, 2)$ B. $(-4, -2)$ C. $(-, -2)$ D. $(4, 2)$ E. $(2, -4)$

6. The point A is rotated through 90° gives the point A^1 . If A $(3, -5)$ and the rotation is counter clockwise, then coordinate of the point A^1 is ...

A. $(5, -3)$ B. $(5, 3)$ C. $(-5, -3)$ D. $(3, 5)$ E. $(-3, 5)$

7. A figure in XY plane is rotated through 45° clockwise then reflected to Y axis. The matrix that represents the result of the two transformations mentioned is ...

A. $\frac{1}{2}\sqrt{2}\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$ B. $\frac{1}{2}\sqrt{2}\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$ C. $\frac{1}{2}\sqrt{2}\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$

D. $\frac{1}{2}\sqrt{2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$ E. $\frac{1}{2}\sqrt{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

8. A plane figure is rotated through 30° counterclockwise then is rotated again through 45° counterclockwise. The matrix that represents the two transformations mentioned is ...

A. $\frac{1}{4}\begin{pmatrix} \sqrt{6}-\sqrt{2} & -\sqrt{6}-\sqrt{2} \\ \sqrt{6}+\sqrt{2} & \sqrt{6}-\sqrt{2} \end{pmatrix}$ B. $\frac{1}{4}\begin{pmatrix} \sqrt{6}-\sqrt{2} & -\sqrt{2}+\sqrt{6} \\ \sqrt{6}+\sqrt{2} & \sqrt{6}-\sqrt{2} \end{pmatrix}$

C. $\frac{1}{4}\begin{pmatrix} \sqrt{6}-\sqrt{2} & -\sqrt{2}-\sqrt{6} \\ \sqrt{6}+\sqrt{2} & \sqrt{6}+\sqrt{2} \end{pmatrix}$ D. $\frac{1}{4}\begin{pmatrix} \sqrt{6}-\sqrt{2} & -\sqrt{2}-\sqrt{6} \\ \sqrt{6}-\sqrt{2} & \sqrt{6}+\sqrt{2} \end{pmatrix}$

E. $\frac{1}{4}\begin{pmatrix} \sqrt{6}+\sqrt{2} & -\sqrt{6}-\sqrt{2} \\ \sqrt{6}+\sqrt{2} & \sqrt{6}-\sqrt{2} \end{pmatrix}$

9. Give the points: A $(3, 2)$, B $(5, 6)$, C $(7, 2)$. The triangle ABC then gets a dilatation of scale factor 4 to the center O. The area of dilatation result is ...

A. 100 B. 120 C. 128 D. 144 E. 168

10. If the point $(2, 3)$ is reflected to the line $x + 4y + 5 = 0$ then the image is ...

A. $(-\frac{2}{17}, 2\frac{3}{17})$ B. $(\frac{2}{17}, 2\frac{3}{17})$ C. $(-2\frac{2}{17}, -1\frac{1}{17})$

D. $(-2\frac{2}{17}, 1\frac{1}{17})$ E. $(-\frac{4}{17}, -5\frac{16}{17})$

Active Note Book