1. COUNTING PRINCIPLES

Addition Principle

Let A and B be two actions that cannot be performed at the same time.
If action *A* can be performed in *m* ways and action *B* can be performed in *n* ways, then action *A or B* can be performed in *m+n* ways.

Example: There are five balls in the first box, three balls in the second box, and four balls in the third box. In how many different ways can you pick a ball from these boxes?

Multiplication Principle

Let a set of tasks containing k parts have equally possible chances.
If the first task can be performed in n_1 ways, second task can be performed in n_2 ways and so on, then the number of ways to perform the entire task is $n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k$ ways.

Example: There are five balls in the first box, three balls in the second box, and four balls in the third box. In how many different ways can you pick three balls from these boxes if one ball must be selected from each boxes?

Listing

The task contains only one part.
Example: List of outcomes as a dice is rolled.

={ }

Product Table

The task contains two parts.
Example: How many different ways can you eat and drink something based on the lists below.

Drink = {Teh Manis, Lemon Tea, Cola, Aqua}

Food = {Nasi Goreng, French Fries, Hamburger}

Example: How many two digit numbers can be formed from the numbers in the set of {1, 2, 3, 4}?

Three Diagram

The task contains two or more parts.
Example: How many different ways can Ali suit with three different pants, 2 different t-shirts, and 3 different shoes?

Example: How many three digit numbers can be formed by using $\{0,1,2,3,\ldots,9\}$?

Example: How many three digit numbers can be formed by using $\{0,1,2,3,\ldots,9\}$ <u>without repetition</u>?

Example: How many three digit odd number can be formed by using $\{1,2,3,4,5,6,7\}$?

Example: How many three digit odd number can be formed by using $\{1,2,3,4,5,6,7\}$ <u>without repetition</u>?

Example: In how many different ways can six letters be delivered to 4 mailboxes?

Example: Find the number of positive factors of 360.

Example: How many four digit numbers can be formed by using $\{1,2,3,4,5,6,7,8\}$?

a) If there is no restriction?

b) If repetition is not allowed?

c) If the numbers must be greater than 4000 and repetition is not allowed?

d) If the numbers must be less than 4000, divisible by 5 and repetition is not allowed?

2. PERMUTATIONS

Factorial

The product of first n natural numbers is called a factorial of n and denoted by $n!$

$$n! = n\cdot(n-1)\cdot(n-2)....3\cdot 2\cdot 1 \ \ \& \ \ 0! = 1$$

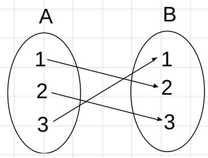
Example: Calculate the expressions below.

- $4! =$
- $3! + 5! =$
- $\dfrac{8!}{4!} =$
- $\left(\dfrac{8}{4}\right)! =$
- $\left(\dfrac{9}{5}\right)! =$
- $\dfrac{8! - 6!}{5!} =$
- $\dfrac{(n+3)!}{(n-1)!}\cdot\dfrac{(n-2)!}{(n+2)!} =$
- $\dfrac{(n+2)! + (n+1)n!}{(n+1)(n-1)!} =$
- $\dfrac{(n+1)!}{n! + (n-1)!} = 3 \Rightarrow n = ?$
- $\dfrac{(n+2)!}{(n^3-n)\cdot(n-3)!} = 6n - 12 \Rightarrow n = ?$

Permutation Functions

Let f be a function defined on $A = \{1,2,3\}$.

If $f = \{(1,2),(2,3),(3,1)\}$, then we can show f in two ways:



or $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

Permutation Function

Each function which is defined on a finite set A and which is one to one & onto is called **permutation on the set A**.

If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, then what about others?

$f_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$, $f_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, $f_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$,

$f_5 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, $f_6 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

We will show permutations on A as

$(2 \ \ 3 \ \ 1),(2 \ \ 1 \ \ 3),....,(1 \ \ 2 \ \ 3)$

Note: We can say that different arrangements of the elements of a finite set A are called **permutation of A.**

Identity Permutation

$I : \begin{pmatrix} a_1 & a_2 & ... & a_n \\ a_1 & a_2 & ... & a_n \end{pmatrix}$ defined on $A = \{a_1, a_2, ..., a_n\}$

Composition of Permutation Functions

Let $f : \begin{pmatrix} a & b & c \\ b & a & c \end{pmatrix}$, $g : \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix}$ on $A = \{a,b,c\}$.

$$\left.\begin{array}{l} (fog)(a) = f(g(a)) = f(c) = c \\ (fog)(b) = b \\ (fog)(c) = a \end{array}\right\} \Rightarrow fog : \begin{pmatrix} a & b & c \\ c & b & a \end{pmatrix}$$

$gof : \begin{pmatrix} a & b & c \\ & & \end{pmatrix}$

Note:

1. $fog \neq gof \rightarrow$ not commutative
2. $(fog)oh = fo(goh) \rightarrow$ associative
3. $foI = Iof = f$

Inverse Permutation

The function f^{-1} satisfying $fof^{-1} = f^{-1}of = I$ is the inverse of f.

Example: Given $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$ on $A = \{1,2,3,4\}$, find f^{-1}.

Example: $f=\begin{pmatrix}1&3&5&7&9\\3&7&9&5&1\end{pmatrix}$ & g are two permutations defined on $K=\{1,3,5,7,9\}$. Find g if it is given that $gof=\begin{pmatrix}1&3&5&7&9\\7&5&3&9&1\end{pmatrix}$.

Example: $f=\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$ & $g=\begin{pmatrix}1&2&3&4\\2&3&1&4\end{pmatrix}$ are two permutations on $A=\{1,2,3,4\}$. Find h such that $f^{-1}=goh$.

A permutation is a selection and ordering of objects.

Number of Permutations of n Distinct Elements

$P(n,n)=n!$

Example: How many different ways can 3 students be seated at a desk?

Example: How many different 8-letter words can you form by using the letters of ISTANBUL?

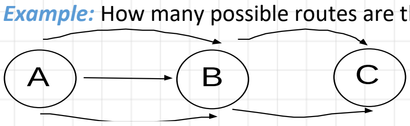
Example: How many two-digit numbers can you form by using the elements of set $A=\{1,2,3,4\}$ one at a time?

Example: How many different arrangements are possible if you have 5 novel, 3 comics, 5 math books,

a) If no restriction?

b) If each subject must be together?

Example: How many possible routes are there from A to C?



Example: How many three-digit numerals can be formed by using the digits 0, 1, 3, 5, 7?

Example: How many numerals of positive even integers less than 1000 can be formed from the elements of $A=\{0,1,2,3,4,5\}$?

Permutations of r Elements Selected from n Elements

$$P(n,r)=n(n-1)...(n-r+1)=\frac{n!}{(n-r)!}$$

Example: Calculate the expressions below.

- $P(5,3)=$
- $P(n,n)=$
- $P(n,0)=$
- If $P(n,4)=20\cdot P(n,2)$, then find n.
- $4P(2n,1)=2P(n,1)+36\Rightarrow n=?$

Example: In a group of 16 person, how many ways can a president and vice president be selected?

Example: From the letters of word ISTANBUL,

a) How many arrangements of 3 letters can be formed?

b) How many of these 3-letter words do not contain T?

c) How many of these 3-letter words begin with B and end with N?

Example: How many 3 digit odd numbers can be formed from 3, 4, 5, 6, 7 if no digit may be repeated in a number?

Example: (UN 2010 PAKET A)
Dari 10 calon pengurus OSIS akan dipilih ketua, sekretaris, dan bendahara. Banyak cara memilih pengurus OSIS adalah …
There are 10 candidates for OSIS. How many different ways can a chairman, secretary, and treasurer be selected?

Permutation with Restrictions

Permutation with Grouped Elements

Example: How many 5-letter words can be formed from the letters of MERAK if A and K must be next to each other?

Example: Find the number of permutations of {1,2,3,4,5,6,7} if 2,3,4 must be together?

Example: In how many ways can 7 students sit together if 2 of them must not sit next to each other?

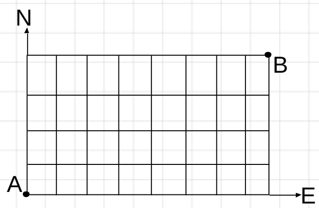
Example: In how many different ways can we place 5 different Math, 3 different Biology, and 4 different Physics books to shelf if

a) Math books must be together?

b) Biology books must be together and Physics books must be together also?

c) All same subject books must be together?

Example: How many different shortest ways are there from A to B in the figure below?



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Permutation with Identical Elements (Distinguishable Permutations)

Distinguishable Permutations of n Elements

If n objects are given with r of them are alike, then

$$\frac{n!}{r!}$$

Example: In how many different ways can the letters of DIFFERENCE be arranged?

Example: (UN 2012/E52)
Banyak susunan kata yang dapat di bentuk dari kata"WIYATA" adalah….
How many different words can be formed by using the letters of "WIYATA"?

If n objects are given with r_1 of them are alike, r_2 of them are alike, …, then

$$\frac{n!}{r_1!\cdot r_2!\cdots}$$

Example: How many five-digit even or odd numerals can be formed by using the digits of the number 22255?

Example: We have 5 identical Math, 3 identical Biology, 4 identical Physics books. Find the number of arrangements on the shelf if

a) If there is no restriction.

b) If same subject books must stay together.

Example: If I toss a coin successively 7 times, in how many ways can I get 4 heads and 3 tails?

Circular Permutations

Example: Show on circle that how many different kinds of placements there are if you have three items.

Circular Permutations of n Elements
$(n-1)!$

Example: In how many ways can seven people be seated around a circular table?

Example: In how many ways can 4 boys and 4 girls be seated around a table if

a) There is no restriction?

b) Two particular girls must be seated next to each other?

c) If the boys and girls must be seated alternately?

Example: In how many ways 3 boys and 4 girls be seated group by group (girls together, boys together)

a) on a row?

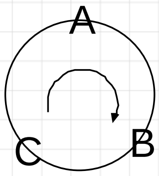
b) around a circular table?

Example: (UN 2012/A13)
Dalam sebuah keluarga yang terdiri dari Ayah, Ibu, dan 5 orang anaknya akan makan bersama duduk mengelilingi meja bundar. Jika Ayah dan Ibu duduknya selalu berdampingan, maka banyak cara mereka duduk mengelilingi meja bundar tersebut adalah....

A family consisting mother, father and 5 children will sit around a circular table. If mother and father sit next to each other, the number of different ways they can sit around the table is …

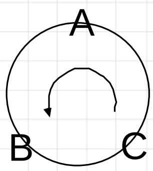
Note: If a circular permutation of objects does not have a definite <u>top</u> or <u>bottom</u> such as bracelets or key rings, we observe that two positions are same by turning over the arrangement.

Look from <u>front</u>
Counter clock-wise direction



ABC

Look from <u>back</u>
clock-wise direction



ABC

Example: In how many different ways can 5 keys be arranged on a ring?

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3. COMBINATION

A combination is a selection of objects without regarding the order.

For permutation: ab & ba are different.

For combination: ab & ba are same.

Combination of n elements taken r at a time

Let A be a set with n elements.
Any subset with r elements of A is called a combination of n elements taken r at a time, shown as $\binom{n}{r}$ or $C(n,r)$ or C_r^n

Example: Show combination and permutation of $A=\{a,b,c\}$ taken 2 at a time.

Remember;
permutation → selecting and ordering.
combination → selecting.
So, select r element from n elements and order them.

$$\underbrace{C(n,r)}_{\text{select r element}} \cdot \underbrace{r!}_{\text{order r element}} = \underbrace{P(n,r)}_{\text{select and order r element}}$$

Then, $C(n,r)\cdot r!=\dfrac{n!}{(n-r)!}$. So,

$$C(n,r)=\frac{n!}{(n-r)!\cdot r!}$$

Example: Calculate the following expressions.

- $C(4,2)=$
- $C(6,4)=$

Example: Find the number of groups of 4 students chosen from a class of 12 students.

Example: A basketball coach will select 6 players out of 15 students and then select one captain out of 6 players. How many different ways can it be done?

Example: A football team will be formed from class 11A and 11B. 11A has 23 students, 11B has 20 students. 5 students from 11A, and 6 students from 11B will be chosen. How many different ways can it be done?

Example: (UN 2011 PAKET 46)
Setiap 2 warna yang berbeda dicampur dapat menghasilkan warna baru yang khas. Banyak warna baru yang khas apabila disediakan 5 warna yang berbeda adalah …
If any two color are mixed, new special color can be obtained. If there are 5 different color, the number of special color that can be obtained is …

Example: (UN 2010 PAKET A)
Sebuah kotak berisi 4 bola putih dan 5 bola biru. Dari dalam kotak diambil 3 bola sekaligus, banyak cara pengambilan sedemikian hingga sedikitnya terdapat 2 bola biru adalah …
There are 4 white and 5 blue balls in a box. If three balls are taken at a time, there are …. ways in which at least two balls are obtained as blue.

Example: In a saloon, there is a circular table for 5 people and a sofa for 3 people. In how many different ways can 8 guests be seated in this saloon?

Example: In how many different ways can 16 people be separated into 2 groups if

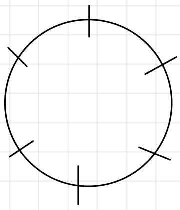
a) one group travels to Yogya and the other to Jakarta?

b) the two groups play basketball together?

Example: How many different ways can 16 people be separated into 4 equal teams?

Example: If a coin is tossed 7 times. How many different ways can we get 4 heads and 3 tails?

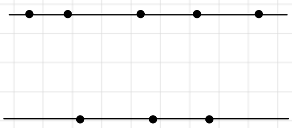
Example: By six points on the circle, how many



a) lines are determined?

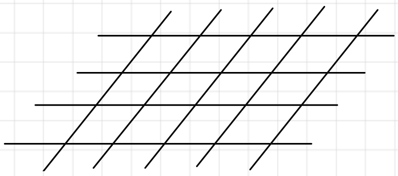
b) triangles are determined?

Example: How many triangles can be formed by using following points?



Example: (UN 2010 PAKET B)
Diketahui 7 titik dan tidak ada 3 titik atau lebih segaris. Banyak segitiga yang dapat dibentuk dari titik–titik tersebut adalah …
If there are 7 point on a plane such that no three or more of them do not form a line, then how many different triangle can be formed by using those points?

Example: Four parallel lines are intersected by another five parallel lines. How many parallelograms can be formed?



Properties

$$\binom{n}{r}=\binom{n}{n-r}$$

Example: Show that $C(9,3)=C(9,6)$.

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Example: If $\binom{n}{5}=\binom{n}{7}$, then $n=?$

Example: The number of 3-element subset of a set is equal to the number of 8-element subset of the same set. Find the 5 element subset of this set.

Example: If the number of subsets with 2 elements & with 3 elements is equal for the set A, then find

a) the number of elements of A.

b) the number of subsets of A with 3 elements.

c) the number of subsets of A with less than 4 elements.

$$\binom{n+1}{r}=\binom{n}{r}+\binom{n}{r-1}$$

Example: Show the property above.

$$\binom{n}{0}+\binom{n}{1}+\cdots+\binom{n}{n}=2^n$$

Each represents the number of subsets with r elements, so if you sum them you find the total number of subsets.

Example: Find the total number of subsets of the set $H=\{a,b,c,d,e,f\}$

Example: How many subsets of the set $P=\{1,2,3,4,5,6,7\}$ contain

a) 2 or 6?

b) both 2 and 6?

Combinations with Identical Elements

In some cases we may choose same element more than once. That is, the set may contain identical elements.

Example: let's consider $K = \{A, B, C\}$.

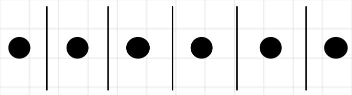
Choose two elements from K with identical selection.

$S = \{\ \{A,A\},\{A,B\},\{A,C\},\{B,B\},\{B,C\},\{C,C\}\ \}$.

How many different ways can it be done?

Example: Let's distribute 6 identical balls into 3 boxes in a way that each box must contain at least one ball.

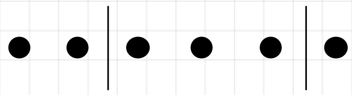
Let's separate balls from each other by lines.



That is, we divide balls into 6 and each contain 1.

To make # of groups 3, enough to choose 2 and erase others.

Let's choose 2nd and 5th



So, how many different ways we can choose 2 lines out of 5 = $\binom{5}{2}$.

Theorem

Let n be a positive integer. Then the number of <u>positive</u> integer solutions (each $x_i > 0$) to $x_1 + x_2 + \cdots\cdots + x_r = n$ is

$$\binom{n-1}{r-1}$$

Proof:

Example: Find the number of positive integer solutions to the equation $a + b + c + d + e + f = 17$.

Example: We want to divide 21 identical marbles among 5 children so that each child takes at least one. How many different shares can be done?

Example: Find the number of integer solutions to equation $x + y + z = 78$ where $x \geq 11,\ \ y \geq 16,\ \ z \geq 4$.

Corollary

Let n be a positive integer. Then the number of <u>non-negative</u> integer solutions (each $x_i \geq 0$) to $x_1 + x_2 + \cdots\cdots + x_r = n$ is

$$\binom{n+r-1}{r-1}$$

Proof:

Example: Find the number of non-negative integer solutions to the equation $a + b + c + d + e = 18$.

Recall the beginning problem and solve it

Example: How many different ways can we choose two elements from $K = \{A, B, C\}$ with identical selection?

That is; $A + B + C = 2$ where $A, B, C \geq 0$.

$$\binom{n+r-1}{r-1} = \binom{2+3-1}{3-1} = \binom{4}{2} = 6$$

Example: There are 15 people on the bus and 8 bus stops left. Driver is making a list of passengers getting of each stop. How many different lists can be done?

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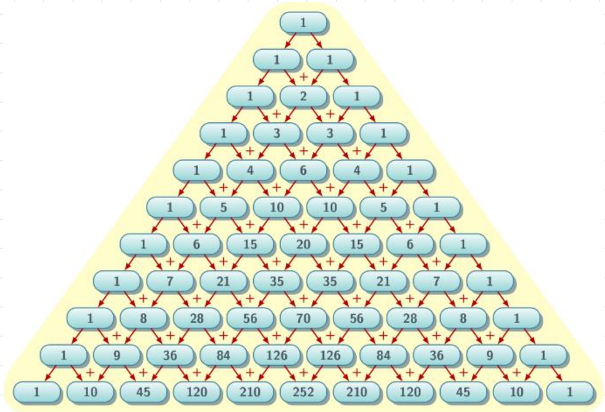
4. BINOMIAL EXPANSION

Pascal Triangle



Above triangle gives coefficients of n[th] power of sum of two terms.

$(a+b)^0 = \underline{1}$

$(a+b)^1 = \underline{1}\cdot a + \underline{1}\cdot b$

$(a+b)^2 = \underline{1}\cdot a^2 + \underline{2}\cdot ab + \underline{1}\cdot b^2$

$(a+b)^5 = \dots$

Results

For $(a+b)^n$;

- There exists $n+1$ terms in the expansion.
- Sum of the exponents in each term is n.
- Sum of coefficients : substitute 1 for each variable.
- Constant term : substitute 0 for each variable.

Example: Given $(x+y)^{12}$ find

- number of terms:
- sum of coefficients:
- constant term:

Example: Expand $(2x+y)^6$.

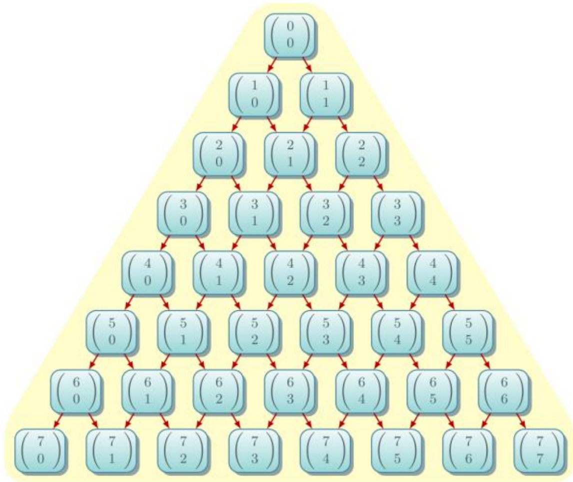
Example: Expand $(x^2-3y)^4$.

Example: Given $\left(n+\frac{1}{n}\right)^7$ find

- Sum of coefficients:
- Constant term:

Finding Binomial Term Using Combination

There exist 1-to-1 relationship between coefficients in Pascal triangle and combination.



Generalization

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

Example: Expand $(x+y)^6$.

Example: Expand $(2a-b)^5$.

r^{th} entry in the expansion of $(x+y)^n$

$$\binom{n}{r-1}x^{n-(r-1)}y^{r-1}$$

Example: Find the 9[th] term in expansion of $(a+b)^{12}$.

Example: Find the coefficient of 4[th] term in the expansion of $(2x-4y)^7$.

Example: If 2^7xy^3 is a term in the expansion of $(ax+2y)^4$, then find a.

Example: Find the constant term of $\left(\frac{2}{x^2}+\frac{x^3}{2}\right)^{10}$.

Example: Evaluate the expression of

$$\binom{5}{0}+2\binom{5}{1}+2^2\binom{5}{2}+2^3\binom{5}{3}+2^4\binom{5}{4}+2^5\binom{5}{5}=$$

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5. PROBABILITY

Experiment: an effort that generates a certain chance.

Example: rolling a dice.

Sample space: set of all outcomes as a result of an experiment.

Example: S = {1,2,3,4,5,6}

Event: each subset of a sample space.

Example:

{1}
{1,2}→ scoring 1 or 2.
{2,3,5}→scoring first three prime numbers.
$\varnothing$

Note: Since each event is a set already, we may apply set operations for events also.

That is, let A and B be two events in a sample space S.

$A \text{ or } B \Leftrightarrow A \cup B \Leftrightarrow$ namely about summation
$A \text{ and } B \Leftrightarrow A \cap B \Leftrightarrow$ namely about multiplication
$\text{not } A \Leftrightarrow A^c = A'$

Example: Consider following events of rolling dice experiment. Find $A \cup B,\ A \cap B,\ A^c$.

$A = \{1,2,3,4\},\ B = \{4,5,6\}$

Mutually Exclusive Events

Two events which cannot occur at the same time are called **mutually exclusive** events.
That is, no common element.

Example: If a dice is rolled, then $A = \{2,4,6\}$ and $B = \{1,3,5\}$ are mutually exclusive.

Probability

If E is an event in the sample space S (or E is a subset of S) then the probability of the event E is defined by

$$P(E) = \frac{n(E)}{n(S)}$$

Example: If a coin is tossed, what is the probability that a head turns up?

Sample space: $S = \{H,T\} \Rightarrow n(S) = 2$
Event: E=head turns up $= \{H\} \Rightarrow n(E) = 1$

$$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}$$

Example: If a coin is tossed, what is the probability that either a head or a tail turn up?

Example: If a coin is tossed, what is the probability that neither a head nor a tail turns up?

Example: Probability that the number is rolled is odd?

Example: A coin is tossed 3 times. Find the probability of getting only one head.

Certain Event & Impossible Event

If $P(E) = 1$, then E is called **certain event.**
If $P(E) = 0$, then E is called **impossible event.**

Example: A player beats on a number from 2 to12 and rolls two dice. If he guessed the sum of the numbers, he wins the game. Which number has the highest possibility?

Rules of Probability

- For any event E, $0 \le P(E) \le 1$.
- For sample space S, $P(S) = 1$.
- For two mutually exclusive events A & B, $P(A \cup B) = P(A) + P(B)$.
- For any two events A & B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- For any event A, $P(A) + P(A') = 1$.

Example: What is the probability of getting prime number or even number when the dice is rolled?

Example: In a bag, there exist 5 blue, 4 red, 6 yellow marbles. What is the probability of getting red or yellow marble if one of them is drawn?

Example: A coin is tossed 4 times. What is the probability of showing at least one tail?

Example: A group of six people is selected. What is the probability that at least two of them have the same birthday?

Counting Principles and Probability

Example: A box contains 12 white and 10 red balls. One ball is drawn and returned then second one is drawn.

a) What is the probability of getting first white and then red?

b) Repeat the same question without returning the balls.

Example: A box contains 12 white and 10 red balls. Two balls are drawn at a time.

a) What is the probability of getting one white and one red balls?

b) What is the probability of getting two white balls?

Example: Two integers are randomly selected from the set {1,2,3,4,5,6,7,8} without repetition. What is the probability that the sum of the numbers is 12?

Example: Ten people want to be seated around a circular table. What is the probability that two particular people will not be seated next to each other?

Example: A box contains 4 red, 5 white, 3 blue marbles. What is the probability of getting

a) one white , two red, and one blue if four marbles are taken at a time?

b) white, red, blue, and red respectively if four marbles are drawn one by one with returning marbles each time?

c) white, red, blue, and red respectively if four marbles are drawn one by one without returning marbles each time?

Example: A bag contains two white balls, three red balls and four black balls.

a) If a ball is drawn from the bag, what is the probability that it is red?

b) If a red ball is drawn and not being returned into the bag, then a second ball is drawn, what is the probability that the second ball is a white ball?

Example: (UN 2012/B25)
Dua buah dadu dilempar undi bersamaan sebanyak satu kali. Peluang kedua mata dadu yang muncul tidak ada yang sama adalah ...
Two dice are rolled together. The probability that both dice do not show the same number is ...

Example: (UN 2012/A13)
Dua buah dadu dilempar undi bersama–sama satu kali. Peluang muncul mata dadu berjumlah 5 atau 7 adalah...
Two dice are rolled together. The probability that the sum of dice is 5 or 7 is ...

Example: (UN 2011 PAKET 12)
Dari dalam kantong berisi 8 kelereng merah dan 10 kelereng putih akan diambil 2 kelereng sekaligus secara acak. Peluang yang terambil 2 kelereng putih adalah ...
Two marbles will be drawn at random at the same time from the bag containing 8 red marbles and 10 white marbles. The probability of getting 2 white marbles is...

Example: (UN 2010 PAKET B)
Sebuah kotak berisi 4 bola merah, 3 bola putih, dan 3 bola hitam. Diambil sebuah bola secara acak, peluang terambil bola merah atau hitam adalah ...
A box contains 4 red balls, 3 white balls and 3 black balls. If a ball is drawn at random, the probability of getting red or black is...

Conditional Probability

Conditional Probability

A & B be two events in a same experiment.
$P(A\backslash B)$ → conditional probability of event A (depends on B)

$$P(A\backslash B) = \frac{probability\ of\ A\&B}{probability\ of\ B} = \frac{P(A\cap B)}{P(B)}$$

Example: Two dice are rolled. What is the probability that sum of the numbers rolled is 8 if it known that one number is 5?

Example: Two dice are rolled. What is the probability that sum of the numbers rolled is greater than 9, given that the first die shows 4?

Example: In a class of 40 students. 15 students join Math Club and 10 students join Physics Club. A student is chosen randomly from the group.

a) What is the probability that the student studies only Math?

b) What is the probability that the student studies Math if it is known that he/she studies Physics?

c) What is the probability that the student is not studying Math if it is known that he/she studies Physics?

Binomial Probability

Example: What is the probability that getting exactly 3 Heads on an experiment of tossing a coin 7 times?

Example: What is the probability of getting 2 exactly four times if a die is rolled 8 times?

Example: For a man, the probability of getting the target is 2/5. If he tries 6 times.

a) What is the probability of getting target exactly 2 times?

b) At least 5 times?

c) At least 2 times?

Exercises 1.4 – Page 89 in Zambak

Active Note Book

Review Test

1) $\frac{14!}{11!}=?$

A) 24 B) 6 C) 1864 D) 3 E) 2184

2) $\frac{14!}{7\cdot 4!\cdot(14-3)!}=?$

A) 7 B) 8 C) 13 D) 18 E) $\frac{156}{7}$

3) $\frac{(n+2)!}{n!}=42$. Find n.

A) 3 B) 4 C) 5 D) 6 E) 7

4) $\frac{(n-1)!\cdot(n-2)!}{n!\cdot(n-3)!}=?$

A) n B) $n-2$ C) $\frac{n-2}{n}$ D) $\frac{n-1}{n-2}$ E) $\frac{n}{n-3}$

5) $\frac{3!\cdot(n-1)!}{n!}=6$. Find n.

A) 1 B) 3 C) 6 D) 30 E) 36

6) $\frac{(n+1)!}{n!+(n-1)!}=3$. Find n.

A) 3 B) 5 C) 7 D) 8 E) None of them

7) Find n if $\frac{P(5,5)}{P(3,3)}=2n$.

A) 10 B) 20 C) 24 D) 30 E) 32

8) How many different 3-digit numbers can be formed by using the set $A=\{1,2,3,4,5,6,7\}$ which are greater than 350? (Without repetition)

A) 100 B) 112 C) 130 D) 135 E) 160

9) Calculate the sum of all 3-digit numbers which can be formed by using the set $A=\{3,4,5\}$. (Without repetition)

A) 2664 B) 1332 C) 720 D) 120 E) 6

10) In how many ways, can 4 physics and 4 chemistry teachers be arranged at a round table if the chemists and physicists alternate seats?

A) 6 B) 8 C) 24 D) 120 E) 144

11) In how many ways, can seven letters be mailed, if there are four mail boxes?

A) $4!\cdot 7!$ B) $4!$ C) $7!$ D) 4^7 E) 7^4

12) In how many ways, can six persons be seated around a table?

A) 720 B) 120 C) 24 D) 6 E) 3

13) In how many different ways can the letters of "SQUARE" be arranged if the arrangements begin with "E" and end with "S"?

A) 6 B) 24 C) 60 D) 120 E) 256

14) In how many ways, can 5 girls and 3 boys be seated around a table if two boys are together?

A) 2880 B) 1844 C) 1440 D) 720 E) 120

15) How many different 6-digit numbers can be formed from the digits of the number 265463?

A) 24 B) 120 C) 144 D) 360 E) 720

16) In how many ways can 4 girl and 5 boys be seated on a desk?

A) 4^5 B) $4!\cdot 5!$ C) $4!$ D) $5!$ E) $9!$

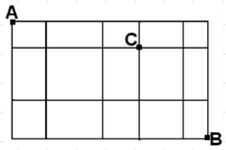
17) In how many ways, can six persons be seated around a table if three of them sit together?

A) 720 B) 144 C) 120 D) 60 E) 36

18) How many different 3-digit numbers can be formed by using the set $A=\{5,6,7,8,9\}$ which are less than 780? (Without repetition)

A) 6 B) 18 C) 24 D) 30 E) 72

19) A person moves from point A to point B via C. If the person uses the shortest way then how many different options does he have?



A) 8 B) 12 C) 16 D) 20 E) 24

20) Find the number of ways in which 6 Algebra and 4 Geometry books can be placed on a shelf so that the arrangement begins and ends with a Geometry book?

A) $6!\cdot 2!\cdot 2!$ B) $6!\cdot 4!$ C) $10!$ D) $2\cdot 8!$ E) $12\cdot 8!$

21) In how many ways can six keys be held on a circular key holder?

A) 720 B) 360 C) 120 D) 60 E) 24

22) How many different 9-letter words can be formed by using the letters of the word "KATAMARAN"?

A) $\frac{9!}{4!}$ B) $\frac{9!}{4!\cdot 2!}$ C) $\frac{9!}{5!}$ D) $\frac{9!}{2!\cdot 3!}$ E) $\frac{9!}{4!\cdot 5!}$

23) How many 4-digit even numbers can be written by using the digits {0, 1, 2, 3, 4, 5} without repetition?

A) 90 B) 108 C) 144 D) 156 E) 180

24) For the set {1, 2, 3, 4, 5}, in how many of the permutations with 3 elements have the element "4"?

A) 6 B) 12 C) 24 D) 36 E) 60

25) By using the digits of the number 6003888, how many different 7-digit numbers can be written?

A) 210 B) 240 C) 300 D) 360 E) 400

26) There are 8 courses in a school, but 3 of them are given at the same time. In how many ways can a student choose 3 courses?

A) 30 B) 40 C) 48 D) 52 E) 55

27) Find n if $C(n,4) = P(n,3)$.

A) 6 B) 12 C) 18 D) 27 E) 32

28) 7 different points on a circle are given. By using these points how many triangles can we draw if the vertices will be on these points?

A) 20 B) 35 C) 50 D) 75 E) 96

29) Between 8 different cards in how many ways can we select at least one card?

A) 255 B) 216 C) 196 D) 169 E) 127

30) The number of subsets with at most 2 elements is 37. How many elements are there in this set?

A) 6 B) 7 C) 8 D) 9 E) 10

31) Between 15 persons we will select 6 people. If 3 of them will certainly join the group then in how many ways can we select the group?

A) 550 B) 455 C) 445 D) 340 E) 220

32) Between 5 women and 4 men we will select 3 persons. If there will be at least 1 man in the group then in how many ways can we select the group?

A) 54 B) 64 C) 72 D) 74 E) 84

33) Find the number of subsets of the set $A=\{a,b,c,d,e,f\}$ which contains a or b, but not both?

A) 16 B) 24 C) 32 D) 40 E) 48

34) If the number of subset of a set with at most two elements is 29, then what is the number of subsets of this set with 5 elements?

A) 144 B) 126 C) 56 D) 21 E) 6

35) $\binom{16}{1}+\binom{16}{3}+\binom{16}{5}+\ldots+\binom{16}{15}=a$ and

$\binom{12}{0}+\binom{12}{2}+\binom{12}{4}+\ldots+\binom{12}{12}=b$ then what is $\frac{a}{b}=?$

A) 2 B) 4 C) 8 D) 16 E) 32

36) What is the coefficient of the third term of the expansion of $\left(x^3-\frac{1}{x}\right)^6$?

A) -18 B) -15 C) 15 D) 18 E) 30

37) Find n if the constant term of the expansion of $\left(x^2-\frac{1}{x}\right)^n$ is the 5th term.

A) 2 B) 6 C) 8 D) 12 E) 20

38) Let A and B be two events. If $P(B)=\frac{1}{4}$ and $P(A-B)=\frac{1}{3}$, what is $P(A\cup B)$?

A) $\frac{1}{9}$ B) $\frac{1}{6}$ C) $\frac{1}{4}$ D) $\frac{5}{12}$ E) $\frac{7}{12}$

39) Of 16 girls, 6 have blue eyes. If three girls are chosen at random what is the probability that at least one has blue eyes?

A) $\frac{3}{8}$ B) $\frac{2}{5}$ C) $\frac{11}{14}$ D) $\frac{9}{14}$ E) $\frac{10}{13}$

40) A coin and dice are thrown at the same time. What is the probability that the coin is head and dice is 4?

A) 1/72 B) 1/24 C) 1/12 D) 1/3 E) 1/2

41) Three marbles are taken one by one out from a bag which contains 5 red, 4 blue and 3 black marbles. What is the probability that they are all blue if none is replaced?

A) $\frac{1}{55}$ B) $\frac{4}{55}$ C) $\frac{5}{11}$ D) $\frac{5}{12}$ E) $\frac{4}{5}$

42) An urn contains 4 red and 6 black balls; the other contains 5 red and 2 black balls. A ball is taken out from the second urn and put in the first urn. If a ball now is taken out from the first urn, what is the probability that it is black?

A) $\frac{4}{7}$ B) $\frac{11}{18}$ C) $\frac{8}{17}$ D) $\frac{30}{77}$ E) $\frac{2}{11}$

43) A and B are two events. If $P(A\cup B)=3/4$ $P(A)=1/2$ and $P(B)=3/8$, then which one of the following is the probability of the event that both A and B occur?

A) 1/24 B) 1/8 C) 3/4 D) 5/8 E) 7/8

44) The probability that man will live 10 more years is 1/3 and the probability that his wife will live 10 more years is 1/4. What is the probability that both will be alive in 10 years?

A) 1/2 B) 2/5 C) 2/12 D) 1/12 E) 1/6

45) A bag contains 4 red, 6 white and 3 brown marbles. If three marbles are chosen at random without replacement, what is the probability that each has different color?

A) $\frac{5}{143}$ B) $\frac{6}{143}$ C) $\frac{12}{143}$ D) $\frac{13}{143}$ E) $\frac{36}{143}$

46) In a school, 25% of the boys and 10% of the girls failed. The boys are 40% of the students. If a student is selected at random, what is the probability that he (or she) is failure?

A) $\frac{4}{25}$ B) $\frac{3}{5}$ C) $\frac{7}{20}$ D) $\frac{2}{5}$ E) $\frac{16}{25}$

47) Three coins are tossed together. What is the probability of obtaining two tails and a head?

A) $\frac{1}{3}$ B) $\frac{3}{8}$ C) $\frac{1}{2}$ D) $\frac{1}{4}$ E) $\frac{1}{8}$

48) A coin is flipped 5 times successively (one after the other). What is the probability of obtaining 4 heads?

A) $\frac{4}{5}$ B) $\frac{1}{5}$ C) $\frac{5}{32}$ D) $\frac{4}{13}$ E) $\frac{3}{13}$

49) A subset is chosen from the subsets of $A=\{a, b, c, d, e, f\}$. What is the probability that it contains b or c?

A) $\frac{4}{5}$ B) $\frac{3}{4}$ C) $\frac{3}{5}$ D) $\frac{2}{3}$ E) $\frac{1}{2}$

50) In a throw of two dice, if one comes out 4 what is the probability that the sum of the numbers is greater than 6 and less than 11?

A) $\frac{7}{36}$ B) $\frac{2}{9}$ C) $\frac{11}{36}$ D) $\frac{7}{11}$ E) $\frac{2}{3}$