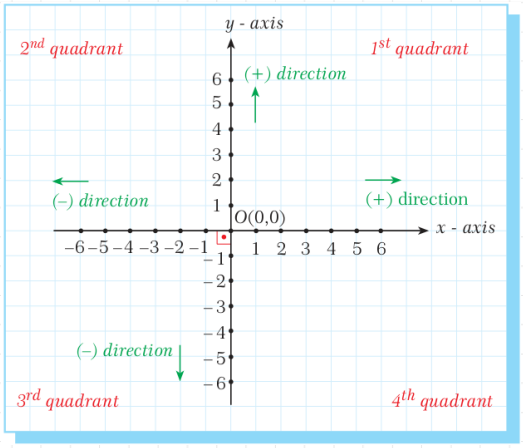
1. THE COORDINATE PLANE

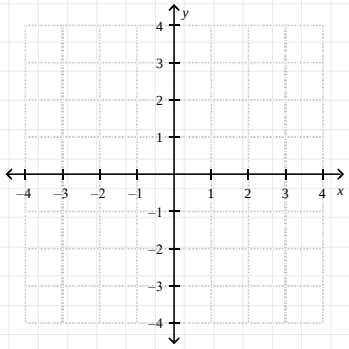
A. Analytic Analysis Of Points

Basic Concepts

A coordinate or analytic plane contains of two perpendicular number lines, the x-axis (**abscissa**) and the y-axis (**ordinate**) that intersect at their origins. The point of inter-section O of the coordinate axes is called the **origin** of the coordinate plane.



Example: Plot the points $A(1,-2)$, $B(-3,-4)$, $C(-3,2)$ on the coordinate plane.

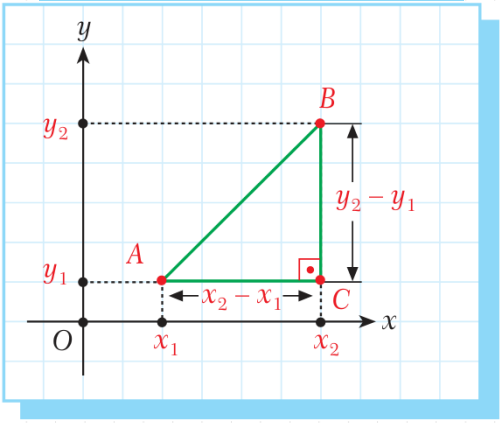


Example: If $A(m,n)$ is in the 4th quadrant, in which quadrant are the points $B(n,m)$ and $C(\frac{m}{n},n)$?

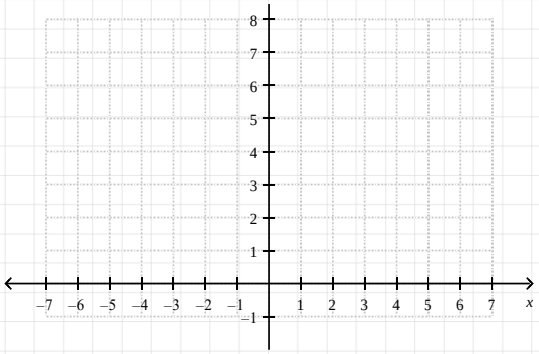
Distance between Two Points

The distance between the points $A(x_1,y_1)$ and $B(x_2,y_2)$ is

$$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



Example: Find the distance between two points $A(-3,7)$ and $A(5,1)$.



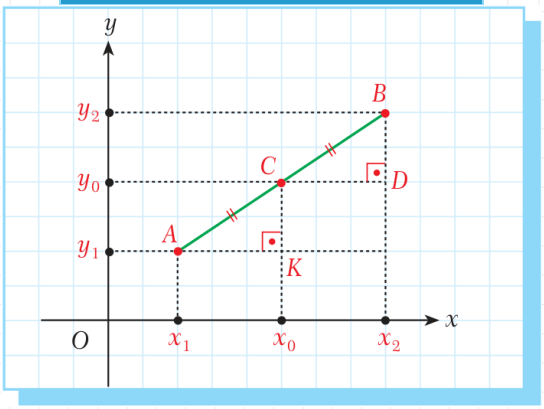
Example: $M(5,7)$, $K(x,4)$, $N(4,8)$ are given. If K is at the same distance from the points M and N , then find x .

Example: Find the point K on the y-axis which is equidistant to the points $A(2,2)$ and $B(-4,0)$.

Midpoint of a Line Segment

The midpoint of line segment $[AB]$ where $A(x_1,y_1)$ and $B(x_2,y_2)$ is

$$C(x_0,y_0) = C\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$



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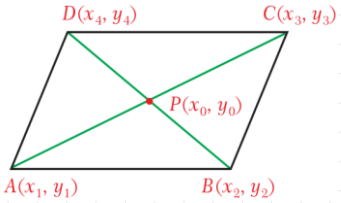
Example: The points $A(1,2)$ and $B(-5,8)$ are given. Find the coordinates of the midpoint of $[AB]$.

Example: The points $A(1,4)$ and $B(3,-10)$ are given. Find the distance from midpoint of $[AB]$ to origin.

Note:

Let A,B,C,D be the vertices of a parallelogram as in the following picture,



then $P\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$ & $P\left(\frac{x_2+x_4}{2},\frac{y_2+y_4}{2}\right)$

Therefore,

$$\boxed{x_1+x_3=x_2+x_4 \text{ and } y_1+y_3=y_2+y_4}$$

Example: If the points $A(-2,-3)$, $B(3,-2)$, $C(x,y)$ and $D(-1,3)$ are the vertices of parallelogram $ABCD$, then find C.

Coordinates of a Point Dividing a Line Segment in a Given Ratio

- **Dividing Internally:**

Example: The points $M(1,2)$ and $N(7,11)$ are given. Find the coordinates of the point P which divides $[MN]$ internally in the ratio of $\frac{|MP|}{|PN|}=2$.

Example: The points $A(-5,0)$ and $B(15,-10)$ are given. Find the coordinates of the point C which divides $[AB]$ internally in the ratio of $\frac{|CA|}{|CB|}=4$.

Example: Find the coordinates of the point R which divides the line segment from $S(-4,13)$ to $T(5,-5)$ internally in the ratio of $\frac{|RS|}{|RT|}=\frac{4}{5}$.

- **Dividing Externally:**

Example: The points $K(2,3)$ and $L(4,7)$ are given. Find the coordinates of the point O which divides $[KL]$ externally in the ratio of $\frac{|KO|}{|LO|}=\frac{5}{3}$.

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Example: The points $M(-5,0)$ and $N(-2,4)$ are given. Find the coordinates of the point P which divides $[MN]$ externally in the ratio of $\frac{|PM|}{|PN|}=\frac{3}{2}$.

Exercises 1.1 – Page 20 in Zambak

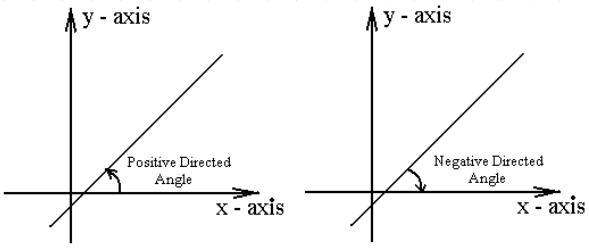
4-a,c,d , 6, 9-a,b, 11, 12, 13, 15

2. ANALYTIC ANALYSIS OF LINES

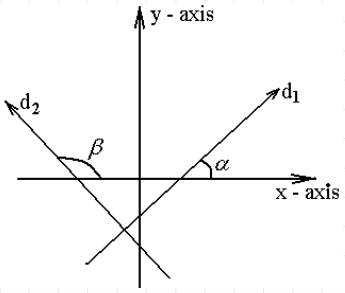
B. Slope of a Line

Inclination and Slope of a Line

Inclination of line is the positive angle formed by the positive side of x-axis and the line itself.



Slope of line is tan(inclination)



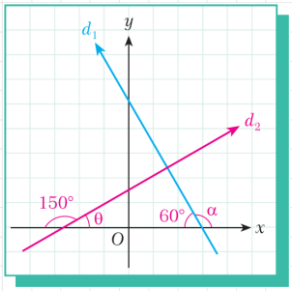
$m_1=\tan\alpha$

$m_2=\tan\beta$

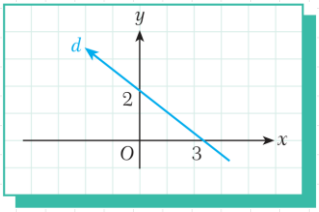
Example: Find the missing parts in following table.

Inclination	Slope
30	
	1
120	
	$\sqrt{3}$
150	

Example: Find the slope of d_1 and d_2



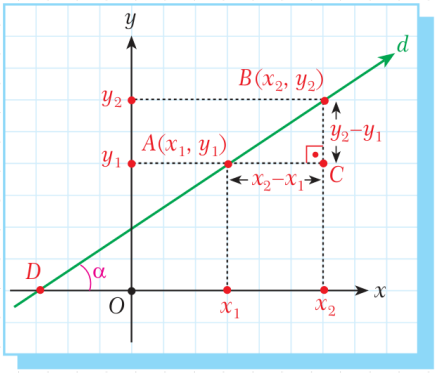
Example: Find the slope of d.



Finding the Slope of a Line

The slope of a line passing through the points $A(x_1, y_1)$, $B(x_2, y_2)$ is

$$m_{AB} = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$



Example: Find the slope of the line passsing through $A(1,7)$ and $B(-1,-3)$.

Example: The inclination of the line passing through the points $A(1,-2)$ and $B(p,3)$ is 45^o. Find the value of p.

Example: The points $A(-1,1)$, $B(2,3)$ and $C(k,5)$ are collinear. Find k.

Observation: If any three points A, B and C are collinear then their corresponding slopes are equal.

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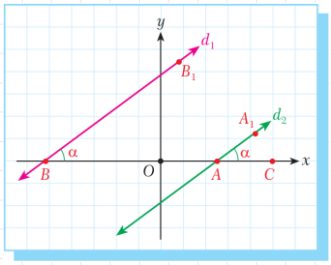
Parallel and Perpendicular Lines

- **Parallel Lines**

Two lines are parallel if and only if they have the same slopes.

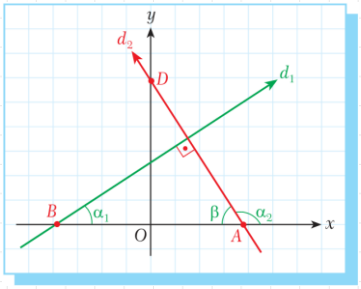


$$d_1 // d_2 \Leftrightarrow m_1 = m_2$$

Example: If $AB // CD$, find the value of *k*, where *A(-k,3), B(-7,13), C(9,1),* and D(-k-1,6).

- **Perpendicular Lines**

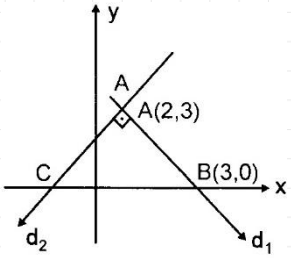
Two lines are perpendicular if they have 90 degrees in between.



$$d_1 \perp d_2 \Leftrightarrow m_1 \cdot m_2 = -1$$

Example: The points $M(1-k,3)$, $N(2,-3)$, $K(3,k)$ are given. If the lines MN and NK are perpendicular to each other, then find *k*.

Example: Find the abscissa of the point *C*.



Exercises 1.2 – Page 60 in Zambak

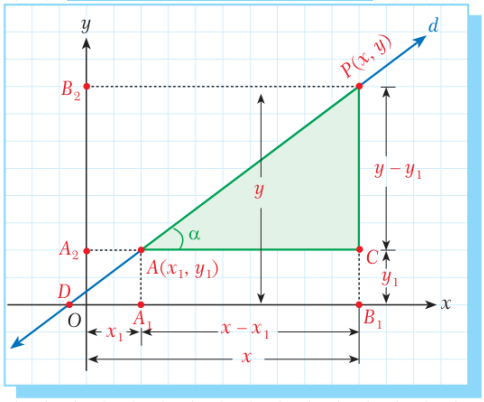
4, 5, 6-a,d, 7, 8, 9, 10, 11

C. Equation of a Line

Equation of a Line in Point-Slope Form

The equation of a line with slope ***m*** passes through the point $A(x_1, y_1)$ is

$$y - y_1 = m(x - x_1)$$



Example: Write equation of the line which passes through the point $A(1,-2)$ with slope $m=3$.

Example: Find the equation of the line which passes through the point $C(-3,-4)$ with inclination of 135^o.

Example: Find the equation of a line which is passing through $A(-2,3)$ and perpendicular to the line with slope 2.

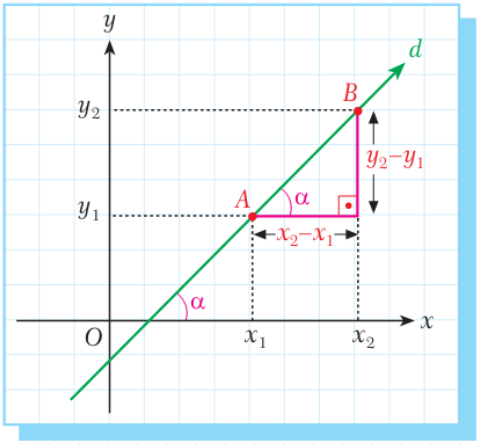
Example: Write the equation of the line passing through the point $A(4,-6)$ which is parallel to the line joining the points $B(-2,-\frac{1}{2})$ and $C(1,-1)$.

ANALYTIC ANALYSIS OF LINE

Equation of a Line in Two-Point Form

The equation of a line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$



Example: Write the equation of the line passing through the points $M(-5,1)$ and $K(7,-1)$.

Example: Find the equation of the line which passes through points $A(-4,2)$ and $B(3,7)$.

Observation: Each equation of line can be written in the form of $y = mx + n$, where m is the slope and n is the $y-$ intercept of the line.

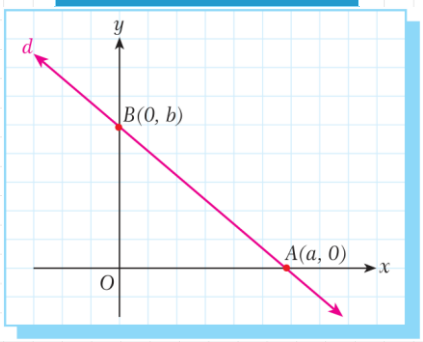
Example: Write the equation of the line which has a slope $m = -4$ and passes through $A(-3,4)$.

Example: Write the equation of the line which is passing through both $P(-2,5)$ and $Q(1,4)$.

Equation of a Line with Known Intercepts on the axes

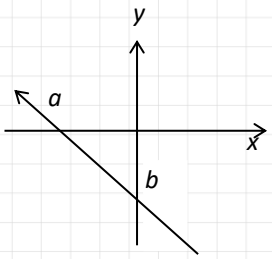
The equation of the line with x-intercept at ***a*** and y-intercept at ***b*** is

$$\frac{x}{a} + \frac{y}{b} = 1$$

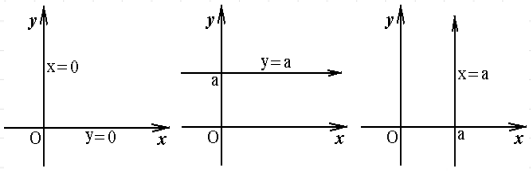


Example: Find the equation of the line whose *x* and *y*-intercepts are -4 and -3 respectively.

Example: If the equation of the line below is $3x + 4y + 12 = 0$, then find $a + b$.



Equation of the Coordinate Axes & Equation of a Line Parallel to a Coordinate Axis



Example: Show the lines passing through $A(2,5)$ which are parallel to the coordinate axes.

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Exercises 1.2 – Page 60 in Zambak

12, 13, 14, 15, 18, 20, 21, 25, 26, 28, 29, 30, 32, 34, 36, 38

D. Finding the Slope of a Line With a Given Equation

Remember that each equation of line can be written in the form of $y = mx + n$, where m is the slope.

So, we can find slope by changing format as above.

Example: Find the slope of each line.

A) $y = 3x + 5$

B) $2x - 3y + 1 = 0$

C) $-x + 3y + 1 = 0$

D) $4 - 2x = 0$

Example: Find the equation of the line passing through the point $P(2,4)$ which has the same slope as the line $2x - y + 3 = 0$.

Example: If the line $(m-2)x + y - 3 = 0$ is perpendicular to the line $x - 2y - m = 0$, then find m.

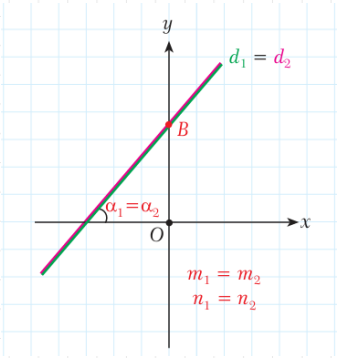
Exercises 1.2 – Page 60 in Zambak

40, 41, 42, 43

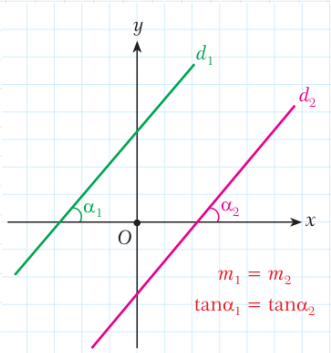
E. Relative Positions of Two Lines

$$\left.\begin{array}{l} d_1 : a_1x + b_1y + c_1 = 0 \\ d_2 : a_2x + b_2y + c_2 = 0 \end{array}\right\}$$

1. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the lines d_1 & d_2 coincide. ($d_1 = d_2$)

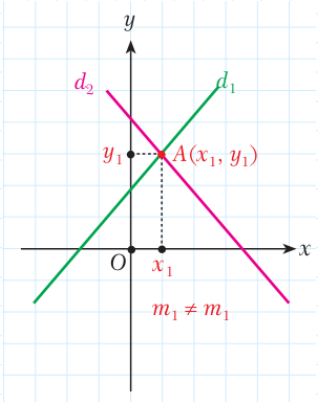


2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the lines d_1 & d_2 are parallel to each other. ($d_1 // d_2$)



3. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the lines d_1 & d_2 intersect at a point.

$d_1 = d_2$ gives intersection point.



Example: Determine the relative positions of the following three lines.

$d_1 : 6x + y + 4 = 0$

$d_2 : 3x + \frac{1}{2}y + 1 = 0$

$d_3 : -2x - \frac{1}{3}y + 1 = 0$

Example: If the lines $kx + 8y - 6 = 0$ and $2x + 4y + c = 0$ are coincident, then find the value of $k + c$.

Example: Find the intersection point of $x + y + 1 = 0$ and $2x - y + 2 = 0$.

Example: Find the intersection point of $2x - y - 4 = 0$ and $3x + y - 3 = 0$.

Example: Find the intersection point of $-2x + y + 1 = 0$ and $2y = 4x + 7$.

Exercises 1.2 – Page 60 in Zambak

47, 48, 49

Active Note Book

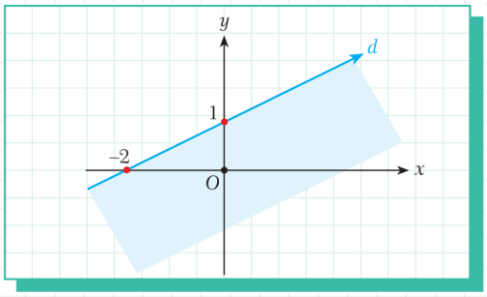
F. First Degree Inequalities in Two Unknowns

Example: Show the graph of equality $2x-3y+12=0$ on the coordinate plane.

Example: Show the solution set of inequality $x+y-2>0$ on the coordinate plane.

Example: Show the solution set of inequality $x-3y\geq 6$ on the coordinate plane

Example: Write the inequality of the graph below.



Example: Draw the graph of system of inequalities given by

$$x+2y\geq 0$$
$$2x-y-4<0$$

Exercises 1.2 – Page 60 in Zambak

52-a,c, 53-a,d, 54, 55, 59

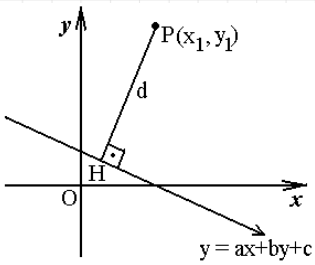
3. FURTHER APPLICATIONS

Distance from a Point to a Line

$P(x_1, y_1)$

$d_1 : ax + by + c = 0$

$$d = \frac{|a \cdot x_1 + b \cdot y_1 + c|}{\sqrt{a^2 + b^2}}$$



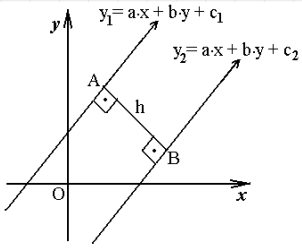
Example: Find the distance from the point $A(4,2)$ to the line $3x - 2y + 4 = 0$.

Example: Find the distance from the point $A(3,5)$ to the line $y = 3x + 5$.

Distance between Two Parallel Lines

$$\left.\begin{array}{l} d_1 : ax + by + c_1 = 0 \\ d_2 : ax + by + c_2 = 0 \end{array}\right\} \Leftrightarrow d_1 // d_2$$

$$h = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$



Example: Find the distance between the lines $-2x + 3y - 4 = 0$ and $-2x + 3y - 17 = 0$.

Example: Find the distance between the lines $y = 2x + 1$ and $2y = 4x - 3$.

Exercises 1.3 – Page 76 in Zambak

6, 7, 10, 12, 14

Active Note Book

Review Test

1) If A(a,b) is on the 3rd quadrant then which of the following is on the 2nd quadrant?

A) (-a,b) B) (-a,-b) C) (a,b) D) (a,-b) E) (b,-a)

2) What is the distance between A(-2,5) and B(10,10)?

A) 13 B) 14 C) 15 D) 16 E) 18

3) If the distance between the points A(3,5) and B(a,2) is 5 units, find the sum of the possible values of a.

A) -6 B) -1 C) 4 D) 6 E) 7

4) Let M be the midpoint of [AB] where A(2,5) and M(5,4). What are the coordinates of B?

A) (7,2) B) (7,4) C) (4,7) D) (8,4) E) (8,3)

5) In analytic plane, $A(4,8)$, $B(2,5)$, $C(7,1)$ and $D(x,y)$ are vertices of a parallelogram ABCD then find the coordinates of point D.

A) $(9,4)$ B) $(8,4)$ C) $(7,3)$ D) $(7,4)$ E) $(9,3)$

6) What is the coordinates of C which divides AB internally in the ratio $\frac{CA}{CB}=3$ if A(-4,8) and B(8,-8) are given?

A) (5,-4) B) (5,4) C) (-4,5) D) (8,4) E) (8,3)

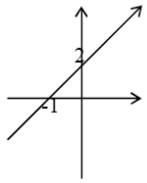
7) What is the coordinates of P which divides MN externally in the ratio $\frac{PM}{PN}=\frac{5}{2}$ if M(5,-10) and N(8,-16) are given?

A) (-10,-10) B) (10,20) C) (10,-20)
D) (-10,20) E) (-10,-20)

8) What is the slope of line $\frac{x}{3}-\frac{2y}{5}=1$?

A) 2/3 B) 3/5 C) 5/6 D) 6/5 E) 5/3

9) What is the slope of the line given in figure?



A) -1 B) -2 C) -1/2 D) 2 E) 1/2

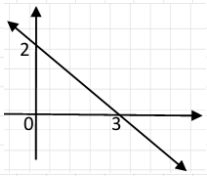
10) Find the intersection of the lines $2x+y=-5$ and $y=-x$.

A) (-5,5) B) (5,5) C) $(-\frac{5}{3},\frac{5}{3})$ D) $(\frac{5}{3},-\frac{5}{3})$ E) (-5,-5)

11) Find the equation of the line that passes through the point A(2,-1) and is perpendicular to the line $y=-2x-1$.

A) $2x-y-1=0$ B) $2x-3y-3=0$ C) $-3x+2y+1=0$
D) $x-2y-4=0$ E) $x-3y-1=0$

12) What is the equation of the given line in the figure?

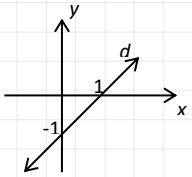


A) $3x+2y=6$ B) $2x+3y=6$ C) $3x-2y=6$
D) $y=6-2x$ E) $x-2y=6$

13) If the point A(2,3) is on the line $y=2x+n$, then find *n*.

A) 3 B) 2 C) 1 D) –1 E) –2

14) Which one of the following is the equation of the line given in the figure?



A) $y=x+1$ B) $y=x-1$ C) $y=-x-1$
D) $y=-x+1$ E) $y=-x-2$

15) The line $y=2x-3$ cuts the *x*-axis at .....

A) 2 B) $-\frac{3}{2}$ C) –2 D) $\frac{3}{2}$ E) $-\frac{1}{2}$

16) The line $2x-3y=6$ cuts the *y*-axis at .....

A) –2 B) –1 C) 2 D) 1 E) 3

17) Which one of the following lines does <u>not</u> pass through the origin?

A) $x-3y=0$ B) $y=5x$ C) $\frac{x+y}{4}=0$
D) $2x-3y=0$ E) $x+3=y$

18) If the lines given by the equations $x+2y=-3$, $2x+y=3$ and $ax+y=5$ intersect at a point, then find the value of a.

A) $\frac{3}{8}$ B) $\frac{3}{5}$ C) $\frac{8}{3}$ D) 1 E) 0

19) If the slope of the line $(2-k)x+(3k+1)y+5=0$ is $\frac{1}{4}$, then find *k*.

A) 1 B) 5 C) 7 D) 9 E) 12

20) If the lines $x-y+5=0$, $px-y+1=0$ and $3x+4y-6=0$ intersect at a point, then find *p*.

A) –2 B) –1 C) 1 D) 2 E) 3

21) If the lines $3x+my+2=0$ and $nx-6y+4=0$ coincide, then find $m+n$.

A) –3 B) 3 C) 9 D) 18 E) 20

22) The points A(-4,-1), B(6,5), and C(0,*a*) are given. If the point C is on the line AB, find *a*.

A) $\frac{7}{5}$ B) $\frac{13}{5}$ C) 5 D) $\frac{2}{3}$ E) 3

23) Find the equation of the line which passes through the intersection of the lines $3x+y-7=0$, $x-2y=0$ and which is parallel to the line $2x-y+3=0$.

A) $y=2x-3$ B) $y=2x+3$ C) $y=-2x+3$
D) $y=2x-1$ E) $y=2x+1$

24) If the line $(m-2)x+y-3=0$ is perpendicular to the line $2x-m=0$, then find the intersection point of the lines.

A) (1,4) B) (1,3) C) (2,4) D) (2,3) E) (0,0)

25) Find the distance from the point $A(4,2)$ to the line $3x-2y+4=0$.

A) $\frac{10}{\sqrt{13}}$ B) $\frac{11}{\sqrt{13}}$ C) $\frac{12}{\sqrt{13}}$ D) $\frac{13}{\sqrt{13}}$ E) $\frac{14}{\sqrt{13}}$

26) P is in the first quadrant. If the distance between the point P(m,2m) and the line $12x+5y=1$ is 5 units, then what is the value of m?

A) $\frac{43}{11}$ B) –3 C) $-\frac{65}{22}$ D) 5 E) 3

27) Find the distance between the lines $-2x+3y-4=0$ and $-2x+3y-17=0$.

A) $\sqrt{11}$ B) $\sqrt{12}$ C) $\sqrt{13}$ D) $\sqrt{14}$ E) $\sqrt{15}$

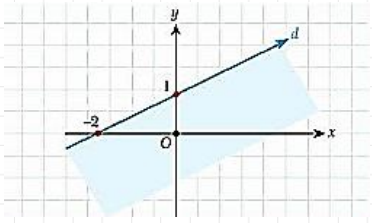
28) The lines $\frac{x}{2}+\frac{y}{6}+1=0$ and $6x+2y+c=0$ are parallel. If the distance between these lines is $\sqrt{10}$ cm, then what is the sum of the possible values of *c*?

A) 18 B) 20 C) 24 D) 32 E) 40

29) The points A(-3,4) and B(7,0) are given. Find the equation of line which is perpendicular to the line segment [AB] and bisects the line segment [AB].

A) $y-5x+6=0$ B) $2y+5x+6=0$ C) $-2y+5x-6=0$
D) $3y+2x-4=0$ E) $y-2x-1=0$

30) Which one of the following is the inequality of graph shown in the figure?



A) $x-2y+2=0$ B) $x-2y+2\geq 0$ C) $x-2y-2\leq 0$
D) $x-2y+2<0$ E) $x+2y-2>0$