



Technical Section

Perceptually inspired afterimage synthesis[☆]

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ABSTRACT

Afterimages comprise a common, recurring perceptual phenomenon experienced on a daily basis. Afterimages are best realized when staring at some high-intensity light source (i.e., a light bulb) and then changing the fixation to other less luminous portions of the scene: a temporary “ghost” image of that strong intensity remains noticeable for a period of time. During the time the afterimage stays active, several color gradations appear and fade, with little resemblance to the color that originally stimulated them. Although research on the topic has been moderately active in the ophthalmology and vision domains, no definitive model has been devised. In computer graphics, research on afterimages remains young. In this paper we propose a technique that addresses the duration as well as the luminance and color transitions of the effect, all inspired by psychophysical and physiological evidence. The main contribution of this paper is a model for color transitions in afterimages (also known as flight-of-colors). To our knowledge, no record exist on the subject in the computer graphics literature. The proposed method is also fast and suitable for real-time applications. Our stance towards afterimages is more than just curiosity on this peculiar effect and we believe that its understanding and proper simulation can assist on relevant tasks such as urban and road engineering for safer pedestrian and vehicle mobility at adverse lighting conditions.

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1. Introduction

Photorealistic rendering is the ultimate goal of computer graphics and is often associated only with the physically accurate transport of light within a scene. Another less acknowledged facet of photorealistic rendering is the simulation of perceptual effects of the Human Visual System (HVS), that is, how light is *perceived* by an individual as opposed to how light *arrives* at the retina of an observer.

A challenge behind perceptual effects is the inability to perform accurate measurements. The mechanism of how the human eye is wired together and its signals interpreted by the brain is complex and remains unknown to a considerable extent, thus narrowing studies to subjective experiments. As a consequence, for years the investigation of perceptual effects remained largely confined to a few committed vision researchers. Today many perceptual effects are already part of graphics engines and production pipelines, as can be seen in recent films, games and simulators. The graphics industry has noticed that the reproduction of such

effects, due to their subjective nature, does not need to be accurate to feel convincing.

One common perceptual effect in contemporary computer graphics is tone-mapping, which attempts to overcome the physical limitations of display devices in order to present better contrasting images; tone-mapping operators are largely inspired by the dynamic light adaptation mechanism of the human visual system. More familiar instances of perceptual effects in graphics include depth-of-field, motion-blur and bloom. Another frequently encountered effect, glare, caused by the diffraction of light at the eyelashes, is often reported as a perceptual effect due to its attachment to the physiology of the eye, although it is mostly a physical consequence of light-matter interaction [1].

A perhaps more curious perceptual effect experienced on a daily basis is the one behind the formation of *afterimages*, which is the focus of study of this paper. Often an observer finds herself staring at strong light intensities in a scene (e.g., a car's headlight at night) and then shifts her attention to a dimmer region in order to avoid the discomforting stance. As a result, a still image of that strong intensity remains imprinted in the retina. The patterns present on this burnt image vary in tonality and duration, with peculiar colors transitioning in succession, afar from the original tone that produced them. It is believed that afterimages are a byproduct of photochemical bleaching within the photoreceptor

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cells of the retina [2]. Several researchers have ventured into the physiological cause of the effect, but the precise mechanism is still unknown.

Afterimages can be categorized as either *negative*, where the bright and dim parts become reversed, or *positive*, with bright and dim parts of the afterimage corresponding in intensity to those of the originally observed scene. Negative afterimages are generally better understood than positive ones. The main cause is attributed to the fatigue of the visual system when overstimulated by some static, average-intensity pattern over an extended period of time (of the order of dozens of seconds). Since the human visual system is built upon an opponent-color system, it responds to this form of visual fatigue by increasing the opponent stimuli of the overstimulated colors, thus producing a negative imprint. A whole category of popular optical illusions is attributed as a consequence of negative afterimages.

Positive afterimages, on the other hand, are perceived after glancing at high intensity features, such as light sources, even after very short periods of exposure. The viewer then experiences some temporal blindness at the observed region where the positive afterimage is formed. In due time, the afterimage fades, but this change does not happen abruptly neither uniformly. Instead, the afterimage gradually changes in color and intensity as it fades, introducing colors that little resemble the original tone that generated them. The innards of positive afterimages in the human visual system are still obscure, and this remains an open research topic for physiologists [3,4].

This paper focuses on the less understood *positive* afterimages, and any subsequent reference to the term afterimage alone in this paper refers to a positive afterimage. The phenomenon investigated in this paper should not be confused with *light trail* shaders which only simulate the fading of *comet-like* trails in motion blur effects without caring on how the color transitions happen in the afterimage [5].

The proposed method deals with the *luminance transition* (duration) and the *color transitions*—also known as *flight-of-colors*—of the afterimage effect. Our approach is inspired by previous psychophysical experiments and physiological evidence, and is capable of generating plausible afterimages even though the exact mechanisms behind the phenomenon remain relatively unclear. The technique is *fast* enough to be incorporated into existing real-time graphics pipeline. We feel that simulating afterimages transcends pure aesthetics and can be a valuable asset on urban and road engineering efforts for safer pedestrian and vehicle mobility at adverse lighting conditions.

The remaining four sections of this paper are organized as follows: [Section 2](#) presents the relevant related work on afterimage physiology and simulation; [Section 3](#) is dedicated to describing our proposed model for digital afterimage synthesis in detail; [Section 4](#) offers results and a discussion on the proposed color transition scheme. Finally, [Section 5](#) concludes the paper and points future research directions.

2. Related work

The material reviewed here is mostly comprised of investigations of subjective matter. The precise neurophysiological mechanism that drives afterimage formation is only partially understood, although recent physiological clues seem to point in the right direction. Nonetheless the research initiatives listed here have motivated us to apply some of these theories and experimental/subjective data to a computer model in an attempt to synthesize believable afterimages. Therefore, the technique described in this paper should by no means be taken as a definitive solution for afterimage simulation, but rather as a

plausible, fast and accessible framework that investigates the phenomenon in the computer graphics domain.

Renowned natural philosopher Sir Isaac Newton was so curious about afterimages to the point of becoming a victim of the phenomenon [6]. The scientist stared directly at the sun for prolonged periods of time, then turning his attention into dark corners of his chamber in order to observe the chromatic transitions. This eventually caused temporary damage to his retina that lasted for many months (clinically, a solar scotoma). Newton is said to have “left the sun alone” after the incident. The German poet and natural scientist Johann Wolfgang von Goethe attempted a detailed description of afterimages in his *Theory of Colours* [7]. After gazing at the reflex of the sunlight upon a sheet of paper, he took notes as he observed the color transitions of the resulting afterimage.

Since then many researchers have performed experiments to retrieve clues regarding afterimage formation. Titchener [8] conjectured that afterimages were caused by excitatory-inhibitory chains in opponent-color systems, inspired by the ideas of Ewald Hering (the actual presence of opponent systems in the HVS was only validated decades later by Hurvich and Jameson [9]). The observations of Titchener paved the ground for explaining negative afterimages, although the same ideas could not be asserted for positive afterimages. Shuey [10] compiled a survey of several experiments on afterimages. Following that, *color naming* methods were used to record chromatic transitions at largely *discrete* steps [11]. For example, a positive afterimage caused by exposition to white light yields to transitions of blue, yellow, green, carmine red and blue-green, but little was recorded regarding the *continuous* transitions between these colors [11]. An important finding in his research was the recurring pattern of a particular set of colors in the afterimage transitions, regardless of the initial input color stimuli that generated them.

More sophisticated studies were later performed by Padgham, using color matching techniques to study the continuous transitions between colors in afterimages [12], as well as luminosity transitions and the overall duration of the phenomenon based on the total exposure time [13]. Padgham, however, only registered his own subjective experience without experimenting with other test volunteers. He was careful nonetheless to consistently validate his own observations through multiple occasions over a period of several months. In his experiments, luminance transition and color transition were measured using a binocular matching technique. The subject (himself) was flashed the stimulus light to his left eye to form an afterimage. Then by adjusting a reference light on the right eye so that it matched the afterimage in his left eye, he was able to record color and intensity transitions in a color diagram. Different color filters were used to change the color of the reference white light.

Reidenbach examined the afterimages resulted by exposing test subjects to different LED lights [14]. He measured the color transition of the afterimages under a combination of several LED color, optical power and exposure time conditions. The subjects were asked to match the experienced color transitions of the afterimages with a color plate on a monitor. An example of the color transition registered from a green LED is shown in [Fig. 9\(a\)](#). The ultimate goal of his research is to manufacture better LEDs for equipments that are less prone of causing uncomfortable afterimages on the user.

The research reviewed so far deals mostly with psychophysical observations on afterimages and are thus subjective in nature. Physiological experiments on afterimages were also developed in an attempt to unveil the exact essence of their occurrence. Although there is still no satisfactory answer, the kinetics of the photoreceptors in the visual system seem to provide the best clues on the formation of afterimages. The concentration of photopigments in photoreceptor

cells is driven by a bi-directional photochemical reaction, causing these pigments to constantly *bleach* and *regenerate* in response to light. When stimulated by light, photopigments are released (bleached) within the photoreceptor cells. This pigment bleaching is capable of stimulating the visual nerves and empowering us with vision. At the same time, new pigments are composed (regenerated), just to eventually bleach again. The rate at which bleaching and regeneration occur depends on the intensity of the light arriving at the photoreceptor. These cells are prone to bleaching pigments much faster than they can regenerate when overstimulated by a strong light [15–17]. One byproduct of bleaching is the change of sensitivity in the photoreceptors, as observed and simulated by Gutierrez et al. [18]; these changes are expected to influence the visual perception of afterimages. Afterimages are likely to appear when the regeneration rate is much lower than the bleaching rate. As the bleach-regenerate reaction restabilizes, the afterimage gradually dissolves [2].

Interestingly, in the computer graphics field, two pioneering attempts to simulate afterimages appeared independently in the same year and in the same conference [4,19]. The present paper extends the results of the short paper of Mikamo et al. [19], and their technique is introduced in greater detail as the paper develops in the subsequent sections.

Ritschel and Eisemann devised a model for synthesizing afterimages based on the photoreceptor kinetics described above [4]. A differential equation is then solved to provide the luminosity changes in the afterimage, but color transition is left unaccounted. Their main purpose is to establish a computational model of afterimages based on chemical reactions in the eyes: bleaching and regeneration. Although their method does not account for color transitions, their model is likely compatible with the flight-of-colors scheme introduced in this paper. The afterimage mask is also blurred to give a fuzzy, low-frequency appearance to the afterimage. In a last step, the computed afterimage is superimposed onto the LDR image to produce the final result. The use of photoreceptor kinetics makes it possible to change the intensity of afterimages as time varies. In addition, the blur size of the afterimage varies with time, further contributing to the fading effect. Since their method is unable to handle color transitions, they compensate this limitation by using the opponent color of the stimulus as the color of the afterimage for each particular pixel.

The color remains the same throughout the entire simulation, only its intensity varies. Finally, through some psychophysical experiments with test subjects, they found that afterimages can substantially alter the perceived brightness of scenes.

3. Proposed method

In this paper we model luminance transitions by fitting the psychophysical data of Padgham [13] into an exponential function. This function behaves according to the rules of the photoreceptor kinetics, while combining subjective elements [11,14] into the model. The function is also more lightweight to compute than the involved differential equations. More importantly, we build upon the initial attempts of Mikamo et al. [19] to simulate color transitions in afterimages. The present method incorporates the subjective results of different sources [11,12,14] into the model to better simulate the *flight-of-colors*—the non-uniform color transitions that characterizes afterimages. Our model is thus capable of delivering a robust way to reproduce all of the important features of afterimages originated by light at any wavelength.

An overview of our entire afterimage reproduction algorithm is shown in Fig. 1, depicting the majority of the symbols and definitions that will be introduced throughout this Section. First, we compute a difference image ΔI_{on} between the original masked color I_{on} with the reference light color used by Padgham [12] in his experiments, I_{on}^R . We then use this difference image to determine the actual pixel colors for the resulting afterimage.

The rationale behind this difference image comes from the fact that Padgham based his experiments on a single isoluminant light pattern, of a characteristic warm yellowish hue. This particular tone most certainly does not match with the high intensities tones contained in the scene captured in I_{on} . In order to compensate for this mismatch, we use Padgham's fixed light color and intensity as a starting *reference* and then incrementally *extrapolate* the synthesized afterimage based on the differences ΔI_{on} between I_{on} and how the same scene *would look like* if using only that reference light color.

Since visualizing afterimages require an observer to stare at high intensity regions and then shifting the focus to dimmer regions, the eye mechanism of light adaptation—from bright to

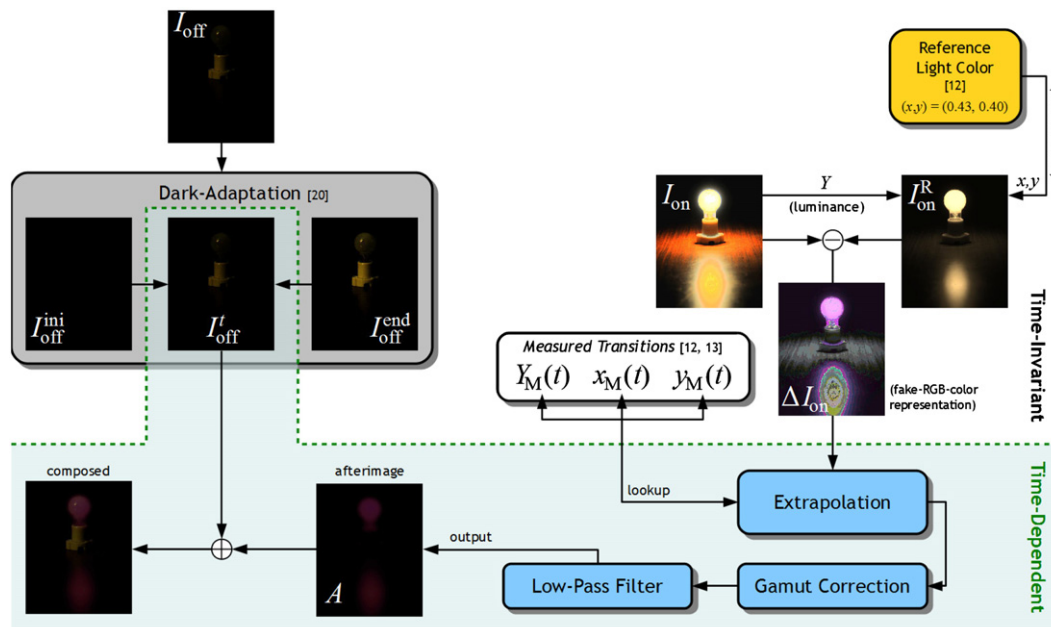


Fig. 1. Overview of the proposed afterimage simulation algorithm.

dark—is also active during the effect of the afterimage. Therefore, for completeness and additional realism, we have also incorporated a dark adaptation mechanism to our method, as described by Durand and Dorsey [20]. This way, while the afterimage is transitioning, the background scene slowly adapts to low lighting conditions on which the afterimage is then superimposed with simple additive color blending.

The following subsections discuss in detail each of the steps involved in the proposed afterimage simulation model, in order.

3.1. Input images

The algorithm requires a single input image I_{on} . High dynamic range (HDR) imagery is preferred, but the process also works with traditional LDR images. Our model ensures that the synthesized afterimage overlay is in a proper displayable range even for HDR inputs. Luminance compression (tone mapping) is still necessary for displaying the background image on which the afterimage is imposed, though. As mentioned before, we also employ a dark adaptation mechanism for improved realism. We used the algorithm proposed by Durand and Dorsey [20] to that end. A snapshot of the dark-adapted background I_{off} is made necessary. In a typical computer graphics simulation, this would be the dimmer background scene on which the afterimage is imposed.

3.2. Computing the reference light mask

At its core, our color transition scheme for afterimages is based on the observations of Padgham [12]. This step concerns on recreating the intensities of the I'_{on} mask in terms of the reference light used by Padgham, producing I^R_{on} . In short, this step requires combining the luminance component of I'_{on} with the chromatic component of that reference light. For this goal, the Yxy color space (which derives from the CIE XYZ color space) is particularly useful since it is capable of decoupling luminance and chrominance information. Therefore, the luminance component Y of each pixel in I^R_{on} is the same as in I'_{on} , but the chromatic components x and y come from Padgham's reference light, and remain constant throughout all pixels of I^R_{on} , namely $(x,y) = (0.43, 0.40)$.

3.3. Computing the color difference mask

A compilation describing the flight-of-colors of afterimages for all possible input light wavelengths is not likely to appear any-time soon. Our method attempts to extrapolate color transitions for arbitrary light colors building upon the recorded experience of Padgham [12] on a particular light hue.

A per-pixel difference between I_{on} and I^R_{on} is computed in the CIE *Lab* color space, as defined below (the *Lab* asterisks are omitted for clarity):

$$\Delta a(i,j) = a_{on}(i,j) - a^R_{on}(i,j) \quad (1)$$

$$\Delta b(i,j) = b_{on}(i,j) - b^R_{on}(i,j) \quad (2)$$

where $a_{on}(i,j)$ and $a^R_{on}(i,j)$ correspond to the a component of the *Lab* triplet of the pixel located at (i,j) in the I_{on} and I^R_{on} images, respectively (notation is analogous for b). The motivation behind this particular color space comes from the fact that afterimages are associated with the opponent-color systems in the HVS: the *Lab* color system is not only an opponent-color space, but also keeps color distances in an HVS-compatible and nearly perceptually uniform fashion.

Basically, this difference image ΔI_{on} is an attempt to make a correspondence between the color of each pixel in the input image I_{on} with those in the reference light color in I^R_{on} . This difference

image is the key for the extrapolation stage that follows later, in which the actual afterimage is synthesized. Our extrapolation scheme is inspired by the results of Weve [11], who observed that the color transitions of afterimages induced by different light color stimuli tend to converge to a common color transition palette.

3.4. The luminance transition

Luminance transitions in our model follow the experimental results of Padgham in [13]. The luminance decay curve is depicted in Fig. 2 and can be approximated by the following function:

$$Y_M(t) = \begin{cases} e^{-0.083t-0.978} & (t \leq 22) \\ e^{-0.055t-1.584} & (t > 22) \end{cases} \quad (3)$$

The exponential nature of the approximation comes from the solution of differential equations which represent the photochemical reaction of the photopigments in the photoreceptor cells. Note that the intensity of afterimages is measured in relative units of retinal illumination (Td). The luminance transition curve is what drives the fading effect of the afterimage over time, as will be explained later.

3.5. The color transition

Color transitions in our afterimages are derived from the experiments of Padgham [12], as plotted in Fig. 3 and in the color diagram of Fig. 5 (in blue).

Expressing these curves analytically would yield an unnecessarily complicated function. We therefore express these transitions as a simple one-dimensional array of values, with a uniform sampling period of 4 s (Table 1). Intermediate values can be

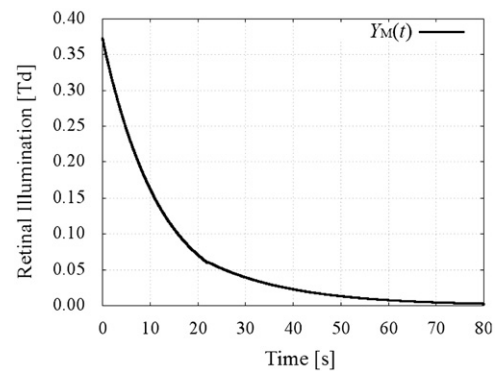


Fig. 2. Luminance transitions of afterimages in the Yxy color space according to the elapsed time as experienced and recorded by Padgham [13].

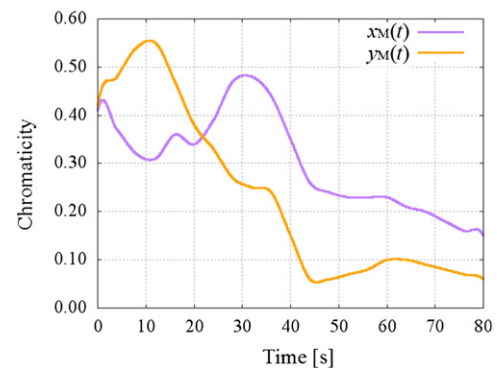


Fig. 3. Color transitions of afterimages in the Yxy color space according to the elapsed time as experienced and recorded by Padgham [12].

Table 1
The color transition.

Time[s]	x_M	y_M	Time[s]	x_M	y_M
0.0	0.41	0.42	44.0	0.26	0.06
4.0	0.37	0.48	48.0	0.24	0.06
8.0	0.32	0.54	52.0	0.23	0.07
12.0	0.31	0.55	56.0	0.23	0.08
16.0	0.36	0.47	60.0	0.23	0.10
20.0	0.34	0.38	64.0	0.21	0.10
24.0	0.39	0.33	68.0	0.20	0.09
28.0	0.47	0.27	72.0	0.18	0.08
32.0	0.48	0.25	76.0	0.16	0.07
36.0	0.44	0.24	80.0	0.15	0.06
40.0	0.35	0.15			

obtained by using some interpolating scheme at the enclosing samples. In our experiments we used Catmull-Rom interpolation to retrieve smoothly interpolated data.

If the light stimuli in I_{on} were identical to that of the reference light of Padgham [12], that is, if it so happens that $I'_{on} = I_{on}^R$, then using the luminance and color transitions of Figs. 2 and 3 would produce the desired afterimage. However, as discussed before, this is very unlikely to be true and information of the distinct colors of I'_{on} must be taken into account somehow.

Before we proceed, an important remark is that in the experiments of Padgham [12] he was unable to specify the color transition in the first 20 s after turning off the light. According to the author, “The eyes were dark-adapted for 10 min prior to each exposure to the stimulus. (...) it is not possible to make measurements for about 20 s after the presentation of the stimulus. This is the order of time necessary to allow the extreme sensation of glare to subside, and for the after-image color to be perceived.” [12].

In our model, however, we assume a less restrictive light adaptation condition, i.e., not requiring the observer to be dark-adapted for such a long period. To that end, the time step zero in Fig. 3 actually corresponds to the time step of 20 s in Padgham’s observations. For consistency, the same applies for the luminance transition previously discussed (see Fig. 2).

3.6. Computing the afterimage

This subsection details the extrapolation strategy we devised to infer the flight-of-colors for input colors that differ from the one of the reference light. The idea is to apply the color transition scheme described above, as if simulating the afterimage only for I_{on}^R instead of I_{on} . We use that result as a base ground on which we will incrementally enhance by adding image-specific details on top of it. These image-specific details are encoded in the ΔI_{on} image computed previously.

An afterimage frame A at some instant t can be obtained, in the Lab color space, as follows:

$$L_A(i,j)_t = L_M(t) \cdot Z(i,j) \quad (4)$$

$$a_A(i,j)_t = a_M(t) \cdot Z(i,j) + \tau_M(t) \cdot \Delta a(i,j) \cdot k \quad (5)$$

$$b_A(i,j)_t = b_M(t) \cdot Z(i,j) + \tau_M(t) \cdot \Delta b(i,j) \cdot k \quad (6)$$

with

$$Z(i,j) = \left(\frac{L_{on}(i,j)}{L_{on}^{max}} \right)^\gamma$$

where $a_M(t)$, $b_M(t)$ and $L_M(t)$ are retrieved from the original measured experiments of Padgham, as depicted in Figs. 2 and 3 (conversion from Yxy to Lab is implicit).

The effect of $Z(i,j)$ in the model is so that high intensity areas brings up strong afterimages. This function is simply defined as a ratio with respect to the maximum luminance L_{on}^{max} of the entire input image. Since it is confined to the range of $[0,1]$, the resulting luminance of the afterimage is guaranteed to be in the displayable range of the display device. The parameter γ offers some flexibility for adjusting the ratio of the relationship in a non-linear fashion, and its effect can be observed in Fig. 4. Higher values for γ

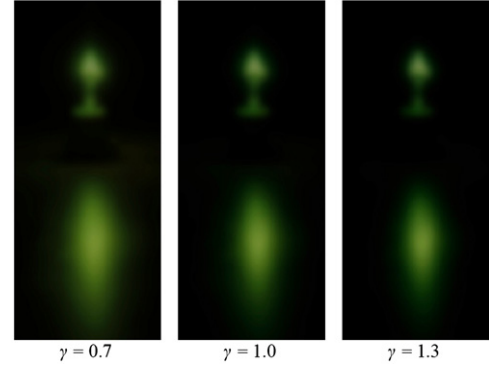


Fig. 4. Visual impact of the parameter γ in the afterimage simulation. In all images, $t=0$ and $k=0.05$.

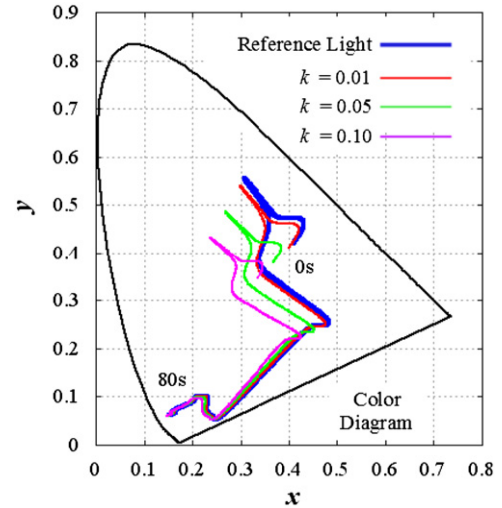


Fig. 5. The afterimage color transition (flight-of-colors) produced from a blue input light stimulus. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

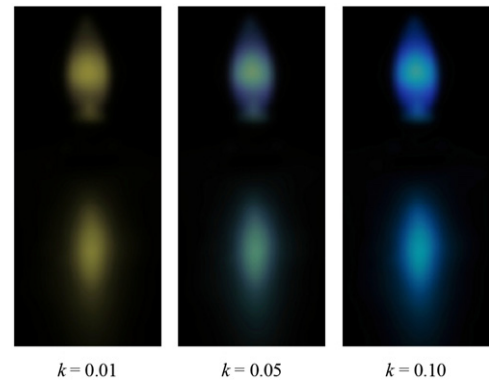


Fig. 6. Visual impact of the parameter k in the afterimage simulation. In all images, $t=0$ and $\gamma=0.7$.

wash out the afterimage color and spread its influence, while lower values make colors more saturated.

The term $\tau_M(t)$ is simply the function $L_M(t)$ normalized to the range (0,1), that is, $\tau_M(t) = L_M(t)/L_M(0)$. Just as $L_M(t)$ controls the gradual fading of the afterimage luminosity, $\tau(t)$ controls the fading

of the chrominance component. To put it in perspective, $\tau(t)$ assumes its maximum value when $t=0$, that is, $\tau(0)=1$. At this instant, $\Delta a(i,j)$ and $\Delta b(i,j)$ will have maximum effect on the colors of the afterimage. The trailing parameter k can be used to control the color transitions. Large values for k make the initial colors of

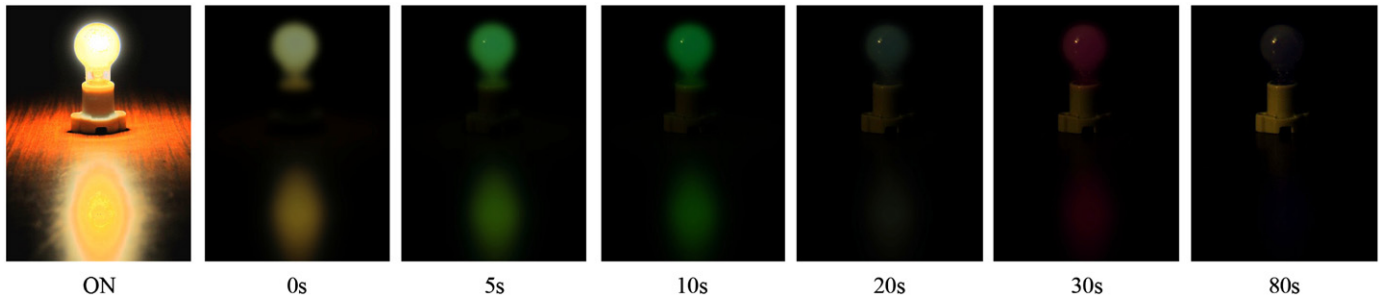


Fig. 7. Afterimage transitions of a light bulb, whose color is similar to the reference light of [12]. The elliptical highlight below the light bulb is due to light reflection on the wooden table on which the socket was placed.

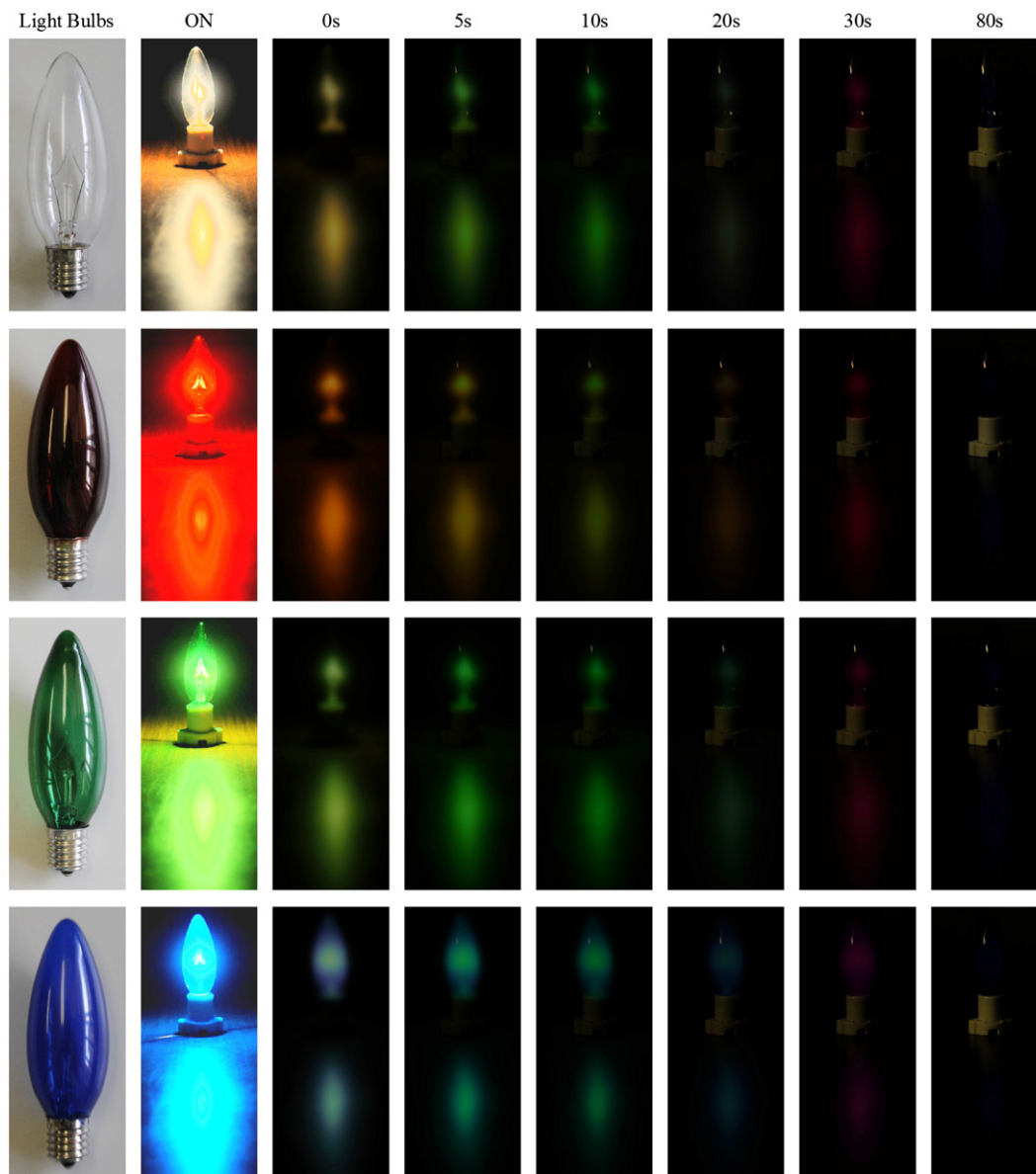


Fig. 8. Afterimage simulation at various discrete time steps of different colored light bulbs simulated according to the proposed technique.

the afterimage to have higher resemblance to the original colors of the stimuli that generated it (Fig. 6). As time passes, the intermediate colors tend to converge to the same set of transitions, independent of the value of k , as demonstrated in Fig. 5.

Note that the parameters γ and k are determined empirically, as we lack substantial evidence to derive an automatic estimator. Until more accurate measurements of afterimages are made available through research, these parameters would serve as compensation factors for the color transition data currently available.

The set of equations defined above has two desirable properties: (i) the original input colors are visible at the early stages of the simulation, as observed by Reidenbach [14]; and (ii) the color transitions of afterimages caused by any color stimulus tend to converge to the same unique intermediate transition, as pointed by Weve [11].

3.7. Gamut correction

Even though the luminances of the afterimage are guaranteed to be in the displayable range, the same cannot be said for its chrominances. When the *Lab* triplet obtained from Eqs. (4)–(6) is converted to *RGB* for displaying purposes, some pixels may lie outside of the supported gamut of the display device. This happens because the *Xy* measurements of Padgham—in particular the *xy* color components—are HVS-related and therefore potentially broader than what display devices are capable of presenting. Consequently, the resulting *RGB* triplet may contain negative values. We fix this issue in two simple steps: first we clamp any negative *RGB* components to zero; second, since clamping affects the luminance of the pixel, we force the pixel luminance to that before the pixel was corrected. This last step is accomplished in the *Lab* color space.

Our model can also be made device-agnostic by using the device-independent color transform scheme proposed by Reinhard et al. [21].

3.8. Blurring

At this point, the afterimage is almost ready to be displayed. The missing task is to apply a low-pass filter to the afterimage frame using a fixed-size Gaussian profile. This is necessary because afterimages do not retain much high-frequency details and tend to be perceived blurry. Gaussian profiles fit well when modeling aspects of the visual phenomena (e.g. lateral inhibition [22]) and are thus welcome to fulfill the perceptually based requirements of our simulation.

3.9. Dark adaptation

Finally, and for completeness only, we have decided to additively blend the synthesized afterimage A on top of a gradually dark-adapting image computed from I_{off} . The process of dark-adaptation that we have employed is described in [20]. In short, dark-adaptation is performed by simple interpolation of two images (derived directly from I_{off}) with an exponentially decaying blending weight that varies according to the elapsed time t . This process is illustrated in Fig. 1 (left).

Durand and Dorsey originally used a rod and cone adaptation model to interpolate chrominances and luminances between a light-adapted image and a dark-adapted one. We, instead, simply interpolate between two luminance images $I_{\text{off}}^{\text{ini}}$ and $I_{\text{off}}^{\text{end}}$. The dark image $I_{\text{off}}^{\text{end}}$ is the same as the light one $I_{\text{off}}^{\text{ini}}$, but scaled by a small constant. Only luminance has to be interpolated since the chrominance is the same for both images.

As for the interpolation function itself, we use the same blending procedure that Durand and Dorsey used. The dark

adaptation process is formulated by the following equation:

$$Y_{\text{off}}^t(i,j) = Y_{\text{off}}^{\text{end}}(i,j) + (Y_{\text{off}}^{\text{ini}}(i,j) - Y_{\text{off}}^{\text{end}}(i,j))e^{-t/T} \quad (7)$$

where, Y_{off}^t , $Y_{\text{off}}^{\text{end}}$, and $Y_{\text{off}}^{\text{ini}}$ are the luminance of I_{off}^t , $I_{\text{off}}^{\text{end}}$, and $I_{\text{off}}^{\text{ini}}$, respectively. Just after the light is turned off ($t=0$), the background is completely dark ($Y_{\text{off}}^{\text{ini}}$). As time passes, the vision starts to adapt to the scene luminance ($Y_{\text{off}}^t(i,j)$), until it eventually fully adapts to the dark lighting condition ($Y_{\text{off}}^{\text{end}}$).

4. Results and discussion

An example of an afterimage simulation at different time steps using our technique is shown in Fig. 7. The color of the light bulb is similar to the color of the reference light used by Padgham [12]. For this example, we set $\gamma = 1.0$ and $k=0.05$. At the beginning of the simulation, the colors of the afterimages are similar to the ones coming from the light bulb; as noted by Reidenbach, this is the expected behavior of afterimages [14]. The simulation then transitions to green, bluish green, purple and red before fading completely. This flight-of-colors is not possible in the afterimage model suggested by Ritschel and Eisemann [4]. Also note the effects of dark-adaptation: at the beginning of the simulation, nothing can be seen on the background, but as the simulation progresses the socket of the light bulb gradually turns visible.

Additional results of the proposed afterimage technique are shown in Fig. 8, for different light bulb colors. The wire filament of each light bulb is the same, which means that the observed light color is a consequence of the surrounding glass paint coating (see the leftmost column of Fig. 8). For printing purposes, the HDR input images (I_{on}) of Fig. 8 were tone mapped. For all of the simulations, we set $\gamma = 0.7$ and $k=0.05$. At the beginning ($t=0$) each afterimage inherits a color similar to the one of the light bulb that originated

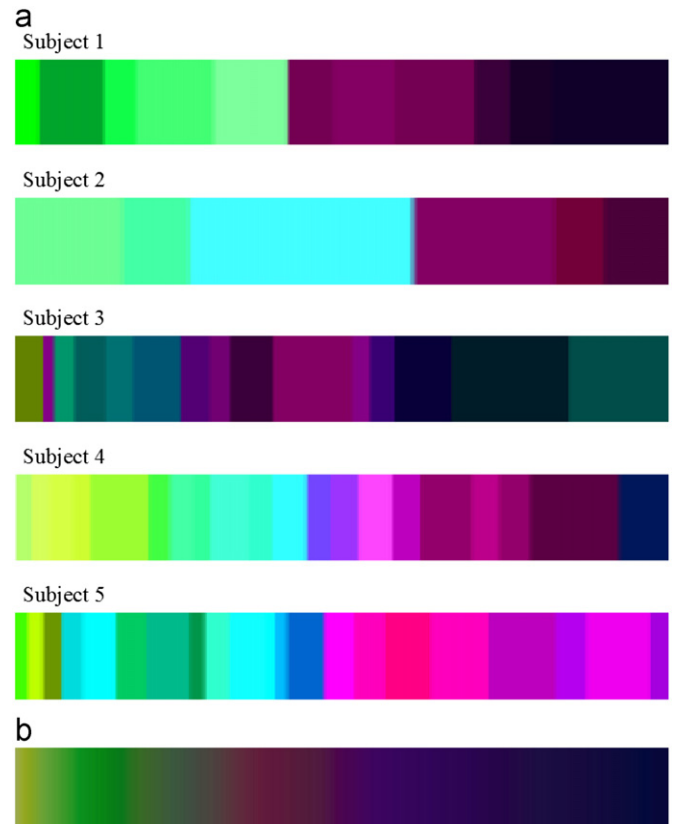


Fig. 9. Comparison of afterimage color transitions between (a) five experimental results of Reidenbach [14] and (b) our model.

them. As the simulation runs, the color transitions between the afterimages become more similar to each other. This convergence behavior is in accordance with the experimental results of Weve [11]. Currently our afterimage algorithm does not take the stimuli exposure time into account.

We have also compared the flight-of-colors obtained with our technique to those registered by Reidenbach [14]. Fig. 9(a) shows the subjective experience reported by each of the five Reidenbach's test subjects when exposed to a 530 nm green LED light. Fig. 9(b) shows the result obtained with our model. Unfortunately we were unable to acquire an equivalent LED light to compare our simulated afterimages against Reidenbach's description. We have instead used a regular green colored light bulb as shown in Fig. 8.

Each flight-of-colors recorded by Reidenbach was assembled based on a multi-color palette presented to the subjects on a computer monitor, from which they were able to describe their experience by pointing to colors on that monitor. From left to right, they represent the color of the afterimage experienced by each subject as time passed. Discontinuities on the luminosity of these transitions are due to the discrete nature of the displayed palette. Although the subjective judgments of each volunteer seem to differ greatly, a few recurring patterns can be observed. For example, all afterimages were reported to start at some green/yellowish tone, and in most of them the occurrence of turquoise, cyan and purple can be verified orderly. Our result in the bottom shares many similarities

with the ones of Reidenbach, despite the apparent luminosity contrast due to his palette. It is possible to identify temporal resemblance in his color transitions to ours.

We have also applied our method to less controlled environments. Fig. 10 presents the results of our simulation when applied to urban traffic lights at nighttime conditions. Afterimages for both the green/blue and red lights of the semaphore are depicted. Fig. 11 shows our technique applied to the popular blinking-lights ornaments of Christmas trees. A total of four blinking states can be produced in the ornament we used: (i) all lights are on; (ii) all lights are off; (iii) blue and green lights are turned off; and (iv) red and yellow lights are turned off. Afterimages for all of these configurations are illustrated in the picture.

5. Conclusion and future work

This paper introduces a technique for rendering positive afterimages, a common perceptual phenomenon experienced on a daily basis. In computer graphics, afterimage simulation remains a fresh research topic, being unveiled just very recently. The proposed method is robust enough to simulate the most prominent features of afterimages—namely, luminance and color transitions—in a believable fashion for arbitrary light wavelengths.

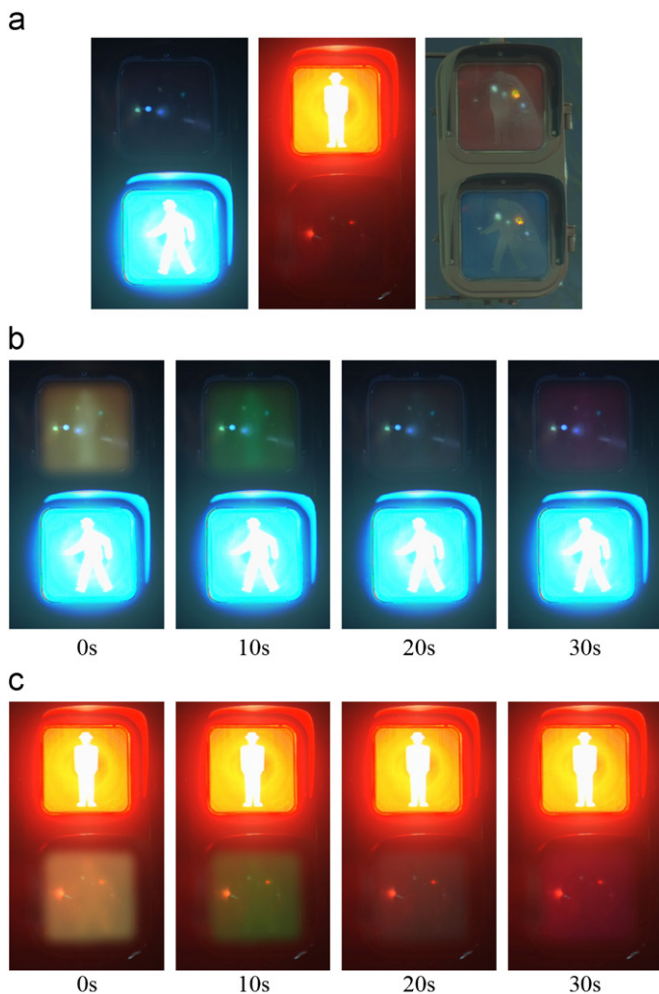


Fig. 10. The proposed afterimage synthesis algorithm applied to real traffic light photographs: (a) input images, (b) simulated afterimages caused by a red traffic light, and (c) simulated afterimages caused by a blue traffic light. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

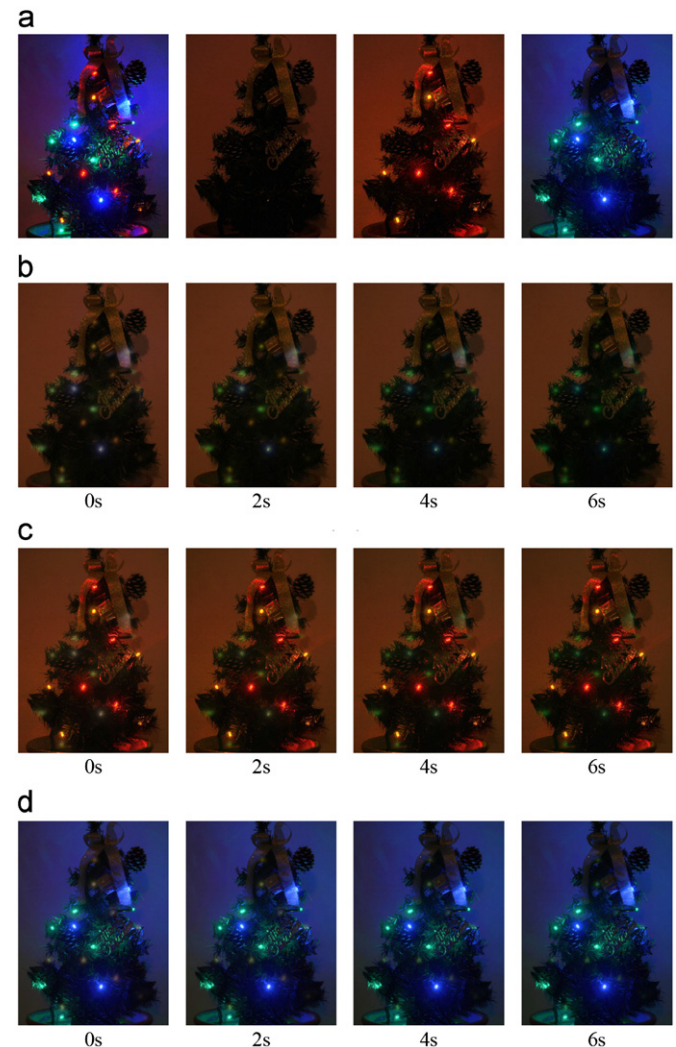


Fig. 11. The proposed afterimage synthesis algorithm applied to ornamental blinking-lights: (a) input images, (b) simulated afterimages made from all lights being turned off, (c) simulated afterimages made from blue and green lights being turned off, and (d) simulated afterimages made from red and yellow lights being turned off.

Our model is also perceptually inspired, building upon previous psychophysical experiments and physiological evidence of several researchers [11–14], thus enabling the technique to reproduce a plausible flight-of-colors. Our algorithm is also orthogonal to other perceptually based techniques, as demonstrated with the dark-adaptation mechanism, meaning that it can coexist non-intrusively with other perceptual effects in the application pipeline.

Another advantage of our model is its independence from any color transition data source. So far, our framework makes use of Padgham's experimental data, but this data could be easily replaced by more accurate measurements as research in the field progresses and more physiological evidence is gathered.

As an application, we believe that a proper simulation of afterimages can benefit urban and road engineering, where safer pedestrian and vehicle mobility at adverse lighting conditions is a constant issue. In addition, since the technique is fast enough to suit the needs of interactive applications, it can also be applied in games and driving simulators to improve immersion and set mood.

As a future work, we would like to investigate the impact of the total light exposure time on the overall afterimage duration. So far we have strictly followed the experimental data of Padgham [12,13], and thus limited our afterimage simulation to exposure times of about 1 s. We also would like to further validate the flight-of-colors through subjective experiments with test subjects.

Additionally, we would like to devise an automatic parameter estimation strategy for the parameters γ and k , which are currently being specified empirically.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version of <http://dx.doi.org/10.1016/j.cag.2013.02.008>.

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