

$$i) \operatorname{erf}(x) = \frac{4}{\pi} \int_0^{x^2} \int_0^{\sqrt{v}} e^{-(u^2+v^2)} du dv$$

$$= \frac{1}{\pi} \int_{-x^2}^{x^2} \int_{-\sqrt{v}}^{\sqrt{v}} e^{-(u^2+v^2)} du dv$$

$$\operatorname{erf}(x) \geq 1 - e^{-x^2}$$

$$x \rightarrow \infty$$

$$\operatorname{erf}(x) \geq 1 - \frac{1}{e^{x^2}} \rightarrow 1$$

$$ii) F_X(x) = \int_{-\infty}^x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

$$= \int_{-\infty}^0 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt + \int_0^x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$

$$\frac{t-\mu}{\sigma\sqrt{2}} = y, \quad \frac{1}{\sigma\sqrt{2}} dt = dy$$

$$= \frac{1}{2} + \frac{1}{\sqrt{\pi}} \int_0^{\frac{x-\mu}{\sigma\sqrt{2}}} e^{-y^2} dy$$

$$= \frac{1}{2} + \frac{1}{2} \frac{1}{\sqrt{\pi}} \int_0^{\frac{x-\mu}{\sigma\sqrt{2}}} e^{-y^2} dy$$

$$= \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right) = \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right)$$