Hw 6

$$f_1(t) = \lambda e^{-\lambda t}$$

$$f_{X_2}(t) = \lambda e^{-\lambda t} * \lambda e^{-\lambda t}$$

$$= \int_0^t \lambda e^{-\lambda(t-\tau)} \lambda e^{-\lambda \tau} \, d\tau$$

$$f_{X_k}(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!} = \frac{(\lambda t)^{k-1} \lambda e^{-\lambda t}}{(k-1)!}$$

$$= \int_0^t \lambda e^{-\lambda(t-\tau)} \cdot \lambda^k e^{-\lambda \tau} \frac{\tau^{k-1}}{(k-1)!} \, d\tau$$

$$= \lambda^{k+1} e^{-\lambda t} \int_0^t \frac{\tau^{k-1}}{(k-1)!} \, d\tau$$

$$= \lambda^{k+1} e^{-\lambda t} \frac{t^k}{k!}$$