

확률통계론

Final_Term_Project

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Generation of RVs

- Generation of three random variables (X, Y1, Y2)
- First, I am going to generate X
- X: Gaussian, $N(\mu_x=4, \sigma_x^2=25)$
 - Going to make 2000 samples
- I used the Python package below

```
1 import numpy as np
2 import pandas as pd
3 import matplotlib.pyplot as plt
4 from random import *
5 import random
```

My own Number

- My own procedure and parameter values
- I'm going to input a and b
- I choose $a = 4$, $b = 8$

```
7 a = input("input a: ")
8 b = input("input b: ")
9 b = int(b)
10 a = int(a)
11
```

My own number

- And going to get Random Number by uniform $N(\mu x=4, \sigma x^2=25)$
- I use an Excel file for related functions and data below

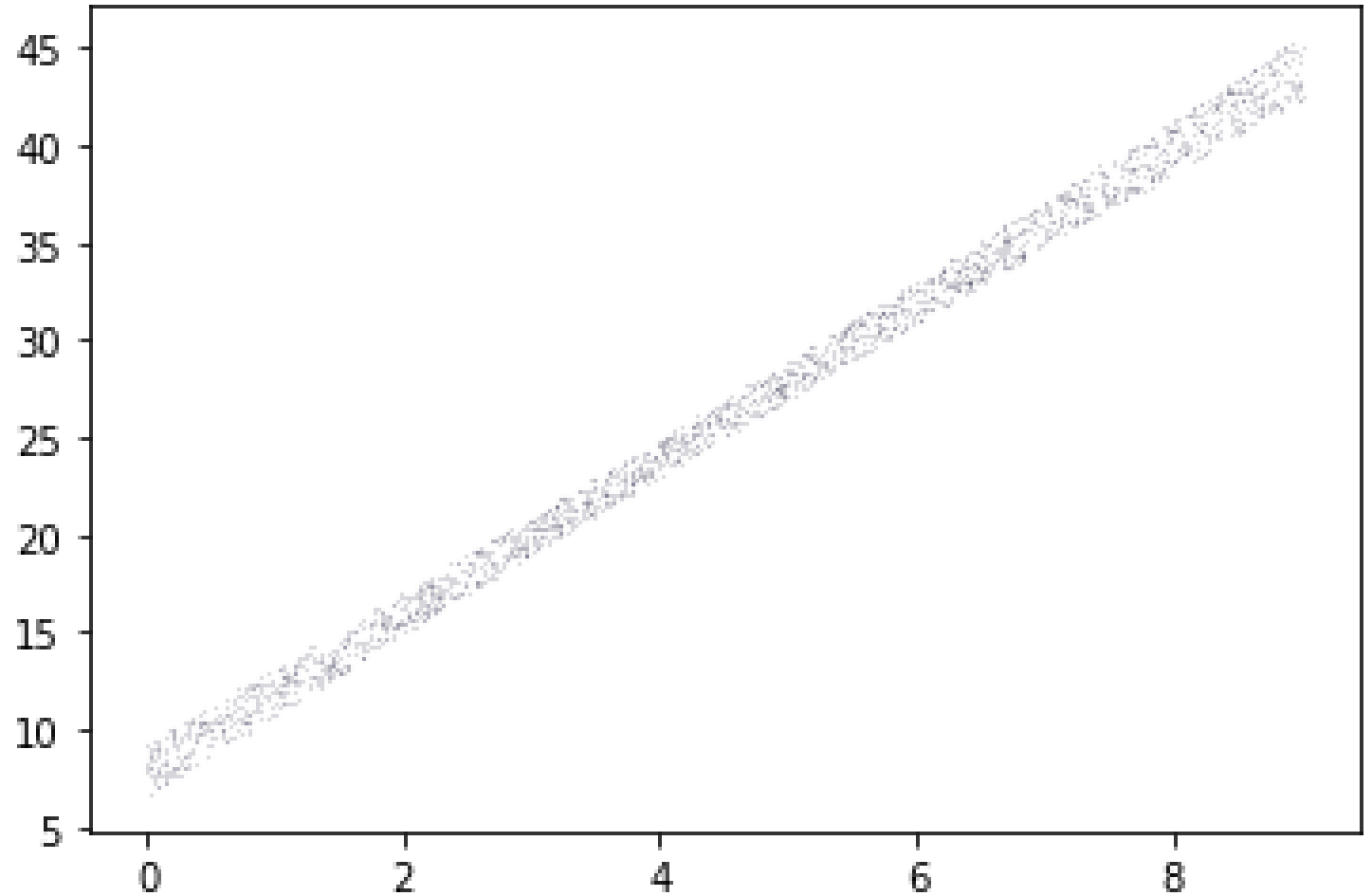
```
12 RandomNumber = np.zeros(2000)
13 for x in range(0, 2000):
14     RandomNumber[x] = uniform(0.0, 9.0) #uniform() float 랜덤 변수 만들어주는 함수
15
16 print(RandomNumber)
17 df = pd.DataFrame(RandomNumber).T
18 df.to_excel(excel_writer = "C:/Users/admin/Desktop/test.xlsx")
19
20 ResultNumber = np.zeros(2000)
```

Excel data of number X

2800124 4.050158 8.356211 4.561949 7.779586 1.811715 3.337341 7.986612 2.852998 3.864758 2.856608 3.632609 3.849471 0.52574 6.956441 1.598077 2.753909 6.100736 4.03967 3.295523 1.51126 6.403282 2.446927 6.439027 3.204186 6.266004 3.866373 5.3837 7.262033
5.806923 0.682547 6.31978 2.642901 2.196887 1.445923 0.234901 3.763398 3.828294 6.188497 3.206833 0.886422 5.491967 4.9285 2.563937 7.566999 1.096203 3.173223 3.835912 7.822746 8.092194 3.782905 3.66861 3.982142 6.912851 4.286876 5.946406 7.826572 3.153898
5.078374 1.017926 4.216192 8.516578 8.327905 2.67427 3.473466 2.912778 6.429843 4.616989 3.426627 7.314486 1.305996 3.534332 8.173712 4.865276 8.129823 1.355165 7.916541 1.99917 0.257632 1.622838 2.967418 4.018498 3.771655 5.107457 8.391757 4.794999 0.289182
1.190146 1.579084 6.05448 5.181546 0.067993 6.683489 1.687299 6.814718 1.552276 1.390641 8.950857 1.828465 0.663458 1.334003 4.820019 8.70282 3.494254 6.621628 3.15983 3.756623 5.967495 3.373957 1.578346 2.893465 7.436183 6.665432 3.603641 6.190326 4.429154
1.363513 0.177986 1.087561 7.293844 4.470354 2.219087 4.607411 4.614922 0.956272 4.234467 4.952445 5.701261 7.503832 0.854812 3.138253 5.898255 7.856467 4.491693 7.964306 3.997129 0.683658 0.912974 4.134805 5.356387 7.117343 8.151725 6.815979 8.749359 1.119074
8.244364 3.1770378 2.919855 0.912907 4.698521 8.73825 8.419305 0.249929 5.747366 1.369453 2.828505 0.202313 3.317662 2.154013 4.578171 1.966873 8.716901 5.882385 7.646311 1.07233 6.895925 0.637034 3.310835 1.94755 0.35035 3.166937 6.800198 0.894997 4.351991
8.864809 3.459766 5.039781 5.985259 1.96833 6.745792 8.192241 2.699568 0.770686 2.546651 5.241141 1.117609 7.820202 5.570573 5.656424 1.741559 4.808089 2.637249 2.288186 4.219371 1.163004 5.973201 4.10797 2.465308 2.536669 7.511948 8.661992 5.948419 8.612283
3.906929 4.925172 4.320592 4.471853 8.956979 2.788059 4.119383 5.360936 5.226367 8.860618 5.832804 7.943849 2.326472 1.375886 4.281972 0.604745 5.847887 8.342104 4.366418 6.184097 3.355211 6.440599 3.693467 8.888252 7.121753 2.485791 1.637901 0.67883 3.873665
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2.085054 2.663921 1.604557 5.842774 4.429967 4.426507 4.029225 7.161068 6.38985 5.761226 0.630313 3.27083 6.892885 3.123125 0.504264 4.290742 1.787927 4.479195 0.999645 1.236238 6.968902 6.992473 6.964636 0.986063 4.599129 4.956246 8.125125 4.38529 4.501159
7.168986 1.724113 0.19958 6.475731 8.626383 8.913244 2.758884 5.291767 2.680272 0.745168 3.030789 2.909166 0.493688 7.368797 7.263338 2.890863 1.166934 7.214932 0.006312 5.320219 6.860493 0.198322 3.533291 0.712797 3.849876 6.018623 3.574883 3.473105 4.889317
3.87417 2.189893 3.671573 7.401218 5.161488 2.310114 7.395847 5.888669 4.802973 5.440582 1.308703 3.058201 7.896394 5.116651 2.161672 4.391409 6.823948 1.081068 3.398549 4.297917 7.808986 0.461169 6.695338 8.635971 6.936812 8.385916 7.999459 4.786808 1.936311
1.958028 4.870857 5.920336 4.621767 6.163329 3.667301 1.535392 7.405367 7.197562 3.540133 6.328389 5.848147 4.17558 2.48965 0.309313 2.156832 7.09469 1.477177 7.302504 6.915359 8.912208 9.597074 2.312054 1.958344 7.81863 1.664611 4.666008 6.708372 5.189041
5.393763 3.166589 0.004934 7.257643 7.652854 1.145618 7.865769 3.888424 7.093009 3.012904 3.406945 8.866499 7.220773 4.860943 5.380647 2.560647 4.402343 1.980036 0.46673 0.254397 5.326105 8.106967 2.694968 0.827198 4.544356 2.499354 3.786966 7.188474 0.232312
7.397404 3.302992 4.929718 4.652719 0.645442 7.611188 4.681528 6.242001 3.713334 4.685567 7.669391 2.307774 5.027807 5.887502 3.265276 5.156965 1.142468 7.167406 0.974306 1.612871 5.071735 3.688547 8.280164 8.909494 1.9787 5.299636 4.21122 1.32507 3.967235
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2.99261 5.186459 4.137659 1.564575 2.956161 4.957686 7.282314 4.465413 8.048158 3.373488 8.678675 3.049082 3.114724 2.942926 3.851162 2.178552 6.207767 4.81378 5.376872 5.169241 4.799581 0.728591 2.352229 7.800168 2.752531 4.838557 4.27808 5.24742 0.076975
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8.029529 0.841172 1.223918 5.804081 4.707377 4.988145 3.257285 3.485734 3.755711 7.316918 3.694592 1.673606 7.804375 6.617061 5.119094 0.23212 0.176492 4.74059 0.769235 1.553545 2.566801 2.713845 4.548781 7.493832 7.483343 2.856161 5.660536 3.293631 8.039179
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4.106858 0.860251 3.255051 0.809316 3.873029 5.87242 3.267158 1.586579 3.765193 2.910925 8.299621 3.477001 6.122786 7.18376 4.496149 5.115237 4.21609 2.250406 4.753419 1.03948 4.245802 7.736899 0.242447 4.566926 2.909093 6.972604 2.813907 0.823359 5.283362
9.61573 6.378828 7.18469 2.272533 2.450881 4.00801 8.967175 7.636782 6.400627 5.287551 2.595204 8.350393 0.158371 2.5652 7.201465 3.330144 8.006657 6.527164 3.073682 5.543928 1.158095 5.18363 2.265237 0.208346 8.577333 3.485217 4.187561 4.30284 3.81763 4.524774
0.126177 5.703401 6.091005 4.621685 2.800176 6.207136 6.399049 8.814388 1.65353 5.389425 5.895481 7.07491 2.700883 2.767669 1.982542 8.728858 8.126986 3.642753 0.326422 5.918819 6.400126 5.120034 2.746403 6.579514 4.364282 0.072673 3.735926 4.563758 8.267989
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0.058853 8.854909 5.093545 2.254365 6.061159 3.194441 4.195867 3.681444 4.259863 6.170212 3.780598 2.398188 5.18018 8.330191 0.270759 1.363755 3.267055 0.916378 8.798428 8.646791 8.71454 8.76784 8.651708 1.436209 4.287335 7.717456 0.234042 5.912357 2.89291
6.130819 0.881806 7.733475 0.893007 2.332893 3.515353 7.815952 5.06613 2.373387 6.186156 8.547652 6.663148 6.665343 0.833565 4.242026 4.467371 6.245952 8.847729 8.28666 2.0047 0.481015 1.590986 0.934279 2.919729 1.445677 4.398166 3.461297 1.524734 1.987318
7.993187 2.751901 1.575133 2.18078 1.097718 1.030592 7.66756 5.061457 7.495711 1.79777 8.078893 2.825502 5.847883 8.872863 4.458469 4.436946 5.938992 7.398252 0.049175 0.924226 0.798363 1.818942 5.052056 8.825407 2.635709 6.624904 5.81948 4.141563 8.108993
5.440645 2.686468 7.56779 8.39363 5.005119 5.431576 5.321875 5.06142 2.052961 1.984741 6.284806 0.83449 1.817835 0.188909 3.442142 6.986369 3.157265 1.953917 1.014111 4.775314 0.723809 4.274758 7.408739 0.517531 4.040501 1.335657 1.500006 2.166026 3.878658
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1.458289 5.248526 3.091408 0.968792 3.19185 0.668792 3.267826 1.974074 3.788102 6.048085 2.4998 3.700256 1.931357 0.497174 4.662337 6.645781 4.381

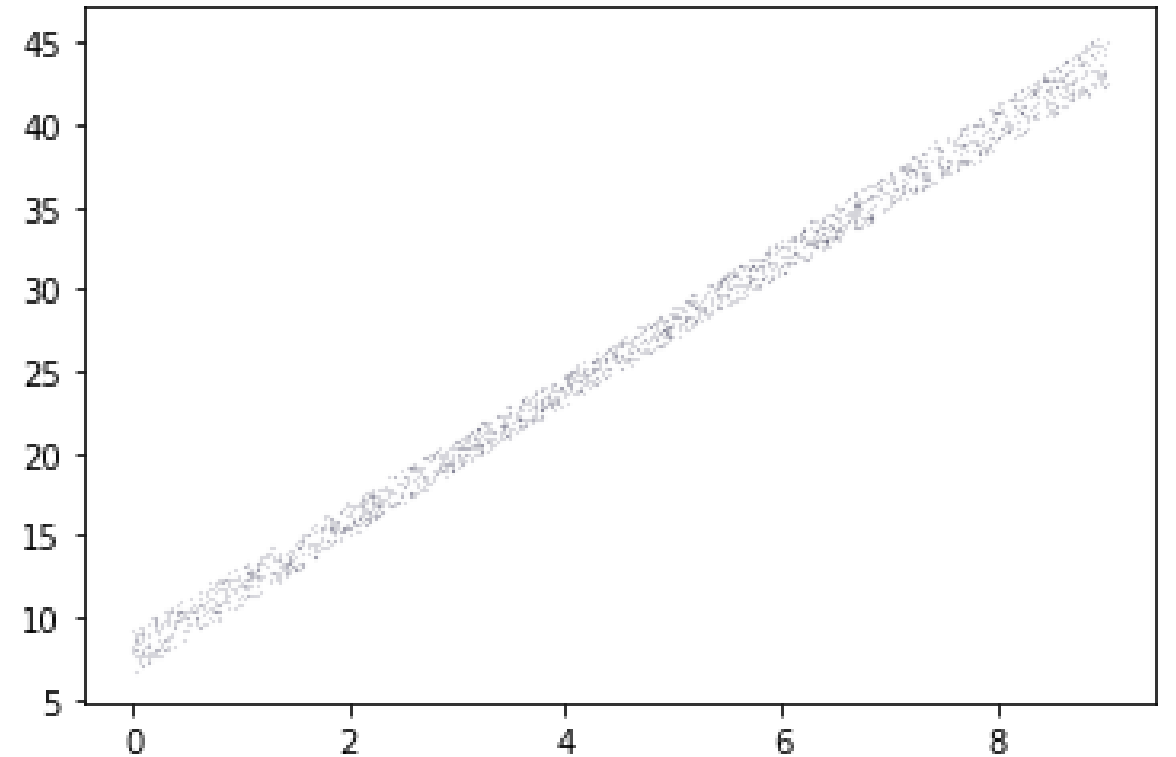
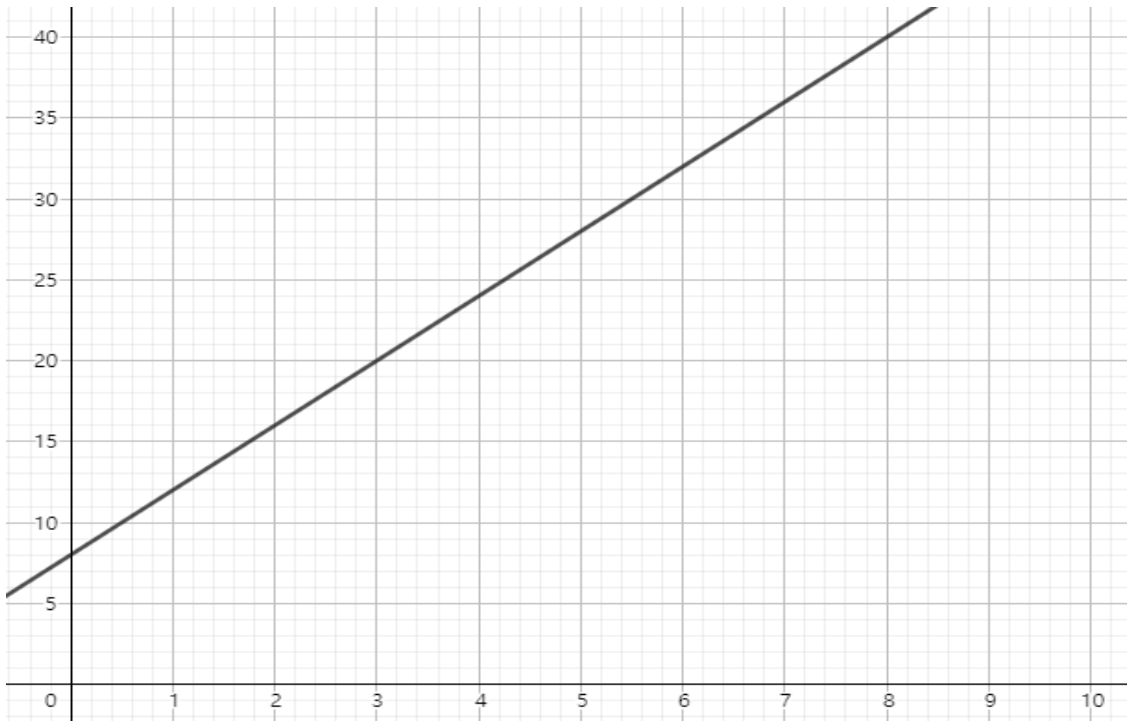
Estimation Y1

- Generate Y1
- When $a = 4$, $b = 8$



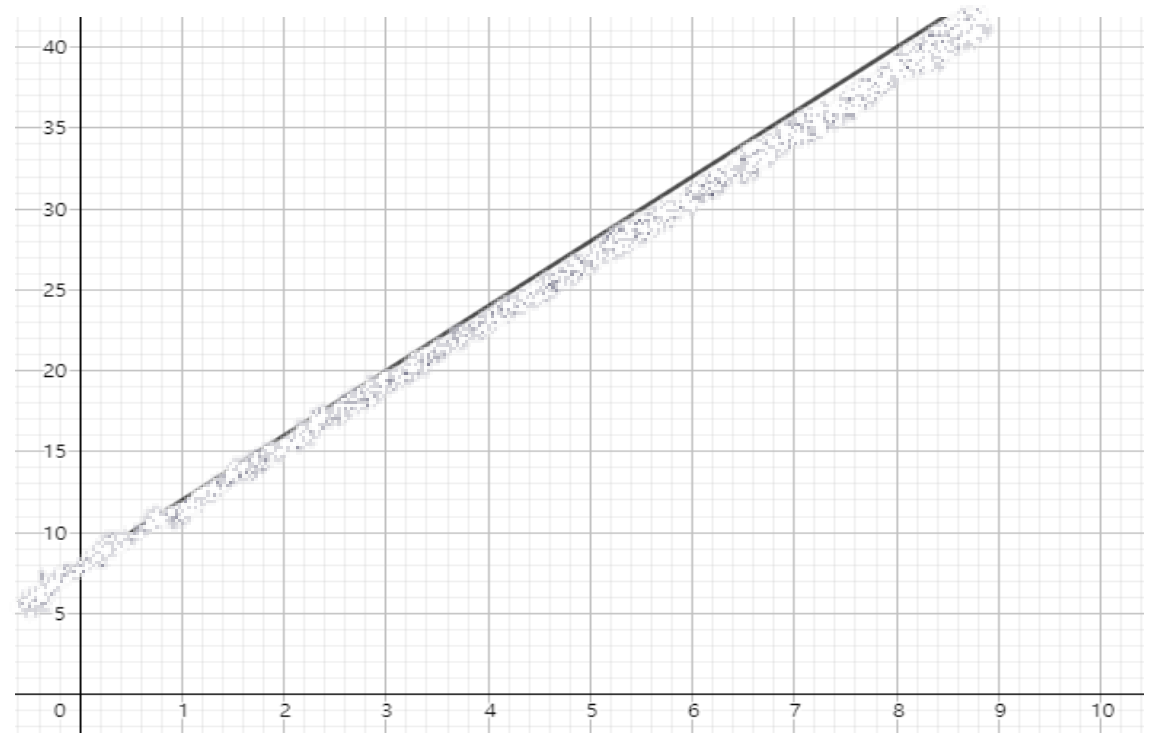
Estimation Y1

- Compare Y1 and Eq.1



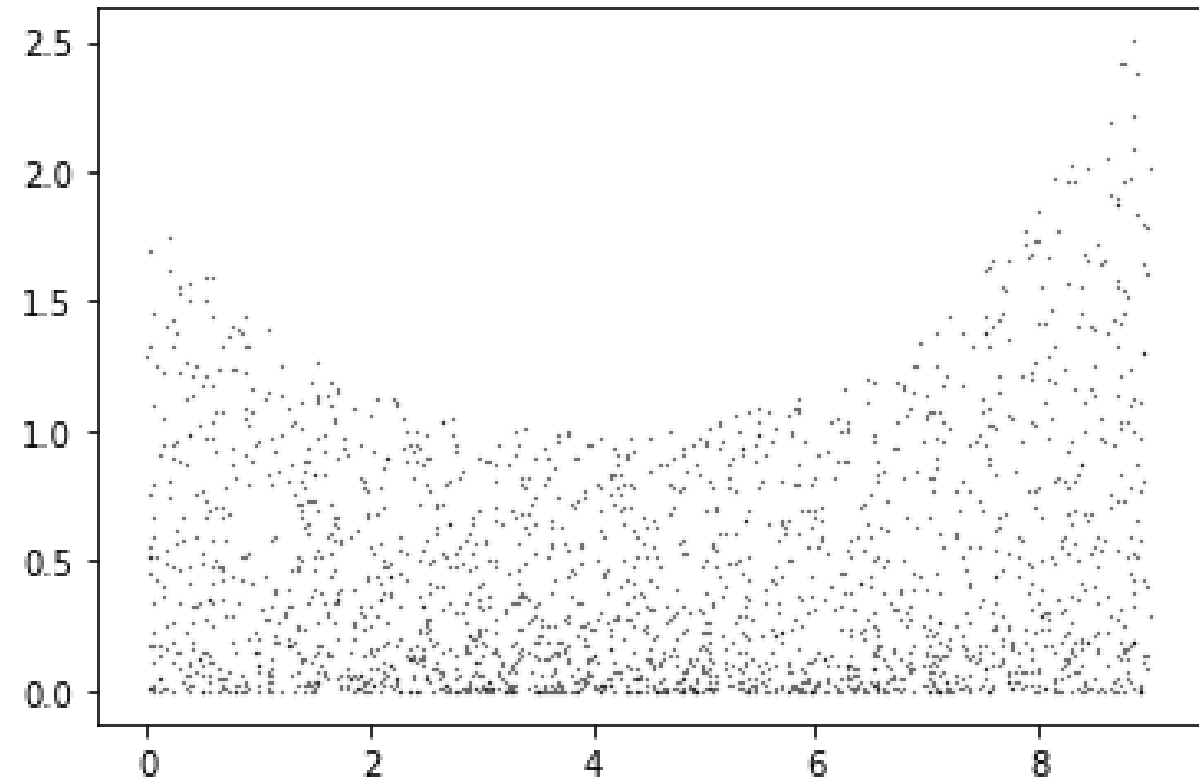
Estimation Y1

- I am going to compare two graphs.
- It seems to similar
- So, I am going to calculate the mean squared difference between two graphs



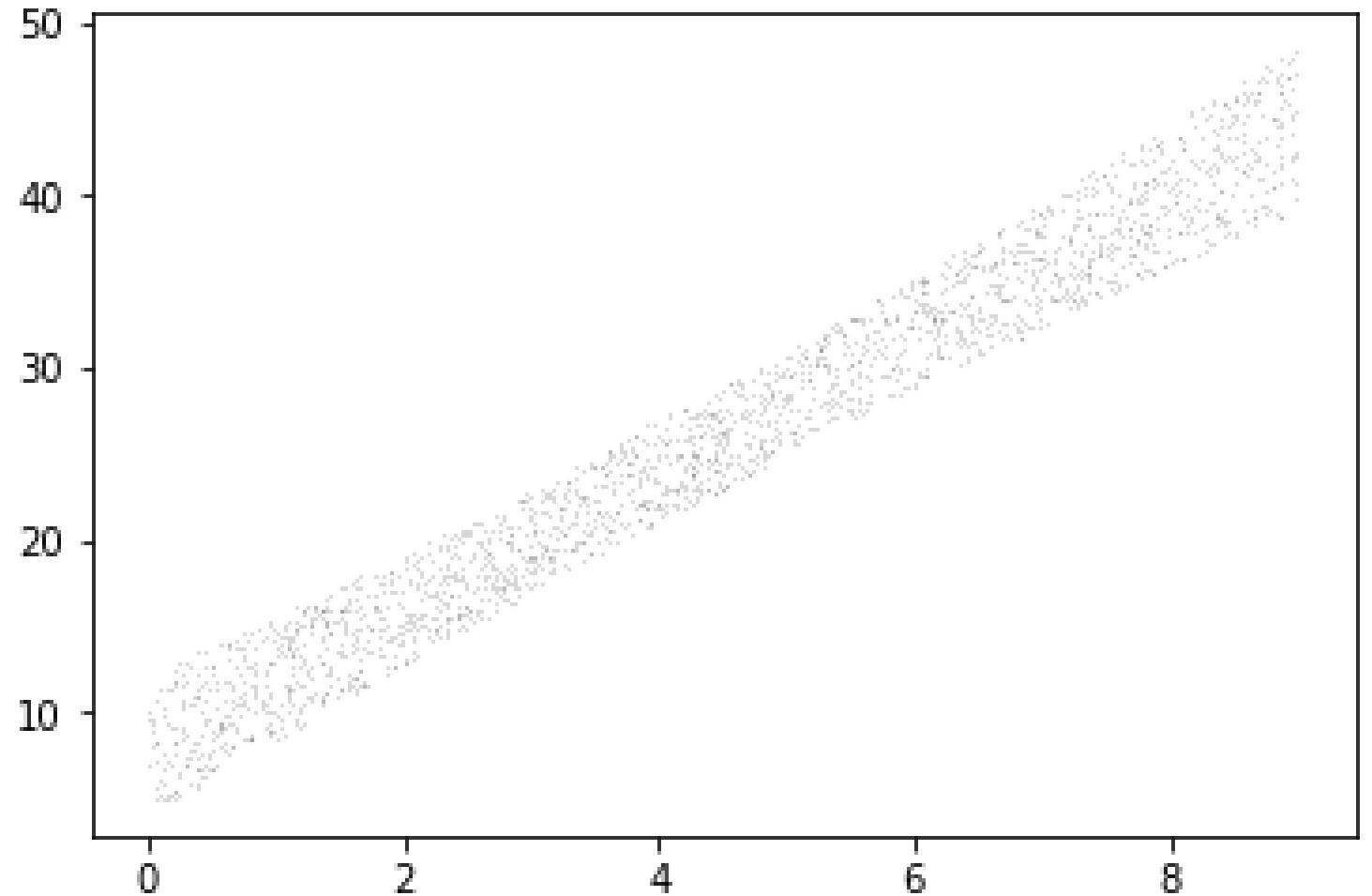
Estimation Y1

- Calculate the mean squared difference between the estimated Y1 and generated Y1



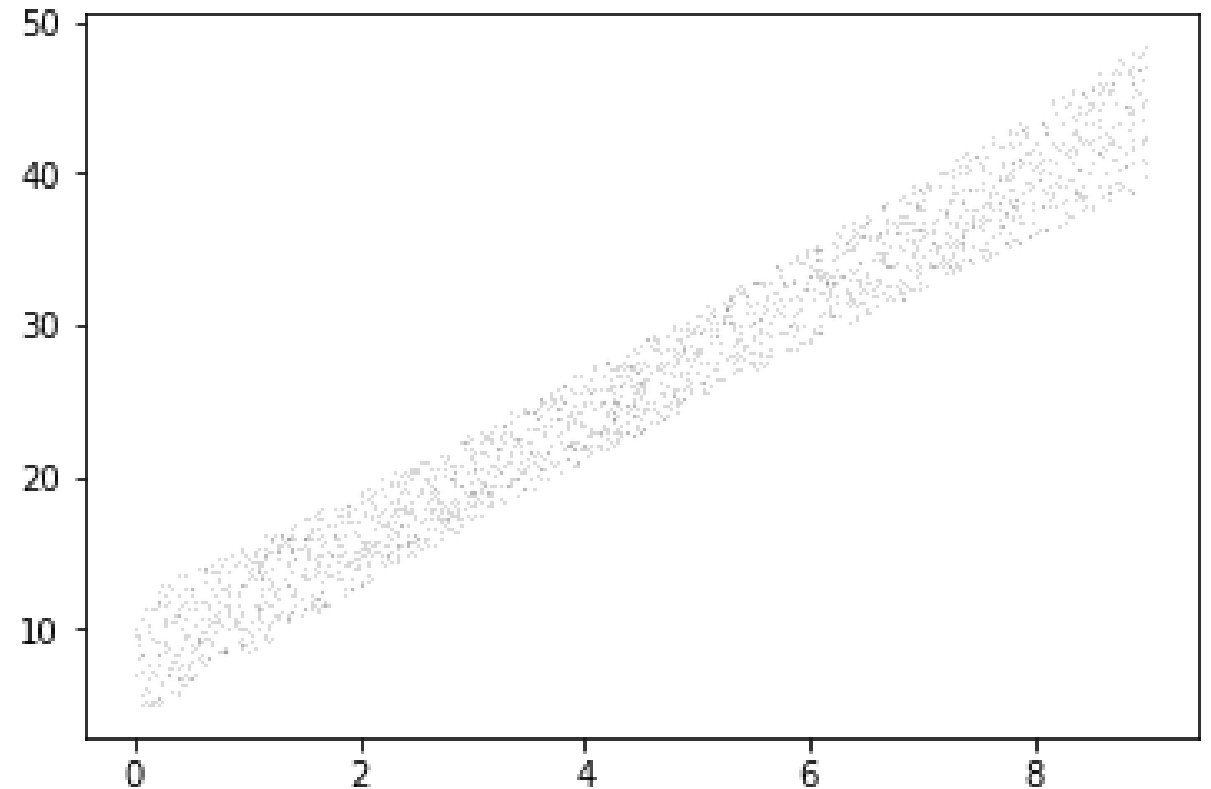
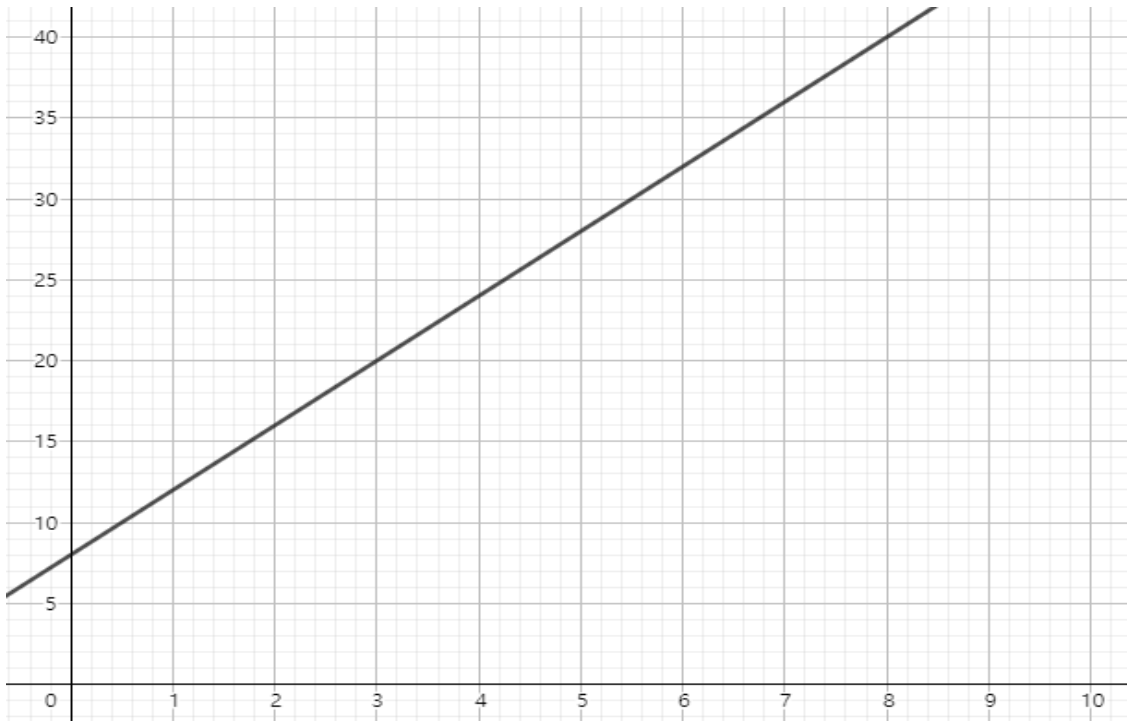
Estimation Y2

- Generate Y2
- When $a = 4$, $b = 8$



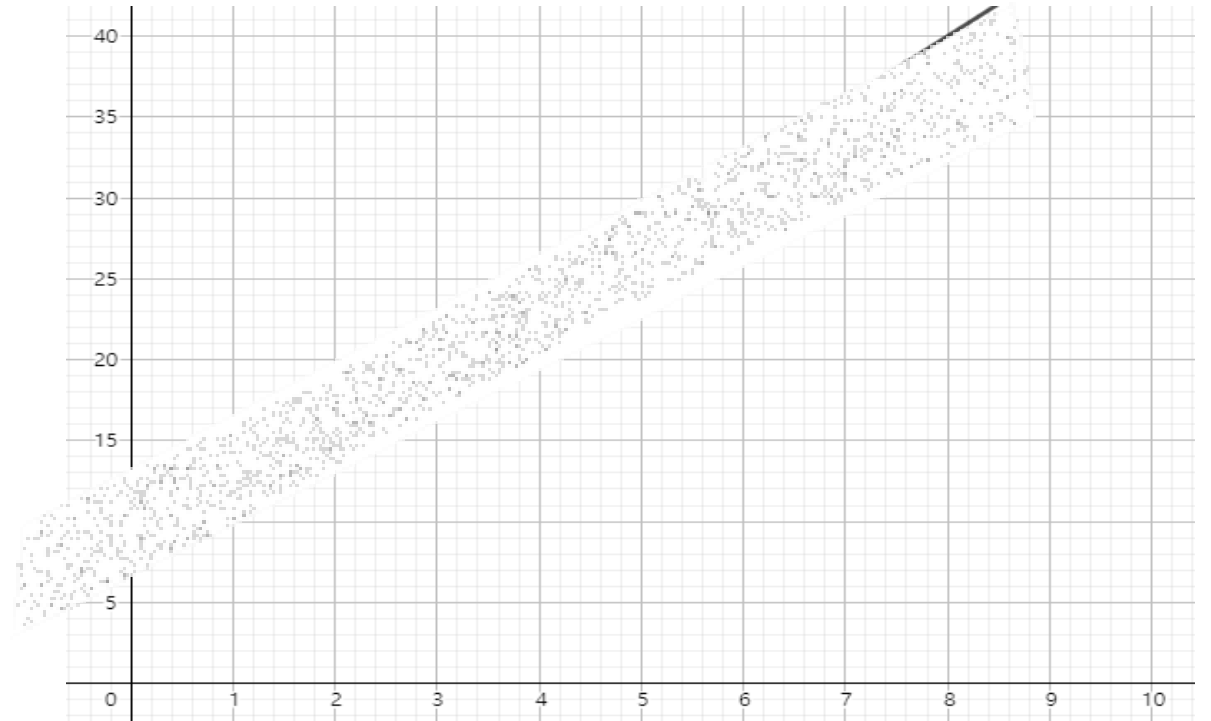
Estimation Y2

- Compare y_2 and Eq.2



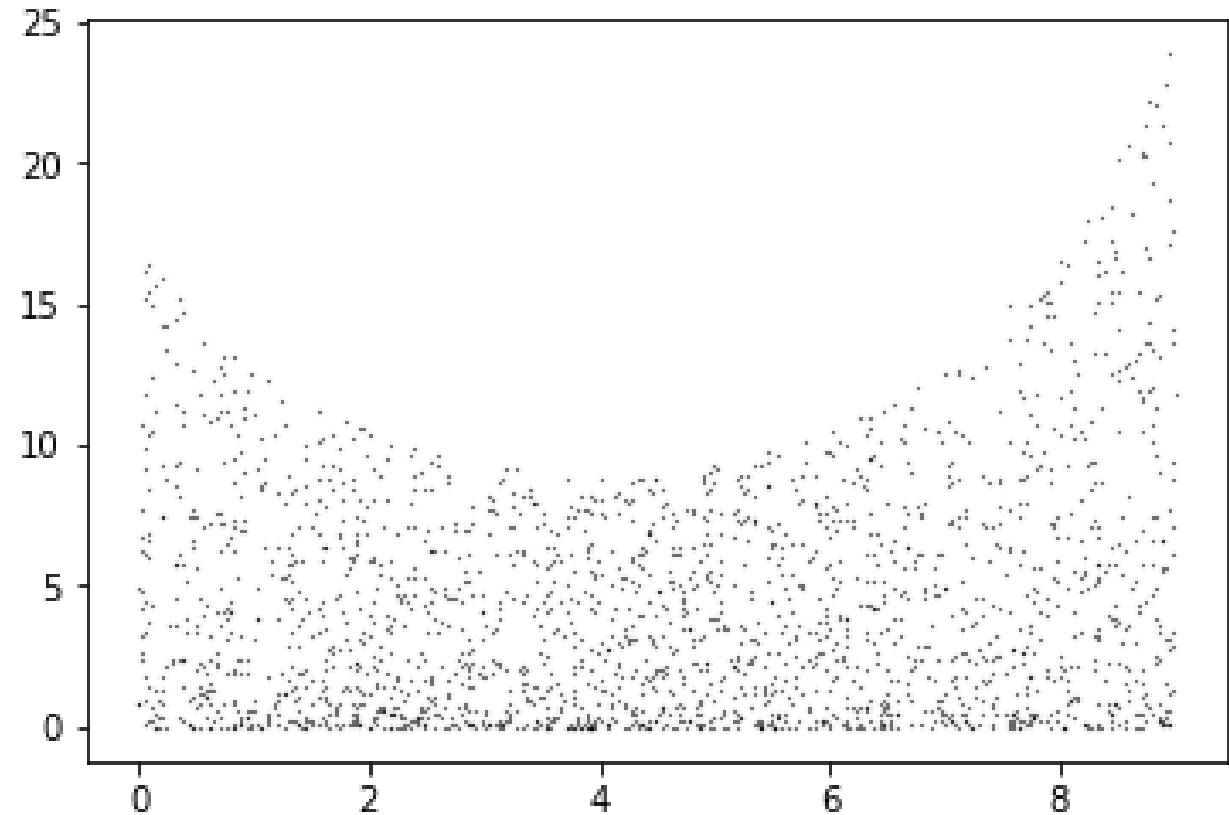
Estimation Y2

- I am going to compare two graphs.
- It seems to similar, but there is little different between two graphs
- So, I am going to calculate the mean squared difference between two graph



Estimation Y2

- Calculate the mean squared difference between the estimated Y2 and generated Y2

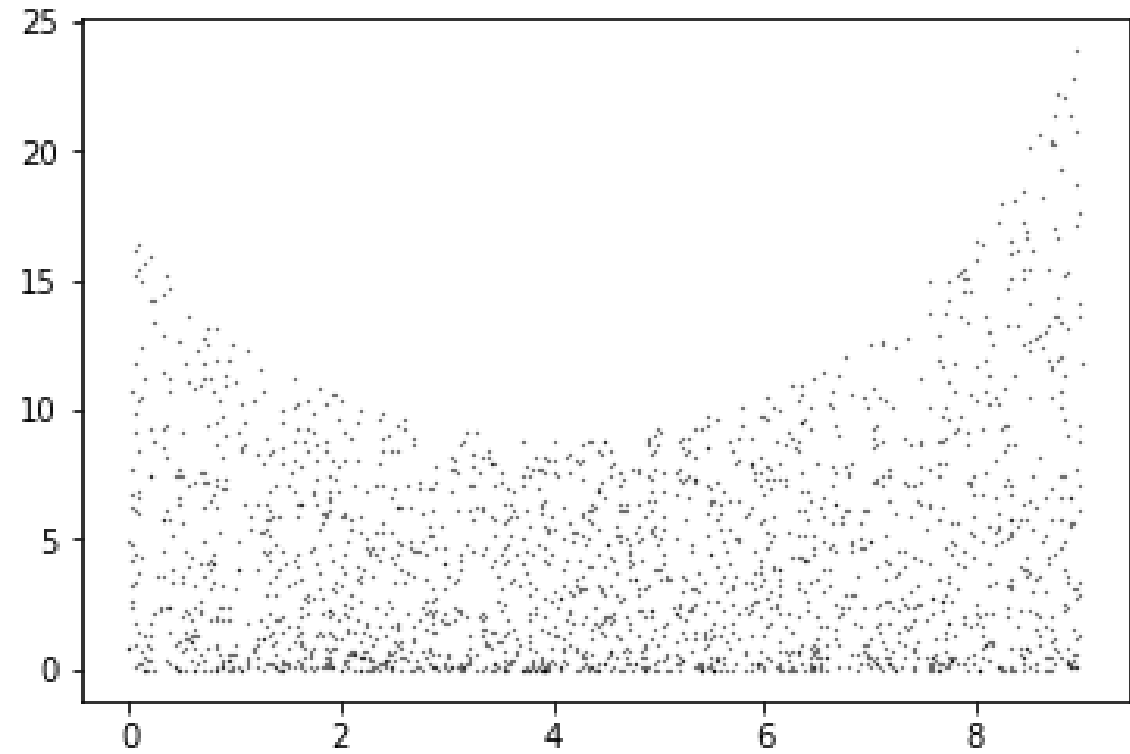
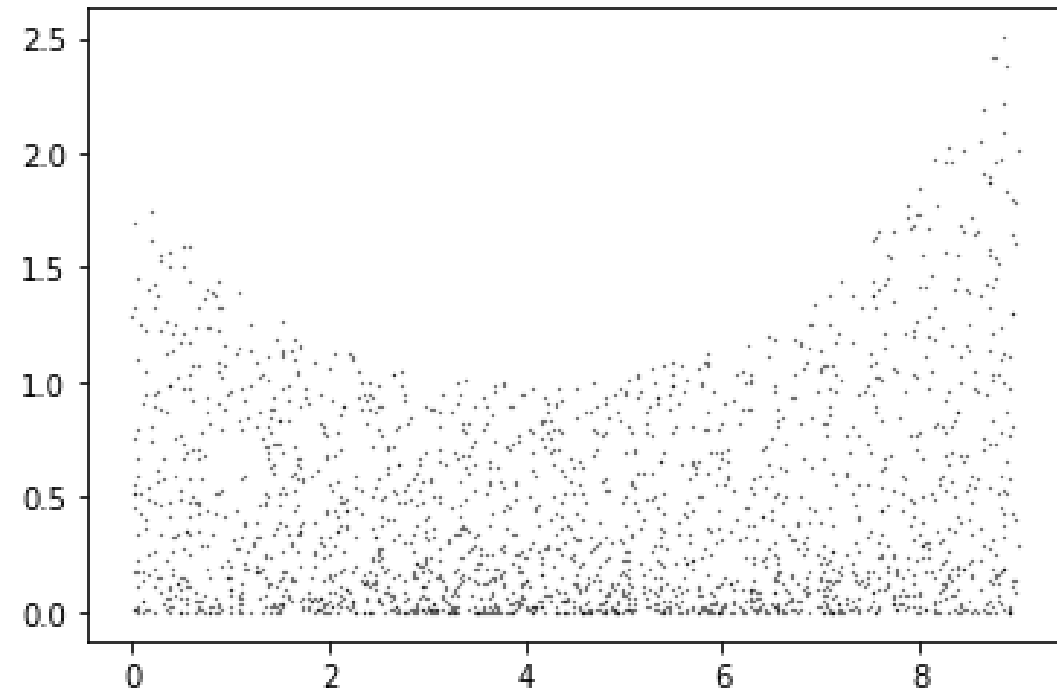


Analysis Y1 and Y2

- I am going to Compare squared difference between Y1 and Y2
- Through this analysis, I will learn about the effects of the graph based on σ_z^2

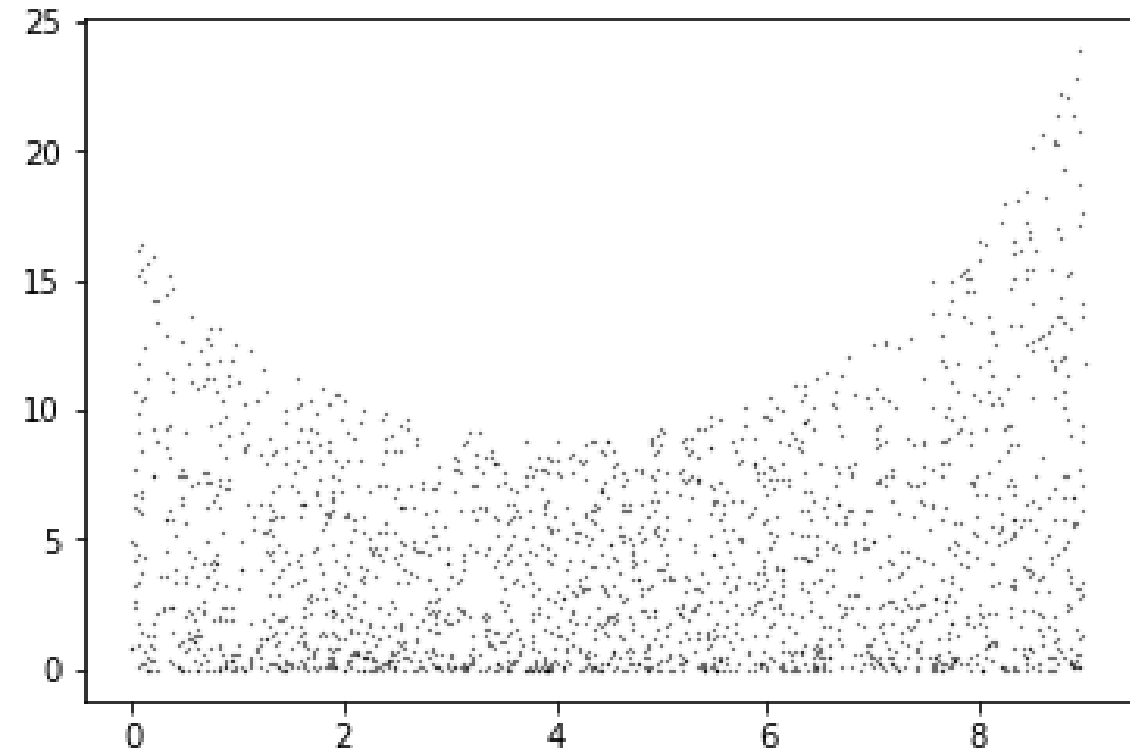
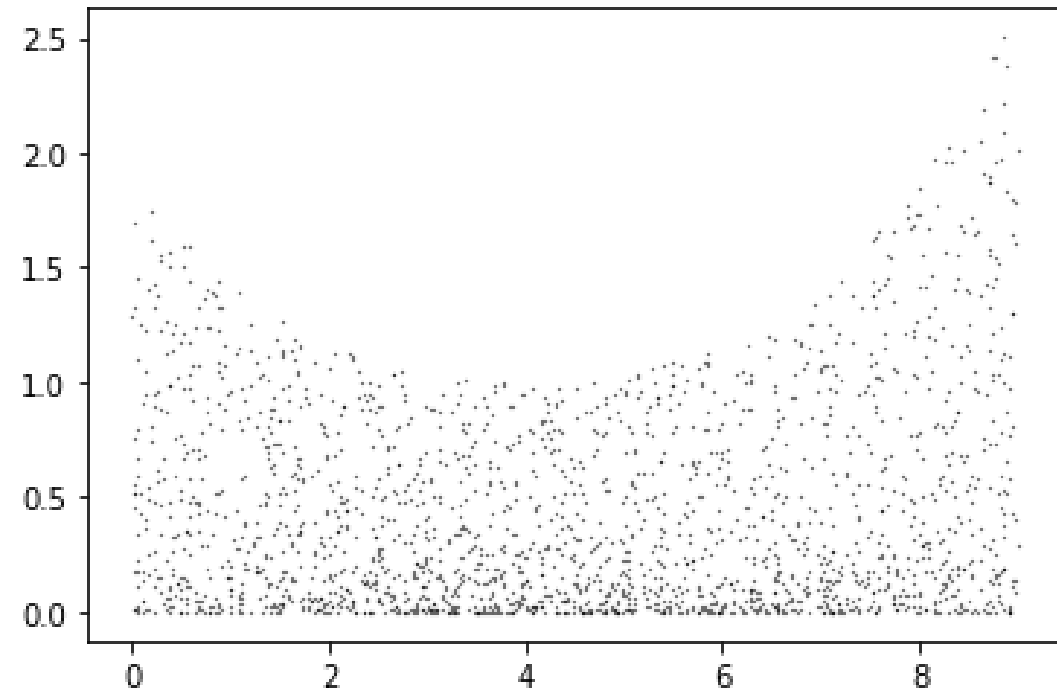
Analysis Y1 and Y2

- Left one is Y1, and right one is Y2.
- The two graphs show very similar shapes to the extent that they are thought to be the same



Analysis Y1 and Y2

- The difference was the smallest in the center, and the difference got bigger as it went to the edge
- Right side is more bigger than left side



My Conclusions

- The estimated Y and generated Y seems very similar
- But as the σ_z^2 grew, the difference became more and more
- When I compare squared difference between Y_1 and Y_2 , the forms of graphs are very similar, of course there is a difference cause of size
- When I analyze squared difference, the difference was the smallest in center of the graph, and the difference got bigger as it went to the edge. And right side is bigger than left side.

Full Code

```
1 import numpy as np
2 import pandas as pd
3 import matplotlib.pyplot as plt
4 from random import *
5 import random
6
7 a = input("input a: ")
8 b = input("input b: ")
9 b = int(b)
10 a = int(a)
11
12 RandomNumber = np.zeros(2000)
13 for x in range(0, 2000):
14     RandomNumber[x] = uniform(0.0, 9.0) #uniform() float 랜덤 변수 만들어주는 함수
15
16 print(RandomNumber)
17 df = pd.DataFrame(RandomNumber).T
18 df.to_excel(excel_writer = "C:/Users/admin/Desktop/test.xlsx")
19
20 ResultNumber = np.zeros(2000)
21
22 for x in range(0, 2000):
23     temp = -1 * uniform(-3.0, 3.0) * (2.718 ** (((RandomNumber[x] - 4) * (RandomNumber[x] - 4)) / 50)) #이게 뒷부분 연산하는 거임
24     ResultNumber[x] = RandomNumber[x] * a + b + temp #y값 계산
25
26 plt.scatter(RandomNumber, ResultNumber, s = 20**-2, c = '#000000') #Generated Y를 구하는 그래프
27 FinalNumber = np.zeros(2000)
28 for x in range(0, 2000):
29     FinalNumber[x] = (RandomNumber[x]*a + b - ResultNumber[x])**2
30
31 plt.scatter(RandomNumber, FinalNumber, s = 20**-1, c = '#000000') #difference를 구하는 그래프
```

Thank You

Use Python, Geogebra
random, numpy, matplotlib, pandas