Optimizing NELHA Economic Output with Linear Programming

Benjamin A. Beasley

Abstract—A simplified model of operations at the Natural Energy Laboratory of Hawaiʻi Authority (NELHA) campus in Kailua-Kona demonstrates the application of linear programming (LP) to maximize total economic output at the park.

1 Introduction

Through the Natural Energy Laboratory of Hawaiʻi Authority (NELHA), the State of Hawaiʻi operates an 870-acre research and commercial campus at Keāhole Point in Kailua-Kona, just south of Kona International Airport [1]. This large development, comprising the Natural Energy Laboratory of Hawaiʻi (NELH) and the Hawaiʻi Ocean Science and Technology (HOST) Park, hosts a variety of tenants, ranging from education and long-range research projects to mature commercial endeavors.

A key feature of NELHA is access to clean, cold, deep sea water (DSW). As the flanks of Hawaiʻi's volcanoes continue beyond the shoreline, the seafloor slopes rapidly into the abyss. This makes it practical to construct offshore pipeline systems with seawater intakes as deep as 3,000 feet.

This report constructs a simplified model of NELHA operations, and explores the application of linear programming to optimize the allocation of pumped DSW to maximize total economic activity.

2 Problem Formulation

To limit the required scope, the simplified model of research and commercial activity at NELHA will focus on pumped DSW, and will consider only certain major consumers.

Let the N uses at the site be $\langle x_1, \ldots, x_N \rangle$, with each quantity in an annual arbitrary unit of production. Let the DSW flow in gallons per minute (gpm) required per annual unit of production be $\langle w_1, \ldots w_N \rangle$, and the corresponding land area in acres be $\langle \ell_1, \ldots \ell_N \rangle$. Finally, let the economic value of each unit of annual production (for commercial uses, defined here as either total wholesale price or gross revenue) be $\langle v_1, \ldots v_N \rangle$.

Since tenants have long-term leases, the model assumes all existing tenants will at least maintain their present land and DSW uses, focusing instead on the optimal allocation of remaining resources. Then the goal is to maximize $\sum_{i=1}^{N} v_i x_i$ for new development, subject to constraints.

2.1 Facility Capabilities

Activities at NELHA are limited by DSW pumping capacity and by available land. The seawater distribution system pulls DSW from depths of 2,000 and 3,000 feet, with a total pumping capacity of 43,400 gallons per minute (gpm) [2]. Average current usage is around 20,000 gpm [3], leaving a free capacity of 23,400 gpm.

The total land area of the park is 870 acres. After conservation easements, shoreline setbacks, and other requirements, 644 acres of leasable land remain; based on published maps, some 322 acres are available for future development [1].

Then for all new uses:

$$\sum_{i=1}^{N} w_i x_i \leq 23400 \text{ gpm} \qquad \sum_{i=1}^{N} \ell_i x_i \leq 322 \text{ acres}$$

2.2 Abalone Aquaculture

Big Island Abalone Corporation (BIAC) operates a 10-acre aquafarm that produces 100 tons or about 91,000 kg of abalone per year, feeding them kelp grown on-site [4]. Because abalone require a constant stream of cool seawater, BIAC is one of the largest users of DSW at NELHA.

Abalone wholesale prices on the international export market are variable, but may range from roughly \$25 to \$80 per kg [5]; thus if the amount of additional abalone production x_1 is in $\frac{\text{kg}}{\text{year}}$, then $v_1 \in [25, 80]\ \frac{\$}{\text{kg}}$. To capture the risk of exceptional market behavior, the model will consider a wider range, $v_1 \in [10, 200]$.

If BIAC uses 10,000 gpm on average, then $w_1 = \frac{10000 \text{ gpm}}{91000 \text{ kg/year}} \approx 0.11 \frac{\text{gpm}}{\text{kg/year}}$. Since the farm is 10 acres, $\ell_1 = \frac{10 \text{ acres}}{91000 \text{ kg/year}} \approx 1.1 \times 10^{-4} \frac{\text{acre}}{\text{kg/year}}$.

2.3 Bottled Water

NELHA hosts at least two large bottled-water plants that desalinate, filter, and bottle deep sea water for export, claiming exceptional purity. One of these plants produces 750,000 bottles per day [6]. Assuming year-round production, and a similar rate from the other plant, this is 2 plants $\times (7.5 \times 10^5) \frac{\text{bottles}}{\text{plant} \times \text{day}} \times 365.24 \frac{\text{days}}{\text{year}} \approx (5.48 \times 10^8) \frac{\text{bottles}}{\text{year}}$.

Based on published maps, the existing bottled-water plants occupy very roughly a fifth of the developed land, or 64 acres [1]; thus if the additional production x_2 is in $\frac{\text{bottles}}{\text{year}}$, then $\ell_2 \approx \frac{64 \text{ acres}}{2 \text{ plants}} \times \frac{2 \text{ plants}}{5.48 \times 10^8 \text{ bottles/year}} \approx 1.17 \times 10^{-7} \frac{\text{acres}}{\text{bottle/year}}$. If one bottle is 16 ounces, then $w_2 = \frac{1 \text{ gal}}{128 \text{ oz}} \times \frac{16 \text{ oz}}{1 \text{ bottle}} \times \frac{1 \text{ year}}{365.24 \text{ days}} \times \frac{1 \text{ day}}{1440 \text{ minutes}} \approx 2.38 \times 10^{-7} \frac{\text{gpm}}{\text{bottle/year}}$. If the average wholesale price of bottled water is \$1.11 per gallon [7], and this specialty water runs 50% over the average, then $v_2 = 1.5 \times \frac{\$1.11}{\text{gal}} \times \frac{1 \text{ gal}}{8 \text{ bottles}} \approx \frac{\$0.208}{\text{bottle}}$.

Since this specialty bottled water sells for above the usual commodity price, we assume the market for it is limited, or at least that it must be created over time by intensive marketing. Furthermore, a "green" research campus should not be dominated by environmentally-unfriendly bottled water production. Hence the additional production is constrained to 50% of the current capacity, or one new plant of similar size:

$$x_2 \leq (2.74 \times 10^8) \frac{\text{bottles}}{\text{year}}.$$

2.4 Nutritional Microalgae Aquaculture

The largest facility at NELHA by area is a 90-acre installation operated by Cyanotech [8], which uses the consistent sunlight of the island's leeward side to cultivate two species of microalgae in the for sale as nutritional supplements. The company's net annual sales are approximately \$34 million [9]. If x_3 is in units of the current annual production, then $v_3 = 3.4 \times 10^7$ and $\ell_3 = 90$.

Cyanotech uses deep sea water to refrigerate their product while drying it. Actual water usage is not readily available; to incorporate this uncertainty, and to analyze the sensitivity of the optimal solution to the DSW efficiency of Cyanotech's drying process, the model will vary DSW usage for current production in the range $w_3 \in [1 \times 10^2, 1 \times 10^5]$.

2.5 Pre-Commercial Research

Because of the potentially large but uncertain future economic impact of basic research, commercial R&D, and startup ventures, it is difficult to quantify their economic value. A representative example will be used to derive one possible answer.

Ocean Thermal Energy Conversion (OTEC) is an experimental electric power generation technology that uses the temperature gradient between sun-warmed surface seawater and cold benthic seawater as an energy source. Scaled to tens or hundreds of megawatts, and deployed on floating offshore platforms, this technology could provide reliable, clean, and carbon-neutral power to island communities and naval bases.

Makai Ocean Engineering and its partners operate an OTEC heat exchanger test facility at NELHA. These experimental heat exchangers have typical design flows of 4,000 gpm; we assume that up to 5,000 gpm may be required at any time [10].

If this is considered one unit of research; if this particular research has a 50% chance of leading to construction of 10 full-scale plants averaging 75 MW each in the next 25 years; and if the construction cost per kW is competitive with solar PV at approximately $\frac{\$3{,}000}{\text{kW}}$ [11]; then the value of plant construction will be $\frac{10 \text{ plants}}{25 \text{ years}} \times \frac{75 \text{ MW}}{1 \text{ plant}} \times \frac{1000 \text{ kW}}{1 \text{ MW}} \times \frac{\$3000}{1 \text{ kW}} = \frac{\$9 \times 10^7}{\text{year}}$ for one unit of research. Thus $v_4 = \$9 \times 10^7$, $w_4 = 5000$ gpm, and $\ell_4 = 2$ acres.

Additionally, to preserve the research nature of the campus, we impose a rule that no less than one quarter of new land or water use must go to research:

$$w_4 x_4 \geq \frac{1}{4} \sum_{i=1}^{N} w_i x_i \qquad \ell_4 x_4 \geq \frac{1}{4} \sum_{i=1}^{N} \ell_i x_i$$

3 Linear Programming Formulation

Maximize

$$v_1 x_1 + v_2 x_2 + v_3 x_3 + v_4 x_4$$

Subject To

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 \leq 23400$$
$$\ell_1 x_1 + \ell_2 x_2 + \ell_3 x_3 + \ell_4 x_4 \leq 322$$
$$x_2 \leq 2.74 \times 10^8$$
$$\frac{w_1}{4} x_1 + \frac{w_2}{4} x_2 + \frac{w_3}{4} x_3 - \frac{3 w_4}{4} x_4 \leq 0$$
$$\frac{\ell_1}{4} x_1 + \frac{\ell_2}{4} x_2 + \frac{\ell_3}{4} x_3 - \frac{3 \ell_4}{4} x_4 \leq 0$$
$$x_1, x_2, x_3, x_4 \geq 0$$

For an initial example solution, let the wholesale cost of abalone be $v_1 = 50\frac{\$}{\text{kg}}$, and let Cyanotech's average DSW usage be $w_3 = 1 \times 10^3$ gpm.

3.1 Standard and Slack Forms

The problem as written is already in standard form: it is a maximization; each variable has a nonnegativity constraint (because it is a change in production, and we assume tenants will not decrease production); there are no equality constraints; and all inequality constraints are $\leq$. The compact standard form is:

$$\text{Maximize } \mathbf{c}^{\mathrm{T}}\mathbf{x} \quad \text{Subject to} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \qquad \mathbf{x} \geq \mathbf{0}$$

where

$$\mathbf{A} = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \\ \ell_1 & \ell_2 & \ell_3 & \ell_4 \\ 0 & 1 & 0 & 0 \\ w_1/4 & w_2/4 & w_3/4 & -3w_4/4 \\ \ell_1/4 & \ell_2/4 & \ell_3/4 & -3\ell_4/4 \end{bmatrix}$$
$$= \begin{bmatrix} 0.11 & 2.38 \times 10^{-7} & 1000 & 5000 \\ 1.1 \times 10^{-4} & 1.17 \times 10^{-7} & 90 & 2 \\ 0 & 1 & 0 & 0 \\ 2.75 \times 10^{-2} & 5.95 \times 10^{-8} & 250 & -3750 \\ 2.75 \times 10^{-5} & 2.925 \times 10^{-8} & 22.5 & -1.5 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} 23400 & 322 & 2.74 \times 10^8 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{c} = \mathbf{v} = \begin{bmatrix} 50 & 0.208 & 3.4 \times 10^7 & 9 \times 10^7 \end{bmatrix}^{\mathrm{T}}$$

In this form, $\mathbf{c}^{\mathrm{T}}$ is the *objective function*; $\mathbf{A}\mathbf{x} \leq \mathbf{b}$ are the *constraints* (together with the nonnegativity constraints $\mathbf{x} \geq \mathbf{0}$ required by standard form); and the *desired outputs* are $\mathbf{x}$, the vector of annual production increases for each use.

The slack form is formed from the standard from by adding slack variables to convert the $\leq$ constraints into equality constraints. The nonnegativity constraints are implicit:

$$\begin{aligned} z &= 50x_1 + 0.208x_2 + 3.4 \times 10^7 x_3 + 9 \times 10^7 x_4 \\ x_5 &= 23400 - 0.11x_1 - 2.38 \times 10^{-7} x_2 - 1000x_3 - 5000x_4 \\ x_6 &= 322 - 1.1 \times 10^{-4} x_1 - 1.17 \times 10^{-7} x_2 - 90x_3 - 2x_4 \\ x_7 &= 2.74 \times 10^8 - x_2 \\ x_8 &= -2.75 \times 10^{-2} x_1 - 5.95 \times 10^{-8} x_2 - 250x_3 + 3750x_4 \\ x_9 &= -2.75 \times 10^{-5} x_1 - 2.925 \times 10^{-8} x_2 - 22.5x_3 + 1.5x_4 \end{aligned}$$

Slack form can be described compactly by the same $\mathbf{A}$, $\mathbf{b}$, and $\mathbf{c}$ from standard from, plus the nonbasic and basic variable sets N and B and the objective-function constant term v:

$$N = \{1, 2, 3, 4\} \qquad B = \{5, 6, 7, 8, 9\} \qquad v = 0$$

4 Solution by Hand

The problem is compact enough that it can be solved by hand-application of the simplex algorithm.

4.1 Initialization

The minimum $b_k \in \mathbf{b}$ occurs at one-based index $k = 4$ (or equivalently $k = 5$, but we apply Bland's Rule). In either case, $b_k = 0 \geq 0$, so the initial basic solution is feasible. This means that $\left[x_i : i \in N\right] = \mathbf{0}$ is in the solution space, so in $\left[x_i : i \in B\right] \geq \mathbf{0}$.[1]

4.2 Iteration

The simplex algorithm transforms the slack form program with repeated pivots until there is no positive cost in the objective function coefficients $\mathbf{c}$.

1) All nonbasic variables have positive coefficients in the objective function. We use Bland's Rule to select x_1 as the entering variable x_e. We consider how much we can increase x_1 without violating any nonnegativity constraint:

$$\mathbf{\Delta} = \begin{bmatrix} \frac{b_1}{a_{1,1}} \\ \frac{b_2}{a_{2,1}} \\ \Longrightarrow \\ \frac{b_4}{a_{4,1}} \\ \frac{b_5}{a_{5,1}} \end{bmatrix} = \begin{bmatrix} \frac{23400}{0.11} \\ \frac{322}{1.1 \times 10^{-4}} \\ a_{3,1} \leq 0 \implies \\ \frac{0}{2.74 \times 10^{-2}} \\ \frac{0}{2.74 \times 10^{-5}} \end{bmatrix} \approx \begin{bmatrix} 2 \times 10^5 \\ 3 \times 10^6 \\ \infty \\ 0 \\ 0 \end{bmatrix}$$

The last two entries are smallest, so we choose x_8 as the leaving variable, again using Bland's rule to choose it over x_9.
Placing $x_e = x_1$ on the left-hand side of the fourth equation yields:

$$x_1 = -36.36x_8 - 2.164 \times 10^{-6} x_2 - 9.091 \times 10^3 x_3 + 1.363 \times 10^5 x_4$$

and substituting in the remaining constraint equations and the objective function yields:

$$\begin{aligned} z &= -1.818 \times 10^3 x_8 + 0.2079x_2 + 3.355 \times 10^7 x_3 + 9.682 \times 10^7 x_4 \\ x_5 &= 23400 + 4x_8 - 2.647 \times 10^{-23} x_2 - 1.137 \times 10^{-13} - 20000x_4 \\ x_6 &= 322 + 4 \times 10^{-3} x_8 - 1.168 \times 10^{-7} x_2 - 89x_3 - 17x_4 \\ x_7 &= 2.74 \times 10^8 - x_2 \\ x_1 &= -36.36x_8 - 2.164 \times 10^{-6} x_2 - 9.091 \times 10^3 x_3 + 1.363 \times 10^5 x_4 \\ x_9 &= 1 \times 10^{-3} x_8 - 2.919 \times 10^{-8} x_2 - 22.25x_3 - 2.25x_4 \end{aligned}$$

$$N = \{8, 2, 3, 4\} \quad B = \{5, 6, 7, 1, 9\} \quad v = 0$$
$$\mathbf{A} = \begin{bmatrix} -4 & 2.647 \times 10^{-23} & 1.137 \times 10^{-13} & 20000 \\ -4 \times 10^{-3} & 1.168 \times 10^{-7} & 89 & 17 \\ 0 & 1 & 0 & 0 \\ 36.36 & 2.164 \times 10^{-6} & 9.091 \times 10^3 & -1.363 \times 10^5 \\ -1 \times 10^{-3} & 2.919 \times 10^{-8} & 22.25 & 2.25 \end{bmatrix}$$
$$\mathbf{b} = \begin{bmatrix} 23400 & 322 & 2.74 \times 10^8 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{c} = \begin{bmatrix} -1.818 \times 10^3 & 0.2079 & 3.355 \times 10^7 & 9.682 \times 10^7 \end{bmatrix}^{\mathrm{T}}$$

1. If this had not been the case, it would have been necessary to generate an auxiliary linear program with an extra nonbasic variable, then apply the simplex algorithm to find a modified slack form with a feasible initial basic solution.

2) With $x_e = x_2$ entering by Bland's Rule,

$$\mathbf{\Delta} \approx \begin{bmatrix} 9 \times 10^{26} & 3 \times 10^9 & 3 \times 10^8 & 0 & 0 \end{bmatrix}^\mathrm{T}$$

and so, again by Bland's rule, $x_l = x_1$ leaves:

$$N = \{8, 1, 3, 4\} \quad B = \{5, 6, 7, 2, 9\} \quad v = 0$$

$$\mathbf{A} = \begin{bmatrix} -4 & -1.223 \times 10^{-17} & 2.469 \times 10^{-15} & 20000 \\ -1.966 & -5.397 \times 10^{-2} & -40.16 & 7.376 \times 10^3 \\ -1.681 \times 10^7 & -4.622 \times 10^5 & -4.202 \times 10^9 & 6.303 \times 10^{10} \\ 1.681 \times 10^7 & 4.622 \times 10^5 & 4.202 \times 10^9 & -6.303 \times 10^{10} \\ -0.4916 & -1.349 \times 10^{-2} & -100.4 & 1.842 \times 10^3 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 23400 & 322 & 2.74 \times 10^8 & 0 & 0 \end{bmatrix}^\mathrm{T}$$

$$\mathbf{c} = \begin{bmatrix} -3.496 \times 10^6 & -9.608 \times 10^4 & -8.399 \times 10^8 & 1.320 \times 10^{10} \end{bmatrix}^\mathrm{T}$$

3) With $x_e = x_4$ entering,

$$\mathbf{\Delta} \approx \begin{bmatrix} 1 & 4 \times 10^{-2} & 4 \times 10^{-3} & \infty & 0 \end{bmatrix}^\mathrm{T}$$

and so $x_l = x_9$ leaves:

$$N = \{8, 1, 3, 9\} \quad B = \{5, 6, 7, 2, 4\} \quad v = 0$$

$$\mathbf{A} = \begin{bmatrix} 1.338 & 0.1465 & 1.090 \times 10^3 & -10.86 \\ 2.135e-3 & 5.860e-5 & 0.4360 & -4.004 \\ 1.369 \times 10^4 & -564.6 & -7.664 \times 10^8 & -3.422 \times 10^7 \\ -1.369 \times 10^4 & 564.6 & 7.664 \times 10^8 & 3.422 \times 10^7 \\ -2.669 \times 10^{-4} & -7.324 \times 10^{-6} & 5.451 \times 10^{-2} & 5.429 \times 10^{-4} \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 23400 & 322 & 2.74 \times 10^8 & 0 & 0 \end{bmatrix}^\mathrm{T}$$

$$\mathbf{c} = \begin{bmatrix} 2.687 \times 10^4 & 591.8 & -1.205 \times 10^8 & -7.166 \times 10^6 \end{bmatrix}^\mathrm{T}$$

4) With $x_e = x_8$ entering by Bland's rule,

$$\mathbf{\Delta} = \begin{bmatrix} 1.7 \times 10^5 & 1.5 \times 10^6 & 2.0 \times 10^5 & \infty & \infty \end{bmatrix}^\mathrm{T}$$

and so $x_l = x_5$ leaves:

$$N = \{5, 1, 3, 9\} \quad B = \{8, 6, 7, 2, 4\} \quad v = 4.700 \times 10^8$$

$$\mathbf{A} = \begin{bmatrix} 0.7476 & 0.1095 & 814.9 & -8.117 \\ 1.596 \times 10^{-3} & -1.752 \times 10^{-4} & -1.304 & -3.987 \\ -1.023 \times 10^4 & -2.063 \times 10^3 & -7.776 \times 10^8 & -3.410 \times 10^7 \\ 1.023 \times 10^4 & 2.063 \times 10^3 & 7.776 \times 10^8 & 3.410 \times 10^7 \\ 1.995 \times 10^{-4} & 2.190 \times 10^{-5} & 0.1630 & -1.623 \times 10^{-3} \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} 1.749 \times 10^4 & 284.7 & 3.458 \times 10^7 & 2.394 \times 10^8 & 4.669 \end{bmatrix}^\mathrm{T}$$

$$\mathbf{c} = \begin{bmatrix} -2.008 \times 10^4 & -2.350 \times 10^3 & -1.424 \times 10^8 & -6.948 \times 10^6 \end{bmatrix}^\mathrm{T}$$

Now the objective function contains only negative coefficients, so the simplex algorithm terminates.

4.3 Interpretation

Setting each basic variable $B = \{8, 6, 7, 2, 4\}$ to the corresponding value from $\mathbf{b}$ and zeroing each nonbasic variable $N = \{5, 1, 3, 9\}$ yields the solution

$$\langle x_1, x_2, x_3, x_4 \rangle = \langle 0, 2.394 \times 10^8, 0, 4.669 \rangle$$

This means that, to maximize the total economic activity subject to constraints based on the current assumptions:

- x_1: Abalone production should not increase.
- x_2: Bottled water production should be increased by $2.394 \times 10^8 \, \frac{\text{bottles}}{\text{year}}$, or a little less than the full production of a large new plant.
- x_3: Microalgae production should not increase.
- x_4: There should be new basic research equivalent to 4.669 OTEC heat exchanger test facilities.

Of course, the recommended development is potentially very sensitive to model assumptions; this will be explored later.

The total economic activity generated by new development under the optimal solution is obtained simply by substituting the chosen $\mathbf{x}$ into the original objective function $\mathbf{c}^\mathrm{T}\mathbf{x}$:

$$z = \$470,005,200 \text{ per year}$$

5 Exploring the Solution

5.1 Validating the Solution by Hand

The following Python 3 script (with `numpy` and `scipy` packages) validates the hand solution:

```
#!/usr/bin/env python3
from itertools import starmap
import numpy as np
from scipy.optimize import linprog

w = np.array([0.11, 2.38e-7, 1e3, 5e3])
l = np.array([1.1e-4, 1.17e-7, 90, 2])
v = np.array([50, 0.208, 34e6, 9e7])

A = np.array([
    w, l, [0, 1, 0, 0], [1, 1, 1, -3] * w / 4,
    [1, 1, 1, -3] * l / 4, ])
b = np.array([23400, 322, 2.74e8, 0, 0])
c = v

# Our standard form is a maximization, but this
# routine is a minimization; simply negate the
# coefficients of the objective function.
result = linprog(
    method='simplex', c=-c, A_ub=A, b_ub=b)
print("z = {z:0.4g}".format(z=-result.fun))
print("; ".join(starmap("x{0} = {1:0.4g}".format,
    enumerate(result.x, 1))))
```

Its output matches the hand solution exactly:

```
z = 4.7e+08
x1 = 0; x2 = 2.394e+08; x3 = 0; x4 = 4.669
```

5.2 Visualization

The feasible region (simplex) is an n-dimensional convex polytope, where n is the number of variables in standard form, or the number of nonbasic variables in slack form. Here $n = 4$, the number of uses being optimized.

The following two-dimensional plots represent several cross-sections of this higher-dimensional structure, using a colormap as a third dimension and holding a fourth dimension at a fixed value.

Each plot is formed by uniformly sampling a chosen volume in parameter space and evaluating the constraints at each parameter tuple; if all are satisfied, the point lies in the feasible region. This is relatively computationally intensive, but much simpler than enumerating the actual simplex vertices, which would be equivalent to the exponential-time worst case of the simplex algorithm.

Figure 1 shows the feasible solution space when there is no additional abalone production ($x_1 = 0$). Figure 2 zooms in on the upper edge of Figure 1. This three-dimensional cross-section of the simplex seems to be a tetrahedron formed by two nonnegativity constraints and two additional planes.

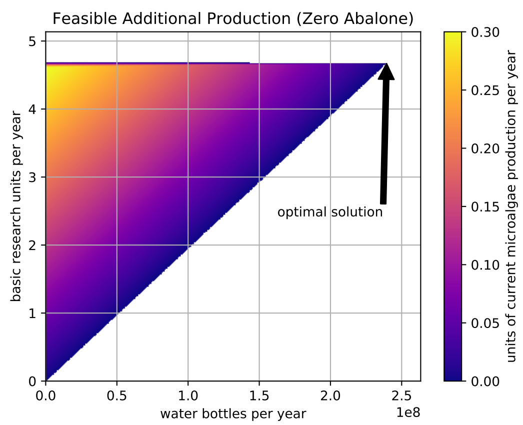

Fig. 1. Feasible region with $x_1 = 0$ (no additional abalone)

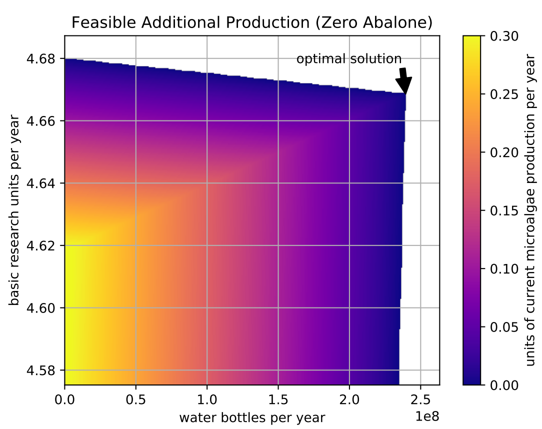

Fig. 2. Feasible region with $x_1 = 0$; x_4 near feasible limit

Figure 3 shows the feasible solution space when there is no additional microalgae production ($x_3 = 0$). This 3D cross-section of the 4D simplex also seems to be tetrahedral.

Since the optimal solution has $\langle x_1, x_3 \rangle = \langle 0, 0 \rangle$, the feasible regions of Figures 1-3 all contain the optimal solution. We could also hold x_2 or x_4 at the (nonzero) optimal value and explore the result.

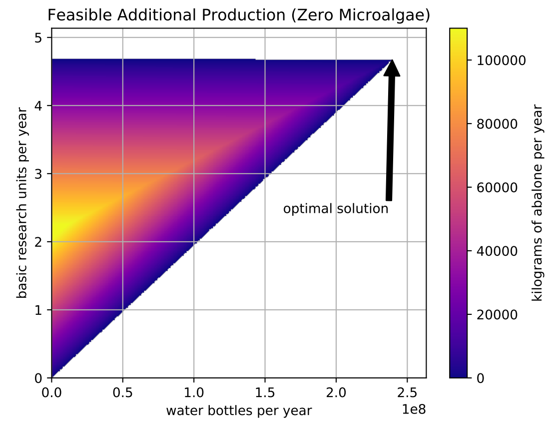

Fig. 3. Feasible region with $x_3 = 0$ (no additional microalgae)

It turns out that water bottling creates the most economic activity per unit of land or water, so resources will be allocated to it until either the bottling expansion limit $x_2 \le 2.74 \times 10^8$ or the basic research land quota $l_4 x_4 \ge \frac{1}{4} \sum_{i=1}^{N} \ell_i x_i$ is reached. The optimal solution reaches the land quota, so with $x_2 = 2.394 \times 10^8$, the land quota determines x_4 and there are no resources for x_1 or x_3; hence the feasible region in this case is a single point.

Finally, holding $x_4 = 4.669$ yields Figure 4. This shows that, with the basic research increase from the optimal solution, the remaining resources only allow a very small increase in abalone or microalgae production; but a huge increase in bottled water production is still possible.

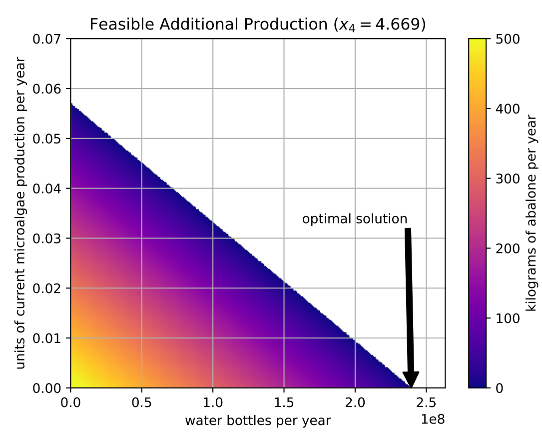

Fig. 4. Feasible region with $x_4 = 4.669$ (optimal basic research)

6 Varying Constraints

By varying the values of constraints, we can analyze the sensitivity of the optimal solution to assumed values and external processes.

6.1 Microalgae Drying Efficiency

As mentioned before, actual numbers are not readily available for Cyanotech's current DSW usage. We vary w_3 in $\left[1 \times 10^2, 1 \times 10^5\right]$ to represent this uncertainty. Figure 5 shows that, within a plausible range, this parameter has no influence on the optimal solution.

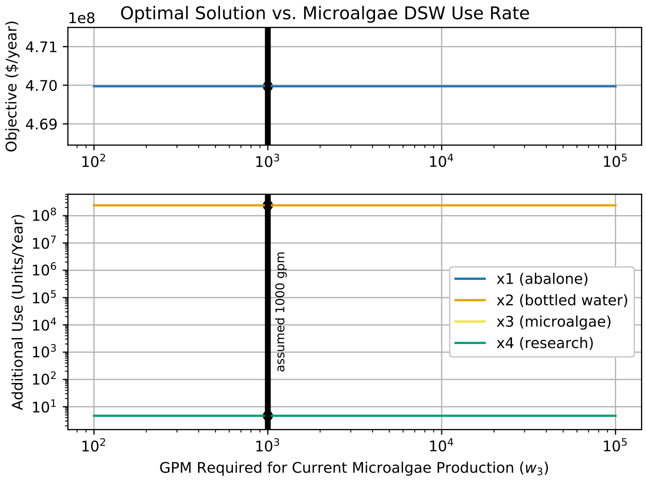

Fig. 5. Optimal solution with $w_3 \in \left[1 \times 10^2, 1 \times 10^5\right]$

6.2 Typical Basic Research DSW Efficiency

While the OTEC heat exchanger experiment is representative of basic research endeavors, its water usage might not be typical. Figure 6 shows how the optimal solution is affected by this assumption via its effect on the constraints.

This plot is much more interesting. There is a certain relatively narrow range of water use for basic research that results in additional microalgae production being part of the optimal solution. We can see that the recommended mix of additional production can be quite sensitive to the actual water efficiency of each use.

6.3 Pumping Capacity

What if additional DSW pumping capacity were constructed? Knowing the increase in total economic activity for a hypothetical capacity increase, considering other constraints like available land, would be very important in deciding whether to spend the money on an expansion.

Figure 7 shows that economic activity continues to increase roughly linearly for plausible

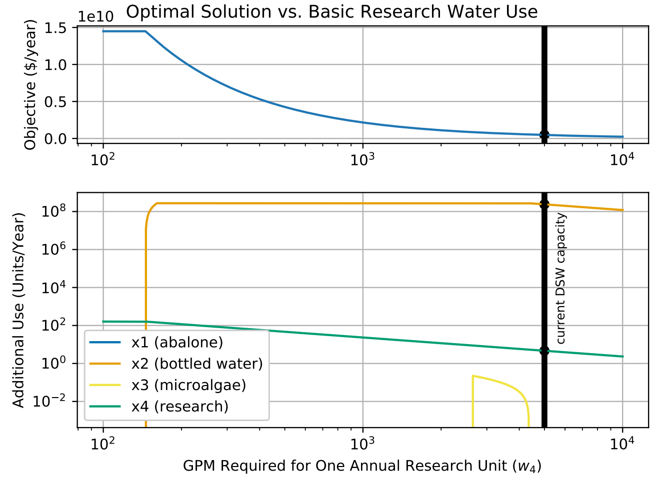

Fig. 6. Optimal solution with $w_4 \in \left[1 \times 10^2, 1 \times 10^4\right]$

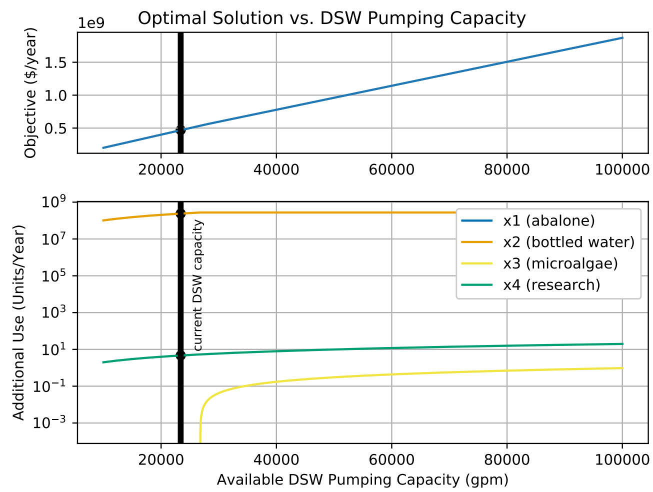

Fig. 7. Optimal solution with $w_4 \in \left[1 \times 10^2, 1 \times 10^4\right]$

pipeline expansions. With sufficient additional capacity, added microalgae production joins the optimal solution.

7 Additional Parameters

7.1 Abalone Wholesale Prices

Originally, we planned to consider abalone wholesale prices across and just outside the plausible market range, $v_1 \in [10, 200]$ per kg. It turns out that no price in this interval brings additional abalone production into the optimal solution. Figure 8 shows the variation over a much wider range.

It turns out that, in the current model, additional abalone production is not worth its extreme DSW usage unless the wholesale price of abalone is well over $2,000 per kg! This is far outside the plausible range.

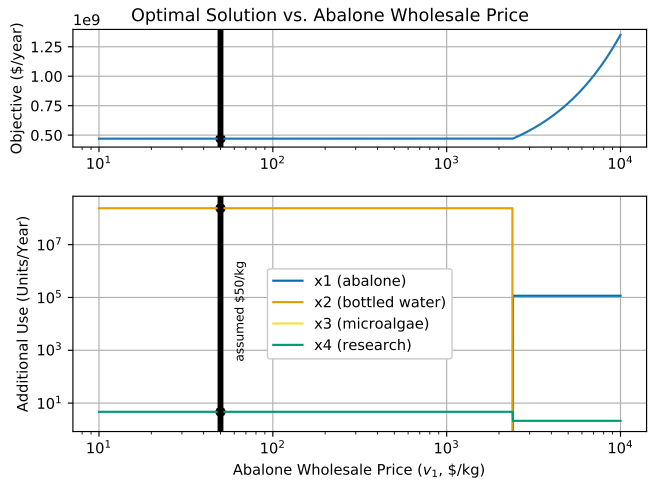

Fig. 8. Optimal solution with $v_1 \in [1 \times 10^1, 1 \times 10^4]$

7.2 Basic Research Payoff

The value of one unit of research is hard to estimate. We used the parameters of the OTEC heat exchanger test facility and some wild assumptions to come up with an annual value of 9×10^7 for one unit of research. Figure 9 considers a broad range around this assumed value.

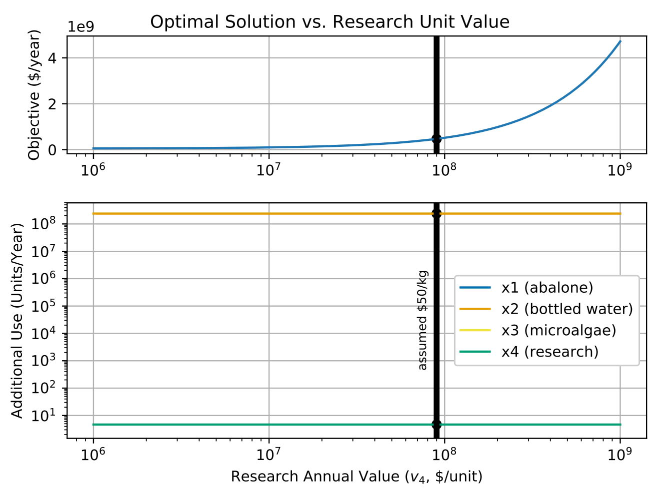

Fig. 9. Optimal solution with $v_4 \in [1 \times 10^6, 1 \times 10^9]$

In this wide plausible range, the economic value of research and pre-commercial activities has a major influence on the objective (total economic activity), but no influence on the optimal allocation of uses.

8 Conclusion

As with most economic models, a key concern about the present model is its potential sensitivity to parameters and assumptions, many of which cannot be easily validated. This report has shown that varying parameters and observing the solution response is a simple way to assess the sensitivity of the model to particular assumptions.

Linear programming provides a viable tool for modeling the economic impact of resource allocation and expansion decisions at NELHA. Even the present greatly simplified model demonstrates complex interactions among constraints and shows how linear programming can be used to recommend, analyze, and justify development and production decisions.

References

[1] NELHA, *Master Plan for Natural Energy Laboratory of Hawai'i Authority*. Kailua-Kona: NELHA, 2011.

[2] "Park assets," http://nelha.hawaii.gov/park-assets/, accessed: 2018-11-05.

[3] NELHA, "Nelha 2015 seawater rate analysis: Final report," http://nelha.hawaii.gov/wp-content/uploads/2013/09/2015.Seawater.Rate_.Analysis.Final_.Report.3.12.15.pdf, NELHA, Tech. Rep., 2015, accessed: 2018-11-05.

[4] "Our story," http://www.bigislandabalone.com/ourstory.html, accessed: 2018-11-05.

[5] GLOBEFISH, "Abalone production continues to grow, coupled with continuing demand, prices high and stable," http://www.fao.org/in-action/globefish/market-reports/resource-detail/en/c/902597/, UN FAO, Tech. Rep., 2017, accessed: 2018-11-05.

[6] D. Shimogawa, "The largest deep sea bottling plant in the world," *Hawaii News Now*, 2010, accessed: 2018-11-05.

[7] "How much does bottled water cost?" https://www.bottledwater.org/economics/real-cost-of-bottled-water, accessed: 2018-11-05.

[8] "Our purpose – cyanotech," https://www.cyanotech.com/our-purpose/, accessed: 2018-11-05.

[9] "Form 10-k: Cyanotech corporation," https://www.sec.gov/Archives/edgar/data/768408/000143774918011915/cyan20180331_10k.htm, accessed: 2018-11-05.

[10] "Ocean thermal energy conversion," https://www.makai.com/ocean-thermal-energy-conversion/, accessed: 2018-11-05.

[11] O. Comstock, "Construction costs for most power plant types have fallen in recent years," *Today in Energy*, 2017, accessed: 2018-11-05.