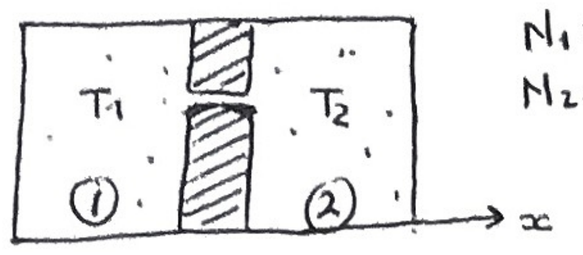
(1.)

N, T_1, T_2

$N_1 = ?$
$N_2 = ?$



* u redu za realnim plinovima možemo zanemariti zbog premalu i Bolcm. konstantu.

v_{1x}, v_{2x}, n_1, n_2

nema sudara, iz suda 1 u sud 2 za neko vrijeme Δt prođe ΔN_1 molekula, a iz suda 2 u sud 1, ΔN_2 molekula:

$$\Delta N_1 = \frac{1}{2} n_1 \cdot S \cdot v_{1x} \cdot \Delta t \qquad \text{ravnoteža} \Rightarrow \Delta N_1 = \Delta N_2$$

$$\Delta N_2 = \frac{1}{2} n_2 \cdot S v_{2x} \cdot \Delta t$$

$$\Rightarrow \quad \cancel{\frac{1}{2}} n_1 \cancel{S} \cdot v_{1x} \cdot \cancel{\Delta t} = \cancel{\frac{1}{2}} n_2 \cancel{S} \cdot v_{2x} \cdot \cancel{\Delta t}$$

$$\Rightarrow \quad n_1 v_{1x} = n_2 v_{2x}$$

koncentracija: $n = \frac{N}{V}$

sr. brzina: $v = \sqrt{\frac{8kT}{\pi \cdot m_0}}$ ← čišći opis

m_0 – masa jedne molekule

ali, bolje koristiti sr. kvadratnu brzinu:

$$\overline{v} = \sqrt{\frac{3kT}{m_0}}$$

$$\overline{v}^2 = \overline{v_x}^2 + \overline{v_y}^2 + \overline{v_z}^2 = 3\overline{v_x}^2$$

$$\Rightarrow \overline{v_x} = \sqrt{\frac{kT}{m_0}}$$

* formula za broj molekula koji prođu kroz kanal:

$$\Delta N = \frac{1}{2} \cdot n \cdot S \cdot \underbrace{v_i \cdot \Delta t}_{\text{izotropnost} \Rightarrow d}$$

$$\Rightarrow \frac{N_1}{V} \sqrt{\frac{kT_1}{m_0}} = \frac{N_2}{V} \sqrt{\frac{kT_2}{m_0}}$$

$$N_1 \sqrt{T_1} = N_2 \sqrt{T_2}$$

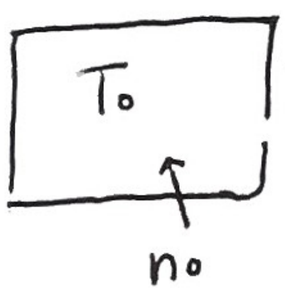
$N = N_1 + N_2 \Rightarrow$

$$N_1 \sqrt{T_1} = (N - N_1)\sqrt{T_2}$$

$$\Rightarrow \quad N_1 = \frac{(N - N_1)\sqrt{T_2}}{\sqrt{T_1}} \Rightarrow \boxed{N_1 = \frac{N\sqrt{T_2}}{\sqrt{T_1} + \sqrt{T_2}}}$$

$$\boxed{N_2 = \frac{N\sqrt{T_1}}{\sqrt{T_1} + \sqrt{T_2}}}$$

(2.) $T_0, p, T, p_0 = ?$



$$\Delta N_0 = \frac{1}{2} n_0 S v_{0x} \cdot \Delta t$$

$$\Delta N = \frac{1}{2} n S v_x \cdot \Delta t$$

ravnoteža: $\Delta N_0 = \Delta N$

* m_0 ne miješati sa drugim parametrima koji u redovnu imaju 0!

$$\Rightarrow \cancel{\frac{1}{2}} n_0 \cancel{S} v_{0x} \cdot \Delta t = \cancel{\frac{1}{2}} n \cancel{S} \cdot v_x \cdot \Delta t$$

$$\Rightarrow \quad n_0 \cdot v_{0x} = n \cdot v_x$$

$$v_x = \sqrt{\frac{3kT}{m_0}} \quad \Rightarrow \quad n_0 \cdot \sqrt{\frac{3kT_0}{m_0}} = n \cdot \sqrt{\frac{3kT}{m_0}}$$

$$v_{0x} = \sqrt{\frac{3kT_0}{m_0}} \quad \Leftrightarrow \quad n_0 \sqrt{T_0} = n\sqrt{T}$$

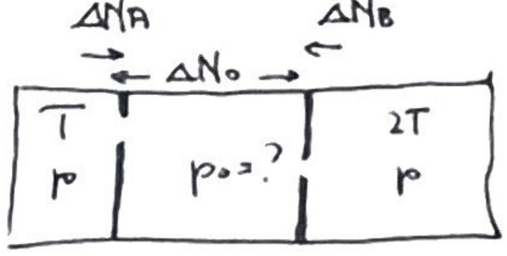
$$\Rightarrow \quad n_0 = n\sqrt{\frac{T}{T_0}}$$

$$\left.\begin{aligned} p_0 \cdot V &= n_0 k T_0 \\ pV &= nkT \end{aligned}\right\} \Rightarrow \quad \frac{p_0}{p} = \frac{n_0 k T_0}{nkT} = \frac{n_0 T_0}{nT}$$

$$\Rightarrow \quad \frac{p_0}{p} = \frac{n\sqrt{\frac{T}{T_0}} \cdot T_0}{nT} = \sqrt{\frac{T}{T_0}} \frac{T_0}{T} = \sqrt{\frac{T_0}{T}}$$

$$\Rightarrow \boxed{p_0 = p\sqrt{\frac{T_0}{T}}}$$

3.)



ravnoteža gor u sudu, dvije pregrade

ravnoteža: $2\Delta N_0 = \Delta N_0 + \Delta N_B$

$$\left.\begin{aligned}\Delta N_A &= \tfrac{1}{2} n_1 v_{1x} \cdot S\Delta t \\ \Delta N_B &= \tfrac{1}{2} n_2 v_{2x} \cdot S\Delta t \\ \Delta N_0 &= \tfrac{1}{2} n_0 \cdot v_{0x} \cdot S\cdot\Delta t\end{aligned}\right\} \Rightarrow \tfrac{1}{2} n_0 \cdot v_{0x} \cdot \cancel{S\Delta t} = \tfrac{1}{2}\left(n_1 v_{1x}\cancel{S\Delta t} + n_2 v_{2x}\cancel{S\Delta t}\right)$$

$$2 n_0 \cdot v_{0x} = n_1 v_{1x} + n_2 v_{2x}$$

$$\left.\begin{aligned} v_{0x} &= \sqrt{\frac{3kT_0}{m_0}} \\ v_{1x} &= \sqrt{\frac{3kT_1}{m_0}} \\ v_{2x} &= \sqrt{\frac{3kT_2}{m_0}}\end{aligned}\right\} \Rightarrow \quad 2\cdot n_0 \sqrt{\frac{3kT_0}{m_0}} = n_1 \cdot \sqrt{\frac{3kT_1}{m_0}} + n_2 \sqrt{\frac{3kT_2}{m_0}}$$

$$\Rightarrow \quad 2 n_0 \sqrt{T_0} = n_1\sqrt{T_1} + n_2\sqrt{T_2}$$

$$\left.\begin{aligned} p_1 V_0 &= n_1 k T_1 \\ p_2 V &= n_2 k T_2 \\ p_0 V &= n_0 k T_0\end{aligned}\right\} \Rightarrow n_i = \frac{p_i V}{kT}$$

$$\Rightarrow \quad 2\cdot p_0 \frac{\sqrt{T_0}}{T_0} = p \frac{\sqrt{T_1}}{T_1} + p \frac{\sqrt{T_2}}{T_2}$$

$T_1 = T$, $T_2 = 2T$ $\Rightarrow$ $2\frac{p_0}{T_0}\sqrt{T_0} = \frac{p}{T}\sqrt{T} + \frac{p}{2T}\sqrt{2T}$ (*)

Natrag:

$$2\frac{p_0}{T_0} T_0^{3/2} = \frac{p}{T} T^{3/2} + \frac{p}{2T}\cdot (2T)^{3/2}$$

$$\Rightarrow \quad 2 p_0 T_0^{1/2} = p T^{1/2} + p (2T)^{1/2}$$

$$\Rightarrow \quad 2 p_0 \sqrt{T_0} = p\sqrt{T} + p\sqrt{2T} \quad (2)$$

(1) i (2) nam daju:

$$\left.\begin{aligned} \frac{p(1+\sqrt{2})}{\sqrt{2T}} &= \frac{2p_0}{\sqrt{T_0}} \quad /\sqrt{T}\end{aligned}\right\} \Rightarrow \frac{1}{\sqrt{2}} p (\sqrt{T} + \sqrt{2T}) = 2p_0 \frac{T}{\sqrt{T_0}}$$

$$\Rightarrow p(\sqrt{T} + \sqrt{2T}) = \frac{2\sqrt{2}\, p_0 T}{\sqrt{T_0}}$$

$$p(\sqrt{T} + \sqrt{2T}) = 2p_0\sqrt{T_0}$$

$$2p_0\sqrt{T_0} = \frac{2\sqrt{2}\, p_0 T}{\sqrt{T_0}}$$

$$\boxed{T_0 = \sqrt{2}\cdot T}$$

$$p = \frac{2p_0\sqrt{T_0}}{\sqrt{T} + \sqrt{2T}} = \frac{2\cdot p_0 \sqrt{\sqrt{2}\cdot T}}{\sqrt{T} + \sqrt{2T}}$$

$$p = \frac{2\cdot 2^{1/4} \cdot p_0 \cancel{\sqrt{T}}}{\cancel{\sqrt{T}}(1+\sqrt{2})} = \frac{2}{1+\sqrt{2}} 2^{5/4} p_0$$

$$\boxed{p_0 = p\cdot(1+\sqrt{2})\cdot 2^{-5/4}}$$

Natrag:

$$\Rightarrow (*) \Rightarrow \quad \frac{p(1+\sqrt{2})}{\sqrt{2T}} = \frac{2p_0}{\sqrt{T_0}} \quad (1)$$

$T_0 = \text{const.}$ $N_0 = \text{const.}$ $\rightarrow$ $E_{k0} = \text{const.}$

$$\overline{E_k} = \frac{m_0 \bar{v}^2}{2} \Rightarrow \bar{v} = \sqrt{\frac{2\overline{E_k}}{m_0}}$$

$$\overline{E_k} = \frac{3}{2} kT$$

Ukupna kin. energija molekula koji u jed. vremena uđu u centralni dio sudа, jednaka je ukupnoj kin. energiji molekula koji izađu iz tog dijela. Modificiramo jednadžbu ravnoteže:

$$2\cdot \Delta N_0 \cdot \frac{3}{2} kT_0 = \Delta N_A \cdot \frac{3}{2} kT_1 + \Delta N_B \frac{3}{2} kT_2$$

$2\cdot\left(\frac{1}{2} n_1 v_{1x}\cdot S\Delta t\right)\frac{3}{2} kT_0 \ldots$ slično kao u prethodnom dijelu, pa slijedi:

$$2\cdot n_0 v_{0x}\cdot \frac{3}{2} kT_0 = n_1 v_{1x}\cdot \frac{3}{2} T_1 k + n_2 v_{2x}\cdot \frac{3}{2} kT_2 \Rightarrow (\ldots)$$

$$\Rightarrow \quad 2\cdot n_0 \sqrt{T_0}\cdot \frac{3}{2} kT_0 = n_1\sqrt{T_1}\cdot \frac{3}{2} kT_1 + n_2\sqrt{T_2}\cdot \frac{3}{2} kT_2$$

$$\Rightarrow \left\{\begin{aligned} T_1 &= T \\ T_2 &= 2T\end{aligned}\right\} \Rightarrow 2 n_0 T_0^{3/2} = n_1 \cdot T^{3/2} + n_2\sqrt{T\cdot 2}\cdot (2T)^{3/2}$$

U slučaju nekih nedoumica, slobodno mi piši :)

"međukoraci" su za samostalno rješavanje...