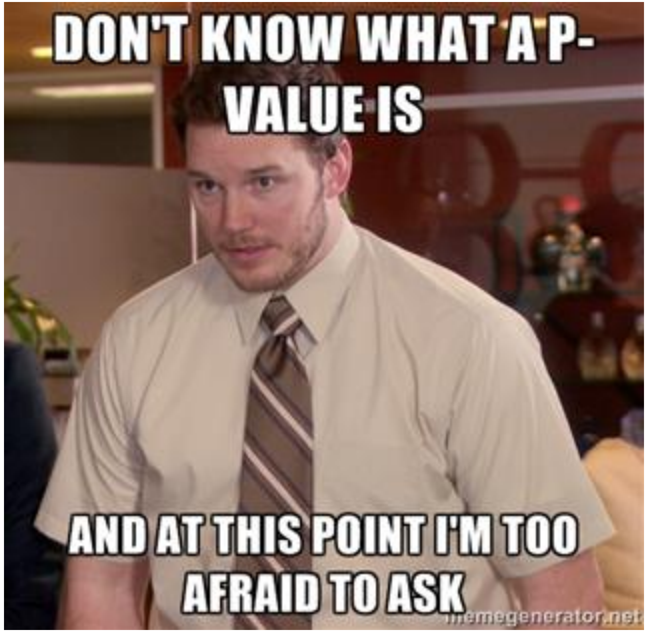
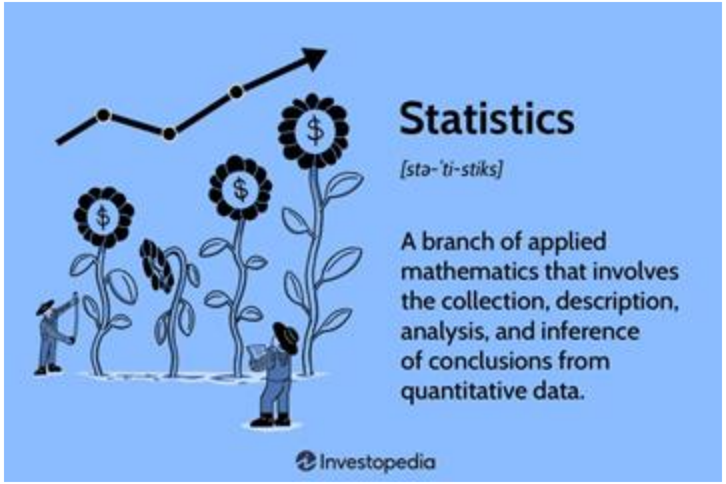
t-statistics





William Sealy Gosset



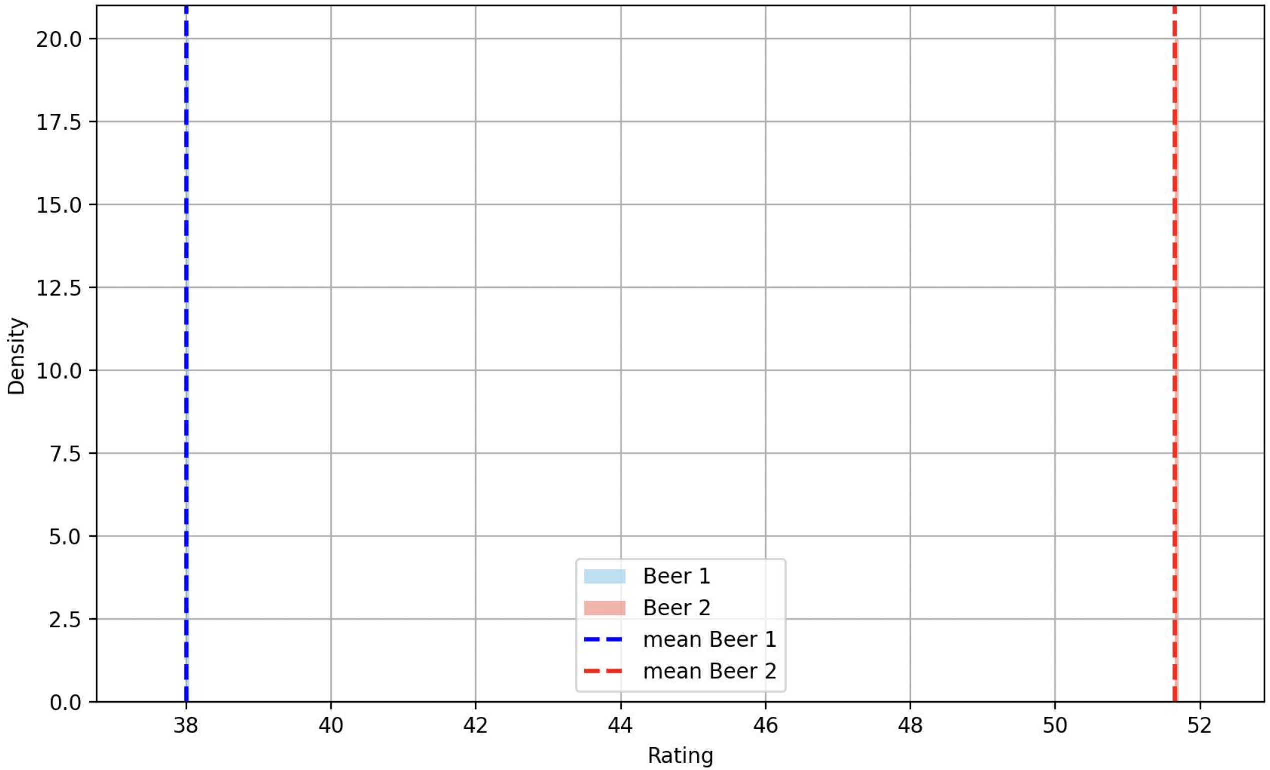
If I add the barley to the beer, its taste is going to improve.

independent variable = CAUSE

If I <u>add the barley</u> to the beer, its <u>taste is going to improve</u>.

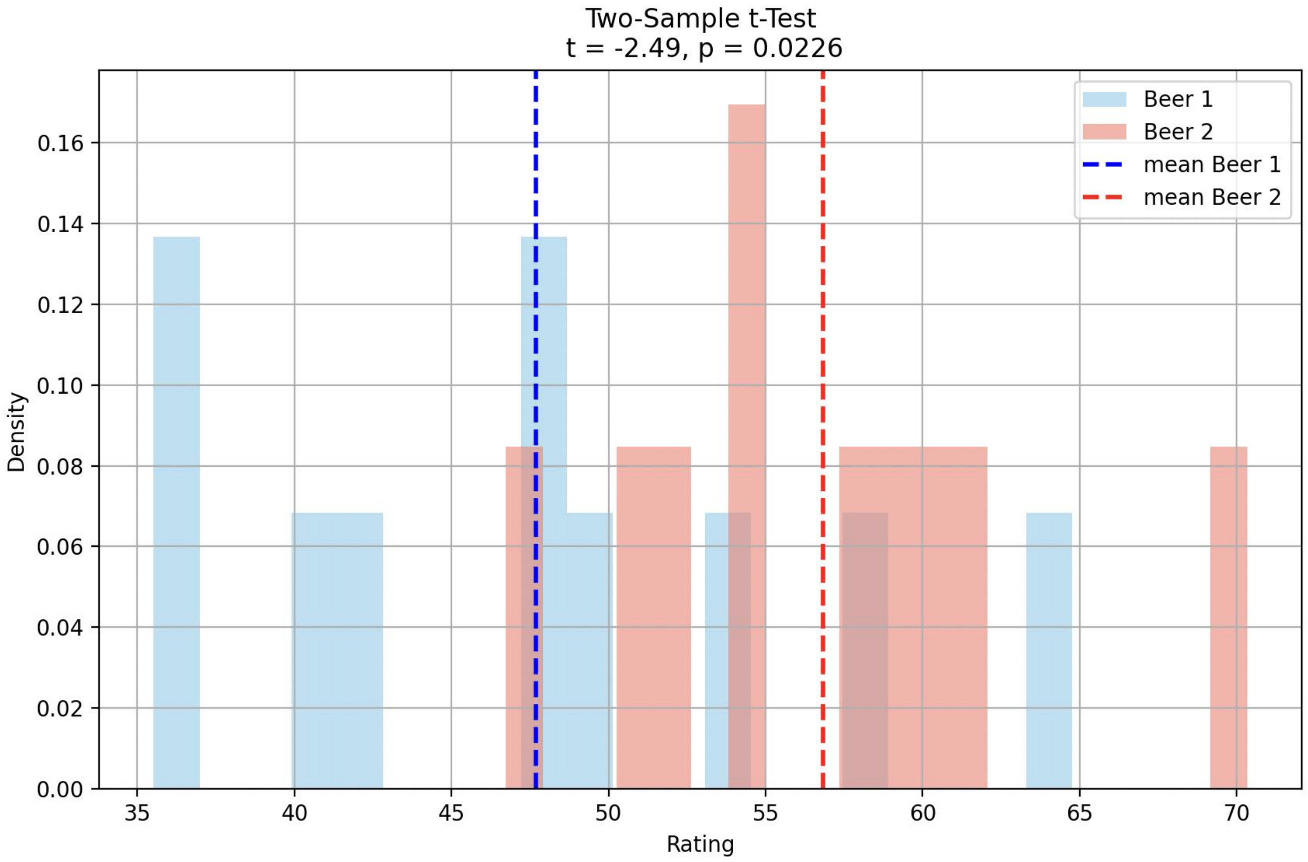
dependent variable = EFFECT

n = 1



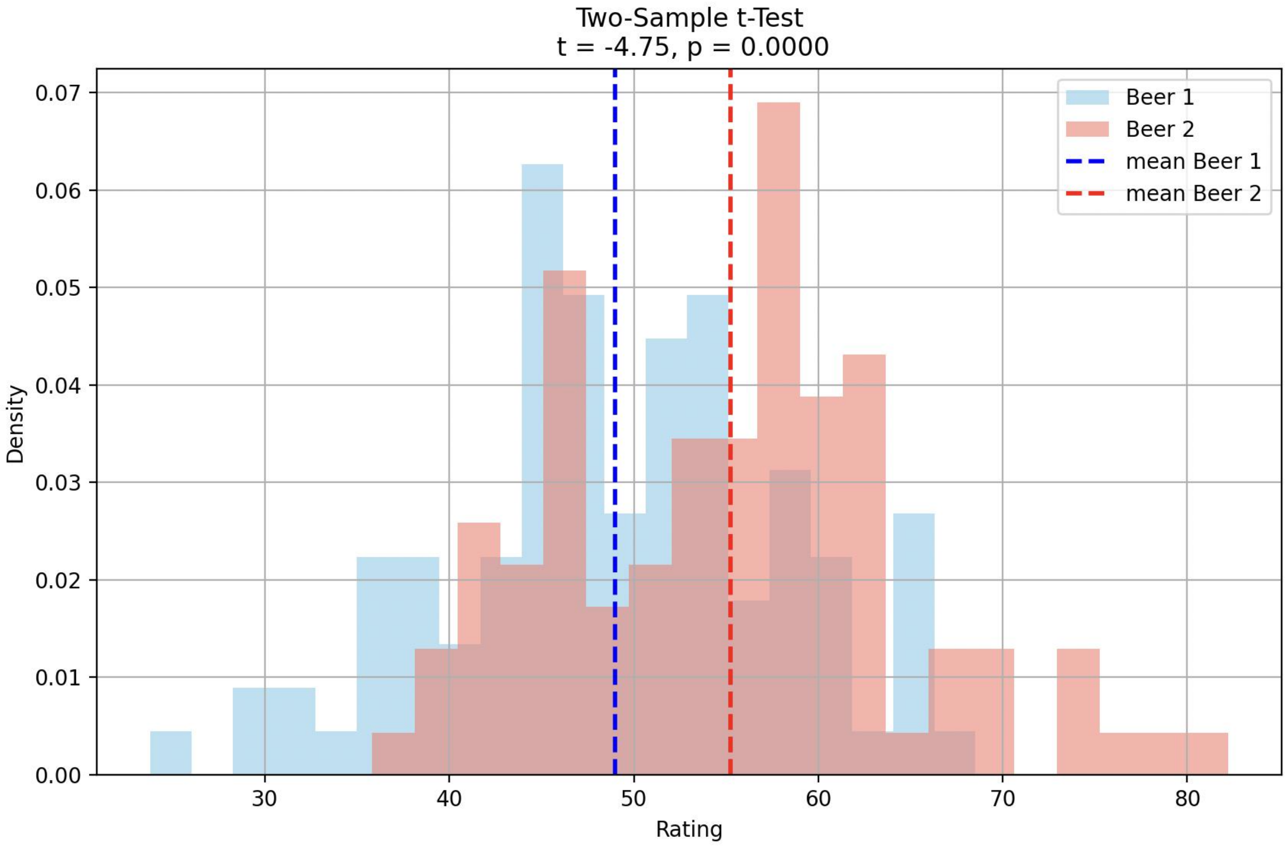
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
Density
38
40
42
44
46
48
50
52
Rating
Beer 1
Beer 2
mean Beer 1
mean Beer 2

n = 10

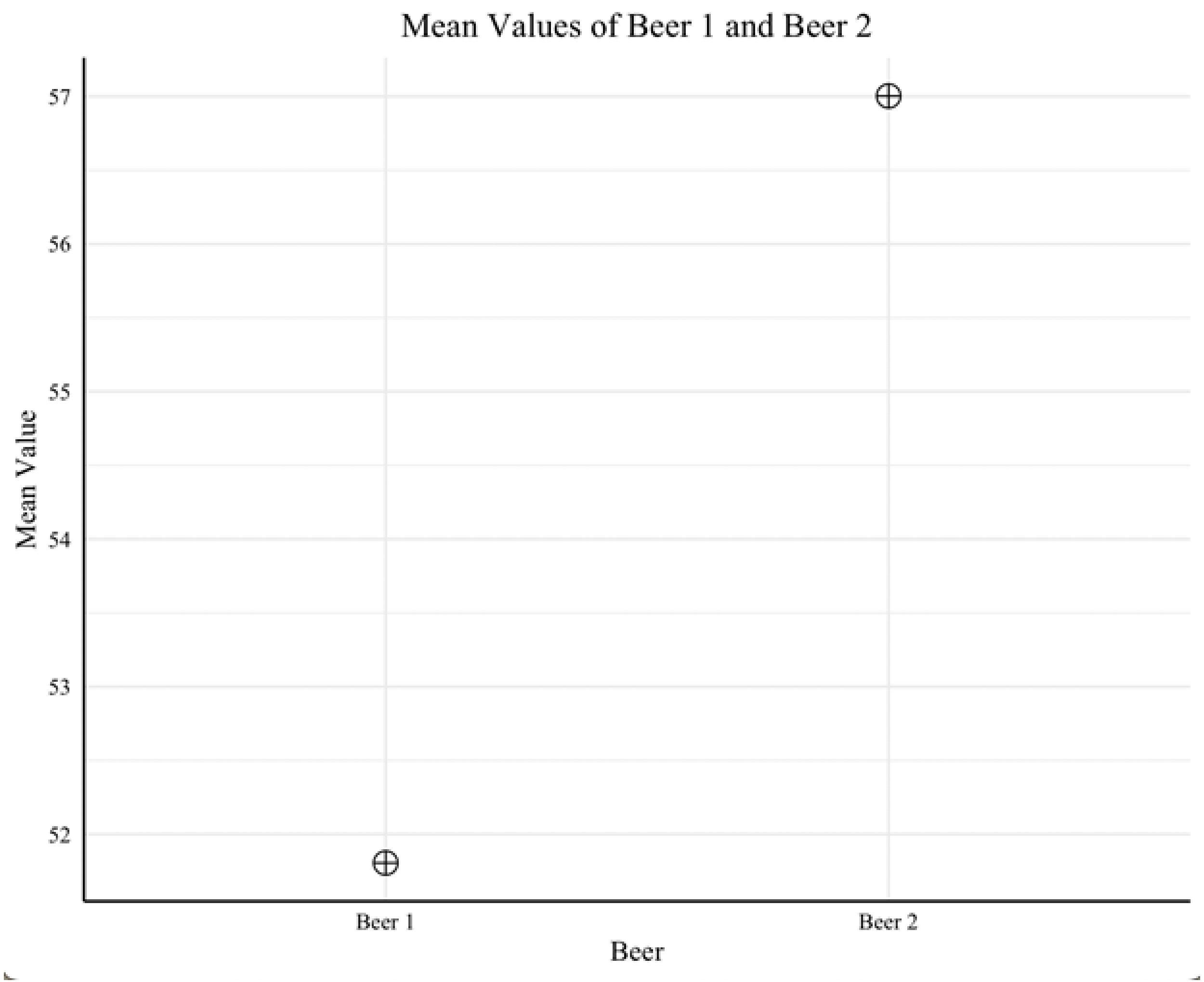


Two-Sample t-Test
t = -2.49, p = 0.0226
Beer 1
Beer 2
mean Beer 1
mean Beer 2
Density
Rating
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
35
40
45
50
55
60
65
70

n = 100

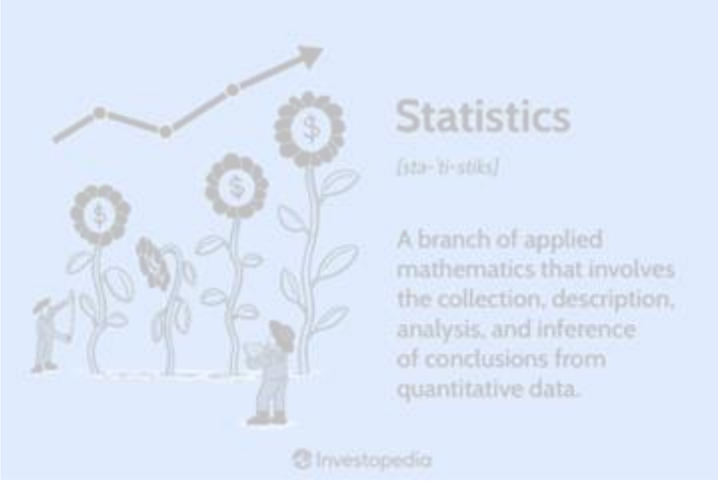


Two-Sample t-Test
t = -4.75, p = 0.0000
Beer 1
Beer 2
mean Beer 1
mean Beer 2
Density
Rating
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
30
40
50
60
70
80



Mean Values of Beer 1 and Beer 2
57
56
55
54
53
52
Mean Value
Beer 1
Beer 2
Beer

Student’s t-test



William Sealy Gosset

What is t-test?

T-test is a statistical test that is used to compare the <u>means</u> of <u>two groups</u>.

Two-Sample T-Test

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\bar{x}_1$ = observed mean of 1st sample
$\bar{x}_2$ = observed mean of 2nd sample
s_1 = standard deviation of 1st sample
s_2 = standard deviation of 2nd sample
n_1 = sample size of 1st sample
n_2 = sample size of 2nd sample

T-test assumptions

The data is continuous

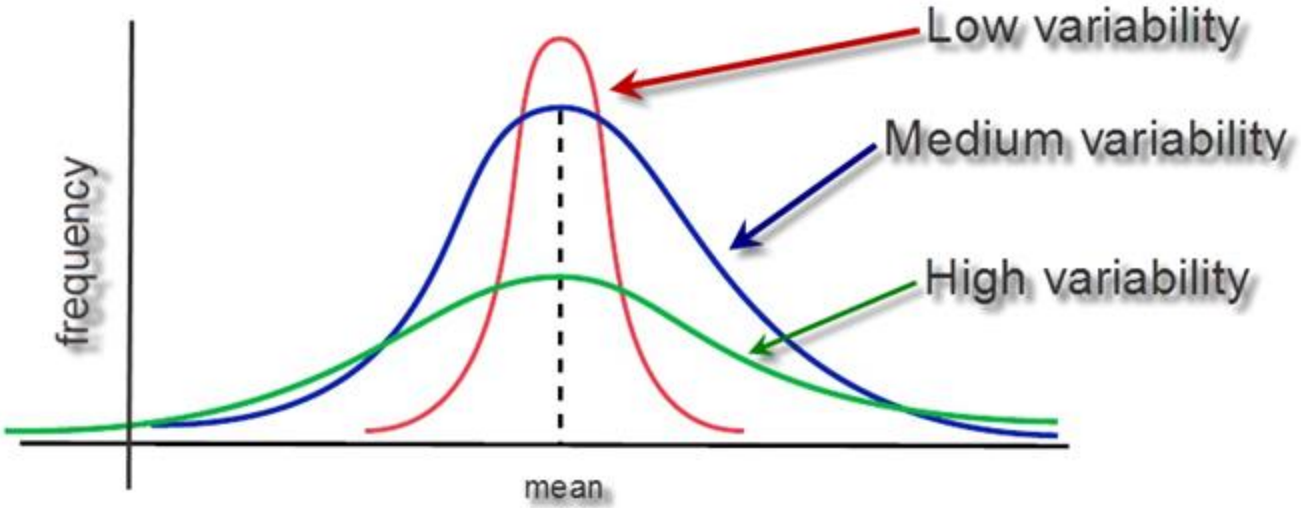
The data is randomly sampled from the population

The variance is homogenous

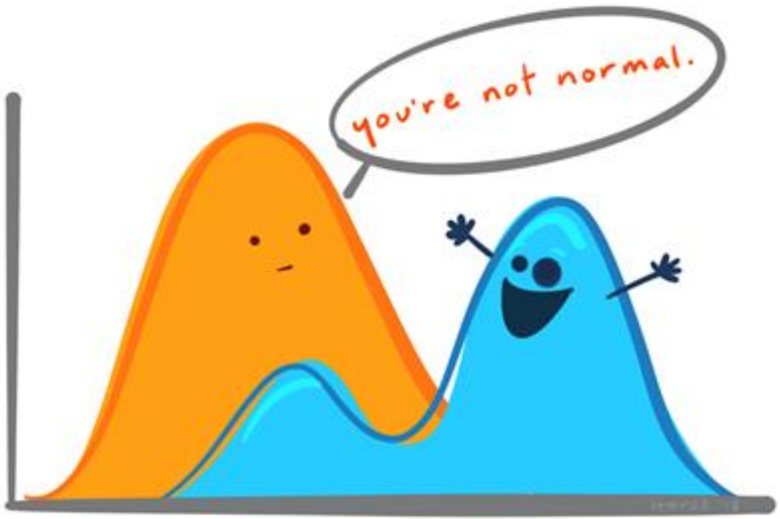
The data distribution is approx. normal

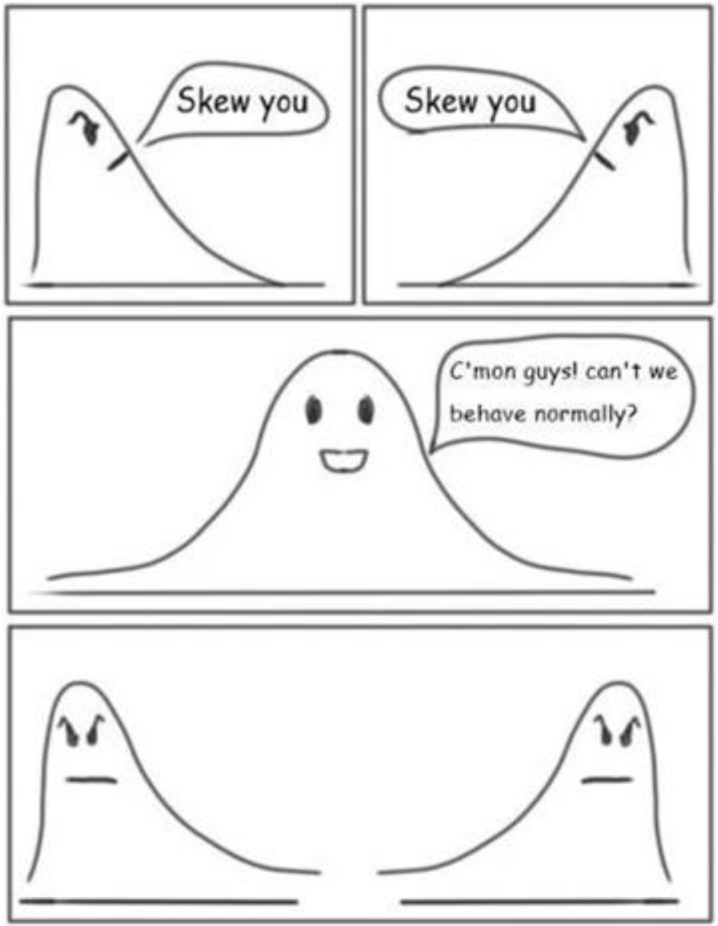
What is variance?

How spreaded are our data points.



Normal distribution





Hypothesis testing

The null hypothesis (H_0) is that the true difference between these group means is zero.

The alternative hypothesis (H_a) is that the true difference is different from zero.

Hypothesis testing

The <u>null hypothesis</u> (H_0):

$\mu_1 = \mu_2$

The <u>alternate hypothesis</u> (H_a):

$\mu_1 \neq \mu_2$ (two-tailed test)

$\mu_1 > \mu_2$ or $\mu_1 < \mu_2$ (one-tailed test)

Hypothesis testing

The null hypothesis (H_0):

Both beers are rated similarly.

The alternate hypothesis (H_a):

There is a difference in rating the taste of beer 1 and beer 2. (two-tailed test)

Beer 1 is better than beer 2. or Beer 2 is better than beer 1. (one-tailed test)

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t = -1.7067, df = 98, p-value = 0.09106

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The t-value measures the difference between the means of the two groups relative to the variance of the data.

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The t-value (-1.7067) indicates the direction of the difference between the means. Since it is negative, it suggests that the mean of group 1 is lower than the mean of group 2.

The results from our data

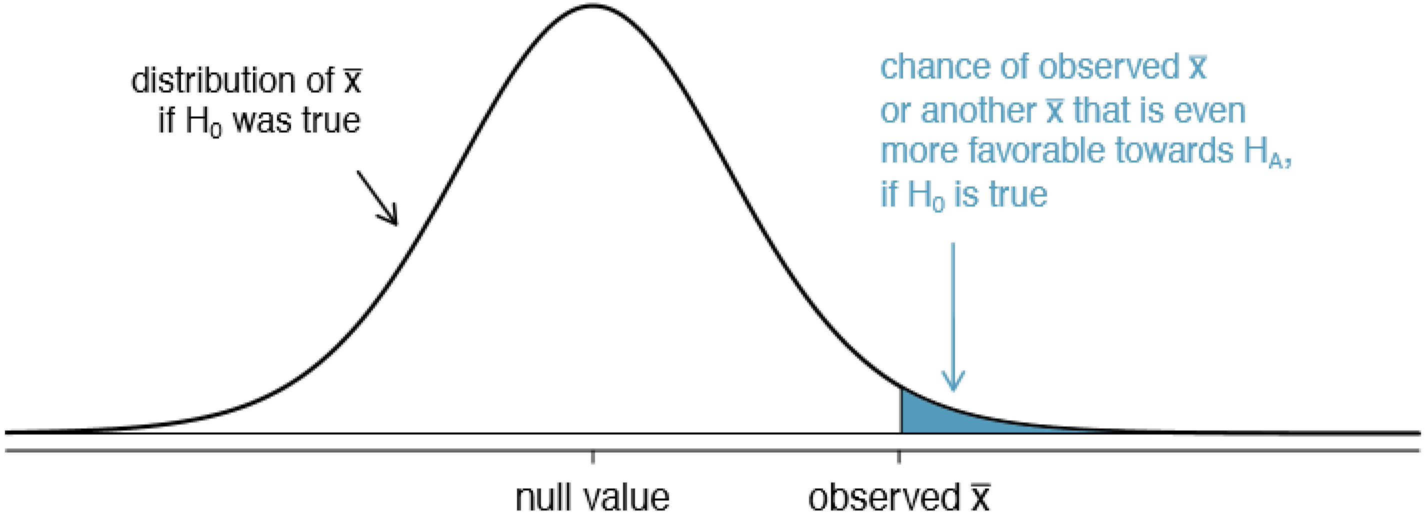
t = -1.7067, **df = 98**, p-value = 0.09106

Degrees of freedom are the maximum number of logically independent values, which may vary in a data sample. Degrees of freedom are calculated by subtracting one from the number of items within the data sample.

$df = n_1 + n_2 - 2$

The results from our data

t = -1.7067, df = 98, **p-value = 0.09106**



conclusion from statistical analysis

the true state of nature

	Accept the Null	Reject the Null
Null hypothesis is **true**	**Correct Conclusion**	**Type I Error** **reject a true null hypothesis**
Null hypothesis is **false**	**Type II Error** **accept a false null hypothesis**	**Correct Conclusion**

Type I Error (false-positive)

Type II Error (false-negative)

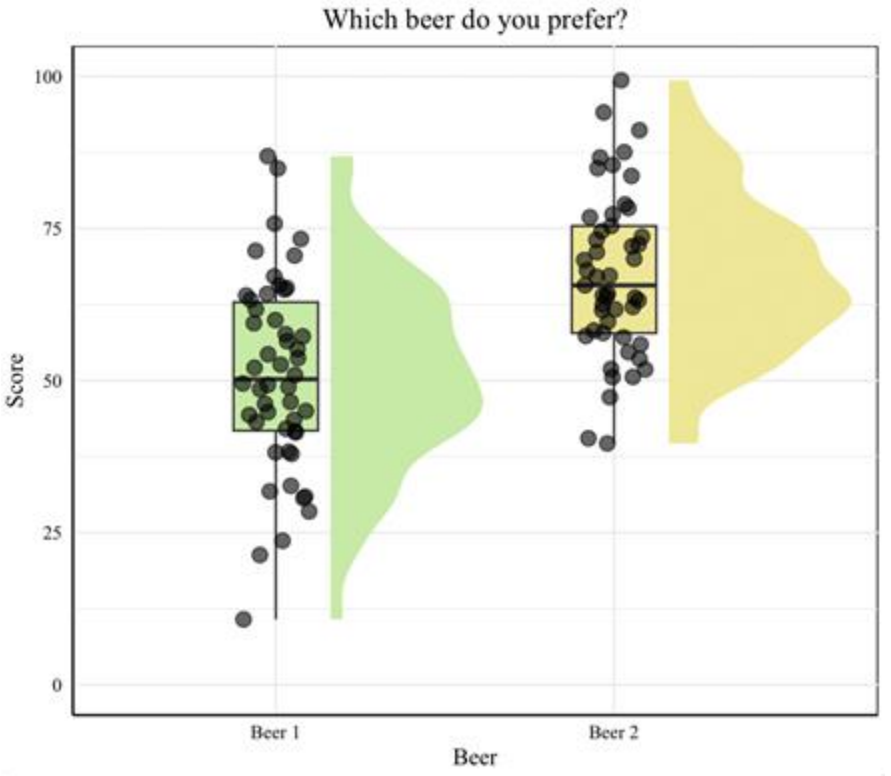
The results from our data

t = -1.7067, df = 98, **p-value = 0.09106**

There is no difference between preferences for any of the beer types - we confirm H_0 and reject H_a.

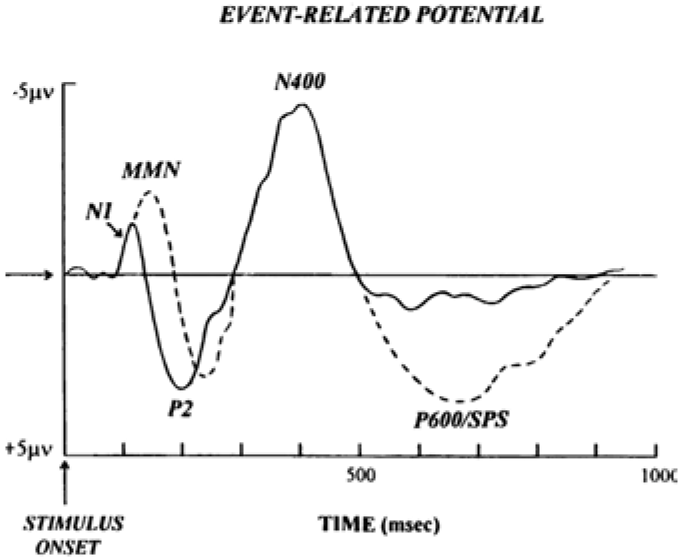
Exercise - interpret the following results

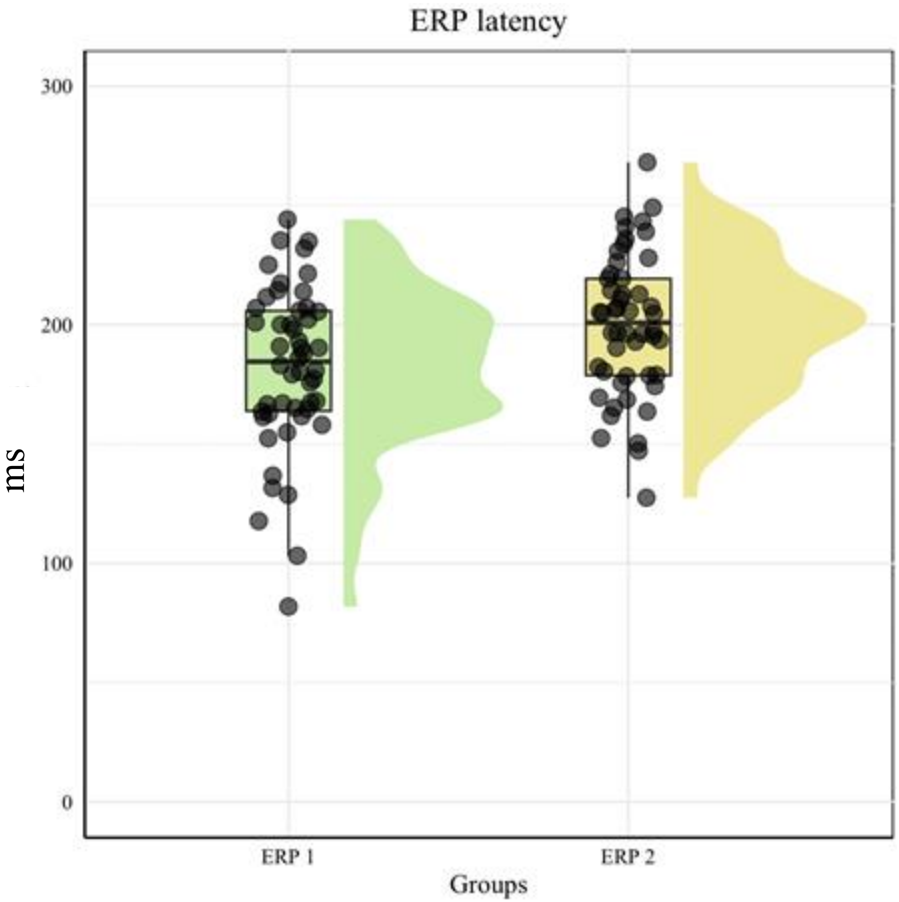
t = -5.63, df = 98, p-value = 0.0000001752



T-test in the EEG world

- Latency of ERP peak

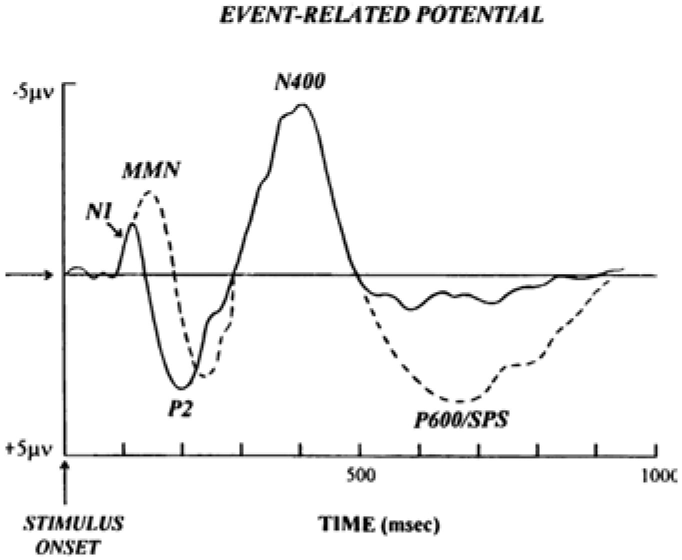


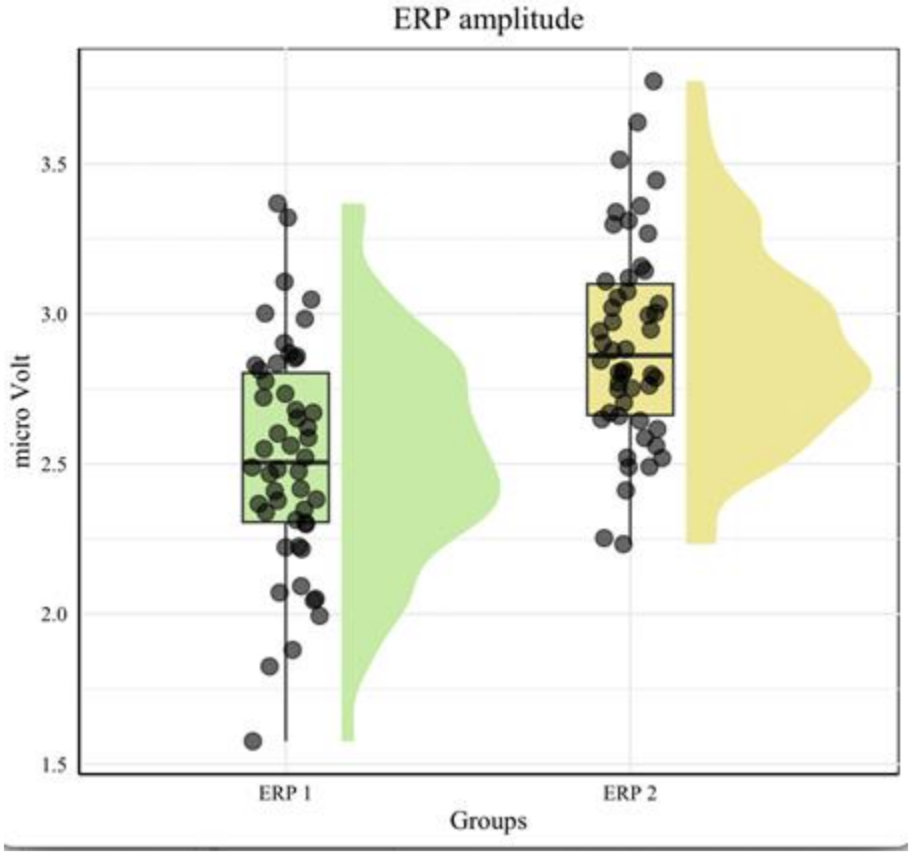


t = -2.8006, df = 98, p-value = 0.006166

T-test in the EEG world

- Amplitude of ERP peak





t = -4.7139, df = 98, p-value = 0.000008064

What about EEG?

EEG data often do not fulfill t-test assumptions.

T-test assumptions

The data is continuous

The data is randomly sampled from the population

The variance is homogenous

The data distribution is approx. normal

What about EEG?

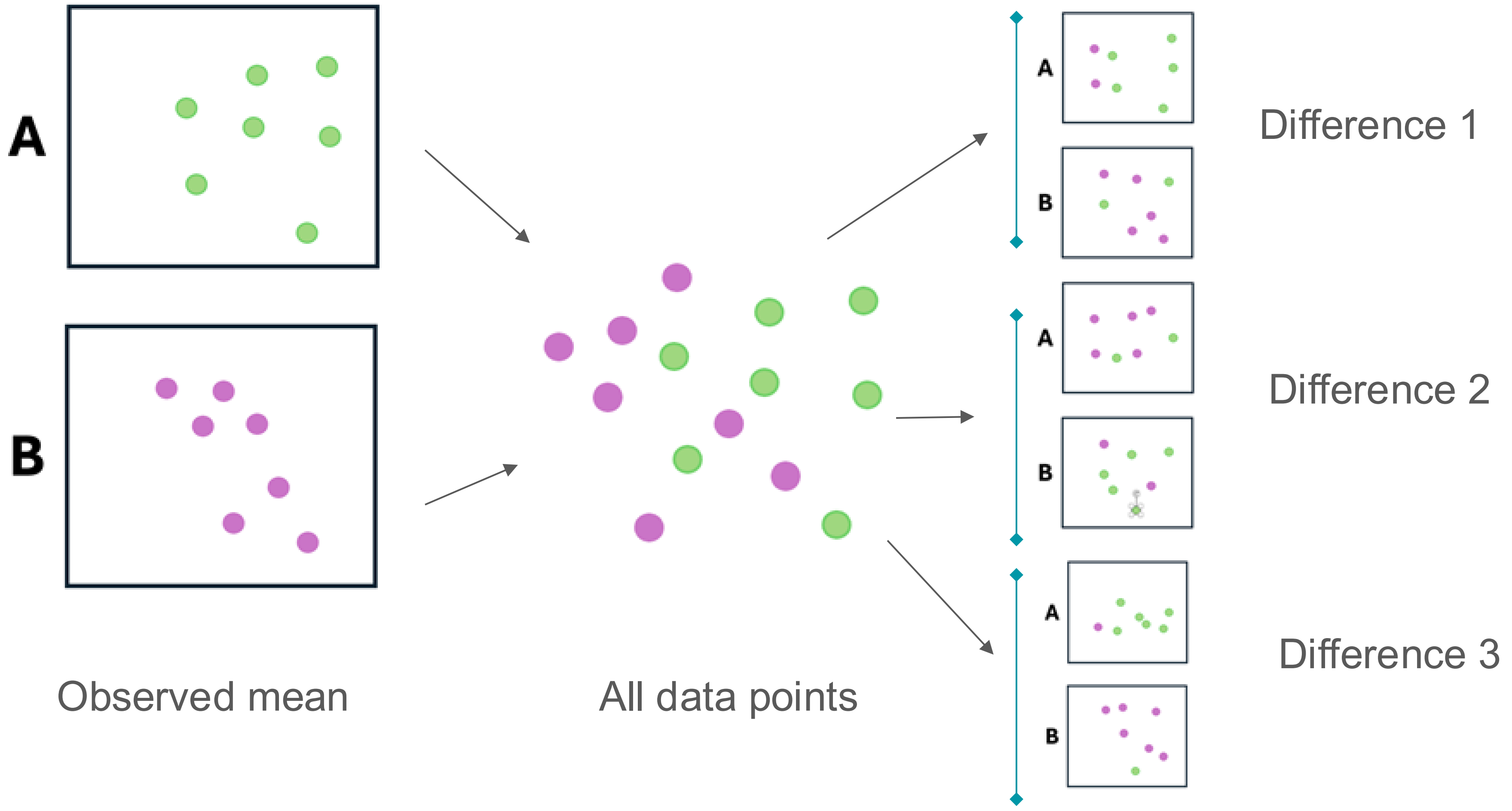
EEG data often do not fulfill t-test assumptions.

Surrogate tests → no assumptions about data distribution.

- Permutation
- Bootstrapping

Bootstrapping

- We assume that the best estimation of the population is our sample
- Recipe:
    - Collect data - there are at least two conditions
    - Check the difference between means
    - Randomly assign data to two groups and check the difference
    - Repeat 1000 times
    - Create a distribution of means
    - Check the probability of the original difference - if it is in the 5% tail - you can assume that the difference is not due to the chance


A
B
Observed mean
All data points
A
B
Difference 1
A
B
Difference 2
A
B
Difference 3

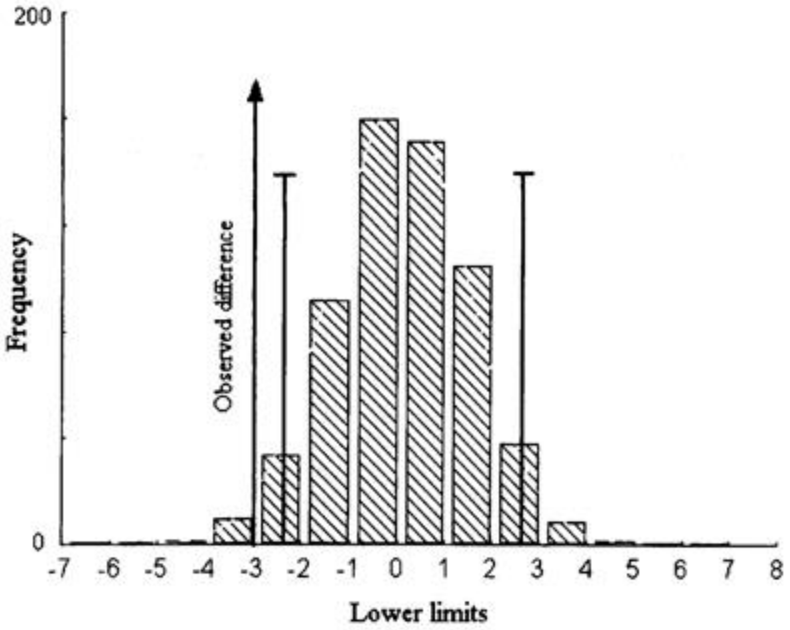


Figure 3. Histogram from 1,000 bootstrap resamplings on the difference between event-related potentials (Cz) to old and new low-imagery words for Subject A. Relative to this distribution, the observed difference (from the experiment) has a probability of less than .05.

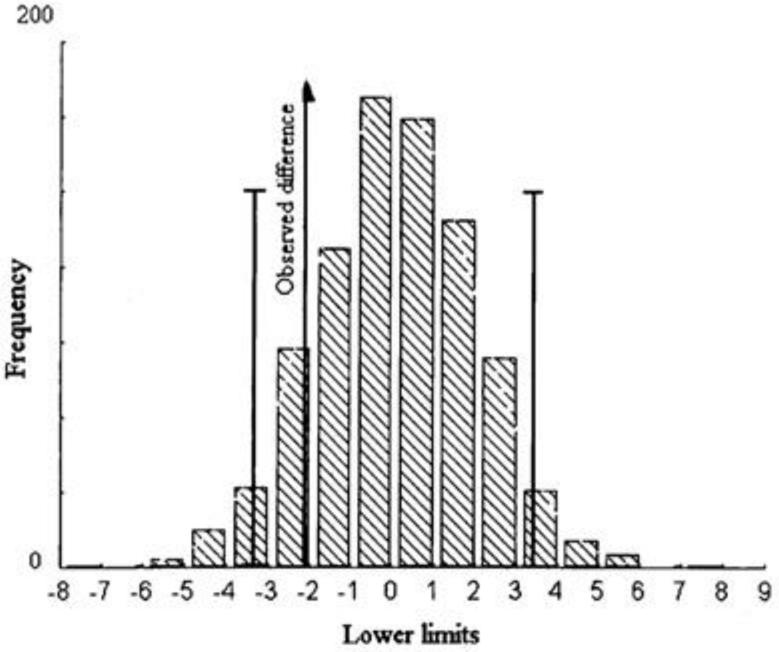
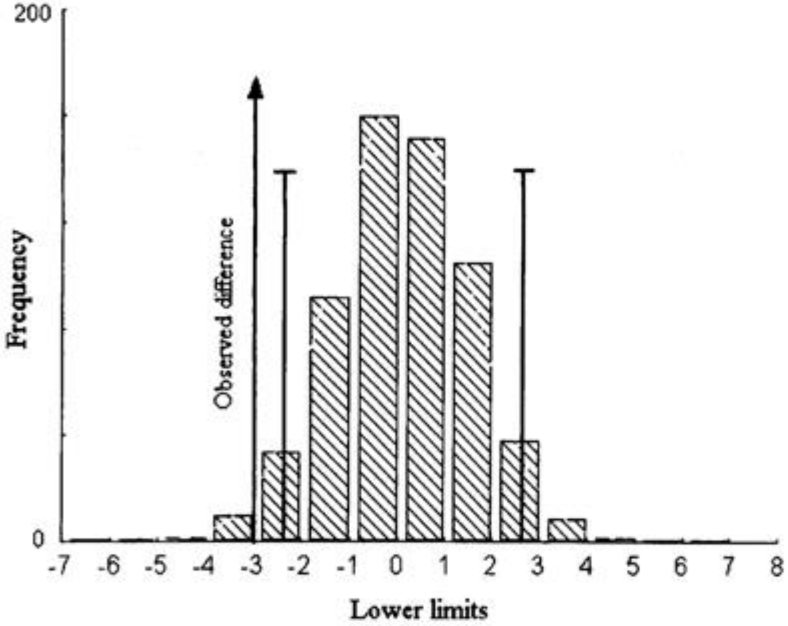


Figure 4. Histogram from 1,000 bootstrap resamplings on the difference between event-related potentials (Pz) to old and new low-imagery words. Relative to this distribution, the observed difference (from the experiment) has a probability of greater than .05.



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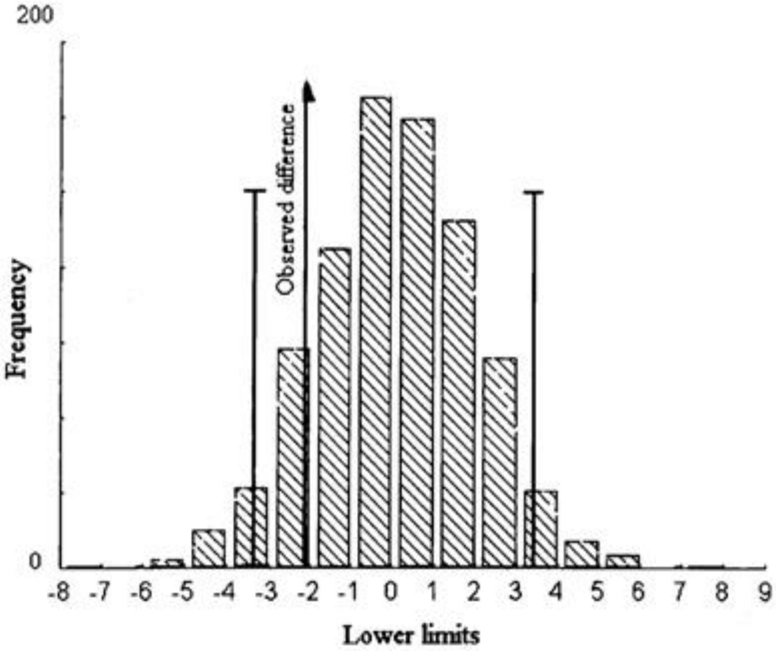


Figure 4. Histogram from 1,000 bootstrap resamplings on the difference between event-related potentials (Pz) to old and new low-imagery words. Relative to this distribution, the observed difference (from the experiment) has a probability of greater than .05.

What should I use for my EEG data analysis?

Steve Luck says:

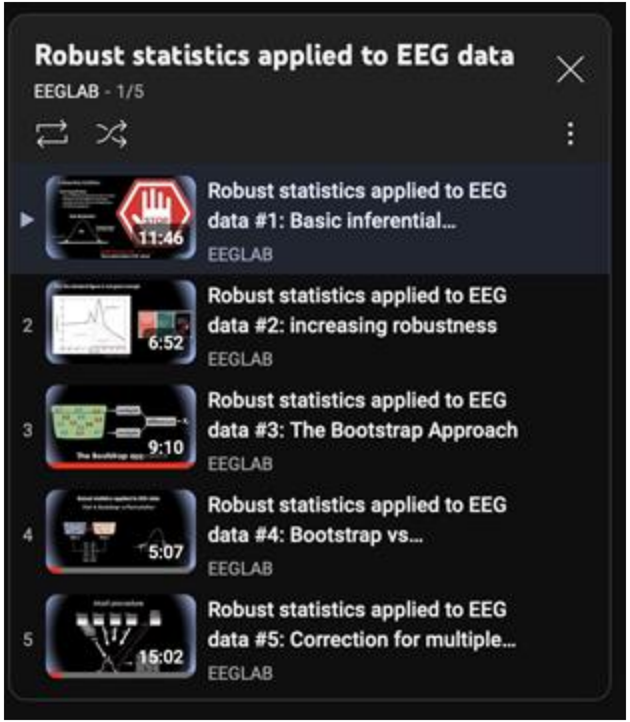
“Use simple t-test for the exploratory analysis and surrogate tests for publications.”

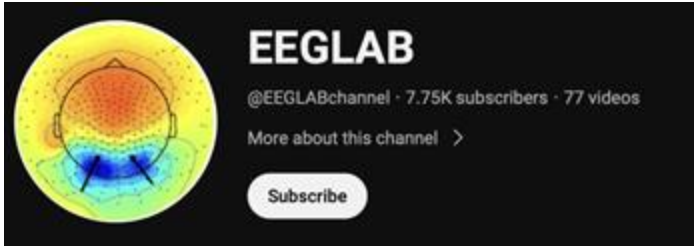
Cluster-based Permutation Testing

Many data points → high chance that we will find something significant

1. **Identify clusters**
2. Compute the size of the cluster
3. Randomly assign data to clusters
4. Check what is the most probable size of a cluster

If you want to know more…





https://www.youtube.com/@EEGLABchannel