Problem Set 5.

Problem 3-6:

$$p_X(x) = \begin{cases} \frac{3}{8}(2-\sqrt{x}), & \text{if } 0 < x < 4 \\ 0, & \text{otherwise} \end{cases}$$

(e.g. Normal p.d.f is positive on $(-\infty, \infty)$, but many other p.d.f are supported (have positive value) on a subset of $\mathbb{R}$, then we need to specify the other parts as value 0).

CDF of a cont. r.v. is the integration of the pdf.

$$F_X(x) = \int_{-\infty}^{x} p_X(t)\,dt. \qquad (*)$$

$\Rightarrow$ $F_X(x) = 0$, when $x < 0$;

$F_X(x) = 1$, when $x > 4$.

When $0 \le x \le 4$, we derive that

$$F_X(x) = \int_0^x p_X(t)\,dt$$

$$= \int_0^x \frac{3}{8}(2-\sqrt{t})\,dt$$

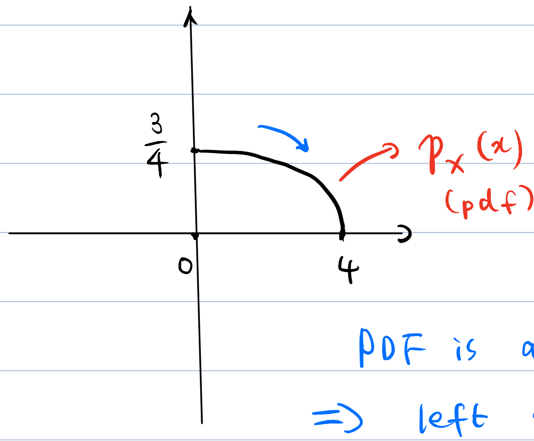
$$= \left(\frac{3}{4}t - \frac{1}{3/2}\cdot\frac{3}{8}t^{3/2}\right)\Big|_0^x$$

$$= \frac{3}{4}x - \frac{1}{4}\cdot\sqrt{x^3}$$

Together,

$$F_X(x) = \begin{cases} 0 & , \quad x < 0 \\ \frac{3}{4}x - \frac{1}{4}\sqrt{x^3} & , \quad 0 \le x \le 4 \\ 1 & , \quad x > 4 \end{cases}$$

(CDF)



right skewed.

PDF is an increasing function

$\Rightarrow$ left skewed

PDF is a decreasing function

$\Rightarrow$ right skewed.

$$\mathbb{P}(X \le 0.3) = F_X(0.3)$$

$$= \frac{3}{4} \cdot (0.3) - \frac{1}{4} \cdot (0.3)^{\frac{3}{2}}.$$

the difference is $\mathbb{P}(X = 0.5)$, which is 0 for a cont. r.v.

$$\mathbb{P}(X \ge 0.5) = 1 - \mathbb{P}(X < 0.5) = 1 - F_X(0.5)$$

in R console: (3/4) * 0.3 - (1/4) * 0.3^(3/2)

[1] 0.1839208

1 - ((3/4) * 0.5 - (1/4) * 0.5^(3/2))

[1] 0.7133883.

$$\mathrm{Var}(X) = E[X^2] - (E[X])^2$$

$$E[x] = \int_0^4 x \cdot \frac{3}{8}(2-\sqrt{x})\,dx$$

$$= \int_0^4 \frac{3}{4}x - \frac{3}{8}x^{\frac{3}{2}}\,dx$$

$$= \left(\frac{1}{2}\cdot\left(\frac{3}{4}x\right) - \frac{1}{5/2}\cdot\left(\frac{3}{8}x^{\frac{3}{2}}\right)\right)\Bigg|_0^4$$

$$= \frac{6}{5}$$

$$E[x^2] = \int_0^4 x^2 \cdot \frac{3}{8}(2-\sqrt{x})\,dx$$

$$= \int_0^4 \frac{3}{4}x^2 - \frac{3}{8}x^{\frac{5}{2}}\,dx$$

$$= \left(\frac{1}{3}\cdot\left(\frac{3}{4}x^2\right) - \frac{1}{7/2}\cdot\left(\frac{3}{8}x^{\frac{5}{2}}\right)\right)\Bigg|_0^4$$

$$= 4 - \frac{12}{7}$$

$$= \frac{16}{7}.$$

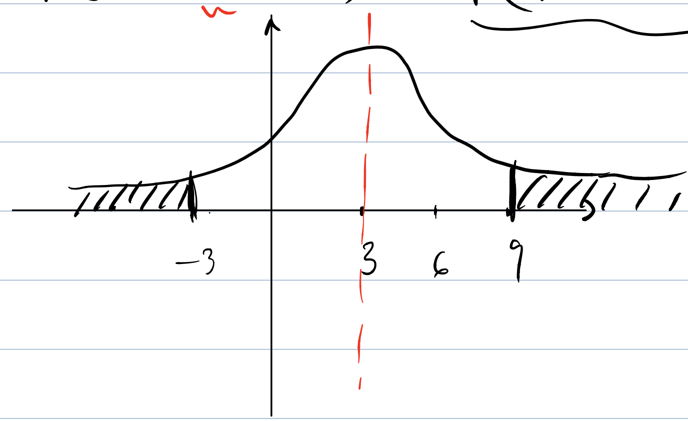
$$\mathrm{Var}(x) = E[x^2] - (E[x])^2$$

$$= \frac{16}{7} - \left(\frac{6}{5}\right)^2$$

$$= \frac{148}{175}$$

23(c). $X \sim N(3, 9)$

$P(|X-3| > 6) = P(X < -3) + P(X > 9)$.



$= 2 \cdot P(X < -3)$.

use R: pnorm(-3, mean = 3, sd = 3) * 2.

[1] 0.0456. (preferred).

another choice:

$X = 3 \cdot Z + 3$, then $Z \sim N(0,1)$.

$P(X < -3) = P(3Z + 3 < -3)$

$= P(Z < -2)$ ✓

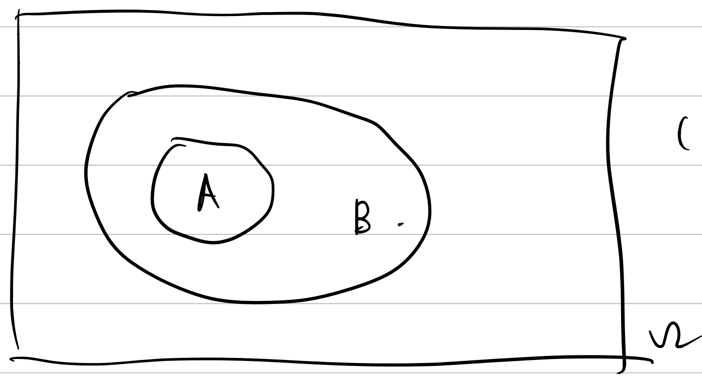
mean − 2·SD

Practice Midterm

P1.

the occurance of event A implies that the event B must also occur.

$A \Rightarrow B$.



(Venn diagram)

Extreme case: A is empty,

$\Rightarrow$ A independent with B

A & B mutually exclusive.

(this is a extreme case, we usually do not consider).

Suppose, A is not empty. B is not a must event.

$A \subseteq B \Rightarrow$ A & B not independent

($p(A) \cdot p(B) = p(A \cap B) = p(A)$ which implies that $p(A) = 0$ or $p(B) = 1$ ✗)

$A \subseteq B \Rightarrow$ A & B not mutually exclusive

($0 = p(A \cap B) = p(A)$ ✗)

Problem set 4. #10.

$X \sim \text{Binomial}(n, p)$, then the investment afterwards

$\uparrow$ follows. $\quad n = 18, \; p = \frac{1}{2}$

equals $28 \cdot \underbrace{X}_{\#\text{ of doubles}} + 0 \cdot \underbrace{(18 - X)}_{\#\text{ of nothing}}$

$= 28 \cdot X$

$E[X] = n \cdot p = 18 \cdot \frac{1}{2} = 9$.

$sd[X] = (n \cdot p \cdot (1-p))^{1/2} = (18 \cdot \frac{1}{2} \cdot \frac{1}{2})^{1/2}$

$= \frac{3}{2} \cdot \sqrt{2}$.

$E[28X] = 28 \cdot E[X] = 28 \cdot 9 = 252$

$sd(28X) = 28 \cdot sd(X) = 28 \cdot \left(\frac{3}{2}\sqrt{2}\right)$

$= 42 \cdot \sqrt{2}$

$= 59.397$.

① What model ?

② specify the model explicitly.
(e.g. n, p for Binomial).

③ What is the relation between the desired result and your r.v in the model ?

• SD of a sample. $(x_1, x_2, \cdots, x_n)$

$$SD^2 = s^2 := \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$ (unbiased)

(preferred).

(in test, we always use $\frac{1}{n-1}$ unless specified).