Q2: Part 1:

$$r_n = \max_{i=1\cdots n}\{P_i + r_{n-i}\}.$$

P_1	P_2	P_3	P_4
47	65	84	95.

$r_0 = 0$

$r_1 = \max\{P_1 + r_0\} = 47$, $s_1 = 1$

$r_2 = \max\{P_1 + r_1, P_2 + r_0\} = \max\{47\times 2, 65\} = 94$, $s_2 = 1$

$r_3 = \max\{P_1 + r_2, P_2 + r_1, P_3 + r_0\} = \max\{141, 112, 84\} = 141$, $s_3 = 1$.

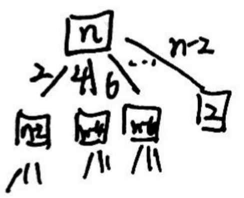
$r_4 = \max\{P_1 + r_3, P_2 + r_2, P_3 + r_1, P_4 + r_0\}$

$= \max\{188, 159, 131, 95\}$

$= 188$, $s_4 = 1$.

Part 2

a.




```
Re(L, M, sign){
   n = L.size()
   if (n == 2){
      return (M-1+ Σ_{k=1}^{2} l_k)^3
   }
   else{
      Total = [ ]
      for (i = [2, 4, 6, ... n]){
         if (M-i+1+ Σ_{k=1}^{i} l_k > 0){
            if (sign == 1){
               Total.append( Re( L[i, ..., n], M, sign+1) )
            }
            else{
               Total.append(Re(L[i, ..., n], M, sign+1) +
                        (M-i+1 + Σ_{k=1}^{i} l_k)^3)
            }
         }
      }
      Return max(Total)
   }
}
```

(b) Define T[j] to record the minimum penalty accrued by print 1-j on page, where word j is the last word of document.

$T[0]=0$. $T[2]=0$

$$T[j] = \begin{cases} \min\limits_{2\le i\le j-2} T[i] + (M-j+i-\sum_{k=i}^{j} l_k)^3 & \text{If } M-j+i+\sum_{k=i}^{j} l_k \ge 0, \text{ words } i \text{ through fit on the line} \\ \min\limits_{2\le i\le j-2} T[i] & \text{If } M-j+i+\sum_{k=i}^{j} l_k \ge 0 \text{ and } j=n, \text{ this is the last line.} \end{cases}$$

The runtime is $O(n^2)$ Because the minimum for each word requires computing the minimum over previous words.

(c) For example: we have words ~~{aaa, bbb, ccc, dd}, M=11~~

The greedy approach is [select as much as words into one line].

result of

For example: {aaa, bbb, ccc, dd, abcdefghijk}. M=11

aaa bbb ccc , b=0
dd , b = 11-2 = 9
abcdefghijk , b=0.

So, this penalty is $9^3 = 729$.

aaa bbb , b=4
ccc dd b=5
abcdefghijk b=0

this penalty is $4^3+5^3 = 189$.

So, greedy approach would not work for this problem.

Scanned with CamScanner

Q3: suboptimality:

f_i, finish time
S_i, start time.
S, events

Prove: A is an optimal solutions, then $A' = A - \{o_1\}$ is an optimal Solution to $S' = \{ i \in S,\ f_i \le S_{o_1} \}$.

If there exists B' to S' such that $|B'| > |A'|$, then let

$$B = B' \cup \{o_1\}$$

we can know that B is an globally optimal Solution and we have

$$|B| > |A|.$$

which is contridiction with the optimality of A, So, B' doesn't exist. A' is the suboptimal solution for S'.