

Q3: W: line 1: Time.
 line 2: 1
 line 3: ~~$T(n)$~~ $O\left(\frac{n}{2}\right)^2$
 line 4: a_4
 line 5: $T(n-2)$

$$\begin{aligned} \text{So, } T(n) &= T(n-2) + O(n^2) + O(1) \\ &= T(n-2) + O(n^2) \end{aligned}$$

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{n}{5}\right) + n$$

$$= 2T\left(\frac{n}{5}\right) + n$$

Assume that $T(n) \leq C \cdot n$ for some $C > 0$ and $n < k$

Prove : $T(n=k) \leq Ck$ when $n=k$.

$$T(n) = 2T\left(\frac{n}{5}\right) + n, \text{ because } \frac{n}{5} < k \text{ when } n=k$$

$$= 2T\left(\frac{k}{5}\right) + k \leq 2 \cdot C \cdot \frac{k}{5} + k \leq C \cdot k$$

$$\Rightarrow \frac{2}{5}Ck + k \leq Ck$$

$$\Rightarrow \frac{7}{5} \leq C$$

So, there must exist a constant C that can make $T(n=k) \leq C \cdot k$ when $C \geq \frac{7}{5}$.