Problem set 4

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Instructions: Here is our fourth set of practice problems.

Practice problems

(1) Suppose that X is random variable with probability distribution given by

$$X = \begin{cases} 0 & \text{with probability } \frac{1}{4} \\ 1 & \text{with probability } \frac{1}{2} \\ \sqrt{2} & \text{with probability } \frac{1}{4} \end{cases}$$

Now let $Y = X^2$. What is the probability distribution of Y?

answer:

$$p_Y(y) = \begin{cases} \frac{1}{4} & \textit{if } y = 0 \\ \frac{1}{2} & \textit{if } y = 1 \\ \frac{1}{4} & \textit{if } y = 2 \end{cases}$$

(2) Let X be a random variable with standard deviation 4. What is the standard deviation of $2 + X$?

answer: 4

(3) Let X be a random variable with variance 10. What is the variance of $3X + 12$?

answer: 90

(4) Suppose $E(X) = 2$ and $SD(X) = 1$. What is the expectation of $4X - 2$?

answer: 6

(5) Consider a box that contains a bunch of tickets: each ticket has a 1 or 0 written on it. Let p denote the proportion of tickets with a 1 written on it. Now randomly pick a ticket from this box and let the number be denoted Y. Find $P(Y = 1)$, EY and $SD(Y)$.

answer: p; p; $\sqrt{p(1-p)}$

(6) If $EX = 1$ and $\text{var}(X) = 2.5$ find EY where $Y = (2 - X)^2$.

answer: 3.5

(7) Over the first hour of trading, you expect the price of beanie babies to rise by \$ 1200 (with a variance of 100). Over the second hour, you expect it to drop by \$2000 (variance 200). Over the third hour, you expect the price to rise by \$10 (variance 3). What do you expect to be the change in prices over the course of the three hours? What's its variance?

answer: 3210; not enough information given

(8) It has been claimed that employees from all parts of the company are equally likely to be chosen for promotion (although you suspect that there is a bias in favor of people from marketing!). Suppose that the proportion of employees that work in marketing is $1/4$. Assuming the promotions to be independent, what is the probability that, out of six promotions, four of them go to people from marketing? What is the probability that no more than four go to people from marketing?

answer: 0.03296; 0.99536

(9) Suppose that each of your cover letters will contain a typo independently with probability 0.2. What is the probability that, out of 20 cover letters, one or more contains a typo?

answer: 0.98847

(10) Suppose that you will invest \$14 in each of 18 independent 'Double-or-nothing' instruments. What is the expected value of your investment afterwards? The standard deviation?

answer: 252; 59.397

(11) Suppose that you will invest \$5 in each of 63 independent 'Double-or-nothing' instruments. What is the expected value of your returns? The standard deviation?

answer: 0; 39.686

(12) Suppose that workplace accidents occur each day independently with probability 0.02. If, over the course of a 30-day month, 5 or more accidents occur, insurance premiums will go up (they won't go up otherwise!). What's the probability that insurance premiums go up?

answer: 0.00030

(13) Your extremely generous friend offers to play a simple game with you. She will allow you to flip a coin until the first time you flip tails, and will pay you 2^n dollars, where n is the number of times you flipped the coin in total. Let X be your winnings from playing this game one time. The probability distribution of X is given by

$$p_X(x) = \begin{cases} \frac{1}{2^n} & \text{if } x = 2^n \text{ with } n = 1, 2, 3, \ldots; \\ 0 & \text{otherwise.} \end{cases}$$

What are your expected winnings from this game?

answer: ∞

(14) Suppose that five hundred investors participate on an internet message board. Each of them will invest in Wolf Cola with probability 0.3. Is this enough information to determine the probability distribution of the number of them that will do so? If so, name the distribution; if not, give an example of information that would be sufficient to do so.

answer: No; E.g. that they are independent

(15) Suppose that the probability distribution of the proportion X of chips in a bag that are broken is given by

$$p_X(x) = \begin{cases} 3x^2 & \text{if } \ 0 < x < 1 \\ 0 & \text{if otherwise} \end{cases}$$

What is the shape of this distribution? What is the CDF for this r.v.? What is the probability that more than 75% of the chips are broken?

answer: Left skewed; $F_X(x) = x^3$*;* $^{37}/_{64}$

(16) Suppose that a random variable has CDF $F_X(x) = \sqrt{x}$ for $0 < x < 1$. What is the probability that $P(X > 0.5)$?

answer: $1 - \sqrt{0.5}$

(17) Suppose that the probability distribution of the time X in seconds until a bus arrives at its stop is given by

$$p_X(x) = \begin{cases} \frac{3\sqrt{x}}{2000} & \text{if } \ 0 < x < 100 \\ 0 & \text{if otherwise} \end{cases}$$

What is the probability that the bus will arrive in less than 25 seconds?

Hint: remember that $\sqrt{x} = x^{1/2}$

answer: $^{1}/_{8}$