STA 200A: Homework 5

1. Let $X \sim \text{Gamma}(\alpha, \lambda)$. Find $E[1/X]$.
2. Let $X \sim \text{Uniform}[0, 1]$, and put $Y = \sqrt{X}$. Find $E[Y]$.
3. Let X be an Exponential(λ) rv. Derive a formula for $E[X^2]$.
4. Find the density function of $Y = e^Z$, where $Z \sim \mathcal{N}(\mu, \sigma^2)$. This is called the log-normal density, since $\log Y$ is normally distributed.
5. Let $f(x) = (1 + \alpha x)/2$ for $-1 \leq x \leq 1$ and $f(x) = 0$ otherwise, where $-1 \leq \alpha \leq 1$. Show that f is a density and find the corresponding cdf. Find the upper quartile of the distribution in terms of α.
6. Suppose X has the density function $f(x) = cx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.
   (a) Find c.
   (b) Find the cdf $F_X(x)$.
   (c) What is $P(0.1 \leq X < 0.5)$?
7. If $X \sim \mathcal{N}(0, \sigma^2)$, find the density of $Y = |X|$.
8. Let $Z \sim N(0, 1)$ be a standard normal rv. Let $g : \mathbb{R} \to \mathbb{R}$ be a differentiable function, with derivative g'. Show that the following identity holds.

$$E[g'(Z)] = E[Zg(Z)].$$

   (You may assume both of these expectations exist. Also, for convenience, you may assume that $g(z)e^{-z^2/2} \to 0$ as $z \to \pm\infty$. In other words, the function g doesn't increase too quickly when $|z|$ is large.)
9. Chapter 5, Theoretical Exercise 11 (just parts b and c)