

## PROBLEM SET 6

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**Instructions:** Here is our sixth set of practice problems.

### Practice problems

- (1) Consider competing firms A and B. The firms will expand into  $X_A, X_B$  new markets, respectively, where  $0 \leq X_A \leq 2$ ,  $0 \leq X_B \leq 2$ . Suppose that each firm has the same marginal distribution, given below. If the two firms expand independently, what is the joint probability distribution of their numbers of expansions? Would you say that it would be an unusual coincidence for them both to expand to 2 new markets?

$x$	$p(x)$
0	$1/2$
1	$1/3$
2	$1/6$

- (2) Suppose that you will roll two fair six-sided dice. Let  $U$  be the first result, and  $V$  the second. Furthermore, let

$$X_1 = \begin{cases} 1 & \text{if } U \geq 4 \\ -1 & \text{if otherwise} \end{cases}$$

and

$$X_2 = \begin{cases} 1 & \text{if } V \geq 4 \\ -1 & \text{if otherwise} \end{cases}$$

What is the joint distribution of  $X_1$  and  $X_2$ ?

- (3) Suppose that you will roll two fair six-sided dice. Let  $U$  be the first result, and  $V$  the second. Furthermore, let

$$X_1 = \begin{cases} 1 & \text{if } U \geq 3 \\ -1 & \text{if otherwise} \end{cases}$$

and

$$X_2 = \begin{cases} 1 & \text{if } V \geq 3 \\ -1 & \text{if otherwise} \end{cases}$$

What is the joint distribution of  $X_1$  and  $X_2$ ?

- (4) Consider r.v.'s  $X$  and  $Y$  with the joint probability distribution given below.  $X$  here indicates the occurrence of a recession in a given quarter, and  $Y$  the number of firms in a sector that declare bankruptcy. Are these independent under this model? If not, find the conditional distributions of  $Y$  given  $X = 1$ , and compare it with the corresponding marginal distribution.

$p_{X,Y}(x, y)$		$y$			
		0	1	2	3
$x$	0	$7/16$	$7/32$	$15/96$	$1/16$
	1	$1/16$	$1/32$	$1/96$	$1/48$

- (5) Are  $X$  and  $Y$ , with the joint distribution given below, independent?

$p_{X,Y}(x, y)$		$y$		
		-1	0	1
$x$	0	$1/28$	$5/56$	$1/56$
	1	$1/7$	$5/14$	$1/14$
	2	$3/28$	$15/56$	$3/56$

- (6) Find the marginal and conditional distributions of  $Y$  given  $X = 0$  and given  $X = 1$  under the joint distribution below.

$p_{X,Y}(x, y)$		$y$			$p_X(x)$
		1	2	3	
$x$	0	$4/9$	$4/27$	$2/27$	$2/3$
	1	$1/27$	$2/27$	$2/9$	$1/3$
$p_Y(y)$		$13/27$	$2/9$	$8/27$	

- (7) What is the missing value in the table of joint probabilities below? Are these r.v.'s independent?

$p_{X,Y}(x,y)$		$y$		
		-1	0	1
$x$	-1	$7/54$	$5/54$	$7/54$
	0	$5/54$	$7/54$	$5/54$
	1	$7/54$	$5/54$	?