STA 200A Homework_1

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1. (a) Because there are 26 letters and 10 digits, 2 positions for letters and 5 positions for digits,

   So, the final answer is : $26^2 \times 10^5 = 67600000$

   (b) Choose 2 letters from 26 letters and there are 2 permutations for those 2 letters and choose 5 numbers from 10 digits and there are 5! permutations for those 5 numbers. The final answer is : $\frac{26!}{24!} \times \frac{10!}{5!} = 19656000$

2. (a) There are 8 selections for the first position and 2 choices for second position and 9 choices for third position. $8 \times 2 \times 9 = 144$

   (b) The first is fixed and the rest reason is as same as the question (a) : $2 \times 9 = 18$

3. (a) If no restrict, there are $8! = 40320$

   (b) Let think that A and B combine to one person and there are 7! permutations for arrangements. And the permutations of A and B is 2! . $7! \times 2! = 10080$

   (c) At the first position, there are 4 choices for man. At the second position, there are 4 choices for women. At the third position, there are 3 choices for man. At the fourth position, there are 3 choices for women. The choices of rest position is as same as regulation above. But we need to multiply 2 because the first position can be man or women.

   $4^2 \times 3^2 \times 2^2 \times 1^2 \times 2 = 1152$

   (d) Let us think that the 5 people combine as a one person. So, there are 4! permutations for those people. But there are 5! permutation for those 5 people. $5! \times 4! = 2880$

   (e) Let us think that each couple combines as one person. There are 4! permutations. There are 4 couples and the permutations of each is 2!. $4! \times 2!^4 = 384$

4. We first choose one person out and shakes hands with the other 19 people and this person has been excluded in next steps. Then, we also choose one person from left 19 people and shakes hands with the other 18 people. Until there are only 2 person left and shake hands with each other. $\sum_{i=1}^{19} i = 190$

5. Only using the formulation which teacher you gave us . $\frac{12!}{3! \times 4! \times 5!} = 27720$

6. Suppose that there are **n** objects and we need to select **k** from those objects. In general, the formula of this goal is $\binom{n}{k}$.

In another thinking is that we first choose **one** object from **n** objects. The next consideration is that we need to choose **k-1** objects from **n-1** objects. This formula is $\binom{n-1}{k-1}$.

Second, because we have selected **one** object from **n** objects set. If we do not exclude this object, we will count some combinations repeat because for this object, those combinations have counted over. So, there are **n-1** objects in this second step. We also choose **one** object from **n-1** objects. Because we have selected **one** object, we only need to choose **k-1** object from **n-2** objects set. This formula is that $\binom{n-2}{k-1}$.

Repeat this flow again and again until there are only **k-1** objects left in this set. So, the ultimate formula is $\binom{k-1}{k-1}$.

Finally, we add those count number together.

(Note: The example of this thinking is at **Chapter 1, Problem 13**, shake hands problem. We first choose one person out and shakes hands with the other 19 people and this person has been excluded in next steps. Then, we also choose one person from left 19 people and shakes hands with the other 18 people. Until there are only 2 person left and shake hands with each other.)

7. When n=1, the formula is : $\sum_{k=0}^{1}\binom{1}{k} = 2^1$. This equation holds.

Suppose that when n = n, the equation

$$\sum_{k=0}^{n}\binom{n}{k} = 2^n \tag{1}$$

holds.

In the next step, when n = n+1, we should prove that the equation

$$\sum_{k=0}^{n+1}\binom{n+1}{k} = 2^{n+1} \tag{2}$$

holds.

We first use the right of (2) to subtract the right of (1). The result is :

$$2^{n+1} - 2^n = 2^n \tag{3}$$

Then, we use the left of (2) to subtract the left of (1). The result is :

$$\sum_{k=0}^{n+1}\binom{n+1}{k} - \sum_{k=0}^{n}\binom{n}{k} = \sum_{k=0}^{n}\left(\binom{n+1}{k} - \binom{n}{k}\right) + 1 = 0 + \sum_{k=1}^{n}\left(\binom{n+1}{k} - \binom{n}{k}\right) + 1 = \sum_{k=1}^{n}\left(\binom{n+1}{k} - \binom{n}{k}\right) + 1 \tag{4}$$

Let's see the one single item

$$\binom{n+1}{k}-\binom{n}{k}$$
$$=\frac{(n+1)!}{(n+1-k)!k!}-\frac{n!}{(n-k)!k!}$$
$$=\frac{(n+1)n!-n!(n+1-k)}{(n-(k-1))!k!}$$
$$=\frac{n!k}{(n-(k-1))!k!}$$
$$=\frac{n!}{(n-(k-1))!(k-1)!}$$
$$=\binom{n}{k-1}$$

So, the (4) formula can be written : $\sum_{k=1}^{n}(\binom{n+1}{k}-\binom{n}{k})+1=\sum_{k=1}^{n}\binom{n}{k-1}+1=\sum_{k=0}^{n}\binom{n}{k}$

According the (1) formula which we suppose, we can get that the formula (4) equals with (3).

So, we have proved that the (2) formula holds. At last the identity $\sum_{k=0}^{n}\binom{n}{k}=2^n$ has been proved.

8. Suppose that the m items are all non-defect. So, the probability is $P=\frac{\binom{n-k}{m}}{\binom{n}{m}}$. So, at least with one defective item is $1\text{-}P$. So, we only need to make $P=0.1$ and we can find out the formulation to calculate the value of m.
The formula of m is :

$$\prod_{i=0}^{k-1}\frac{n-m-i}{n-i}=0.1$$

If k << n, k-1 << n. So, this formula approximates :

$$(\frac{n-m}{n})^k=0.1$$

So, the m is :

$$m\approx(1-\sqrt[k]{0.1})n$$

When n=1000, k=10, the m=206
When n=10000, k=100, the m=227

9. (a) For each person, he or she has 2 choices. The first person can go any class, but the rest people must go to class as the same as the first person. $(\frac{1}{2})^4=\frac{1}{16}$
(b) Random selects 4 people for 5 people. Let us consider that the 4 people as one person. And the left of person must have the different class between the 4 people. So, there are 2

permutations. The total numbers of combinations are 32. $\frac{\binom{5}{4}\times 2}{32}=\frac{5}{16}$

(c) There are only 2 choices for Marcelle, because we do not need to select 4 people from 5 people. $\frac{2}{32}=\frac{1}{16}$