Practice Final

STA 200B
Winter 2020
University of California, Davis

Exam Rules: This exam is closed book and closed notes. Use of calculators, cell phones or any other communication or electronic devices is not allowed. You must show all of your work to receive credit. You are allowed to use 2 sheets of notes, two-sided. Your exam will be conducted online. The exam will be provided at the same time through piazza to all. You will then display it on your screen. We will open a zoom session in which you need to participate and where you enable your camera on laptop or cell phone to record your test taking, in lieu of the proctoring and monitoring that is happening in an in-class final. As if it was an in-class exam, books and cell phones need to be packed away (except if you use the camera on the cell phone to monitor your exam taking, in which case you need to have the zoom app on your cell phone). With your signature you ascertain that you abide by these rules and the UC Davis Code of Academic Conduct.

Name : ______________________

ID : ______________________

Signature : ______________________

1. Let $X_1, \dots, X_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. Suppose you only observe $\sum_{i=1}^n X_i$ and $\sum_{i=1}^n X_i^2$.
   (a) Derive a γ confidence interval for μ with the shortest length and interpret the meaning of this confidence interval.
   (b) Derive a γ confidence interval for σ^2.
   (c) Derive jointly sufficient statistics for μ and σ.

2. Suppose X_1 and X_2 are independent normal random variables with the same mean μ and variance σ^2. Let
$$Y_1 = X_1 + 2X_2$$
$$Y_2 = 2X_1 - X_2.$$
   (a) Prove that Y_1 and Y_2 are independent and find their joint distribution.
   (b) Let X_3 be another normal random variable that is i.i.d of X_1 and X_2. Find the constant c such that
$$\frac{c(\overline{X}_2 - \mu)}{\sqrt{(X_1 - X_2)^2 + 2(X_3 - \mu)^2}}$$
   follows a t-distribution, where $\overline{X}_2$ is the average of X_1 and X_2. Also find the degrees of freedom of the t-distribution.

3. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution on the interval $[\theta, \theta + 1]$, with unknown $\theta > 0$.

   (a) Assuming that the prior distribution of θ has the pdf $f(x|\beta) = \frac{1}{\beta} e^{-x/\beta}$. Find the Bayes estimator of θ for the squared error loss function. Just provide an expression but do not work out the final answer.

   (b) Derive a 95% Bayes confidence interval for θ.

   (c) Derive a 95% upper Bayesian confidence bound. How does this confidence bound compare with the upper bound of the two-sided ci?

   (d) When would you use the upper confidence bound?

4. Let $X_1, \ldots, X_n$ be a random sample from a Negative Binomial(r, θ) distribution, where r is known and $0 < \theta < 1$ is unknown. Assume θ has a prior distribution Beta(α, β), where $\alpha, \beta > 0$ are known.

   (a) Find the posterior distribution of θ given $X_1, \ldots, X_n$.

   (b) Find the Bayes estimator for θ under squared error loss.

   (c) Show $\bar{X}$ is a minimal sufficient statistic for θ, by using the Bayes estimator found in the last part.

   (d) Obtain a 90% Bayesian ci for θ. What is the interpretation of this ci and how does it differ from the interpretation of a frequentist ci?

5. Assume we have a sample from a Poisson(λ) distribution. Obtain the UMVUE of $P(X = 0)$.