STA 200A: Homework 4

Note, the "Problems" and "Theoretical Exercises" are listed in separate sections at the end of the chapter.

The problem numbers are based on the **9th edition**. (A copy of these problems is available on the course webpage under the folder 'book problems'.) Below the notation 4.T10 means Chapter 4, Theoretical Exercise 10. Similarly, the notation 4.P23 means Chapter 4, Problem 23.

1. Let X be a Poisson rv with parameter λ. Calculate $E\left[\frac{1}{X+1}\right]$ in terms of λ.

2. 4.T10

3. There are n distinct items arranged in a random order. (All orderings are equally likely). The items are searched sequentially until a desired item is found. What is the expected number of items searched?

4. The university administration assures a mathematician that he has only 1 chance in 10,000 of being trapped in a much-maligned elevator in the mathematics building. If he goes to work 5 days a week, 52 weeks a year, for 10 years, and always rides the elevator up to his office when he first arrives, what is the probability that he will never be trapped? That he will be trapped once? Twice? Assume that the outcomes on all the days are mutually independent (a dubious assumption in practice).

   Finally, compute the same probabilities by approximating the Binomial distribution with Poisson distribution, and see how close the numbers are.

5. An urn contains balls numbered 1 to N. Let X be the largest number drawn in n drawings when random sampling with replacement is used. (The event $X \leq k$ means that each of n numbers drawn is less than or equal to k.)

   Show that when N is large,
   $$E[X] \approx \frac{n}{n+1} N.$$

6. Let X be a binomial rv with parameters (n, p). Let t be a fixed real number. Derive a formula for the expectation $E[e^{tX}]$.

7. Let X be a binomial rv with parameters (n, p). Show that $E[X] = np$ by directly evaluating the sum $\sum_{x=0}^{n} xP(X = x)$.

8. 4.P23