(a) Always reject H_0, no matter what data are obtained (equivalent to the practice of choosing the α level to force rejection of H_0).

(b) Always accept H_0, no matter what data are obtained (equivalent to the practice of choosing the α level to force acceptance of H_0).

8.17 Suppose that $X_1, \ldots, X_n$ are iid with a beta$(\mu, 1)$ pdf and $Y_1, \ldots, Y_m$ are iid with a beta$(\theta, 1)$ pdf. Also assume that the Xs are independent of the Ys.

(a) Find an LRT of $H_0: \theta = \mu$ versus $H_1: \theta \neq \mu$.

(b) Show that the test in part (a) can be based on the statistic

$$T = \frac{\Sigma \log X_i}{\Sigma \log X_i + \Sigma \log Y_i}.$$

(c) Find the distribution of T when H_0 is true, and then show how to get a test of size $\alpha = .10$.

8.18 Let $X_1, \ldots, X_n$ be a random sample from a n(θ, σ^2) population, σ^2 known. An LRT of $H_0: \theta = \theta_0$ versus $H_1: \theta \neq \theta_0$ is a test that rejects H_0 if $|\bar{X} - \theta_0|/(\sigma/\sqrt{n}) > c$.

(a) Find an expression, in terms of standard normal probabilities, for the power function of this test.

(b) The experimenter desires a Type I Error probability of .05 and a maximum Type II Error probability of .25 at $\theta = \theta_0 + \sigma$. Find values of n and c that will achieve this.

8.19 The random variable X has pdf $f(x) = e^{-x}, x > 0$. One observation is obtained on the random variable $Y = X^\theta$, and a test of $H_0 : \theta = 1$ versus $H_1 : \theta = 2$ needs to be constructed. Find the UMP level $\alpha = .10$ test and compute the Type II Error probability.

8.20 Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x\|H_0)$	.01	.01	.01	.01	.01	.01	.94
$f(x\|H_1)$	.06	.05	.04	.03	.02	.01	.79

Use the Neyman–Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = .04$. Compute the probability of Type II Error for this test.

8.21 In the proof of Theorem 8.3.12 (Neyman–Pearson Lemma), it was stated that the proof, which was given for continuous random variables, can easily be adapted to cover discrete random variables. Provide the details; that is, prove the Neyman–Pearson Lemma for discrete random variables. Assume that the α level is attainable.

8.22 Let $X_1, \ldots, X_{10}$ be iid Bernoulli(p).

(a) Find the most powerful test of size $\alpha = .0547$ of the hypotheses $H_0: p = \frac{1}{2}$ versus $H_1: p = \frac{1}{4}$. Find the power of this test.

(b) For testing $H_0 : p \leq \frac{1}{2}$ versus $H_1 : p > \frac{1}{2}$, find the size and sketch the power function of the test that rejects H_0 if $\sum_{i=1}^{10} X_i \geq 6$.

(c) For what α levels does there exist a UMP test of the hypotheses in part (a)?

8.23 Suppose X is one observation from a population with beta$(\theta, 1)$ pdf.

(a) For testing $H_0: \theta \leq 1$ versus $H_1: \theta > 1$, find the size and sketch the power function of the test that rejects H_0 if $X > \frac{1}{2}$.

(b) Find the most powerful level α test of $H_0: \theta = 1$ versus $H_1: \theta = 2$.

(c) Is there a UMP test of $H_0: \theta \leq 1$ versus $H_1: \theta > 1$? If so, find it. If not, prove so.