Q3: ① $\lim_{n\to\infty} \frac{n^3 \log n}{n^4} = \lim_{n\to\infty} \frac{\log n}{n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$

So, $L = 0 \Rightarrow n^3 \log n = f(n) = O(n^4)$

② Assume that we can find a x that $k(n) = n^3 \log(n) = \theta(n^x)$

It can have:

$$\lim_{n\to\infty} \frac{n^3 \log n}{n^x} = L = C$$, where C is a constant.

~~$\lim_{n\to\infty} \frac{\log n}{n^{3-x}} \Rightarrow \lim_{n\to\infty} \frac{1}{n}(3-x)n$~~

$$\Rightarrow \lim_{n\to\infty} \frac{\log n}{n^{x-3}} = C \Rightarrow \lim_{n\to\infty} \frac{\frac{1}{n}}{(x-3)n^{x-3-1}} = C$$

$$= \lim_{n\to\infty} \frac{1}{x-3} \cdot \frac{1}{n} \cdot \frac{1}{n^{x-3-1}} = C$$

$$= \lim_{n\to\infty} \frac{1}{x-3} \frac{1}{n} \cdot n^{4-x} = C$$

If C is a constant, which means that $\frac{1}{n} \cdot n^{4-x} = 1 \Rightarrow x = 3$.

However, x can't be 3 because It is no sence that $\frac{1}{x-3}$ when $x = 3$.

So, there is no x that can make $k(n) = \theta(n^x)$.