Q_1: $\sum_{i=1}^{n} 4^i = 4 \cdot \frac{1-4^n}{1-4} = 4 \cdot \frac{4^n-1}{3} = \frac{4}{3}(4^n-1)$

when $n=1$, $\sum_{i=1}^{1} 4^i = 4$. $\frac{4}{3}(4-1) = 4$. Left equals right

suppose when $n=k$, $\sum_{i=1}^{k} 4^i = \frac{4}{3}(4^k-1)$

prove: $\sum_{i=1}^{k+1} 4^i = \frac{4}{3}(4^{k+1}-1)$

$$
\begin{aligned}
\sum_{i=1}^{k+1} 4^i &= \sum_{i=1}^{k} 4^i + 4^{k+1} \\
&= \frac{4}{3}(4^k-1) + 4^{k+1} \\
&= \frac{1}{3}4^{k+1} - \frac{4}{3} + 4^{k+1} \\
&= \frac{4}{3}4^{k+1} - \frac{4}{3} \\
&= \frac{4}{3}(4^{k+1}-1)
\end{aligned}
$$

So, Left equals right. by using induction rule,

we have $\sum_{i=1}^{n} 4^i = \frac{4}{3}(4^n-1)$

Q_2 : For outer loop:

$$T(n) = \sum_{i=1}^{\log n} I$$

For medium loop

$$I = \sum_{j=1}^{\log i} J$$

For inner loop

$$J = \sum_{k=1}^{\frac{n}{2}} 1.$$

So, $T(n) = \sum_{i=1}^{\log n} \sum_{j=1}^{\log n} \sum_{k=1}^{\frac{n}{2}} 1$

$$= \sum_{i=1}^{\log n} \frac{n}{2} \log n$$

$$= \frac{n}{2} \log^2 n = O(n \log^2 n)$$

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Q3: $T(n) = \frac{n}{2} + \frac{(1+\frac{n}{2}-1)\cdot(n-1)}{2} + 2T(\frac{n}{2})$

$= \frac{n(n-1)}{4} + \frac{n}{2} + 2T(\frac{n}{2})$

$T(n) = 2T(\frac{n}{2}) + f(n)$ where $f(n) = \frac{n^2}{4} + \frac{n}{4}$

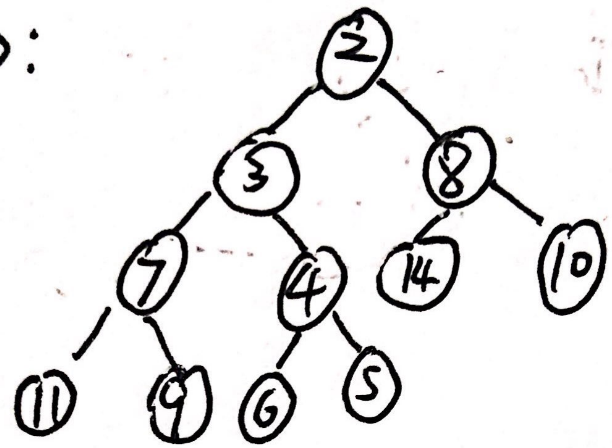
$f(n) = \Theta(n^2)$

According to master method.

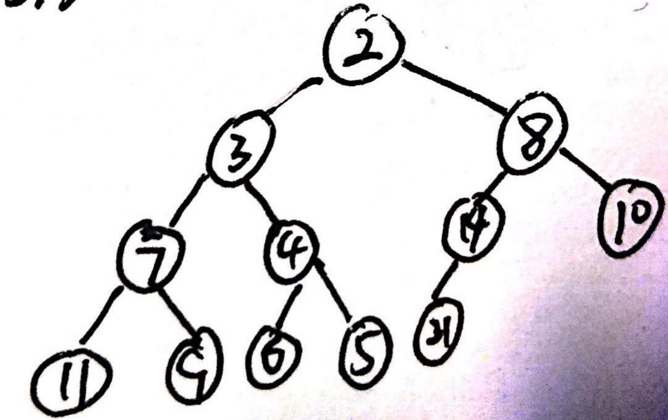
$a=2,\ b=2,\ d=2. \Rightarrow a < b^d$

So, $T(n) = \Theta(n^d) = \Theta(n^2)$

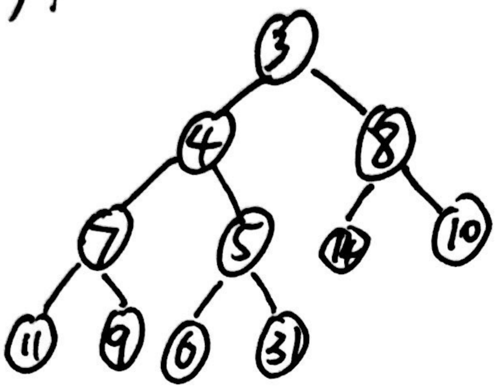
Q4: Push (2):



Push (31):

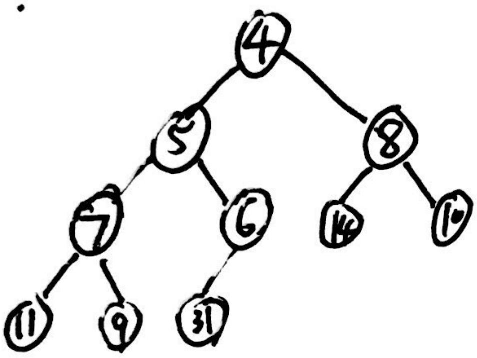


POP():



)

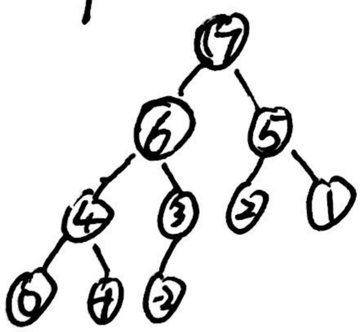
Pop():



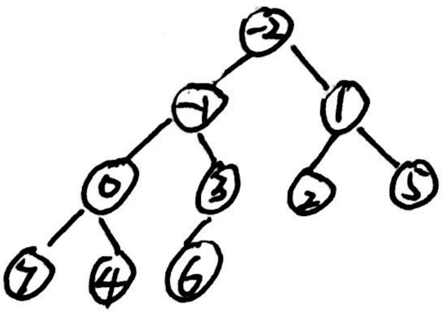
d,

7, 6, 5, 4, 3, 2, 1, 0, −1, −2.

initial heap:



Final heap:



(b) $T(n) = 1 + 1 + \log 2 + 1 + \log 3 + \cdots + 1 + \log n$

$$= n + \sum_{i=1}^{n} \log i$$

$$\leq n\log n + \sum_{i=1}^{n} \log n$$

$$= n\log n + n\log n = 2n\log n$$

So, $T(n) = O(n\log n)$

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Q5: a. $O(n)$

b. $O(n^2)$

c. $O(n^3)$

d. $O(n^2)$

e. $T(n) = \sum_{i=1}^{n} \sum_{j=1}^{i^2} \sum_{k=1}^{j} 1$

$= \sum_{i=1}^{n} \left(\frac{(1+i^2)\,i^2}{2}\right)$

$\leq \frac{n(1+n^2)n^2}{2} = O(n^5)$

Q5. f: $T(n) = \sum_{i=1}^{n} I$

$$I = \sum_{j=1}^{i^2} J$$

$$J = 1\{j\%i == 0\} \cdot \sum_{k=1}^{j} 1 + 1\{j\%i \neq 0\} 1.$$

So, $T(n) = \sum_{i=1}^{n} \sum_{j=1}^{i^2} \left[1\{j\%i==0\} \sum_{k=1}^{j} 1 + 1\{j\%i \neq 0\} 1 \right]$

$$= \sum_{i=1}^{n} \left[\sum_{l=1}^{i-1} \cdot i l + \sum_{l=0}^{i^2-i-1} \cdot 1 \right]$$

$$= \sum_{i=1}^{n} \left[i \sum_{l=1}^{i-1} l + i^2 - i - 1 \right]$$

$$= \sum_{i=1}^{n} \left[\underbrace{i \cdot \frac{(1+i-1)(i-1)}{2}}_{n^3} + \underbrace{i^2}_{n^2} - \underbrace{i}_{n} - \underbrace{1}_{c} \right] = O(n^4) + O(n^3) + O(n^2) + O(n)$$

$$= O(n^4)$$

Q6: $\log_4 n = \dfrac{\log_{16} n}{\log_{16} 4} = \dfrac{1}{\log_{16} 4} \log_{16} n$

$$\lim_{n \to \infty} \frac{\log_4 n}{\log_{16} n} = \frac{1}{\log_{16} 4},$$

So, $\log_4 n = \Theta(\log_{16} n) = O(\log_{16} n)$

For the same reason,

$\log_{16} n = O(\log_4 n)$

Q7: $3n^4 + 6n = \theta(n^4)$

$n\log n^{1000} = 1000n\log n = \theta(n\log n)$

$7n^3\log n + 1000 = \theta(n^3\log n)$

$3^n = \theta(3^n)$

$6^n = \theta(6^n) = 2^n \cdot 3^n$

$1024n^2 + 4n + 460 = \theta(n^2)$

$$\begin{cases} f_1(n) = n\log n^{1000} = \theta(n\log n) \\ f_2(n) = 1024n^2 + 4n + 460 = \theta(n^2) \\ \cancel{f_3(n) = 3n^4 + 6n = \theta(n^4)} \\ f_3(n) = 7n^3\log n + 1000 = \theta(n^3\log n) \\ f_4(n) = 3n^4 + 6n = \theta(n^4) \\ f_5(n) = 3^n \\ f_6(n) = 6^n \end{cases}$$

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