Problem Set 6, P3.4.5

Problem 3.

X_1 only depend on U

X_2 only depend on V.

- U, V are independent. $\Rightarrow$ X_1, X_2 are independent.

Independence $\Leftrightarrow$ outer product of marginal distribution is the joint distribution.

$$\mathbb{P}(X_1 = 1) = \mathbb{P}(U \geq 3) = \frac{4}{6} = \frac{2}{3}.$$

1, 2, 3, 4, 5, 6

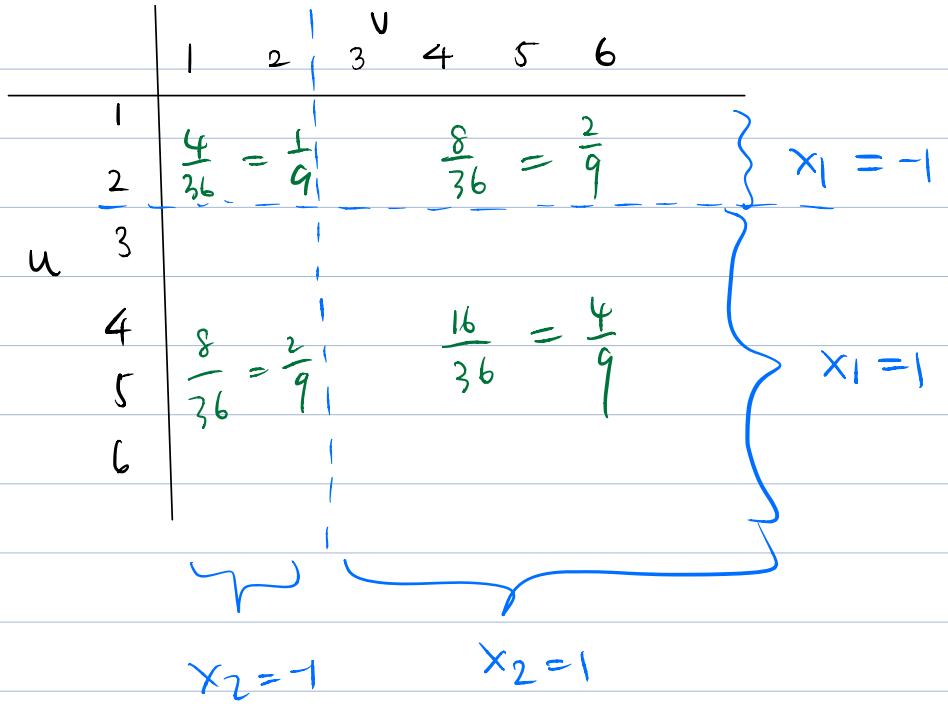
4 out 6.

$$\mathbb{P}(X_1 = -1) = 1 - \mathbb{P}(X_1 = 1) = 1 - \frac{2}{3} = \frac{1}{3}.$$

For X_2, the marginal distribution is the same.

X_1 \ X_2	-1 $\frac{1}{3}$	1 $\frac{2}{3}$
-1 $\frac{1}{3}$	1/9	2/9
1 $\frac{2}{3}$	2/9	4/9

(independent case).


v
1 2 3 4 5 6
u
1
2
3
4
5
6
4/36 = 1/9
8/36 = 2/9
8/36 = 2/9
16/36 = 4/9
x1 = -1
x1 = 1
x2 = -1
x2 = 1

Problem 4

x \ y	0	1	2	3	marginal dist. of x
0	42	21	15	6	$\frac{7}{8}$
1	6	3	1	2	$\frac{1}{8}$
marginal dist. of y	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	

(×7, ×7, ×15, ×3)

96.

(causal)

outer product:

	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$
$\frac{7}{8}$	$\frac{7}{16}$	$\frac{7}{32}$	$\frac{7}{48}$	$\frac{7}{96}$
$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{48}$	$\frac{1}{96}$

this is not identical to the joint dist.,

$\Rightarrow$ they are not independent.

e.g. $\mathbb{P}(Y=0 \mid X=1) = \dfrac{\mathbb{P}(Y=0, X=1)}{\mathbb{P}(X=1)}$ ← row sum.

$$\left(\frac{1/16}{1/8}, \frac{1/32}{1/8}, \frac{1/96}{1/8}, \frac{1/48}{1/8}\right)$$

$$\| $$

$$\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{12}, \frac{1}{6}\right)$$

this is the conditional dist. of Y given $X=1$.

It is different from the marginal distribution.

Problem 5.

		y = −1	y = 0	y = 1	
x	0	2	5	1	$\frac{1}{7}$
	1	8	20	4	$\frac{4}{7}$
	2	6	15	3	$\frac{3}{7}$
		$\frac{2}{7}$	$\frac{5}{7}$	$\frac{1}{7}$	

(×2, ×5, ×5, ×2)

$\Rightarrow$ independent. if the rows / columns are proportional.

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The probabilities sum up to $\frac{8}{7}$.

Which means that this is not a probability distribution.