Problem set 3

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Instructions: Here is our third set of practice problems. The TAs will go over some of these in discussion, but please feel free to ask about anything that's difficult or unclear in office hours or on Piazza.

Practice problems

(1) Consider a Laplace probability model for rolling a single, six-sided die. What is the probability that the result of the roll is two or smaller?

answer: $1/3$

(2) Consider a probability model for rolling a single, six-sided die with

$$P(A) = \begin{cases} 1 & \text{if } \boxdot \text{ is in } A \\ 0 & \text{if } \boxdot \text{ is not in } A \end{cases}.$$

What is the probability that the result of the roll is two or smaller?

answer: 0

(3) Suppose that events A and B in a probability model have

$$P(A) = 0.3,$$
$$P(B) = 0.4,$$

and

$$P(A \text{ and } B) = 0.15 .$$

What is $P(A \text{ or } B)$?

answer: 0.55

(4) Suppose that events A and B in a probability model have

$$P(A) = 0.1,$$

$$P(B) = 0.8,$$

and

$$P(A \text{ and } B) = 0.1 .$$

Are A and B mutually exclusive?

answer: no!

(5) Suppose that $P(A) = 0.9$ and $P(B) = 0.8$. Do you have enough information to compute $P(A \text{ and } B)$? If so, do this; if not, determine its smallest possible value. Is there enough information to determine whether A and B are mutually exclusive?

answer: No; 0.7. Yes: they are not!

(6) Roll two six-sided dice. Let A be the event that the total number of dots (sum of the two results) is even, and B be the event that the total is odd. Are A and B mutually exclusive?

answer: yes!

(7) Roll two six-sided dice. Let A be the event that the total number of dots is twelve, and B be the event that the first roll is six. What is $P(A \text{ and } B)$?

answer: 1/36

(8) Suppose that A and B are mutually exclusive events with $P(A) = 0.3$ and $P(B) = 0.4$. What is $P(A \text{ or } B)$?

answer: 0.7

(9) Suppose that you will roll two fair, six sided dice. What is the probability that the sum of the two results is less than or equal to four?

answer: 1/6

(10) The SEC will investigate two different transactions for insider trading. If the probability that each is found to involve insider trading is 0.47, and the probability that both are found to involve insider trading is 0.22, What is the probabilty that at least one of the two transactions is found to involve insider trading?

answer: 0.72

(11) Suppose that a complex stochastic (i.e. probability) model for stock prices assigns a probability of 0.12 that the Dow Jones Industrial Average attains a value of 36,000 for the first time by January 1st, 2026. What is the smallest possible probability that can be assigned by this model to the event that the Dow Jones Industrial Average attains a value of 36,000 for the first time by January 1st, 2024? *Hint: draw a Venn diagram.*

answer: 0.12

(12) Suppose that A and B are events in some probability model that are such that $P(A) = 0.5$ and $P(B) = 0.7$. What is the smallest possible value of $P(A \text{ and } B)$? (Hint: draw a Venn diagram!)

answer: 0.2

(13) Suppose that $P(A) = 0.2$, $P(B) = 0.4$, and that A and B are mutually exclusive. What is the conditional probability that B occurs given that A does not occur?

answer: $1/2$

(14) Suppose that $P(A) = 0.2$, $P(B) = 0.4$, and that A and B are mutually exclusive. What is the conditional probability that B occurs given that A occurs?

answer: 0

(15) Suppose that $P(A) = 0.2$, $P(B) = 0.4$, and that whenever A occurs, B occurs as well. What is the conditional probability that B occurs given that A does not occur?

answer: $1/4$

(16) Suppose that $P(A) = 0.2$, $P(B) = 0.4$, and that whenever A occurs, B occurs as well. What is the conditional probability that B occurs given that A occurs?

answer: 1

(17) Suppose that the probability that a boy cries wolf, when there is a wolf present, is 0.8. Suppose also that, when there is not a wolf present, the probability that the boy cries wolf is 0.3. Furthermore, suppose that the probability that a wolf is present is 0.2. If you hear the boy cry wolf, what is the probability that there is a wolf present?

answer: 0.4

(18) If Janet Yellen is in a good mood, there is a probabilty of 0.62 that the weather is good. If she is not in a good mood, there is a probability of 0.48 that the weather is good. The probability that she is in a good mood, without taking into account the weather, is 0.7. If the weather is good, what is the probability that she is in a good mood?

answer: ≈ 0.75

(19) Suppose that a medical test has a sensitivity of 0.8 and a specificity of 0.7. For a disease occuring otherwise with probability 0.25, what is the probability that a patient who tests positive has the disease?

answer: ≈ 0.47

(20) Suppose that a medical test has sensitivity 0.8. If the disease occurs otherwise with probability 0.1, and a patient who tests postitive has the disease with probability 0.7, what is the specificity of the test?

answer: ≈ 0.962

(21) A crooked gambler in a saloon reaches into her pocket, grasps a die, takes it out, and rolls it. Her pocket contains five dice: one of them is weighted to come up ⚄ with certainty, and the other four are fair (giving equal chances for any of the results ⚀, ⚁, ⚂, ⚃, ⚄, and ⚅). Assume each of the five dice is equally likely to be grasped.

What is the probability that the die rolled yields ⚄? If the die rolled yields ⚄, what is the probability that it was the weighted die?

answer: 1/3; 3/5

(22) Let X be the number obtained from the die roll in problem (21). What is the distribution of X?

answer:

x	$p(x)$
1	*2/15*
2	*2/15*
3	*2/15*
4	*2/15*
5	*1/3*
6	*2/15*

(23) Let X be the number obtained from the die roll in problem (21). What is the expected value of X?

answer: 3.8

(24) Fill in the mising value in the probability distribution below.

x	$p(x)$
-2	0.08
-1	0.14
0	0.15
1	0.31
2	?

answer: 0.32

(25) What is $E(X)$ for a random variable X with the probability distribution in problem (24)?

answer: 0.65

(26) Maria is given a choice between lotteries I and II (below). Which has higher expected payout?

I	
x	$p_X(x)$
0	0.1
25	0.1
50	0.3
75	0.2
100	0.3

II	
y	$p_Y(y)$
0	0.3
25	0
50	0.1
75	0.2
100	0.4

answer: (I) $E(X) = 62.5 > 60 = E(Y)$ *(II)*

(27) Suppose that lottery I yields a payoff of \$10 with certainty, while lottery II yields a loss of \$10200 with probability 0.8 and a gain of \$41000 with probability 0.2. Which lottery has higher expected payoff?

answer: (II) $40 > 10$ *(I)*