Problem 1, 3, 6a, 11, 22 (Notability)

Problem 1. $\bar{x} = 11$, $\underbrace{y_i = 2 \cdot x_i - 5}$.

scaling & shift: $\begin{cases} z_i = 2 \cdot x_i & \Rightarrow \bar{z} = 2 \cdot \bar{x} \\ y_i = z_i - 5 & \Rightarrow \bar{y} = \bar{z} - 5 \end{cases}$

$\underbrace{\bar{y} = 2 \cdot \bar{x} - 5} = 2 \cdot 11 - 5 = 17$.

$$\boxed{\begin{array}{l} \cdot \quad y_i = a \cdot x_i + b, \quad a, b \text{ are constants, for all } i = 1, 2, \cdots, n. \\ \begin{cases} \Rightarrow \underbrace{\bar{y} = a \cdot \bar{x} + b}. & \text{(scaling \& shift, linear transform)} \\ \Rightarrow sd(y) = a \cdot sd(x). \end{cases} \end{array}}$$

we could try to prove it by definition.

suppose Sample size $= n$. $(x_i) = (x_1, x_2, x_3, \cdots, n)$.

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i ;$$

$$\bar{y} = \frac{1}{n} \cdot \sum_{i=1}^{n} y_i = \frac{1}{n} \cdot \sum_{i=1}^{n} (2 \cdot x_i - 5)$$

$$= \frac{1}{n} \cdot \left(2 \sum_{i=1}^{n} x_i - 5n \right)$$

$$= 2 \cdot \left(\frac{1}{n} \sum_{i=1}^{n} x_i \right) - 5$$

$$= 2\bar{x} - 5.$$

Problem 3 suppose last week's average temperature is $\bar{x}$°F.

$\bar{x} = 65$.

x: sample

x: $(x_1, x_2, x_3, \cdots, x_n)$.

- $n = 7$, w.r.t daily temp.
- $n = 7 \times 24$, w.r.t hourly temp.

...

°F vs. °C.

$$a°C = \left(\frac{9}{5}a + 32\right)°F$$

$$\underbrace{b}_{x_i}°F = \underbrace{\frac{5}{9}(b - 32)}_{y_i}°C$$

- $b = \frac{9}{5}a + 32$

$\Leftrightarrow b - 32 = \frac{9}{5}a$

$\Leftrightarrow \frac{5}{9}(b - 32) = a$.

suppose last week's average temperature is $\bar{y}$°C,

where $(y_i) = (y_1, y_2, y_3, \cdots, y_n)$

$$y_i = \frac{5}{9}(x_i - 32). \quad \checkmark$$

$$\Rightarrow \bar{y} = \frac{5}{9}(\bar{x} - 32) = \frac{5}{9}(65 - 32) = 18.33.$$

Problem 6a)

- relative area.
- area of a (circular) sector is

$$A = \frac{\theta}{2} \cdot r^2 \quad \left(\text{for a circle, } A = \pi \cdot r^2 = \frac{(2\pi)}{2} \cdot r^2 \right).$$

θ: angle, r = radius.

hence, if $\theta = 40° = \frac{2}{9} \cdot \pi$, then $A = \frac{(2\pi/9)}{2} \cdot r^2$

$$= (\pi/9) \cdot r^2.$$

- In country A,

$$\text{percentage of men} = \frac{\#\{\text{men in country } A\}}{\#\{\text{all people in country } A\}}.$$

$$= \frac{\cancel{(\pi/9)} \cdot 4^2}{\cancel{(\pi/9)} \cdot 8^2} = \frac{16}{64} = 25\%.$$

- In country B,

$$\text{percentage of men} = \frac{\cancel{(\pi/9)} \cdot 6^2}{\cancel{(\pi/9)} \cdot 10^2} = \frac{36}{100} = 36\%.$$

Problem 11.

squares. Area : $A = h^2$, h = height.

(Area of) ones : fives : tens : twenties : hundreds

= 78% : 10% : 7% : 4% : 1%.

(height of) " : " : " : " : "

= $\sqrt{78} : \sqrt{10} : \sqrt{7} : \sqrt{4} : \sqrt{1}$.

this is not the unique solution; the proportions could be multiplied by any constants.

Problem 22.

standard deviation : (sample size n)

sample variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad (1)$$

$$= \frac{1}{n-1}\left((x_1-\bar{x})^2 + \cdots + (x_n - \bar{x})^2\right)$$

$$s^2 = \frac{1}{n-1}\left((x_1^2 + \cdots + x_n^2) - \frac{1}{n}(x_1 + \cdots + x_n)^2\right)$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right) \qquad (2).$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\cdot\bar{x}\right)$$

Sample size $= 10 + 1 = 11 = \tilde{n}$.

sample $(x_1, x_2, \cdots, x_{10}, y)$, $\quad y = 9$.

new data point.

$$s^2 = \frac{1}{11-1}\left((x_1^2 + \cdots + x_{10}^2 + y^2) - \frac{1}{11}(x_1 + \cdots + x_{10} + y)^2\right)$$

$$= \frac{1}{11-1}\left((100 + 9^2) - \frac{1}{11}(5+9)^2\right)$$

$$= \frac{1}{10}\left(181 - \frac{1}{11}\cdot 196\right)$$

$\Rightarrow s = 4.04$.

Note: for real number $x_1, \cdots, x_n$.

inequality:

$$\left(\sum_{i=1}^{n} x_i^2\right) \geq \frac{1}{n} \cdot \left(\sum_{i=1}^{n} x_i\right)^2 \quad \checkmark$$

(contradict to the settings in this problem).