Homework 5

1. Find the shortest path tree from every node to node 1 for the graph of Fig.1 using the Bellman-ford and Dijkstra algorithms.

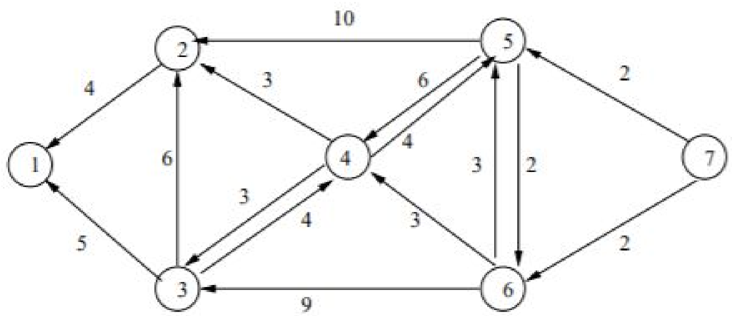

Figure 1: Graph for Problem 1

Describe the algorithmic change necessary to answer this question.

2. Prove

Edge $e = (v, w)$ does not belong to a minimum spanning tree of G if and only if v and w can be joined by a path consisting entirely of edges that are cheaper than e.

3.

Suppose you are given a directed graph $G = (V, E)$ with costs on the edges c_e for $e \in E$ and a sink t (costs may be negative). Assume that you also have finite values $d(v)$ for $v \in V$. Someone claims that, for each node $v \in V$, the quantity $d(v)$ is the cost of the minimum-cost path from node v to the sink t.

(a) Give a linear-time algorithm (time $O(m)$ if the graph has m edges) that verifies whether this claim is correct.

(b) Assume that the distances are correct, and $d(v)$ is finite for all $v \in V$. Now you need to compute distances to a different sink t'. Give an $O(m \log n)$ algorithm for computing distances $d'(v)$ for all nodes $v \in V$ to the sink node t'. (*Hint:* It is useful to consider a new cost function defined as follows: for edge $e = (v, w)$, let $c'_e = c_e - d(v) + d(w)$. Is there a relation between costs of paths for the two different costs c and c'?)