Q4: Part I: $T(n) = 9T(\frac{n}{3}) + n\log n$ $\quad a=9, b=3, \log_b a = \log_3 9 = 2$.

$f(n) = n\log n = O(n^{1.5})$ $\leftarrow$ $\lim_{n\to\infty} \frac{n\log n}{n^{1.5}} = 0$

$C_i = \log_b a = 2, \quad C = 1.5$

$C < C_i$

So, we can use master $T(n) = \theta(n^2)$

Part II: $T(n) = 2T(\frac{n}{5}) + n$

Assume $T(n) \leq C \cdot n$ for all $n < K$.

Prove $T(n) \leq cn$ when $n = K$.

$T(K) = 2T(\frac{K}{5}) + K$

$\leq 2C\frac{K}{5} + K \leq CK$

$K \leq \frac{4}{5}CK$

$1 \leq \frac{4}{5}C$

$\frac{5}{4} \leq C$.

So, there exist C that can make the majority correct.