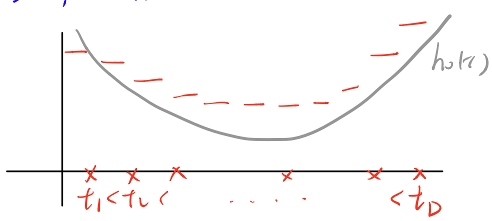
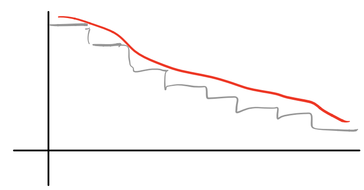
Hypothesis Testing $\begin{cases} \text{One-Sample Test} \\ \text{Two/More Samples} \\ \text{Test for Trend} \\ \text{Stratified Tests} \end{cases}$

interested in: comparing $\underbrace{\text{survival experience}}_{\substack{h(t) \\ S(t) \\ H(t)}}$ of populations.

One-Sample Test:



or



H_0: $h(t) = \underbrace{h_0(t)}_{\text{prespecified}}$ for all $t < t_D$

vs H_1: $h(t) \neq h_0(t)$ for some $t < t_D$

H_0: $S(t) = S_0(t)$.

H_1: $S(t) \neq S_0(t)$.

For each subinterval: $t_k \leq t < t_{k+1}$

d_k: # failure at t_k

Y_k: # under study / at risk just prior to t_k

Event: Yes	No	Total
d_k	$Y_k - d_k$	Y_k

$$d_k \sim Bin\Big(Y_k,\ \underbrace{P(\text{event} \in [t_k, t_{k+1}) \mid \text{event} \geq t_k)}_{\substack{\text{total risk of failure during } [t_k, t_{k+1}) \\ \text{given survival prior } t_k.}}\Big)$$

$=$ cumulative hazard between $[t_k, t_{k+1})$

idea similar to binning data χ^2 tests.

define observed rate: $\frac{d_k}{Y_k}$ expected rate: $\int_{t_k}^{t_{k+1}} h_0(t)dt$

but we have extended time period, with each having some evidence, so take a weighted vote

$$O(t_D) = \sum_{k=1}^{D} W(t_k) \frac{d_k}{Y_k}$$

$$E(t_D) = \int_0^{t_D} W(s)\, h_0(s)\, ds$$

$\Rightarrow$ test stat:

$$Z(t_D) = O(t_D) - E(t_D)$$

if H_0 is true:

$$Z(t_D) \approx 0.$$

$$Var(Z(t_D)) = \int_0^{t_D} W^2(s) \frac{h_0(s)}{Y(s)}\, ds$$

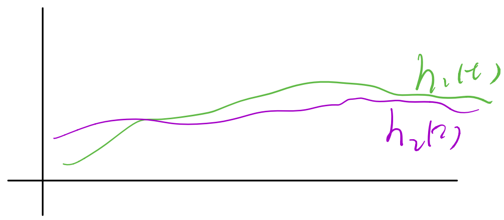
Then, standardized version

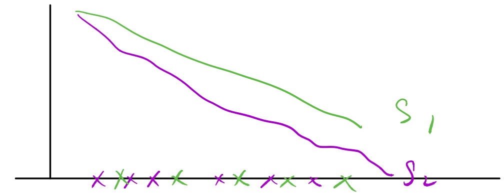
$$\frac{Z(t_D)^2}{V(Z(t_D))} \sim \chi^2_{df=1}$$

or $$\frac{Z(t_D)}{\sqrt{V(Z(t_D))}} \sim N(0,1)$$

Choose: weight.

Two-Sample Test:





$H_0: h_1(t) = h_2(t)$ for $t \le t_D$

vs $H_1: h_1(t) \ne h_2(t)$ for some $t \le t_D$

Let distinct event time of pooled sample:

$0 < t_1 < t_2 < \cdots \; t_D$

Y_k : across all grps $\quad Y_k = Y_k^1 + Y_k^2$

d_k : across all grps. $\quad d_k = d_k^1 + d_k^2$

The info. at t_k can be summarized:

	Event: Yes	Event: No	
grp 1	d_k^1		Y_k^1
grp 2	d_k^2		Y_k^2
	d_k		Y_k

Suppose H_0 is true:

Grp 1: $d_k^1 \sim$ Hypergeometric$(N = Y_k, K = d_k, n = d_k, k = d_k^1)$

Draw n times without replacement from a population of size N with K reds, the number of reds k is of interest.

$$E[d_k^1] = Y_k^1 \frac{d_k}{Y_k}.$$

$$\mathrm{Var}[d_k'] = \underbrace{Y_k' \frac{d_k}{Y_k} \frac{Y_k - d_k}{Y_k} \frac{Y_k - Y_k'}{Y_k - 1}}_{\text{lead to complicated expression}}$$

So expect $\dfrac{d_k'}{Y_k'} - \dfrac{d_k}{Y_k} \approx 0$ if H_0 is true

and $\underbrace{\sum_{k=1}^{D} W(t_k) \left(\dfrac{d_k'}{Y_k'} - \dfrac{d_k}{Y_k} \right)}_{Z(t_D)\,:\ \text{sum of weight diff between obs vs exp.}} \approx 0.$

along with $\mathrm{Var}(Z)$

$$= \sum_{k=1}^{D} W(t_k)^2 \frac{Y_k'}{Y_k} \left(1 - \frac{Y_k'}{Y_k}\right) \left(\frac{Y_k - d_k}{Y_k - 1}\right) d_k$$

$$\Rightarrow \frac{Z}{\sqrt{\mathrm{Var}(Z)}} \overset{\text{approx.}}{\sim} N(0,1)$$