Q3: Part 1: $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{5n^3+n^2+3}{\log n + n^3 + n}$

$= \frac{15n^2+2n}{\frac{1}{n}+3n^2+1} = \frac{30n+2}{-n^{-2}+6n} = \frac{30}{n^{-3}+6} = 5$ when $n\to\infty$

So $\lim_{n\to\infty} \frac{5n^3+n^2+3}{\log n + n^3 + n} = 5$.

By using limit lemma:

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = L$, $L\neq 0$ and it is finite,

So, $f(n) = \Theta(g(n)) = O(g(n))$

Part 2: $f(n) = 5n^3 + n^2 + 3$

$f(n) \leq 5n^6 + n^6 + 3n^6$ when $n \geq 1$

$\leq 9n^6$ when $n \geq 1$

So, There exist $c=9$ and $n_0 = 1$, such that for all $n \geq n_0$, we have $f(n) \leq c \cdot n^6$

, So, $f(n) = O(n^6)$