Q2: First, we must determine that How many times will the "while" loop has.

~~So~~ suppose k:

$$3 + 2\cdot 3 + 3\cdot 3 + \cdots + 3\cdot k = n.$$

$$3\cdot \frac{(1+k)k}{2} = n \Rightarrow k \simeq \sqrt{n}.$$

Second, sum the inner loop:

$$T(n) = \sum_{i=1}^{\sqrt{n}} 3\cdot i = 3\cdot\frac{(1+\sqrt{n})\cdot\sqrt{n}}{2} = O(n).$$

So, it is a linear time.

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Q3. Part 1:

10. 17, 1, 11, 4. 12, $\xrightarrow{\text{Sort}}$ start time. 1, 4, 10, 11, 12, 17.

finish time 4, 7, 13, 14, 15, 20

Select 1: 17.

select 2: 12.

Select 3: 4

Select 4: 1.

Part 2: $S = \{1, 2, \cdots, n\}$ of n jobs.

s_i, start of activity i

f_i, finish of activity i

Greedy strategy: start the last.

Assume: $S_1 \geq S_2 \geq S_3 \cdots \geq S_n$.

Property ①. There exists an optimal solution A that greed strategy choice the first "1" in A.

proof: $A: \{a_1, a_2 \cdots a_k\}$. the greedy choice first is G_1.

If $a_1 = G_1$, ~~there is~~ begin with a greedy choice.

If $a_1 \neq G_1$:

we have $f_{a_2} \leq S_{a_1}$

Because the Greedy strategy is selected the start last. we have,

$S_{G_1} \geq S_{a_1}$

So we have $S_{G_1} \geq S_{a_1} \geq f_{a_2}$, So, it is Compatible.

So, there exists an optimal solution A that statisify those Condition.

property ②: optimal solution Contains optimal subsolutions.

If A is an optimal solution which contains G_1, then $A' = A - \{G_1\}$ is an optimal solution to $S' = \{i \in S, f_i \leq S_{G_1}\}$.

proof: If there exists B' to S' such that $|B'| > |A'|$, then, let

$$B = B' \cup \{G_1\}.$$

then B is an globally optimal solution,

$$|B| > |A|$$

which is contradicting to the optimality of A, So, there doesn't exist B'

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