where b and c are positive constants. For example, if an experimenter is testing whether a drug lowers cholesterol level, H_0 and H_1 might be set up like this with θ_0 = standard acceptable cholesterol level. Since a high cholesterol level is associated with heart disease, the consequences of rejecting H_0 when θ is large are quite serious. A loss function like (8.3.13) reflects such a consequence. A similar type of loss function is advocated by Vardeman (1987).

Even for a general loss function like (8.3.13), the risk function and the power function are closely related. For any fixed value of θ, the loss is either $L(\theta, a_0)$ or $L(\theta, a_1)$. Thus the expected loss is

$$\begin{aligned} R(\theta,\delta) &= L(\theta,a_0)P_\theta(\delta(\mathbf{X})=a_0)+L(\theta,a_1)P_\theta(\delta(\mathbf{X})=a_1) \\ &= L(\theta,a_0)(1-\beta(\theta))+L(\theta,a_1)\beta(\theta). \end{aligned} \tag{8.3.14}$$

The power function of a test is always important when evaluating a hypothesis test. But in a decision theoretic analysis, the weights given by the loss function are also important.

8.4 Exercises

8.1 In 1,000 tosses of a coin, 560 heads and 440 tails appear. Is it reasonable to assume that the coin is fair? Justify your answer.

8.2 In a given city it is assumed that the number of automobile accidents in a given year follows a Poisson distribution. In past years the average number of accidents per year was 15, and this year it was 10. Is it justified to claim that the accident rate has dropped?

8.3 Here, the LRT alluded to in Example 8.2.9 will be derived. Suppose that we observe m iid Bernoulli(θ) random variables, denoted by $Y_1, \ldots, Y_m$. Show that the LRT of $H_0\colon \theta \le \theta_0$ versus $H_1\colon \theta > \theta_0$ will reject H_0 if $\sum_{i=1}^m Y_i > b$.

8.4 Prove the assertion made in the text after Definition 8.2.1. If $f(x|\theta)$ is the pmf of a discrete random variable, then the numerator of $\lambda(\mathbf{x})$, the LRT statistic, is the maximum probability of the observed sample when the maximum is computed over parameters in the null hypothesis. Furthermore, the denominator of $\lambda(\mathbf{x})$ is the maximum probability of the observed sample over all possible parameters.

8.5 A random sample, $X_1, \ldots, X_n$, is drawn from a Pareto population with pdf

$$f(x|\theta,\nu)=\frac{\theta\nu^\theta}{x^{\theta+1}}I_{[\nu,\infty)}(x), \quad \theta>0, \quad \nu>0.$$

(a) Find the MLEs of θ and ν.

(b) Show that the LRT of

$$H_0\colon \theta=1,\ \nu \text{ unknown}, \qquad \text{versus} \qquad H_1\colon \theta\ne 1,\ \nu \text{ unknown},$$

has critical region of the form $\{\mathbf{x}\colon T(\mathbf{x}) \le c_1 \text{ or } T(\mathbf{x}) \ge c_2\}$, where $0 < c_1 < c_2$ and

$$T=\log\left[\frac{\prod_{i=1}^n X_i}{(\min_i X_i)^n}\right].$$