Practice Midterm I

STA 200B

University of California, Davis

Exam Rules: This exam is closed book and closed notes. You may bring one page of notes, double-sided. Use of calculators, cell phones or any other electronic or communication devices is not allowed. You must show all of your work to receive credit. You will have 50 minutes to complete the exam.

Note: You do not need to show that the second derivative is negative when deriving MLEs. If needed, you may use that for the Beta(α, β) distribution we have $EX = \alpha/(\alpha+\beta)$, $\quad \text{var}(X) = \alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1)]$ and for the Gamma(α, β) distribution $EX = \alpha/\beta$, $\quad \text{var}(X) = \alpha/\beta^2$.

Name : _____

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1. Suppose that the number of defects in a 1200-foot roll of magnetic recording tape has a Poisson distribution for which the value of the mean (θ) is unknown.

   a) Suppose five rolls of this tape are selected at random. Determine the joint distribution $f(x_1, \ldots, x_5|\theta)$ of the five rolls.

   b) Suppose the number of defects found on the rolls are $2, 2, 6, 0$ and 3. If the prior distribution of θ is the gamma distribution with parameters $\alpha = 3$ and $\beta = 1$, find the posterior distribution of θ.

   c) Find the Bayes estimator with respect to the squared error loss function.

d) Write an equation from which the Bayes estimator with respect to the absolute error loss function can be calculated.

e) Find a method of moments estimator for θ^2.

2. Suppose that $X_1, \ldots, X_n$ form a random sample where each of the X_i is the number of successes in N i.i.d. Bernoulli trials with parameter θ, i.e., θ is the probability of success.

   a) Derive the MLE for θ.

   b) Derive the MLE for EX_1^2.

   c) Obtain a method of moments estimator for $\text{var}(X_1)$.

3. Suppose that the proportion θ of defective items in large shipment is unknown and that the prior distribution of θ is the beta distribution with parameters 1 and 10. Assume in a random sample of 20 items one find that 1 item is defective.

   a) What is the expected value and variance of the prior distribution?

   b) What is the posterior distribution?

   c) What is the Bayes estimator for θ if one uses the quadratic loss function?

   d) Find the MLE for θ. Is it the same as the Bayes estimator?

   e) Suppose that you change the sampling plan and will keep on sampling until you find 3 defective items. Let X be the number of non-defective items until this happens. Note the negative binomial distribution with parameters θ and k has the pmf $g(x|\theta) = \binom{x+k-1}{k-1}\theta^k(1-\theta)^x$.
   Derive the MLE and Bayes estimator again.

4. Let $X_1, \ldots, X_n$ be a random sample from a distribution with pdf

$$f(x|\theta_1, \theta_2) = \frac{1}{\theta_2} e^{-(x-\theta_1)/\theta_2},$$

for $x \geq \theta_1$, $-\infty < \theta_1 < \infty$, and $\theta_2 > 0$.

a) Find jointly sufficient statistics (T_1, T_2) where θ_1 and θ_2 are both unknown.

b) If θ_2 is known, find a sufficient statistic for θ_1.