Practice Midterm II

STA 200B
Winter 2020
University of California, Davis

Exam Rules: This exam is closed book and closed notes. You may bring one page of notes, double-sided. Use of calculators, cell phones or any other electronic or communication devices is not allowed. You must show all of your work to receive credit. You will have 50 minutes to complete the exam. This exam has 4 pages, make sure you have all four pages.

Note: You do not need to show that the second derivative is negative when deriving MLEs. If needed, you may use that for the Beta(α, β) distribution we have $EX = \alpha/(\alpha+\beta)$, $\text{var}(X) = \alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1)]$ and for the Gamma(α, β) distribution $EX = \alpha/\beta$, $\text{var}(X) = \alpha/\beta^2$.

Name : ____________________

ID : ____________________

Signature : ____________________

1. Let $X_1, \dots, X_n$ be a random sample from a distribution with p.d.f.

$$f(x|\theta_1, \theta_2) = \frac{1}{\theta_2} e^{-(x-\theta_1)/\theta_2},$$

for $x \geq \theta_1$, $-\infty < \theta_1 < \infty$, and $\theta_2 > 0$.

   (a) Find jointly sufficient statistics (T_1, T_2) when θ_1 and θ_2 are both unknown.
   (b) Find a method of moments estimator for for $\theta_1+\theta_2$ and obtain the MSE for this estimator.
   (c) If θ_1 is known, find the UMVUE for θ_2.
   (d) If θ_2 is known, find the Fisher information in the sample.

2. Suppose $X_1, \dots, X_n$ form a random sample from a Poisson distribution with parameter $\theta > 0$.

   (a) Obtain the UMVUE by first showing efficiency of a suitable statistic.
   (b) Obtain the UMVUE without using Fisher information or the log-density derivative.
   (c) Obtain method of moments estimators based on first and second moments. Which of these is preferred? State your reasons.
   (d) Show that there is no unbiased estimator of $1/\theta$.

3. Suppose we draw a sample $X_1, \ldots, X_n$ of size n from the distribution $N(\mu_1, \sigma_1^2)$ and a sample $Y_1, \ldots, Y_m$ of size m from the distribution $N(\mu_2, \sigma_2^2)$. Assume $\sigma_1^2 = 4\sigma_2^2$ and $\mu_1 = \mu_2 =: \mu$. We aim to estimate $\theta = \mu$ and use the estimator $\theta_\alpha = \alpha \bar{X}_n + (1-\alpha)\bar{Y}_m$, where $\bar{X}$ denotes the sample means.

   (a) For what value of α is the MSE minimized? What is the value of the MSE at the minimum?

   (b) How does this MSE compare to that of the estimator that is obtained when you pool the two samples into one and take the sample average as estimator?

4. Suppose $X_1, \ldots, X_n$ form a random sample from a distribution with p.d.f. $f(x|\theta) = \theta x^{\theta-1}$, for $0 < x < 1$ and $\theta > 0$.

   (a) Find a minimal sufficient statistics for θ.

   (b) Is the sample mean admissible for estimating θ under square error loss? Provide your reasoning.

   (c) Use that fact that $E(-\log X_i) = \frac{1}{\theta}$ to find an UMVUE of $\frac{1}{\theta}$.