Q_1: $Y_{ij} \sim B(\mu_i)$ $f(y|\mu) = \mu^y (1-\mu)^{1-y}$

$$f_n(\underline{Y}|\mu_i) = \prod_{i=1}^{p} \mu_i^{\sum_j^m Y_{ij}} (1-\mu_i)^{m-\sum Y_{ij}}$$

$$L = \log f_n(\underline{Y}|\mu_i) = \sum_{i=1}^{p} \sum_{j=1}^{m} Y_{ij} \log \mu_i + (m - \sum_j^m Y_{ij}) \log(1-\mu_i)$$

$$\frac{\partial L}{\partial \mu_i} = \frac{1}{\mu_i} \cdot \sum_{j=1}^{m} Y_{ij} - (m - \sum_{j=1}^{m} Y_{ij}) \frac{1}{1-\mu_i} = 0$$

$$= \sum_{j=1}^{m} Y_{ij} (1-\mu_i) - (m - \sum_{j=1}^{m} Y_{ij}) \cdot \mu_i = 0$$

$$\hat{\mu}_i = \frac{1}{m} \sum_{j=1}^{m} Y_{ij}$$

under null: $H_0: \mu_1 = \mu_2 \cdots = \mu_p = \mu_0$

$$f_n(\underline{Y}|\mu_0) = \mu_0^{\sum_i \sum_j Y_{ij}} (1-\mu_0)^{mp - \sum\sum Y_{ij}}$$

$$\hat{\mu}_0 = \frac{1}{mp} \cdot \sum_i \sum_j Y_{ij}$$

$$\lambda(\underline{Y}) = \frac{L(\hat{\theta}_0|\underline{Y})}{L(\hat{\theta}|\underline{X})} = \frac{\prod_{i=1}^{p} \left(\frac{1}{m}\sum_{j=1}^{m} Y_{ij}\right)^{\sum_{j=1}^{m} Y_{ij}} \cdot \left(1 - \frac{1}{m}\sum_{j=1}^{m} Y_{ij}\right)^{m - \sum_{j=1}^{m} Y_{ij}}}{\left(\frac{1}{mp}\sum_i^p \sum_j^m Y_{ij}\right)^{\sum\sum Y_{ij}} \cdot \left(1 - \frac{1}{mp}\sum_i \sum_j Y_{ij}\right)^{mp - \sum_i \sum_j Y_{ij}}} < C, \text{ reject } H_0$$

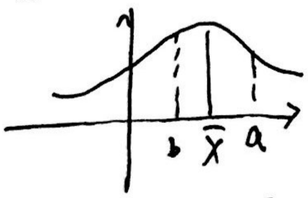
Q_2: $X_1 \cdots X_n \overset{iid}{\sim} N(\mu, 1)$, $H_0: a \le \mu \le b$

$$f(x|\mu) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x-\mu)^2\right)$$

$$f_n(\underline{x}|\mu) = \left(\frac{1}{\sqrt{2\pi}}\right)^n \exp\left(-\frac{1}{2}\sum(x_i-\mu)^2\right)$$

no restricted: $\hat{\mu} = \frac{\sum x_i}{n} = \bar{x}$

under $H_0: a \le \mu \le b$



$$\begin{cases} \text{If } b \le \bar{x} & \hat{\mu}_0 = b. \\ \text{If } a \ge \bar{x} & \hat{\mu}_0 = a. \\ \text{If } a \le \bar{x} \le b & \hat{\mu}_0 = \bar{x} \end{cases}$$

If $b \le \bar{x}$

$$\lambda(\underline{x}) = \frac{L(\hat{\theta}_0|\underline{X})}{L(\hat{\theta}|\underline{X})} = \frac{\exp\left(-\frac{1}{2}\sum(x_i-b)^2\right)}{\exp\left(-\frac{1}{2}\sum(x_i-\bar{x})^2\right)} < C.$$

$$= \exp\left[-\frac{1}{2}\left[\sum(x_i-b)^2 - \sum(x_i-\bar{x})^2\right]\right] < C$$

$$= \exp\left[-\frac{1}{2} n \cdot (\bar{x}-b)^2\right] < C \quad \text{for } b \le \bar{x}$$

$$= -\frac{1}{2} n (\bar{x}-b)^2 < \log C$$

$$(\bar{x}-b)^2 > (-2\log C)/n$$

$$\bar{x} - b > \sqrt{(-2\log C)/n} \quad \text{for } b \le \bar{x}$$

If $a \geq \bar{x}$ $\hat{\mu_0} = a$ if $a < \bar{x} < b$ $\hat{\mu_0} = \bar{x}$

$\lambda(x) = 1$

$\lambda(x) = \exp\{-\frac{1}{2} n (\bar{x} - a)^2\} < C.$

$= -\frac{1}{2} n (\bar{x} - a)^2 < \log C$

$(\bar{x} - a)^2 > (-2\log C)/n$

$-(\bar{x} - a) > \sqrt{(-2\log C)/n}$

$(\bar{x} - a) < -\sqrt{(-2\log C)/n}.$

So, reject Ho when $\bar{x} - b > \sqrt{(-2\log C)/n}$ or $\bar{x} - a < -\sqrt{(-2\log C)/n}$

b) $H_0: a \leq \mu \leq b.$

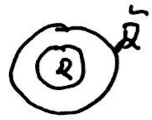
Consider two simple test: $H_0: a \leq \mu$ and $H_0: \mu \leq b.$

The rejection region for $H_0: a \leq \mu$ is $\{x, \bar{x} - a < -\sqrt{(-2\log C)/n}\}$

the rejection region for H_0 $\mu \leq b$ is $\{x, \bar{x} - b > \sqrt{(-2\log C)/n}\}$

The union interaction is $\{\bar{x} - a < -\sqrt{(-2\log C)/n}\} \cup \{\bar{x} - b > \sqrt{(-2\log C)/n}\}.$

Q3: $H_0: \theta = \theta_0$ $H_1: \theta = \theta_1$ $\alpha \in (0,1)$



reject region R $P_{\theta_0}(X \in R) < \alpha$ $\theta \in \theta_0 \exists R \subseteq \tilde{R}$, $P_{\theta_0}(X \in \tilde{R}) = \alpha.$

R $P_{\theta_0}(X \in R) > 0 \Rightarrow P_{\theta_1}(X \in R) > 0$

~~Consider a UMP test, For any most powerful level test.~~

$\underset{\sim}{X \in R}$ $f(X|\theta_1) > k f(X|\theta_0)$ $k > 0$

$\int_R f(x|\theta_1) dx > k \int_R f(x|\theta_0) dx$

$A_1 > k A_0$

$A_1 = \int_R f(x|\theta_1) dx = P_{\theta_1}(X \in R)$

$A_0 = \int_R f(x|\theta_0) dx = P_{\theta_0}(X \in R) < \alpha$

$\tilde{A} = \int_{\tilde{R}} f(x|\theta_0) dx = P_{\theta_0}(X \in \tilde{R}) = \alpha.$

This shows that if $P_{\theta_0}(X \in R) \neq 0$, than there exists a most powerful test with some rejection region R. Then: $P_{\theta_0}(X \in R) \leq \alpha$. it is a level α test.

$P_{\theta_0}(X \in R) = b \leq \alpha.$

if $b < \alpha$. $P_{\theta_0}(X \in R) < \alpha$, then, there exists $\tilde{R} \supseteq R$, $P_{\theta_0}(X \in \tilde{R}) = \alpha.$

it is impossible because $\tilde{R}$ is bigger than R.

So, $b = \alpha.$

So, it is a size α test.

Q4: $P_y(\vec{x})$ is a valid p-value. $P_\theta(P_y(\vec{x}) \leq \alpha) \leq \alpha$ If $\theta \in \Theta_y$.

$P(\vec{x}) = \sup_{y \in P} P_y(\vec{x}) \geq P_y(\vec{x}) \quad \forall y \in P$

For $\forall \theta_0 \in \Theta_0$ since $\Theta_0 = \bigcup_\gamma C P$

Q5: Since $C(\vec{x})$ be a $1-\alpha$ confidence

$P_\theta(\vec{x} \in R) = P_\theta(C(\vec{x}) \cap \Theta_0 = \phi)$

$= 1 - P_\theta(C(\vec{x}) \cap \Theta_0 = \phi)$

$\leq 1 - P_\theta(\theta \in C(\vec{x}))$

$\leq 1-(1-\alpha)$

$\leq \alpha$

$\forall \theta \in \Theta_0 \quad P_\theta(\vec{x} \in R) \leq \alpha.$

$\sup_{\theta \in \Theta_0} \beta(\theta) = \sup_{\theta \in \Theta_0} P_\theta(\theta \in \Theta_0$

thus is a α level test