Feedback-Coordinated Ramp Control of Consecutive On-Ramps Using Distributed Modeling and Godunov-Based Satisfiable Allocation

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***Abstract*—This paper presents a feedback control design for a coordinated ramp metering problem for two consecutive on-ramps. We design a traffic allocation scheme for ramps based on Godunov's numerical method and using a distributive model. Most of the previous work for designing feedback control for ramp metering is based on either the discretized linear methods or nonlinear methods based on the traffic ordinary differential equations (ODEs). We utilize the distributive model to construct a control condition for regulating the traffic density at critical density. Then, we design a Godunov-method-based satisfiable allocation scheme that gives us the actual control for each ramp individually. We show the stability properties of the closed-loop system and validate the effectiveness of the feedback control law by running a simulation using real traffic flow measurements with parameter estimation.**

***Index Terms*—Coordinated ramp metering, distributed traffic model, feedback control, Godunov method.**

I. Introduction

RAMP meters are used to control the inflow into the freeway so that better flow conditions can be achieved on the freeway. One of the methods to optimize and control the flow on the freeways is coordinated ramp metering [1]. Ramp metering can be designed as fixed periodic cycles using historical data or time of day schedules, or it can also be designed based on sensor measurements and feedback control in real time. Coordinated ramp metering problem refers to a freeway system that has entry and exit ramps on it at various points. The question for the design then becomes, how should the ramp metering be designed taking into account mutual interactions of various ramps and their overall effect on performance of the freeway. Fig. 1 illustrates a ramp system that can be controlled by metering to effect the traffic conditions on the freeway.

Ramp metering techniques can be categorized as, pre-timed; predictive and traffic responsive. Ramp metering controllers can be categorized mainly into isolated ramp meters and coordinated ramp meters. [2]. Isolated traffic-responsive ramp metering was first implemented in 1960s and is still used at a lot of locations today. An isolated or local ramp controller is based on the real time traffic information in the neighborhood of a particular ramp. This technique is popular due to its simplicity and works well in optimizing the local traffic conditions on the freeway stretch. The main disadvantage of this approach is the lack of coordination between different ramp controls towards the common goal of optimizing the freeway conditions throughout [3]. For instance, isolated ramp metering strategies applied independently to multiple ramps on a freeway network would work well in case of unconstrained ramp queues. However in reality, ramps have limited capacity to hold the vehicles and ramp queues must be restricted so that they do not interfere with the traffic on arterial roads. In this case, local control might lead to a congested or even a jammed freeway and coordinated strategy must be deployed [4]. Coordinated ramp metering was first used in 1970s and is gradually gaining popularity for making system wide coordinated control decisions. A coordinated traffic responsive ramp control is designed with real time traffic information throughout the freeway section such that ramp controllers work in tandem to optimize traffic conditions on the freeway. One of the main disadvantages of this approach is that often it can become very complex and expensive in terms of its design and implementation.

Various types of dynamics have been used to design control laws for isolated ramps. These dynamics include, simple inflow outflow based steady state model in [5], linearized discrete time model in [6], fuzzy logic based controller in [7], and a neural network based controller in [8]. Many lumped parameter based models and distributed models have been studied in [9]. Lumped parameter models used previously had a limitation in them as they did not produce vanishing viscosity weak solutions in the limit. In order to address this issue, enhanced Godunov based hybrid models have been presented in [10]–[12] for isolated ramp metering problem. The Godunov based model treats the problem as a hybrid system with discrete states, where in each state there are different inflow and outflow conditions. When there are no uncertainties, then the controller knows

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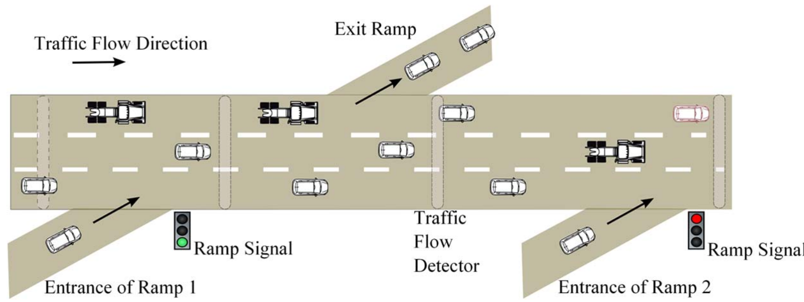

Fig. 1. Coordinated ramp metering.

exactly which discrete state the system is in, and hence the nonlinear dynamics can be canceled using techniques such as in feedback linearization [10] and sliding mode control [13].

Most of the coordinated control strategies proposed in the literature, use either an approach involving linearization of nonlinear dynamics and then time discretization or involve use of fuzzy logic and artificial neural networks in which design is not explicitly based on the stability of the closed loop system. The approach proposed here is based on feedback linearization of the coupled macroscopic non-linear dynamics of the traffic model (without any space or time discretization). Hence our approach is inherently different.

This work uses the distributive model to derive a control constraint [9] and then design the actual control using the Godunov based numerical scheme (see [11] and [12]). Our contribution to the topic of feedback ramp controls is that we use entropy consistent distributed model to come up with a control constraint law for regulating the traffic flow at the critical density. Then we combine this constraint law along with the Godunov based numerical technique to design an satisfiable allocation scheme, which provides the coordinated control law for the individual ramps. Combining the distributed model with the Godunov's scheme for designing the coordinated feedback control law is completely new, and hence the control design is a novel contribution to this area.

Structure of rest of the paper is as follows: Section II presents a literature survey of the topic, Section III provides a mathematical background of the traffic flow models used in this paper, Section IV formulates the coordinated ramp problem, and designs a feedback control law and finally Sections V and VI present the simulation results and concluding remarks respectively.

II. Literature Survey

Ramp metering is one of the main methods to control flow conditions on a freeway segment. It can be used to influence the amount of traffic on a freeway by controlling the inflow from connected streets. Literature containing ramp metering techniques can be found dated more than 45 years, see [5], [14]–[16]. Early researchers have mainly used optimization based methods for ramp metering such as [17]–[19]. A decentralized ramp control design is presented in [20]. One of the first feedback control theory based control law for ramp metering problem is ALINEA (see [6]) which uses concepts of linearization and time discretization. Intelligent control methods such as neural network based controller is presented in [8], [21] and [22] and fuzzy logic based in [7], [23] and a combined approach using Fuuzy-Neuro algorithm in [24]. Many researchers assessed the effectiveness of proposed ramp metering methods (see [25], and [26]) and their cost benefits [27]. Many countries, such as Italy [28], U.S.A [29], Germany [30], France [31], New Zealand [32], Netherlands [33] and U.K. [34] have been utilizing ramp metering for controlling traffic flows on the freeway, for a good amount of time. A general overview of ramp metering problem is covered in the Traffic Control Systems Handbook [35]. Nonlinear lumped parameter model based feedback control is detailed in [9]. Various model formulations such as the distributed model, lumped model, and their continuous and discrete time versions are shown in [36]–[39]. Recently, there has been an interest in developing control law for ramp metering using the distributed LWR model (such as in [9] and [40]. The work in [9] is based on feedback linearization in the distributed setting, while the methodology in [40] uses adjoint based optimization. Godunov's numerical method based discretization and feedback linearization has been used in [10] and [11] to model the ramp metering problem, assuming no uncertainties in the system parameters. The paper [12] considers a Godunov approximation based dynamics with uncertainties in the system parameters, and presents a way to design robust controllers. The paper [13] uses sliding mode control for the same problem assuming perfect knowledge of the model by the controller, and the paper [41] deals with the problem of uncertain free flow speed using sliding mode control.

Many ideas pertaining to the isolated ramp metering problem have been extended to design the coordinated ramp metering system. These include use of nonlinear state feedback control in [42] and use of microscopic mathematical model in [43]. A nonlinear model-predictive hierarchical control approach is presented for coordinated ramp metering of freeway networks in [4] and nonlinear optimal control concepts are applied to the same problem in [44]–[47]. An evolutionary fuzzy system is presented for coordinated and traffic responsive ramp metering in [3], [48], an adaptive fuzzy systems is used for the same in [49], and a neuro-fuzzy algorithm in [50]. A multilayer control structure and PSO (Particle Swarm Optimization) algorithm are used for coordinated ramp control on freeways in [2]. Heuristic ramp-metering coordination strategy is implemented at Monash freeway, Australia [51].

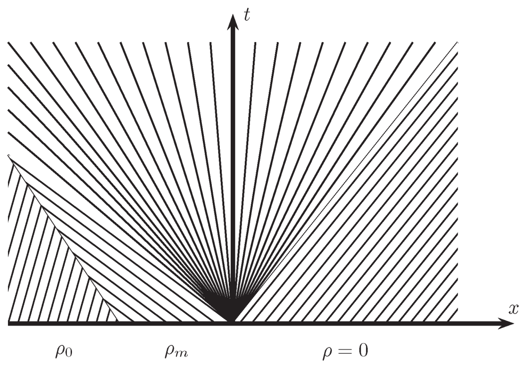

Fig. 2. Traffic characteristics.

III. Mathematical Background

A. LWR and Greenshield's Models for Traffic

The macroscopic traffic flow model formulates the relationship among the key traffic flow parameters such as density, flow etc. The classic LWR (Lighthill-Whitham-Richards) model was proposed in 1956. It is a one-dimensional macroscopic traffic model named after the authors in [52]–[54]. The dynamics of traffic flow using this model is given by

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}f(t,x) = 0 \tag{1}$$

where, ρ is the traffic density and f is the flux. Traffic flux is defined as the product of traffic density and the traffic speed v, i.e., $f = \rho \times v$. There are many models that link traffic density to traffic speed. One of them is Greenshield's model which proposes a linear relationship between traffic density and traffic speed (see [55]). This model is given by

$$v(\rho) = v_f \left(1 - \frac{\rho}{\rho_m}\right) \tag{2}$$

where v_f is the free flow speed and ρ_m is the maximum possible density or jam density. Free flow speed is the traffic speed when there is no traffic, i.e., when the traffic density is zero. Traffic jam density is the density at which there is a traffic jam, i.e., when the traffic speed is zero.

Traffic flow using Greenshields model is given by

$$f(t) = v_f \rho(t) \left(1 - \frac{\rho(t)}{\rho_m}\right). \tag{3}$$

B. Godunov Based Model

To study the traffic characteristics and to gain an insight into the system evolution we consider a Riemann problem with a piecewise constant initial condition. Consider an initial value problem for a Riemann's problem where the upstream traffic density is lower (see [56]–[58]). Discretizations of macroscopic first order traffic flow models using Godunov's scheme are discussed in [59]–[61]. Fig. 2 shows the characteristics of traffic where the initial traffic data are shown on the x-axis, traffic density is piecewise constant. The middle section has the jam density ρ_m, the upstream has a lower density ρ_0 and the downstream has zero density. As time increases, the shock wave travels upstream and at the same time the jam dissipates as a rarefaction onto the downstream. This can been observed on the y-axis.

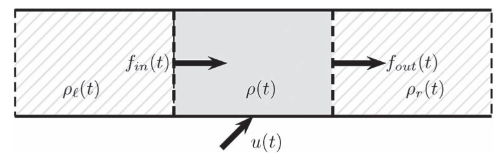

Fig. 3. Godunov dynamics.

The Godunov method is based on solving the Riemann problem where the initial condition is a piecewise constant function with two values ρ_ℓ and ρ_r for the upstream (left) and downstream (right) densities (see [62]). Either a shockwave or a rarefaction wave originates from the junction of the two densities. A shockwave develops if $f'(\rho_\ell) > f'(\rho_r)$ (see [63]).

The speed of the shockwave is given by Equation (4). In this equation, $x_s(t)$ is the position of the shockwave as a function of time. If the shock speed is positive then the inflow at junction between the two traffic densities will be a function of upstream traffic density, whereas if the shock speed is negative then the inflow at junction between the two traffic densities will be a function of downstream traffic density

$$s = \frac{dx_s(t)}{dt} = \frac{[f(\rho_\ell) - f(\rho_r)]}{\rho_\ell - \rho_r}. \tag{4}$$

A rarefaction develops if $f'(\rho_\ell) < f'(\rho_r)$. The rarefaction can be entirely to the left, or to the right or in the middle.

The analysis of the shockwave and rarefaction conditions gives us the Godunov based ODE model for traffic. ODE for this method is derived from the conservation law (see Fig. 3), and is give by Equation (5), where we have assumed unit length for the section

$$\frac{d\rho(t)}{dt} = f_{in}(t) - f_{out}(t) + u(t). \tag{5}$$

Now, the inflow $f_{in}(t)$ will be a function of upstream density ρ_ℓ and downstream density ρ_r. Here upstream and downstream are with respect to the left junction. Hence we have the relationship given by Equation (6) where we have used the function $F(\cdot,\cdot)$ that will be obtained from the Godunov method

$$f_{in}(t) = F(\rho_\ell, \rho). \tag{6}$$

Similarly, for the right junction, the outflow $f_{out}(t)$ is given by

$$f_{out}(t) = F(\rho, \rho_r). \tag{7}$$

The function $F(\rho_\ell, \rho_r)$ in terms of its arguments is given by the Godunov method as follows (see [62, section 13.5, pages 143–145]).

$$F(\rho_\ell, \rho_r) = f\left(\rho^*(\rho_\ell, \rho_r)\right). \tag{8}$$

Here, the flow-dictating density ρ^* is obtained from the following (see [62]):

1) $f'(\rho_\ell), f'(\rho_r) \geq 0 \Rightarrow \rho^* = \rho_\ell$

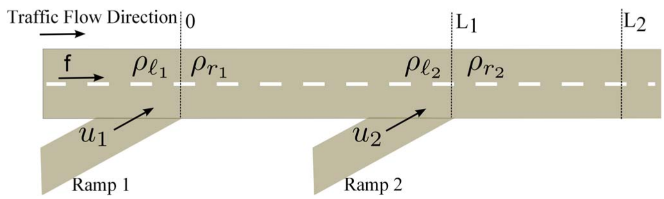

Fig. 4. Coordinated ramp metering.

2) $f'(\rho_\ell), f'(\rho_r) \le 0 \Rightarrow \rho^* = \rho_r$
3) $f'(\rho_\ell) \ge 0 \ge f'(\rho_r) \Rightarrow \rho^* = \rho_\ell$ if $s > 0$, otherwise $\rho^* = \rho_r$
4) $f'(\rho_\ell) < 0 < f'(\rho_r) \Rightarrow \rho^* = \rho_c$

where, ρ_c is obtained as the solution to $f'(\rho_c) = 0$. ρ_c is called the critical traffic density and is equal to $\rho_m/2$.

Hence depending on the traffic densities on the left and right side of the junction, flow at the junction can have three possible values, i.e., $F_q(\rho_\ell, \rho_r)$ can have three distinct values, $f(\rho_\ell)$, $f(\rho_r)$, or $f(\rho_c)$, where $q \in \{1, 2, 3\}$.

IV. Coordinated Ramp Metering

A. Problem Formulation

Fig. 4 illustrates the coordinated ramp model, where main freeway section has an inflow given by f, the inflows controlled by the first and second ramps are given by u_1 and u_2. The traffic density on the main freeway section just before the first and second ramp is given by ρ_{ℓ_1} and ρ_{ℓ_2}, traffic density just after the first and second ramp is given by ρ_{r_1} and ρ_{r_2}. Length of the freeway section from the start up to the second ramp is L_1 and upto the end of freeway section (containing both ramps) is L_2.

The aim is to keep the aggregate traffic density on the freeway section containing the two ramps equal to the critical density, hence we define an error function as in [9]

$$e(t) = \frac{1}{2}\int_0^{L_1} \left(\rho(t,x) - \rho_{c_1}\right)^2 dx + \frac{1}{2}\int_{L_1}^{L_2} \left(\rho(t,x) - \rho_{c_2}\right)^2 dx. \quad (9)$$

The limits of integral for the problem are from the start to the end of the mainline that includes both ramps. The function $e(.)$ is a mapping at each time t from the space of functions on $[0, \mathrm{L}]$ to the space of real numbers. The objective for the control law is to make the error term go to zero asymptotically. We will try to achieve the closed-loop dynamics represented by

$$\dot{e}(t) + k_1 e(t) + k_2 \int_0^t e(s) ds = 0 \quad (10)$$

which will enable us to obtain

$$\lim_{t \to \infty} e(t) = 0. \quad (11)$$

B. Feedback Control Design

In order to design a desired control law that makes the error term go to zero asymtotically, we start differentiating the error term with respect to time to get

$$\dot{e}(t) = \frac{d}{dt}\frac{1}{2}\left(\int_0^{L_1} (\rho(t,x) - \rho_{c_1})^2 dx + \int_{L_1}^{L_2} (\rho(t,x) - \rho_{c_2})^2 dx\right). \quad (12)$$

Simplifying further using Leibniz integral rule, we get

$$\dot{e}(t) = \frac{1}{2}\left(\int_0^{L_1} \frac{\partial}{\partial t}\left(\rho(t,x) - \rho_{c_1}\right)^2 dx + \int_{L_1}^{L_2} \frac{\partial}{\partial t}\left(\rho(t,x) - \rho_{c_2}\right)^2 dx)\right) \quad (13)$$

which further simplifies to

$$\dot{e}(t) = \int_0^{L_1} \left(\rho(t,x) - \rho_{c_1}\right) \frac{\partial}{\partial t}\left(\rho(t,x) - \rho_{c_1}\right) dx + \int_{L_1}^{L_2} \left(\rho(t,x) - \rho_{c_2}\right) \frac{\partial}{\partial t}\left(\rho(t,x) - \rho_{c_2}\right) dx). \quad (14)$$

Using the conservation equation (1), $\frac{\partial \rho}{\partial t} = -\frac{\partial q}{\partial x}$, for further simplification gives

$$\dot{e}(t) = \int_0^{L_1} \left(\rho_{c_1} - \rho(t,x)\right) \frac{\partial}{\partial x} q(t,x) dx + \int_{L_1}^{L_2} \left(\rho_{c_2} - \rho(t,x)\right) \frac{\partial}{\partial x} q(t,x) dx \quad (15)$$

further simplification gives

$$\dot{e}(t) = \rho_{c_1} \int_0^{L_1} \frac{\partial}{\partial x} q(t,x) dx - \int_0^{L_1} \rho(t,x) \frac{\partial}{\partial x} q(t,x) dx + \rho_{c_2} \int_{L_1}^{L_2} \frac{\partial}{\partial x} q(t,x) dx - \int_{L_1}^{L_2} \rho(t,x) \frac{\partial}{\partial x} q(t,x) dx \quad (16)$$

solving the integrals yield

$$\dot{e}(t) = \rho_{c_1}\left(q\left(t, L_1^-\right) - q(t,0)\right) - \int_0^{L_1} \rho(t,x) \frac{\partial}{\partial x} q(t,x) dx + \rho_{c_2}\left(q(t,L_2) - q\left(t, L_1^+\right)\right) - \int_{L_1}^{L_2} \rho(t,x) \frac{\partial}{\partial x} q(t,x) dx. \quad (17)$$

The flow at the left most boundary is produced by the freeway and ramp in flows. Therefore, we have

$$q(t,0) = u_1 + f(t). \quad (18)$$

From the boundary condition at the second ramp, we also have

$$q\left(t, L_1^+\right) = q\left(t, L_1^-\right) + u_2. \quad (19)$$

We have showed the steps to derive the control law assuming different ρ_c at the two sections. However for simplicity and for getting a compact expression for the control law, we now assume $\rho_{c_1} = \rho_{c_2} = \rho_c$. Substituting (18) and (19) in (17) gives

$$\dot{e}(t) = \rho_c \left(q\left(t, L_1^-\right) - u_1 - f(t) + q(t, L_2) - q\left(t, L_1^-\right) - u_2\right) - \int_0^{L_2} \rho(t,x)\frac{\partial}{\partial x}q(t,x)dx. \quad (20)$$

Hence, we can write

$$u_1 + u_2 = q(t, L_2) - f(t) - k_1 e(t) - k_2 \int_0^t e(s)ds - \frac{1}{\rho_c}\int_0^{L_2} \rho(t,x)\frac{\partial}{\partial x}q(t,x)dx. \quad (21)$$

Equation (21) provides a constraint on control that would ensure the maximum possible flow on the stretch of freeway including the two ramps. Replacing the value of $q(t, L_2)$ we obtain

$$u_1 + u_2 = v_f \rho(t, L_2)\left(1 - \frac{\rho(t, L_2)}{\rho_m}\right) - f(t) - k_1 e(t) - k_2 \int_0^t e(s)ds - \frac{1}{\rho_c}\int_0^{L_2} \rho(t,x)\frac{\partial}{\partial x}q(t,x)dx. \quad (22)$$

Equation (22) establishes a relationship between the control variable of two ramps (u_1, u_2) and traffic flow conditions. We call this equation as the control constraint equation. For further analysis, we will be using this equation, which is derived using the distributive model.

C. Satisfiable Allocation Using Godunov Scheme

Analysis in the previous section provides with a constraint on $u_1 + u_2$, but not a way to calculate individual u_1 and u_2. For this, we use a Godunov based scheme to design the individual u_1 and u_2, while maintaining the control constraint from equation (22). Design of control law based on Godunov based scheme is discussed below and illustrated in Fig. 5 as well.

- Case (i): $f'(\rho_\ell) \geq 0$ and $f'(\rho_r) \geq 0$
  $\Rightarrow \rho_l \leq \rho_m/2$ and $\rho_r \leq \rho_m/2$
  $\Rightarrow \rho^* = \rho_\ell$
  Control Action: We try to make $\rho_\ell \to \rho_m/2$ by judiciously choosing u.
- Case (ii): $f'(\rho_\ell) \leq 0$ and $f'(\rho_r) \leq 0$
  $\Rightarrow \rho_l \geq \rho_m/2$ and $\rho_r \geq \rho_m/2$
  $\Rightarrow \rho^* = \rho_r$
  Control Action: We choose $u = 0$.
- Case (iii): $f'(\rho_\ell) \geq 0$ and $f'(\rho_r) \leq 0$
  $\Rightarrow \rho_l \leq \rho_m/2$ and $\rho_r \geq \rho_m/2$
  — if $s > 0 \Rightarrow \rho^* = \rho_\ell$
  Control Action: We try to make $\rho_\ell \to \rho_m/2$ by judiciously choosing u.
  — if $s \leq 0 \Rightarrow \rho^* = \rho_r$
  Control Action: is we choose $u = 0$.
- Case (iv): $f'(\rho_\ell) \leq 0$ and $f'(\rho_r) \geq 0$
  $\Rightarrow \rho_l \geq \rho_m/2$ and $\rho_r \leq \rho_m/2$
  $\Rightarrow \rho^* = \rho_c$ (critical density)
  Control Action: We choose $u = 0$.

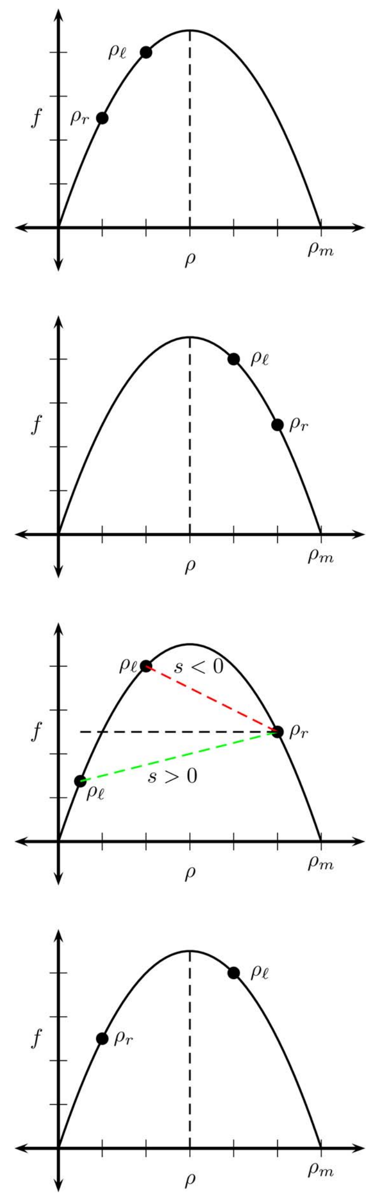

Fig. 5. Cases i, ii, iii and iv.

Summary of the control law is that there are total five cases. Three of them require u to be zero (call this case as A) and two of them require $u > 0$ (call this case as B). Now we define a variable K such that, $K = u_1 + u_2$ [see equation (22)], define a flow function $F(\rho)$ such that $F(\rho) = v_f \rho(1 - \rho/\rho_m)$ and define α such that

$$\alpha = F\left(\left(\frac{\rho_m}{2} - \rho_{\ell_1}\right) + \left(\frac{\rho_m}{2} - \rho_{\ell_2}\right)\right). \quad (23)$$

Let a variable p such that, $K = p \times \alpha$, then the control law for coordinated ramp control using the Godunov based allocation scheme is given in Table I.

V. Simulation Results

The simulations are performed using the data obtained from the ramps at the intersection of I-15 NB and Tropicana and at

TABLE I
CONTROL LAW FOR COORDINATED RAMP CONTROL

Ramp 1	Ramp 2	Control Law
A	A	$u_1 = 0$ $u_2 = 0$
A	B	$u_1 = 0$ $u_2 = min(K, F(\frac{\rho_m}{2} - \rho_{\ell 2}))$
B	A	$u_1 = min(K, F(\frac{\rho_m}{2} - \rho_{\ell 1}))$ $u_2 = 0$
B	B	$\left.\begin{array}{l} u_1 = F(\frac{\rho_m}{2} - \rho_{\ell 1}) \\ u_2 = F(\frac{\rho_m}{2} - \rho_{\ell 2}) \end{array}\right\}$ if $K \geq \alpha$ $\left.\begin{array}{l} u_1 = pF(\frac{\rho_m}{2} - \rho_{\ell 1}) \\ u_2 = pF(\frac{\rho_m}{2} - \rho_{\ell 2}) \end{array}\right\}$ if $K < \alpha$

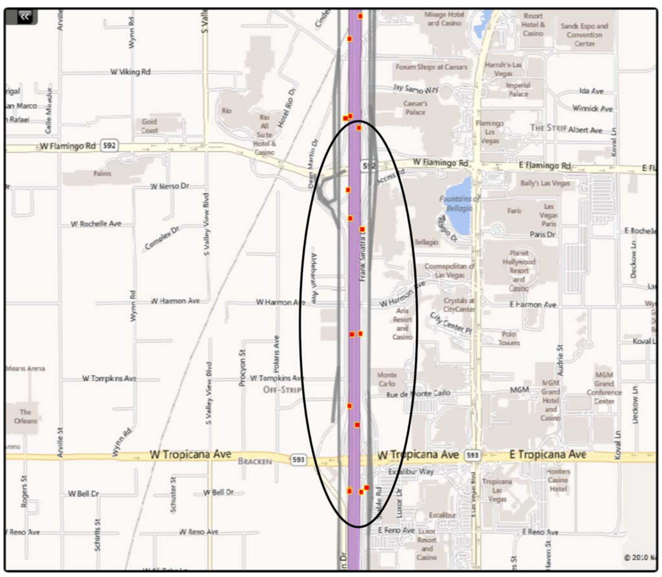

Fig. 6. Ramp metering problem location.

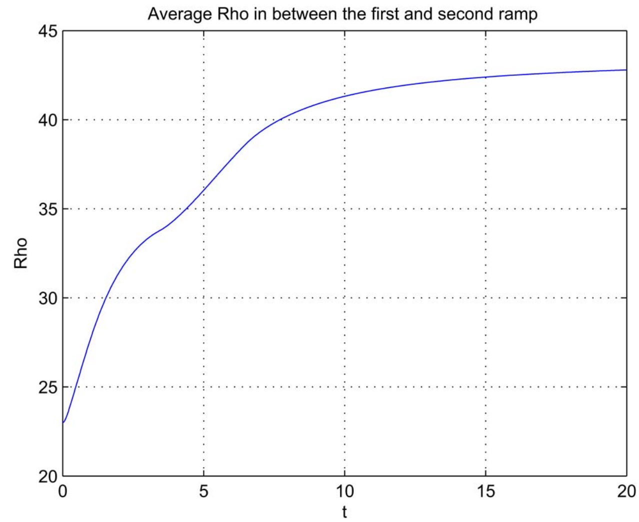

Fig. 7. Traffic density in between the two ramps.

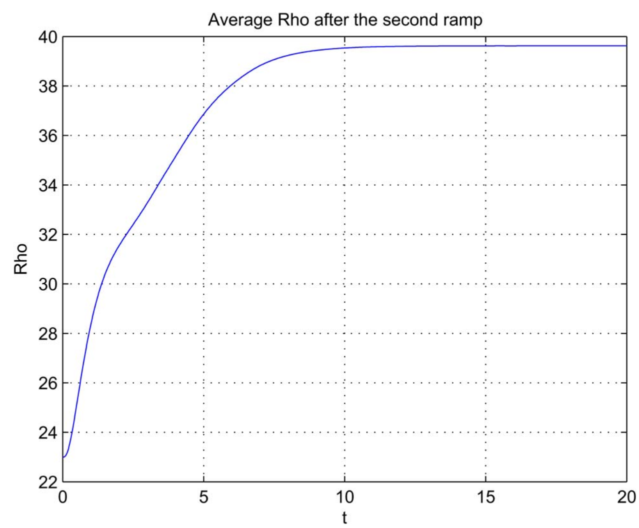

Fig. 8. Traffic density after the second ramp.

the intersection of I-15 NB and Flamingo in Las Vegas as shown in Fig. 6. This location is chosen based on inputs from Freeway and Arterial System of Transportation (FAST). Criterion for selection of freeway involved several factors including, that both the on ramps (at Tropicana and Flamingo) have a ramp meter that is controlled based on freeway sensors. Further the location ensures a wide range of data in terms of density, speed and traffic flow which is important for testing robustness in the model.

The data was collected from freeway detectors between roadway id : 59, segment id : 2 and roadway id : 72, segment id : 1. It was collected from 6 A.M. to 12 P.M. on a Thursday. The detectors at the location are loop detectors and the counts are polled every 5 min. This data is aggregated and then reported every 15 min. Counts on the ramp are obtained through video based detection. Application of the least square estimator to the flow density relationship gave us the values of v_f and ρ_m approximately 70 miles/hour, and 86 vehicles/mile respectively [10]. We use these values of traffic parameters in our simulation. Although, theoretically we have a lot of freedom to select the gains k_1 and k_2, in practice they will have to be estimated through simulation of an actual scenario by deploying the controller and experimenting with different values. They would have to be fine tuned for a particular location and might vary from location to location based on other conditions. For our simulation we have used $k1 = 50$ and $k2 = 100$.

We analyze the results of the coordinated ramp control law by observing the traffic densities in the freeway section between the first and second ramp and in the section after the second ramp. Profile of traffic density between the two ramps is shown in Fig. 7 and traffic density profile after the second ramp is shown in Fig. 8. For simulation, the highway stretch was discretized into 10 sections, such that stretch between the two ramps has five sections and the stretch after second ramp also has five sections. Density profiles of these ten sections are shown in Fig. 9.

Fig. 10 shows the inflow into the first section and outflow after the last section of highway. It also shows the input profiles of both the ramps. Error profile defined in equation (9) is shown in Fig. 11. Ramp controls are shown in Fig. 12. Based on the

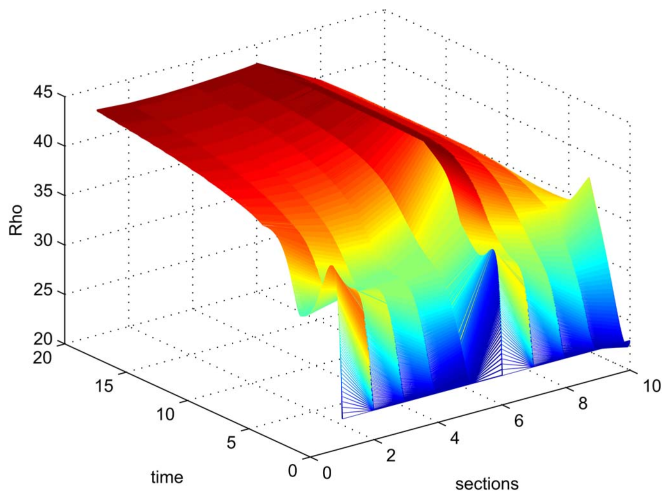

Fig. 9. Traffic density in each of the 10 sections on the highway.

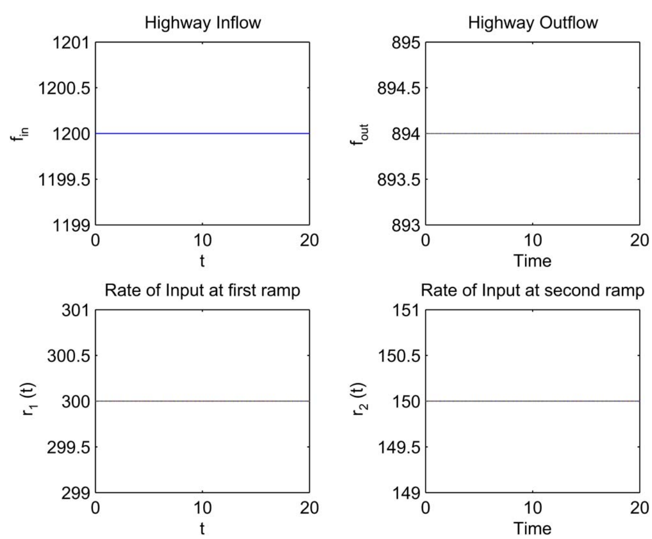

Fig. 10. Demand profiles.

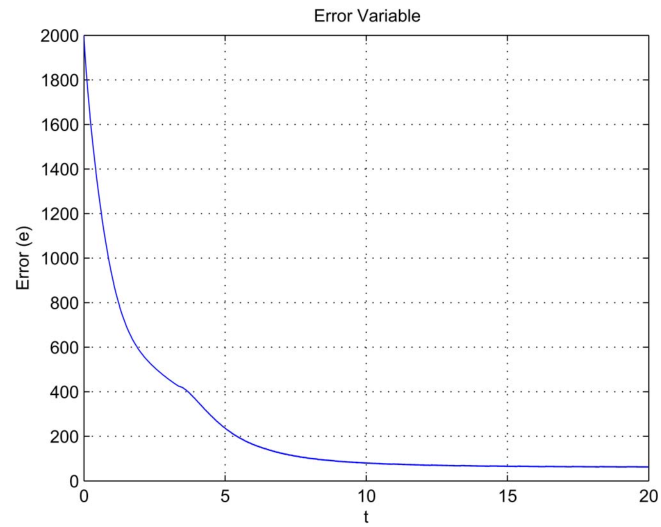

Fig. 11. Error profile.

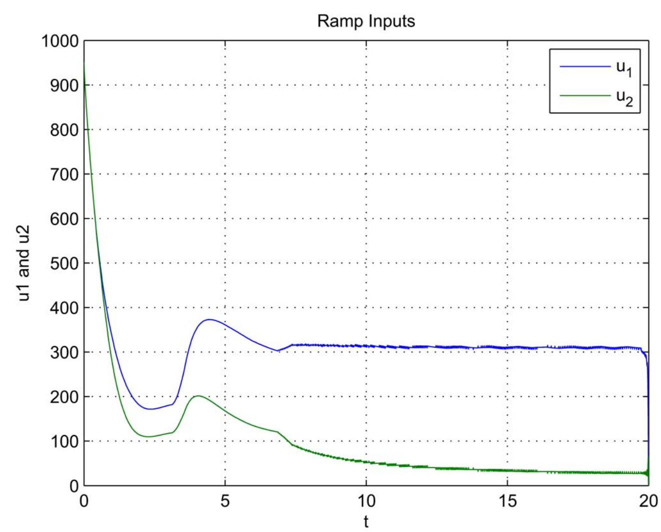

Fig. 12. Control inputs at ramp.

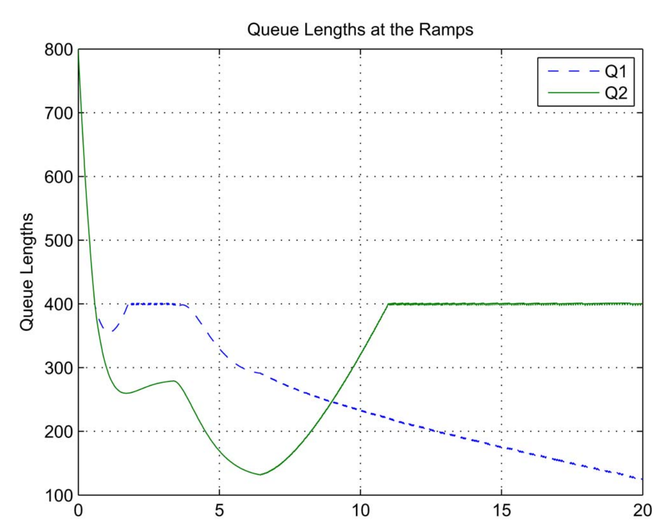

Fig. 13. Queue lengths at the two ramps.

input profiles into the ramps and control inputs from ramp to the highway, queue lengths at the two ramps are shown in Fig. 13.

We observe that density profile in both sections of freeway converge very close to the desired value of critical density (43 in this case). While ρ_1 converges to 42.8, ρ_2 stays between 39.5 to 40, values which are very close to the critical density. Simulation results show that the proposed control law tries to make the error term go to zero asymptotically.

VI. Conclusion

This paper presented a novel feedback control law using distributive modeling and the Godunov based control allocation for coordinated ramp metering problem. The paper used a coupled control law condition derived from distributive modeling and then developed Godunov based satisfiable allocation strategy for optimizing the performance of the system. The study presented a step by step theoretical derivation of the control law constraint as well as the design details of the control allocation algorithm. Finally, the paper presented simulation results for the designed control laws, which showed the density on the freeway section stabilizing gradually around the critical or desired density.

Scenario considered in this paper involved two consecutive on-ramps. For a generalized setup where the infrastructure involves multiple off-ramps, on-ramps and multiple lanes, control law would have to be derived using enhanced dynamical model. For instance, if we have an off ramp in between the two on ramps, we would have to use an enhanced dynamical model which incorporates the outflow at the off ramp and then we can use the feedback linearization technique to design the control law. Steps in the design would be similar to those described in this paper.

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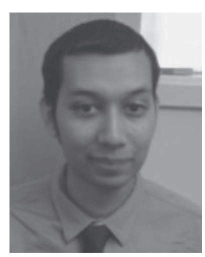

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