

# 1

a

$$l - \text{Link} \\ \forall x : \neg \text{scared}(l, x)$$

b

$$\forall p : \text{princess}(p) \Rightarrow [\forall s : \text{saved}(s, p) \Rightarrow \text{loves}(p, s)]$$

Though the phrasing *loves her saviour* perhaps makes a presumption that there is at most one saviour and also that there exists one, resulting in a more complex version which requires the equality operator (hidden in the unique exists operator):

$$\forall p : \text{princess}(p) \Rightarrow [(\forall s : \text{saved}(s, p) \Rightarrow \text{loves}(p, s)) \wedge \exists! s : \text{saved}(s, p)]$$

c

Here we again require equality.

$$m - \text{Master Sword}, a - \text{Agahnim} \\ \text{sword}(m) \wedge \text{can\_defeat}(m, a) \wedge [\forall s : (\text{sword}(s) \wedge s \neq m) \Rightarrow \neg \text{can\_defeat}(s, a)]$$

d

We can either define the leader of the Dark World as a constant:

$$d - \text{leader of the Dark World}, lw - \text{Light World} \\ \text{is\_defeated}(d) \Rightarrow \text{is\_free}(lw)$$

Or by searching the universum of entities which however also requires the usage of equality since the article *the* is used. In prepositional logic we would use a specific function.

$$dw - \text{Dark World}, lw - \text{Light World} \\ (\exists! d : \text{is\_leader}(d, dw)) \wedge (\forall d : (\text{is\_defeated}(d) \wedge \text{is\_leader}(d, dw)) \Rightarrow \text{is\_free}(lw))$$

## 2

### 2.1

a

$$\dots = 1 \text{ iff } \llbracket R(x', x'') \rrbracket^{M_1, g_1} = 1 \text{ and } \llbracket R(x'', b) \rrbracket^{M_1, g_1} = 1 \\ \dots = 1 \text{ iff } (e_2, e_3) \in V_M(R) \text{ and } (e_5, e_6) \in V_M(R) \\ \dots = 1 \text{ iff } 1 \text{ and } 1 \Rightarrow 1$$

b

$$\dots = 1 \text{ iff there is } d \in U_M \text{ such that } \llbracket A(x'') \rightarrow R(x'', j) \rrbracket^{M_1, g_1[x''/d]} = 1 \\ \dots = 1 \text{ iff there is } d \in U_M \text{ such that } \llbracket A(d) \rightarrow R(d, j) \rrbracket^{M_1, g_1} = 1 \\ \dots = 1 \text{ iff there is } d \in U_M \text{ such that } \llbracket \neg A(d) \vee R(d, j) \rrbracket^{M_1, g_1} = 1 \\ \text{set } d = e_1 \\ \dots = 1 \text{ iff } \llbracket \neg A(e_1) \vee R(e_1, j) \rrbracket^{M_1, g_1} = 1 \\ \dots = 1 \text{ iff } \llbracket \neg 0 \vee R(e_1, j) \rrbracket^{M_1, g_1} = 1 \\ \dots = 1 \text{ iff } \llbracket \neg 0 \rrbracket^{M_1, g_1} = 1 \text{ or } \llbracket R(e_1, j) \rrbracket^{M_1, g_1} = 1 \\ \Rightarrow 1$$

**c**

$$\begin{aligned} \dots = 1 \text{ iff for all } d \in U_M : \llbracket B(x) \rightarrow (A(x) \vee \forall x'' \neg R(x'', x)) \rrbracket^{M_1, g_1[x/d]} &= 1 \\ \dots = 1 \text{ iff for all } d \in U_M : \llbracket B(x) \rrbracket^{M_1, g_1[x/d]} = 0 \text{ or } \llbracket (A(x) \vee \forall x'' \neg R(x'', x)) \rrbracket^{M_1, g_1[x/d]} &= 1 \end{aligned}$$

for  $d = e_4$  :

$$\begin{aligned} \llbracket B(x) \rrbracket^{M_1, g_1[x/e_4]} &= 1 \quad (e_4 \in V_M(B)) \\ \llbracket (A(x) \vee \forall x'' \neg R(x'', x)) \rrbracket^{M_1, g_1[x/e_4]} &= \\ &= \llbracket A(x) \rrbracket^{M_1, g_1[x/e_4]} = 1 \text{ or for all } f \in U_M : \llbracket R(x'', x) \rrbracket^{M_1, g_1[x/e_4, x''/f]} = 0 \\ &\text{for } f = e_2 : \llbracket A(x) \rrbracket^{M_1, g_1[x/e_4]} = 0 \wedge \llbracket R(x'', x) \rrbracket^{M_1, g_1[x/e_4, x''/e_2]} = 1 \\ &\Rightarrow 0 \end{aligned}$$

**2.2**

$$g(x) = e_1, g(x') = e_2$$

