Neural Networks Exercise Sheet 1

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1 Mathematical objects

The following terms could be defined in an algebraic way. We assume the most common field, **R**.

a) scalar: single number, $x \in \mathbb{R}$
Example:
$$x' = 5$$

b) vector: ordered array, $\mathbf{x} \in \mathbb{R}^n, n \in \mathbb{N}$
Example:
$$\mathbf{x'} = (4, 5)$$

c) matrix: two-dimensional array, $\mathbf{A} \in \mathbb{R}^{n \times m}, n, m \in \mathbf{N}$
Example:
$$\mathbf{A'} \in \mathbb{R}^{2\times 2}$$
$$\mathbf{A'} = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$$

d) tensor: multi-dimensional array, $\mathbf{T} \in \mathbb{R}^{\mathbf{v}}, \mathbf{v} \in \mathbb{N}^m$
Example:
$$\mathbf{T}' \in \mathbb{R}^{2\times 2\times 2}$$
$$\mathbf{T}'_{1,*,*} = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$$
$$\mathbf{T}'_{2,*,*} = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

2 Vectors in machine learning

Every datapoint x represents a student and the features are the following:

$$\textbf{features} = \begin{bmatrix} \text{no. finished assignments} \\ \text{points in the exam} \end{bmatrix}$$

Vector representation:

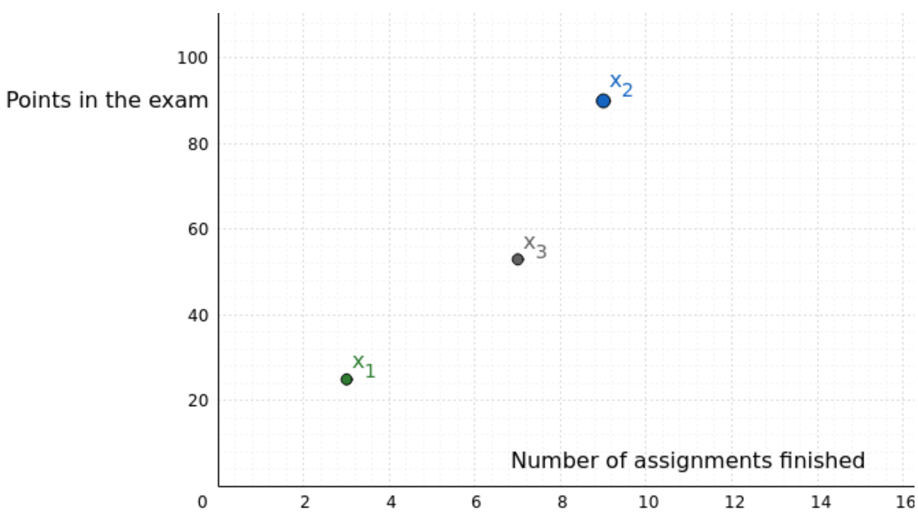
$$\mathbf{x}_1 = \begin{bmatrix} 3 \\ 25 \end{bmatrix}$$

$$\mathbf{x}_2 = \begin{bmatrix} 9 \\ 90 \end{bmatrix}$$

$$\mathbf{x}_3 = \begin{bmatrix} 7 \\ 53 \end{bmatrix}$$

Matrix representation:

$$\mathbf{X} = \begin{bmatrix} 3 & 25 \\ 9 & 90 \\ 7 & 53 \end{bmatrix}$$



3 Operations on vectors and matrices

a)

$$\begin{bmatrix} 1.5 & 2 & 0.5 \\ 0 & 1 & 1.5 \end{bmatrix}$$

b)

$$\begin{bmatrix} 15 \\ 17 \end{bmatrix}$$

c)

$$\begin{bmatrix} 4 & 2 & -9 \\ -3 & 8 & 0 \end{bmatrix}$$

d)

$$\begin{bmatrix} 8 & -2 \\ 14 & -4 \end{bmatrix}$$

e) Impossible, because the operation is not defined on the matricies of these dimensions.

4 Multiplication properties, types of matrices

4.1

a)

$$\mathbf{A} \in \mathbf{R}^{n \times m} \text{ is symmetric} \Leftrightarrow \mathbf{A}^T = \mathbf{A}$$

b)

$$\mathbf{A} \in \mathbf{R}^{n \times m} \text{ is orthogonal} \Leftrightarrow \mathbf{A}^T = \mathbf{A}^{-1}$$

c)

$$\mathbf{v} \in \mathbb{R}^n \text{ is a unit vector} \Leftrightarrow |v| = 1$$
$$|\cdot| \text{ is a vector norm}$$

d)

$$\mathbf{v} \perp \mathbf{u} \Leftrightarrow \mathbf{v} \cdot \mathbf{u} = 0$$

4.2

a)

$$(B\lambda I)^T C = \lambda(BI)^T C = \lambda I^T B^T C = \lambda BC \qquad \Rightarrow c), m \times n$$

b)

$$A^{-1}(CA^{-1})^T\lambda = A^{-1}(A^{-1})^T C^T \lambda = A^{-1}AC^T\lambda = IC^T\lambda = C^T\lambda \qquad \Rightarrow a), n \times m$$

c)

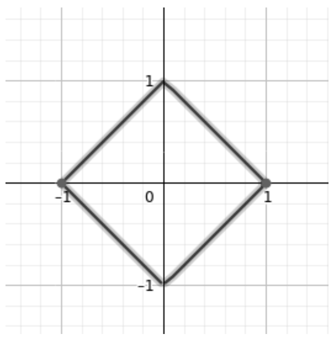
$$AA^T B^T C = IB^T C = BC \qquad \Rightarrow c), m \times n$$

5 Vector norms

$$|\mathbf{a}|_1 = 6$$

$$|\mathbf{a}|_2 = \sqrt{14}$$

a) L1



b) L2