1

a

$$\llbracket \text{every wizard but Harry} \rrbracket^M = \{P \subseteq U_M | \llbracket \text{wizard} \rrbracket \subseteq P \wedge \llbracket \text{harry} \rrbracket \not\subseteq P\}$$
$$\llbracket \text{fear Voldemort} \rrbracket^M \in \llbracket \text{every wizard but Harry} \rrbracket^M$$

b

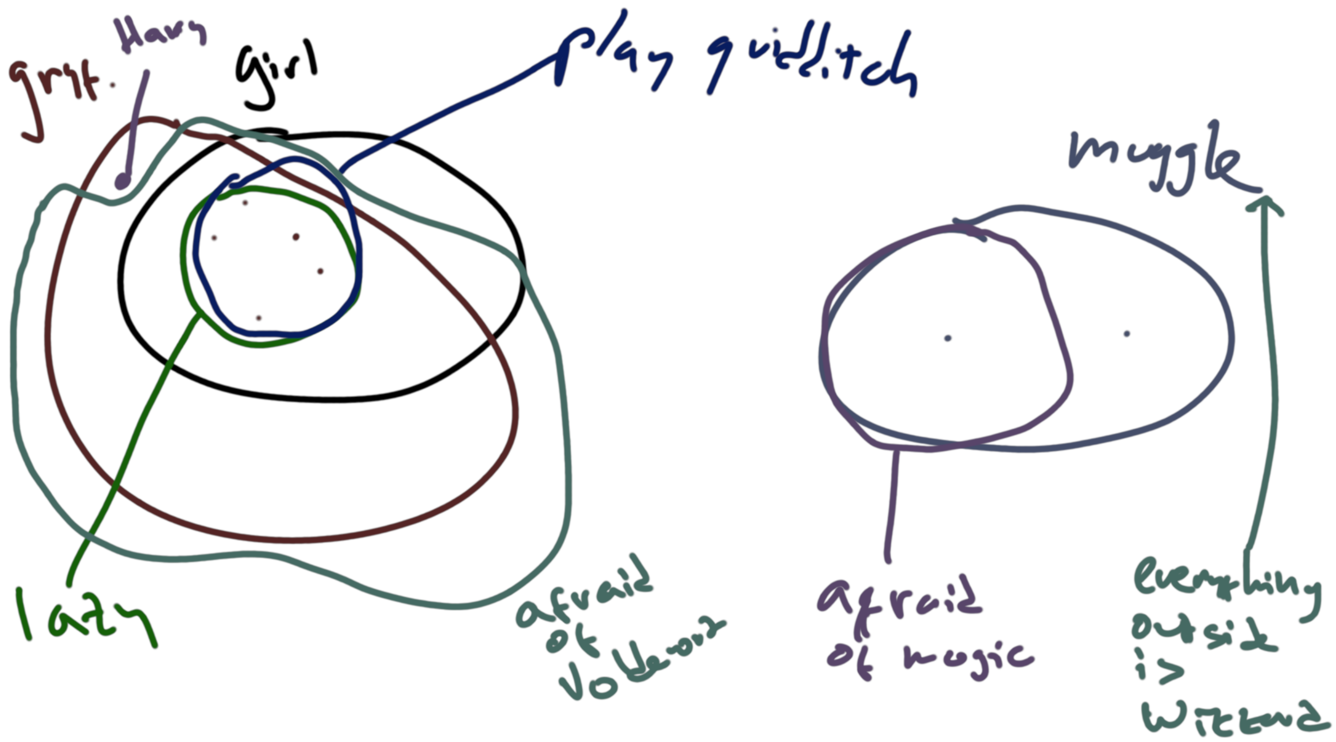
$$\llbracket \text{some but not all muggles} \rrbracket^M = \{P \subseteq U_M | \text{card}(\llbracket \text{muggle} \rrbracket \cap P) > 0 \wedge \llbracket \text{muggle} \rrbracket \cap P \neq \llbracket \text{muggle} \rrbracket\}$$
$$\llbracket \text{afraid of magic} \rrbracket^M \in \llbracket \text{some but all not muggles} \rrbracket^M \Leftrightarrow$$

c

$$\llbracket \text{at most five girls} \rrbracket^M = \{P \subseteq U_M | \text{card}(\llbracket \text{girl} \rrbracket \cap P) \leq 5\}$$
$$\llbracket \text{play quidditch} \rrbracket^M \in \llbracket \text{at most five girls} \rrbracket^M$$

d

$$\llbracket \text{five Gryffindor} \rrbracket^M = \{P \subseteq U_M | \leq \text{lower\_boundary\_few}(C) \leq \text{card}(\llbracket \text{Gryffindor member} \rrbracket \cap P) \leq \text{upper\_boundary\_few}(C)\}$$
$$\llbracket \text{is lazy} \rrbracket^M \in \llbracket \text{five Gryffindor} \rrbracket^M$$



2

a

Some but not all men *walked rapidly.* ⇒ *Some but not all* men *walked.* ⇒ Left upward
Some but not all men walked. ⇒ *Some but not all humans* walked. ⇒ Right upward (persistent)

b

At most five men walked. ⇒ *At most five* men walked rapidly. ⇒ Left downward
At most five humans walked. ⇒ *At most five* men walked. ⇒ Right downward (antipersistent)

c

Exactly five men <u>walked</u>. $\Rightarrow$ *Exactly five* men <u>walked rapidly</u>. $\Rightarrow$ Left downward
(*Exactly five* <u>men</u> walked. $\not\Rightarrow$ *Exactly five* <u>humans</u> walked.)$\wedge$(*Exactly five* <u>humans</u> walked. $\not\Rightarrow$ *Exactly five* <u>men</u> walked.)
$\Rightarrow$ Right non-monotonous

3

Let Q be an upward monotonous quantifier. Therefore:

$$\forall X, Y : X \in Q \wedge X \subseteq Y \Rightarrow Y \in Q$$

Then the following P: holds:

$$\forall X', Y' : X' \notin Q \wedge X' \subseteq Y' \Rightarrow Y' \notin Q$$

For contradiction assume that it is not the case, therefore:

$$\forall X', Y' : X' \notin Q \wedge X' \subseteq Y' \wedge Y' \in Q$$

Which is in contradiction with the premise if we substitute $X = Y'$ and $Y' = X$.
Since $Y \in Q \Leftrightarrow Y \notin \neg Q$. The statement P can therefore be rewritten as:

$$\forall X', Y' : X' \in \neg Q \wedge X' \subseteq Y' \wedge Y' \in \neg Q$$

Which describes the downward monotonicity of $\neg Q$