1

$$U = \{\text{mike}, \text{will}, \text{elli}\}$$
$$\text{atomic prep.} = \{\text{ride\_bike}(x) | x \in U\} \cup \{\text{sleep}(x) | x \in U\} \cup \{\text{tease}(x, y) | x, y \in U \times U\} =$$
$$\Big\{\text{ride\_bike}(\text{mike}), \text{ride\_bike}(\text{will}), \text{ride\_bike}(\text{elli}), \text{sleep}(\text{mike}), \text{sleep}(\text{will}), \text{sleep}(\text{elli}), \text{tease}(\text{mike}, \text{mike}), \text{tease}(\text{mike}, \text{will}),$$
$$\text{tease}(\text{will}, \text{mike}), \text{tease}(\text{will}, \text{will}), \text{tease}(\text{will}, \text{elli}), \text{tease}(\text{elli}, \text{will}), \text{tease}(\text{elli}, \text{elli}), \text{tease}(\text{elli}, \text{mike}), \text{tease}(\text{mike}, \text{elli})\Big\}$$

2

Using the preposition ordering from the above.

$$M = \Big\{$$
$$(1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0),$$
$$(1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1),$$
$$(0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0),$$
$$(0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0),$$
$$(1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0),$$
$$\Big\}$$

3

3.1

This probability/cuount should be 0. And indeed, in no model does anyone sleep and ride at the same time $\to 0$.

3.2

Mike rides in 3 models and sleeps in 2 of them. From that, $\frac{3}{5} > \frac{2}{5}$.
However, I would disagree with using the frequency of an action as a proxy for liking something. E.g. in every model the boys brush their teeth but that does not mean that they enjoy it more than riding a bike.

3.3

Elli sleeps in 5 models and rides the bike in 0 of them. From that, $\frac{5}{5} > \frac{0}{5}$.

3.4

Mike teases Will in two models and Elli in no model. From that, $\frac{2}{5} > \frac{0}{5}$.

3.5

Will never teases anybody, therefore the implication is trivially fulfilled.

Further inferences

- Nobody can tease while asleep.
- Nobody teases themselves.
- Elli doesn't tease anyone and is not teased by anyone.

4

a) $(1, 1, 0, 0, 0)^T$

b) $(1, 1, 0, 0, 1)^T$

c) $(0, 0, 0, 0, 0)^T$

d) $(0, 0, 0, 0, 0)^T$

e) $(0, 0, 1, 1, 0)^T$

5

(a) *Mike is sleeping*, (b) *A boy is sleeping.*

$p(a) = \frac{1}{5}, p(a|b) = \frac{1}{2}$, inference$(a, b) = \frac{\frac{1}{2} - \frac{1}{5}}{1 - \frac{1}{5}} = \frac{\frac{3}{10}}{\frac{8}{10}} = \frac{3}{8}$

The score is higher than 0, therefore is a boy sleeps, then it suggests that Mike is also sleeping. However, it is not 1, therefore there are some cases in which some boy is sleeping but not Mike. With more samples the inference would probably be higher.