# Adaptive Differential Evolution by Adjusting Subcomponent Crossover Rate for High-Dimensional Waveform Inversion

Zhibin Pan, Junjie Wu, Zhaoqi Gao, and Jinghuai Gao

***Abstract*—In this letter, a new adaptive differential evolution (DE) for high-dimensional waveform inversion is proposed. In conventional DE algorithms, individuals are treated as a whole and share the same fitness function and parameters. However, conventional DE algorithms have ignored the huge difference among the subcomponents in an individual and are not effective for high-dimensional problems. Therefore, for high-dimensional problems, we expand the unit of crossover rate from the whole individual to its subcomponents and propose a new adaption algorithm by adjusting the crossover rate of each subcomponent. In our algorithm, both kinds of crossover rate, including individual crossover rate and subcomponent crossover rate, play important roles in crossover operation. Based on local fitness function, the subcomponent crossover rate is adaptively obtained to improve the efficiency of crossover operation. On the other hand, the individual crossover rate is used to prevent the population diversity from decreasing in crossover operation. We embed the adaption algorithm into cooperative coevolutionary DE (CCDE) and propose a new adaptive DE by adjusting the subcomponent crossover rate named CRsADE. We have conducted experiments on waveform inversion to test the performance of the proposed algorithm. The results show that CRsADE performs better than CCDE significantly both on convergence speed and accuracy. In order to estimate the validity of CRsADE, we have also applied it to real seismic data.**

***Index Terms*—Cooperative coevolutionary differential evolution (CCDE), differential evolution (DE), individual crossover rate, subcomponent crossover rate, waveform inversion.**

## I. Introduction

Waveform inversion is an effective technique widely used to investigate the properties of the shallow subsurface in Earth geophysics. Waveform inversion aims to find the most probable Earth model matching with the observed data; thus, it can be considered as an optimization problem. Two basic optimization approaches (local and global search algorithms) can be used to solve the inversion problem.

Local search recovers the practical Earth model by means of gradient-based optimization algorithms. They are fast realized, but their performance strongly relies on the selection of the initial model [1]. On the other hand, the global search algorithms, such as evolutionary programming [2], genetic algorithm [3], and particle swarm optimization [4], are computationally robust and do not need an accurate initial model, but they are restricted by their large amount of calculations. Due to the rapid development of computer technology, global search methods are becoming more and more popular in waveform inversion.

Differential evolution algorithm (DE) [5], [6] is one kind of global search methods successfully combining optimization and biological evolution. It is prominent for its simple implementation and relatively fast convergence. Recently, DE has been successfully applied in the waveform inversion problem and performed well [7]. However, when applied to high-dimensional problems, the conventional DE could hardly obtain global minima and the best matching model. Therefore, cooperative coevolutionary approach was proposed by Potter and De Jong [8] as an explicit means of problem decomposition in evolutionary algorithms. It was proven to be efficient for separable or weak nonseparable problems in [9] but was still poor in strong nonseparable problems.

To solve high-dimensional waveform inverse problem effectively, Wang and Gao proposed a novel cooperative coevolutionary DE algorithm (CCDE) by adopting a new mutation operator based on the introduction of local fitness function which could well represent the quality of subcomponents after decomposing [10]. It was proved that CCDE could solve the high-dimensional waveform inverse problem effectively. However, the performance of CCDE was not really satisfactory and was still time-consuming. To accelerate the speed, Gao *et al.* [11] introduced a new population evolution strategy into CCDE, which was used for tuning the population size. In this letter, we propose a new adaptive DE by adjusting the subcomponent crossover rate in high-dimensional problems to enhance the efficiency of crossover operation in the evolution and accelerate convergence further.

The remainder of this letter is organized as follows. The theory of waveform inversion and DE is reviewed in Section II. Section III describes our proposed algorithm named as adaptive DE by adjusting the subcomponent crossover rate (CRsADE). Experiments and applications are presented in Section IV. Section V concludes this letter.

## II. Review of Waveform Inversion and CCDE

### A. Waveform Inversion

Given the observed data from a seismic survey, waveform inversion aims to find the most probable Earth model matching with the data. Waveform inversion can be regarded as the

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inverse process of wavefield modeling. The process of wavefield modeling is generally denoted as follows:

$$\mathbf{d} = \mathbf{g}(\mathbf{m}) \tag{1}$$

where $\mathbf{d}$ denotes the synthetic data, $\mathbf{m}$ is the model physical parameter under the surface, and $\mathbf{g}$ is the forward operator.

Waveform inversion can also be considered as to find the best Earth model $\mathbf{m}$ whose synthetic data best matching with the observed data. It is often treated as an optimization problem whose objective function measures the difference between the synthetic data and the observed data.

For simplicity, we restrict the study to the problem of a 1-D model, which is divided into $N$ layers. The objective function of waveform inversion can be written as

$$f(\mathbf{m}) = \sum_{r-1}^{NR} \int |d_{o,r}(t) - d_{c,r}(t, \mathbf{m})| \, dt \tag{2}$$

where $d_{o,r}(t)$ denotes the real observed data in the $r$th receiver; $d_{c,r}(t, \mathbf{m})$ denotes the synthetic data in the $r$th receiver according to the Earth model $\mathbf{m}$ ($\mathbf{m} = [v_1, v_2, \ldots, v_N]$, with $v_i$ being the velocity of the $i$th layer); NR is the number of receivers; and $t$ is the travel time.

Due to the strong nonlinear of waveform inversion, it is promising for global optimization methods to solve this problem. In addition, the number of layers in the Earth model is always very large, so high-dimensional optimization algorithms are becoming the focus in waveform inversion.

### B. DE

DE is a population-based metaheuristic algorithm that has earned wide publicity for its simple structure with fewer parameters and its excellent performance in numerical optimization [12].

DE involves a population of NP individuals, and each individual is a $D$-dimensional vector as shown in the following:

$$X_i^G = \left\{x_{i,1}^G, x_{i,2}^G, \ldots, x_{i,D}^G\right\}, \quad i = 1, 2, \ldots, NP \tag{3}$$

where $G$ denotes the generation. The main flow of DE involves four operations.

1) Initialization: The initial population should cover the entire search space. Thus, the $j$th parameter of the $i$th individual at $G = 0$ is given by

$$x_{i,j}^0 = x_{\min,j} + \text{rand}(0,1) \times (x_{\max,j} - x_{\min,j}) \tag{4}$$

where rand(0,1) represents a uniformly distributed random variable within the range [0,1] and $x_{\min,j}$ and $x_{\max,j}$ are, respectively, the upper and lower search bounds for the $j$th parameter.

2) Mutation: After initialization, DE creates mutant individuals by imposing disturbance on the population

$$V_i^G = X_{r1}^G + F \times \left(X_{r2}^G - X_{r3}^G\right) \tag{5}$$

where the indices $r_1$, $r_2$, and $r_3$ are mutually exclusive integers randomly generated within the range [1, NP], which are also different from the index $i$. The mutation strategy showed previously is called DE/rand/1.

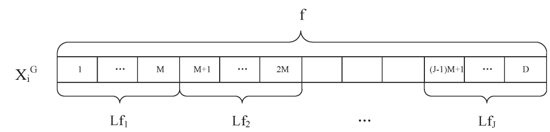


Fig. 1. Illustration of decomposition strategy and local fitness function of CCDE. In the figure, $f$ denotes the global fitness function of individual $X_i^G$, and $Lf_j$ denotes the local fitness function associated to the subcomponent $X_{i,j}^G$.

3) Crossover: Crossover operation is to obtain the candidate individual $U_i^G$ by getting its variable from $V_i^G$ or $X_i^G$:

$$u_{i,j}^G = \begin{cases} v_{i,j}^G, & \text{if } (\text{rand}(0,1) < CR) \text{ or } (j = j_{\text{rand}}) \\ x_{i,j}^G, & \text{otherwise} \end{cases} \tag{6}$$

where $u_{i,j}^G$ is the $j$th parameter of the $i$th candidate individual at generation $G$ and $j_{\text{rand}}$ is a randomly chosen integer from [1, D]. The crossover rate CR is chosen from [0, 1].

4) Selection: For a given objective function $f(\mathbf{m})$ to be minimized, the selection operation is described as

$$X_i^{G+1} = \begin{cases} U_i^G, & \text{if } f\left(U_i^G\right) \le f\left(X_i^G\right) \\ X_i^G, & \text{otherwise.} \end{cases} \tag{7}$$

Steps 2–4 are repeated until a stopping criterion is reached.

There are three control parameters of DE: $F$, CR, and NP. The parameters $F$ and CR largely affect the algorithm's searching ability and convergence speed. Therefore, some adaptive DE algorithms, like SaDE [13], have been proposed. In SaDE, the value of CR is gradually self-adapted by learning from previous experiences in generating promising solutions. These algorithms can perform excellent in low-dimensional problem but still work poorly in high-dimensional problem.

### C. CCDE

DE handles an individual as a whole and fails to perform well for large-scale problems. To decompose the high-dimensional problem into several small ones, Potter and De Jong firstly introduced the strategy of cooperative coevolutionary (CC) into evolutionary algorithms [10]. Recently, Wang and Gao proposed CCDE [10] by introducing the concept of local fitness function. As shown in Fig. 1, a $D$-dimensional individual is decomposed into $J$ $M$-dimensional subcomponents ($D = J \cdot M$). The local fitness function $Lf_j$ is expected to effectively represent the quality of the $j$th subcomponent. Based on the local fitness function, CCDE adopted a new mutation operation and was very effective when solving high-dimensional waveform inversion problem.

In the application of CCDE in waveform inversion, the local fitness function is defined as follows:

$$Lf_j(\mathbf{m}) = \sum_{r=1}^{NR} \int |[d_{o,r}(t) - d_{c,r}(t, \mathbf{m})] \times \text{win}_j(t)| \, dt \tag{8}$$

where $\text{win}_j(t)$ denotes a window function and is defined according to the specific problem [10].

## III. Proposed CRSADE Algorithm

As shown in Section II, the performance of DE is very sensitive to the value of $F$ and CR. In SaDE, the value of CR is

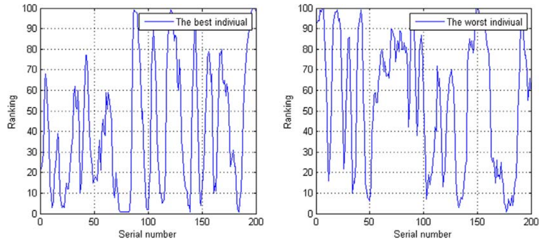


Fig. 2. Performance of all of the subcomponents in the best and worst individuals at generation 10. The $x$-axis is the serial number of the subcomponents. The $y$-axis is the ranking of the subcomponents, which is based on local fitness function among the whole population.

gradually self-adapted according to the previous experiences. However, the algorithm just treats the individual as a whole in the crossover operation, and each subcomponent of an individual shares the same CR. For high-dimensional problems, the difference among the subcomponents of an individual may be very large. It is ineffective and improper to adopt the same CR for all of the subcomponents in an individual. We have conducted a verification experiment to show the large difference among the subcomponents in an individual.

In the experiment, we adopt CCDE to solve the waveform inversion problem [10]. The dimension of the problem is set to 200, and the population size NP is 100. The $j$th subcomponent in the $i$th individual is denoted as $X_{i,j}$, and its local fitness value is $Lf(i,j)$. We construct a set $Lf(j) = \{Lf(1,j), Lf(2,j), \ldots, Lf(NP,j)\}$ as the $j$th subcomponent's local fitness set in the whole population. The local fitness set is sorted by the local fitness values in an increasing order, and the performance of $X_{i,j}$ is represented by the ranking of $Lf(i,j)$ in the set $Lf(j)$. That is to say, superior subcomponents have upper rankings. We picked two most representative individuals, the best and worst individuals, to observe their subcomponents in the evolution. The performance of all of the subcomponents in the two individuals at generation 10 is plotted in Fig. 2. From Fig. 2, we can find that the difference among the subcomponents in an individual is very large. The best individual may have a worst subcomponent, and the worst individual may have a best subcomponent. Therefore, if we adopt evolution strategy and parameters that best fit each subcomponent, the algorithm would become more effective.

### *A. Adaption Strategy*

In a conventional DE, the crossover operation is a combination of mutant individuals and target individuals. CR determines the average number of subcomponents inheriting from the target individuals in the candidate individuals and is equal for each subcomponent in an individual. As shown in Fig. 2, the difference among the subcomponents of an individual is actually very large. Taking the exploitation ability of the algorithm into consideration, in a target individual, we prefer those superior subcomponents with better local fitness values to enter the candidate individual rather than inferior ones with worse local fitness values. Therefore, it is unreasonable to assign all of the subcomponents the same crossover rate.

For superior subcomponents in the target individual, a lower CR can enlarge their probabilities of entering the candidate individual. It is beneficial for obtaining better candidate individual and improving the exploitation ability of the algorithm. On the other hand, for inferior subcomponents, their performance can hardly exceed the corresponding ones' in the mutant individual. At this point, a higher CR can give the subcomponents in the mutant individual larger opportunity and thus improve the quality of candidate individual.

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However, it is worth noting that, if the choice of subcomponent crossover rate excessively benefits for exploitation, the loss of population diversity would happen. As the loss goes on, it may probably lead to premature convergence, which is a fatal defect for the global optimization algorithm.

To avoid the shortcoming described previously, we simultaneously use two kinds of crossover rate: individual crossover rate (CRi) and subcomponent crossover rate (CRs). The parameter CRs is the crossover rate of the subcomponent in an individual. The parameter CRi is defined the same as CR in a conventional DE except that CRi does not directly control the crossover operation but control the value of CRs. In our algorithm, each individual obtains its CRi according to the process of evolution using parameter adaption strategy like SaDE, while CRs is adaptively controlled by both CRi and local fitness function. The role of CRi is to control the average number of subcomponents inheriting from mutant individual, thus ensuring sufficient population diversity. The local fitness function is used to ensure that subcomponents will obtain their crossover rates based on their quality. CRi and CRs are combined to cooperatively control the process of evolution.

Based on the aforementioned idea, we propose a new adaption strategy for CRs here. The value of CRi corresponds to the $i$th individual $X_i$ that is produced using the SaDE algorithm and is denoted as $\mathrm{CRi}(i)$. The value of CRs corresponds to subcomponent $X_{i,j}$ that is denoted as $\mathrm{CRs}(i,j)$ and produced by the following equation:

$$\mathrm{CRs}(i,j) = \mathrm{CRi}(i) \times \frac{Lf(i,j) - Lf(j)_{\min}}{Lf(j)_{\max} - Lf(j)_{\min}} \times F_c(i,j) \quad (9)$$

where $Lf(j)_{\min}$ and $Lf(j)_{\max}$, respectively, denote the minimum and maximum values of the local fitness function in the set $Lf(j) = \{Lf(1,j), Lf(2,j), \ldots, Lf(NP,j)\}$; $(Lf(1,j) - Lf(j)_{\min})/(Lf(j)_{\max} - Lf(j)_{\min})$ is the proportion of the local fitness of $X_{i,j}$ in $Lf(j)$ and denotes the relative quality of subcomponent $X_{i,j}$; and $F_c(i,j)$ is the controllable gain factor selected based on subcomponents' local fitness. Of course, superior subcomponents are expected to have smaller gain factors, while inferior ones have larger ones.

We partition the subcomponents in an individual into three segments: inferior part, normal part, and superior part. These three segments are divided by the ranking of local fitness function values. For subcomponent $X_{i,j}$, the ranking is obtained by sorting $Lf(j)$ and denoted as $r_{i,j}$.

We define two parameters $r_{\min}$ and $r_{\max}$ to be the lower and upper limits of the normal segment. Subcomponents whose ranking are lower than $r_{\min}$ or higher than $r_{\max}$ are, respectively, classified into superior and inferior segments. The choice of $r_{\min}$ and $r_{\max}$ should comply with the following principle: the number of subcomponents in the normal segment is expected to be approximately equal to the sum of the other two segments. We define $F_c(i,j)$ in (9) according to the three segments

$$F_c(i,j) = \begin{cases} \alpha, & \text{if } (r_{i,j} < r_{\min}) \\ \beta, & \text{if } (r_{i,j} > r_{\max}) \\ \gamma, & \text{otherwise.} \end{cases} \quad (10)$$

The aforementioned equation is a piecewise function. $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ are different gain factors for the subcomponents in three

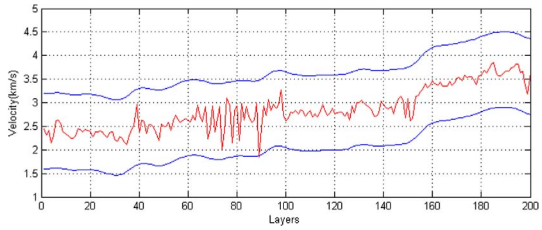


Fig. 3. Real velocity model (red) and the upper and lower bounds of the search space (blue).

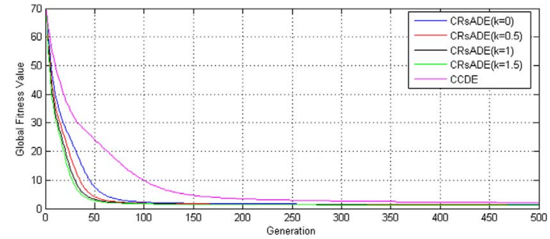


Fig. 4. Best global fitness values in each generation in the evolution process of CRsADE ($k = 0$, $k = 0.5$, $k = 1$, and $k = 1.5$) and CCDE on waveform inversion.

segments. These gain factors are set to reflect the difference among three segments. The values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ generally satisfy the rule $\boldsymbol{\alpha} \leqslant \boldsymbol{\gamma} \leqslant \boldsymbol{\beta}$ so that superior subcomponents can have smaller CRs and inferior subcomponents have larger CRs.

The proposed algorithm shares the same mutation and selection operations with CCDE. In the following tests, we will show that the CRsADE algorithm prominently improves both the convergence speed and evolution quality.

## IV. Waveform Inversion Experiments and Its Applications on Real Seismic Data

In this letter, all of the experiments use the same objection function and local fitness function as defined in (2) and (8). The inverse problem involves 200 layers, and each subcomponent contains one layer in our experiment. The length of window $\text{win}_j(t)$ in (8) is chosen to be three times of the length of the subcomponent plus the width of the wavelet. In the evolution, the population size NP is set to 100. The mutation scale $F$ and individual crossover rate CRi are produced by SaDE. We use $G_{\max} = 500$ as the stopping criterion. All experiments are run 35 times independently.

There are five extra parameters: $\alpha$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, $r_{\min}$, and $r_{\max}$ in CRsADE. $r_{\min}$ and $r_{\max}$ are chosen to define the boundary of the three segments: inferior part, normal part, and superior part. We hope that the number of subcomponents in the normal part and that in the other two parts will be equal. Therefore, we set $r_{\min} = 0.25 \times NP$ and $r_{\max} = 0.75 \times NP$ here. For $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, because the roles of the inferior part and superior part are absolutely equal in the evolution, we set $\boldsymbol{\alpha} = \boldsymbol{\gamma} - k$, $\boldsymbol{\beta} = \boldsymbol{\gamma} + k$ in the experiments. To rationally control the average number of subcomponents in the mutant individual entering the next generation, we set $\boldsymbol{\gamma} = 2$ to ensure that the subcomponent whose value of local fitness is at the average level $(Lf(i,j) = (Lf(j)_{\min} + Lf(j)_{\max})/2)$ is assigned the same value with CRi(i).

The value of $k$ represents how much CRs relies on the quality of subcomponent in the target individual, i.e., the choice of $k$ affects the algorithm's exploitation ability. In order to find the best parameter $k$, we have conducted the following tests.

The proposed algorithm CRsADE has been compared with CCDE using a velocity model taken from a real well log as shown in Fig. 3. To test the influence of parameter $k$, we set $k = 0, 0.5, 1, \text{and } 1.5$ in the experiments. The convergence speed and final quality of the objective function value are both tested. The performance is shown in Fig. 4 and Table I.

It can be observed from Fig. 4 that the convergence speed of CRsADE is prominently faster than CCDE. Furthermore, the data in Table I show that the final quality of the object function value of CRsADE is also better than CCDE to a large extent. The highest improvement ratio of CRsADE compared to CCDE is 42.01%, and the lowest is 32.86%. The best choice of $k$ for CRsADE is 1.5. Moreover, in this case, CCDE costs 500 generations to make the global fitness function value reach 2.11, while CRsADE only costs 66 generations. It is a significant improvement for the convergence speed of the waveform inversion.

TABLE I
Performance Comparison of CCDE and CRsADE With Different $k$ at the Final Generation 500 [GF: Global Fitness Value; IR: Improvement Ratio (Compared to CCDE)]

| | CCDE | k=0 | k=0.5 | k=1 | k=1.5 |
|---|---|---|---|---|---|
| GF | 2.1100 | 1.4167 | 1.3436 | 1.2370 | 1.2236 |
| IR(%) | 0 | 32.86 | 36.32 | 41.37 | 42.01 |

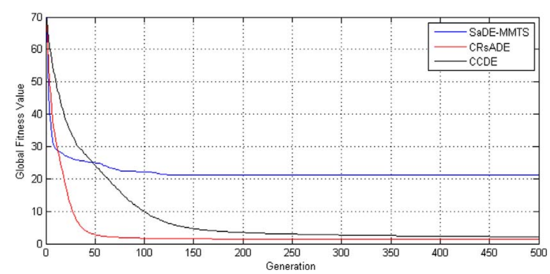


Fig. 5. Best global fitness values in each generation in the evolution process of SaDE-MMTS, CRsADE, and CCDE on waveform inversion.

We have also compared CRsADE with other recent adaptations of DE for the high-dimensional problem like SaDE-MMTS [14]. It is a prominent adaptive large-scale DE algorithm by incorporating SaDE, JADE mutation strategy, and multitrajectory search. The experimental condition is the same as the previous experiment. The convergence behaviors are shown in Fig. 5. The convergence speeds of SaDE-MMTS and CRsADE are similar, but CRsADE performs superior to SaDE-MMTS on the solution quality.

To intuitively observe the inversion performance, we have shown the comparison of the velocity model inverted by CCDE and CRsADE (terminated at generations 500 and 100) in Fig. 6(a) and (b). The inverted velocity model by CCDE and CRsADE both match well with the real data. However, it is obvious that the model inverted by CRsADE which is terminated at generation 500 is more accurate than that of CCDE in the same condition. Furthermore, the performance of CRsADE at generation 100 can achieve nearly the same performance as that of CCDE at generation 500.

To conform the validity of the proposed algorithm further, CRsADE with $k = 1.5$ is applied to real seismic data as shown in Fig. 7(a). The data are stacked data whose trace number is 430 and time length is 210 ms. The Earth model is divided into 67 layers, and each layer contains 3-ms travel time. The density is assumed to be constant. The population size NP is

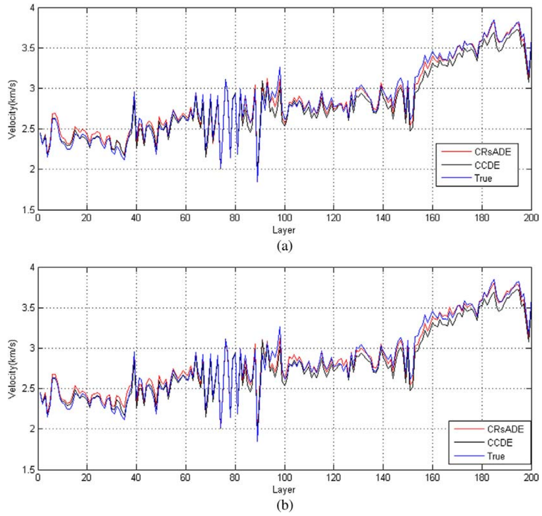


Fig. 6. Comparison of the inverted velocity by CCDE and CRsADE with the real velocity under different termination conditions. (a) CCDE and CRsADE are all terminated at generation 500. (b) CCDE is terminated at generation 500, while CRsADE is terminated at generation 100.

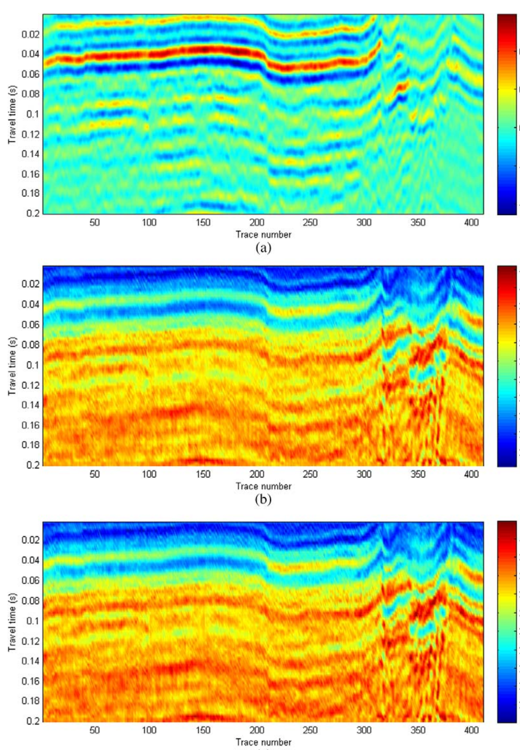


Fig. 7. (a) Real seismic data. (b) Inverted velocity by CRsADE terminated at generation 100. (c) Inverted velocity by CCDE terminated at generation 500.

set to 50, and the evolution stops at generation 100. We also adopt CCDE for comparison. The parameters $F$ and CRi are both produced by SaDE in the two algorithms. In particular, CCDE is terminated at generation 500. The results are shown in Fig. 7(b) and (c).

From Fig. 7, we can find that both CCDE and CRsADE well exhibit the Earth geological features. In addition, CRsADE costs only 100 generations to reach the performance of CCDE at generation 500. Thus, in this simulation, the CRsADE method outperforms CCDE.

## V. Conclusion

In this letter, a new adaptive DE algorithm named CRsADE has been proposed for high-dimensional waveform inversion. In consideration of the huge difference among the subcomponents in an individual, the proposed algorithm expands the unit of crossover rate from individual to subcomponent and introduces a new adaptive strategy for subcomponent crossover rate. Based on the local fitness function, the adaptive strategy improves the algorithm's exploitation ability by assigning the superior components a smaller crossover rate and the inferior components a larger crossover rate. In addition, to ensure sufficient population diversity in the evolution, the strategy of cooperation control of individual and subcomponent crossover rate is introduced. The individual crossover rate is assigned for each individual according to the process of evolution. This cooperation strategy aims to control the average number of subcomponents inheriting from the mutant individuals.

Experiments show that CRsADE is very effective and has prominent advantages of both convergence speed and accuracy for waveform inversion. Compared to CCDE, CRsADE can obtain inversion results with an improvement of approximately 40% in precision and reduce the convergence time by 80%. Furthermore, compared to other high-dimensional adaption DE algorithms like MMTS-SaDE, CRsADE can also perform better.

## References

[1] F. A. Belina, J. R. Ernst, and K. Holliger, "Inversion of crosshole seismic data in heterogeneous environments: Comparison of waveform and ray-based approaches," *J. Appl. Geophys.*, vol. 68, no. 1, pp. 85–94, May 2009.

[2] C. D. Groot-Hedlin and F. L. Vernon, "An evolutionary programming method for estimating layered velocity structure," *Bull. Seismol. Soc. Amer.*, vol. 88, no. 4, pp. 1023–1035, Aug. 1998.

[3] R. Govindan, R. Kumar, S. Basu, and A. Sarkar, "Altimeter-derived ocean wave period using genetic algorithm," *IEEE Geosci. Remote Sens. Lett.*, vol. 8, no. 2, pp. 354–358, Mar. 2011.

[4] R. Shaw and S. Srivastava, "Particle swarm optimization: A new tool to invert geophysical data," *Geophysics*, vol. 72, no. 2, pp. 75–83, Mar. 2007.

[5] R. Storn and K. Price, "Differential evolution—A simple and efficient heuristic for global optimization over continuous spaces," *J. Global Optim.*, vol. 11, no. 4, pp. 341–359, Dec. 1997.

[6] S. Das and P. N. Suganthan, "Differential evolution: A survey of the state-of-the-art," *IEEE Trans. Evol. Comput.*, vol. 15, no. 1, pp. 4–31, Feb. 2011.

[7] Z. W. Li, T. Y. Hao, Y. Xu, Y. Xu, and S. Roecker, "A global optimizing approach for waveform inversion of receiver functions," *Comput. Geosci.*, vol. 36, no. 7, pp. 871–880, Jul. 2010.

[8] M. Potter and K. De Jong, "A cooperative coevolutionary approach to function optimization," in *Proc. 3rd Conf. Parallel Probl. Solving Nature*, 1994, vol. 2, pp. 249–257.

[9] Y. Liu, X. Yao, Q. Zhao, and T. Higuchi, "Scaling up fast evolutionary programming with cooperative coevolution," in *Proc. IEEE Congr. Evol. Comput.*, May 2001, vol. 2, pp. 1101–1108.

[10] C. Wang and J. H. Gao, "High-dimensional waveform inversion with cooperative coevolutionary differential evolution algorithm," *IEEE Geosci. Remote Sens. Lett.*, vol. 9, no. 2, pp. 297–301, Mar. 2012.

[11] Z. Gao, Z. Pan, and J. H. Gao, "A new highly efficient differential evolution scheme and its application to waveform inversion," *IEEE Geosci. Remote Sens. Lett.*, vol. 11, no. 10, pp. 1702–1706, Oct. 2014.

[12] P. Rakshit *et al.*, "Realization of an adaptive memetic algorithm using differential evolution and Q-learning: A case study in multirobot path planning," *IEEE Trans. Syst., Man, Cybern. A, Syst.*, vol. 43, no. 4, pp. 814–831, Jul. 2013.

[13] A. K. Qin, V. L. Huang, and P. N. Suganthan, "Differential evolution algorithm with strategy adaptation for global numerical optimization," *IEEE Trans. Evol. Comput.*, vol. 13, no. 2, pp. 398–417, Apr. 2009.

[14] S. Z. Zhao, P. N. Suganthan, and S. Das, "Self-adaptive differential evolution with multi-trajectory search for large-scale optimization," *Soft Comput.*, vol. 15, pp. 2175–2185, Nov. 2011.