Modeling Market Dynamics with Nondeterministic Finite Automata: An Agent-Based Approach to Bounded Rationality Under Uncertainty

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***Abstract*—Traditional equilibrium-based economic models often fail to capture the complexity of real-world markets characterized by uncertainty, bounded rationality, and emergent behavior. Our project presents an agent-based model where decision-making is governed by nondeterministic finite automata (NFA), enabling agents to probabilistically transition between strategic states. Implemented using the MESA framework, our model explores how nondeterministic decision processes affect wealth distribution, cooperation levels, and inequality in a simulated market economy. Comparative experiments between deterministic and nondeterministic methods reveal significant differences in emergent outcomes, demonstrating that uncertainty fundamentally shapes population-level dynamics. Time-series analysis, spatial visualizations, and animations illustrate clustering patterns, strategy diffusion, and inequality formation. While the model provides insights into adaptive behavior under uncertainty, challenges remain in scalability, cognitive enrichment, and empirical validation.**

***Index Terms*—agent-based modeling, nondeterministic finite automata, bounded rationality, market dynamics, computational economics**

I. Introduction

Real-world economic systems exhibit complex dynamics that defy simple analytical characterization. Markets are populated by agents operating under uncertainty and bounded rationality, making decisions based on incomplete information and limited computational capacity [1], [2]. Traditional economic models, with equilibrium assumptions and perfect rationality, struggle to explain phenomena such as market volatility, wealth inequality, emergent cooperation, and strategic adaptation.

Agent-based modeling (ABM) has emerged as a powerful paradigm for studying such systems, enabling researchers to simulate interactions among heterogeneous agents and observe macro-level patterns arising from micro-level behaviors [3]. Prior work has demonstrated that decentralized agent interactions can generate complex outcomes including business cycles, asset bubbles, and spontaneous cooperation [4]. However, most existing agent-based economic models employ deterministic strategies, limiting their ability to capture the inherent uncertainty and probabilistic nature of human decision-making.

Our project addresses the gap by incorporating nondeterministic finite automata (NFA) as the cognitive architecture for agents in a market economy simulation. Each agent's decision process is presented as an NFA, allowing probabilistic transitions between strategic states such as cooperation, defection, negotiation, and withdrawal based on interaction outcomes. This approach bridges automata theory and computational economics, providing a rigorous framework for modeling bounded rational agents under uncertainty.

Our contributions include: (1) a formal NFA-based agent model for economic decision-making, (2) implementation of a multi-agent market simulation using the MESA framework, (3) comparative analysis of deterministic versus nondeterministic transition regimes, and (4) visualization of emergent spatial and temporal patterns in wealth distribution and strategic behavior.

II. Background and Related Work

A. Agent-Based Modeling in Economics

Agent-based computational economics (ACE) studies economic processes as dynamic systems of interacting agents [3]. Foundational models such as Sugarscape [2] and the El Farol Bar problem [1] demonstrated how simple agent rules can produce complex aggregate behavior. ABM has been applied to diverse domains including market microstructure, industrial organization, and macroeconomic policy analysis.

B. Bounded Rationality and Finite-State Models

The concept of bounded rationality, that was introduced [5], recognizes that decision-makers face cognitive limitations and information constraints. Finite-state automata provide a natural formalization of bounded rationality, representing agents with limited memory and computational capacity [6]. Deterministic finite automata (DFA) have been used to model repeated game strategies, but such models lack the flexibility to represent uncertain or exploratory behavior.

C. Nondeterminism in Decision Models

Nondeterministic finite automata extend DFA by allowing multiple possible transitions from a given state on the same input, introducing probabilistic choice into the decision process. While NFA are well-studied in formal language theory, their application to economic agent modeling remains relatively unexplored. Recent work in multi-agent systems has shown that stochastic strategies can lead to different equilibrium outcomes and robustness properties [7].

III. Model Design and Formalization

A. Agent Architecture

Each agent in our model is endowed with a nondeterministic finite automaton defined by the 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$ where:

- $Q = \{C, D, N, W\}$ is a finite set of states representing Cooperation, Defection, Negotiation, and Withdrawal
- $\Sigma = \{\text{high}, \text{medium}, \text{low}\}$ is the input alphabet representing payoff signals from interactions
- $\delta : Q \times \Sigma \to 2^Q$ is the nondeterministic transition function mapping state-input pairs to sets of possible next states
- $q_0 \in Q$ is the initial state
- $F = \emptyset$ (no accepting states, reflecting ongoing adaptive behavior)

The nondeterministic transition function allows agents to probabilistically select among multiple successor states, modeling uncertainty and exploration in strategic choice. For example, an agent in state C receiving a *low* payoff signal might transition to D with probability 0.6 or remain in C with probability 0.4.

B. Interaction Mechanism

At each simulation step, agents are randomly paired for mutual interactions. The payoff from an interaction depends on the strategic states of both participants, with the following structure:

- Mutual cooperation (C, C): both receive high payoffs
- Cooperation vs. defection (C, D): cooperator receives low payoff, defector receives very high payoff
- Mutual defection (D, D): both receive medium-low payoffs
- Negotiation and withdrawal states introduce intermediate outcomes

Agents update their wealth based on interaction payoffs and use payoff signals as inputs to their NFA, determining strategic state transitions for subsequent rounds.

C. Simulation Environment

The model is implemented using the MESA framework for agent-based modeling in Python. Agents are placed on a spatial grid, enabling visualization of geographic clustering and strategy diffusion. Key simulation parameters include population size, initial wealth distribution, NFA transition probabilities, and interaction frequency.

IV. Experimental Design and Results

A. Experimental Setup and Data Collection

Experiments were conducted comparing deterministic and nondeterministic transition rules in the NFA-based market simulation. The main comparison involved 50 agents over 200 time steps, with each agent starting with 100 units of resources. A separate spatial visualization experiment was run with 30 agents on a 10×10 grid over 100 time steps to demonstrate emergent spatial patterns and agent interactions. Metrics collected include wealth distribution (Gini coefficient), cooperation rates (fraction of agents in each of the four NFA states: COOPERATE, DEFECT, NEGOTIATE, WITHDRAW), resource trajectories, and spatial autocorrelation of strategies.

B. Wealth Distribution and Inequality

Analysis of wealth trajectories reveals stark differences between deterministic and nondeterministic regimes. Both models began with perfectly equal resource distribution (initial Gini = 0.0000, initial average resources = 100.00 for both). Deterministic Model Wealth Dynamics:

- Final average resources: 1294.00 units
- Final Gini coefficient: 0.0208 (extremely low inequality)
- Resource volatility (standard deviation): 346.41
- Resource range: 100.00 to 1294.00

Nondeterministic Model Wealth Dynamics:

- Final average resources: 815.06 units
- Final Gini coefficient: 0.0363 (moderate inequality, 1.74× higher than deterministic)
- Resource volatility (standard deviation): 205.80
- Resource range: 100.00 to 815.06

Key Finding: The deterministic model accumulated significantly more aggregate wealth (478.94 units higher, or 59% more than nondeterministic), while the nondeterministic model exhibited higher inequality (Gini difference of +0.0155). Despite higher individual volatility in the deterministic regime (Std Dev = 346.41 vs 205.80), the deterministic model's superior resource accumulation is driven by sustained cooperation. The nondeterministic model's lower aggregate wealth suggests that probabilistic strategy switching inhibits mutually beneficial cooperation cycles, resulting in less efficient resource generation over the 200-step horizon.

C. Cooperation Dynamics and State Distribution

Time-series analysis reveals fundamental differences in how the two regimes evolve behaviorally over 200 time steps.

Deterministic Regime - Convergence to Pure Cooperation:

- Initial cooperation rate: 0% (random initialization)
- Final cooperation rate: 100.00%
- Average cooperation over entire run: 100.00%
- Final state distribution: 100% Cooperate, 0% Defect, 0% Negotiate, 0% Withdraw
- Convergence pattern: Rapid convergence to all-cooperate equilibrium by early simulation steps

Nondeterministic Regime - Persistent Oscillation and Diversification:

- Initial cooperation rate: 0% (random initialization)
- Final cooperation rate: 4.00%
- Average cooperation over entire run: 10.32%
- Final state distribution: 4% Cooperate, 2% Defect, 34% Negotiate, 60% Withdraw
- Convergence pattern: No convergence; agents remain distributed across all four states with dominant withdrawal behavior

Critical Observation: The 96.00 percentage-point difference in cooperation rates (-96.00%) demonstrates that nondeterministic transitions prevent convergence to the high-cooperation equilibrium. Instead of settling into stable cooperation like deterministic agents, nondeterministic agents exhibit persistent state oscillations. The dominance of the WITHDRAW state (60% of agents) in the final nondeterministic distribution suggests that probabilistic strategy selection allows agents to repeatedly explore low-engagement states, preventing them from locking into mutually beneficial cooperation. The maintenance of 34% NEGOTIATE agents further indicates ongoing exploration and lack of equilibrium settlement. This contrasts sharply with the deterministic regime's complete fixation on cooperation, indicating that determinism enables a form of "lock-in" to profitable strategies while nondeterminism maintains behavioral diversity at the cost of economic efficiency.

D. Spatial Patterns and Emergent Behavior

Spatial visualization on a 10×10 grid with 30 agents over 100 time steps reveals how nondeterministic transitions affect agent clustering and strategy diffusion. Spatial Model Final State Distribution (30 agents, 100 steps):

- Cooperate agents: 2 (6.7%)
- Defect agents: 6 (20.0%)
- Negotiate agents: 7 (23.3%)
- Withdraw agents: 15 (50.0%)
- Final average resources: 457.37 units
- Final cooperation rate: 6.67%

Visual analysis of agent positions and state colors reveals that nondeterministic agents produce significantly more heterogeneous spatial distributions compared to the static patterns expected in deterministic regimes. The spatial model replicates key findings from the 50-agent non-spatial experiments: cooperation collapses to 6.67% (compared to 4% in the non-spatial nondeterministic run), while withdrawal dominates at 50% of the population. The presence of 20% defectors and 23.3% negotiators indicates that agents explore diverse behavioral strategies across space rather than converging to monolithic regions. Animation sequences demonstrate that nondeterministic agents exhibit persistent waves of strategy transitions propagating through the spatial grid. The heterogeneous final state distribution suggests that spatial proximity does not promote convergence to cooperation in the nondeterministic case, contrary to typical spatial game theory expectations. The distributed pattern of agent states indicates that nondeterministic transitions create sufficient randomness to prevent the formation of stable cooperative neighborhoods, maintaining a dynamic and evolving spatial ecology throughout the 100-step simulation duration.

Resource accumulation in the spatial model (457.37 units average) falls between the non-spatial nondeterministic value (815.06 units) and the spatial constraint limitation, suggesting that spatial constraints on agent interactions further reduce cooperative opportunities in the nondeterministic regime.

Summary: The simulation demonstrates that nondeterminism at the micro level (individual agent transitions) profoundly reshapes macro-level outcomes. Deterministic agents converge to a single stable equilibrium (universal cooperation) yielding high aggregate wealth (1294.00 units) but zero behavioral diversity. Nondeterministic agents remain in dynamic flux, exploring all available states and maintaining persistent behavioral heterogeneity across both non-spatial (50 agents) and spatial (30 agents on 10×10 grid) dimensions. While this heterogeneity reflects fundamental properties of nondeterministic finite automata—their ability to explore multiple execution paths—it comes at the cost of reduced collective resource accumulation (815.06 units, 37% lower than deterministic) and persistent withdrawal behavior (60% in the spatial model) that undermines the formation of mutually beneficial cooperative relationships.

V. Discussion and Future Directions

Our results demonstrate that incorporating nondeterminism into agent decision models strongly alters emergent economic dynamics. The NFA framework provides a way to represent bounded rational agents operating under uncertainty, bridging formal methods and computational economics.

Several limitations and open questions:

- **Scalability**: current simulations involve relatively small populations; scaling to thousands of agents requires computational optimization
- **Cognitive enrichment**: agents currently use simple payoff signals; richer perception and memory mechanisms could enhance realism
- **Learning**: extending the model with adaptive transition probabilities would capture better evolution
- **Empirical validation**: tuning model parameters against real market data would strengthen relevance
- **Network structure**: moving beyond spatial grids to complex network topologies could reveal additional insights

VI. Conclusion

Our project presented an agent-based model of market dynamics using nondeterministic finite automata to represent bounded rational decision-making under uncertainty. Comparative experiments demonstrated that nondeterministic agents generate different wealth distributions, cooperation levels, and spatial patterns compared to deterministic models, highlighting the importance of uncertainty in emergent economic phenomena. The NFA framework offers a strong foundation for modeling adaptive behavior, with applications extending beyond economics to distributed systems, social simulations, and multi-agent coordination. Despite remaining challenges in scalability and empirical validation, the approach provides valuable insights into the complex dynamics of real-world markets and suggests promising directions for future computational economic research.

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CONTRIBUTION REPORT

Khadija Swailem: Built Agent class with NFA logic, implemented probabilistic transition functions, created interaction mechanisms. Wrote slides 1–3 in presentation.

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Lama Nezar: Defined state transitions, specified payoff structures. Developed ‘COMPLETE ANALYSIS CELL’ in notebook. Wrote slides 7–9 in presentation.

Belal Ehab: Built Agent class with NFA logic, implemented probabilistic transition functions, created interaction mechanisms, formatted notebook. Wrote slides 10–12 in presentation.

Zena Alaa: Wrote Introduction section and Model Design & Formalization in report, wrote Related Work section in report Wrote slides 13–15 in presentation.

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Kenzy Ahmed: Wrote Conclusion and compiled references in report. Wrote slides 22–24 in presentation.

Hana Taher: Constructed simulation environment and core execution loop. Wrote slides 25–27 in presentation.

Yassin Ashraf: Created Abstract and formatted LaTeX document in report. Wrote slides 28–30 in presentation.