图像压缩
Image Compression

图像压缩原理:

减小数据冗余(data redundancy)

编码冗余
像素间相关性冗余
视觉冗余

方法:

编码冗余

变长编码

像素间相关性冗余

行程编码

预测编码

变换域表达(PCA,COSINE,WAVELET)

视觉冗余

有损处理

压缩比

$$C_R = \frac{n_{img}}{n_{cod}}$$

n_{img} : 图像数据量
n_{cod} : 编码后数据量

压缩误差评估

均方误差(rms)

$$e_{rms} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x,y) - f(x,y)]^2 \right]^{1/2}$$

信噪比

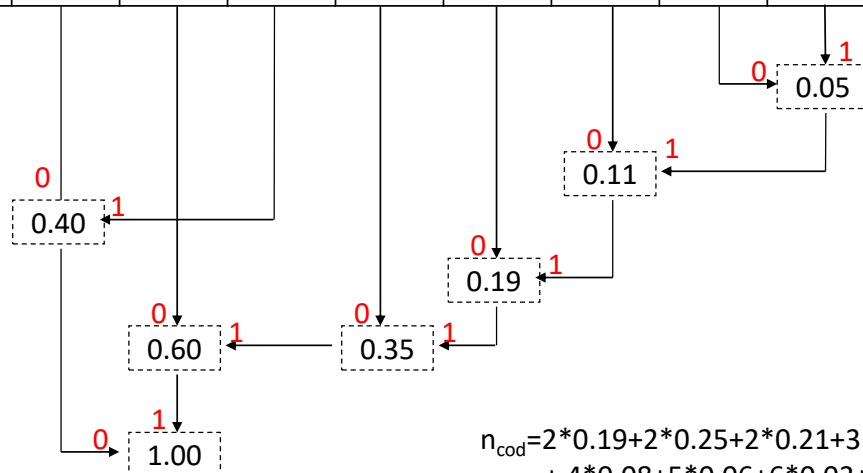$$SNR = \frac{\sum \sum [\hat{f}(x,y)]^2}{\sum \sum [\hat{f}(x,y) - f(x,y)]^2}$$

变长编码

高概率图像值→ 低字长 码

香农第1定理(Shannon's 1st theorem):
    编码最小平均字长–熵(entropy) :

$$H = - \sum_r p(r) \, log_2 \, p(r)$$

熵编码 (Huffman code)

	00	10	01	110	1110	11110	111110	111111
Gray	0	1	2	3	4	5	6	7
Prob	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02

0 → 0.05 ← 1

0 → 0.11 ← 1

0 → 0.40 ← 1

0 → 0.19 ← 1

0 → 0.60 ← 1   0 → 0.35 ← 1

0 → 1.00 ← 1

$n_{cod} = 2*0.19 + 2*0.25 + 2*0.21 + 3*0.16 + 4*0.08 + 5*0.06 + 6*0.03 + 6*0.02 = 2.7$

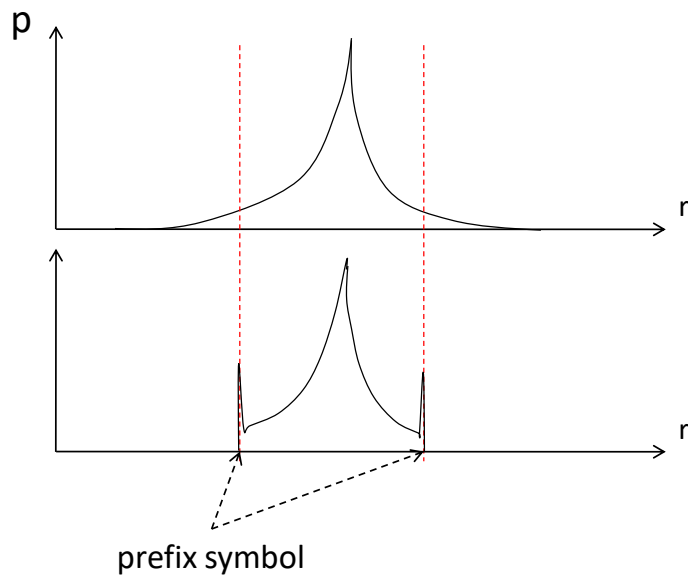| Code	00	10	01	110	1110	11110	111110	111111
Gray	0	1	2	3	4	5	6	7

Pixels: 1, 0, 1, 2, 1, 0, 2, 4, 3, 7, 6, 2, 0, …

Codes : 10, 00, 10, 01, 10, 00, 01, 1110, 110, 111111,
111110, 01, 00, …

Code stream: 100010011000011110110111111111100100…

Decoding: 10,00,10,01,10,00,01,1110,110,111111,
111110,01,00…

移位Huffman编码

prefix symbol

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算术编码(Arithmetic Coding)

Huffman coding :

Symbol → Code

Symbol Sequence → Code Stream

Arithmetic Coding :

Symbol → Interval

Symbol Sequence → Subdivided real interval [0, 1]

Symbol	Probability	Subinterval
a_1	0.2	[0.0, 0.2)
a_2	0.2	[0.2, 0.4)
a_3	0.4	[0.4, 0.8)
a_4	0.2	[0.8, 1.0)

LIST   11
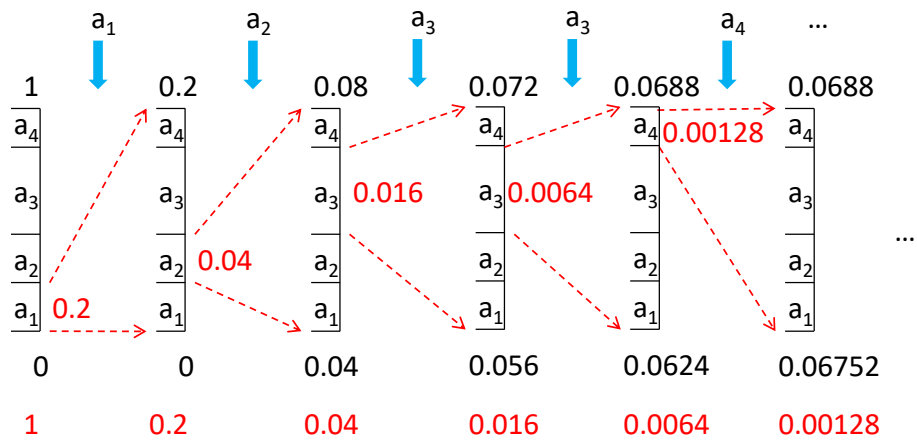
行程编码（Run-length coding)

a, a, a, ..., a → (a, n)

n

a : Symbol (block of symbols)

行程编码

Symbol: 0, 0, 0, 0, 15, 15, 15, 15, 15, 15, 15, 15, 15, 27, 27, 62, 62, 62, 62, 62, 62, 62, 62,
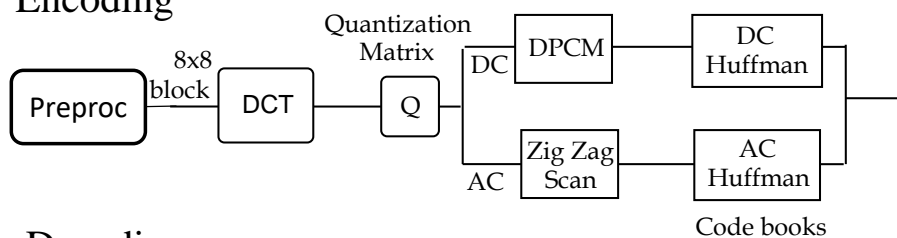
Code: 0, 4, 15, 9, 27, 2, 62, 8

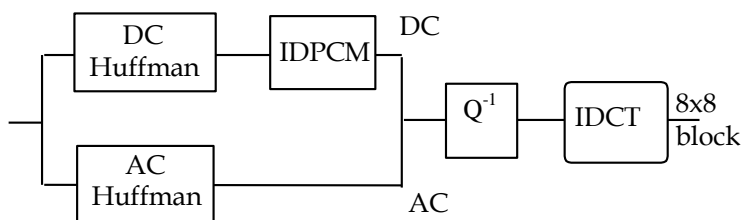JPEG (Joint Photographic Experts Group)
图像压缩标准

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JPEG Lossy
JPEG Lossless
JPEG 2000

JPEG Encoding and Decoding

Encoding

8x8 block Quantization Matrix

Preproc — DCT — Q — DC → DPCM → DC Huffman

AC → Zig Zag Scan → AC Huffman

Code books

Decoding

DC Huffman — IDPCM → DC

AC Huffman — AC

Q⁻¹ — IDCT — 8x8 block
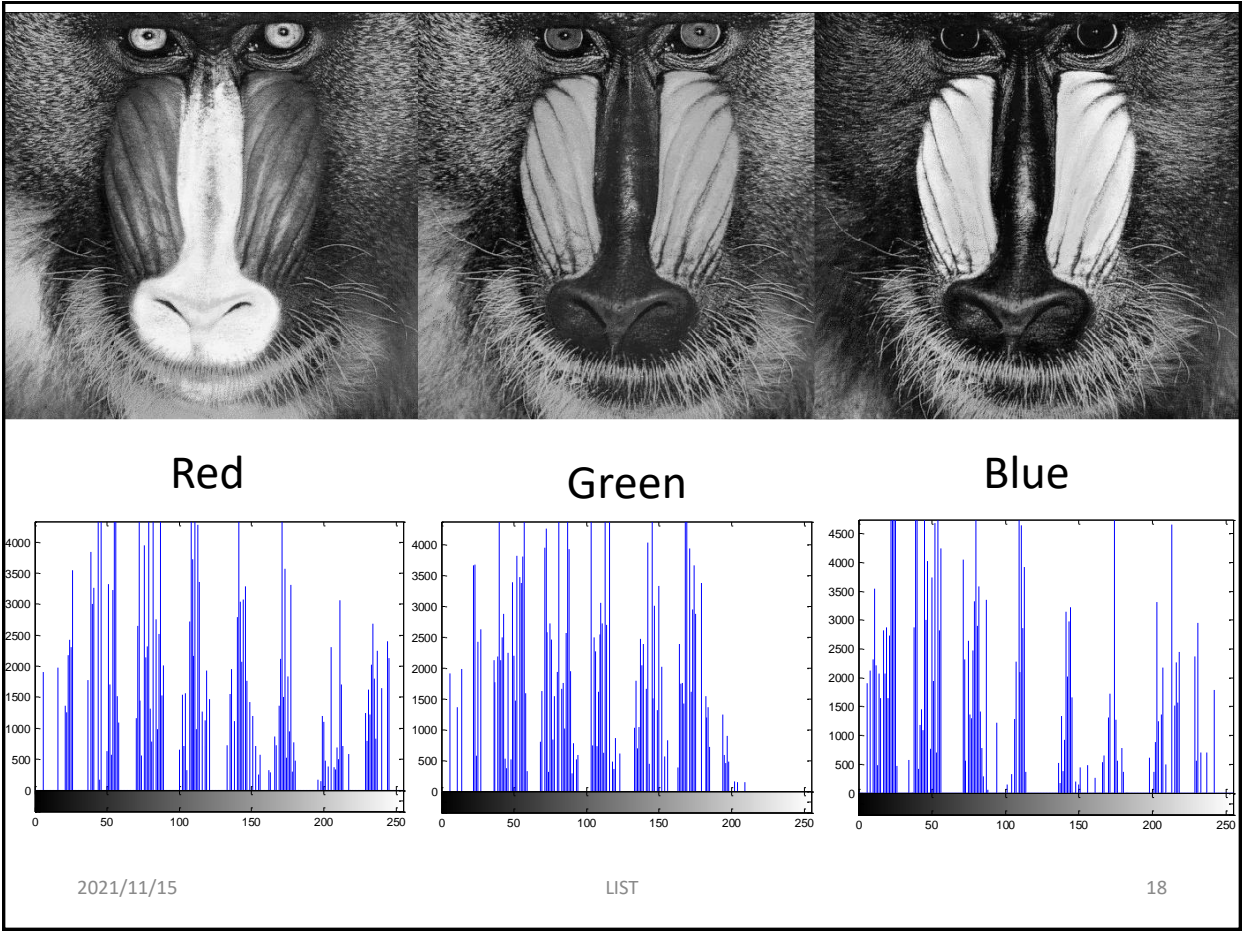
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预处理模块
颜色空间变换、降采样、分块
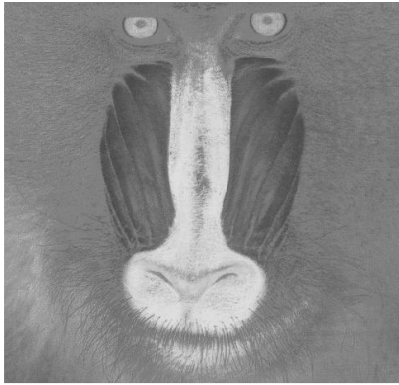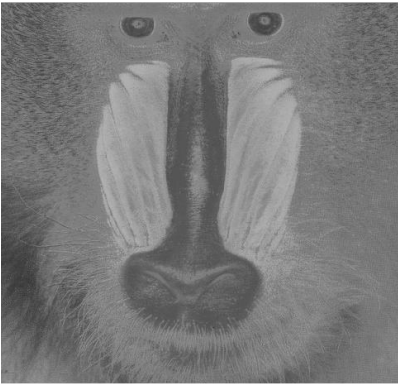
YUV (YCrCb) Color Space

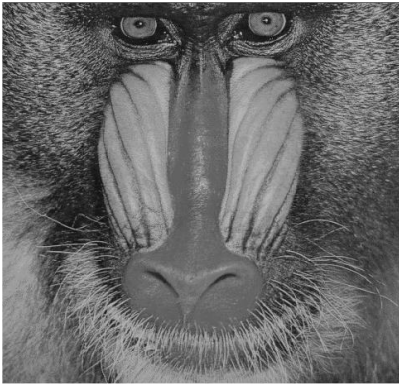$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & -0.100 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$(R,G,B) \rightarrow (Y, C_b, C_r)$

Color Image

Red   Green   Blue
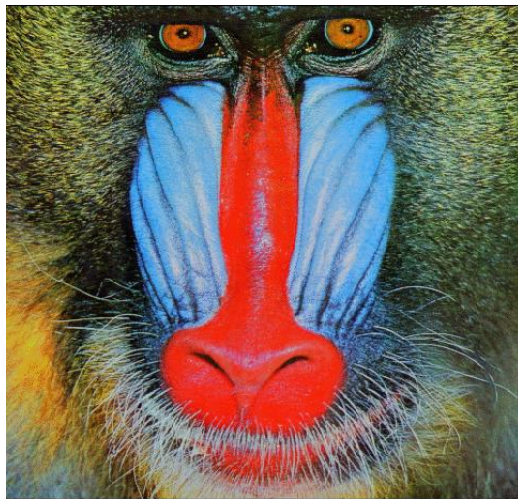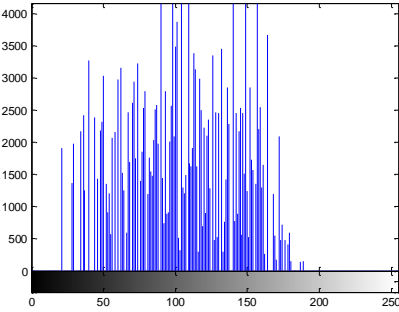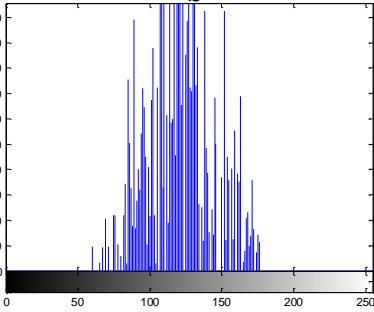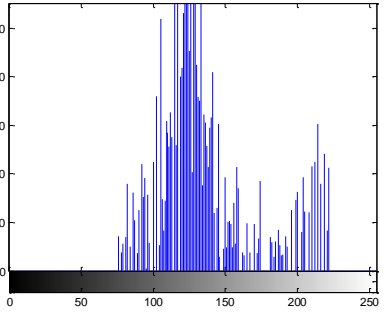
Original image

RGB down-sampled image

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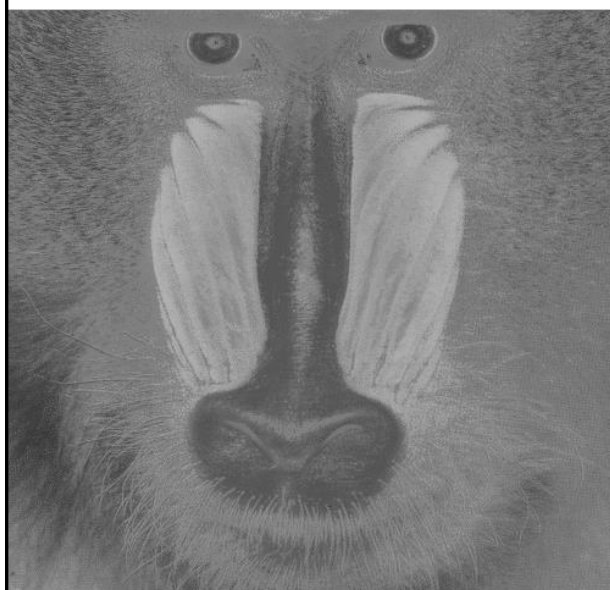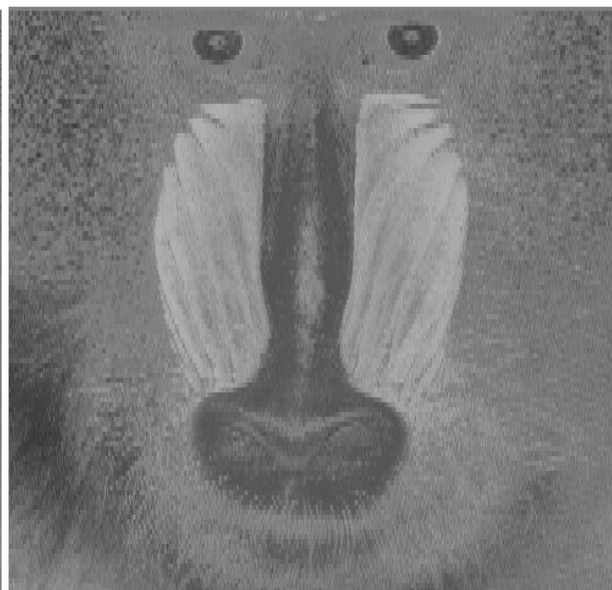
original C$_b$　　　　　　　　　　Down-sampled C$_b$
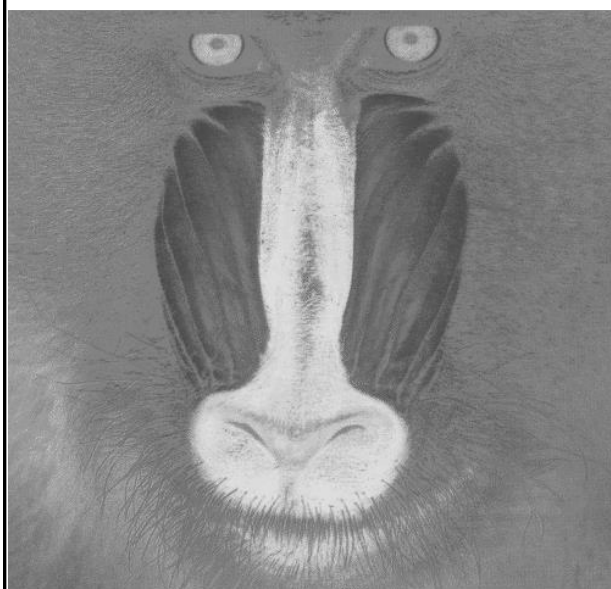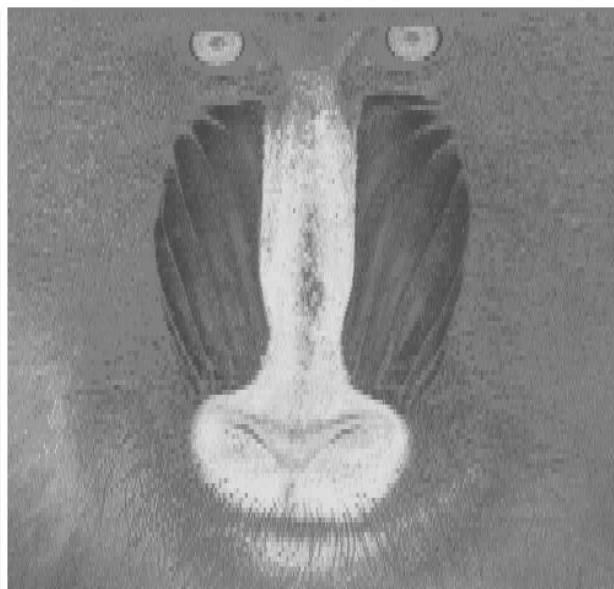
Original C$_r$ Down-sampled C$_r$
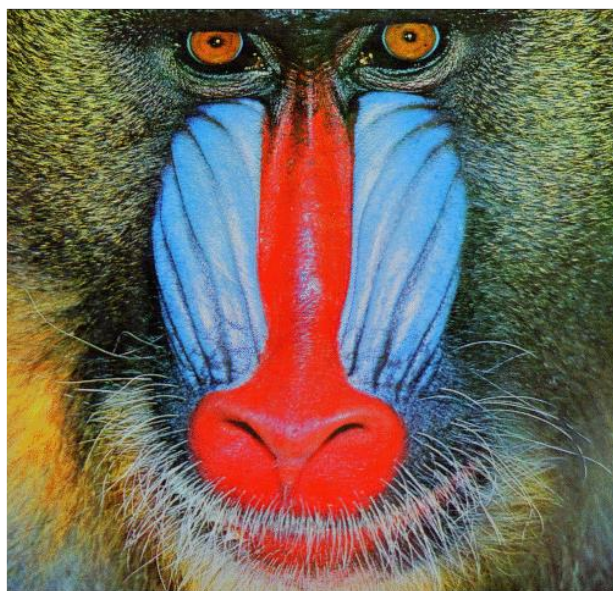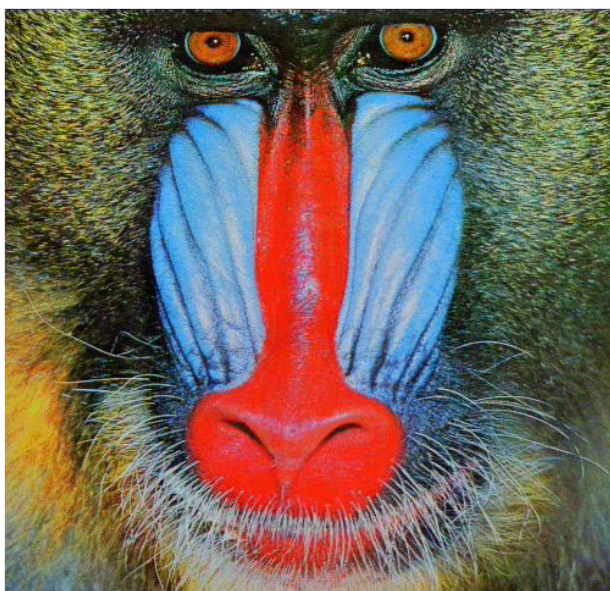
Original Image   Down-sampled C_b, C_r

Down-sampled RGB
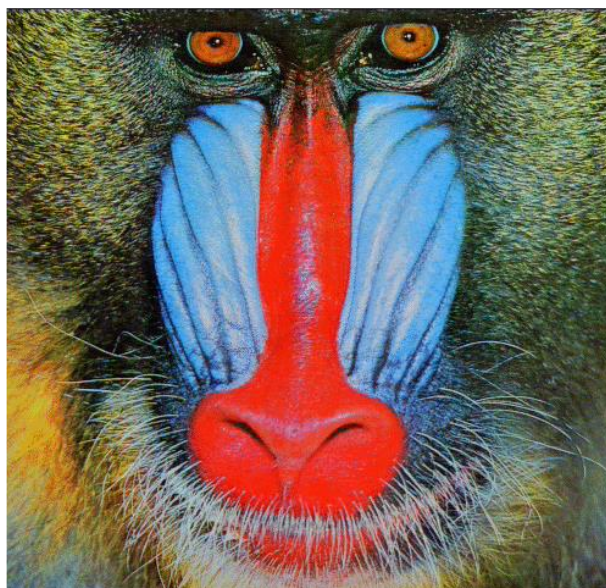
Down-sampled C_b , C_r

YUV (YCbCr) 降采样

4:4:4 → [Y0(dc), Y0(ac)],
[Cb(dc), Cb(ac)],
[Cr(dc), Cr(ac)]

4:2:2 → [Y0(dc), Y0(ac)]
[Y1(dc), Y1(ac)],
[Cb(dc), Cb(ac)],
[Cr(dc), Cr(ac)]

○ pixel with Y, Cr and Cb value

○ pixel with only Y value

● pixel with only Cr and Cb value

DCT on 8x8 blocks

8 pixels
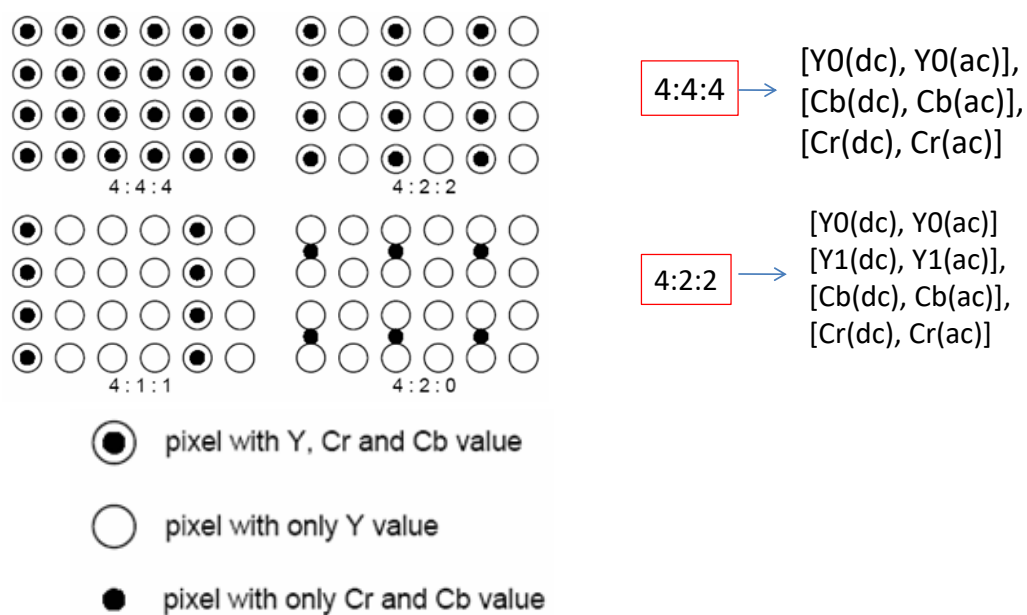
8 pixels

- We will break the image into non-overlapping 8x8 blocks.

- For each block u(m,n), we will take an 8x8 DCT

64 pixels

64 pixels
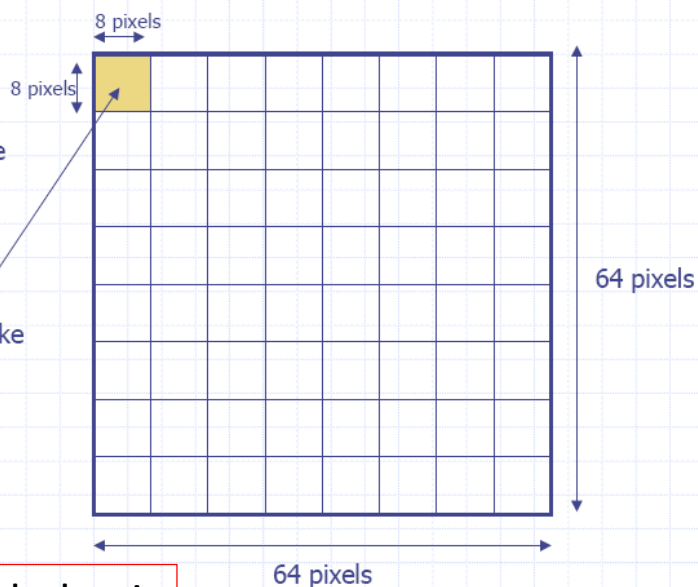
MCU: Minimum coded unit

Digital Cosine Transform

$$C(u) = \alpha(u) \sum_{x=0}^{N-1} f(x) \cos\left[\frac{\pi(2x+1)u}{2N}\right]$$

$$f(x) = \sum_{u=0}^{N-1} \alpha(u) C(u) \cos\left[\frac{\pi(2x+1)u}{2N}\right]$$

$$C(0) = \alpha(0) \sum_{x=0}^{N-1} f(x)$$

$$\alpha(u) = \begin{cases} \sqrt{\dfrac{1}{N}} & (u = 0) \\ \sqrt{\dfrac{2}{N}} & (u > 0) \end{cases}$$

DC Coefficient / AC Coefficients

DCT Basic Functions
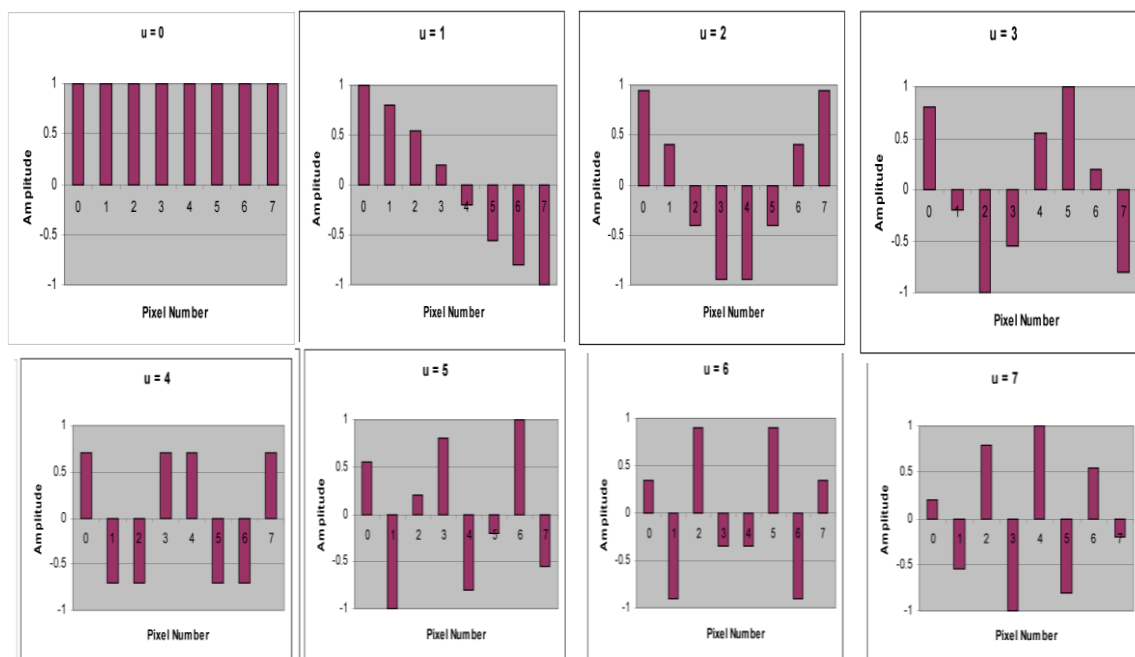
8x8 2D DCT Basic Functions
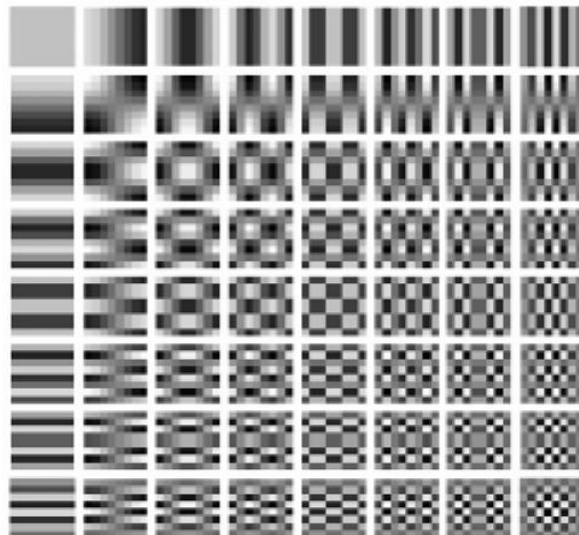
Cosine频域系统量化

Quantization Ta

$$scale\ factor\ (\%) = \begin{cases} \dfrac{5000}{Q} & 1 \le Q \le 50 \\ 200 - 2*Q & 50 \le Q \le 99 \\ 1 & Q = 100 \end{cases}$$

$$F_q(u,v) = F(u,v)/Q_{uv}$$

The Luminance Quantization Table

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

The Chrominance Quantization Table

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99

编码

DC → Difference Coding
AC → RLE, Huffman Encoding

DC (Y, Cb, Cr): 差分预测编码

DC$_{i-1}$ DC$_i$

· · · Block$_{i-1}$ | Block$_i$ · · ·

DIFF = DC$_i$ - DC$_{i-1}$

(SIZE, AMPLITUDE)
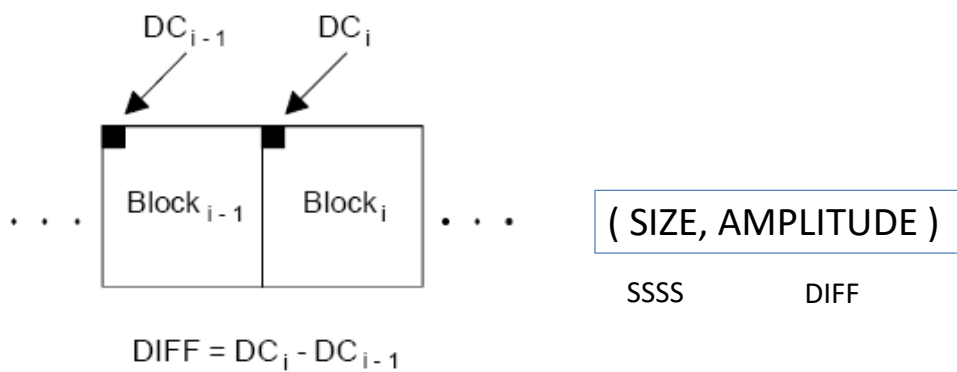
SSSS   DIFF

SSSS	DIFF values
0	0
1	−1,1
2	−3,−2,2,3
3	−7..−4,4..7
4	−15..−8,8..15
5	−31..−16,16..31
6	−63..−32,32..63
7	−127..−64,64..127
8	−255..−128,128..255
9	−511..−256,256..511
10	−1 023..−512,512..1 023
11	−2 047..−1 024,1 024..2 047

Table for luminance DC coefficient differences

Category	Code length	Code word
0	2	00
1	3	010
2	3	011
3	3	100
4	3	101
5	3	110
6	4	1110
7	5	11110
8	6	111110
9	7	1111110
10	8	11111110
11	9	111111110

Table for chrominance DC coefficient differences

Category	Code length	Code word
0	2	00
1	2	01
2	2	10
3	3	110
4	4	1110
5	5	11110
6	6	111110
7	7	1111110
8	8	11111110
9	9	111111110
10	10	1111111110
11	11	11111111110

AC (Y, C_b, C_r) : 行程编码+Huffman编码

DC AC$_{01}$ AC$_{07}$

AC$_{70}$ AC$_{77}$

Zig-zag order
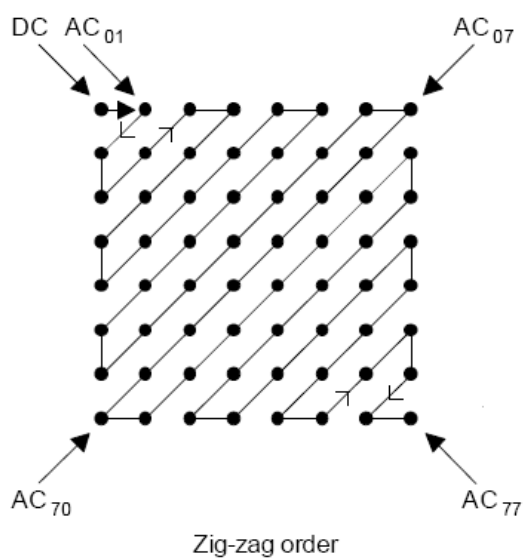
-38	18	1	-3	-1	0	0	0
-9	-8	1	3	1	0	0	0
-3	-2	0	1	0	0	0	0
4	4	0	-1	0	0	0	0
-2	-1	0	0	0	0	0	0
-1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Cnt of 0	Bits	Value
4 RRRR	4 SSSS	

SSSS

	0	1	2	.	.	.	9	10

		0	EOB				
RRRR	.	N/A	COMPOSITE VALUES
	.	N/A	
	.	N/A	
	15	ZRL	

SSSS	AC coefficients
1	−1,1
2	−3,−2,2,3
3	−7..−4,4..7
4	−15..−8,8..15
5	−31..−16,16..31
6	−63..−32,32..63
7	−127..−64,64..127
8	−255..−128,128..255
9	−511..−256,256..511
10	−1 023..−512,512..1 023

Table for luminance AC coefficients

Run/Size	Code length	Code word
0/0 (EOB)	4	1010
0/1	2	00
0/2	2	01
0/3	3	100
0/4	4	1011
0/5	5	11010
0/6	7	1111000
0/7	8	11111000
0/8	10	1111110110
0/9	16	1111111110000010
0/A	16	1111111110000011
1/1	4	1100
1/2	5	11011
1/3	7	1111001
1/4	9	111110110
1/5	11	11111110110

. . .

Table for chrominance AC coefficients

Run/Size	Code length	Code word
0/0 (EOB)	2	00
0/1	2	01
0/2	3	100
0/3	4	1010
0/4	5	11000
0/5	5	11001
0/6	6	111000
0/7	7	1111000
0/8	9	111110100
0/9	10	1111110110
0/A	12	111111110100
1/1	4	1011
1/2	6	111001
1/3	8	11110110

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Marker code assignments

Code Assignment	Symbol	Description
X'FFC0'	SOF_0	Baseline DCT
X'FFC4'	DHT	Define Huffman table(s)
X'FFD8'	SOI*	Start of image
X'FFD9'	EOI*	End of image
X'FFDA'	SOS	Start of scan
X'FFDB'	DQT	Define quantization table(s)
X'FFDC'	DNL	Define number of lines
X'FFDD'	DRI	Define restart interval
X'FFDE'	DHP	Define hierarchical progression
X'FFDF'	EXP	Expand reference component(s)
X'FFE0' through X'FFEF'	APP_n	Reserved for application segments
X'FFF0' through X'FFFD'	JPG_n	Reserved for JPEG extensions
X'FFFE'	COM	Comment

```
00000000  FF D8 FF E0 00 10 4A 46   49 46 00 01 01 01 00 C8   ......JFIF......
00000010  00 C8 00 00 FF E1 00 62   45 78 69 66 00 00 49 49   .......bExif..II
00000020  2A 00 08 00 00 00 01 00   69 87 04 00 01 00 00 00   *.......i.......
00000030  1A 00 00 00 00 00 00 00   01 00 86 92 02 00 2D 00   ..............-.
00000040  00 00 2C 00 00 00 00 00   00 00 49 6E 74 65 6C 28   ..,.......Intel(
00000050  52 29 20 4A 50 45 47 20   4C 69 62 72 61 72 79 2C   R) JPEG Library,
00000060  20 76 65 72 73 69 6F 6E   20 5B 31 2E 35 31 2E 31    version [1.51.1
00000070  32 2E 34 34 5D 00 00 8B   FF FE 00 2E 49 6E 74 65   2.44].......Inte
00000080  6C 28 52 29 20 4A 50 45   47 20 4C 69 62 72 61 72   l(R) JPEG Librar
00000090  79 2C 20 76 65 72 73 69   6F 6E 20 5B 31 2E 35 31   y, version [1.51
000000a0  2E 31 32 2E 34 34 5D 00   FF DB 00 43 00 0C 11 11   .12.44]....C....
000000b0  14 14 14 21 21 21 21 2A   25 27 25 2A 35 2D 2E 2E   ...!!!!*%'%*5-..
000000c0  2D 35 43 38 37 3B 37 38   43 56 47 43 48 48 43 47   -5C87;78CVGCHHCG
000000d0  56 5A 53 56 5A 56 53 5A   67 67 6E 6E 67 67 7D 83   VZSVZVSZggnngg}.
000000e0  89 83 7D 9D A7 A7 9D CA   D1 CA FF FF FF FF DB 00   ..}.............
000000f0  43 01 0C 18 18 18 18 18   18 1A 1D 27 39 29 22 1E   C..........'9)".
```