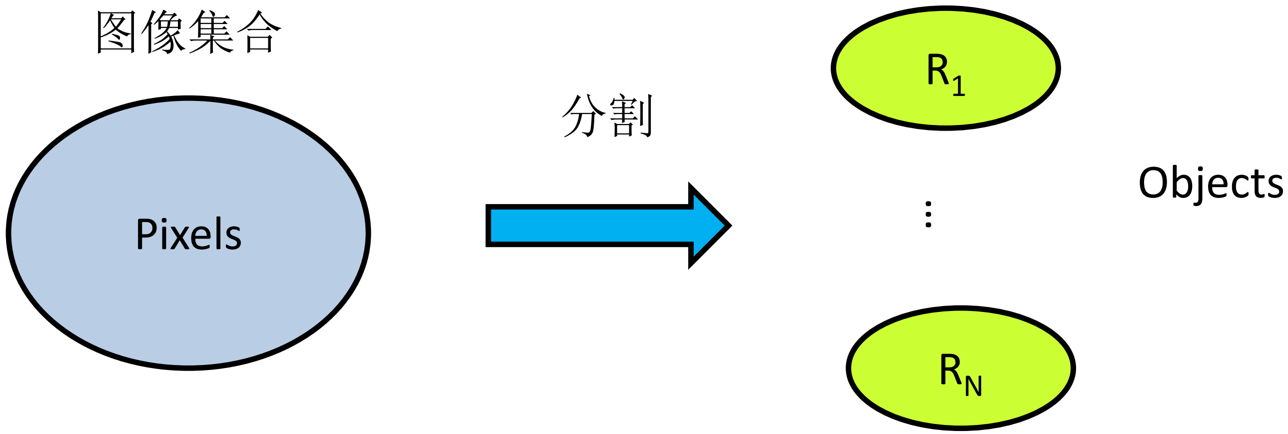
图像分割
Image Segmentation

图像分割目标

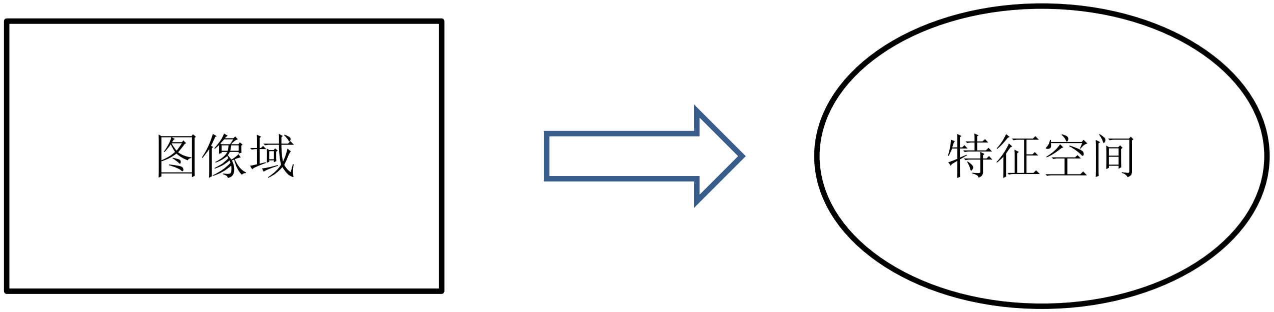


$$\sum_{i=1}^{N} R_i = I$$

核心问题

- 特征空间(Feature Space)
  - 有效特征组成的表达域
- 分类器(Classifier)
  - 实现像素分类

特征提取(Feature Extraction)



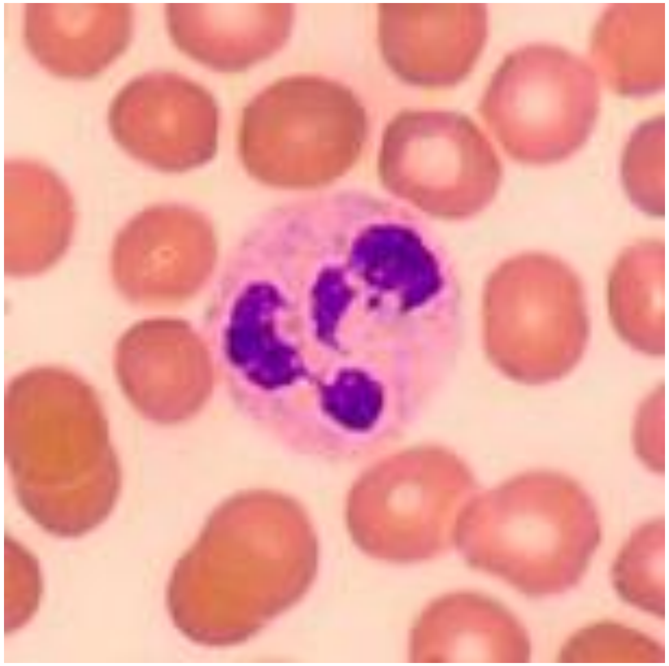
有效特征(Significant Features)

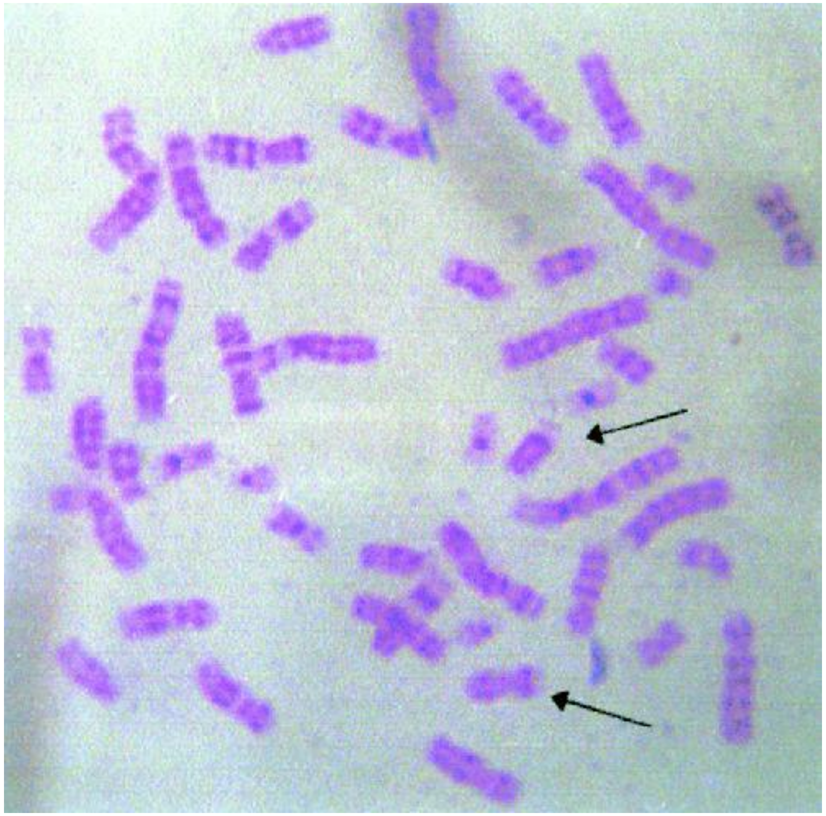
- 类间的差异性
- 类内的相似性

特征(Features)

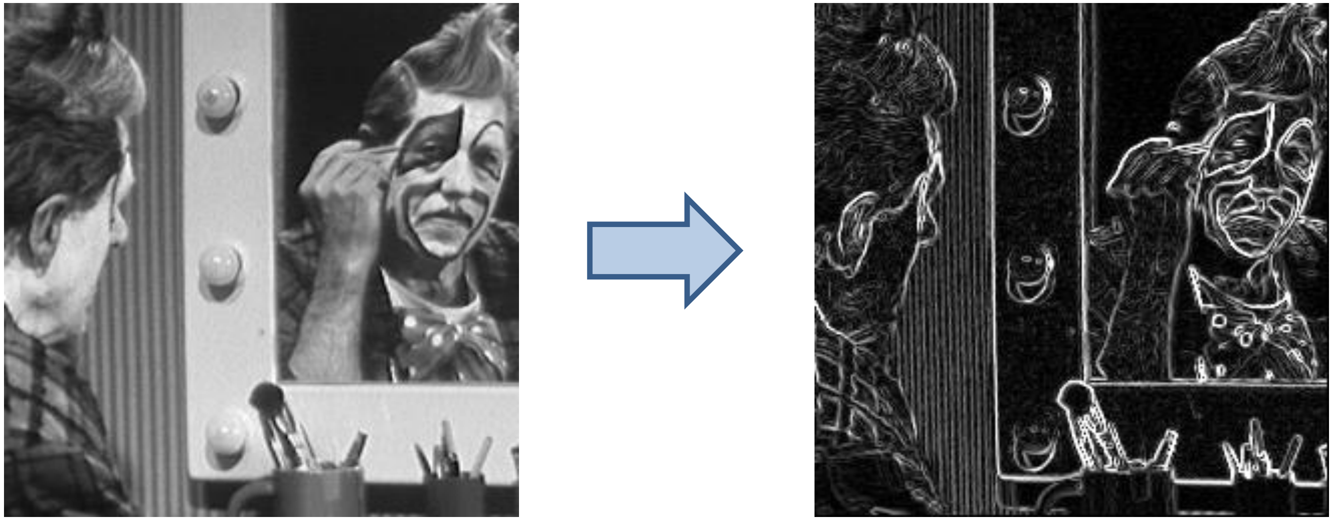
- 图像值
- 边界特征
- 纹理特征(Texture)

图像值为有效特征的例子

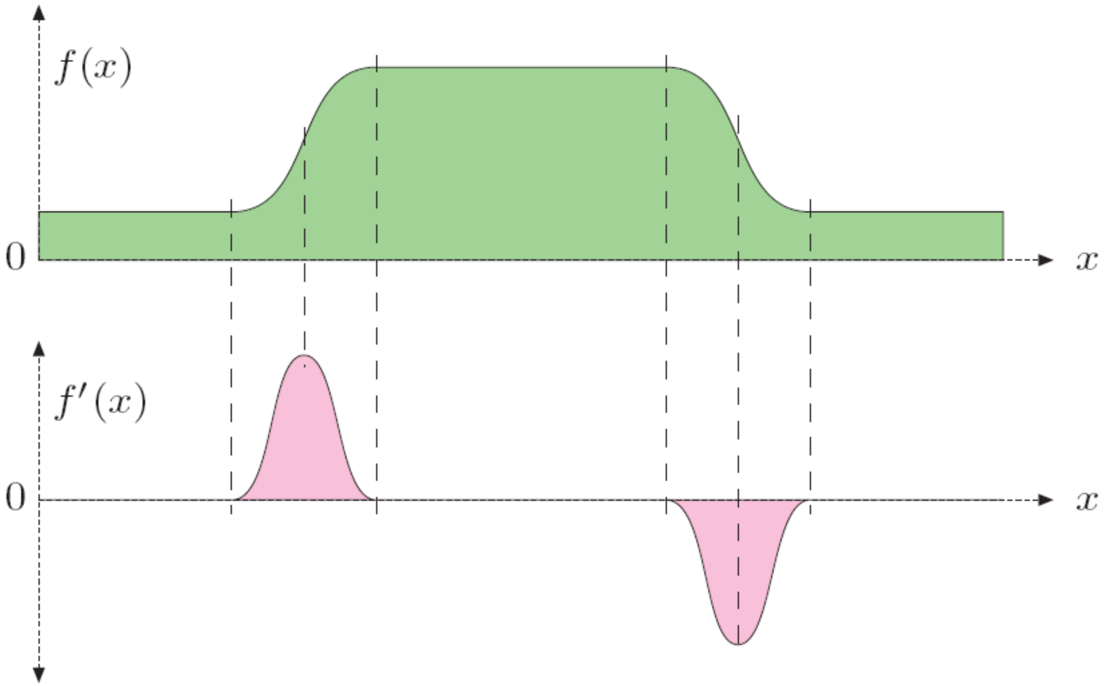




边界特征：高通滤波



梯度（Gradient）



$$\nabla f(x,y)=\begin{bmatrix}\dfrac{\partial f(x,y)}{\partial x}\\ \dfrac{\partial f(x,y)}{\partial y}\end{bmatrix}$$

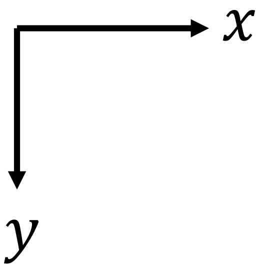
幅值：边界的强度

方向：边界的方向（法向量）

梯度滤波器（Gradient Filters）

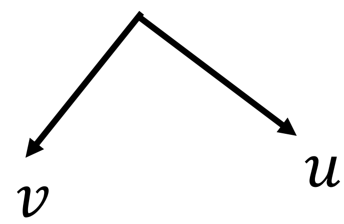
$$h_x = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$h_y = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



$$h_u = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$h_v = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



Prewitt

$$h_x^P = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$h_y^P = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$f * h_x^P = f * \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Sobel

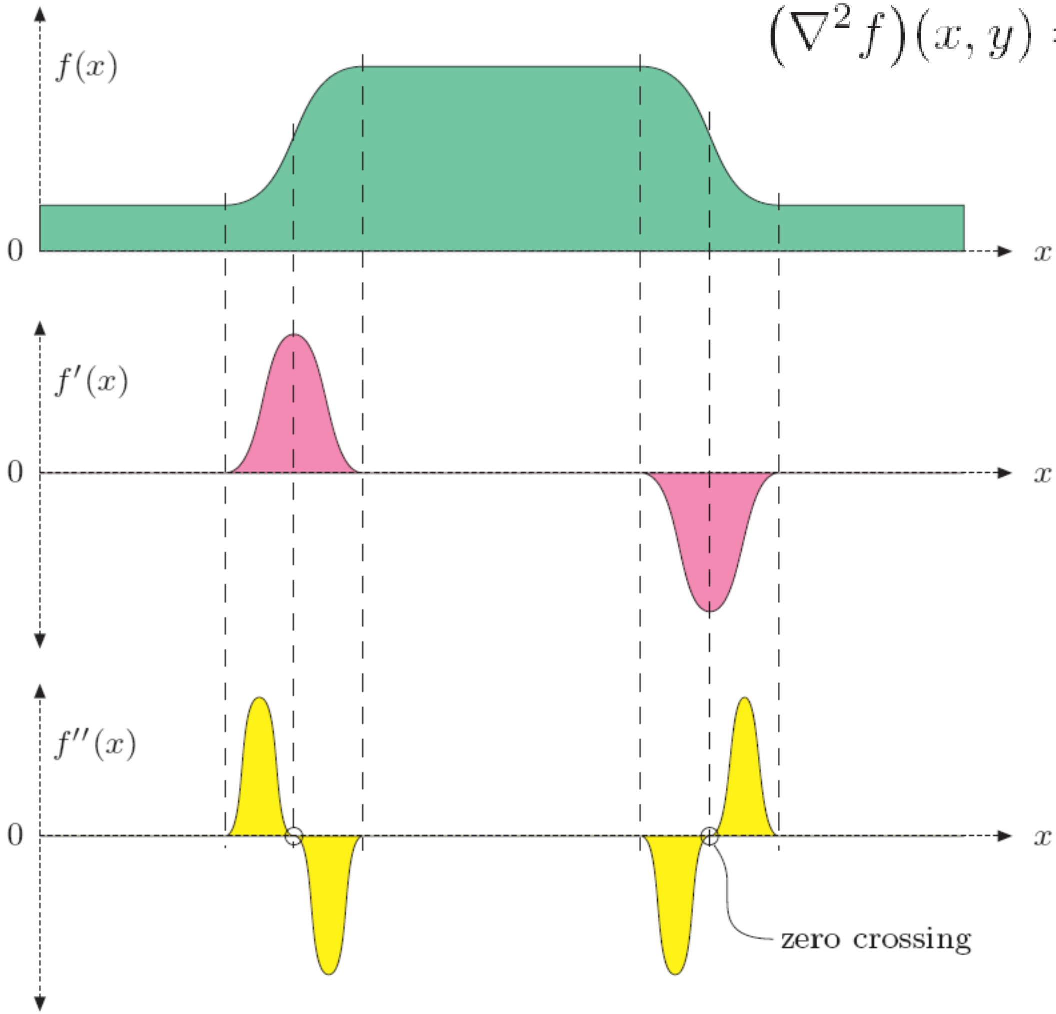
$$h_x^S = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$h_y^S = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$f * h_x^S = f * \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Laplacian

$$(\nabla^2 f)(x, y) = \frac{\partial^2 f}{\partial^2 x}(x, y) + \frac{\partial^2 f}{\partial^2 y}(x, y)$$



LoG (Laplacian of a Gaussian)

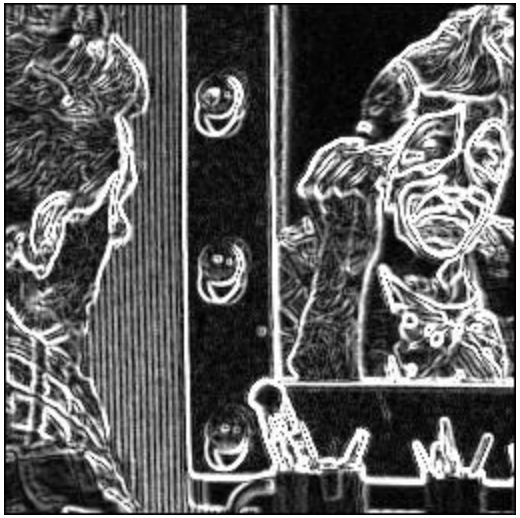
$$f(x,y) \longrightarrow \boxed{f * G_\sigma} \longrightarrow \boxed{\nabla^2} \longrightarrow$$

$$f(x,y) \longrightarrow \boxed{f * \mathrm{LoG}} \longrightarrow$$

$$G_\sigma(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$LoG = \nabla^2 G_\sigma(x,y)$$

Original Image



Sobel (Magnitude)

LoG (σ=1.0)

统计特征(statistical features)

p(z) : normalized histogram

矩(moment)

$$\boxed{\mu_n(z) = \sum_i (z_i - m)^n p(z_i)} \qquad m = \sum_i z_i p(z_i)$$

$$\mu_0(z) = \sum_i p(z_i) = 1$$

$$\mu_1(z) = \sum_i z_i p(z_i) - m \sum_i p(z_i) = 0$$

$$\mu_2(z) = \sum_i (z_i - m)^2 p(z_i) = \sigma^2(z)$$

…

熵(entropy)

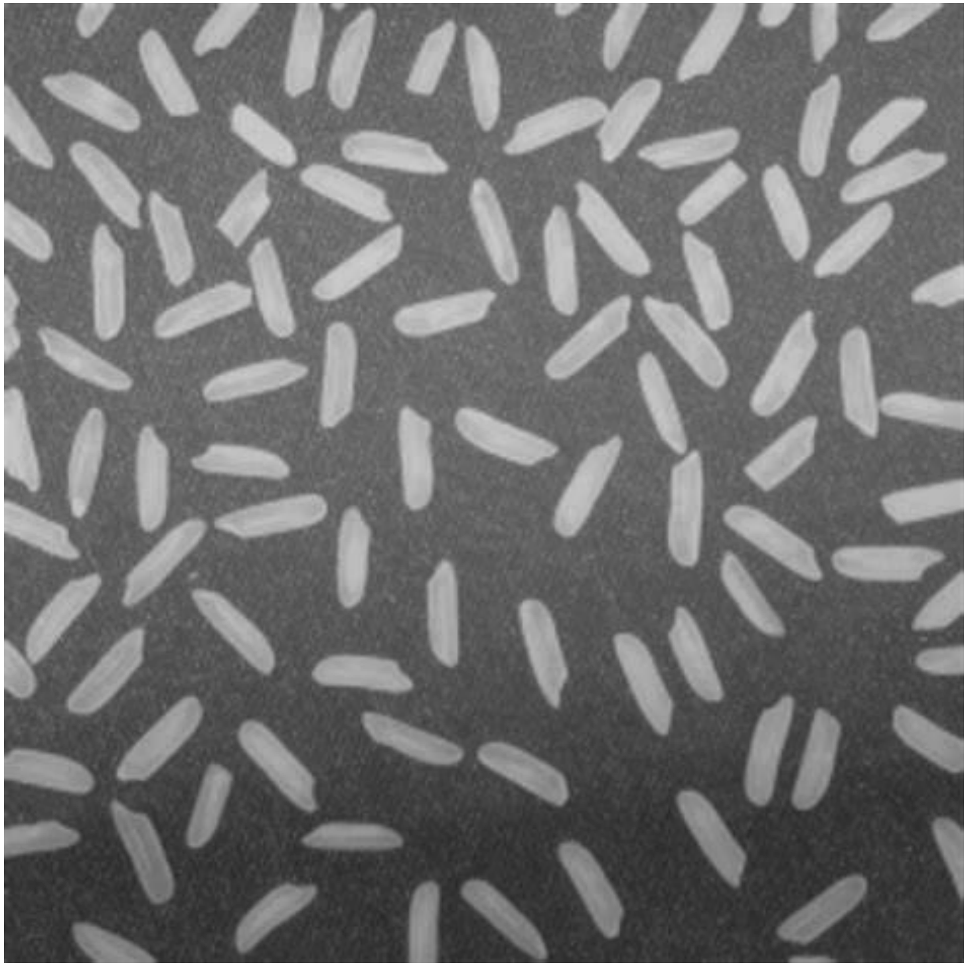
$$\boxed{E(z) = -\sum_i p(z_i) \ln p(z_i)}$$

分类器（Classifier）

- 线性分类器（Linear Classifier）
- 非线性分类器（Nonlinear Classifier）

- 0-1分类器（0-1 Classifier）
- 模糊分类器（Fuzzy Classifier）

面积分割 vs 边界分割



目标对象

边界与非边界

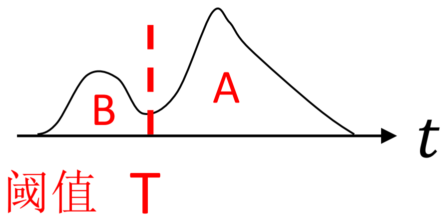
有限面积→封闭边界

封闭边界 → 区域面积

面积分割与边界分割等价

阈值法(Thresholding)

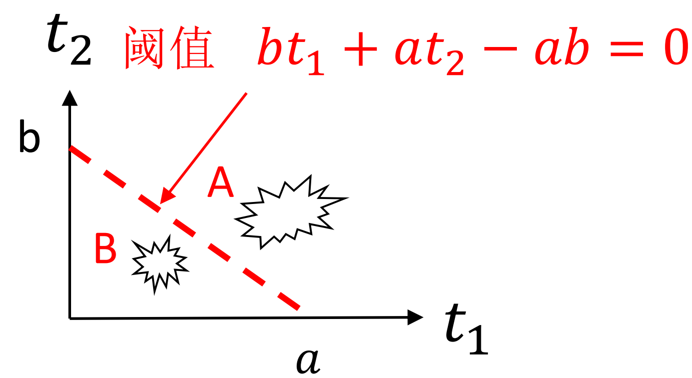
1D 特征空间



样本至阈值的距离

$$D(t) = t - T$$

2D 特征空间



样本至阈值的距离

$$D(t_1, t_2) = \frac{bt_1 + at_2 - ab}{\sqrt{b^2 + a^2}}$$

记：像素$f(x,y)$的特征值为$\vec{t}(x,y)$

$$g(x,y) = \begin{cases} 1 & \left(D\left(\vec{t}(x,y)\right) > 0\right) \\ 0 & otherwise \end{cases}$$

模糊分类

$$\mu_A(x,y) = \frac{1}{1 + e^{-D(\vec{t}(x,y))}}$$

分类误差(Misclassification Error)

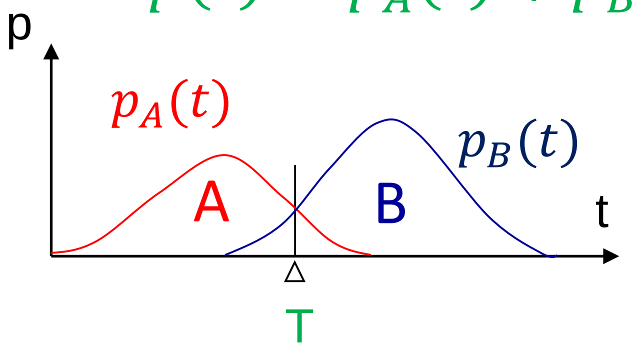
$$e = (e_{A\to B}+e_{B\to A})/N \ \ \to min$$

$e_{A\to B}$ ： A类像素错分到B

$e_{B\to A}$ ： B类像素错分到A

N ： 图像面积

$$p(t) = p_A(t) + p_B(t)$$

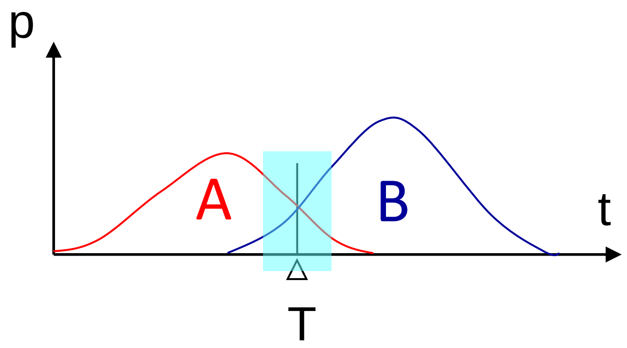


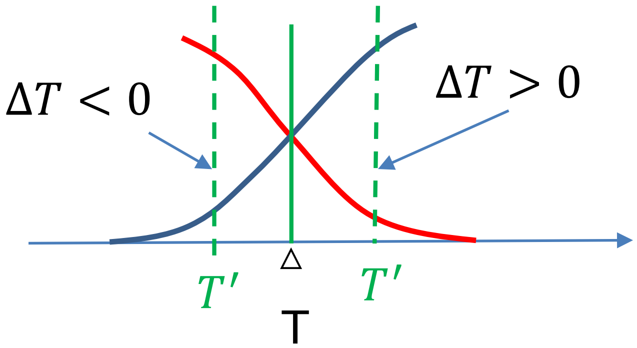
$$e_{A\to B}(T) = \int_T^{\infty} p_A(t)dt$$

$$e_{B\to A}(T) = \int_{-\infty}^{T} p_B(t)dt$$

$$e(T) = e_{A\to B}(T) + e_{B\to A}(T) = \int_T^{\infty} p_A(t)dt + \int_{-\infty}^{T} p_B(t)dt$$

最佳阈值 $\min_T e(T)$





$$T' = T + \Delta T \qquad (T' \in \Omega(T))$$

$$\boxed{e(T') > e(T) \quad (\Delta T \neq 0)}$$

$$e(T') = \int_{T'}^{\infty} p_A(t)dt + \int_{-\infty}^{T} p_B(t)dt$$

$$\boxed{\Delta T > 0}$$

$$e(T') = e(T) - \int_{T}^{T'} p_A(t)dt + \int_{T}^{T'} p_B(t)dt$$

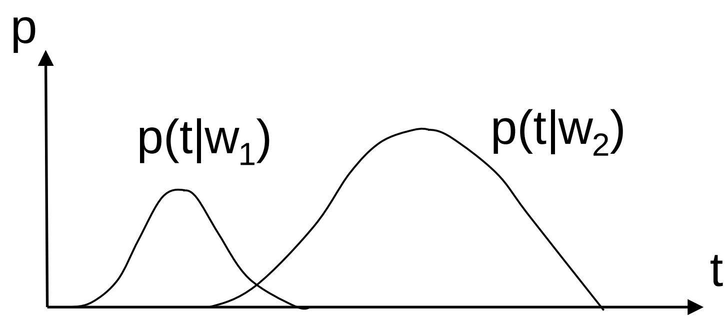
$$\int_{T}^{T'} p_A(t)dt < \int_{T}^{T'} p_B(t)dt$$

$$\boxed{\Delta T < 0}$$

$$e(T') = e(T) + \int_{T'}^{T} p_A(t)dt - \int_{T'}^{T} p_B(t)dt$$

$$\int_{T}^{T'} p_A(t)dt > \int_{T}^{T'} p_B(t)dt$$

贝叶斯分类器（Bayes Classifier）

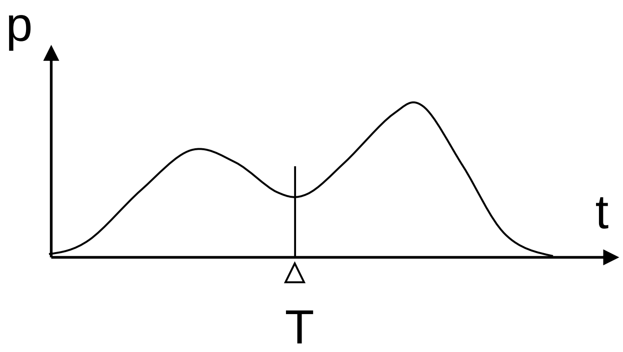


$$p(t\,|\,w_i)P(w_i) = p(w_i\,|\,t)P(t)$$

$$p(w_i\,|\,t) = p(t\,|\,w_i)P(w_i)\,/\,P(t)$$

t→w_k, if $p(w_k\,|\,t) = \max_i \left\{ p(w_i\,|\,t) \right\}$

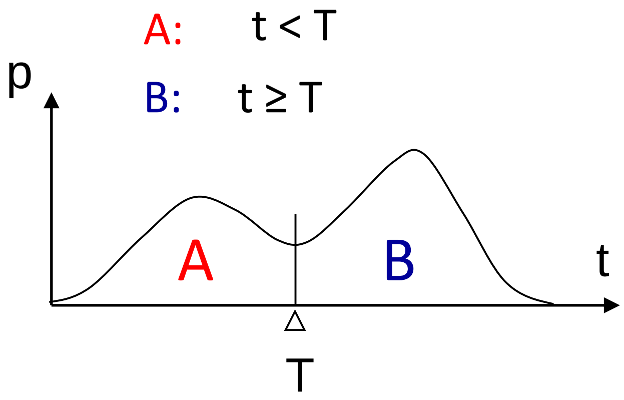
谷点阈值（Valley Thresholding）



条件：双峰直方图

关键步骤：　直方图平滑

大津阈值（Otsu's Thresholding Method）



$$\sigma(T) = P_A(T)\sigma_A(T) + P_B(T)\sigma_B(T)$$

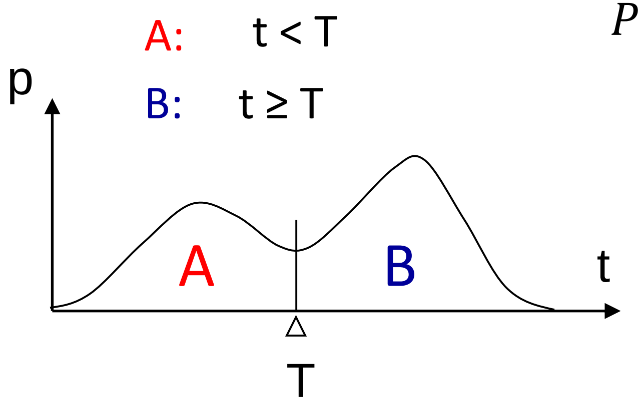
$$\min_T \sigma(T)$$

$$P_A(T) = \int_{-\infty}^{T} p(t)\,dt \qquad P_B(T) = 1 - P_A(T)$$

$$\mu_A(T) = \frac{1}{P_A(T)}\int_{-\infty}^{T} t \cdot p(t)\,dt \qquad \mu_B(T) = \frac{1}{P_B(T)}\int_{T}^{\infty} t \cdot p(t)\,dt$$

$$\sigma_A^2(T) = \frac{1}{P_A(T)}\int_{-\infty}^{T} \big(t - \mu_A(T)\big)^2 p(t)dt \qquad \sigma_B^2(T) = \frac{1}{P_B(T)}\int_{T}^{\infty} \big(t - \mu_B(T)\big)^2 p(t)dt$$

最大熵阈值



$$P(T) = \int_{-\infty}^{T} p(t)dt$$

$$p_A(t) = p(t)/P(T)$$

$$p_B(t) = p(t)/(1 - P(T))$$

$$H_A(T) = -\int_{-\infty}^{T} p_A(t) ln(p_A(t))dt$$

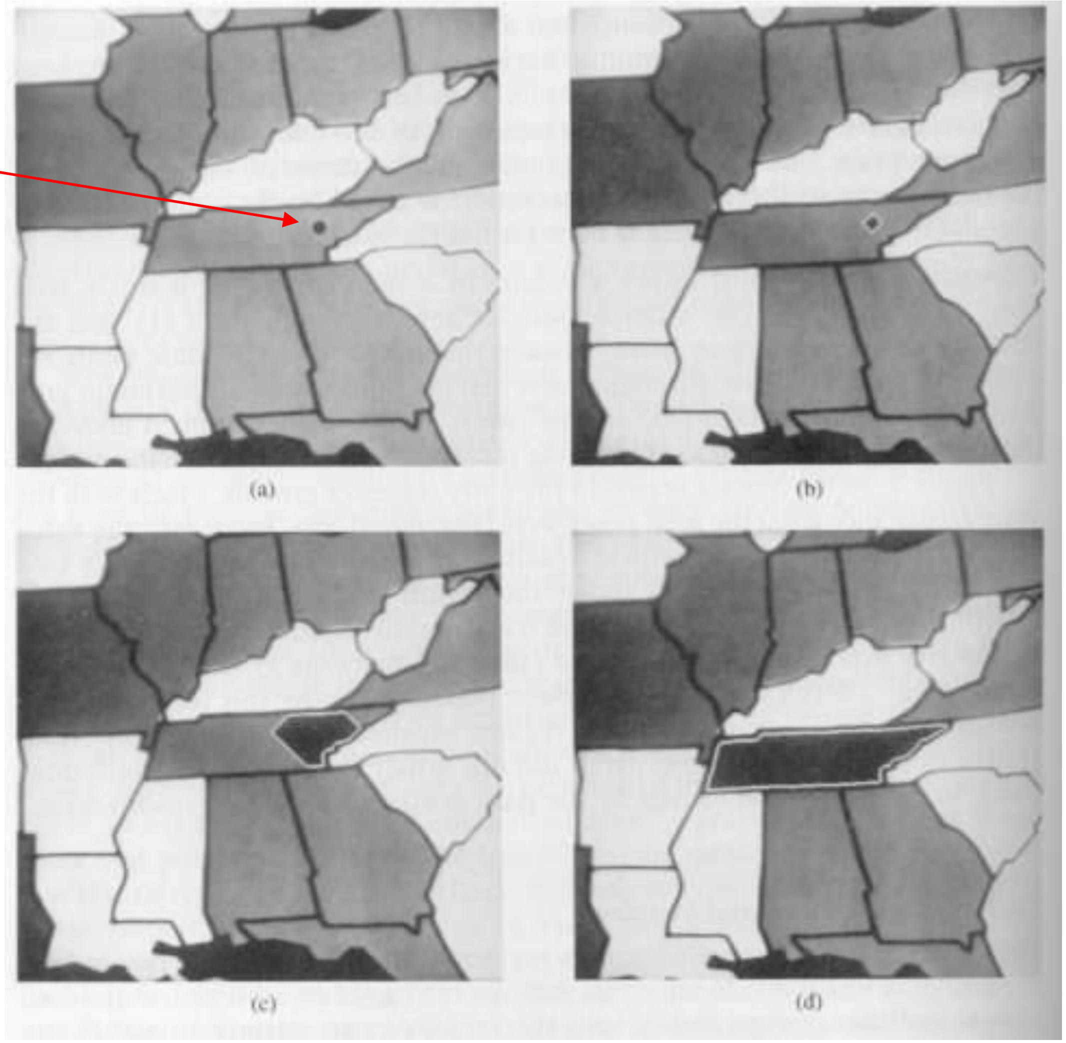
$$H_B(T) = -\int_{T}^{\infty} p_B(t) ln(p_B(t))dt$$

$$\Psi(T) = H_A(T) + H_B(T)$$

$$\max_{T} \Psi(T)$$

区域增长

seed



- 设置种子

  一个种子提取一个连通域

- 定义一致性条件

  类内特征相似性测度及阈值

- 使用栈实现增长

  1. 初始种子入栈
  2. 从栈中弹出种子
  3. 标记种子点
  4. 遍历所有未标记邻点

     判断邻点与种子点同类

     是：压栈

     否：继续
  5. 重复2-4，直到栈空