Report For Exercise Branch & Bound

Formal Lower bounds

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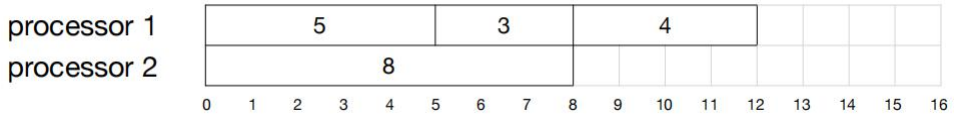
1. Problem description / demand analysis 问题描述

Use $LB = \max\{LB1, LB2\}$ for the design of a Branch & Bound algorithm
$E = \{5,3,8,4\}$ and $m = 2$
Start from the following feasible solution: $x1 = 1, x2 = 1, x3 = 0, x4 = 1$



1) Complete the tree of the Branch & Bound, and find the optimal solution
2) Give the order of the nodes you explored

2. Results and analysis 结果和分析

解：$\mathrm{LB1} = \max\{\min_i\{\mathrm{load(processor\ i)} + \max_i p_i\}, \max_i\{\mathrm{load(processor\ i)}\}\}$,

$\mathrm{LB2} = \max\{\frac{1}{m}\sum_{i=1}^{n} p_i, \max_i\{\mathrm{load(processor\ i)}\}\}$, $\mathrm{LB} = \max\{\mathrm{LB1}, \mathrm{LB2}\}$,

Where $\max_i\{\mathrm{load(processor\ i)}\}$ is the load of the current solution,

$\frac{1}{m}\sum_{i=1}^{n} p_i = \frac{5+3+8+4}{2} = 10$;

1) 本题从 $x1 = 1, x2 = 1, x3 = 0, x4 = 1$ 该解开始，得到

$x1 = 1$ 时, the remaining jobs is $\{3,8,4\}$, $\max_i p_i = 8$,

$\max_i\{\mathrm{load(processor\ i)}\} = 5, \min_i\{\mathrm{load(processor\ i)} + \max_i p_i\} = 8$,

$\mathrm{LB1} = 8, \mathrm{LB2} = 10, \mathrm{LB} = 10$;

$x1 = 1, x2 = 1$ 时, the remaining jobs is $\{8,4\}$, $\max_i p_i = 8$,

$\max_i\{\mathrm{load(processor\ i)}\} = 8, \min_i\{\mathrm{load(processor\ i)} + \max_i p_i\} = 8$,

$\mathrm{LB1} = 8, \mathrm{LB2} = 10, \mathrm{LB} = 10$;

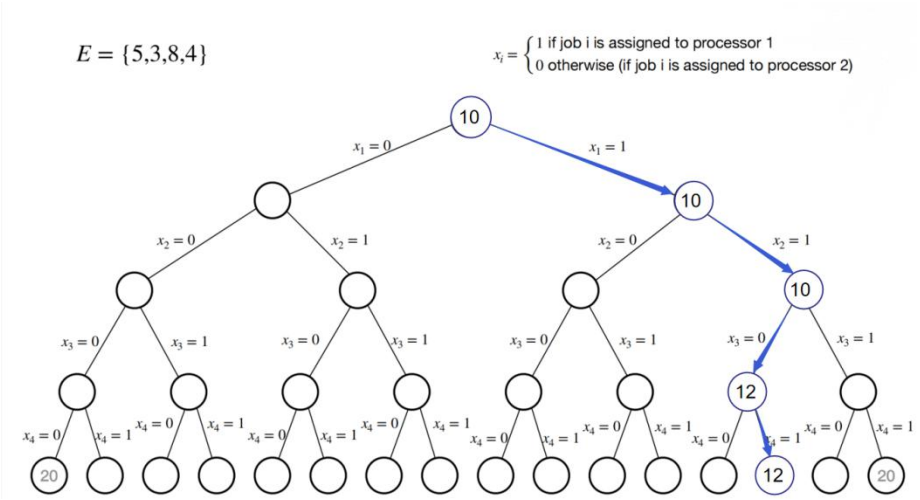
$x1 = 1, x2 = 1, x3 = 0$ 时, the remaining jobs is $\{4\}$, $\max_i p_i = 4$,

$\max_i\{\mathrm{load(processor\ i)}\} = 8, \min_i\{\mathrm{load(processor\ i)} + \max_i p_i\} = 12$,

$\mathrm{LB1} = 12, \mathrm{LB2} = 10, \mathrm{LB} = 12$;

$x1 = 1, x2 = 1, x3 = 0, x4 = 1$ 时，

$\max_i\{\mathrm{load(processor\ i)}\} = 12$, $LB = 12$;

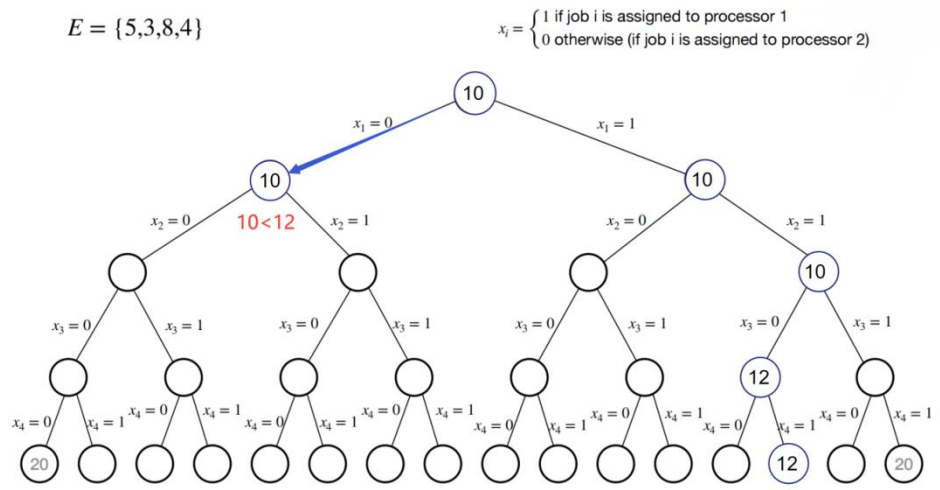


接着，看 $x1 = 0$ 的 branch，

$x1 = 0$ 时, the remaining jobs is $\{3,8,4\}, \max_i p_i = 8,$

$\max_i \{\text{load(processor i)}\} = 5, \min_i \{\text{load(processor i)} + \max_i p_i\} = 8$

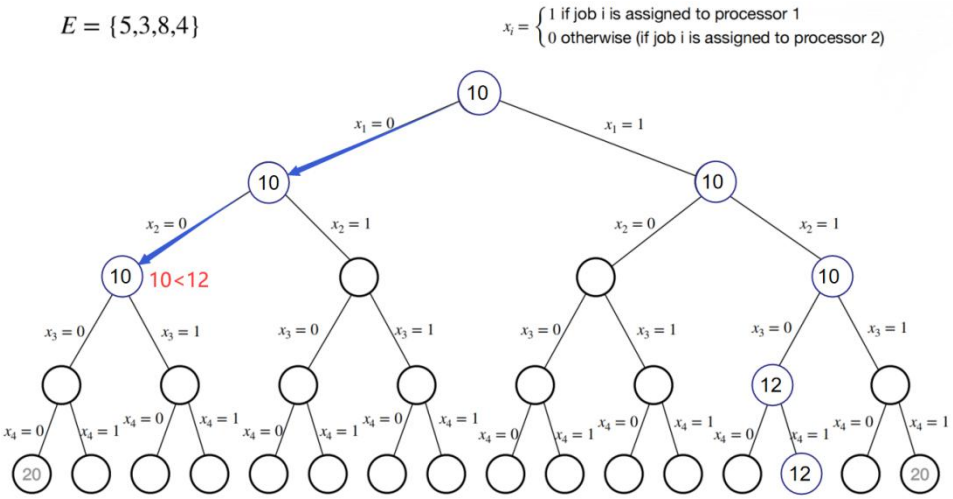
$LB1 = 8, LB2 = 10, LB = 10;$



10<12 继续深度遍历,看 $x1 = 0, x2 = 0$ 时,the remaining jobs is $\{8,4\}, \max_i p_i = 8,$

$\max_i \{\text{load(processor i)}\} = 8, \min_i \{\text{load(processor i)} + \max_i p_i\} = 8,$

$LB1 = 8, LB2 = 10, LB = 10;$

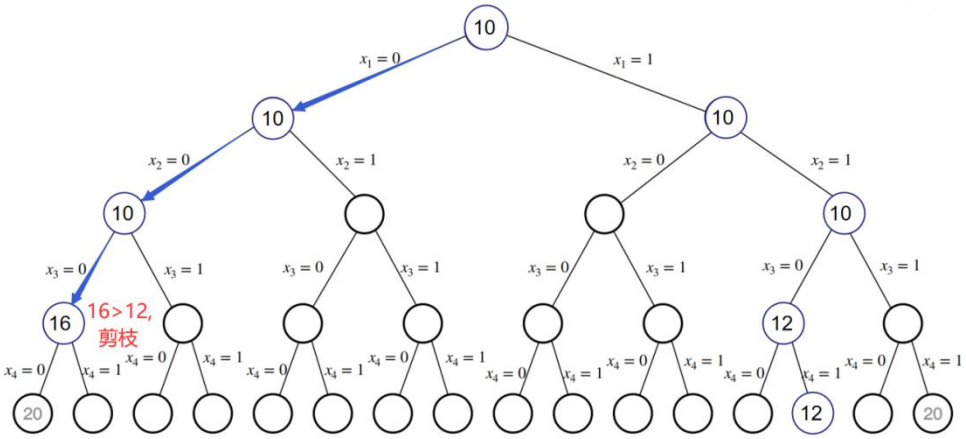


10<12 继续深度遍历,

看 $x1=0, x2=0, x3=0$ 时,the remaining jobs is $\{4\}$, $\max_i p_i = 4$,
$\max_i \{\text{load(processor i)}\} = 16, \min_i \{\text{load(processor i)} + \max_i p_i\} = 4$,
$LB1 = 16, LB2 = 16, LB = 16$;

$E = \{5,3,8,4\}$

$x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$

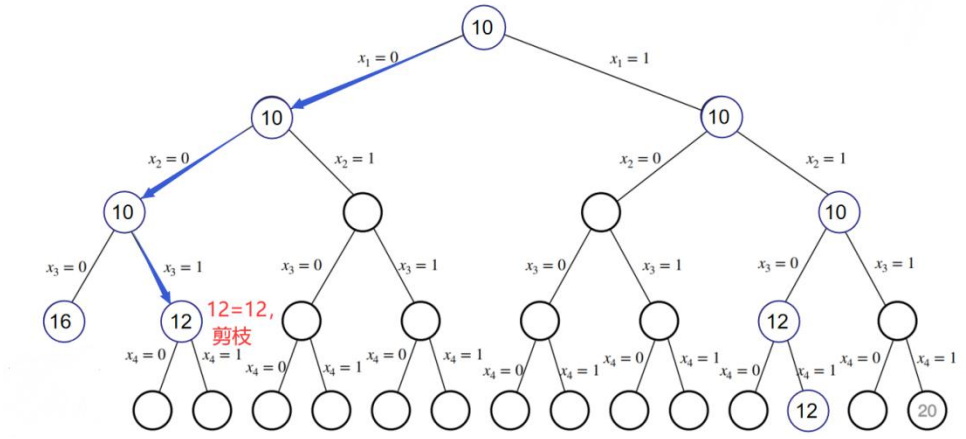


16>12 从而进行广度遍历,

看 $x1=0, x2=0, x3=1$ 时,the remaining jobs is $\{4\}$, $\max_i p_i = 4$,
$\max_i \{\text{load(processor i)}\} = 8, \min_i \{\text{load(processor i)} + \max_i p_i\} = 12$,
$LB1 = 12, LB2 = 10, LB = 12$;

$E = \{5,3,8,4\}$

$x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$

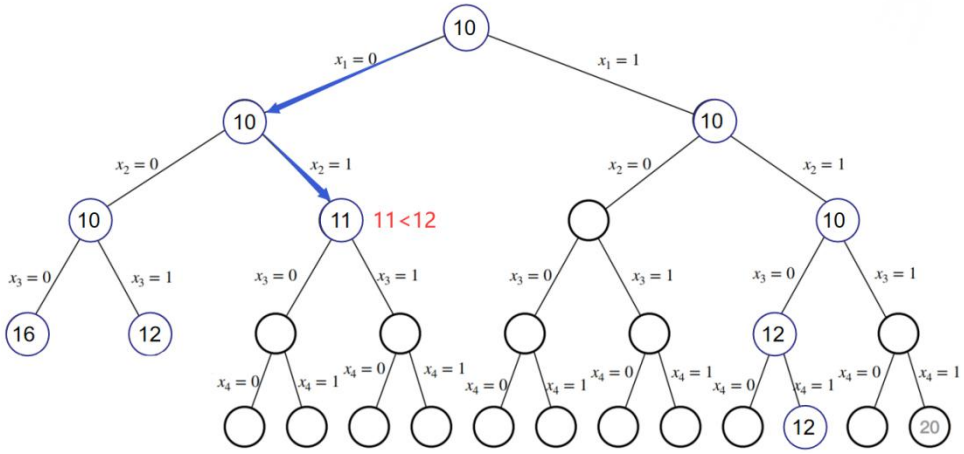


12=12 从而进行广度遍历,

看 $x1=0, x2=1$ 时,the remaining jobs is $\{8,4\}$, $\max_i p_i = 8$,
$\max_i \{\text{load(processor i)}\} = 5, \min_i \{\text{load(processor i)} + \max_i p_i\} = 11$,
$LB1 = 11, LB2 = 10, LB = 11$;

$E = \{5,3,8,4\}$

$x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$



11<12 继续深度遍历,

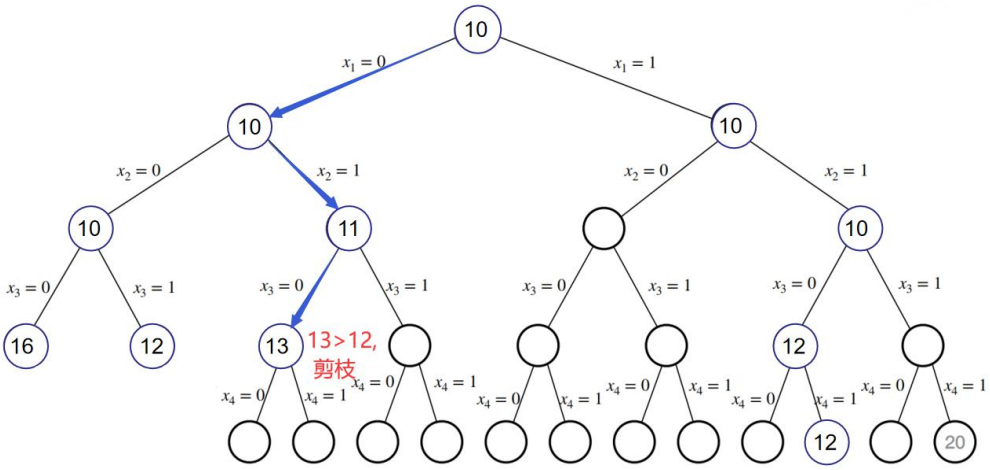
看 $x1 = 0, x2 = 1, x3 = 0$ 时,the remaining jobs is $\{4\}$, $\max_i p_i = 4$,

$\max_i \{\text{load(processor i)}\} = 13, \min_i \{\text{load(processor i)} + \max_i p_i\} = 7$,

$LB1 = 13, LB2 = 13, LB = 13;$

$E = \{5,3,8,4\}$

$x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$



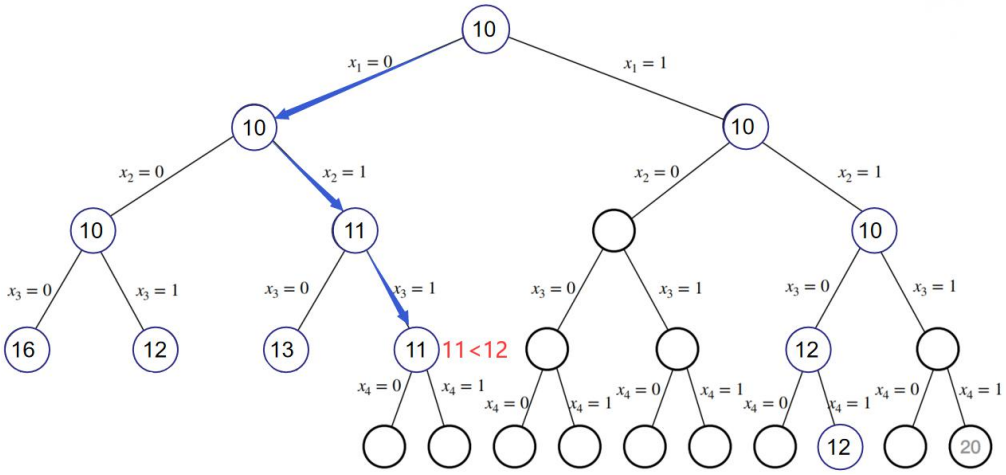
13>12 从而进行广度遍历,

看 $x1 = 0, x2 = 1, x3 = 1$ 时,the remaining jobs is $\{4\}$, $\max_i p_i = 4$,

$\max_i \{\text{load(processor i)}\} = 11, \min_i \{\text{load(processor i)} + \max_i p_i\} = 9$,

$LB1 = 11, LB2 = 11, LB = 11;$

$E = \{5,3,8,4\}$ $x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$

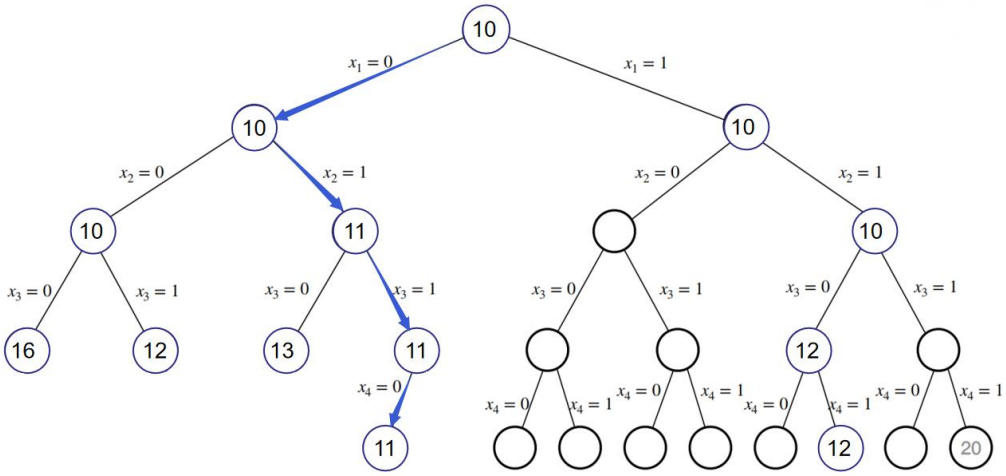


11<12 继续深度遍历,

看 $x1 = 0, x2 = 1, x3 = 1, x3 = 0$ 时,

$\max_i\{\text{load(processor i)}\} = 11, \text{LB} = 11;$

$E = \{5,3,8,4\}$ $x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$



从而得到一个左子树的最优解 $x1 = 0, x2 = 1, x3 = 1, x3 = 0$;

下面，看 $x1 = 1$ 的 branch，

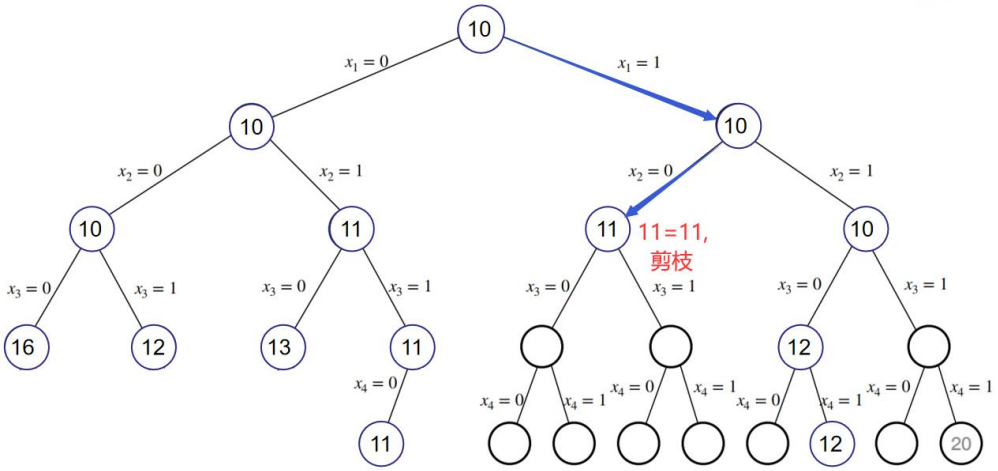
看 $x1 = 1, x2 = 0$ 时,the remaining jobs is $\{8,4\}$, $\max_i p_i = 8$,

$\max_i\{\text{load(processor i)}\} = 5, \min_i\{\text{load(processor i)} + \max_i p_i\} = 11,$

$\text{LB1} = 11, \text{LB2} = 10, \text{LB} = 11;$

$E = \{5,3,8,4\}$

$x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$



11=11 从而进行广度遍历,

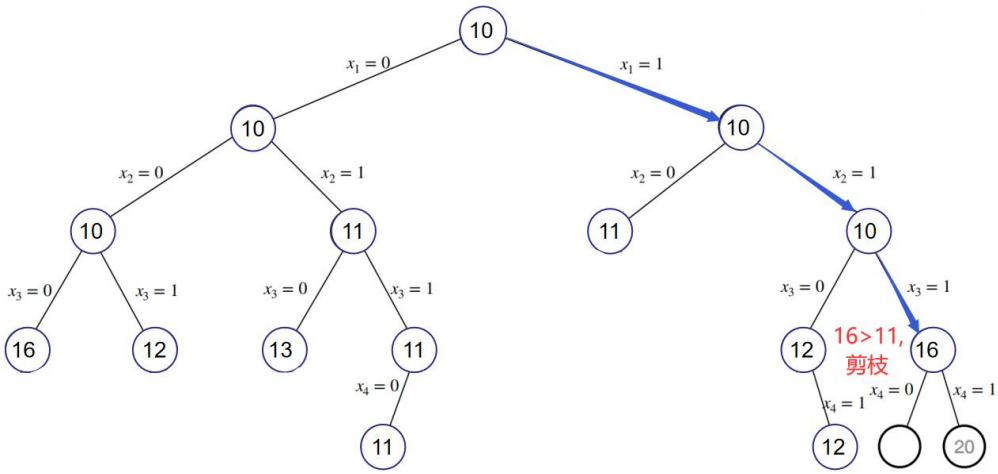
看 $x1 = 1, x2 = 1, x3 = 1$ 时,the remaining jobs is $\{4\}$,$\max_i p_i = 4$,

$\max_i \{\text{load(processor i)}\} = 16, \min_i \{\text{load(processor i)} + \max_i p_i\} = 4$,

$\text{LB1} = 16, \text{LB2} = 16, \text{LB} = 16$;

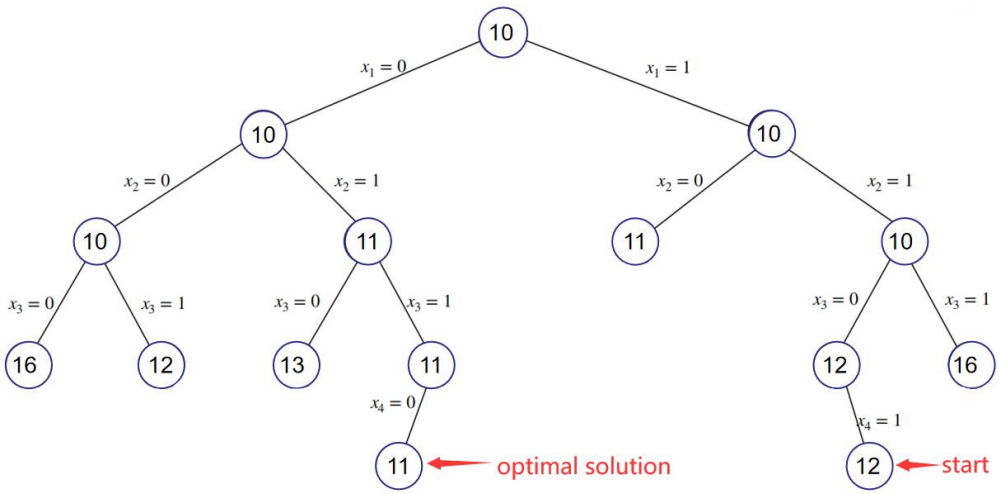
$E = \{5,3,8,4\}$

$x_i = \begin{cases} 1 \text{ if job i is assigned to processor 1} \\ 0 \text{ otherwise (if job i is assigned to processor 2)} \end{cases}$



从而得到最优解为 $x1 = 0, x2 = 1, x3 = 1, x4 = 0$;

The tree of the Branch & Bound:



The optimal solution:

processor 1 : 3 8

processor 2 : 5 4

即 minimum load=11;

2) The order of the nodes I explored:

$x1 = 0, x2 = 1, x3 = 1, x4 = 0$