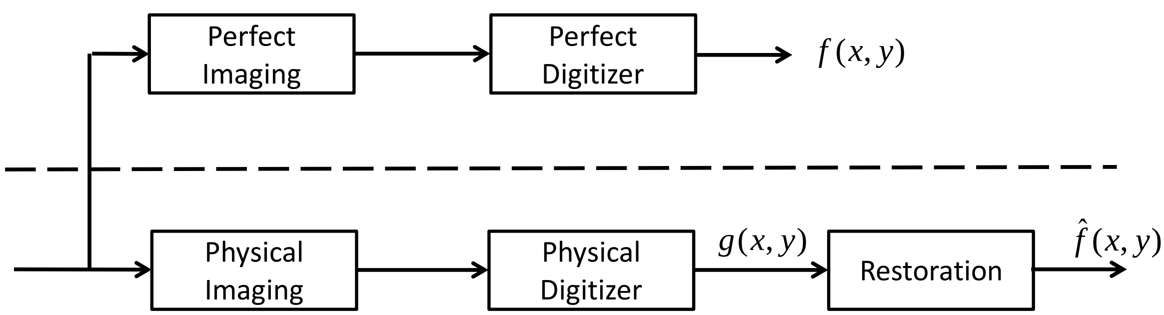
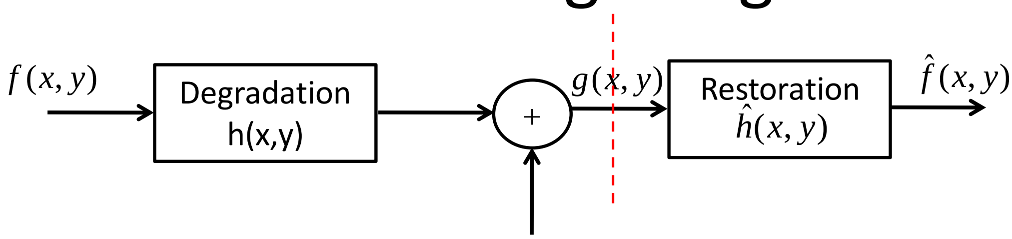
图像复原
Image Restoration

Imaging Modelling



线性模糊模型
Linear Model of Image Degradation



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v) \cdot F(u,v) + N(u,v)$$

$$\hat{f}(x,y) = \hat{h}(x,y) * g(x,y) = \hat{h}(x,y) * [h(x,y) * f(x,y) + \eta(x,y)]$$

$$\hat{F}(u,v) = \hat{H}(u,v) \cdot G(u,v) = \hat{H}(u,v) \cdot [H(u,v) \cdot F(u,v) + N(u,v)]$$

逆滤波（Inverse Filter）

$$\hat{F}(u,v)=\hat{H}(u,v)\cdot G(u,v)=\hat{H}(u,v)\cdot[H(u,v)\cdot F(u,v)+N(u,v)]$$

$$if \quad \hat{H}(u,v)\cdot H(u,v)=1 \Longrightarrow \hat{H}(u,v)=\frac{1}{H(u,v)}$$

$$\boxed{\hat{F}(u,v)=F(u,v)+\frac{N(u,v)}{H(u,v)}}$$

Wiener Filter

$$E\{|\,f(x,y)-\hat{f}(x,y)\,|^2\}\rightarrow \min$$

$$\hat{F}(u,v)=\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2+S_n(u,v)}G(u,v)$$

$$\boxed{\hat{F}(u,v)=\frac{H^*(u,v)}{\left|H(u,v)\right|^2+S_n(u,v)/S_f(u,v)}G(u,v)}$$

$$S_n(u,v)=\left|N(u,v)\right|^2$$

$$S_f(u,v)=\left|F(u,v)\right|^2$$

$$K=S_n(u,v)/S_f(u,v)$$

运动模糊（匀速直线运动）
Model of Uniform Linear Motion

$$g(x) = \int_0^{\tau} f(x - vt)dt$$

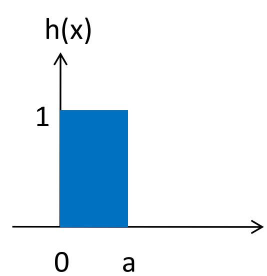
τ: exposure time / shutter opened
v : velocity of motion

$$g(x) = \int_0^{a} f(x - t)dt$$

a : distance

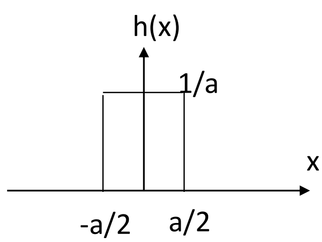
$$g(x) = h(x) * f(x)$$

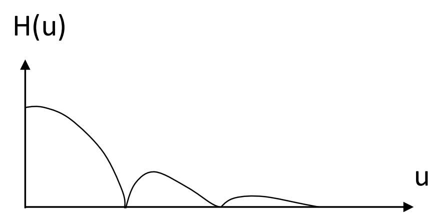
$$h(x) = \begin{cases} 1 & (0 \le x \le a) \\ 0 & \text{otherwise} \end{cases}$$



运动参数估计
Estimation of Motion Distance

Cannon Method





$$H(u)=\int_{-\mathrm{a}/2}^{\mathrm{a}/2}\frac{1}{\mathrm{a}}e^{-j2\pi\mathrm{ux}}dx=\frac{1}{-j2\pi\pi\mathrm{u}}e^{-j2\pi\mathrm{ua}}\Big|_{-a/2}^{a/2}=\frac{e^{-j\pi\mathrm{ua}}-e^{j\pi\mathrm{ua}}}{-j2\pi\pi\mathrm{u}}$$

$$H(u)=\frac{\sin(\pi\mathrm{ua})}{\pi\mathrm{ua}}=\mathrm{sinc}(\pi\mathrm{au})$$

Identification of blur parameters from motion blurred images

Y. Yitzhaky and N.S. Kopeika

Graphical Models and Image Processing, 59(5), 1997:310-320

Estimation of a correct point spread function (PSF)

f(x,y) : given ideal image
g(x,y) : the motion blurred image
h(x,y) : linear shift-invariant PSF
n(x,y) : random noise

degradation model:

$$g(x,y) = \iint h(x-x',y-y')\,f(x',y')\,dx'\,dy' + n(x,y)$$

assumption:

Exposure time : about 1/30 s
→ no extreme changes in motion velocity

Identification of Motion Direction

Motion Blurring → Low pass filter

⇒ intensity at low frequencies increased
resolution decreased in motion direction

derivative → High pass filter

⇒ intensity at low frequencies suppressed
intensity at high frequencies increased

⇒ Derivative in motion direction should suppress more of image intensity than that in other directions

motion direction → lower power spectrum of derivative

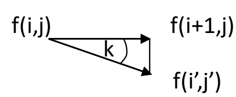
implementation of derivative

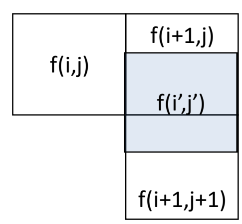
$$\Delta f(i,j)_{[0°]} = f(i+1,j) - f(i,j)$$

$$\Delta f(i,j)_{[k°]} = f(i',j') - f(i,j)$$

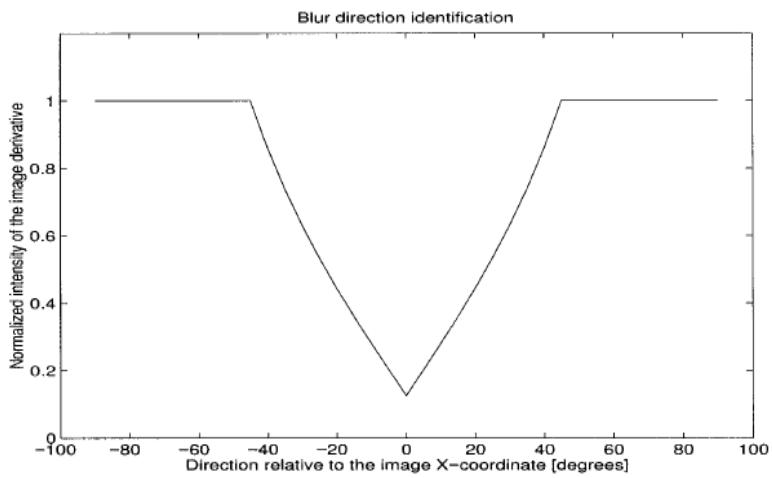
$0 \le k \le 45°$

$$f(i',j') = f(i+1,j)\cdot(1-\tan(k)) + f(i+1,j+1)\cdot\tan(k)$$



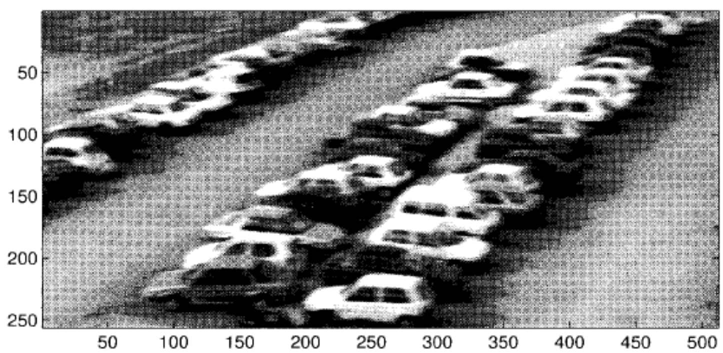


$$I(\Delta f)_{[k°]} = \sum_i \sum_j \left|\Delta f(i,j)_{[k°]}\right|$$

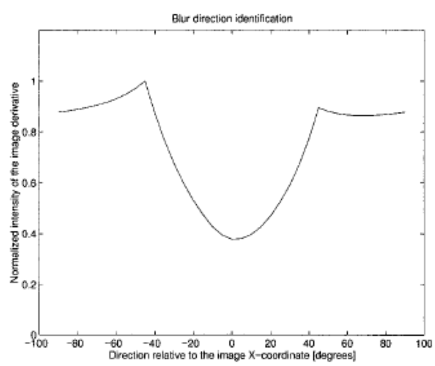


$$I(\Delta f)_{[k^\circ]} = \sum_i \sum_j \left| \Delta f(i, j)_{[k^\circ]} \right|$$

blurred image



50
100
150
200
250
50 100 150 200 250 300 350 400 450 500



Blur direction identification
Normalized intensity of the image derivative
Direction relative to the image X-coordinate [degrees]