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Reconstruction of Complex Sparse Signals in Compressed Sensing with Real Sensing Matrices

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Abstract The existing greedy algorithms for the reconstruction in compressed sensing were designed no matter which type the original sparse signals and sensing matrices have, real or complex. The reconstruction algorithms definitely apply to real sensing matrices and complex sparse signals, but they are not customized to this situation so that we could improve those algorithms further. In this paper, we elaborate on the compressed sensing with real sensing matrices when the original sparse signals are complex. We propose two reconstruction algorithms by modifying the orthogonal matching pursuit to include some procedures specialized to this setting. It is shown via analysis and simulation that the proposed algorithms have better reconstruction success probability than conventional reconstruction algorithms.

Keywords Complex sparse signals · Compressed sensing · Orthogonal matching pursuit (OMP) · Real sensing matrices

1 Introduction

Compressed sensing [1–9] simultaneously performs sampling and compression for sparse signals and thus results in a small number of samples compared to the samples acquired at the Nyquist rate. The resulting compressed signal is composed of linear combinations of elements in the original sparse signal. Since the number of linear equations is less than the number of unknown values, perfectly reconstructing the original signal from the

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compressed signal is impossible in general, but this ill-posed problem can be solved when the signal has sparsity. Accordingly, one of the major issues in compressed sensing is how to accurately and efficiently reconstruct the original sparse signal from the compressed signal.

Since compressed sensing appeared for the first time in the mid 2000's [1–4], many reconstruction algorithms have been being developed. Basis pursuit (BP) [1] was proposed to reconstruct the original signal by casting the reconstruction problem into an ℓ_1-minimization problem and then finding the solution with the linear programming. As a greedy algorithm, the orthogonal matching pursuit (OMP) algorithm was proposed [10], which uses correlation values between the compressed signal and each column in the sensing matrix to find out the location of nonzero components in the original signal. Also, many variants of OMP have been proposed such as regularized OMP (ROMP) [11], compressive sampling matching pursuit (CoSaMP) [12], subspace pursuit (SP) [13], and so on [14, 15]. Note that these greedy algorithms were basically designed no matter which type the original sparse signals and sensing matrices have, real or complex.

Real sparse signals and real sensing matrices were usually considered in most initial works on compressed sensing. Also, complex sparse signals frequently arise in many practical applications such as terahertz imaging [16], synthetic aperture radar [17] and sonar [18], and magnetic resonance imaging [19]. Accordingly, compressed sensing for complex sparse signals has been intensively investigated so that ℓ_p-minimization can be also used in this setting like the BP for real sparse signals [20–24]. As pointed out in the above, the greedy reconstruction algorithms can be directly used for complex sparse signals and any modification is not needed. It is noted that all these reconstruction algorithms for complex sparse signals generally assume complex sensing matrices such as the partial Fourier matrices and the partial complex Hadamard matrices.

On the other hand, the advantages of real sensing matrices over complex sensing matrices are low computational complexity in reconstructing the original signals and small memory space to store the sensing matrices in the system. Many researches have been also done to construct good real sensing matrices and some good examples of real sensing matrices are random matrices whose elements are driven from the Gaussian or the Bernoulli distribution [3], deterministic matrices constructed from binary error-correcting codes or binary pseudo-random sequences [25, 26], and ternary deterministic matrices [27].

In this paper, we investigate the compressed sensing with real sensing matrices for complex sparse signals. For this situation, the existing greedy algorithms can be directly used but they are not customized and need improvement to have better performance. Two reconstruction algorithms are proposed by making the conventional OMP algorithm improve toward our situation. The basic idea of the proposed algorithms is that with real sensing matrices, the compressed sensing for complex sparse signals can be seen as the compressed sensing for two independent real sparse signals with the common locations of nonzero elements unlike [20–24], which is called jointly sparse [28]. More detailedly, the real (imaginary) part of the complex compressed signal is generated from only the real (imaginary) part of the complex sparse signal because the sensing matrix is real. Hence, reconstructing the original complex sparse signal from the complex compressed signal can be viewed as separately reconstructing two real sparse signals sharing the common nonzero positions from two real compressed signals. In reconstructing both real sparse signals, information on the nonzero positions can be shared between the two reconstructions, which is the key factor to make the reconstruction performance improve. We implement this idea into two proposed algorithms and they are justified through a brief analysis. Simulation results show that the proposed algorithms have better performance than conventional reconstruction algorithms for real sensing matrices and complex sparse signals.

2 Preliminaries

Let $x \in \mathbb{C}^N$ denote a complex sparse signal of dimension N, which has at most K nonzero elements. We will call this a *K-sparse* signal. Let $y \in \mathbb{C}^M$ denote a complex compressed signal of dimension M, where $M < N$. Each element of the compressed signal corresponds to a linear combination of elements in x, that is,

$$y = \Phi x,$$

where Φ is called a sensing matrix. In this paper, we assume that all elements of $\Phi \in \mathbb{R}^{M \times N}$ are real.

Algorithm 1. The conventional OMP algorithm; $(r_K, x_K, \Lambda_K) = \mathrm{COMP}(y, K)$

1. Initialize : $r_0 = y$, $\Lambda_0 = \emptyset$, $t = 1$.
2. Correlation : $h_{t-1} = \Phi^H r_{t-1}$.
3. Identification : $\lambda_t = \arg\max_{j=1,\ldots,N} |h_{t-1}(j)|$.
4. Augment the index set : $\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}$.
5. Construct Φ_t : $\Phi_t = \Phi S_{\Lambda_t}^T$, where S_{Λ_t} is the selection matrix consisting of rows (indices from Λ_t) of I.
6. Least squares : $x_t = (\Phi_t^H \Phi_t)^{-1} \Phi_t^H y$.
7. Update current residual : $a_t = \Phi_t x_t$, $r_t = y - a_t$.
8. return to 2) and $t = t + 1$, if $t < K$.

Along with constructing good sensing matrices, reconstructing x from a given y is the most important research topic in compressed sensing. OMP [10] is a greedy reconstruction algorithm which is one of the most widely used in compressed sensing. It uses correlation values between y and each column of Φ when it guesses which positions of x the nonzero elements located in and it is generally faster than BP even with not worse reconstruction accuracy. Algorithm 1 shows the detailed procedure of the OMP algorithm. Using the correlation information, the OMP algorithm pursuits the indices of nonzero elements of x one by one in a greedy manner and then the corresponding nonzero values are calculated by using the least squares. In Algorithm 1, Λ_t represents the estimated nonzero indices of x at the t-th iteration. Also, r_t and a_t represent the residual signal and the estimated compressed signal at the t-th iteration, respectively. At each iteration, the current nonzero index λ_t is determined by choosing the maximum element of the correlation vector h_{t-1}. Note that the OMP algorithm consistently operates no matter if the original sparse signals and sensing matrices are real or complex.

3 New Reconstruction Algorithms with Real Sensing Matrices for Complex Sparse Signals

For complex sparse signals and complex sensing matrices, the equation $y = \Phi x$ can be represented using the real and the imaginary parts as the following [20].

$$\Re\{y\} = \begin{bmatrix} \Re\{\Phi\} & -\Im\{\Phi\} \end{bmatrix} \begin{bmatrix} \Re\{x\} \\ \Im\{x\} \end{bmatrix}$$
$$\Im\{y\} = \begin{bmatrix} -\Im\{\Phi\} & \Re\{\Phi\} \end{bmatrix} \begin{bmatrix} \Re\{x\} \\ \Im\{x\} \end{bmatrix}$$

With this formulation, the complex-valued compressed sensing is transformed into two real-valued compressed sensing problems as mentioned in [20] and we can see that each part of y is related to both the real and imaginary parts of x, which is different from our case.

Algorithm 2. The proposed OMP algorithm 1; $(r_K, x_K, \Lambda_K) = \mathrm{POMP1}(y, K)$

1. Individual reconstruction for real part : $(r_K^{\Re}, x_K^{\Re}, \Lambda_K^{\Re}) = \mathrm{COMP}(\Re\{y\}, K)$.
2. Individual reconstruction for imaginary part : $(r_K^{\Im}, x_K^{\Im}, \Lambda_K^{\Im}) = \mathrm{COMP}(\Im\{y\}, K)$.
3. Compare two residuals :
   $\Lambda_K = \Lambda_K^{\Re}$, if $||r_K^{\Re}||_2^2/||\Re\{y\}||_2^2 < ||r_K^{\Im}||_2^2/||\Im\{y\}||_2^2$.
   $\Lambda_K = \Lambda_K^{\Im}$, if $||r_K^{\Re}||_2^2/||\Re\{y\}||_2^2 \geq ||r_K^{\Im}||_2^2/||\Im\{y\}||_2^2$.
4. Construct Φ_K : $\Phi_K = \Phi S_{\Lambda_K}^T$.
5. Least squares : $x_K = (\Phi_K^H \Phi_K)^{-1} \Phi_K^H y$.
6. Update current residual : $a_K = \Phi_K x_K, r_K = y - a_K$.

Under our assumption, that is, with real sensing matrices and complex sparse signals, the above equations reduce to the following form.

$$\Re\{y\} = \Phi \Re\{x\}$$
$$\Im\{y\} = \Phi \Im\{x\}$$

Since the sensing matrix is real, the real (imaginary) part of y is related to only the real (imaginary) part of x, which ultimately differentiates our work from the setting in [20–24]. Based on this perfect separation of the real and imaginary parts, it is possible to think of this problem as the compressed sensing with the real sensing matrix for two independent real sparse signals which have the common positions of nonzero elements. It is noted that this joint sparsity originates from the fact that the real and imaginary are also K-sparse if the complex original signal is K-sparse. Thus, when reconstructing the original signal, this joint sparsity can be exploited to improve the accuracy of the reconstruction unlike [20–24] and we propose two reconstruction algorithms as follows.

Our first algorithm is proposed in Algorithm 2 and it involves two separate reconstructions of $\Re\{x\}$ and $\Im\{x\}$ from $\Re\{y\}$ and $\Im\{y\}$, respectively, and each reconstruction uses the conventional OMP algorithm with real sensing matrices for real sparse signals. Then, we fully trust the estimated index set of nonzero elements in only one part with a smaller energy of the final residual signal. The fact that a smaller residual energy implies more reliable reconstruction motivates this algorithm.

In Steps 1 and 2 of Algorithm 2, $\Re\{x\}$ and $\Im\{x\}$ are reconstructed from $\Re\{y\}$ and $\Im\{y\}$, respectively. In Step 3, we compare both residuals after suitably normalizing them for fairness. Then, the remaining Steps 4–6 are performed to complete our reconstruction.

Algorithm 3. The proposed OMP algorithm 2; $(r_K, x_K, \Lambda_K) = \mathrm{POMP2}(y, K)$

1. Initialize : $r_0 = y$, $\Lambda_0 = \emptyset$, $t = 1$.
2. Correlation : $h_{t-1} = \Phi^H r_{t-1}$.
3. Refine correlation : $\tilde{h}_{t-1}(j) = \max\{|\Re\{h_{t-1}(j)\}|/||\Re\{y\}||_2, |\Im\{h_{t-1}(j)\}|/||\Im\{y\}||_2\}$
4. Identification : $\lambda_t = \arg\max_{j=1,\ldots,N} |\tilde{h}_{t-1}(j)|$.
   (Remaining steps are same with Algorithm 1.)

Our second reconstruction algorithm is proposed in Algorithm 3. If the conventional OMP algorithm, Algorithm 1, is still used for our settings, the correlation vector h_{t-1} generally has complex values, where its real part $\Re\{h_{t-1}\}$ (imaginary part $\Im\{h_{t-1}\}$) is obtained only from the real part of the current residual (the imaginary part of the current residual). The correlation values are shared along with the two parts and we introduce a new measure $\tilde{h}_{t-1}(j)$ refined from the correlation h_{t-1}. To select the most likely nonzero index, the proposed measure separately takes real and imaginary components of the complex correlation values into consideration while the conventional OMP uses the magnitude of each complex correlation value. It is noted that each component should be normalized before selecting the maximum to correctly guess a nonzero index.

Although there are many advanced greedy algorithms extended from the conventional OMP algorithm such as CoSaMP, ROMP, and SP, we consider only the original OMP algorithm as a basis of our algorithms, which is enough to show the effectiveness of our proposed algorithms because the key idea and the effectiveness of our approach can straightforwardly apply to the other algorithms.

In [28–34], many reconstruction algorithms for jointly sparse signals have been proposed assuming sparse signals and sensing matrices are generally complex and they can be also applied to real signals and matrices. Our proposed reconstruction algorithms apparently exploit the joint sparsity of the two real sparse signals and our setting can be actually interpreted as a special case of the compressed sensing of jointly sparse signals. In [31], p-simultaneous OMP (p-SOMP) is proposed and it uses the p-norm of the multiple correlation values of each coordinate when we choose the most likely nonzero position in the algorithm. It is noted that Algorithm 3 is similar to the case of p-SOMP with $p = \infty$ for two real jointly sparse signals but they are different in that we normalize the correlation values in the refining process. As we will see in Sect. 5, this normalization is very effective when the complex signals are asymmetrically generated, that is, the real and imaginary parts of the sparse signals have quite different distributions. Similarly, the conventional OMP with real sensing matrices for complex sparse signals can be also viewed as the case of p-SOMP with $p = 2$ for two real jointly sparse signals because the ℓ_2-norms of complex correlation values are compared in choosing the maximum.

4 Justification of the Proposed Algorithms

To justify the effectiveness of the proposed algorithms, this section briefly gives analytic explanations why the proposed algorithms have better reconstruction performance than the conventional OMP algorithm. For p-SOMP, analysis on the relation of sparsity and reconstruction success probability is shown in [31] in terms of the maximum correlation value between the columns in Φ. Here, we adopt a different approach which just compares the reconstruction success probabilities of the proposed algorithms and the conventional

OMP algorithm without deriving them in a closed form. For simplicity, we assume that the real and imaginary parts of y have the same distribution.

4.1 The Proposed OMP Algorithm 1

Intuitively, the proposed OMP Algorithm 1 is expected to have superior reconstruction performance to the conventional one because of the principle of *diversity*. We can simply show this as follows.

Let p_{CONV} and p_{PROP1} denote the reconstruction success probabilities of the conventional OMP algorithm and the proposed Algorithm 1, respectively. It is observed that the conventional OMP algorithm for complex sparse signals has almost the same reconstruction success probability with that for real sparse signal. Then, the reconstruction success probability of the proposed OMP Algorithm 1 is expressed as

$$p_{PROP1} \simeq 1 - (1 - p_{CONV})^2 > p_{CONV}, \tag{1}$$

because the real and imaginary parts are independently generated. In Fig. 1, we can see that the success probabilities of the proposed OMP Algorithm 1 and the conventional OMP algorithm actually follow the Eq. (1).

4.2 The Proposed OMP Algorithm 2

Let $x_i \in \mathbb{C},\ i = 1, \ldots, N$, denote the i-th component of x. Then, it is expressed as $x_i = c_i + jd_i$, where $c_i, d_i \in \mathbb{R}$. Also, let $\varphi_i \in \mathbb{R}^M$ denote the i-th column of Φ. Without loss of generality (WLOG), we can assume that the K nonzero elements of x locate on the first K positions of x and then a complex compressed signal y is expressed as

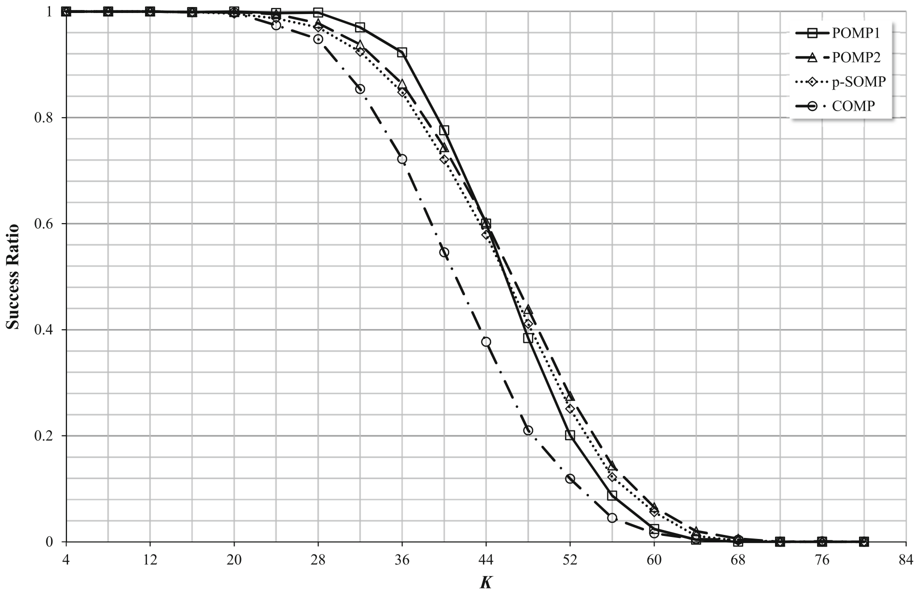

Fig. 1 The reconstruction success probability with the 128 × 256 random sensing matrix drawn from the Bernoulli distribution

$$y = \sum_{i=1}^{K} c_i \varphi_i + j \sum_{i=1}^{K} d_i \varphi_i.$$

Now, we assume that each element of Φ is real-valued and i.i.d. randomly generated with zero mean from any distribution, and every column is normalized, i.e., $\varphi_i^H \varphi_i = 1$. By the central limit theorem, $\varphi_i^H \varphi_j$, $i \neq j$, can be thought of as a random variable drawn by a zero-mean Gaussian distribution for large M. For most good sensing matrices, the values of $\varphi_i^H \varphi_j$, $i \neq j$, are much smaller than one.

As we explained, the conventional OMP algorithm pursuits the indices of nonzero elements of x one by one using correlation information between the residual vector r_{t-1} and the columns of Φ. That is, the probability of correctly finding the first nonzero component (WLOG assumed to locate in the first position of x) can be written as

$$p_{CONV}^1 = P\left(\left|\varphi_j^H y\right| < \left|\varphi_1^H y\right|\right)$$

for $j \notin \{1, 2, \ldots, K\}$. It can be rewritten as

$$p_{CONV}^1 = P\left(\left|\sum_{i=1}^{K} c_i \varphi_j^H \varphi_i + j \sum_{i=1}^{K} d_i \varphi_j^H \varphi_i\right| < \left|c_1 + j d_1 + \sum_{i=2}^{K} c_i \varphi_1^H \varphi_i + j \sum_{i=2}^{K} d_i \varphi_1^H \varphi_i\right|\right).$$

In the RHS of the above equation, the magnitude of $c_1 + jd_1$ are much larger than those of the remaining terms. Then, we have

$$p_{CONV}^1 \simeq P\left(\left|\sum_{i=1}^{K} c_i \varphi_j^H \varphi_i + j \sum_{i=1}^{K} d_i \varphi_j^H \varphi_i\right| < |c_1 + j d_1|\right).$$

By letting $b_i = c_i + jd_i$, it reduces to

$$p_{CONV}^1 \simeq P\left(\left|\sum_{i=1}^{K} b_i \varphi_j^H \varphi_i\right| < |b_1|\right).$$

The proposed OMP Algorithm 2 compares the correlation values of real and imaginary parts and the probability of correctly finding the first nonzero component can be written as

$$p_{PROP2}^1 \simeq P\left(\max\left(\frac{\left|\sum_{i=1}^{K} c_i \varphi_j^H \varphi_i\right|}{\sqrt{\sum_{i=1}^{K} c_i^2}}, \frac{\left|\sum_{i=1}^{K} d_i \varphi_j^H \varphi_i\right|}{\sqrt{\sum_{i=1}^{K} d_i^2}}\right) < \max\left(\frac{|c_1|}{\sqrt{\sum_{i=1}^{K} c_i^2}}, \frac{|d_1|}{\sqrt{\sum_{i=1}^{K} d_i^2}}\right)\right)$$

for $j \notin \{1, 2, \ldots, K\}$. Since in the LHS $\left|\sum_{i=1}^{K} c_i \varphi_j^H \varphi_i\right| / \sqrt{\sum_{i=1}^{K} c_i^2}$ and $\left|\sum_{i=1}^{K} d_i \varphi_j^H \varphi_i\right| / \sqrt{\sum_{i=1}^{K} d_i^2}$ are correlated each other while $|c_1| / \sqrt{\sum_{i=1}^{K} c_i^2}$ and $|d_1| / \sqrt{\sum_{i=1}^{K} d_i^2}$ in the RHS are independent, we obtain

$$p^1_{PROP2} > P\left(\left|\sum_{i=1}^{K} c_i \varphi_j^H \varphi_i\right| < |c_1|\right) \simeq p^1_{CONV},$$

where the observation in the previous subsection is also used. The inequalities for the remaining $K-1$ nonzero components can be derived in the same manner, which result in $p^2_{PROP2} > p^2_{CONV}, \ldots, p^K_{PROP2} > p^K_{CONV}$. Clearly, the reconstruction success probability of the proposed OMP Algorithm 2 is larger than that of the conventional OMP algorithm.

5 Simulation Results

To verify the effectiveness of our proposed algorithms, we via simulations compare their reconstruction success probabilities with the conventional OMP algorithm and the p-SOMP with $p = \infty$. Simulation is performed with three kinds of real sensing matrices. The first sensing matrix has the size 128×256 and its elements are randomly generated from the values $\{+1, -1\}$ with equal probability. The second sensing matrix has the size 256×512 and its elements are randomly generated from the the Gaussian distribution $\mathcal{N}(0, 1^2)$. The third sensing matrix has the size 63×512 and its elements have $\{+1, -1\}$ values by mapping binary Kasami sequence of length 63 to columns of the sensing matrix [26]. To generate K-sparse signals x, we first choose the positions of nonzero elements uniformly at random and then both real and imaginary values of each position are independently drawn from the Gaussian distribution $\mathcal{N}(0, 1^2)$. We perform 1000 independent trials at each simulation point to evaluate the reconstruction success probability. Reconstruction success is declared when

$$\frac{|x - \hat{x}|^2}{|x|^2} < 10^{-5}.$$

Figure 1 shows the reconstruction success probability with respect to the sparsity K when the 128×256 random sensing matrix drawn from the Bernoulli distribution. In the legend, POMP1, POMP2, p-SOMP, and COMP are used to represent the proposed OMP Algorithm 1, the proposed OMP Algorithm 2, the p-SOMP with $p = \infty$, and the conventional OMP algorithm, respectively. In the plot, POMP1 and POMP2 show better reconstruction performance than COMP and the performance of POMP2 is slightly better than that of p-SOMP due to the normalization in the correlation computation.

Figure 2 shows the reconstruction success probability with respect to K when the 256×512 random sensing matrix generated from the Gaussian distribution. Similarly to Fig. 1, POMP1 and POMP2 show better reconstruction performance than COMP and the performance of POMP2 is slightly better than that of p-SOMP.

Figure 3 shows the reconstruction success probability with respect to K when the 63×512 sensing matrix constructed from Kasami sequences is used. Similarly to Figs. 1 and 2, POMP1 and POMP2 show better reconstruction performance than COMP and the performance of POMP2 is slightly better than that of p-SOMP.

Finally, to show the validity of the normalization in Algorithm 3, we also simulate to evaluate the reconstruction success probability of Algorithm 3 without the normalization for the complex sparse signals drawn from some asymmetric complex Gaussian distributions, which means that the variance of the real value is different from that of the imaginary value. Figure 4 shows the reconstruction success probability for this case and

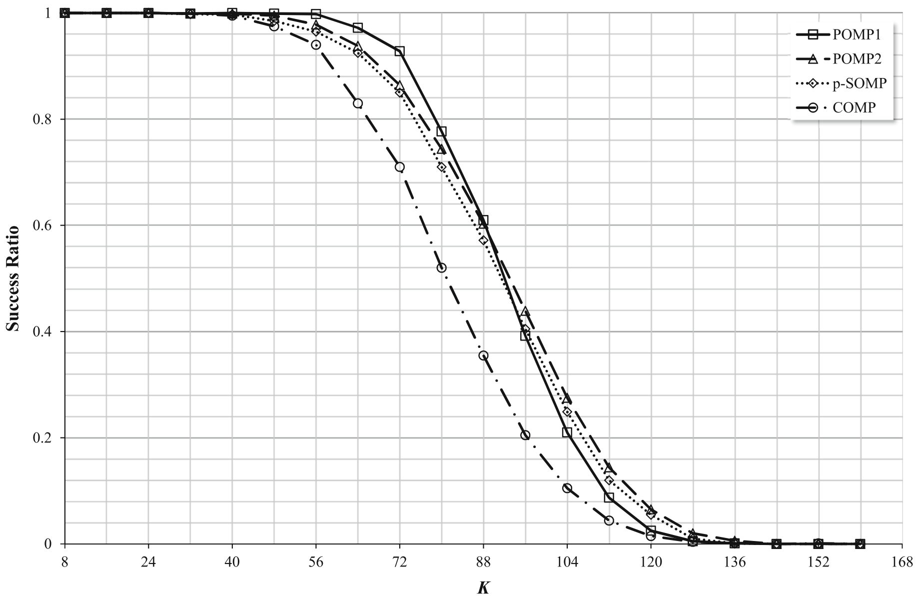

Fig. 2 The reconstruction success probability with the 256 $\times$ 512 random sensing matrix drawn from the Gaussian distribution

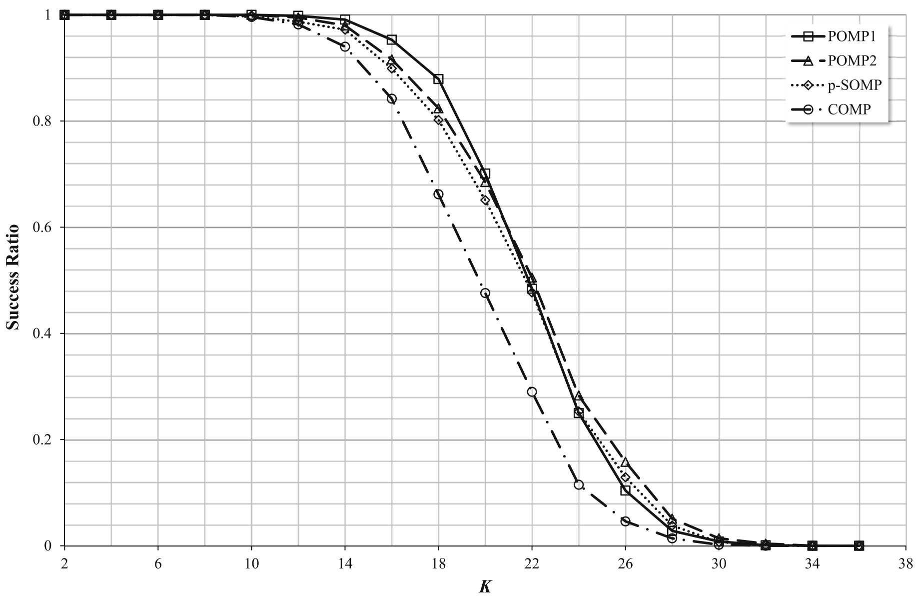

Fig. 3 The reconstruction success probability with the 63 $\times$ 512 sensing matrix constructed from Kasami sequences

POMP2_1, POMP2_4, POMP2_16, and POMP_64 represent the proposed Algorithm 2 without the normalization for the complex sparse signals whose imaginary values are drawn from $\mathcal{N}(0, 1^2)$ in common and whose real values are drawn from $\mathcal{N}(0, 1^2)$,

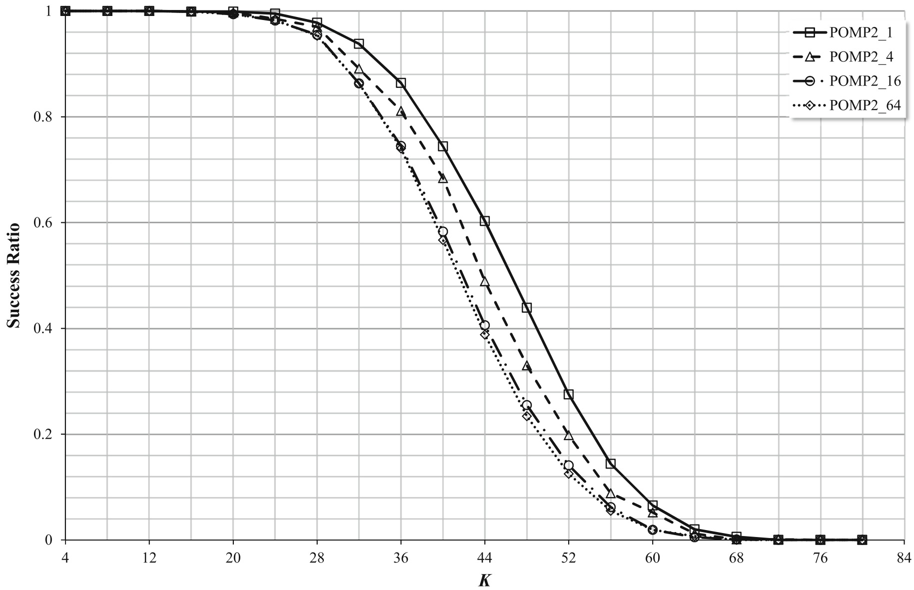

Fig. 4 The reconstruction success probability with the 128 × 256 random sensing matrix drawn from the Bernoulli distribution for the asymmetric complex sparse signals

$\mathcal{N}(0, 2^2)$, $\mathcal{N}(0, 4^2)$, and $\mathcal{N}(0, 8^2)$, respectively. We can see that the asymmetry gives rise to performance degradation when the normalization is not used. Note that when using the normalization in Algorithm 3, the reconstruction performance for all the asymmetric complex sparse signals is the same as that of POMP2_1. Therefore, the proposed Algorithm 2 can work very well for asymmetric complex sparse signals with the aid of the normalization process.

6 Conclusions

We propose two reconstruction algorithms modified from the conventional OMP so that they can have better reconstruction performance for complex sparse signals when real sensing matrices are used. The proposed algorithms are based on the ideas that the complex compressed signals are easily divided into real and imaginary parts and can be regarded as two real compressed signals generated from two independent real sparse signals which share the same nonzero positions. Simulation results show that the proposed algorithms show better reconstruction accuracy than the conventional OMP algorithm and the p-SOMP with $p = \infty$. Also, the normalization of the correlation values in the proposed algorithms are verified through the simulation for the asymmetric complex sparse signals.

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References

1. Donoho, D. L. (2006). Compressed sensing. *IEEE Transactions on Information Theory*, *52*(4), 1289–1306.
2. Candes, E. J., & Tao, T. (2005). Decoding by linear programming. *IEEE Transactions on Information Theory*, *51*(12), 4203–4215.
3. Candes, E. J., & Tao, T. (2006). Near optimal signal recovery from random projections: Universal encoding strategies? *IEEE Transactions on Information Theory*, *52*(12), 5406–5425.
4. Candes, E. J., Romberg, J., & Tao, T. (2006). Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information. *IEEE Transactions on Information Theory*, *52*(2), 489–509.
5. Dai, L., Cui, C., Yu, J., & Liang, H. (2015). Sensing matrix restriction method for compressed sensing radar. *Wireless Personal Communications*, *84*(1), 605–621.
6. Zhang, Y., Fu, X., Zhang, Q., & Liu, X. (2015). Novel compressed sensing algorithm based on modulation classification and symbol rate recognition. *Wireless Personal Communications*, *80*(4), 1717–1732.
7. Angayarkanni, V., & Radha, S. (2016). Design of bandwidth efficient compressed sensing based prediction measurement encoder for video transmission in wireless sensor networks. *Wireless Personal Communications*, *88*(3), 553–573.
8. Xu, W., Lin, J., Niu, K., & He, Z. (2013). On the performance of compressed sensing with partially correct support. *Wireless Personal Communications*, *69*(4), 1877–1884.
9. Zhao, Q., Li, X., & Wu, Z. (2014). Quickest detection of multi-channel based on STFT and compressed sensing. *Wireless Personal Communications*, *77*(3), 2183–2193.
10. Tropp, J. A. (2004). Greed is good: Algorithmic results for sparse approximation. *IEEE Transactions on Information Theory*, *50*(10), 2231–2242.
11. Needell, D., & Vershynin, R. (2010). Signal recovery from incomplete and inaccurate measurements via regularized orthogonal matching pursuit. *IEEE Journal of Selected Topics in Signal Processing*, *4*(2), 310–316.
12. Needell, D., & Tropp, J. A. (2009). CoSaMP: Iterative signal recovery from incomplete and inaccurate samples. *Applied and Computational Harmonic Analysis*, *26*(3), 301–321.
13. Dai, W., & Milenkovic, O. (2010). Subspace pursuit for compressive sensing signal reconstruction. *IEEE Transactions on Information Theory*, *55*(5), 2230–2249.
14. Huang, H., & Makur, A. (2011). Backtracking-based matching pursuit method for sparse signal reconstruction. *IEEE Signal Processing Letters*, *18*(7), 391–394.
15. Zeng, L., Zhang, X., Chen, L., & Yang, W. (2014). SegOMP: Sparse recovery with fewer measurements. *IEICE Transactions on Fundamentals*, *E97–A*(3), 862–864.
16. Mittleman, D. (2003). *Sensing with terahertz radiation*. Berlin: Springer.
17. Cumming, I., & Wong, F. (2005). *Digital processing of synthetic aperture radar data*. Norwood, MA: Artech House.
18. Hayes, M., & Gough, P. (2009). Synthetic aperture sonar: A review of current status. *IEEE Journal of Oceanic Engineering*, *34*(3), 207–224.
19. Webb, A. G. (2003). *Introduction to Biomedical Imaging*. Hoboken, NJ: Wiley-IEEE Press.
20. Ender, J. H. G. (2010). On compressive sensing applied to radar. *Signal Processing*, *90*(5), 1402–1414.
21. Yu, S., Khwaja, A. S., & Ma, J. (2012). Compressed sensing of complex-valued data. *Signal Processing*, *92*(2), 357–362.
22. Yang, Z., Zhang, C., & Xie, L. (2012). On phase transition of compressed sensing in the complex domain. *IEEE Signal Processing Letters*, *19*(1), 47–50.
23. Tivive, F. H. C. & Bouzerdoum, A. (2013). A compressed sensing method for complex-valued signals with application to through-the-wall radar imaging. In *Proceedings of IEEE international conference on acoustics, speech and signal processing*. Vancouver (pp. 2144–2148).
24. Moghadam, A. A. & Radha, H. (2010). Complex sparse projections for compressed sensing. In *Proceedings of annual conference on information sciences and systems*. Princeton (pp. 1–6).
25. Calderbank, R., Howard, S., & Jafarpour, S. (2010). Construction of a large class of deterministic sensing matrices that satisfy a statistical isometry property. *IEEE Journal of Selected Topics in Signal Processing*, *4*(2), 358–374.
26. Hong, S., Park, H., No, J.-S., Helleseth, T., & Kim, Y.-S. (2014). Near-optimal partial Hadamard codebook construction using binary sequences obtained from quadratic residue mapping. *IEEE Transactions on Information Theory*, *60*(6), 3698–3705.
27. Amini, A., & Marvasti, F. (2011). Deterministic construction of binary, bipolar, and ternary compressed sensing matrices. *IEEE Transactions on Information Theory*, *57*(4), 2360–2370.

28. Duarte, M. F., Sarvotham, S., Baron. D., Wakin, M. B., & Baraniuk, R. G. (2005). Distributed compressed sensing of jointly sparse signals. In *Proceedings of Asilomar conference on signals, systems and computers* (pp. 1537–1541).
29. Cotter, S. F., Rao, B. D., Engan, K., & Kreutz-Delgado, K. (2005). Sparse solutions to linear inverse problems with multiple measurement vectors. *IEEE Transactions on Signal Processing*, *53*(7), 2477–2488.
30. Tropp, J. A., Gilbert, A. C., & Strauss, M. J. (2006). Algorithms for simultaneous sparse approximation. Part I: Greedy pursuit. *Signal Processing*, *86*(3), 572–588.
31. Gribonval, R., Rauhut, H., Schnass, K., & Vandergheynst, P. (2008). Atoms of all channels, unite! Average case analysis of multi-channel sparse recovery using greedy algorithms. *Journal of Fourier Analysis and Applications*, *14*(5–6), 655–687.
32. Sundman, D., Chatterjee, S. & Skoglund, M. (2011). Greedy pursuits for compressed sensing of jointly sparse signals. In *Proceedings of European signal processing conference*, Barcelona (pp. 368–372).
33. Lee, K., Bresler, Y., & Junge, M. (2012). Subspace methods for joint sparse recovery. *IEEE Transactions on Information Theory*, *58*(6), 3613–3641.
34. Blanchard, J. D., Cermak, M., Hanle, D., & Jing, Y. (2014). Greedy algorithms for joint sparse recovery. *IEEE Transactions on Signal Processing*, *62*(7), 1694–1704.

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