

Compressive Sensing: From Theory to Applications, a Survey

Saad Qaisar, Rana Muhammad Bilal, Wafa Iqbal, Muqaddas Naureen, and Sungyoung Lee

Abstract: Compressive sensing (CS) is a novel sampling paradigm that samples signals in a much more efficient way than the established Nyquist sampling theorem. CS has recently gained a lot of attention due to its exploitation of signal sparsity. Sparsity, an inherent characteristic of many natural signals, enables the signal to be stored in few samples and subsequently be recovered accurately, courtesy of CS. This article gives a brief background on the origins of this idea, reviews the basic mathematical foundation of the theory and then goes on to highlight different areas of its application with a major emphasis on communications and network domain. Finally, the survey concludes by identifying new areas of research where CS could be beneficial.

Index Terms: Compressive imaging, compressive sensing (CS), incoherence, sparsity, wireless sensor networks (WSNs).

I. INTRODUCTION

Compressive sensing (CS) has witnessed an increased interest recently courtesy high demand for fast, efficient, and in-expensive signal processing algorithms, applications, and devices. Contrary to traditional Nyquist paradigm, the CS paradigm, banking on finding sparse solutions to underdetermined linear systems, can reconstruct the signals from far fewer samples than is possible using Nyquist sampling rate. The problem of limited number of samples can occur in multiple scenarios, e.g., when we have limitations on the number of data capturing devices, measurements are very expensive or slow to capture such as in radiology and imaging techniques via neutron scattering. In such situations, CS provides a promising solution. CS exploits sparsity of signals in some transform domain and the incoherency of these measurements with the original domain. In essence, CS combines the sampling and compression into one step by measuring minimum samples that contain maximum information about the signal: This eliminates the need to acquire

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and store large number of samples only to drop most of them because of their minimal value. CS has seen major applications in diverse fields, ranging from image processing to gathering geophysics data. Most of this has been possible because of the inherent sparsity of many real world signals like sound, image, video, etc. These applications of CS are the main focus of our survey paper, with added attention given to the application of this signal processing technique in the communication and networks domain.

This article starts with presenting a brief historical background of CS during last four decades. It is followed by a comparison of the novel technique with conventional sampling technique. A succinct mathematical and theoretical foundation necessary for grasping the idea behind CS is given. It then surveys major applications of CS specifically in the communications and networks domain. In the end, open research areas are identified and the article is concluded.

II. HISTORICAL BACKGROUND

The field of CS has existed for around four decades. It was first used in Seismology in 1970 when Claerbout and Muir gave attractive alternative of least square solutions [1], Kashin [2] and Gluskin [3] gave norms for random matrices. In mid eighties, Santosa and Symes [4] suggested l_1 -norm to recover sparse spike trains. In 1990s, Rudin, Osher, and Fatemi [5] used total variation minimization in Image Processing which is very close to l_1 minimization. Some more contributions of this era are [6]–[11]. The idea of CS got a new life in 2004 when David Donoho, Emmanuel Candes, Justin Romberg, and Terence Tao gave important results regarding the mathematical foundation of CS. A series of papers have come out in last six years and the field is witnessing significant advancement almost on a daily basis.

A. Nyquist Sampling Theorem

In 1949, Shannon presented his famous proof that any band-limited time-varying signal with ' n ' Hertz highest frequency component can be perfectly reconstructed by sampling the signal at regular intervals of at least $1/2n$ seconds. In traditional signal processing techniques, we uniformly sample data at Nyquist rate, prior to transmission, to generate ' n ' samples. These samples are then compressed to ' m ' samples; discarding $n - m$ samples.

At the receiver end, decompression of data takes place to retrieve ' n ' samples from ' m ' samples. The paradigm of Shannon's sampling theory is cumbersome when extended to the emerging wide-band signal systems since high sampling rates may not be viable for implementation in circuitry: High data-rate analog-to-digital converters are computationally expensive

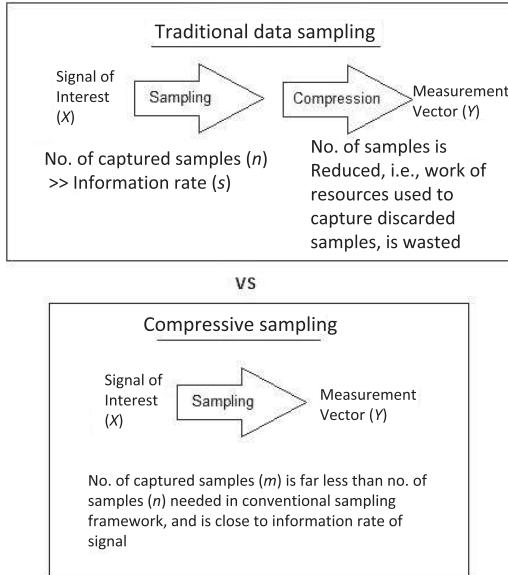


Fig. 1. Traditional data sampling and compression versus CS.

and require more storage space. After reviewing the conventional sampling theorem one may wonder: Why should we go through all computation when we only need ' m ' samples in the end for transmission? Are the real world signals always band limited? How can we get ' n ' samples efficiently, especially if we need a separate hardware sensor for each sample? The alternative theory of CS [12], [13] by Candes, Tao, Romberg, and Donoho have made a significant contribution to the body of signal processing literature, by giving sampling theory a new dimension, as described in subsequently.

Fig. 1 represents concept of traditional data sampling and CS. Further elaboration follows in subsequent sections.

III. CS PARADIGM

CS theory asserts that we can recover certain signals from fewer samples than required in Nyquist paradigm. This recovery is exact if signal being sensed has a low information rate (means it is sparse in original or some transform domain). Number of samples needed for exact recovery depends on particular reconstruction algorithm being used. If signal is not sparse, then recovered signal is best reconstruction obtainable from s largest coefficients of signal. CS handles noise gracefully and reconstruction error is bounded for bounded perturbations in data. Underneath are some definitions that are later used to discuss acquisition/reconstruction models and behaviour of CS to non-sparse signals and noise.

Sparsity: Natural signals such as sound, image or seismic data can be stored in compressed form, in terms of their projection on suitable basis. When basis is chosen properly, a large number of projection coefficients are zero or small enough to be ignored. If a signal has only s non-zero coefficients, it is said to be s -sparse. If a large number of projection coefficients are small enough to be ignored, then signal is said to be compressible. Well known compressive-type basis include 2 dimensional (2D) wavelets for images, localized sinusoids for music, fractal-type waveforms for spiky reflectivity data, and curvelets

for wave field propagation [14].

Incoherence: Coherence measures the maximum correlation between any two elements of two different matrices. These two matrices might represent two different basis/representation domains. If Ψ is a $n \times n$ matrix with Ψ_1, \dots, Ψ_n as columns and Φ is an $m \times n$ matrix with Φ_1, \dots, Φ_m as rows. Then, coherence μ is defined as

$$\mu(\Phi, \Psi) = \sqrt{n} \max |\Phi_k, \Psi_j| \quad (1)$$

for $1 \leq j \leq n$ and $1 \leq k \leq m$. It follows from linear algebra that

$$1 \leq \mu(\Phi, \Psi) \leq \sqrt{n}. \quad (2)$$

In CS, we are concerned with the incoherence of matrix used to sample/sense signal of interest (hereafter referred as measurement matrix Φ) and the matrix representing a basis, in which signal of interest is sparse (hereafter referred as representation matrix Ψ). Within the CS framework, low coherence between Φ and Ψ translates to fewer samples required for reconstruction of signal. An example of low coherence measurement/representation basis pair is sinusoids and spikes that are incoherent in any dimension [15], and can be used for compressively sensing signals having sparse representation in terms of sinusoids.

Restricted isometry property (RIP): RIP has been the most widely used tool for analysing the performance of CS recovery algorithms [16] as illustrated below through CS acquisition and reconstruction models and illustrated in Fig. 2.

A. Acquisition Model

Signal acquisition model of CS is quite similar to conventional sensing framework. If \mathfrak{X} represents the signal to be sensed, then sensing process may be represented as

$$\mathbf{Y} = \Phi \mathfrak{X} \quad (3)$$

where, $\mathfrak{X} \in \mathbb{R}^n$, is the signal to be sensed; Φ is m -by- n measurement matrix and $\mathbf{Y} \in \mathbb{R}^m$ is measurement vector. Under conventional sensing paradigm ' m ' must be at least equal to ' n '. However CS states that ' m ' can be far less than ' n ', provided signal is sparse (accurate reconstruction) or nearly sparse/compressible (approximate reconstruction) in original or some transform domain. Lower values for ' m ' are allowed for sensing matrices that are more incoherent within the domain (original or transform) in which signal is sparse. This explains why CS is more concerned with sensing matrices based on random functions as opposed to Dirac delta functions under conventional sensing. Although, Dirac impulses are maximally incoherent with sinusoids in all dimensions [15], however data of interest might not be sparse in sinusoids and a sparse basis (original or transform) incoherent with Dirac impulses might not exist. On the other hand, random measurements can be used for signals s -sparse in any basis as long as Φ obeys the following condition [17]

$$m = s \log\left(\frac{n}{s}\right). \quad (4)$$

As per available literature, Φ can be a Gaussian [18], Bernoulli [19], Fourier, or incoherent measurement matrix [12]. For a class of reconstruction algorithms known as basis pursuit

(BP), [20] states that most s -sparse signals can be exactly recovered just by ensuring

$$m \geq 4s. \quad (5)$$

Equations (6) and (7) quantify ' m ' with respect to incoherence between sensing matrix and sparse basis. Other important consideration for robust compressive sampling is that measurement matrix well preserves the important information pieces in signal of interest. This is typically ensured by checking RIP of reconstruction matrix Θ (product of measurement matrix with representation basis) [16]. RIP is defined on isometry constant δ_S of a matrix, which is the smallest number such that

$$(1 - \delta_S) \|x\|_{l_2}^2 \leq \|\Theta x\|_{l_2}^2 \leq (1 + \delta_S) \|x\|_{l_2}^2 \quad (6)$$

holds for all s -sparse vectors ' x '. We will loosely say that a matrix obeys the RIP of order s if δ_S is not too close to one. RIP insures that all subsets of s columns taken from matrix are nearly orthogonal and sparse signal is not in null space of matrix being used to sense it (as otherwise it cannot be reconstructed). Similarly, if δ_{2S} is sufficiently less than one, then all pair wise distances between s -sparse signals must be well preserved in the measurement space, as shown by

$$(1 - \delta_{2S}) \|x_1 - x_2\|_{l_2}^2 \leq \|\Theta(x_1 - x_2)\|_{l_2}^2 \leq (1 + \delta_{2S}) \|x_1 - x_2\|_{l_2}^2 \quad (7)$$

for s -sparse vectors x_1 and x_2 .

B. Reconstruction Model

A nonlinear algorithm is used in CS, at receiver end to reconstruct original signal. This nonlinear reconstruction algorithm requires knowledge of a representation basis (original or transform) in which signal is sparse (exact recovery) or compressible (approximate recovery). Signal of interest \mathfrak{X} , can be expressed in representation basis as

$$\Psi x = \mathfrak{X} \quad (8)$$

where x is s -sparse vector, representing projection coefficients of \mathfrak{X} on Ψ . Measurement vector Y , can now be rewritten in terms of x as

$$Y = \Theta x \quad (9)$$

where $\Theta = \Phi\Psi$ is $m \times n$ dimensional, reconstruction matrix.

Reconstruction algorithms in CS, try to solve (9), and exploit the fact that solution is sparse, usually by minimizing l_0 , l_1 , or l_2 norm over solution space. According to classical least square solution (minimization of l_2 norm), reconstructed solution \hat{x} may be expressed as

$$\hat{x} = \min_{x: \Theta x = Y} \|x\|_{l_2} = \Theta^T (\Theta \Theta^T)^{-1} Y \quad (10)$$

where

$$\|x\|_{l_2} = \sqrt{\left(\sum_{i=1}^N |x_i|^2\right)}. \quad (11)$$

Similarly, by using l_1 minimization or BP, as it is known in CS literature, signal can be exactly recovered from ' m ' measurements by solving a simple convex optimization problem [21] through linear programming.

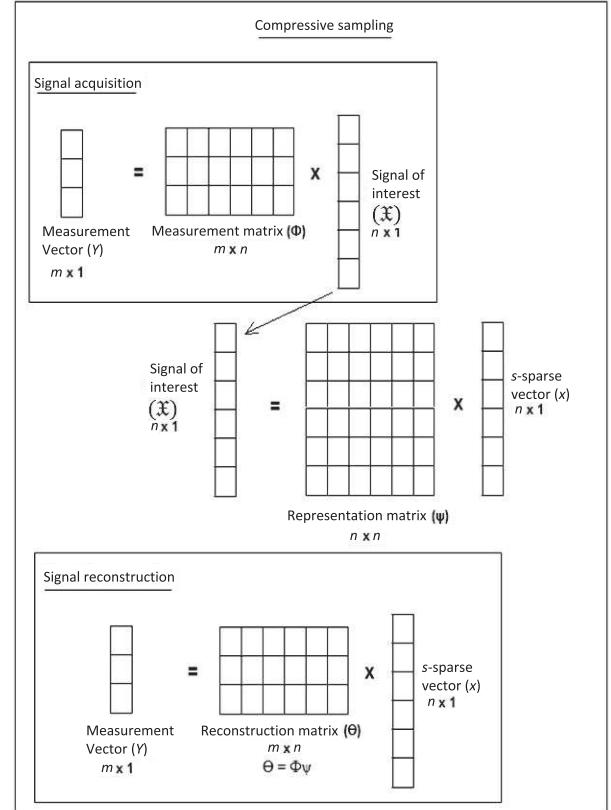


Fig. 2. Compressive acquisition and reconstruction.

$$\hat{x} = \min_{x: \Theta x = Y} \|x\|_{l_1} \quad (12)$$

where

$$\|x\|_{l_1} = \sqrt{\left(\sum_{i=1}^N |x_i|\right)}. \quad (13)$$

Some reconstruction techniques are based on l_0 minimization.

$$\hat{x} = \min_{x: \Theta x = Y} \|x\|_{l_0} \quad (14)$$

where

$$\|x\|_{l_0} = \sqrt{\left(\sum_{i=1}^N |x_i|^0\right)}. \quad (15)$$

l_2 minimization mostly gives unsatisfactory results with non-sparse signals. Since real world signals are usually compressible rather than sparse, l_2 minimization is not an attractive option for reconstruction. On the other hand, l_0 minimization though gives accurate results; however, has computational disadvantage of being a NP hard problem. To address this issue, [13]–[23] use l_1 norm, as it gives same results as l_0 minimization under certain circumstances. Specifically, [13] shows that reconstruction error of linear programming (l_1 norm minimization) has upper bound, provided Θ obeys uniform uncertainty principle and x is sufficiently sparse. Fig. 3 shows l_2 and l_1 minimizations for 3 dimensional data. Plane is the set of all x vectors that satisfy $Y = \Theta x$. l_2 minimization is equivalent to blowing up a hypersphere and picking point where it touches the solution plane. Since l_2

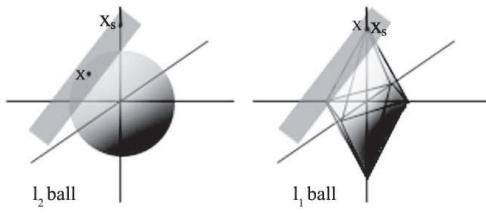


Fig. 3. Geometry of CS minimization.

ball is spherical, usually it picks points away from coordinate axis (non-sparse members of solution plane), whereas l_1 ball has axis aligned shape which helps to introduce a preference for sparse members of solution set.

Though basic sense of energy minimization is common to all solution frameworks, yet variants exist in approach to solve norm minimization equation. Section IV summarizes categories of various reconstruction algorithms in present literature.

C. CS for Non-Sparse Signals

As shown in [13], if

$$\delta_{2S} \leq \sqrt{2} - 1 \quad (16)$$

then, solution \times to l_1 minimization problem (10), obeys

$$\|\times - x\|_{l_2} \leq C_0 \frac{\|x - x_S\|_{l_1}}{\sqrt{s}} \quad (17)$$

and

$$\|\times - x\|_{l_1} \leq C_0 \|x - x_S\|_{l_1} \quad (18)$$

for some constant C_0 , where x is the original signal, x_S is the signal x with all but the largest s components set to 0. If x is s -sparse, then $x = x_S$ and, thus the recovery is exact. If x is not s -sparse, then this asserts that quality of reconstruction is as good as reconstruction obtainable from s largest coefficients of x (positions of which were unknown at time of acquisition). So, while conventional sensing paradigm needs more sensing resources and fancy compression stage for compressible signals, CS provides a simpler acquisition model to sense and compress implicitly.

D. Noise Robustness in CS

Practically every sensed signal will at least have quantization noise owing to finite precision of sensing device. If, noisy measurement signal is expressed as

$$Y = \Theta x + E \quad (19)$$

where E is error signal, with energy bounded as

$$\|E\|_{l_2} \leq \varepsilon \quad (20)$$

where ε is a finite constant, then, solution to relaxed l_1 minimization problem may be expressed as

$$\times = \min_{x: \|\Theta x - Y\|_{l_2} \leq \varepsilon} \|x\|_{l_1}. \quad (21)$$

According to [13], solution to (17) obeys

$$\|\times - x\|_{l_2} \leq C_0 \frac{\|x - x_S\|_{l_1}}{\sqrt{s}} + C_1 \varepsilon \quad (22)$$

provided

$$\delta_{2S} \leq \sqrt{2} - 1 \quad (23)$$

for some constants C_0 and C_1 .

IV. RECONSTRUCTION ALGORITHMS

Many efficient algorithms exist in literature, which, instead of finding ' m ' largest coefficients at the same time, attempt to find these coefficients iteratively [24]–[26]. To present an overview of reconstruction algorithms for sparse signal recovery in CS, these algorithms may be broadly divided into six types as shown in Fig. 4 and elaborated as follows.

A. Convex Relaxation

This class of algorithms solves a convex optimization problem through linear programming [27] to obtain reconstruction. The number of measurements required for exact reconstruction is small but the methods are computationally complex. BP [28], BP de-noising (BPDN) [28], modified BPDN [29], least absolute shrinkage and selection operator (LASSO) [30], and least angle regression (LARS) [31] are some examples of such algorithms. Recent works show matrix versions of signal recovery called $\|M\|_1$ nuclear norm minimization [32]. Instead of reconstructing norm minimization tries to recover a low rank matrix M from Θx . Since rank determines the order, dimension and complexity of the system, low rank matrices correspond to low order statistical models.

B. Greedy Iterative Algorithm

This class of algorithms solve the reconstruction problem by finding the answer, step by step, in an iterative fashion. The idea is to select columns of Θ in a greedy fashion. At each iteration, the column of Θ that correlates most with Y is selected. Conversely, least square error is minimized in every iteration. That row's contribution is subtracted from Y and iterations are done on the residual until correct set of columns is identified. This is usually achieved in M iterations. The stopping criterion varies from algorithm to algorithm. Most used greedy algorithms are matching pursuit [11] and its derivative orthogonal matching pursuits (OMP) [24] because of their low implementation cost and high speed of recovery. However, when the signal is not much sparse, recovery becomes costly.

For such situations, improved versions of OMP have been devised like regularized OMP [33], stagewise OMP [34],

compressive sampling matching pursuits (CoSaMP) [35], subspace pursuits [36], gradient pursuits [37], and orthogonal multiple matching pursuit [38].

C. Iterative Thresholding Algorithms

Iterative approaches to CS recovery problem are faster than the convex optimization problems. For this class of algorithms, correct measurements are recovered by soft or hard thresholding [25], [39] starting from noisy measurements given the signal is

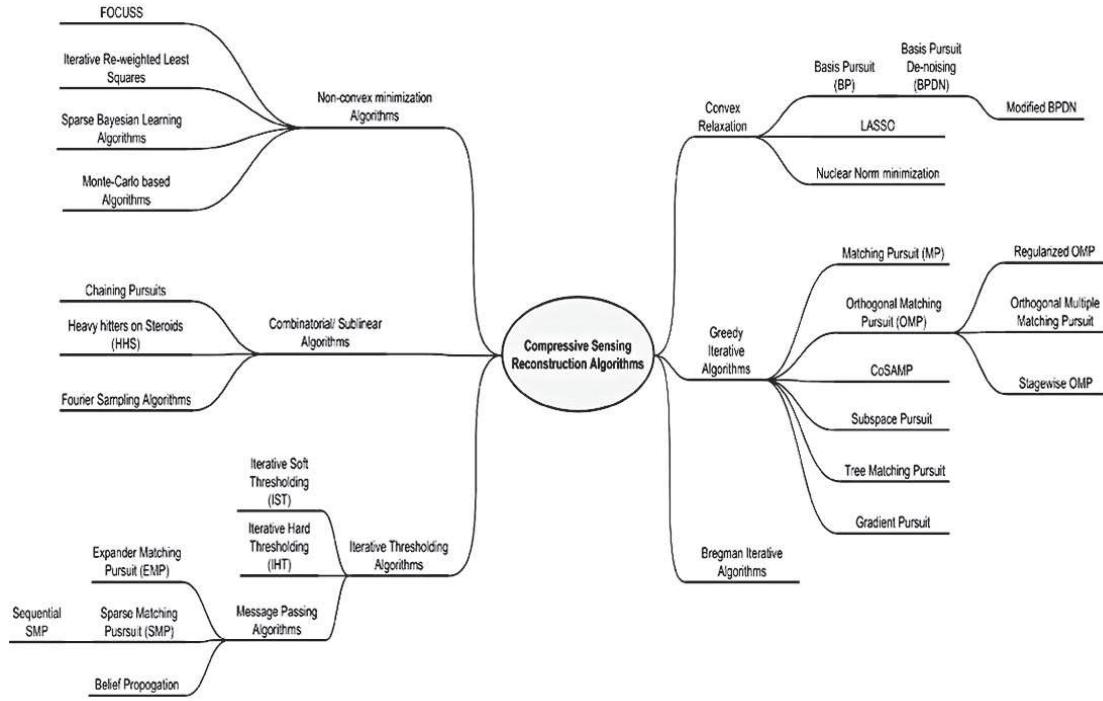


Fig. 4. Compressing sensing reconstruction algorithms and their classification.

sparse. The thresholding function depends upon number of iterations and problem setup at hand. Message passing (MP) algorithms [26] are an important modification of iterative algorithms in which basic variables (messages) are associated with directed graph edges. A relevant graph in case of CS is the bipartite graph with ' n ' nodes on one side (the variable nodes) and ' m ' nodes on the other side (the measurement nodes). This distributed approach has many advantages like low computational complexity and easy implementation in parallel or distributed manner. Expander matching pursuits [40], sparse matching pursuit [41], and sequential sparse matching pursuits [42] are recently proposed algorithms in this domain that achieve near-linear recovery time while using $O(s \log(n/s))$ measurements only. Recently, proposed algorithm of belief propagation also falls in this category [43].

D. Combinatorial/Sublinear Algorithms

This class of algorithms recovers sparse signal through group testing. They are extremely fast and efficient, as compared to convex relaxation or greedy algorithms but require specific pattern in the measurements; Φ needs to be sparse. Representative algorithms are Fourier sampling algorithm [44], chaining pursuits [45], heavy hitters on steroids (HHS) [46], etc.

E. Non Convex Minimization Algorithms

Non-convex local minimization techniques recover CS signals from far less measurements by replacing l_1 -norm by l_p -norm where $p \leq 1$ [47].

Non-convex optimization is mostly utilized in medical imaging tomography, network state inference, streaming data reduction. There are many algorithms proposed in literature that use this technique like focal underdetermined system solution (FOCUSS) [48], iterative re-weighted least squares [49], sparse

bayesian learning algorithms [50], Monte-Carlo based algorithms [51], etc.

F. Bregman Iterative Algorithms

These algorithms provide a simple and efficient way of solving l_1 minimization problem. [52] presents a new idea which gives exact solution of constrained problems by iteratively solving a sequence of unconstrained sub-problems generated by a Bregman iterative regularization scheme. When applied to CS problems, the iterative approach using Bregman distance regularization achieves reconstruction in four to six iterations [52]. The computational speed of these algorithms is particularly appealing compared to that available with other existing algorithms.

Table 1 lists down the complexity and minimum measurement requirements for CS reconstruction algorithms. For instance, as shown in [36], BP can reliably recover signals with $n = 256$ and sparsity level upto 35, from only 128 measurements. Conversely, OMP and ROMP can only be reliable up to sparsity level of 19 for same n and m . Performance of Basis pursuit appears promising as compared to OMP derivatives from minimum measurements perspective.

However, [34] shows that for $n = 10000$, $m = 1000$, and signal sparsity of 100, BP takes approximately 482 seconds for recovery. For the same problem, OMP takes only 8 seconds.

V. APPLICATIONS OF COMPRESSING SENSING

A. Compressive Imaging

A.1 CS in Cameras

CS has far reaching implications on compressive imaging systems and cameras. It reduces the number of measurements,

Table 1. Complexity and minimum measurement requirement of CS reconstruction algorithms.

Algorithm	Complexity	Minimum measurement (m)
BP [28],[36]	$O(n^3)$	$O(s \log n)$
OMP [24],[33],[36]	$O(smn)$	$O(s \log n)$
StOMP [34]	$O(n \log n)$	$O(n \log n)$
ROMP [33],[35]	$O(smn)$	$O(s \log^2 n)$
CoSAMP [35]	$O(mn)$	$O(s \log n)$
Subspace pursuits [36]	$O(smn)$	$O(s \log(n/s))$
EMP [40]	$O(n \log(n/s))$	$O(s \log(n/s))$
SMP [41]	$O(n \log(n/s) \log R)$	$O(s \log(n/s))$
Belief propagation [43]	$O(n \log^2 n)$	$O(s \log n)$
Chaining pursuits [45]	$O(s \log^2 n \log^2 s)$	$O(s \log^2 n)$
HHS [46]	$O(s \text{poly} \log(n))$	$O(\text{poly}(s, \log n))$

hence, power consumption, computational complexity and storage space without sacrificing the spatial resolution. With the advent of single pixel camera (SPC) by Rice University, imaging system has transformed drastically. The camera is based on a single photon detector adaptable to image at wavelengths which were impossible with conventional charge coupled device (CCD) and complementary metal oxide semiconductor (CMOS) images [53]–[55].

CS allows reconstruction of sparse $n \times n$ images by fewer than n^2 measurements. In SPC, each mirror in digital micromirror device (DMD) array performs one of these two tasks: Either reflect light towards the sensor or reflect light away from it. Therefore, light received at sensor (photodiode) end is weighted average of many different pixels, whose combination gives a single pixel. By taking m measurements with random selection of pixels, SPC acquires recognizable picture comparable to an n pixels picture.

In combination with Bayer colour filter, single pixel camera can be used for colour images (hyperspectral camera) [56]. Data captured by single pixel camera can also be used for background subtraction for automatic detection and tracking of objects [57], [58]. The main idea is to foreground objects from background, in a sequence of video frames. However, it is quite expensive for wavelengths other than visible light. CS solves the problem by making use of the fact that in vision applications, natural images can be sparsely represented in wavelet domains [59]. In random projections of a scene with incoherent set of test function and reconstruct it by solving convex optimization problem or OMP algorithm. CS measurements also decrease packet drop over communication channel.

Recent works have proposed the design of tera hertz imaging system. In this system, image acquisition time is proportional to speed of the THz detector [60]. The proposed system eliminates

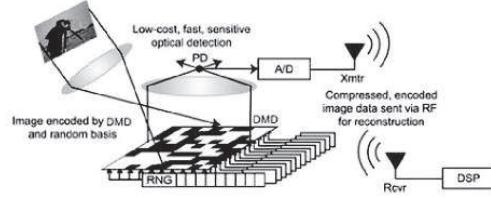


Fig. 5. Block diagram of compressive imaging camera [54].

the need of tera hertz beam and faster scanning of object.

A.2 Medical Imaging

CS is being actively pursued for medical imaging, particularly in magnetic resonance imaging (MRI). MR images, like angiograms, have sparsity properties, in domains such as Fourier or wavelet basis. Generally, MRI is a costly and time consuming process because of its data collection process which is dependent upon physical and physiological constraints. However, the introduction of CS based techniques has improved the image quality through reduction in the number of collected measurements and by taking advantage of their implicit sparsity. MRI is an active area of research for CS community and in recent past, a number of CS algorithms have been specifically designed for it [52], [61]–[64].

A.3 Seismic Imaging

Seismic images are neither sparse nor compressible in strict sense but are compressible in transform domains e.g., in curvelet basis [65]. Seismic data is usually high-dimensional, incomplete and very large. Seismology exploration techniques depend on collection of massive data volume which is represented in five dimensions; two for sources, two for receivers and one for time. However, because of high measurement and computational cost, it is desirable to reduce the number of sources and receivers which could reduce the number of samples. Therefore, sampling technique must require fewer number of samples while maintaining quality of image at the same time. CS solves this problem by combining sampling and encoding in one step, by its dimensionality reduction approach. This randomized sub sampling is advantageous because linear encoding does not require access to high resolution data. A CS based successful reconstruction theory is developed in this sense known as curvelet-based recovery by sparsity-promoting inversion (CRSI) [66].

B. Biological Applications

CS can also be used for efficient and inexpensive sensing in biological applications. The idea of group testing [67] is closely related to CS. It was used for the first time in World War II to test soldiers for syphilis [68]. Since the test for syphilis antigen in blood is costly, instead of testing blood of each and every soldier, the method used to group the soldiers and pool blood samples of whole group and test them simultaneously. Recent works show usage of CS in comparative deoxyribonucleic acid (DNA) microarray [69]. Traditional microarray bio-sensors are useful for detection of limited number of micro organisms. To

detect greater number of species large expensive microarrays are required. However, natural phenomena are sparse in nature and easily compressible in some basis. DNA microarrays consist of millions of probe spots to test a large number of targets in a single experiment. In traditional microarrays, single spot contains a huge number of copies of probes designed to capture single target and hence collects data of a single data point. On the contrary, in comparative microarrays test, sample is measured relative to test sample. As a result, it is differentially expressed - as a fraction of the total number of reference genes and test samples. CS gives an alternative design of compressed microarrays [70] in which each spot contains copies of different probe sets reducing the overall number of measurements and still efficiently reconstructing from them.

C. Compressive Radio Detecting and Ranging (RADAR)

CS theory contributes to RADAR system design by eliminating the need of pulse compression matched filter at receiver and reducing the analog to digital conversion bandwidth from Nyquist rate to information rate, simplifying hardware design [71]. It also offers better resolution over classical RADARs whose resolution can be limited by time-frequency uncertainty principles [72], [73]. Resolution is improved by transmitting incoherent deterministic signals, eliminating the matched filter and reconstructing received signal using sparsity constraints. CS has successfully been demonstrated to enhance resolution of wide angle synthetic aperture RADAR [74]. The CS techniques can effectively be used for monostatic, bistatic and multistatic RADAR. The information scalability property of CS allows it to detect, classify and recognize target directly from incoherent measurements without performing reconstruction or approximate computation. CS imaging is also applicable in sonar and ground penetrating RADARs (GPRs) [75], [76], [77]. Similarly, [78] presents an interesting combination of CS and change detection for human motion identification imaging.

D. Analog-to-Information Converters (AIC)

Communication systems utilizing high bandwidth radio frequency (RF) signals face an inherent problem in the rates required for sampling these signals. In most applications, information content of the signal is much smaller than its bandwidth; it maybe a wastage of precious hardware and software resources to sample the whole signal. Compressive sampling solves the problem by replacing ‘analog to digital conversion (ADC)’ by ‘AIC’. The approach of random non-uniform sampling used in ADC is bandwidth limited with present hardware devices [79] whereas AIC utilizes random sampling for wideband signals for which random non-uniform sampling fails. AIC is based on three main components: demodulation, filtering and uniform sampling [80], [81]. The initially developed random demodulator was limited to discrete multi-tone signals and incurred high computational load [81]. Random filtering utilized in AIC requires less storage and computation for measurement and reconstruction [82]. Recent works propose a converter whose three main components are multiplication of analog signal with bank of periodic waveforms, low pass filtering and uniform sampling at uniform sampling at low rate [83].

VI. CS IN COMMUNICATIONS AND NETWORKS

CS is an attractive tool to acquire signals and network features in networked and communication systems. Below, we have sampled few interesting CS applications in communication and networking domain.

A. Sparse Channel Estimation

CS has been used in communications domain for sparse channel estimation. Adoption of multiple-antenna in communication system design and operation at large bandwidths, possibly in gigahertz, enables sparse representation of channels in appropriate bases. Conventional technique of training-based estimation using least-square (LS) methods may not be an optimal choice. Various recent studies have employed CS for sparse channel estimation. Compressed channel estimation (CCS) gives much better reconstruction using its non-linear reconstruction algorithm as opposed to linear reconstruction of LS-based estimators. In addition to non-linearity, CCS framework also provides scaling analysis. CCS based sparse channel estimation has been shown to achieve much less reconstruction error while utilizing significantly less energy and, in some cases, less latency and bandwidth as well [84].

The estimation of underwater acoustic channels, which are inherently sparse, through CS technique yields results better than the conventional ‘least square estimator’. It also gives approximately equal and in some cases better than the channel estimation through subspace methods from array-processing literature [85]. The use of high time resolution over-complete dictionaries further enhances channel estimation. BP and OMP are used to estimate multipath channels with Doppler spread ranging from mild, like on a normal day, to severe, like on stormy days. Only CS based estimators can handle significant Doppler spread efficiently by exploiting inter-carrier interference explicitly.

Recently, Taubock *et al.* [86] have presented a mechanism of estimating doubly-selective channels within multicarrier (MC) systems such as orthogonal frequency division multiplexing (OFDM). The work builds on an earlier technique proposed by same investigators in which sparsity of MC systems is exploited in delay-Doppler domains through CS. Sparsity of the signals is severely affected by inter-symbol interference (ISI) and inter-carrier interference (ICI) in MC communications. [86] focuses on determining and then overcoming such leakage effects. A basic compressive channel estimator estimates ‘diagonal’ channel coefficients for mildly dispersive channels. In order to combat effects of strongly dispersive channels and to enhance sparsity, the transform basis that had been conventionally used until now - discrete Fourier transform - is changed to a more suitable sparsity-enhancing basis developed explicitly through an iterative basis design algorithm. As a result, the novel compressive channel estimator can predict off-diagonal channel coefficients also, that are an outcome of ISI/ICI.

B. Spectrum Sensing in CR Networks

CS based technique is used for speedy and accurate spectrum sensing in cognitive radio technology based standards and systems [87], [88]. IEEE 802.22 is the first standard to use the concept of cognitive radio, providing an air interface for wireless

communication in the TV band. Although, no spectrum sensing method is explicitly defined in the standard, nonetheless, it has to be fast and precise. Fast Fourier sampling sampling (FFS) - an algorithm based on CS - is used to detect wireless signals as proposed in [87]. According to the algorithm only ' m ' (where most energetic frequencies of the spectrum are detected and the whole spectrum is approximated from these samples using non-uniform inverse fast Fourier transform (IFFT). Using fewer samples FFS results in faster sensing, enabling more spectrum to be sensed in the same time window.

In [88], a wideband spectrum sensing scheme using distributed CS is proposed for CR networks. This work uses multiple CR receivers to sense the same wide-band signal through AICs, produce the autocorrelation vectors of the compressed signal and send them to a centralized fusion center for decision on spectrum occupancy. The work exploits joint sparsity and spatial diversity of signals to get performance gains over a non-distributed CS system.

In past, work has also been done in which CS is applied in parallel to time-windowed analog signals [89]. By using CS, load of samples on the digital signal processor reduces but the ADC still has to digitize analog signal to digital signal. In order to overcome this problem, CS is applied directly on analog signals. This is done on segmented pieces of signal; each block is compressively sensed independent of the other. At the receiver, however, a joint reconstruction algorithm is implemented to recover the signal. The sensing rate is greatly reduced using 'parallel' CS while reconstruction quality improves.

[90] proposes a cyclic feature detection framework based on CS for wideband spectrum sensing which utilizes second order statistics to cope with high rate sampling requirement of conventional cyclic spectrum sensing.

Signal sparsity level has temporal variation in CR networks, and thus optimal CS rate is not static. [91] introduces a framework to dynamically track optimal sampling rate and determine unoccupied channels in a unified way.

C. Ultra Wideband (UWB) Systems

In the emerging technology of UWB communication, CS plays a vital role by reducing the high data-rate of ADC at receiver [92]. CS moves hardware complexity towards transmitter by exploiting the channel itself: Channel is assumed to be part of UWB communication system. The work proposed in [92] has enabled a 3 GHz-8 GHz UWB system to be implemented which otherwise, using Nyquist rate ADCs, would have taken years to reach industry. CS, as used in pulse-based UWB communication utilizes time sparsity of the signal through a filter-based CS approach applied on continuous time signals.

D. Wireless Sensor Networks (WSNs)

CS finds its applications in data gathering for large WSNs, consisting of thousands of sensors deployed for tasks like infrastructure or environment monitoring. This approach of using compressive data gathering (CDG) helps in overcoming the challenges of high communication costs and uneven energy consumption by sending ' m ' weighted sums of all sensor readings to a sink which recovers data from these measurements [93], as shown in Fig. 4. Although, this increases the number of signals

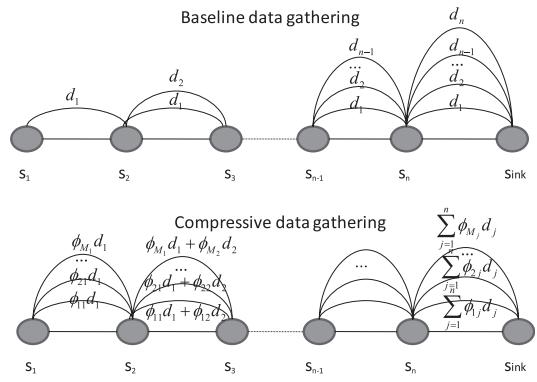


Fig. 6. Baseline data gathering and compressive data gathering (adapted from [93]).

sent by the initial ' m ' sensors, but the overall reduction in transmissions and energy consumption is significant since $m \ll n$ (where n is the total number of sensors in large-scale WSN). This also results in load balancing which in turn enhances lifetime of the network. Lou *et al.* [93] propose a scheme that can detect abnormal readings from sensors by utilizing the fact that abnormalities are sparse in time-domain.

CS is also used, in a decentralized manner, to recover sparse signals in energy efficient large-scale WSNs [94]. Various phenomena monitored by large scale WSNs usually occur at scattered localized positions, hence, can be represented by sparse signals in the spatial domain. Exploiting this sparsity through CS results in accurate detection of the phenomenon. According to the proposed scheme [94], most of the sensors are in sleeping whereas only a few are active. The active sensors sense their own information and also come up with optimum sensor values for their sleeping one-hop neighbours through 'consensus optimization' - an iterative exchange of information with other active one-hop neighbours. Using a sleeping WSN strategy not only makes the network energy efficient but also ensures detection of a physical phenomenon with high accuracy.

The task of localization and mapping of the environment as quickly as possible, for reliable navigation of robots in WSNs has utilized CS technique for its benefit [95]. A mobile robot, working in an indoor WSN for event detection application, may need to know its own position in order to locate where an incident has happened. The conventional approach of using navigation system to build a map requires estimation of features of the whole surrounding. This result in data coming to mobile robot from all sensors in the network, most of which is highly correlated - computational load may increase substantially, especially in case of large scale sensor networks. CS enables the making of high quality maps without directly sensing large areas. The correlation amongst signals renders them compressible. The nodes exploit sparse representation of parameters of interest in order to build localized maps using compressive cooperative mapping framework, which gives superior performance over traditional techniques [95].

E. Erasure Coding

CS can be utilized for inexpensive compression at encoder; making every bit even more precious as it carries more infor-

mation. To enable correct recovery of the compressed data after passing through erasure channels, CS is again utilized as a channel coding scheme [96]. Such CS erasure coding (CSEC) techniques are not a replacement of channel coding schemes; rather they are used at the application layer, for added robustness to channel impairments and in low-power systems due to their computational simplicity. In CSEC, CS not only compresses the required samples to $m << n$, it also disperses the information contained in these m samples to a larger number of k samples, where still $k << n$. The sensing matrix Φ in such schemes is augmented with an additional number of e rows, where e is the number of erasures. At the receiver side, if any of the e or more samples are lost, signal can still be reconstructed using CS reconstruction techniques with high probability unlike conventional erasure coding techniques which discard corrupt symbol completely.

F. Network Management

Network management tasks usually use ‘traffic matrices (TM)’. However, these matrices have many empty spaces as direct observation of TM for large networks may not be possible. Therefore, interpolation from the available values is essential. Internet TMs mostly do not satisfy the conditions necessary for CS but by exploiting the spatio-temporal structure, TM can be recovered from as less as 2% values [97]. The interpolation of missing values is achieved by using spatio-temporal CS technique called sparsity regularized matrix factorization (SRMF), which finds a global low-rank approximation of TM and then a local interpolation technique like k -nearest neighbours is augmented with it to fully recover the traffic matrix.

To obtain overlay network traffic and delay information between two hosts is important for network management, monitoring, design, planning and assessment. Traffic matrix and delay matrix represent the traffic and delay information between two hosts, so introduce the concept of the overlay network traffic matrix and delay matrix. CS theory restores traffic matrix and delay matrix but is not suitable for overlay network. In [98], authors propose a framework which improves CS algorithm to make it more applicable to overlay network traffic matrix and delay matrix restoration. After calculating the traffic matrix and delay matrix this paper quantifies overlay network congestion, which reflect the current network security situation. The experimental results show the restoration effect of traffic matrix and delay matrix is well and the congestion degree reflects the actual network state.

In [98], authors estimate the missing round trip time (RTT) measurements in computer networks using doubly non-negative (DN) matrix completion and compressed sensing. The major contribution of this work is systematic and detailed experimental comparison of DN matrix completion and compressed sensing for estimating missing RTT estimation in computer networks. Results indicate that compressed sensing provides better estimation in networks with sporadic missing values.

G. Multimedia Coding and Communication

CS has great potential to be used in multimedia communication in applications such as wireless multimedia sensor networks (WMSNs). In recent years, various studies have focused

on WMSNs and novel techniques for video transmission are under investigation. CS provides an attractive solution as CS encoded images provide an inherent resilience to random channel errors. Since the samples transmitted have no proper structure, as a result, every sample is equally important and the quality of reconstruction depends on the number of correctly received samples only [99]. Furthermore, video quality can be improved by using a low complexity ‘adaptive parity-based channel coding’ [99] which drops the samples that have error as opposed using conventional forward error correction which entails additional overhead. A compressive sampling based video encoder as discussed by Pudlewski and Melodia, exploits redundancy in video frames by transmitting the encoded difference frame and recreating it, using correlation between the difference frame and a reference frame.

To enhance the error resilience of images, joint source channel coding (JSCC) using CS is also an active area of research. Inflation of CS measurements in order to add error robustness is proposed by Mohseni *et al.* [100]. The authors propose a real time error-correction coding step in analog domain. This makes the image resilient to spiky noise such as salt and pepper noise in images. The samples are precoded using a suitable encoding matrix on the hardware side. An additional coding step follows on the digital side. This approach inherently corrects the errors that arise due to faulty pixel sensors. The novelty of performing this JSCC technique is that it is done in analog hardware.

Recent work by Deng *et al.* propose a CS based codec that enhances error resilience of images [101] as illustrated in Fig. 7. The codec makes non-sparse image signal sparse by applying multi-level 2D discrete wavelet transform (DWT). The advantage of applying CS on DWT coefficients is its ability to spread energy of measurements over all samples. Hence, every sample carries equal information. Novelty introduced in this work over previous works utilizing wavelet based CS [102] is the proposed ‘multi-scale CS’ allocating more measurements to coarser level as coefficients of low frequency sub-band containing maximum information. This scheme increases error robustness at high packet loss rates without any explicit error resilience method i.e., better performance with reduced complexity than contemporary JSCC schemes.

CS has been in use for acquisition, sampling, encoding and analysis of multimedia data for quite some time now and works have been done to optimize the efficiencies of these systems. A recent work combines a number of these systems to get an overall optimized ‘joint CS video coding and analysis’ setup [103].

Instead of performing compressive sampling and encoding of input video signal and then decoding the complete length of signal, to be passed on to an analysis block, the proposed setup performs joint encoding and analysis. This eliminates the need for decoder to decode the whole video sequence. It in effect reduces much of the complexity of CS based decoder. The paper takes ‘object tracking’ as a specific example of analysis. Object tracking through joint CS coding and analysis is done in two ways. Firstly, only the foreground objects are decoded and the background is subtracted in the projected domain. Secondly, predicted position and sizes of the boxes bounding the object, determined by tracking algorithm, are known *a priori* to the decoder. The scheme greatly reduces the number of measurements

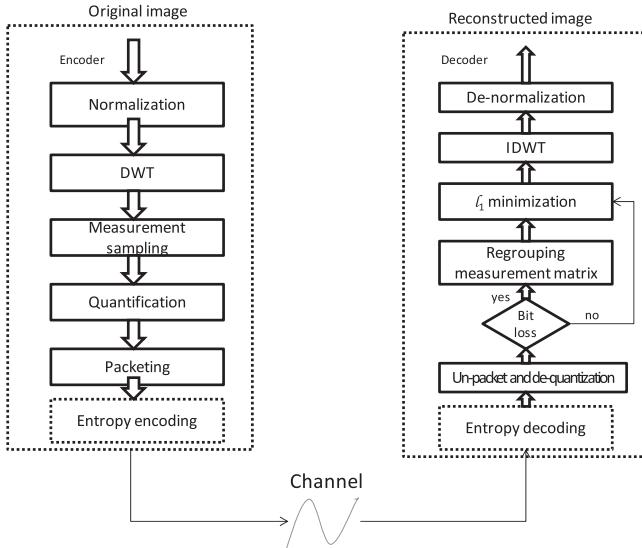


Fig. 7. Robust image compression and transmission using CS [101].

required as compared to traditional techniques.

H. CS based Localization

Device localization is an important requirement in wireless environments. It is chief component in context-aware services and geometric routing schemes. Feng *et al.* [104] describe a CS based localization solution implemented on a mobile device with 20% and 25% localization error improvement over k_{NN} and kernel based methods respectively. In order to mitigate received signal strength variation effects due to channel impediments, the scheme utilizes CS theory to fine localize and enhance access point selection accuracy. Similarly, [105] demonstrates application of CS to recover wireless node position in an n -reference point grid, from only m ($\ll n$) available measurements from other devices.

I. CS based Video Scrambling

Privacy protection is an imperative aspect in context of video surveillance, especially when streaming such data over shared communication medium. [106] discusses application of block based CS to scramble privacy regions in video data. The scheme uses block based CS sampling on quantized coefficients during compression to protect privacy. In order to ensure security, key controlled chaotic sequence is used to construct measurement matrix. The approach provides significant coding efficiency and security improvements over conventional alternatives.

J. Network Traffic Monitoring and Anomaly Detection

Many basic network engineering tasks (e.g., traffic engineering, capacity planning, and anomaly detection) rely heavily on the availability and accuracy of traffic matrices. However, in practice it is challenging to reliably measure traffic matrices. Missing values are common. This observation brings us into the realm of CS, a generic technique for dealing with missing values that exploits the presence of structure and redundancy in many real-world systems. Despite much recent progress made in CS, existing CS solutions often perform poorly for traffic ma-

trix interpolation, because real traffic matrices rarely satisfy the technical conditions required for these solutions [107].

To address the problem of traffic metrics, authors in [107] propose a spatio-temporal CS framework with two key components: (i) A new technique called SRMF that leverages the sparse or low-rank nature of real-world traffic matrices and their spatio-temporal properties and (ii) a mechanism for combining low-rank approximations with local interpolation procedures. Authors claim to have superior performance in problems involving interpolation with real traffic matrices where we can successfully replace upto 98% of the values. The proposed framework is evaluated in applications such as network tomography, traffic prediction, and anomaly detection to confirm the flexibility and effectiveness [107].

Another application domain for CS is traffic volume anomaly detection. In the backbone of large-scale networks, origin-to-destination (OD) traffic flows experience abrupt unusual changes known as traffic volume anomalies, which can result in congestion and limit the extent to which end-user quality of service requirements are met [108]. Given link traffic measurements periodically acquired by backbone routers, the goal is to construct an estimated map of anomalies in real time, and thus summarize the network ‘health state’ along both the flow and time dimensions. Leveraging the low intrinsic-dimensionality of OD flows and the sparse nature of anomalies, a novel online estimator is proposed based on an exponentially-weighted least-squares criterion regularized with the sparsity-promoting norm of the anomalies, and the nuclear norm of the nominal traffic matrix. After recasting the non-separable nuclear norm into a form amenable to online optimization, a real-time algorithm for dynamic anomalography is developed and its convergence established under simplifying technical assumptions. Comprehensive numerical tests with both synthetic and real network data corroborate the effectiveness of the proposed online algorithms and their tracking capabilities, and demonstrate that they outperform state-of-the-art approaches developed to diagnose traffic anomalies.

K. Network Data Mining

A major challenge in network data mining applications is when the full information about the underlying processes, such as sensor networks or large online database, cannot be practically obtained due to physical limitations such as low bandwidth or memory, storage, or computing power. In [109], authors propose a framework for detecting anomalies from these large-scale data mining applications where the full information is not practically possible to obtain. Exploiting the fact that the intrinsic dimension of the data in these applications are typically small relative to the raw dimension and the fact that compressed sensing is capable of capturing most information with few measurements, authors show that spectral methods used for volume anomaly detection can be directly applied to the CS data with guarantee on performance.

L. Distributed Compression in WSNs

Distributed source coding is a compression technique in WSNs in which one signal is transmitted fully and rest of the signals are compressed based on their spatial correlation with

main signal. DSC performs poorly when sudden changes occur in sensor readings, as these changes reflect in correlation parameters and main signal fails to provide requisite base information for correct recovery of side signals. Only spatial correlation is exploited in DSC, while under no-event conditions, sensor readings usually have a high temporal correlation as well.

[110] presents a distributed compression framework which exploits spatial as well as temporal correlation within WSN. CS is used for spatial compression among sensor nodes. Temporal compression is obtained by adjusting number of measurements as per the temporal correlation among sensors. When sensor readings are changing slowly, few measurements are generated. In case, significant changes are detected in sensor readings measurements are generated more rapidly. To allow traction of changing compression rate at receiving station, recently developed rateless codes are used for encoding. Rateless codes are based on the idea that every receiving station continues collecting encoded data until decoding can be finished successfully. Proposed framework features low complexity single stage encoding and decoding schemes as compared to two stage encoding and decoding in previous state-of-the-art, while keeping the compression rate same. Compression rate of $\sum_{i=1}^m H(X_i)$ is achievable, where X_i is the reading from sensor i and $H(\cdot)$ represents the entropy of signal.

M. Network Security

CS can be used as an effective tool for provision of network security with vast potential to contribute. As one example application, clone detection, aiming to detect the illegal copies with all of the credentials of legitimate sensor nodes, is of great importance for sensor networks because of the substantial impact of clones on network operations like routing, data collection, and key distribution, etc [111]. Authors in [111] propose a novel clone detection method based on CS claiming to have the lowest communication cost among all detection methods. They exploit a key insight in designing their technique that the number of clones in a network is usually very limited. Based on the sparsity of clone appearance in the network, they propose a compressed sensing-based clone identification (CSI) scheme for static sensor networks.

VII. PROSPECT OF CS IN FUTURE

The idea of CS application to real-time systems is still in its infancy but one can fairly expect to see CS applied in many communication and networking systems in future, as the demand for cheaper, faster and efficient devices is on the rise. A prospective field of application of CS can be the broadband systems where there is a need to bring innovations in the physical layer technologies to achieve capacity increase and improved network security through resilient architectures and security mechanism that are proactive to threats. These demands make CS a prospective candidate as it increases capacity by reducing the number of samples required to be stored and adds error resilience. The delivery of video over mobile broadband systems is currently an active area of research. The use of CS to exploit spatio-temporal sparsity in 3D video frames can lead to efficient 3D video coding and transmission on mobile hand-held devices. With the

use of CS as a joint source channel coding scheme, complexity of the system can be greatly reduced. Next phase of wireless networks is expected to experience an increase in video demand. WSNs can benefit from the knowledge and application of CS. Another flourishing field of interest for researchers is wearable computing with one application being wireless body sensor networks. For example, wearable body sensors used to transmit vital signals for monitoring can make use of CS in order to reduce size and complexity of wearable devices. CS has the potential to be a paradigm shifter in RF architecture. Work is under way by researchers at University of Michigan [112], to develop a novel RF architecture employing concepts of CS in order to get lower power alternatives to existing devices. Basic premise behind their contribution is a significant power saving achieved courtesy sub-Nyquist sampling of RF signals.

VIII. CONCLUSIONS

In this review article, we have provided a comprehensive survey of the novel CS paradigm and its applications. We have traced origins of this technology and presented mathematical and theoretical foundations of the key concepts. Numerous reconstruction algorithms aiming to achieve computational efficiency and high speeds are surveyed. The applications of CS in diverse fields such as imaging, RADARs, biological applications and analog to information converters are discussed. There has been a surge recently to apply CS to communications and networks domain. We have captured this interest through a set of sample applications and have identified few potential research areas where CS can act as a paradigm shifter.

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