

# Index

## A

Adaptive  
  basis, 620  
  grid, 457, 466  
  smoothing, 563  
Adjoint operator, 758  
  frame, 156  
Admissible tree, 381, 427, 431, 624  
Affine invariance, 147  
*Algorithme à trous*, 176, 240  
Aliasing, 61, 69, 81, 303  
Ambiguity function, 98, 146  
Amplitude modulation, 57, 117  
Analog digital conversion, 7, 65,  
  168, 742  
Analytic  
  discrete signal, 88, 108  
  function, 108, 116  
  wavelet, 109, 129  
Approximation  
  adaptive grid, 457  
  bounded variation, 463, 468  
  image, 464  
  in wavelet bases, 442, 455  
  linear, 8, 436, 442, 468, 551  
  nonlinear, 9, 451, 512, 551  
  support, 9, 23, 455, 615  
  thresholding, 9  
  uniform grid, 442  
Arithmetic code, 491, 494,  
  510, 527  
Atom  
  time-frequency, 15, 89  
  wavelet, 92, 102, 109  
  windowed Fourier, 92

## Audio

  masking, 502  
  scaling, 117, 124, 132  
  transposition, 118, 125, 132

## B

Backprojection, 163  
   $l^1$  pursuit, 672  
  matching pursuit, 644  
  Radon, 55  
Balian-Low theorem, 185, 410  
Banach space, 754  
Bandlets, 631  
Basis  
  biorthogonal, 161, 306, 309, 757  
  orthogonal, 757  
  pursuit, 660  
  Riesz, 161, 265, 757  
Basis pursuit, 25  
  Lagrangian, 25, 664, 665, 684  
  wavelet packets, 663  
Battle-Lemarié wavelet, 281, 291,  
  457  
Bayes  
  estimation, 12, 536, 545  
  risk, 12, 536, 545  
Bernoulli random matrix, 732  
Bernstein inequality, 454  
Besov  
  norm, 460  
  space, 455, 459, 514  
Best approximation, 612  
Best basis, 393, 431, 504, 662  
  approximation, 622  
  compression, 623

- Best basis (*continued*)
    - local cosine, 629
    - search, 623
    - wavelet packet, 626
  - Bezout theorem, 174, 293
  - Binary tree, 624
  - Biorthogonal wavelets
    - basis, 310
    - fast transform, 311
    - lifting, 350
    - ordering, 312
    - regularity, 312
    - splines, 314, 369
    - support, 311
    - symmetry, 312, 369
    - two-dimensional, 345
    - vanishing moments, 311
  - Blind source separation, 744
  - Block basis, 401, 519
    - cosine, 404, 407
    - Fourier, 401
    - two-dimensional, 402
  - Block thresholding, 576
    - risk, 577
  - Boundary conditions, 442, 444
  - Boundary wavelets, 317, 322, 369
  - Bounded variation
    - discrete signal, 47
    - function, 46, 440, 446, 460, 461, 601, 711
    - image, 50, 467, 514, 603, 712
  - Box spline, 70, 174, 266
  - Butterworth filter, 56
- C**
- Canny edge detector, 230
  - Cantor
    - measure, 245
    - set, 242
    - spectrum, 251
  - Capacity dimension, 243
  - CART algorithm, 623
  - Cartoon image, 570
  - Cauchy-Schwarz inequality, 755
  - Chambolle algorithm, 675
  - Channel coding, 744
  - Chirp
    - hyperbolic, 129, 134
    - linear, 94, 126, 133
    - quadratic, 94
  - Choi-William distribution, 148
  - Co-area formula, 50
  - Coarse to fine, 340
  - Code
    - adaptive, 492, 507
    - arithmetic, 491
    - block, 490
    - conditional, 527
    - embedded, 516, 527
    - Huffman, 488
    - prefix, 485
    - Shannon, 488
    - variable length, 485, 506
  - Coding gain, 500
  - Cohen's class, 145
    - discrete, 150
    - marginals, 146
  - Coherent
    - matching pursuit, 658
    - structure, 656
  - Coiflets, 296
  - Color image(s), 478, 689, 692
  - Compact support, 286, 292, 311
  - Compression
    - audio, 482
    - dictionary, 614
    - image, 482, 506
    - speech, 482
    - video, 483, 654

Compressive sensing, 29, 728  
 Concave function, 754  
 Concentration inequality, 618  
 Conditional expectation, 537  
 Cone of influence, 215, 457  
 Conjugate gradient, 165  
 Conjugate mirror filters, 4,  
   276, 306  
   choice, 284, 505  
   Daubechies, 292  
   Smith-Barnwell, 298  
   Vaidyanath-Hoang, 298  
 Continuous wavelet transform,  
   102  
 Convex  
   function, 754  
   hull, 587, 592, 754  
   quadratic, 587  
 Convolution  
   circular, 76, 83, 301, 320, 394,  
     450, 541  
   continuous, 34  
   discrete, 71  
   fast FFT algorithm, 79  
   fast overlap-add, 80  
   integral, 34  
   separable, 83  
 Convolution theorem  
   circular, 77  
   discrete, 74  
   Fourier integral, 37  
 Cosine I basis, 403, 418, 519  
   discrete, 406, 425  
 Cosine IV basis, 404, 418  
   discrete, 407, 422  
 Cost function, 642  
 Covariance, 447, 762  
   operator, 447, 539, 762  
 Cubic spline, 270, 277

Curvelets, 194, 476  
   denoising, 570  
   tight frame, 197

## D

Daubechies wavelets, 292  
 DCT-I, 406, 409  
 DCT-IV, 407, 408  
 Decibels, 99, 541  
 Deinterlacing, 724  
 Devil's staircases, 246  
 DFT, *see* Discrete Fourier transform  
 Diagonal estimation,  
   552  
 Dictionary, 612  
   denoising, 616  
   Gabor, 650  
   local cosine, 646, 663  
   orthonormal bases, 621  
   wavelet packet, 646, 663  
 Dirac, 33, 40, 94, 719, 763  
   comb, 41, 60, 764  
 Discrete Fourier transform, 76, 540,  
   589  
   inversion, 77  
   Plancherel formula, 77  
   two-dimensional, 83  
 Discrete wavelet basis, 308, 563  
 Distortion rate, 11, 484, 517, 520  
 Dolby, 504  
 Dominated convergence, 274, 753  
 Dual  
   analysis, 22  
   frame, 159  
   synthesis, 22, 162  
 Dyadic wavelet transform, 170, 190,  
   568  
   maxima, 224  
   splines, 174  
   two-dimensional, 189

**E**

## Edges

- curve, 232, 471
- detection, 230
- illusory, 236
- image reconstruction, 235, 236
- multiscales, 230

## Eigenvector, 37, 71, 76

## Embedded code, 516, 527

## Energy conservation

- discrete Fourier transform, 77
- discrete windowed Fourier, 101
- Fourier integral, 39
- Fourier series, 73
- matching pursuit, 643
- tight frame, 155
- wavelet transform, 105, 111
- windowed Fourier, 96

## Entropy, 486

- differential, 495

## Error correcting code, 744

## Estimation, 12

- adaptive, 544
- block thresholding, 578
- multiscale edges, 236
- noise variance, 565
- oracle, 550, 551, 590, 705
- orthogonal projection, 550
- thresholding, 553
- Wiener, 539

## Exact Recovery Criteria, 25, 679

**F**

## Fast Fourier transform, 78

- two-dimensional, 85

## Fast wavelet transform

- biorthogonal, 310
- continuous, 114

## dyadic, 175

## initialization, 301

## multidimensional, 349

## orthogonal, 298

## two-dimensional, 346

## Fatou lemma, 753

FFT, *see* Fast Fourier transform

## Filter, 34

- analog, 37
- causal, 34, 71
- discrete, 71
- interpolation, 337
- low-pass, 40, 74
- recursive discrete, 74, 87
- separable, 83
- stable, 34, 71
- two-dimensional discrete, 82
- varying, 351

## Filter bank, 4, 176, 298

- perfect reconstruction, 304
- separable, 346, 399

## Finite elements, 361, 442, 471

## Fix-Strang condition, 286, 330, 370

## Folded wavelet basis, 320

- lifting, 369

## Fourier integral, 2

- amplitude decay, 42
- convolution theorem, 37
- in  $\mathbf{L}^2(\mathbb{R})$ , 38
- in  $\mathbf{L}^1(\mathbb{R})$ , 35
- inverse, 36
- Parseval formula, 39
- Plancherel formula, 39
- properties, 38
- rotation, 53
- sampling, 60
- slice theorem, 54, 726
- support, 45

- two-dimensional, 51
- uncertainty principle, 44
- Fourier series, 72, 438
  - approximation, 438
  - inversion, 73
  - Parseval formula, 73
  - pointwise convergence, 73
  - random measurements, 733
- Fractal
  - dimension, 243
  - noise, 258
- Fractional Brownian, 254, 261
- Frame
  - algorithm, 164
  - analysis, 155
  - definition, 22, 156
  - dual, 160, 187
  - dual wavelet, 180
  - projector, 166
  - synthesis, 156
  - tight, 156, 183, 197, 476
  - wavelet, 178
  - windowed Fourier, 182
- Frequency modulation, 117
- Frequency ridges, 17
- Frobenius norm, 688
- Fubini's theorem, 754

## G

- Gabor, 14
  - dictionary, 650
  - wavelet, 111, 190
- Gaussian
  - function, 41, 45, 126, 137
  - matrix, 731
  - process, 484, 499, 501, 540
  - white noise, 548
- Geometry, 510
- Gibbs oscillations, 47, 69, 440

- Gram matrix, 157
- Gram-Schmidt orthogonalization, 648
- Gray code, 386

## H

- Hölder
  - exponent, 206
  - norm, 464
  - space, 445, 464
- Haar wavelet, 2, 3, 291
- Hard thresholding, 668
- Hausdorff dimension, 243
- Heat diffusion, 221
- Heisenberg
  - box, 16, 90, 109, 388, 420, 628
  - uncertainty, 15, 43, 89, 90, 98
- Hilbert space, 755
- Histogram, 491, 506, 509
- Huffman code, 488, 494
- Hurst exponent, 254
- Hyperrectangle, 587, 590

## I

- Illusory contours, 236
- Impulse response, 34, 82
  - discrete, 70, 82
- Incoherence, 730
- Inpainting, 722
- Instantaneous frequency, 94, 115, 138
- Interpolation, 61, 472, 725
  - Deslauriers-Dubuc, 332, 337
  - function, 328
  - Lagrange, 337
  - spline, 331
  - wavelets, 335
- Inverse problem, 700
  - compressive sensing, 728
  - super-resolution, 713

Inverse problem (*continued*)  
 thresholding, 27  
 Iterative thresholding, 668

**J**

Jackson inequality, 454  
 Jensen inequality, 754  
 JPEG, 11, 519  
 JPEG-2000, 11, 523

**K**

Karhunen-Loève  
 approximation, 447  
 basis, 447, 450, 499, 539, 762  
 Kraft inequality, 486, 507

**L**

Lagrangian  
 approximation, 612, 665  
 basis pursuit, 664, 665, 684  
 Lapped  
 fast transform, 424  
 frequency transform, 418  
 orthogonal basis, 416  
 orthogonal transform, 410  
 projector, 411  
 Lazy wavelet, 352  
 Least favorable distribution, 547  
 Left inverse, 159  
 Legendre transform, 248  
 Level set, 50, 232, 467, 471, 728  
 Lifting, 350  
 dual, 355  
 factorization, 367  
 prediction, 353  
 update, 355  
 Linear  
 Bayes risk, 543

estimation, 12, 537  
 programming, 662

**Lipschitz**

exponent, 205, 456, 460  
 Fourier condition, 206  
 in two dimensions, 230  
 regularity, 206  
 wavelet condition, 211, 212  
 wavelet maxima, 219

Littlewood-Paley sum, 212

**Local cosine**

basis, 20, 418, 440, 501  
 discrete, 423, 429  
 quad-tree, 430  
 tree, 426, 429  
 two-dimensional, 630

Local stationarity, 501

Loss function, 536

LOT, *see* Lapped

**M**

*M*-band wavelets, 390  
 Mallat algorithm, 298  
 Markov chain, 532  
 Masking noise, 561, 570  
 Matching pursuit, 24, 642, 679  
 denoising, 656  
 fast calculation, 645  
 orthogonal, 648  
 wavelet packets, 646  
 Maxima  
 curves, 232  
 of wavelet transform, 218, 231, 245  
 propagation, 221  
 Median filter, 565  
 Mesh, 361, 472  
 Mexican hat wavelet, 103, 180  
 Meyer  
 wavelet, 289  
 wavelet packets, 418

Minimax, 7  
     estimation, 12, 544  
     risk, 12, 544, 545, 586, 590, 606  
     theorem, 545  
 Mirror wavelet basis, 711  
 Missing data, 722  
 Model selection, 617  
 Modulus maxima, 218, 230  
 Modulus of continuity, 334  
 Mother wavelet, 92  
 Moyal formula, 139  
 MP3, 503  
 MPEG, 483  
 MRI imaging, 743  
 Multichannel signals, 688  
 Multifractal, 19, 242  
     partition function, 248  
     scaling exponent, 248  
 Multiresolution approximations  
     definition, 264  
     piecewise constant, 265, 277, 339  
     Shannon, 265, 266, 277, 339  
     splines, 266, 277, 340  
 Multiscale derivative, 208  
 Multiwavelets, 287, 373  
 MUSICAM, 502  
 Mutual coherence, 678

## N

Neural network, 645  
 Norm, 754  
      $L^2(\mathbb{R})$ , 756  
      $\ell^2(\mathbb{Z})$ , 756  
      $\ell^p$ , 454, 460, 755  
      $l^1$ , 660  
      $l^0$ , 665  
     sup for operators, 758  
     weighted, 498, 520  
 NP-hard, 613

## O

Operator  
     adjoint, 758  
     projector, 759  
     sup norm, 758  
     time-invariant, 33, 70  
 Oracle  
     attenuation, 550, 590  
     estimation, 549  
     projection, 551, 557  
 Orthogonal  
     basis, 757  
     projector, 759  
 Orthosymmetric set, 592, 606

## P

Parseval formula, 39, 757  
 Partition function, 248  
 Penalized estimation, 617  
 Piecewise  
     constant, 265, 277, 339  
     polynomial, 543  
 Piecewise regular  
     in 1D, 456, 599  
     in 2D, 471  
 Pixel, 80  
 Plancherel formula, 39, 757  
 Poisson formula, 41, 285  
 Polynomial  
     approximation, 330  
     spline, *see* Spline  
 Posterior distribution, 536  
 Power spectrum, 541, 763  
 Pre-echo, 502  
 Prediction, 606  
 Prefix code, 485  
 Principal directions, 449, 762  
 Prior distribution, 536  
 Prior set, 544

Pseudo inverse, 159

PSNR, 508

Pursuit

  basis, 660

  matching, 642, 679

  orthogonal matching, 648

## Q

Quad-tree, 396, 430, 624

Quadratic

  convex hull, 587, 592

  convexity, 587

Quadrature mirror filters, 302, 371

Quantization, 11, 483, 493

  adaptive, 502

  bin, 493

  high resolution, 493, 496, 507

  low resolution, 510

  uniform, 494

  vector, 484

  weighted, 526

Quincunx

  sampling, 359

  wavelets, 359

## R

Radon transform, 53, 726, 743

Random sensing, 731

Random shift process, 449, 542

Rate distortion, 529

Real wavelet transform, 103

  energy conservation, 105

  inverse, 105

Regularization

  Tikhonov, 700, 722

  total variation, 728

Reproducing kernel

  frame, 167

  wavelet, 106

  windowed Fourier, 97

Residue, 643, 648

Restoration, 700

Restricted isometry constant, 730

Richardson iteration, 163

Ridges

  wavelet, 129

  windowed Fourier, 122

Riemann function, 260

Riemann-Lebesgue lemma, 56

Riesz basis, 22, 65, 161, 265, 757

Rihaczek distribution, 147

Risk, 12, 536

Run-length code, 519

## S

Sampling

  Block, 69

  generalized theorems, 69, 328

  irregular, 158

  redundant, 168

  spline, 70

  two-dimensional, 81

  Whittaker, 68

  Whittaker theorem, 61, 81

Sampling theorems, 7

Satellite image, 712

Scaling equation, 270, 330

Scaling function, 106, 267

Scaling images, 724

Scalogram, 109

Segmentation, 192

Seismic imaging, 719

Self-similar

  function, 19, 244

  set, 242

Separable

  basis, 84, 760

  block basis, 402



- convolution, 83
- decomposition, 84
- filter, 83
- filter bank, 399
- local cosine basis, 431
- multiresolution, 338
- wavelet basis, 338, 341
- wavelet packet basis, 399
- Shannon
  - code, 488
  - entropy theorem, 486
  - multiresolution, 266
  - sampling theorem, 61
- Sigma-Delta, 168
- Signal to Noise Ratio, 541
- Significance map, 510, 516, 519, 526
- Singular value decomposition, 27, 701
- Singular values, 156, 759
- Singularity, 19, 205
  - spectrum, 246
- SNR, 541
- Sobolev
  - differentiability, 438, 443
  - space, 439, 443, 459
- Soft thresholding, 553
- Sonar, 126
- Sound
  - model, 117, 744
  - separation, 744
- Source separation, 29, 687, 744
- Sparse spike deconvolution, 719, 733
- Spectrogram, 92
- Spectrum
  - of singularity, 246
  - operator, 759
  - power, 763
- Speech, 117, 482
- Spline
  - approximation, 457
  - multiresolution, 266

- sampling, 70
- wavelet basis, 281
- Stationary process, 450
  - circular, 540
  - locally, 501
- Stein Estimator, 559
- Super-resolution, 28, 713, 724
- Support
  - approximation, 23
  - recovery, 25
- Sure threshold, 558, 566
- Symmetric filters, 313
- Symmetric operator, 758
- Symmlets, 294, 565

## T

- Tensor product, 339, 760
- Texture discrimination, 191
- Thresholding
  - block, 576
  - estimation, 14, 568, 705
  - hard, 552, 565, 668
  - inverse problem, 27
  - iterative, 668
  - risk, 552, 592
  - soft, 553, 565
  - Sure, 558, 566
  - threshold choice, 556, 705
  - translation invariant, 561, 566
  - wavelets, 566, 606
- Tikhonov regularization, 701, 723
- Time-frequency
  - atom, 15, 89
  - plane, 15, 90
  - resolution, 90, 98, 109, 124, 135, 140, 146, 388
- Tomography, 53, 726, 743
  - backprojection, 55
- Tonality, 502

Total variation, 440, 461, 728

discrete signal, 47

function, 46

image, 50

Transfer function, 83

analog, 37

discrete, 71

Transform code, 11, 482, 483

JPEG, 11, 519

with wavelets, 11, 514

Transient, 628

Translation invariance, 168, 226, 422,

561, 566, 589, 646

Transposition, 118, 125, 132

Triangulation, 361, 472

Delaunay, 475

Turbulence, 258

## U

Uncertainty principle, 16, 43, 89, 90, 98

Uniform sampling, 60

## V

Vanishing moments, 208, 284, 330, 342,

352, 358, 443, 455, 524

Variance estimation, 565

Video compression, 483, 654

Vision, 189

Von Koch fractal, 244

## W

Walsh wavelet packets, 387

Wavelet

directional, 189

seismic, 719

Wavelet basis, 278, 281

Battle-Lemarié, 291, 457

boundary, 301, 322

choice, 284, 524

Coiflets, 296

Daubechies, 3, 292

discrete, 306

folded, 320

graphs, 302

Haar, 291

interval, 317, 369, 442

lazy, 352

lifting, 350

M-band, 390, 504

Meyer, 289

mirror, 711

non-separable, 359

on surfaces, 361

orthogonal, 3

periodic, 318

quincunx, 359

regularity, 287

separable, 341

Shannon, 289

spherical, 365

Symmlets, 294

Wavelet packet basis, 19, 382, 504,

626, 710

quad-tree, 430

tree, 379

two-dimensional, 395

Walsh, 387

Wavelet transform, 17

admissibility, 106, 179

analytic, 109

continuous, 17, 102

decay, 211, 212

dyadic, 170

frame, 178

lifting, 356

- maxima, 218, 232
- multiscale differentiation, 208
- real, 103
- ridges, 129, 216
- Weak convergence, 763
- White noise, 540, 548
- Wiener estimator, 538, 539, 542, 589
- Wigner-Ville
  - cross terms, 140
  - discrete, 149
  - distribution, 89, 136, 140, 651
  - instantaneous frequency, 138
  - interferences, 140
  - marginals, 139
  - positivity, 143
- Window
  - Blackman, 99
  - design, 75, 99, 419
  - discrete, 75

- Gaussian, 99
- Hamming, 99
- Hanning, 75, 99
- rectangle, 75
- scaling, 98
- side-lobes, 75, 99, 125
- Windowed Fourier transform, 16, 92
  - discrete, 101
  - energy conservation, 96
  - frame, 182
  - inverse, 96
  - reproducing kernel, 97
  - ridges, 122

## **Z**

- Zak transform, 203
- Zero-tree, 526
- Zygmund class, 212