DMS639: Analytics in Transport and Telecom

Project Report

Coach Trip with Shuttle Service Problem

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Problem Description:

The Coach Trip with Shuttle Service Problem is a transportation problem, where passengers have to be transported to a central hub. The objective is minimizing the overall transportation costs.

Each vehicle k in the fleet belongs either to the set of coaches *C* or the set of shuttles *S*.

Every coach *k* starts at a specific location and has a fixed capacity. A coach trip can only visit a certain number of bus stops *Sk* max (not counting the hub) and it must always end at the hub.

Each group of passengers starts at a bus stop *bi* and has a travel time limitation *Di* that restricts the acceptable travel time of the group on its way to the hub.

Additionally, to coaches we can also rent shuttles to transport passengers either directly to the hub or to transfer points. A transfer point can be a bus stop. After a shuttle has dropped off its passengers at a transfer point the shuttle trip

ends there and the passengers need to be picked up by exactly one coach. Passenger groups cannot be split.

The coach costs consist only of variable costs per distance traveled while for the shuttles there are additional fixed usage costs as they are booked extra. The vehicle categories “coach” and “shuttle” differ in speed and costs per traveled distance unit.

• The set B+ of bus stops that need to be visited.
• The set B0 of empty bus stops. These do not have passengers and can, but do not need to be visited.
• The set C of coaches.
• The set S of shuttles.
• The location H is the terminal hub.

Nomenclature:

$n \; - \; number\ of\ Bus\ stops$

$m \; - \; number\ of\ coaches$

$k \; - \; number\ of\ shuttles$

$H \; - \; Hub$

$s_k \; - \; starting\ location\ of\ k'th\ coach$

$s_{k'} - \; starting\ location\ of\ k'th\ shuttle$

$cap_k \; - \; capacity\ of\ k'th\ coach$

$cap_{k'} \; - \; capacity\ of\ k'th\ shuttle$

$D^i_{max} \; - \; transportation\ time\ for\ ith\ passenger\ to\ the\ hub$

$S^k_{max} \; - \; Max\ number\ of\ bus\ stop\ on\ a\ route\ for\ kth\ coach$

$q_i \; - \; Number\ of\ passenger\ waiting\ at\ bus\ stop\ b_i$

$D_i \; - \; Maximum\ Travel\ time\ from\ bus\ top\ b_i\ to\ H$

$vc_k \; - \; cost\ per\ unit\ distance\ traveled\ for\ \ k'th\ coach$

$vc_{k'}$ $-$ $cost\ per\ unit\ distance\ traveled\ for\ \ k'th\ shuttle$

$fc_{k'}$ $-$ $fixed\ cost\ \ for\ \ k'th\ shuttle$

$d^{c}_{i,j}\ /\ d^{s}_{i,j}$ $-$ $distance\ between\ pair\ of\ location\ for\ coach/shuttle$

$t^{c}_{i,j}\ /\ t^{s}_{i,j}$ $-$ $travel\ time\ between\ pair\ of\ location\ for\ coach/shuttle$

Constraints :

• All passengers are transported from locations in B+ to the hub H.
• All coaches must travel to hub H and cannot exceed stop limitations.
• Shuttles can drop off their passengers at any stop, but those must be picked up by a coach afterward.
• The time limit of each passenger group must not be exceeded.
• Passenger groups cannot be split.
• Shuttles must not return to their start location.
• Shuttles can do at most one trip.
• The vehicle capacities must not be exceeded.

Objective Function:

Find feasible routing with minimal possible transportation cost.

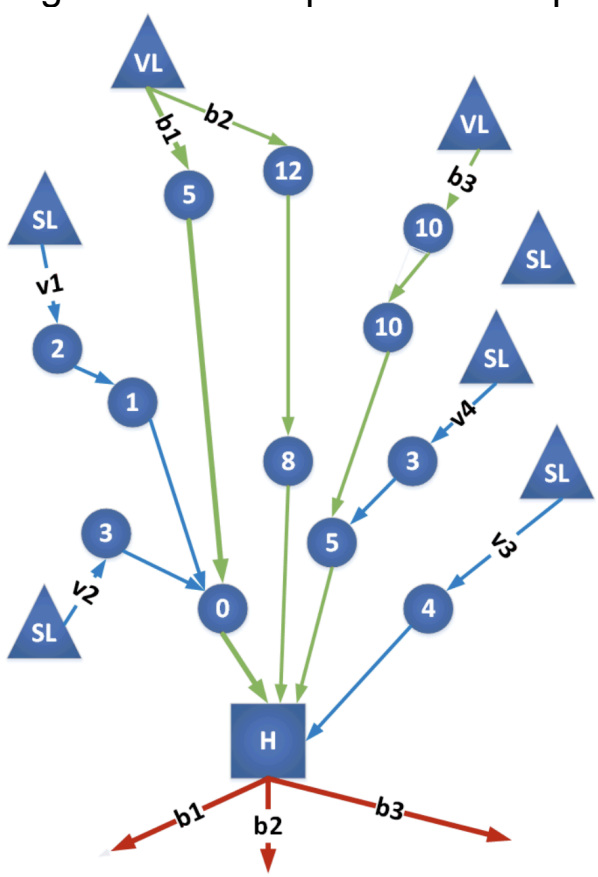

Figure 1: typical trip structures for coach trips

Mathematical Model:

In this section, we formulate a mathematical model for the CTSSP. We add two dummy locations H′ and TP′ to the set of locations V . When a vehicle k uses less than the maximum number of its stops S k max it is allowed to circle in these dummy locations for the remaining stops. H′ is a dummy hub and has a travel time and travel distance of 0 from and to the hub H. It is not reachable from any other location and cannot be left once entered. TP′ is a dummy location needed for the transfer points and has a travel time and travel distance of 0 from all bus stops. Again, it is not reachable from any other location and cannot be left.

We use the decision variables as defined below:

$x_{i,j}^{k,l} \in \{0, 1\}$ is defined for vehicle k entering location j as the l-th stop on its trip after visiting location i before.

$y^{k} \in \{0, 1\}$ indicates s for each shuttle k if it is used in the routing or not

$q^{k} \in Z^{+}$ indicates no. of passengers transported by vehicle k

$t_{l}^{k} \in Z^{+}$ time required from stop l to last stop by vehicle k

$z_{i,l}^{k,k'} \in \{0, 1\}$ indicates if there is a transfer point at location i for shuttle k ′ and coach k and the coach enters location i at the l-th position of its trip

$\tilde{q}^{k,k'} \in Z^+$ is set to 0 if shuttle k ′ and coach k do not share a transfer point and are set to the number of passengers transported by shuttle k if they do

$t^{k',k} \in Z^+$ is set to 0 if shuttle k ′ and coach k do not share a transfer point and are set to the travel time from the transfer point to the hub of the coach if they do

Objective function:

$$min \sum_{k \in C \cup S} vc_k \cdot \left(\sum_{i,j \in V}^{S^k_{max}} x^{k,l}_{i,j} \cdot d^k_{i,j} \right) + \sum_{k \in S} y^k \cdot fc_k$$

Path Constraints:

$$\sum_{j \in B} x^{k,1}_{s_k,j} = 1 \qquad \forall k \in C$$

It ensures that all coaches start from their start location and visit a bus stop location at their first stop

$$\sum_{j \in V} x^{k,1}_{i,j} = 0 \qquad \forall k \in S, \forall i \in V \setminus \{s_k\}$$

It prohibits shuttle using any other location than its start location for a start

$$\sum_{j \in V \setminus B} x^{k,l}_{i,j} = 0 \qquad \forall k \in C \cup S, \; l = 1, \cdots, S^k_{max}, \; \forall i \in V$$

It describes that no vehicle can visit the start locations of other vehicles (these are the locations in V \ B)

$$\sum_{i,j \in V} x^{k,l}_{i,j} \leq y^{k} \quad \forall k \in S$$

It forces the variable y^{k} to 1 if shuttle k is used

$$\sum_{i,j \in V} x^{k,l}_{i,j} \leq 1 \qquad \forall\, k \in C \cup S,\ l = 1, \cdots, S^{k}_{max}$$

It ensurs that each vehicle k can only go at most from one location to one other location at each position l of its trip

$$\sum_{j \in V} x^{k,l}_{i,j} = \sum_{j \in V} x^{k,l+1}_{i,j} \qquad \forall k \in C \cup S,\ l = 1, \cdots,\ S^{k}_{max} - 1,\ \forall i \in V$$

If the vehicle enters a location i at the l-th position of its trip, it has to leave this location i when going to the (l + 1) th position of its trip.

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^{k}_{max}} \sum_{j \in V,\, j \neq H', TP'} x^{k,l}_{i,j} \leq 1\ \forall\, i \in V$$

For every location i at most one vehicle can leave this location to another location j except for the two dummy locations.

Dummy locations constraints:

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^{k}_{max}} \sum_{j \in V \setminus \{H'\}} x^{k,l}_{H,j} = 0$$

It ensures that no vehicle can travel from the hub location H to any other location except the dummy hub location H

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^k_{max}} \sum_{j \in V \setminus \{H, H'\}} x^{k,l}_{j,H'} = 0$$

It prohibits by that the dummy hub location H′ can be reached by any other location than the actual hub location H or the dummy hub location itself

$$\sum_{k \in C \cup S} \sum_{l=1}^{S^k_{max}} \sum_{j \in V \setminus \{H'\}} x^{k,l}_{H',j} = 0$$

from the dummy hub location H′ vehicles can only travel to the dummy hub location

$$\sum_{k \in S} \sum_{l=1}^{S^k_{max}} \sum_{j \in V \setminus \{TP'\}} x^{k,l}_{TP',j} = 0$$

shuttles that enter the transfer point dummy location to cycle there and prohibits them to travel to another location.

End Trip constraints:

$$\sum_{l=1}^{S^k_{max}} \sum_{i \in V} x^{k,l}_{i,H} = 1 \quad \forall k \in C$$

It forces every coach k to visit the hub location H on its trip

$$\sum_{l=1}^{S^k_{max}} \sum_{i \in V} x^{k,l}_{i,H} + \sum_{k' \in C} \sum_{l'=1}^{S^{k'}_{max}} \sum_{j \in B_+ \cup B_0} z^{k,l}_{j,l'} = y^k \qquad \forall k \in S$$

If a shuttle k is used, the r.h.s is equal to 1 and forces the left hand side of the equation to be 1 as well: Either the shuttle travels to the hub at some point of its trip (left hand sum) or the shuttle shares a transfer point with some coach k ′ (right hand sum)

$$\sum_{k \epsilon CUS} \sum_{l=1}^{S^k_{max}} \sum_{j \epsilon V \setminus \{TP'\}} x_{i,j}^{k,l} = 1 \quad \forall i \in B_+$$

It ensures that for each bus stop i with customers waiting there is exactly one vehicle leaving the bus stop to another location differing from the transfer point dummy location

Transfer locations constraints:

$$\sum_{j \in V} x^{k,l}_{j,i} \geq z^{k,k'}_{i,l} \quad \forall k \in C, \forall k' \in S, \forall i \epsilon B_+ \cup B_0, \; l = 1, 2.. S^k_{max}$$

It ensures $z^{k,k'}_{i,l}$ to be equal to 0 if coach k does not enter location i as its l-th stop of the trip

$$\sum_{l'=1}^{S^k_{max}} \sum_{j \in V} x^{k',l'}_{j,i} \geq z^{k,k'}_{i,l} \quad \forall k \in C, \forall k' \in S, \forall i \epsilon B_+ \cup B_0, \; l = 1, 2.. S^k_{max}$$

It ensures $z^{k,k'}_{i,l}$ to be equal to 0 if shuttle k′ does not enter location i at any position of its trip

$$\sum_{j \in V} x^{k,l}_{j,i} + \sum_{l'=1}^{S^k_{max}} \sum_{j' \in V} x^{k',l'}_{j',i} - 1 \leq z^{k,k'}_{i,l}$$

$$\forall k \in C, \forall k' \in S, \forall i \in B_+ \cup B_0, \; l = 1, 2.. S^k_{max}$$

If both shuttle k′ and coach k visit location i, then it is used as a transfer point and the corresponding $z^{k,k'}_{i,l}$ =1

Travel Time Constraints:

$$\sum_{l'=l+1}^{S^k_{max}} \sum_{i,j \in B} x^{k,l}_{i,j} t^c_{i,j} = t^k_l \qquad \forall k \in C, \; l = 1,..., S^k_{max}$$

It sets the value of t^k_l to the travel time needed from the l'th stop on the trip of coach k to the last stop

$$\sum_{l'=l+1}^{S^k_{max}} \sum_{i,j \in B} x^{k,l}_{i,j} t^s_{i,j} = t^k_l \qquad \forall k \in S, \; l = 1,..., S^k_{max}$$

It ensures the travel times for shuttles from their l'th stop to the last stop are set

$$t^k_l \leq \sum_{i \in V} \sum_{j \in B_+ \cup B_0} x^{k,l}_{i,j} D_j \qquad \forall k \in C \cup S, \; l = 1,..., S^k_{max}$$

It ensures that for each bus stop j with customers waiting on the trip of vehicle k the maximum travel time Dj is not exceeded

$$\tilde{t}^{k',k} \leq M \cdot \sum_{l=1}^{S^k_{max}} \sum_{i \in B_+ \cup B_0} z^{k,k'}_{i,l} \quad \forall k \in C, \forall k' \in S$$

the $\tilde{t}$ variable is forced to be 0 if there is no transfer point for coach k and shuttle k′. (M = is an integer number greater than the maximum travel time for any vehicle)

$$\tilde{t}^{k',k} - t^k_l \leq M \cdot (1 - \sum_{i \in B_+ \cup B_0} z^{k,k'}_{i,l}) \quad \forall k \in C, \forall k' \in S, \; l = 1,..., S^k_{max}$$

$$t^{k',k} - t^k_l \geq - M \cdot (1 - \sum_{i \in B_+ \cup B_0} z^{k,k'}_{i,l}) \quad \forall k \in C, \forall k' \in S, \; l = 1,..., S^k_{max}$$

Above three constraints are used to set the $t^{k',k}$ variables to the correct value

Capacity constraints:

$$\sum_{l=1}^{S^k_{max}} \sum_{l \in B_+} \sum_{j \in V \setminus \{TP'\}} x^{k,l}_{i,j} q_i = q^k \quad \forall k \in CUS$$

The variable q^k is set to the amount of passengers waiting at locations that are visited by vehicle k. Only the connections that do not end in the dummy transfer point location TP′ are counted.

$$q^{k} \leq cap_{k} \qquad \forall k \in C \cup S$$

it is ensured that the capacity of the vehicles is not exceeded

$$\tilde{q}^{k,k'} \leq cap_{k'} \cdot \sum_{l=1}^{S^{k}_{max}} \sum_{l=B+\cup B_{0}}^{S^{k}_{max}} z_{i,l}^{k,k'} \quad \forall k \in C, \forall k' \in S$$

$$\tilde{q}^{k,k'} - q^{k'} \leq (1 - \sum_{l=1}^{S^{k}_{max}} \sum_{l=B_{+}\cup B_{0}}^{S^{k}_{max}} z_{i,l}^{k,k'}) . cap_{k'} \quad \forall k \in C, \forall k' \in S$$

$$\tilde{q}^{k,k'} - q^{k'} \geq (1 - \sum_{l=1}^{S^{k}_{max}} \sum_{l=B+\cup B_{0}}^{S^{k}_{max}} z_{i,l}^{k,k'}) . (- cap_{k'}) \quad \forall k \in C, \forall k' \in S$$

Above three constraints set the value of $\tilde{q}^{k,k'}$ correctly.If coach k and shuttle k ′ do not share a transfer point, by first of the above three constraints the variable $\tilde{q}^{k,k'}$′ is forced to be 0. However, if they use a transfer point (i.e., the corresponding z variable is 1 for some location i and some position l), we enforce $\tilde{q}^{k,k'} = q^{k'}$ by 2nd and 3rd constraints, i.e., the q˜ variable is set to the amount of passengers on the shuttle

$$\tilde{q}^{k,k'} \leq q^{k'} \; \forall k \in C, \forall k' \in S$$

It ensures that the value of the variable $\tilde{q}^{k,k'}$can not exceed the value of the variable $q^{k'}$ for a shuttle k ′

$$q^{k} + \sum_{k' \in S} \tilde{q}^{k,k'} \leq cap_{k} \quad \forall k \in C$$

It ensures that for all coaches also the total number of passengers must not violate the capacity of the coach

Heuristic Algorithm:

1st algorithm:

1. Initially, use all the shuttles
2. Solve VRP using the coaches and these shuttles
3. Compute Δ_s for each shuttle s:
    - Δ_s:= change in objective value after removing the shuttle s and solving VRP again
4. Let $s_0 = \text{argmin}_s \Delta_s$
5. If $\Delta_{s0} > 0$, terminate
6. If $\Delta_{s0} <= 0$, remove the shuttle s_0, and Repeat from (2)

The heuristic algorithm solves the VRP subroutine at max $(\#\text{Shuttles})^2$ number of times.
Therefore, if we can solve VRP in polynomial time (using a heuristic algorithm for example), then our heuristic algorithm is also polynomial time, which is a lot faster than the time that CPLEX would take to solve an MIP with so many binary variables.

2nd algorithm:
Clustering-based approach

1. Form clusters of bus stations and starting locations
2. Solve the clusters independently
3. Check if the clusters are 1-opt — is it possible to transfer one coach/shuttle from one cluster to the other such that the overall cost is minimized

Observation: Both the algorithms return locally optimal solutions, they are not guaranteed to return globally optimal solutions. Hence, neither of these algorithms are exact algorithms.

References:

[1] On an effective approach for the coach trip with shuttle service problem of the VeRoLog solver challenge 2015, https://onlinelibrary.wiley.com/doi/full/10.1002/net.21733
[2] A Mathematical Model for the Coach Trip with Shuttle Service Problem, https://openhsu.ub.hsu-hh.de/handle/10.24405/4297