Dynamic Multi-Asset allocation under regime shifts

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Abstract

This paper investigates asset allocation decision in the presence of two volatility regimes and four economic phases. Three portfolio construction strategies will be evaluated in the paper. Firstly assets will be allocated based on Markowitz mean-variance optimization. Secondly, to capture financial data specific features and increase predictability of returns, Markov regime- switching models with two volatility states will be used to calculate assets expected returns, variances, and covariance and to construct a regime dependent portfolio. Lastly, based on Black-Litterman approach, assets will be allocated and rebalanced considering the changing market conditions and economic cycles. The paper mainly attempts to answer the questions how regimes affect asset allocations, and what are the advantages of dynamic portfolio management approach in practical applications.

A good portfolio is more than a long list of good stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies. – Harry Markowitz

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Introduction

Expected returns are one of the most important aspects while making investment decision. The usual problem that investors face is to maximize expected returns of the portfolio subject to some target level of risk. The asset allocation decision is the primary determinant of portfolio performance (D. Boos, J. Koller, O. Schmid April 2004). Assets prices change behavior correspondingly to business cycle phase of the economy, and returns of assets follow the sophisticated process with multiple "regimes," each of which has a very different distribution. Generally, recession periods are characterized by highly volatile bear markets, while recovery periods are low-volatile bullish markets (W. Lihovodov 1999). Investors should choose the optimal portfolio with minimum risk mainly during recession periods, but during recovery, they may switch on the portfolio with higher risk and correspondingly higher return (H.Pedersen 2015). This paper characterizes Dynamic Asset Allocation (hereafter DAA) with monthly rebalancing under a Markov regime-switching model with two volatility regimes and with investors' views on returns. DAA is active management at the macro level that allows capturing shorter-term opportunities in the markets. The period of the time dedicated to dynamic strategies usually varies from one month to a year. To predict the assets returns the Markov Regime Switching models have been used. These models reflect the cyclical changes in the financial markets and represent the behavior of much return series (A. Ang, A. Timmermann June 2011). After the consideration of the regime-dependant returns and the variance-covariance matrix, the results have been used for the portfolio optimization purpose.

There are three strategies considered in this paper. Firstly, the portfolio has been formulated based on Markowitz mean-variance optimization with the historical returns. Secondly, based on the Markov regime-switching model new ex-ante returns have been projected, and the regime dependent variance-covariance matrix and standard deviations have been estimated. Then the new estimates have been used for the Markowitz portfolio optimization. Lastly, the Black-Litterman model approach has been introduced with the investor's own views on the asset returns. The investors' views will alter accordingly for four economic states that are characterized as (i) rising growth and rising inflation, (ii) falling growth and rising inflation, (iii) rising growth and falling inflation, (iv) falling growth and falling inflation states. The underlying economic states suggest different investment opportunities and asset allocation vary significantly over time as investors reconsider their beliefs about the underlying states and regimes. The final model in this study has distributed the investments between the bonds, stocks, commodities, and cash effectively in terms of return and risk of a portfolio. Finally, newly formulated portfolios have been tested on the actual data in order to determine the benefits of regime based assets allocation.

Regime –switching model

Financial markets exhibit cyclical behavior, while some changes can be unique such as breaks, often the changed behavior of markets is recurring such as recessions and expansions (A. Ang, A. Timmermann June 2011). During recessions and expansions, mean, volatility and correlation of assets change dramatically, however, conventional linear asset pricing models imply a positive and monotonic risk-return relation (A. Ang, A. Timmermann June 2011). Conditional on a particular forecasting model, investors usually face with the parameter uncertainty (D. Pettenuzzo, A. Timmermann 2010). Instability of the model parameters during the return generating process is crucial especially for long investment horizon because of the increasing likelihood of regime change and occurring breaks (A. Ang, A. Timmermann June 2011). Changes between regimes can lead to increasing, decreasing and non-monotonic risk-return relations. The Markov switching model of Hamilton (2005), is one of the most popular nonlinear time series models in the literature. The application of this model was to capture business cycle recessions and expansions (D. Hamilton 2005). An essential feature of regime switching models are their capability to capture the behavior of much financial return series including fat tails, periods of high volatility and low volatility, skewness, and time-varying correlations (A. Ang, A. Timmermann June 2011). Regimes introduced into linear asset pricing models can often be solved in closed form because conditional on the underlying regime, normality is recovered (A. Ang, A. Timmermann June 2011). Regime-switching models even catch the phenomenon of regulatory policy changes and the new dynamics of prices. Thus, regime switching models can be used for ex-ante real-time forecasting, optimal portfolio choice, and other economic applications. In the scope of this study regime switching autoregressive (AR) model of order k will be used to predict assets returns. The regime-switching models are applied only for stationary data. At each time t, Δy_t will be drawn from a different distribution, depending on which regime is dominating at time t (A. Ang, A. Timmermann June 2011). The general model has the form of: $\Delta y_t = \mu_{s_t} + \sum_j^k \varphi_{j\, s_t,} \Delta y_{t-j} + \sigma_{s_t} \varepsilon_t \;\; \varepsilon_t \sim N(0, \sigma_{s_t}^2)$

(1)

Where Δy_t $(N \times 1)$ donates returns and $s_t = i, i = 1, \dots, N$ denotes the regime at time t. Markov-switching model also assumes that s_t is unobserved and follows a specific stochastic process, namely an N-state Markov chain which implies that the probability of the current state only depends on the previous state. The model also estimates the regime transition probabilities.

$$p_{ij,t} = p(s_t = j | s_{t-1} = i, I_{t-1}) \qquad (2)$$

Conditional on a value of j $\sum_{i=1}^N p_{ij} = 1$. The existing literature has focused merely on likelihood-based methods for estimation, and commonly the number of regimes is fixed to two. So a two-regime model where $s_t = 1 \; or \; 2$ describe low and high volatility regimes or expansion and recession phases. The general (benchmark) model has constant regime transition probabilities.

$$\mathrm{P} = \mathrm{p}(S_t = 1 \,| S_{t-1} = 1; I_{t-1}) = p_{11} \; (2.1)$$

$$\mathrm{Q} = p(S_t = 2 \,| S_{t-1} = 2; I_{t-1}) = p_{22} \; (2.2)$$

Here is the regime transition probability matrix of the model with two regimes:

$$\begin{pmatrix} P & 1-P \\ 1-Q & Q \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \qquad (3)$$

The extension of the general model is the model with non-perfectly correlated regime states in the assets return (A. Ang, G. Bekaert 1999). If conditional information z_{t-1} affects the mean or volatility, the general regime switching process takes the form of (A. Ang, A. Timmermann June 2011): $y_t = \mu(s_t, y_{t-1}, z_{t-1}) + u_{y_t}$ (4)

where $u_{yt} = \sigma_{s_t}\varepsilon_t$ and z_t is the conditional information such as an interest rate or a leading economic indicator. The transition probabilities may be non-linear functions of the instruments and can be specified as logistic functions of the instruments (A. Ang, G. Bekaert 1999).

$$p(s_t = i|s_{t-1} = i, I_{t-1}) = f_i(z_{t-1}) \quad (5)$$

The distribution of asset returns depends on the regime s at time t and the previous realization of the instruments z_{t-1} that are thought to influence on the realization of the regime. For example, the Federal Reserve short term interest rates ($z_t = r_t$) can be used as the predicting variables for the American assets, because the interest rates serve both as determinants of the discount rate used in calculating present values, and also reflect the expectations of future cash flows (e.g., the higher interest rates may reflect beliefs of strong future growth).

Description of data and assets selection criteria

Financial markets are too sensitive to changes in macroeconomic indicators, and asset prices immediately correct their direction after new information release. The correction of assets prices happen instantly, in the short-term. Therefore investors should choose well-diversified portfolios and minimize the risk of investment. Theory specifies four economic states that correspond to specific business cycle. During each economic state different type of assets behave differently. For example during high growth and high inflation period commodities show the best performance while the Treasury bonds perform poorly because of the inflationary pressures and the rise in the interest rates. During low growth and high inflation periods, the rising inflation negatively affects the Treasury bonds performance and at the same time the falling economic activity negatively affects the stock performance, meanwhile, the prices of commodities grow. During the recession periods, the unfavorable economic conditions force the investors to take minimum risks and switch to low-risk and low-income assets such as the Treasury bonds. During the recovery, the consumers' confidence level increases due to positive growth expectations stimulating the investors to switch their investments from risk-free assets (bonds) to riskier assets (stokes). The paper that is in tune with this work considers the asset allocation with regime changes (A. Ang, G. Bekaert 2002) where the portfolios represent the cases of a few developed countries. They used to maneuver with the regime-switching model to get time-varying correlations during the market downturns and upturns and reveal more substantial benefits from the international diversification. The portfolios which have been used in this study are largely diversified and consist of the developed and emerging markets stocks and bonds, currencies (EUR/USD and USD/JPY) and commodities (Oil and Gold). The diversification in this way will allow gain income from financial markets during different economic states. Moreover, based on portfolio theory, diversification should result in higher expected returns and lower overall volatility as adding low correlation portfolios to an optimization strengthen the reward to risk profile by shifting the mean-variance frontier to the left (M. Engels 2004).

The data reflect 215 monthly return observations, in total, covering the period from March 2000 to January 2018[1]. Further, the assets have been divided into the U.S Stock market with Stock market index S&P 500, the US Treasury Bonds, Germany Stock market index DAX and Germany Government Bonds, the Japan Stock market index Nikkei and Japanese Government Bonds, the China Stock market index Shanghai Composite and China Government Bonds. Adding a new market to the portfolio gives benefits in term of the traditional asset allocation. According to global volatility index data, currently the global level of volatility is decreasing and giving a positive signal to invest in risky assets. The rising correlations across the stocks and the bonds, on the other

[1] Source (Investing 2017) https://www.investing.com/, summery statistics of data are represented in Appendix (Table 1)

hand, make the traditional diversification less efficient. So the investors view the emerging markets as a new opportunity in their portfolio. The emerging markets historically outperform the developed markets` counterparts regarding the assets growth and the investment returns (J. Bambaci, Ch. Chia, B. Ho 2012). For the last 25 years, China has demonstrated some static economic growth[2]. That is why Chinese stocks and bonds have been included in the list of the assets. The risk-free rate for all portfolios in this study is equal to 1.46% which is one month USA Treasury Yield.[3] The DAA will be implemented for two investment periods more precisely for January and February (hereafter Period 1 and Period 2).

Regime-switching model estimation

Firstly asset returns should be predicted; therefore equation (1) has been calculated for each asset: $\Delta y_t = (\Delta y^{Country}_{stock\ index}, \Delta y^{Country}_{T-bill}, \Delta y^{Currency}, \Delta y^{Commodity})'$

The model with switching autoregressive teams as well as switching variances has been used as benchmark model in further analysis for both investment periods (hereafter the calculated benchmark model will be called Model I). The regimes in the Model I correspond to the periods of low and high volatility[4]. The variances change from low to higher levels and the errors alter, correspondingly from small to the bigger ones. To introduce the Model II[5] that is estimated based on equations (4) and (5), we assume that $z_t = r_t^{USA}$ where r_t^{USA} is Federal Funds short-term interest rates. Model I and Model II predicted high volatility periods for S&P 500 that are

[2] Source (NASDAQ 2017) http://www.nasdaq.com/article/7-reasons-to-buy-emerging-markets-now-cm825317

[3] Source (Bloomberg. 2017) https://www.bloomberg.com/markets/rates-bonds

[4] Graph 2 and 3 in Appendix illustrates smoothed probabilities of high volatility regime for S&P 500 based on Model I and Model II estimates.

[5] Model II uses conditional information and is estimated only for S&P 500. While in the scope of this study we are going to use the results from Model I investors also can introduce conditional information for each asset and estimate the returns based on equation (4). Changes in regime predictor variables can change the results significantly.

consistent with the contraction periods in the USA. The results for investment Period 1 of the regime switching Model I revealed that in December 2017 most of the assets had high probability of being in low volatility regime (e.g. the US Stock market and China Stock market have shown the best performance with the 99% probability of being in low volatility regime). The regime transition probability matrix for the US Stock market is the following: $\begin{pmatrix} 97.1 & 2.9 \\ 3.05 & 96.9 \end{pmatrix}$. The likelihood that during next month US stock market will stay in low volatility regime during January is 97.1%[6].

The next step is the prediction of the one-period-ahead returns based on the regime-switching Model I implementation. $R_{t+1} = \left(R_{t+1}^{\substack{Country \\ stock\ index}}, R_{t+1}^{\substack{County \\ T-bell}}, R_{t+1}^{Currency}, R_{t+1}^{Commodity}\right)' = \Delta y_{t+1}^{asset}$

The most important input in this paper is the calculation of state-dependent returns and standard deviations. Authors who used the regime-switching model to forecast returns and standard deviations of asset prices divided the results into two regimes (expansion and recession periods) (A. Ang, G. Bekaert 2002). They calculated the standard deviation and variance-covariance matrix for these regimes separately. After calculation of state-dependent variance-covariance matrix, the portfolio has been optimized for each regime accordingly. In literature, the regime-switching models had been used only for limited amounts of assets and mainly for developed markets' equities. These assets often behave similarly and have similar underlying regimes (A. Ang, G. Bekaert 2002). However, in a well-diversified portfolio which consists of different types of assets the underlying regimes of assets can differ. The Model I results for investment Period 1 determined that in the portfolio all the assets except Gold, Japanese Bonds, and Germany Bonds are in low volatility regime. To overcome this problem a new calculation method for the state-dependent

[6] Regime switching Model I results, current regimes and regime transition probabilities for all assets are represented in Appendix in Table 2.1 and Table 2.2.

returns and variance-covariance matrix will be used. H. Mostafaei, M. Safaei (2012) proofed that for a two-state Markov chain the regime probabilities are steady-state probabilities. and one-period-ahead forecast of returns have been determined by following equation (C. Yuan 2011): $\Delta y_{t+1}^{asset} = P_{s1}\left(\Delta y_{s1_{(t)}}^{asset}\right) + P_{s2}\left(\Delta y_{s2_{(t)}}^{asset}\right)$ (6)

Where: $\Delta y_{s_1(t)/s_2(t)}^{asset} = P_{s_1/s_2}(const + \beta_{s_1(t)/s_2(t)}\Delta y_{t-1} + \sigma_{s_1(t)/s_2(t)}\varepsilon_{(t)})$ (6.1)

Following the same logic, the state-dependent variables can be calculated based on the probability of being in the specific regime, and the new formula will take the following form:

$$\sigma_t^{2\ asset} = P_{s1}\left(\sigma_{s1_{(t)}}^{2\ asset}\right) + P_{s2}\left(\sigma_{\ s2_{(t)}}^{\ 2\ asset}\right) \quad (7)$$

Lastly, based on the calculated variances the variance-covariance matrix has been estimated.

$$\begin{pmatrix} \sigma_{ti}^2 & \cdots & \rho_{\mathrm{i},j}\sigma_{tj}\sigma_{tj} \\ \vdots & \ddots & \vdots \\ \rho_{\mathrm{i},j}\sigma_{ti}\sigma_{tj} & \cdots & \sigma_{tj}^2 \end{pmatrix} \quad (8)$$

Portfolio construction

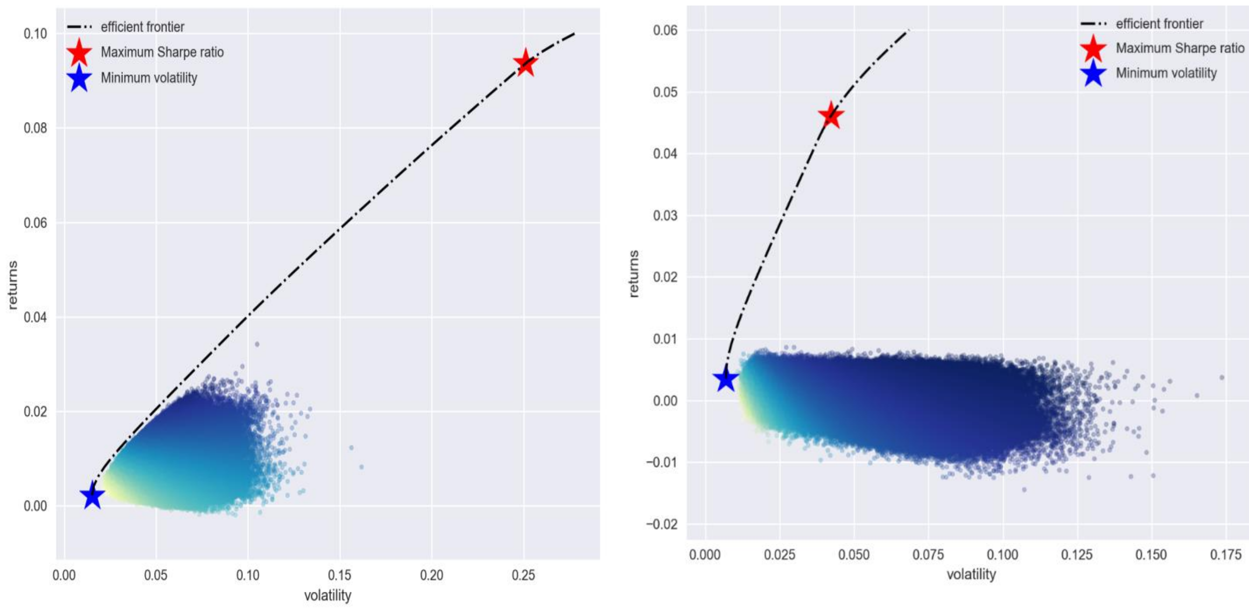
The fundamental goal for an investor is to allocate investments between different assets optimally. Markowitz mean-variance optimization allows investors to allocate their assets considering the trade-off between the risk of the portfolio and its return by selecting the portfolios that maximize the expected portfolio return subject to target level of risk or, equivalently, minimize the variance subject to achieving a target level of expected return (T. Lai, H. Xing, Z.Chen 2011). $\min w' \sum w$ s.t $R_p = \sum w_i \mu_i$ (9)

Where R_p is the end of period return of a portfolio and $w_0, \ldots, w_{T-1}$ are the portfolio weights which sum to 1. Σ is a covariance matrix of the assets returns. Short selling is allowed, because it enlarges the opportunity to gain more returns. In the ideal world, every investor is a mean-variance investor, and all follow the same requirements, everyone buys the same portfolio. In theory, if everyone follows the mean-variance methodology with the same parameters, the efficient fund of risky assets will turn to the market portfolio (M. Engels 2004). The market portfolio also called tangency portfolio and has a maximum Sharpe ratio from among all other portfolios: $Sharpe\ ratio = \frac{Return_p - r_f}{Risk_p}$. The ratio describes how much excess return is received for the extra volatility that investors endure for holding a riskier asset. The problem with Markowitz portfolio optimization is that it relies on historical returns, while the portfolio optimization problem requires ex-ante expected returns and variance-covariance matrix to produce weights of assets which will work in the future (R. Harvey 2017). To solve the problem of ex-post returns (historical) in Markowitz model, we selected the regime-switching Model I and predicted the returns. With the introduction of regimes to the Markowitz model, the optimal portfolio weights become functions of the regimes $w_t^* = w_t^*(s_t)$ (A. Ang, G. Bekaert 1999).

Asset Allocation Results

Based on the theoretical application of tangency (market) portfolio two portfolios have been constructed, first with historical average returns and constant variance-covariance matrix (hereafter the portfolio will be called Portfolio 1), the other with regime-switching Model I projected returns and time-varying variance-covariance matrix (hereafter the portfolio will be called Portfolio 2). The investment opportunities are characterized by substantially lower expected returns and higher risks for Portfolio 2. Obtained results that are represented in Graph 5 are the best illustration that the regime-switching model can capture patterns of financial data series.

Graph 5. Portfolio 2 and Portfolio 1 efficient frontiers



To prove the assumptions which have been presented above both portfolios should be tested on actual returns for January 2018. The test results are the following: The projected return from Portfolio 1 is 9.4%, and the risk is 25.1%[7]. That means that with 95% probability the return of Portfolio 1 will be in the range of [-31.9% 50.7%][8]. The actual return for Portfolio 1 for investment Period 1 would be $\sum_{i=1}^{12} w_i \times r_i = -1.16\%$. The expected return of Portfolio 2 is 4.6% and risk is 4.4%, so with 95% probability the results will be in the range of [-2.56% 11.82%]. The test result for Portfolio 2 showed that the actual return of Portfolio 2 would be $\sum_{i=1}^{12} w_i \times r_i = 3.9\%$.

[7] Variance of the portfolio has been calculated with following equation: $\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}$.

[8] Calculation has been done based on Two-sigma rule $\Pr(\mu - 2\sigma \leq x \leq \mu + 2\sigma)$

Table 4. Test results on actual returns for January

Assets	Actual returns in January	Portfolio 1		Portfolio 2	
		weights	returns	weights	returns
Gold	2%	100%	2.2%	66.0%	1.5%
Oil	7%	27.5%	2.0%	-14.0%	-1.0%
USA bond	0%	-19.1%	0.1%	-1.4%	0.0%
S&P 500	6%	29.0%	1.6%	23.4%	1.3%
Japanese bond	-12%	7.6%	-0.9%	2.4%	-0.3%
Nikkei 225	-2%	-4.0%	0.1%	-100.0%	2.1%
JPY/USD	-3%	-100%	3.1%	16.9%	-0.5%
Germany bond	16%	-15.3%	-2.5%	-1.2%	-0.2%
DAX	2%	73.1%	1.5%	62.1%	1.3%
EUR/USD	4%	-100%	-3.5%	48.6%	1.7%
Shanghai Composite	5%	1.1%	0.1%	-21.0%	-1.1%
China Bond	-5%	100%	-4.9%	18.2%	-0.9%
Portfolio total return			**-1.16%**		**3.89%**

Black- Litterman model results and effect of economic cycles on portfolio performance

Optimal portfolios are very sensitive to the returns behavior. If two asset classes have the same risk but one has a slightly higher forecasted return, then the mean-variance optimizer will allocate the funds mainly in the asset with the highest expected return (G. He, R. Litterman 1999). Black-Litterman model takes the Markowitz model one step further. The model uses a Bayesian approach to combine the investor's subjective views (T. Idzorek 2005). The model takes the expected returns that are implied by the tangency portfolio (prior distribution) and based on them estimates new returns (posterior distribution) and formulates a new portfolio (G. He, R. Litterman 1999). Black-Litterman formula has the following form:

$$\mathrm{E}\,[R] = [(\tau\Sigma)^{-1} + P'\Omega P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega PQ]^{-1} \quad (10)$$ [9]

[9] $M = [(\tau\Sigma)^{-1} + P'\Omega P]^{-1}$ is uncertainty of returns and new covariance matrix takes the form of: $\Sigma_{\mathrm{p}} = \Sigma + \mathrm{M}$

Implied returns are calculated with reverse optimization process that gives the most significant improvement in return estimation procedure. The equation for calculation of implied returns is: $\Pi = \delta \Sigma w_{mkt}$ (11) where $\mathrm{w_{mkt}}$ are the current market capitalization weights from Markowitz mean-variance optimization. The investor who reviews the portfolio over N assets every month maximizes the end of period utility.

$$\max_{\omega} U = w'\Pi - \frac{\delta}{2} w' \sum w \ (12) \qquad w_{new} = (\delta\Sigma_p)^{-1}\Pi \ (12.1)$$

Where $\delta = \frac{(E(r)-r_f)}{\sigma^2}$ is the parameter of absolute risk aversion. In this paper, the risk aversion coefficients for Black- Litterman approach based constructed portfolios, have been fixed equal to five that is a common benchmark in literature. P is a (K x N) view matrix and Ω is a diagonal (K x K) covariance matrix of error terms from the expressed views representing the uncertainty in each views[10] (T. Idzorek 2005). Q is the (N x 1 column) vector of expected returns of the assets from the matrix P. τ is a scalar number indicating the uncertainty in returns distribution. Blamont and Firoozye determined that scalar (τ) is approximately 1 divided by the number of observations (T. Idzorek 2005).

Interdicting views: With the time-varying investment opportunities, the investors should adjust their portfolio weights per new information received. New views will be formulated periodically based on the received macroeconomic data. For specifying the values in the view matrix an equal weighting scheme method of Satchell and Scowcroft (T. Idzorek 2005) has been used. Thus, each row of view matrix sums to 0 there the nominally outperforming assets receive positive weightings, while the nominally underperforming assets receive negative weightings (T. Idzorek 2005). Each row in Matrix P represents one view. The views are based on the actual economic situations

[10] The variance of an individual view portfolio is $\mathrm{p_k} \sum \mathrm{p'_k}$.

created. The analysis of growth and inflation indicators for last two quarters of 2017 showed that economies of the selected counties are characterized by low growth and low inflation with some minor recovery signs and transition trends to high growth and low inflation state. Japan's macroeconomic indicators defiantly showed poor performance. Regime switching Model I also predicted that Japanese bonds are in high volatility regime with 57% probability rate and will remain in the same regime for the next period with 60.6% probability rate. Thus, Nikkei and Japanese bonds will be underweighted against their counterparts. The commodities due to the positive inflation expectations and accommodating economic activity will show improvement against bonds but will underperform the stock markets. The view matrix represents a total of four relative views.

$$P_{Period1} = \begin{pmatrix} -0.5 & -0.5 & 0.0 & 0.3 & 0.0 & 0.0 & 0.0 & 0.3 & 0.0 & 0.0 & 0.3 & 0.0 \\ 0.5 & 0.5 & -0.3 & 0.0 & -0.3 & 0.0 & 0.0 & -0.3 & 0.0 & 0.0 & 0.0 & -0.3 \\ 0.0 & 0.0 & 0.0 & 0.3 & 0.0 & -1.0 & 0.0 & 0.0 & 0.3 & 0.0 & 0.3 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.0 & -1.0 & 0.0 & 0.0 & 0.3 & 0.0 & 0.0 & 0.0 & 0.3 \end{pmatrix}$$

Moreover, we are going to assume that stocks will overperform commodities by 10%, the commodities will overperform bonds by 12%, Nikkei 225 stock market index will underperform all other stocks by 6% and Japanese bonds will underperform the rest of bonds by 19%[11]. Expected returns of view matrix have the form of: $Q_1{}' = (10\%\ 12\%\ 6\%\ 19\%)$. The same method has been used to generate the portfolio weights for investment Period 2. In the fourth quarter the growth rates were down in all the countries. Taking into consideration the fact that the inflation indicators are in normal range, the stocks will be underweighted against commodities and bonds. Because of the negative growth and positive inflation indicators bonds will be overweighed against stocks. The Japanese stocks and bonds will be underweighted against the USA bonds, Germany bonds

[11] The assumption has been made based on historical data analysis on underperformance and overperformance of assets against each other (on average).

and China bonds. Expected returns of view matrix for investment Period 2 is $Q_2{}' = (7\%\ 6\%\ 19\%)$ and the view matrix for Period 2 has the following form:

$$P_{Feb} = \begin{pmatrix} 0.5 & 0.5 & 0.0 & -0.3 & 0.0 & -0.3 & 0.0 & 0.0 & -0.3 & 0.0 & -0.3 & 0.0 \\ 0.0 & 0.0 & 0.3 & -0.3 & 0.0 & 0.0 & 0.0 & 0.3 & -0.3 & 0.0 & -0.3 & 0.3 \\ 0.0 & 0.0 & 0.3 & 0.0 & -0.5 & -0.5 & 0.0 & 0.3 & 0.0 & 0.0 & 0.0 & 0.3 \end{pmatrix}$$

By using the Black-Litterman model approach on Portfolio 1 and Portfolio 2, two new portfolios have been formulated[12]. Correspondingly the first portfolio is with historical inputs (hereafter Portfolio 3) and the second one is with regime-switching Model I inputs (hereafter Portfolio 4). With the introduction of the Black-Litterman model on Portfolio 3 and Portfolio 4, the returns of portfolios become higher. Portfolio 3 expected return is 28%, and the risk of the portfolio is 23.1%. The probability that the return of portfolio will be in the range of [-10.7% 65.7%] for next period is 95%. The expected return from the Portfolio 4 is 15% and risk is 17.3%. Moreover, there is 95% probability that returns from the portfolio will be in the range of [-13.6% 48.6%]. The introduction of the Black–Litterman model and the state-dependent views improved the portfolio performance in term of the returns. At the same time, it increased the risk as the assets are riskier in a Bayesian framework. The uncertainty parameter is viewed as an additional source of risk considered in the portfolio decision (D. Avramov, G. Zhou 2010). We should again measure the portfolio performances based on the actual returns for two investment periods. So the results are the following: the return of the Portfolio 3 would be 16.1% in January against the expected 28%. Meantime it is obvious that the Black-Litterman model helped to improve the predictability of returns as Portfolio 1 return in January was -1.16%. Of course, a regime based Portfolio 4 showed the best performance and would have 28.7% return for investment Period 1. The same model has

[12] Source www.blacklitterman.org/code/bl.xls

been estimated for investment Period 2. The expected return of portfolio is 20% and the risk of portfolio is 20%. And the actual returns that an investor would gain for the Period 2 if he holds the Portfolio 4 is 21.29%.

Table 6. Black-Litterman model results on actual returns data

Assets	Portfolio 3 weights	Portfolio 3 Returns for January	Portfolio 4 weights January	Portfolio 4 returns for January	Portfolio 4 weights February	Portfolio 4 returns for February
Gold	80%	1.8%	62.3%	1.4%	219.9%	19.2%
Oil	8%	0.6%	-17.4%	-1.2%	184.7%	-8.8%
USA bond	-49%	0.2%	-22.4%	0.1%	8.5%	0.5%
S&P 500	106%	5.9%	138.4%	7.7%	-169.5%	6.7%
Japanese bond	-20%	2.4%	-20.8%	2.6%	-28.7%	12.7%
Nikkei 225	-78%	1.6%	-351.6%	7.4%	-205.0%	-9.6%
JPY/USD	-100%	3.1%	16.8%	-0.5%	11.8%	-0.2%
Germany bond	7%	1.1%	8.8%	1.4%	11.7%	-0.7%
DAX	97%	2.1%	145.8%	3.1%	18.2%	-1.0%
EUR/USD	-100%	-3.5%	48.4%	1.7%	99.5%	-1.8%
Shanghai Composite	78%	4.1%	94.2%	4.9%	-81.2%	5.2%
China Bond	69%	-3.4%	-2.9%	0.1%	29.6%	-0.7%
Portfolio total return		**16.05%**		**28.72%**		**21.29%**

Conclusion

In this paper, in order to determine how regimes affect the portfolio performance, the regime dependent returns and standard deviations have been predicted. Based on the assets' underlying probabilities of being in particular regime a new method has been developed to calculate assets returns, variances and variance-covariance matrix. New inputs have been used in Markowitz mean-variance portfolio optimization problem. The result proves the assumption that high and low volatility regimes drastically influence the returns distribution and consequently portfolio performance. Meantime the inputted new calculation technique increased the predictability of variables and hence improved weights optimization process. The newly developed technique considers the underlying regimes of all assets, thus produces more reliable forecasts. The final step in the portfolio construction was the introduction of Black-Litterman model and the views that have been formulated based on the market information. The main attainment is that by producing the state-dependent portfolio weights, the investors can generate portfolios with significantly higher expected returns and gain alpha from financial markets. The critical aspect of this work is its practical usage. While in the scope of this study only market (tangency) portfolio performance has been examined because of its theoretical importance, in reality, investors may not choose portfolios with highest sharp ratio and risk aversion coefficient equal to 5. Investors are free to select any portfolio from the efficient frontier with a target level of risk depending on the level of risk tolerance. For example, based on regime switching model results we know that currently, most assets are in low volatility regime, thus a portfolio with higher expected return and higher risk compared with the market portfolio can be selected.

Appendix

Table 1 Summery statistics

	Gold	Oil	USA bond	S&P 500	Japanese bond	Nikkei 225	JPY/USD	Germany Bond	DAX	EUR/USD	Shanghai Composite	China Bond
count	215	215	215	215	215	215	215	215	215	215	215	215
mean	0.8%	0.8%	-0.1%	0.4%	1.2%	0.1%	0.0%	-1.3%	0.4%	0.2%	0.6%	7.7%
std	5.0%	9.2%	8.6%	4.2%	37.9%	5.8%	2.8%	28.8%	6.0%	3.0%	7.7%	24.2%
min	-18.0%	-32.6%	-26.4%	-17.3%	-238.9%	-11.4%	-7.1%	-235.8%	-25.4%	-9.7%	-24.6%	-36.5%
25%	-2.4%	-5.2%	-5.0%	-1.9%	-12.2%	-3.9%	-1.8%	-5.1%	-2.7%	-1.4%	-4.3%	-3.0%
50%	0.7%	1.6%	-0.3%	1.0%	-4.2%	-0.4%	0.1%	-1.5%	1.0%	0.2%	0.7%	0.9%
75%	3.8%	7.1%	4.0%	2.9%	7.5%	2.8%	1.6%	4.0%	4.0%	2.0%	4.9%	6.0%
max	13.9%	29.7%	31.0%	10.9%	340.0%	31.3%	9.2%	110.1%	21.4%	10.1%	27.4%	79.2%
skw	-0.10	-0.20	0.37	-0.60	2.80	1.06	0.36	-3.05	-0.60	-0.13	-0.16	2.18
kurt	0.61	0.64	2.00	1.35	37.58	2.96	0.64	29.40	2.44	1.09	1.51	3.98

Regime switching model's results

Table 2.1 Regime transition probabilities

Period 1	Gold	Oil	USA bond	S&P 500	Japanese bond	JPY/USD	Germany Bond	DAX	EUR/USD	Shanghai Composite	China Bond
p[0->0]	19.6%	64.8%	86.7%	97.1%	91.4%	0.0%	98.1%	96.4%	96.5%	98.5%	91.9%
p[1->0]	59.7%	11.7%	32.9%	3.1%	39.4%	35.2%	6.2%	5.3%	6.2%	4.1%	27.0%
p[0->1]	80.4%	35.2%	13.3%	2.9%	8.6%	100.0%	1.9%	3.6%	3.5%	1.5%	8.1%
p[1->1]	40.3%	88.4%	67.1%	97.0%	60.6%	64.8%	93.8%	94.7%	93.8%	95.9%	73.0%

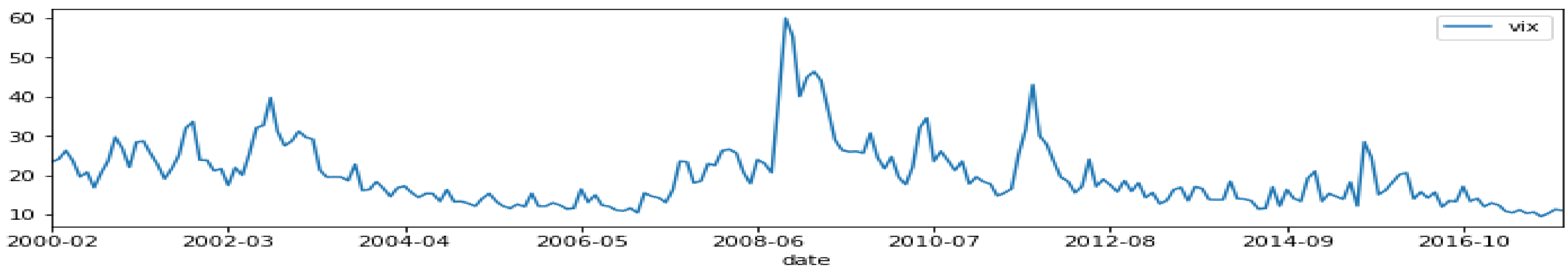
Period 2	Gold	Oil	USA bond	S&P 500	Japanese bond	JPY/USD	Germany Bond	DAX	EUR/USD	Shanghai Composite	China Bond
p[0->0]	19.4%	65.5%	86.7%	97.1%	91.1%	54.5%	73.3%	98.1%	96.5%	96.5%	98.5%
p[1->0]	59.9%	11.4%	32.6%	3.1%	-3.6%	61.3%	47.6%	5.4%	5.2%	6.1%	4.1%
p[0->1]	80.6%	34.5%	13.3%	2.9%	8.9%	45.5%	26.7%	1.9%	3.5%	3.5%	1.5%
p[1->1]	40.1%	88.6%	67.4%	96.9%	103.6%	38.7%	52.4%	94.6%	94.8%	93.9%	95.9%

Table 2.2 Estimations

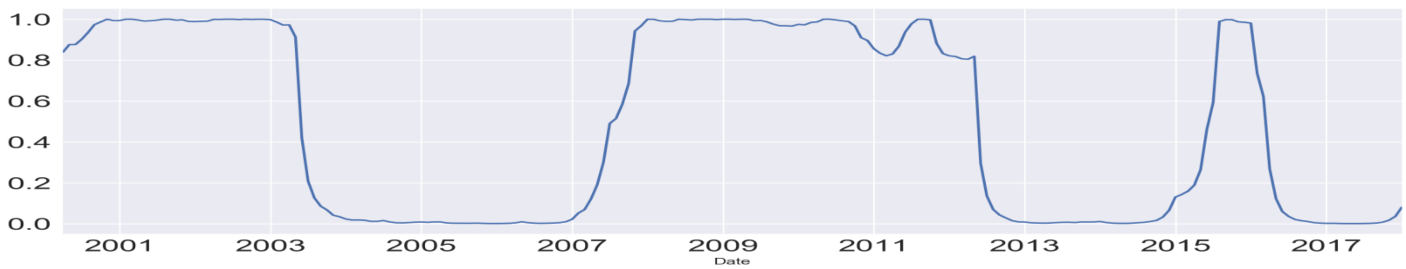
Assets	Period 1					Period 2			
	Coefficient	**Regime 0**	**Stand. Error**	**Regime 1**	**Stand. Error**	**Regime 0**	**Stand. Error**	**Regime 1**	**Stand. Error**
Gold	const	0.0184	(0.007)***	-0.0002	(0.006)	0.0183	(0.007)***	0.0000	(0.006)
	sigma2	0.0019	(0.000)***	0.0017	(0.000)***	0.0019	(0.000)***	0.0017	(0.000)***
	ar.L1	0.4940	(0.231)**	-0.4866	(0.127)***	0.4956	(0.229)**	-0.4862	(0.127)***
Oil	const	0.0399	(0.009)***	-0.0054	(0.011)	0.0410	(0.008)***	-0.0054	(0.011)
	sigma2	0.0020	(0.001)*	0.0085	(0.001)***	0.0020	(0.001)**	0.0085	(0.001)***
	ar.L1	-0.6463	(0.158)***	0.2843	(0.092)***	-0.6471	(0.152)***	0.2823	(0.091)***
USA bond	const	-0.0062	(0.006)	0.0131	(0.023)	-0.0062	(0.006)	0.0129	(0.022)
	sigma2	0.0029	(0.001)***	0.0179	(0.006)***	0.0029	(0.001)***	0.0178	(0.006)***
	ar.L1	-0.0287	(0.091)	0.0694	(0.147)	-0.0294	(0.090)	0.0694	(0.146)
S&P 500	const	0.0114	(0.002)***	-0.0041	(0.007)	0.0118	(0.002)***	-0.0041	(0.007)
	sigma2	0.0005	(0.000)***	0.0028	(0.000)***	0.0005	(0.000)***	0.0028	(0.000)***
	ar.L1	-0.2061	(0.090)**	0.1366	(0.099)	-0.2028	(0.092)**	0.1364	(0.099)
Japanese bond	const	-0.0356	(0.010)***	0.1984	(0.115)*	-0.0355	(0.010)***	0.2122	(0.126)*
	sigma2	0.0149	(0.002)***	0.4994	(0.087)***	0.0153	(0.002)***	0.5175	(0.095)***
	ar.L1	-0.0349	(0.065)	-0.7341	(0.169)***	-0.0388	(0.062)	-0.7533	(0.183)***
Nikkei 225	const	-0.0280	(0.006)***	0.0391	(0.038)	-0.0278	(0.006)***	0.0398	(0.038)
	sigma2	0.0011	(0.000)***	0.0036	(0.001)***	0.0011	(0.000)***	0.0036	(0.001)***
	ar.L1	0.2470	(0.153)*	0.2799	(0.245)	0.2472	(0.151)*	0.2757	(0.244)

JPY/USD	const	0.0035	(0.003)	-0.0003	(0.003)	-0.0039	(0.004)	0.0083	(0.010)
	sigma2	0.0001	(0.000)*	0.0010	(0.000)***	0.0004	(0.000)**	0.0013	(0.001)**
	ar.L1	0.2273	(0.084)***	-0.0415	(0.113)	0.1088	(0.116)	-0.0471	(0.245)
Germany Bond	const	-0.0072	(0.005)	-0.0453	(0.077)	-0.0072	(0.005)	-0.0419	(0.076)
	sigma2	0.0037	(0.000)***	0.3209	(0.045)***	0.0037	(0.000)***	0.3139	(0.043)***
	ar.L1	0.0615	(0.045)	-0.2828	(0.138)**	0.0642	(0.047)	-0.2821	(0.134)**
DAX	const	0.0153	(0.003)***	-0.0111	(0.011)	0.0153	(0.003)***	-0.0109	(0.011)
	sigma2	0.0012	(0.000)***	0.0067	(0.001)***	0.0011	(0.000)***	0.0067	(0.001)***
	ar.L1	-0.0631	(0.109)	0.0649	(0.126)	-0.0641	(0.108)	0.0651	(0.125)
EUR/USD	const	0.0049	(0.002)**	-0.0047	(0.005)	0.0052	(0.002)**	-0.0048	(0.005)
	sigma2	0.0005	(0.000)***	0.0015	(0.000)***	0.0005	(0.000)***	0.0015	(0.000)***
	ar.L1	0.0684	(0.111)	-0.0624	(0.146)	0.0688	(0.111)	-0.0632	(0.145)
Shanghai Composite	const	-0.0012	(0.004)	0.0217	(0.021)	-0.0008	(0.004)	0.0217	(0.022)
	sigma2	0.0027	(0.000)***	0.0126	(0.003)***	0.0027	(0.000)***	0.0127	(0.003)***
	ar.L1	-0.0235	(0.092)	0.1856	(0.158)	-0.0227	(0.092)	0.1856	(0.159)
China Bond	const	0.0032	(0.005)	0.1037	(0.055)*	0.0028	(0.005)	0.1039	(0.055)*
	sigma2	0.0034	(0.001)***	0.0258	(0.004)***	0.0034	(0.000)***	0.0258	(0.004)***
	ar.L1	-0.0931	(0.050)*	0.9663	(0.205)***	-0.0930	(0.050)*	0.9663	(0.205)***
P-values are reported in the parenthesis ***,**,* denotes significance of the coefficient at the 1%, 5%, 10%									

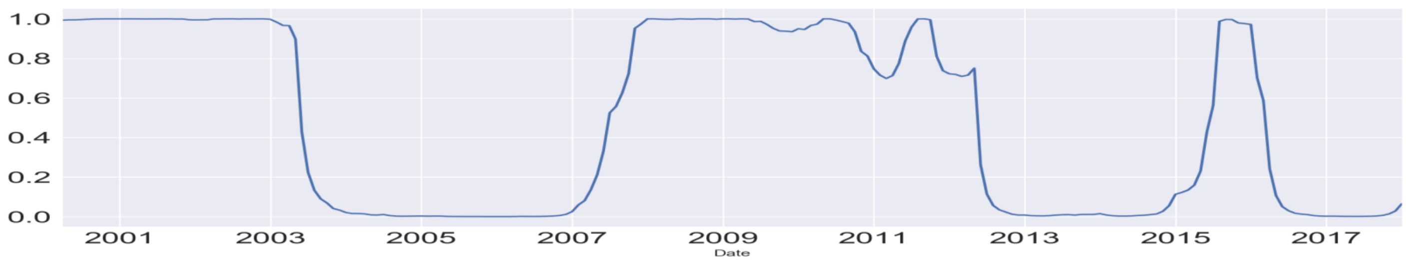
Graph 1 Volatility index



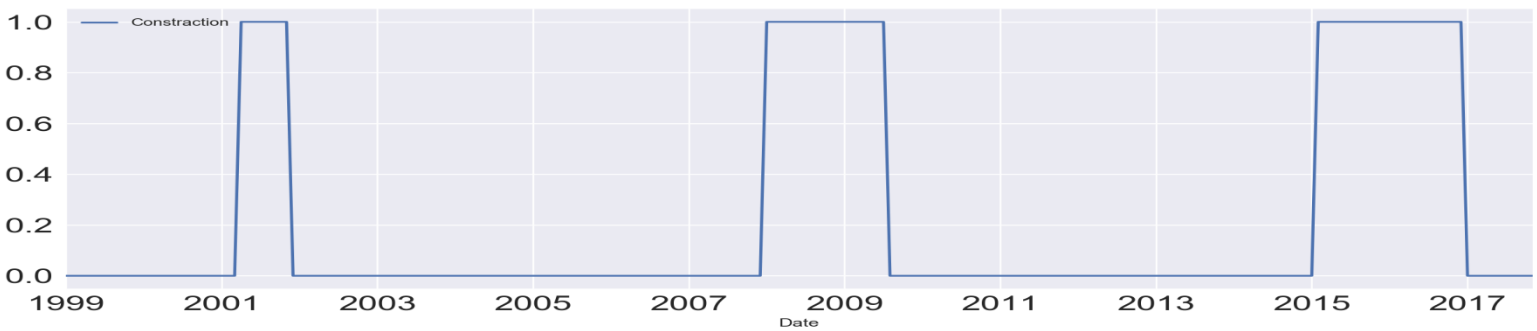
Graph 2. Smoothed probabilities of unconditional high volatility regime for S&P 500 (Model I)



Graph 3. Smoothed probabilities of high volatility regime conditional on federal fund rates for S&P 500



Graph 4. NBER contraction periods



Markowitz optimization results

Table 3.1 Portfolio 1

Assets	Historical returns	Risk	Weights	Expected return	Expected risk
Gold	0.8%	5.0%	100.0%	9.4%	25.1%
Oil	0.7%	9.2%	27.5%		
USA bond	-0.1%	8.6%	-19.1%		
S&P 500	0.4%	4.2%	29.0%		
Japanese bond	1.3%	38.0%	7.6%		
Nikkei 225	0.1%	5.9%	-4.0%		
JPY/USD	0.0%	2.8%	-100.0%		
Germany bond	-1.4%	28.9%	-15.3%		
DAX	0.4%	6.0%	73.1%		
EUR/USD	0.1%	3.0%	-100.0%		
Shanghai Composite	0.6%	7.7%	1.1%		
China Bond	7.8%	24.3%	100.0%		

Table 3.2 Portfolio 2

Assets	Projected returns	Risk	Weights	Return	Risk
Gold	0.8%	4.3%	66.0%	4.6%	4.4%
Oil	1.0%	6.5%	-14.0%		
USA bond	-0.3%	6.4%	-1.40%		
S&P 500	0.9%	2.2%	23.4%		
Japanese bond	7.2%	45.3%	2.4%		
Nikkei 225	-2.5%	3.6%	-100%		
JPY/USD	0.2%	2.0%	16.9%		
Germany bond	-2.8%	38.4%	-1.20%		
DAX	1.4%	3.6%	62.1%		
EUR/USD	0.5%	2.3%	48.6%		
Shanghai Composite	-0.1%	5.3%	-21.0%		
China Bond	0.7%	6.2%	18.2%		

Analyzing growth and inflation indicators for selected countries

Table 5.1 III Quarter 2017

Country	Growth					Inflation				
	High	Normal	Low	Increasing speed	Decreasing speed	High	Normal	Low	Increasing speed	Decreasing speed
	> 2 %	0.5% -2%	< 0.5%	> 0 %	< 0%	> 2 %	0.5% -2%	< 0.5%	> 0 %	< 0%
CHN	-	1.80	-	FALSE	TRUE	-	1.60	-	TRUE	FALSE
USA	-	0.78	-	TRUE	FALSE	-	1.97	-	TRUE	FALSE
DEU	-	0.74	-	TRUE	FALSE	-	1.73	-	TRUE	FALSE
JPN	-	0.55	-	TRUE	FALSE	-	0.60	-	TRUE	FALSE

Table 5.2 IV Quarter 2017

Country	Growth					Inflation				
	High	Normal	Low	Increasing speed	Decreasing speed	High	Normal	Low	Increasing speed	Decreasing speed
	> 2 %	0.5% -2%	< 0.5%	> 0 %	< 0%	> 2 %	0.5% -2%	< 0.5%	> 0 %	< 0%
CHN	-	1.60	-	FALSE	TRUE	-	1.80	-	TRUE	FALSE
USA	-	0.63	-	FALSE	TRUE	2.12	-	-	TRUE	FALSE
DEU	-	0.61	-	FALSE	TRUE	-	1.68	-	FALSE	TRUE
JPN	-	-	0.11	FALSE	TRUE	-	-	0.37	FALSE	TRUE

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