Machine Learning 1 – Fundamentals

Inductive Learning

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Learning System – Basics

- Every machine learning method requires data from an input space X
- An input instance $x_i \in X$ consists of attributes $x_i = \begin{pmatrix} x_{i_1} \\ \cdots \\ x_{i_m} \end{pmatrix}$
- All input instances should be mapped to the solution space Y
  - Often classification or regression tasks
  - For a true/false classification, $Y = \{0, 1\}$

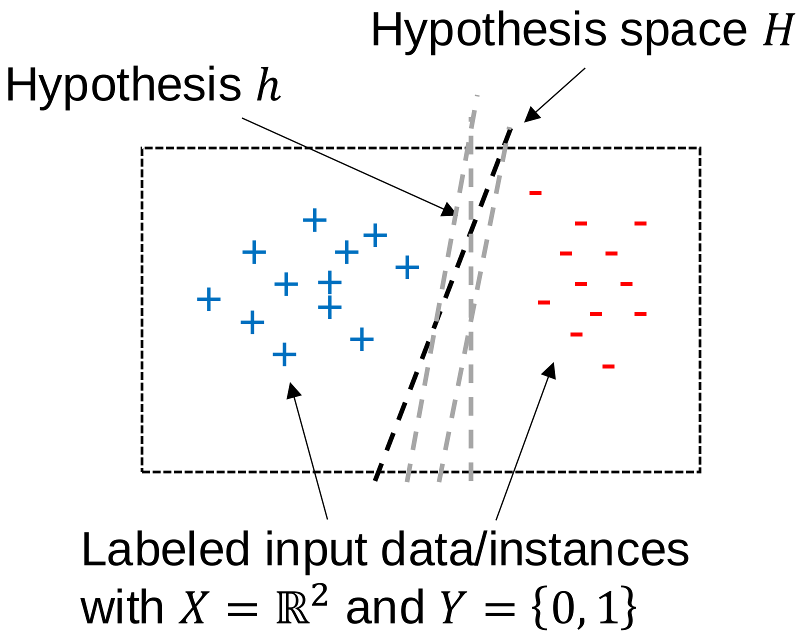
- **Assumption**: There exists a target function $t : X \rightarrow Y$, which can map all instances $x_i \in X$ perfectly to the solution space Y
  - t is unknown
  - The number of all possible input instances x_i is usually extremely high

Learning System

- A hypothesis h is an „arbitrary“ mapping $h\colon X \rightarrow Y$

- The hypothesis space H, consists of all possible $h \in H$, which can be represented by the model

- **Goal**: The machine learning method should find the best hypothesis $h \approx t$ in the large hypothesis space, that best fits the observed data.



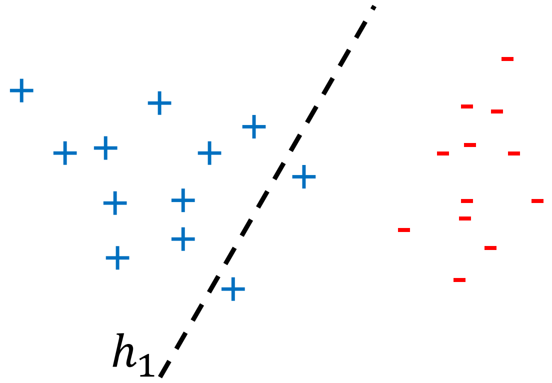
Learning System

- **Consistent**
  - No negative examples are classified positive

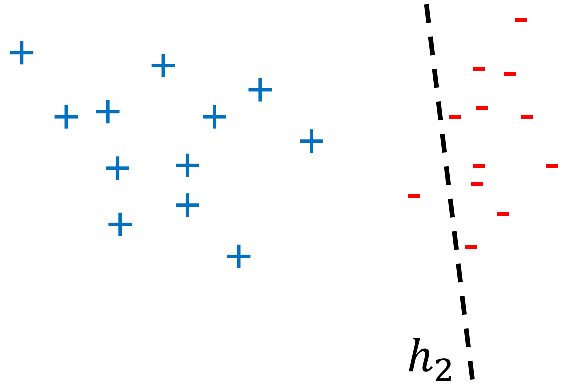
all negativ

- **Complete**
  - All positive examples are classified positive

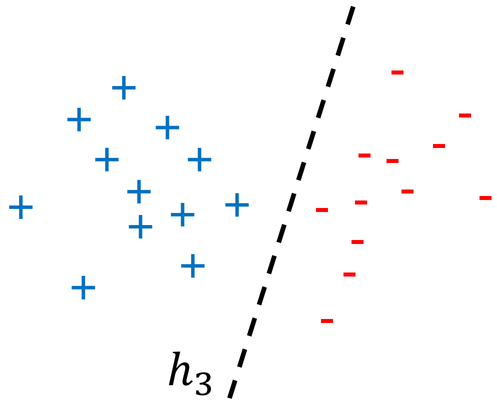
all positive



Consistent hypothesis h_1
(not complete)



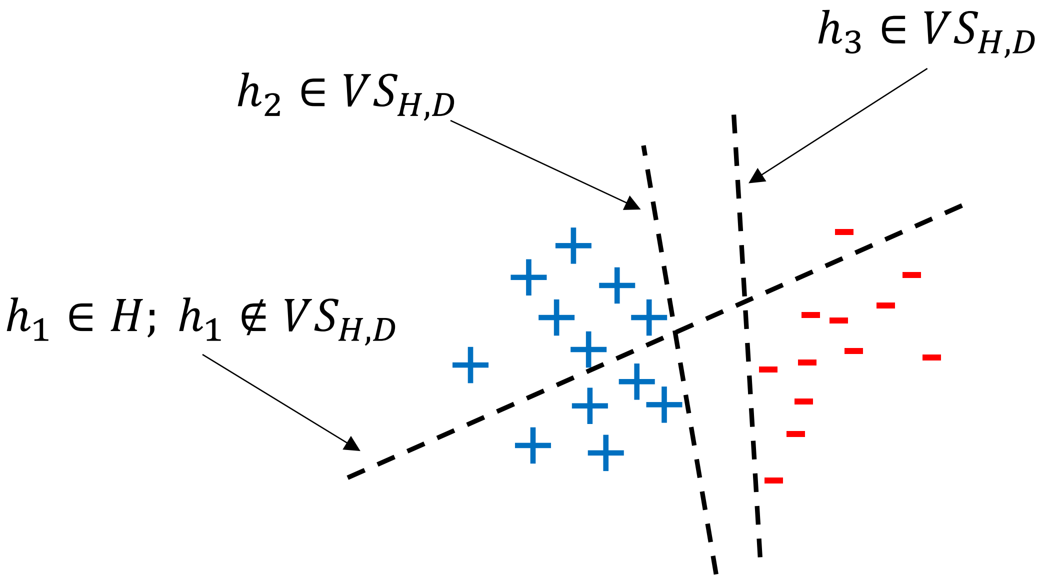
Complete hypothesis h_2
(not consistent)



Complete and **consistent** hypothesis h_3

Version Space

- **Definition**: The version space $VS_{H,D}$, with respect to the hypothesis space H and the set of training examples D, is the subset of hypotheses of H that are consistent and complete with the training examples in D.



What is Induction? 归纳法

- Inductive reasoning is a method of reasoning in which a general principle is derived from a body of specific observations.
  - Example:

    „Socrates is a human being", *„Socrates is mortal"*, *„Caesar is a human being"*, *„Caesar is mortal"*

    => „All human beings are mortal."

- **Definition**: Given a body of specific observations D. Hypothesis h follows inductively from D and prior knowledge B if …

$$B \cup h \mapsto D, B \nrightarrow D, B \cup D \nrightarrow \neg h$$

- **Caution**: Individual examples can be derived from hypotheses, but not the other way around!

What is Deduction? 演绎法

- Deductive reasoning is a method of reasoning that starts with a general principle and and examines the possibility to reach a specific, logical conclusion.
  - Uses mainly prior knowledge.
  - „All humans are mortal.“, „Socrates is a human being.“
    => „Sokrates is mortal.“

- **Definition**: From a set of principles $A,\ B$ follows $\Leftrightarrow$ There is a sequence of principles, from which B follows.

- **Examples**:

$$\frac{\mathrm{A},\ \mathrm{A} \rightarrow B}{\mathrm{B}}$$

Modus Ponens

$$\frac{\forall x P(x)}{\mathrm{P(a)}}$$

Instantiation

Induction vs. Deduction

添加两种方法的对比表格（gpt）

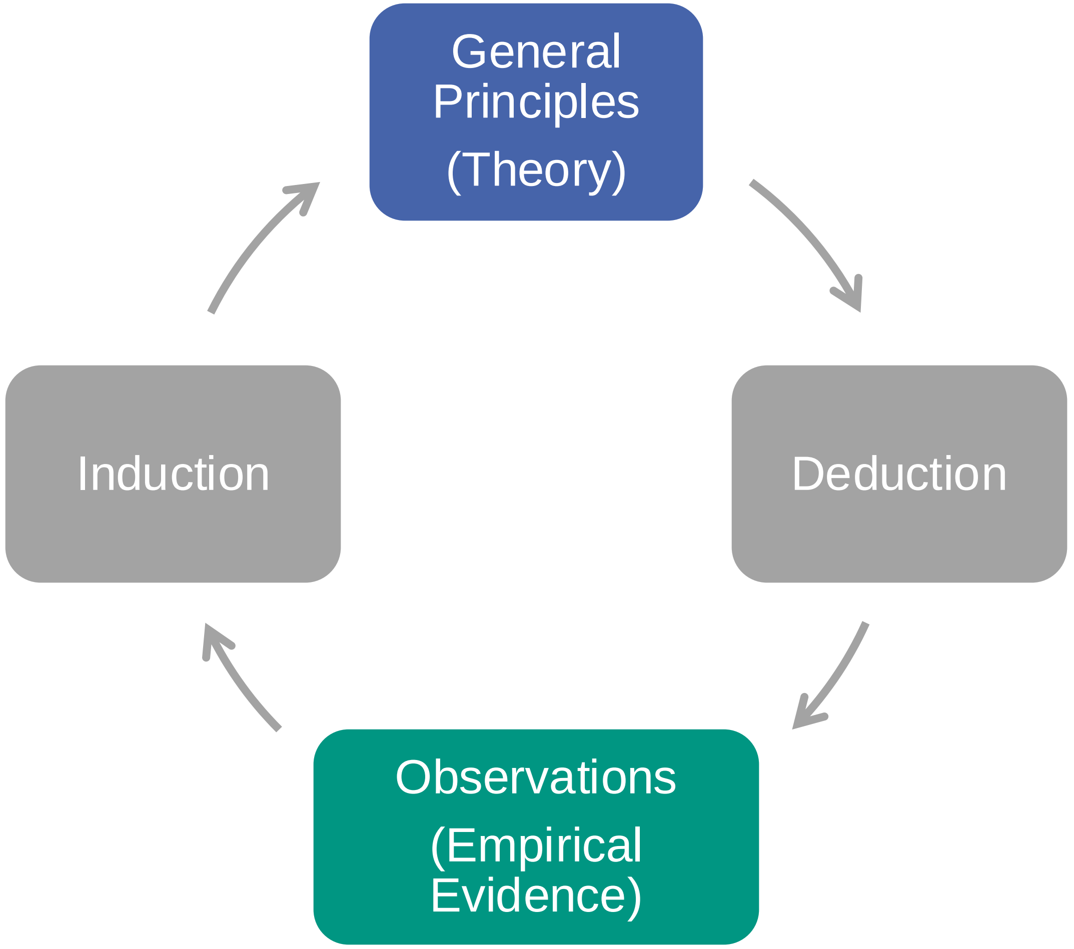
Induction

- Truth-expanding
  (generation of new hypotheses)
- Basic approach based on the so-called inductive learning hypothesis
- Plausibility
- Very widespread learning approach
  - (based on a biological approach; makes living beings capable of surviving)

Deduction

- Truth-preserving
  (Deriving new rules / facts)
- Logical Conclusion
- Correctness

Induction vs. Deduction



Inductive Learning Hypothesis

- **Challenge**:
  - The dimensionality of input space X is often very high
    - Example: Input instances are "normal" 8-bit RGB images with a size of 20×20 pixels
      - Space X contains $\dim(X) = (256 * 256 * 256)^{20*20} \approx 10^{2900}$ different images
        - There are 10^{80} atoms in the universe
  - Target function t is unknown

- **Solution**:
  - Create dataset $D \subseteq X$
  - Create loss function $\ell(t, h)$, which estimates the distance of a hypothesis h to the target function t.
  - Learn hypothesis h for which ℓ is minimal

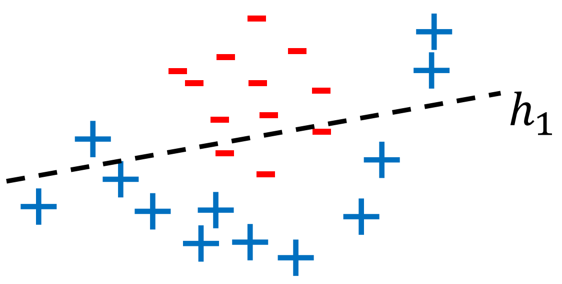
Inductive Learning Hypothesis

- **Definition**: Any hypothesis h that approximates the target function t well enough over a sufficiently large set of training instances $x_i \in D$ will also approximate the target function t well over unknown examples.
- If a learning system makes good predictions even on unknown data, we call this **generalization**.

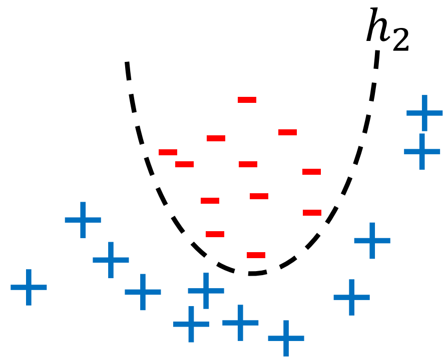
那些从训练数据中学到的假设或规则，能够适用于尚未见过的测试数据。

Hypothesis Space and Version Space

- Problems with models with small hypothesis space
  - The hypothesis space H_{lin} of a linear model contains only linear hypotheses.
  - In example 1, there is no consistent and complete linear hypothesis.
    - Version space $VS_{H,D} = \emptyset$
  - Target function t not included in H_{lin}!
- Model with larger hypothesis space required, e.g. space of polynomials H_{pol}.
  - See example 2
  - Are models with infinite hypothesis space always best?



Example 1:
Linear hypothesis space H_{lin} does not contain a consistent and complete hypothesis to map the target function.
$h_1 \in H_{lin}$



Example 2:
Polynomial hypothesis space H_{pol} contains the target function.
$h_2 \notin H_{lin}$
$h_1, h_2 \in H_{pol}$

Inductive Bias

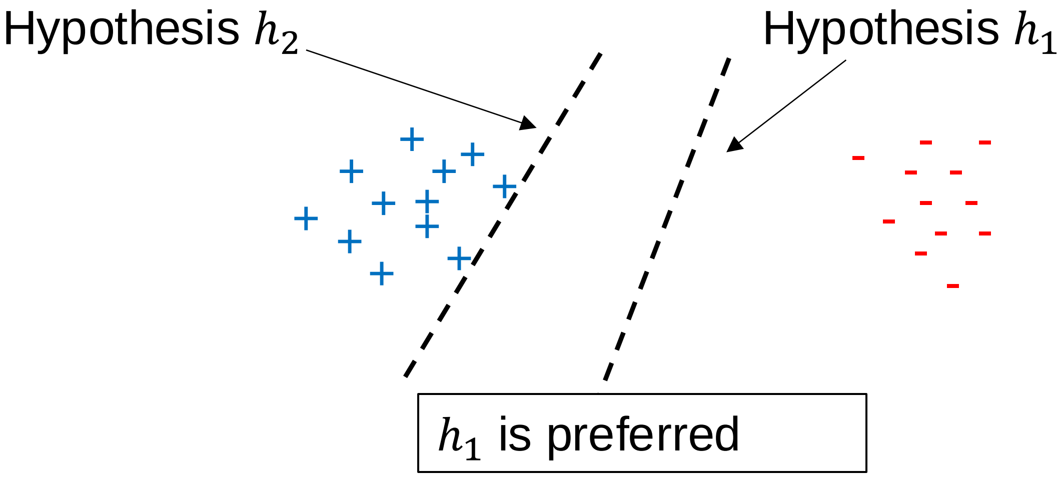
- What does "Sufficiently large number of training instances" mean?
  - The number of training examples required correlates with the size of the hypothesis space $|H|$ of a model
  - Constrain the hypothesis space of a model by means of prior assumptions, but keep $h \approx t$ in the reduced hypothesis space

- **Inductive bias**:
  - Set of assumptions or prior knowledge that a learning system incorporates to generalize from data
  - Certain hypotheses are preferred over other hypotheses in the hypothesis space

- All machine learning techniques have an inductive bias to facilitate an efficient learning process and generalization
  - Different methods have different biases

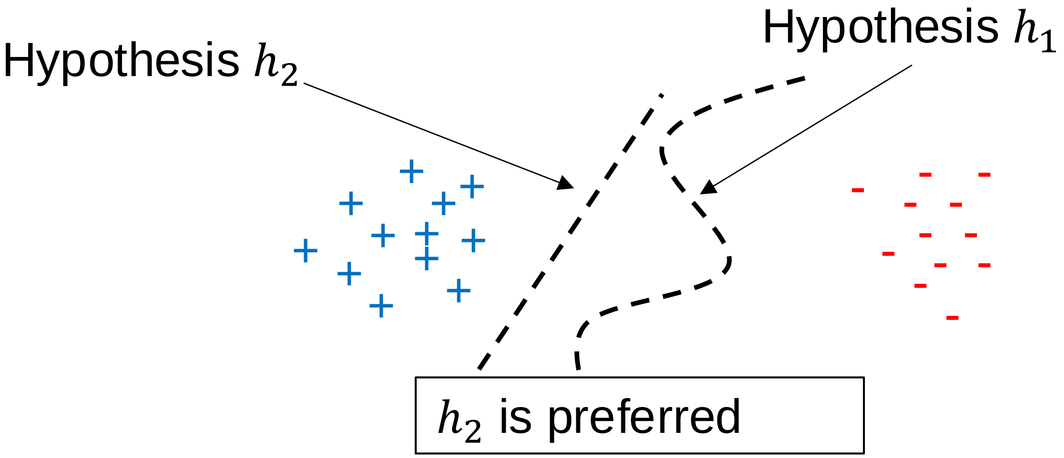
Typical Inductive Biases

- The hypothesis that maximizes the distance between input instances of different classes is preferred.
(See SVM in later lecture)



- If a simple hypothesis and a complex hypothesis both minimize the loss function, the simpler hypothesis is preferred. (See Occam‘s Razor)

奥卡姆剃刀原则



Next Lecture

- **Learning theory**
  - Deepen the concepts addressed in this lecture and demonstrate them with examples.
  - More fundamentals about machine learning