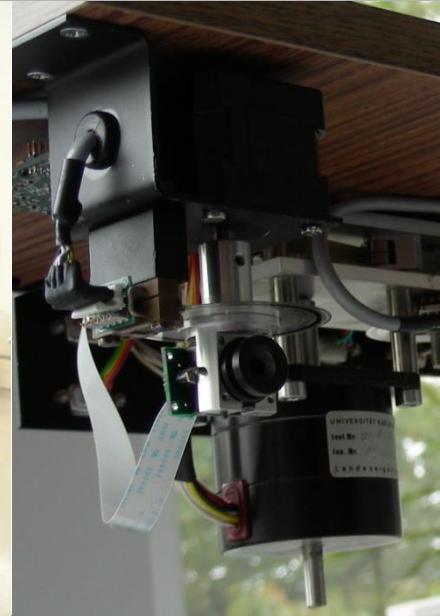
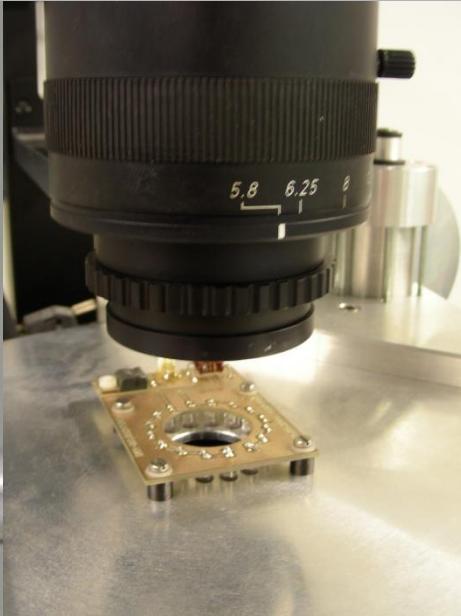


Machine Vision

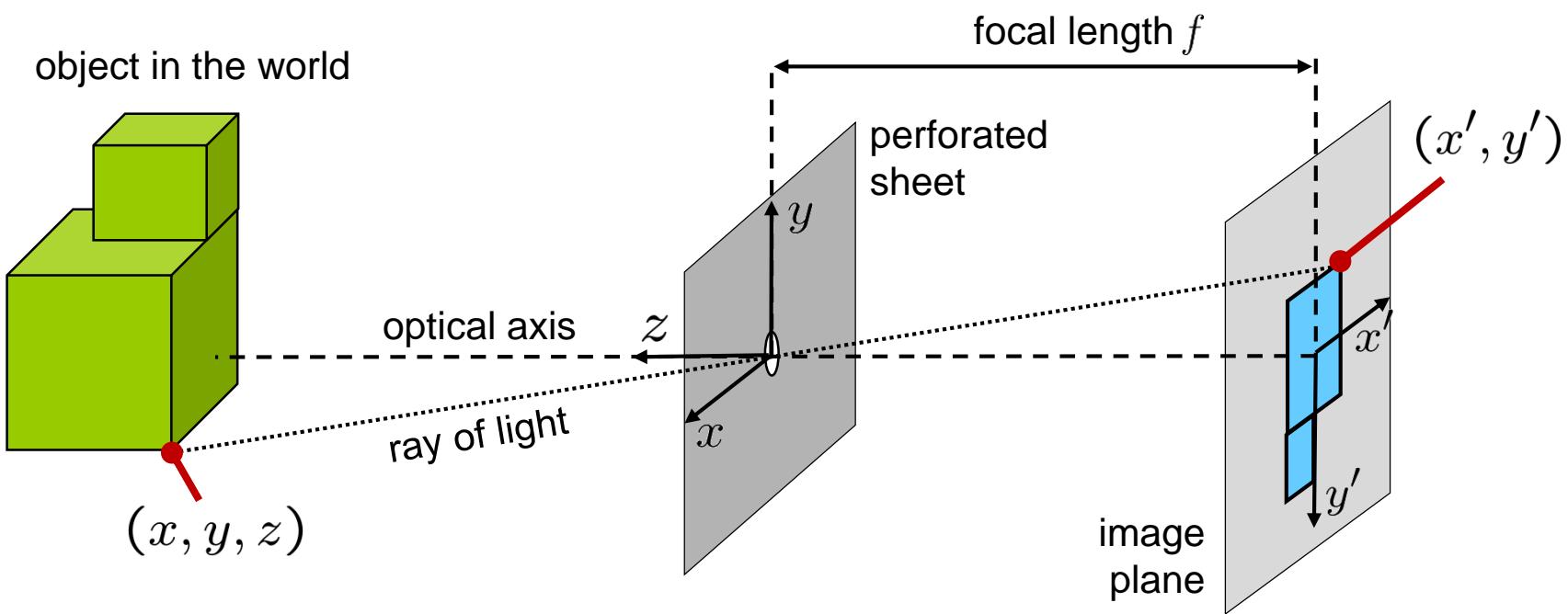
Chapter 7: Camera Optics

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und Regelungstechnik



Pinhole Camera

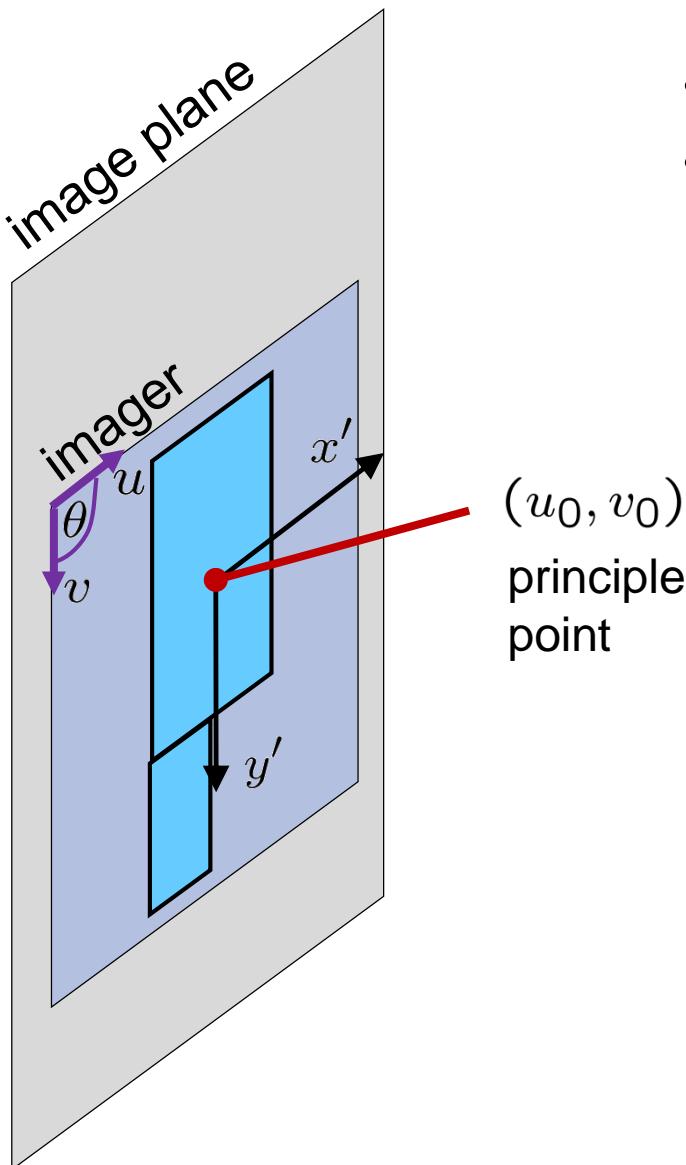


point (x, y, z) is projected onto (x', y')

intercept theorem:

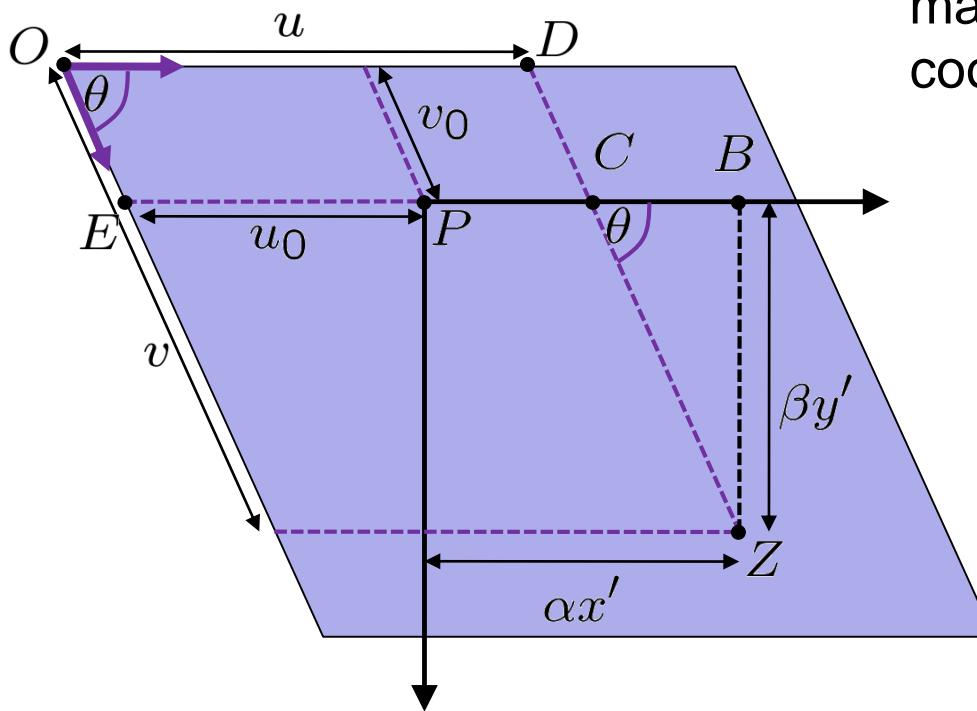
$$\frac{x}{z} = \frac{x'}{f}, \quad \frac{y}{z} = \frac{y'}{f} \quad \Rightarrow \quad z \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f & 0 \\ 0 & f \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

World-to-Image-Mapping



- camera coordinate system
- image coordinate system
 - u-direction parallel to x'-direction
 - v-direction might be skewed
 - θ=angle between u- and v-direction
- principle point=origin of camera coordinate system in image coordinates (u_0, v_0)
- length of unit vector u and v differ from length of unit vector x', y'
scaling factors α, β

World-to-Image-Mapping cont.



map point Z from camera coordinates to image coordinates:

- triangle ZBC:

$$\sin \theta = \frac{\beta y'}{v - v_0}$$

$$\Rightarrow v = \frac{\beta}{\sin \theta} y' + v_0$$

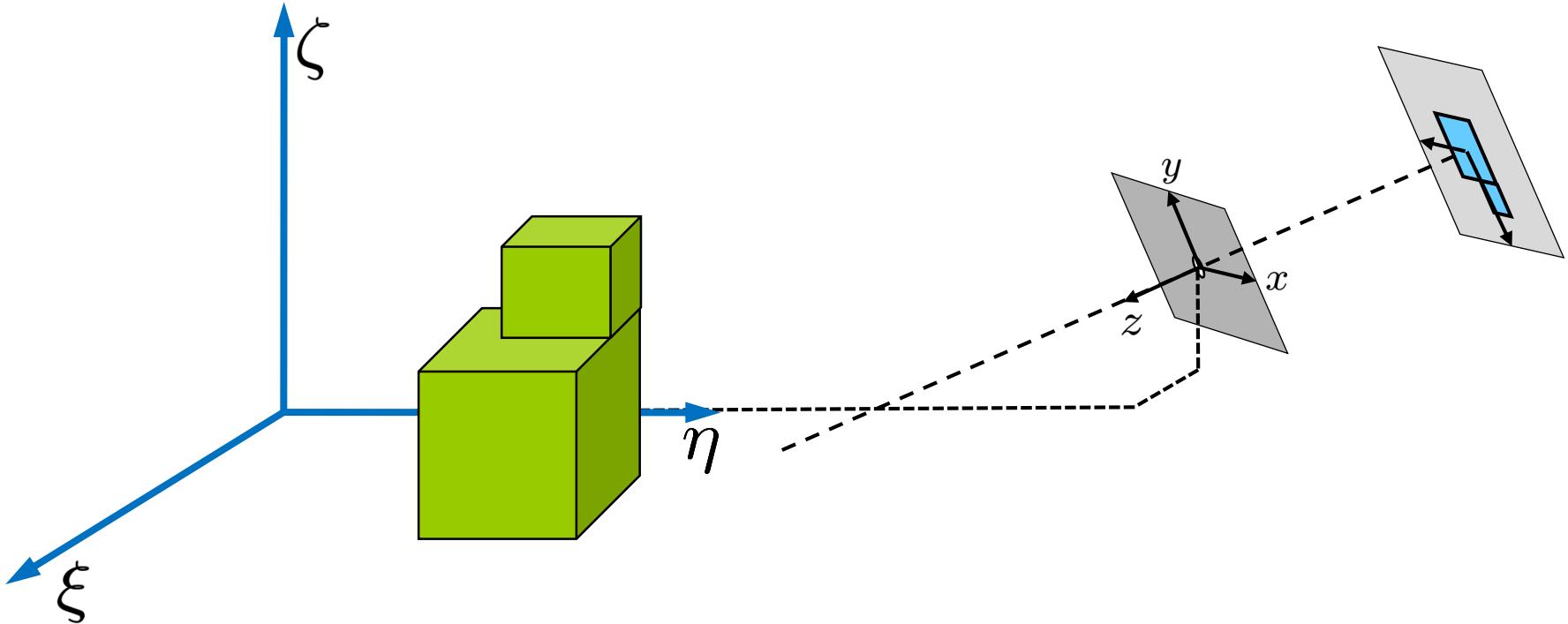
$$\cot \theta = \frac{\alpha x' + u_0 - u}{\beta y'}$$

$$\Rightarrow u = \alpha x' - (\cot \theta) \cdot \beta y' + u_0$$

mapping from camera to image frame:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha & -\beta \cot \theta \\ 0 & \frac{\beta}{\sin \theta} \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

World-to-Image-Mapping cont.



- position of objects on camera coordinates usually unknown
- external coordinate system (“world frame”) (ξ, η, ζ)
- mapping:

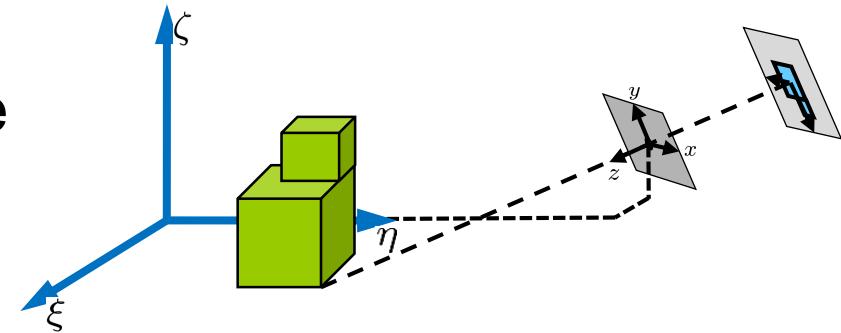
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \vec{t}$$

R : rotation matrix
 \vec{t} : translation vector

World-to-Image-Mapping cont.

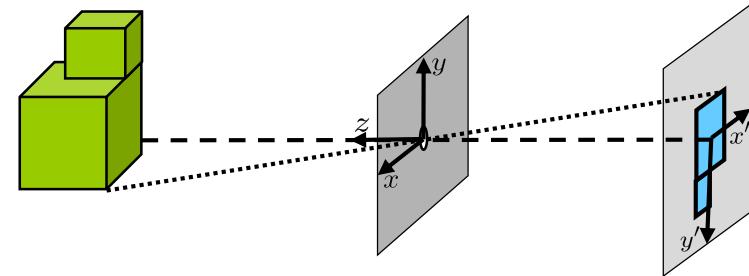
1. coordinate transformation
world frame → camera frame

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \vec{t}$$



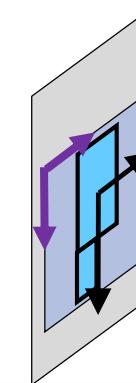
2. perspective projection

$$z \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} f & 0 \\ 0 & f \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$



3. coordinate transform
camera frame → image frame

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha & -\beta \cot \theta \\ 0 & \frac{\beta}{\sin \theta} \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} + \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$



World-to-Image-Mapping cont.

- rewriting step 3:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & -\beta \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

- rewriting step 2:

$$z \cdot \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

World-to-Image-Mapping cont.

- combining step 2 and 3:

$$\begin{aligned} z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} &= \begin{pmatrix} \alpha & -\beta \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ &= \begin{pmatrix} f\alpha & -f\beta \cot \theta & u_0 \\ 0 & \frac{f\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ &= \underbrace{\begin{pmatrix} \alpha' & -\beta' \cot \theta & u_0 \\ 0 & \frac{\beta'}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{=:A} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{aligned}$$

with $\alpha' = f\alpha$, $\beta' = f\beta$

World-to-Image-Mapping cont.

- rewriting step 1:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = (R \mid \vec{t}) \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$

- combining 1 with previous result:

$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = A \cdot (R \mid \vec{t}) \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$

World-to-Image-Mapping cont.

- given (ξ, η, ζ) , how can we calculate (u, v) ?

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = A \cdot \left(R \mid \vec{t} \right) \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \frac{1}{\tilde{z}} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}$$

- given (u, v) , how can we calculate (ξ, η, ζ) ?

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = z R^T A^{-1} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} - R^T \vec{t} \quad \text{with } z \geq 0$$

(ξ, η, ζ) is not unique but element of a ray

World-to-Image-Mapping cont.

- coordinates of camera origin:

$$(\xi, \eta, \zeta)^T = -R^T \vec{t}$$

- parameters:

- *intrinsic parameters*: describe the camera (5 parameters)

- $u_0, v_0, \alpha', \beta', \theta$

- *extrinsic parameters*: the pose of the camera (6 parameters)

- R, \vec{t}

- sometimes, the model is simplified assuming

- $\theta = 90^\circ, \alpha' = \beta'$

Measuring Volume

- measuring volume:

- is a rectangular pyramid

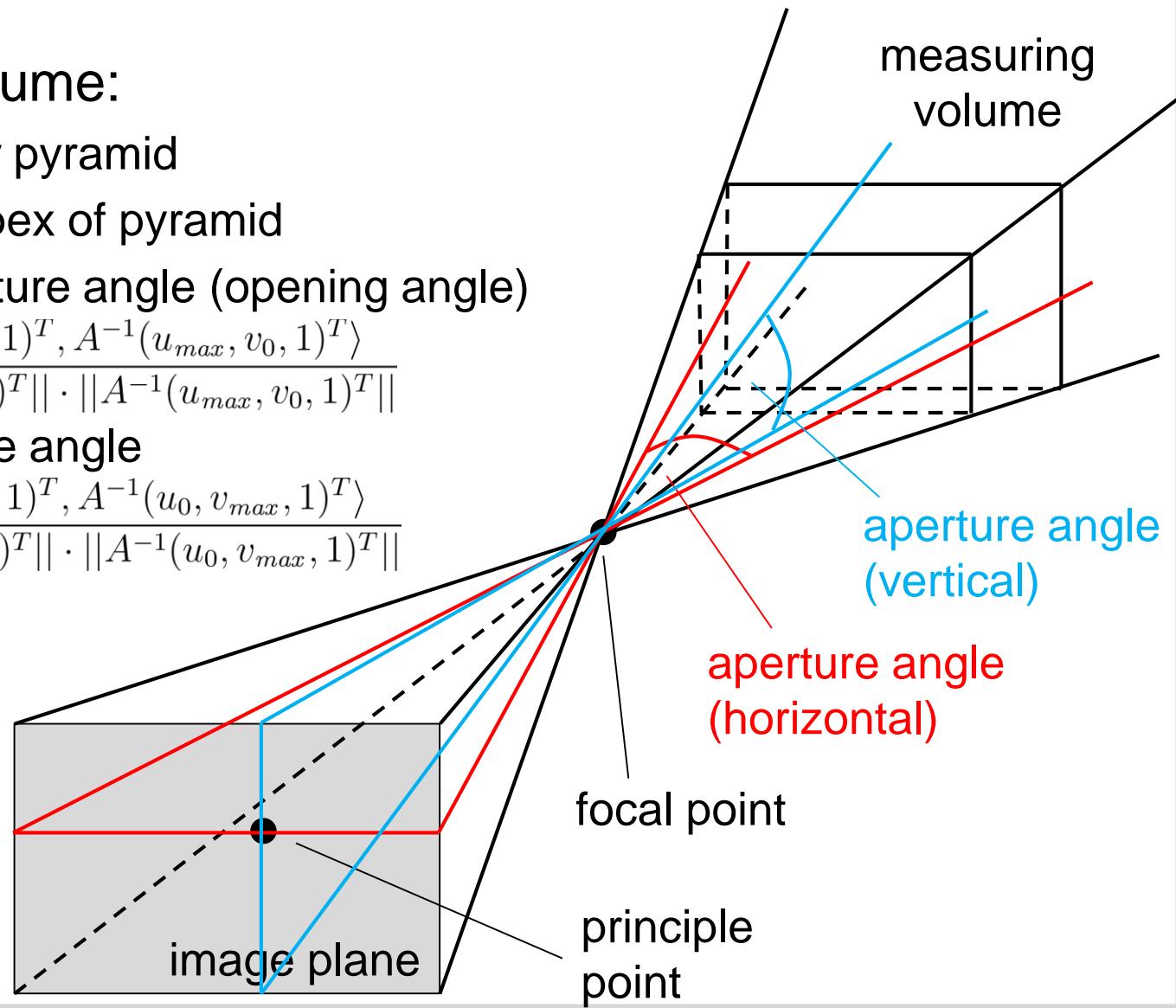
- focal point is apex of pyramid

- horizontal aperture angle (opening angle)

$$\arccos \frac{\langle A^{-1}(0, v_0, 1)^T, A^{-1}(u_{max}, v_0, 1)^T \rangle}{\|A^{-1}(0, v_0, 1)^T\| \cdot \|A^{-1}(u_{max}, v_0, 1)^T\|}$$

- vertical aperture angle

$$\arccos \frac{\langle A^{-1}(u_0, 0, 1)^T, A^{-1}(u_0, v_{max}, 1)^T \rangle}{\|A^{-1}(u_0, 0, 1)^T\| \cdot \|A^{-1}(u_0, v_{max}, 1)^T\|}$$



Perspective Projection

world (3D)

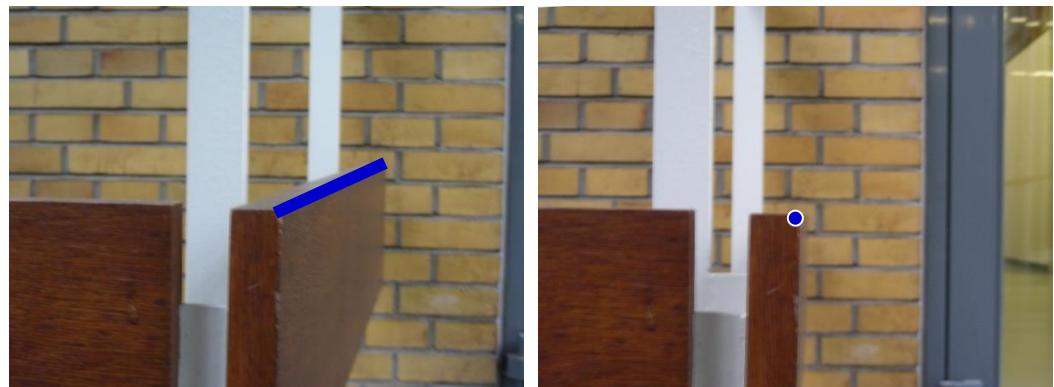
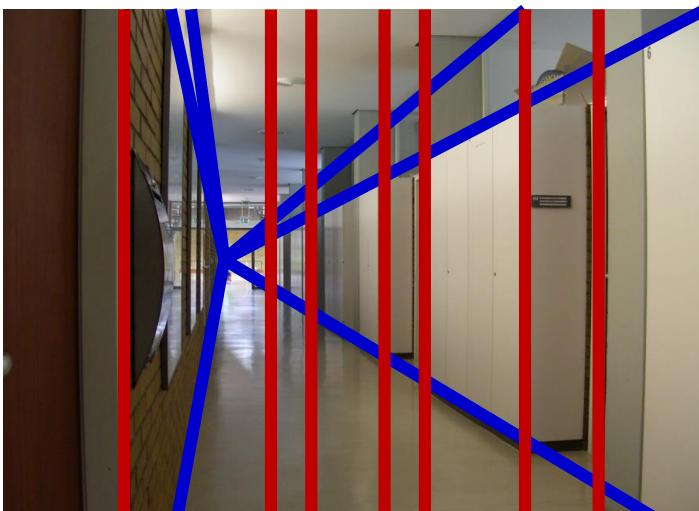
lines →

image (2D)

- lines
- points
(if line meets focal point)

parallel lines →

- lines intersecting
in one point
- parallel lines
(if lines are orthogonal to
optical axis)
- line and point
(if one line meets focal
point)



Perspective Projection cont.

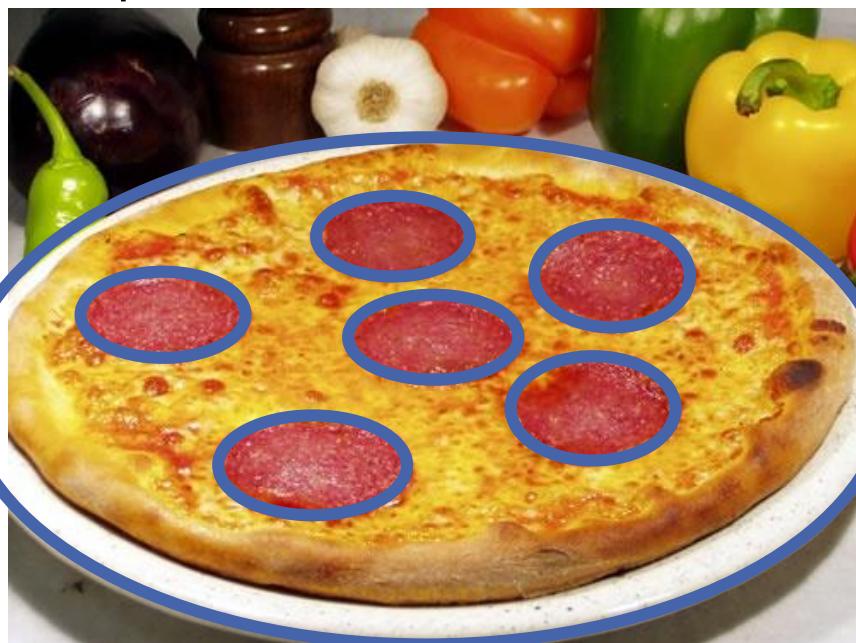
world (3D)

circles and ellipses

image (2D)

ellipses
(or line segments)

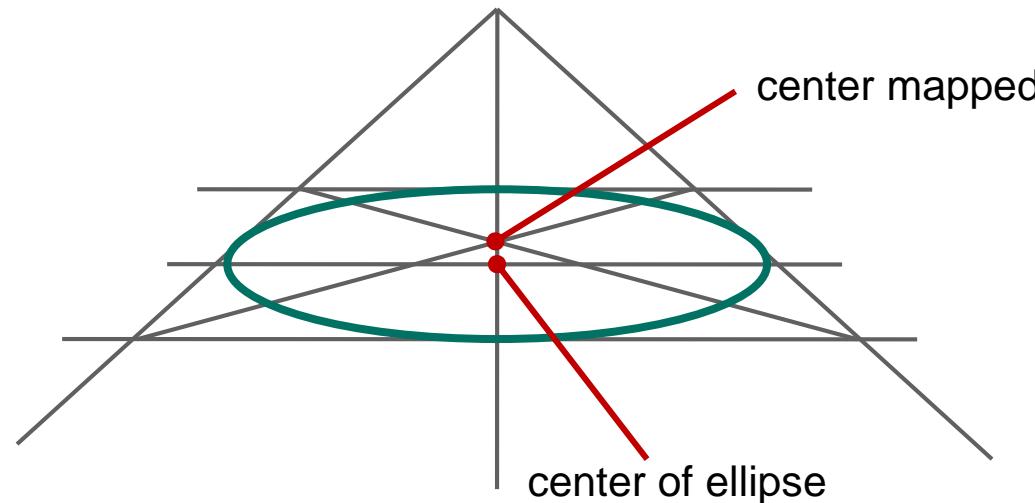
circles in a plane orthogonal
to optical axis



Perspective Projection cont.

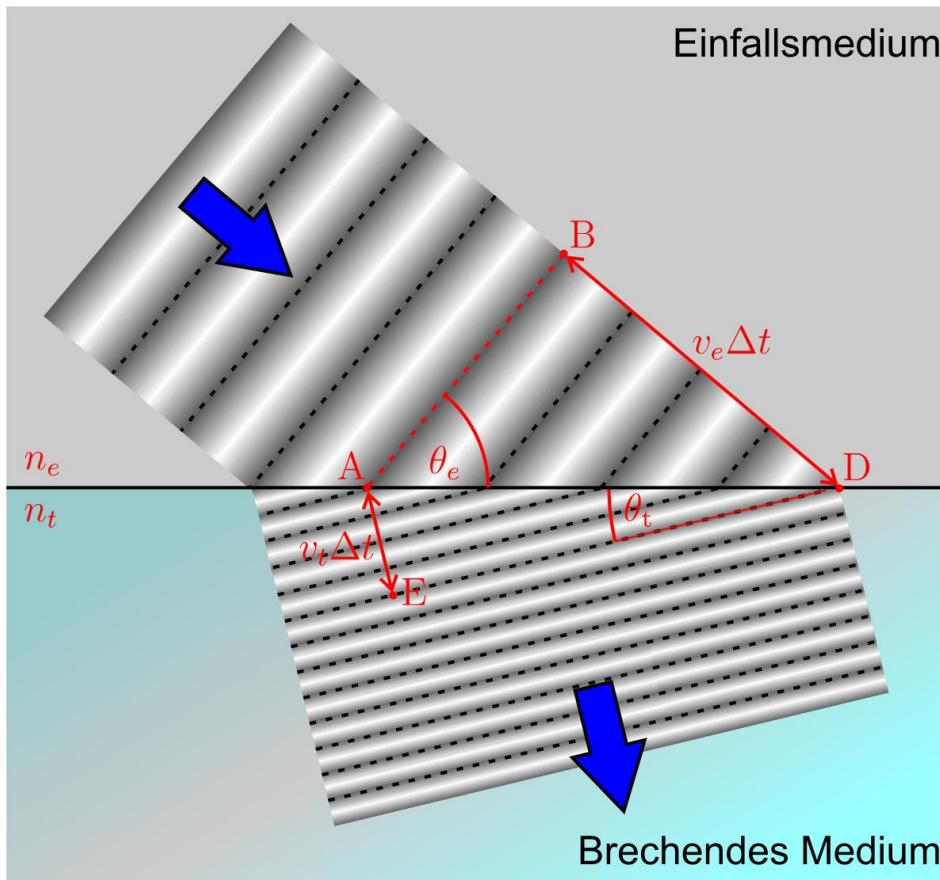
- perspective projection does **not**:

- preserve angles
- preserve lengths
- preserve area
- preserve ratio of lengths
- map the center of a circle/ellipse onto the center of the ellipse mapped
(except: if the plane is orthogonal to the optical axis)



Lenses

- pinhole cameras poorly let light through → lenses
- Snell's law of refraction



Snell's law:

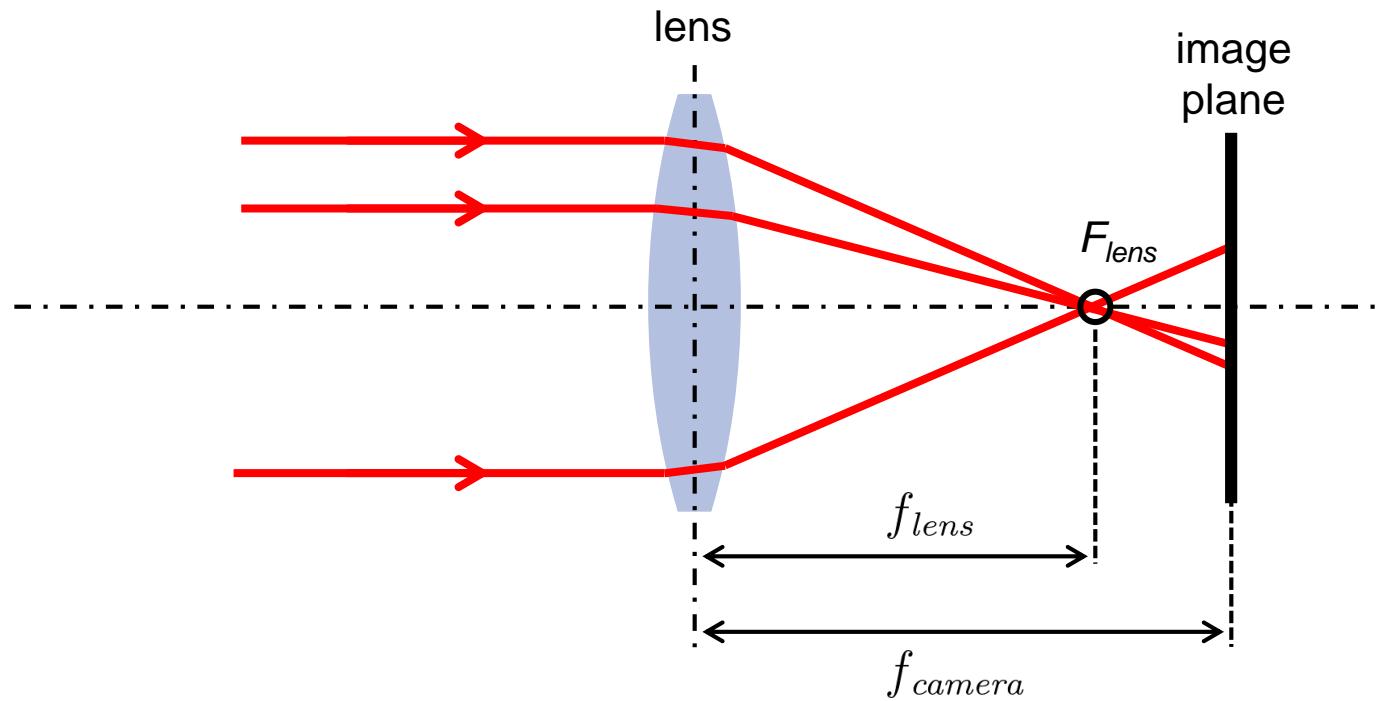
$$n_e \sin \theta_e = n_t \sin \theta_t$$

$$n_{\text{medium}} = \frac{v_{\text{vacuum}}}{v_{\text{medium}}}$$

Lenses cont.

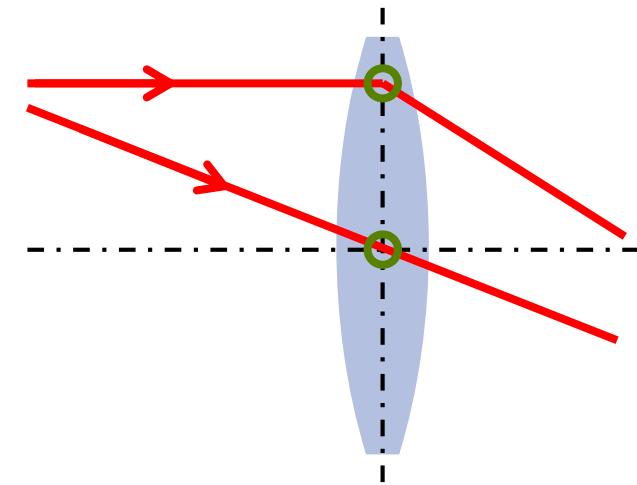
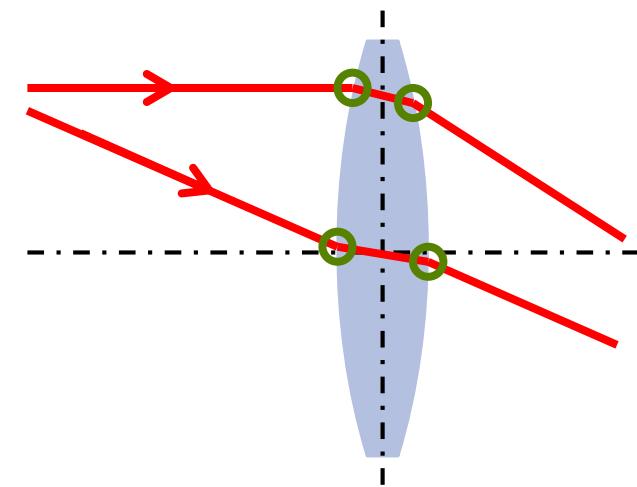
- focal length of a lens:

the distance between lens and focal point, the point where rays of light parallel to the optical axis meet after being refracted



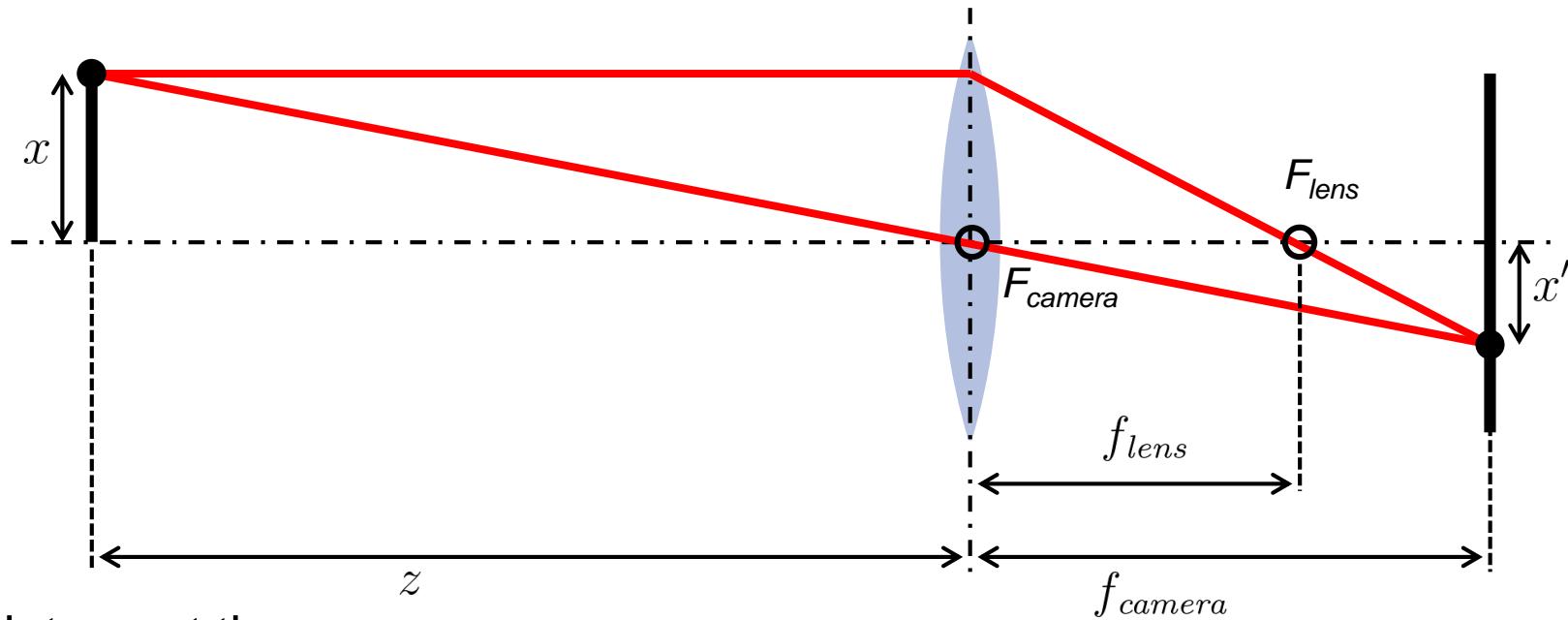
Thin Lenses

- refraction in lenses
 - surface air/glass
 - surface glass/air
- thin lenses
 - negligible thickness
 - double refraction can be approximated by single refraction at the center line
 - simpler geometric modeling



Thin Lenses cont.

- which condition must hold for a sharp image?

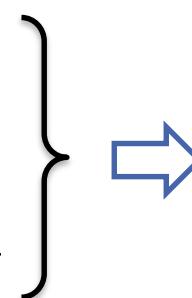


– intercept theorem:

$$\frac{x'}{x} = \frac{f_{camera}}{z}$$

– intercept theorem:

$$\frac{x'}{x} = \frac{f_{camera} - f_{lens}}{f_{lens}}$$

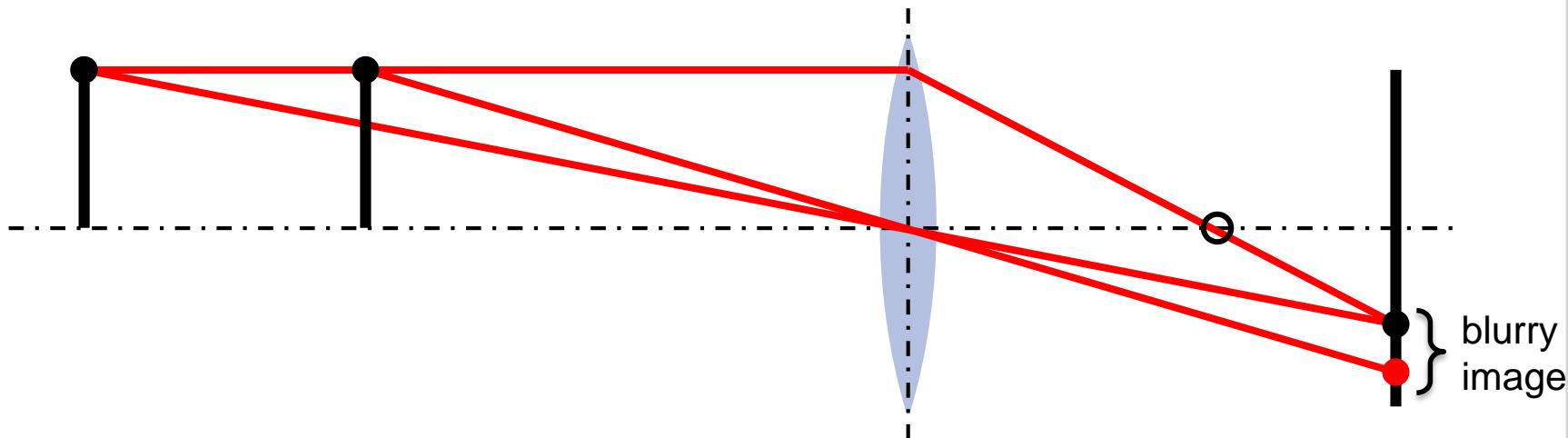


$$\frac{1}{f_{lens}} = \frac{1}{f_{camera}} + \frac{1}{z}$$

lens equation

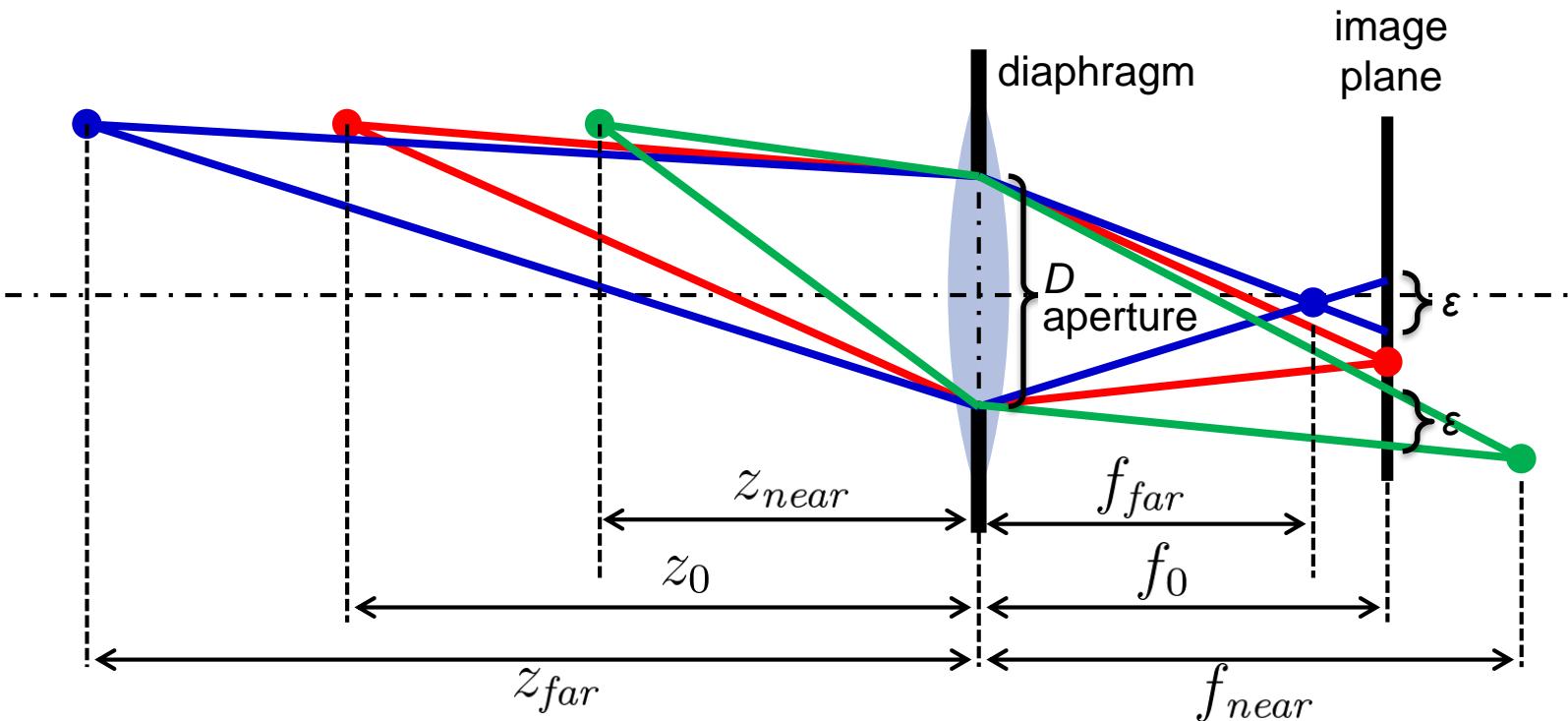
Thin Lenses cont.

- What happens when lens equation is violated?



- How much can we vary z with little effect?

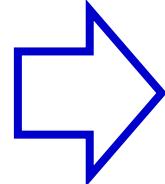
Depth of Field



- intercept theorem: $\frac{\epsilon}{D} = \frac{f_0 - f_{far}}{f_{far}} = \dots = \frac{f_{lens} \cdot (z_{far} - z_0)}{z_{far} \cdot (z_0 - f_{lens})}$
- intercept theorem: $\frac{\epsilon}{D} = \frac{f_{near} - f_0}{f_{near}} = \dots = \frac{f_{lens} \cdot (z_0 - z_{near})}{z_{near} \cdot (z_0 - f_{lens})}$

Depth of Field cont.

$$\frac{\epsilon}{D} = \frac{f_{lens} \cdot (z_{far} - z_0)}{z_{far} \cdot (z_0 - f_{lens})}$$



$$\frac{\epsilon}{D} = \frac{f_{lens} \cdot (z_0 - z_{near})}{z_{near} \cdot (z_0 - f_{lens})}$$

$$z_{far} = \frac{z_0 \cdot d_h}{d_h - (z_0 - f_{lens})}$$

$$z_{near} = \frac{z_0 \cdot d_h}{d_h + (z_0 - f_{lens})}$$

$$d_h = \frac{D \cdot f_{lens}}{\epsilon} \quad (\text{hyperfocal distance})$$

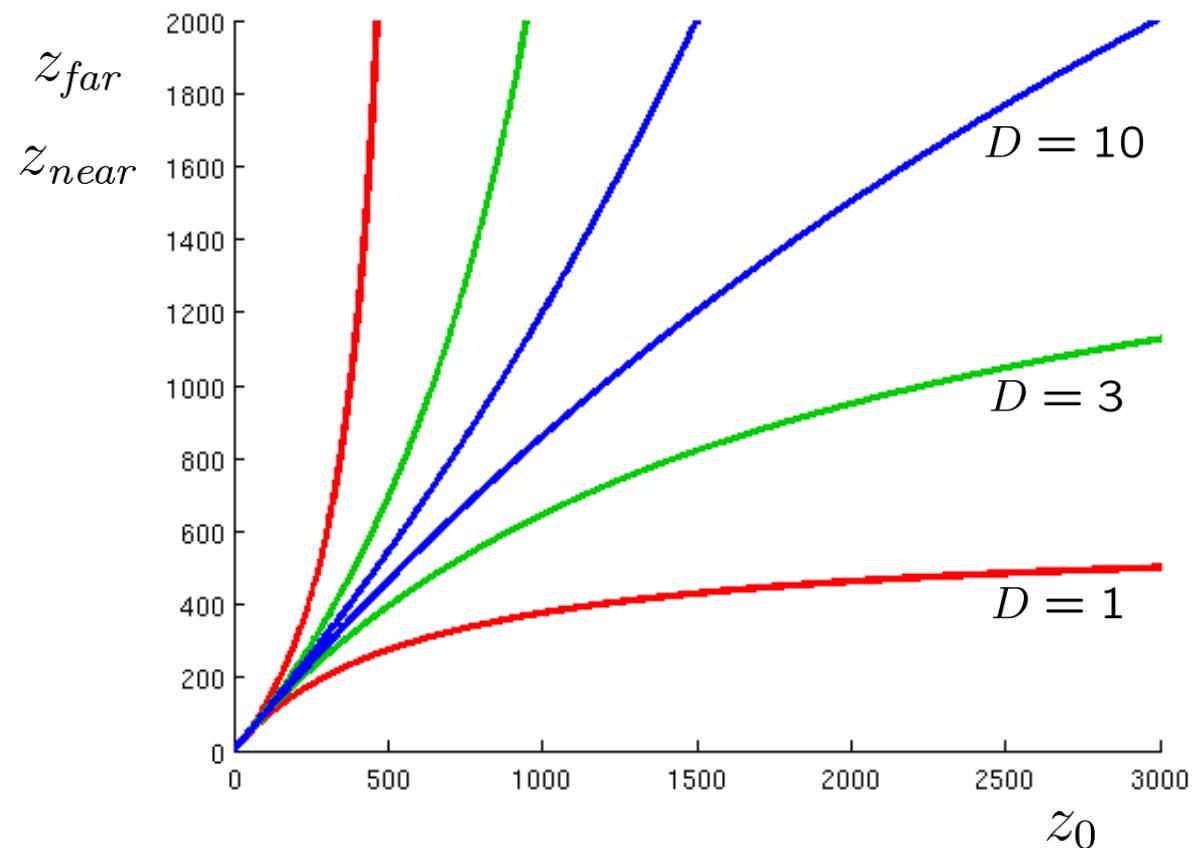
$$\Delta z = z_{far} - z_{near} = 2 \frac{z_0 \cdot d_h \cdot (z_0 - f_{lens})}{d_h^2 - (z_0 - f_{lens})^2}$$

– observation:

for $z_0 \rightarrow d_h + f_{lens}$ holds: $z_{far} \rightarrow \infty$

$$\Delta z \rightarrow \infty$$

Depth of Field cont.



$$f_{lens} = 6$$
$$\epsilon = 0.01$$

Focus Series

- focus bracketing/focus stacking

image series with varying distance between lens and image plane to overcome a limited depth of field

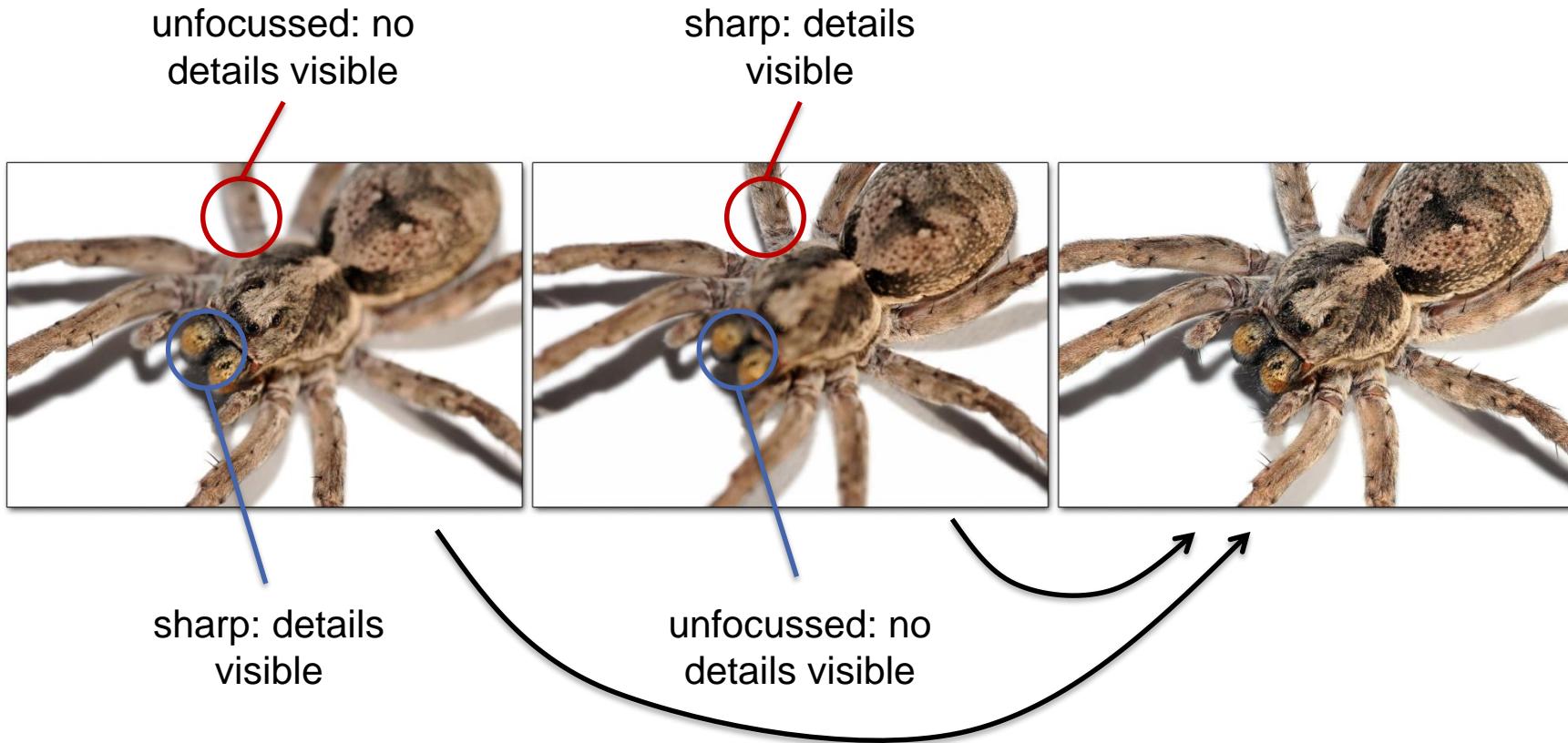


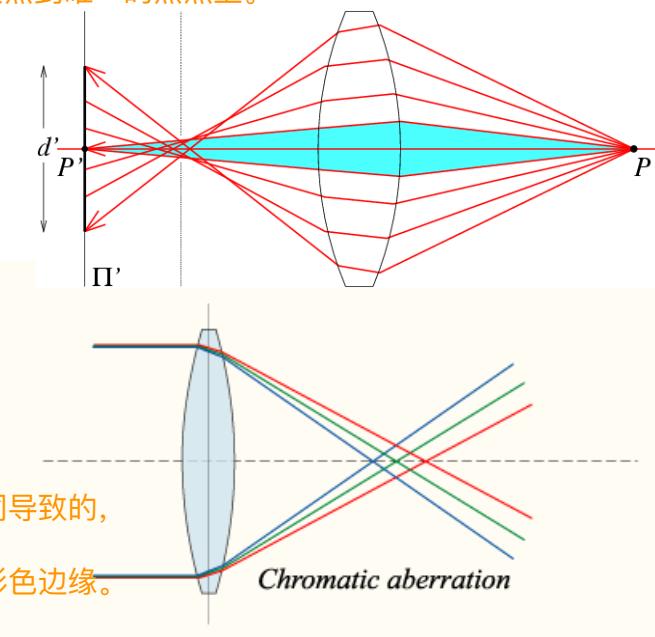
image source: wikipedia

Lens Aberrations

几何像差

几何像差是由于镜头的几何形状不完美导致的，主要表现为镜头无法将光线准确聚焦到唯一的焦点上。
这种问题可能会影响图像的清晰度和细节。

- geometric aberrations:
no unique focal point due to imperfect lens geometry
 - spherical aberration, astigmatism, coma
- chromatic aberrations:
dispersion caused by different refraction index for different wavelength (“rainbow effect”)
- vignetting: 渐晕
reduced light intensity and saturation in the image periphery



渐晕是指图像的边缘部分亮度和饱和度降低。其原因包括镜头设计、光线遮挡等。

Image Distortion

- image distortion:

perspective projection should map lines to lines.

But most cameras do not → distortion

- radial distortion

- suboptimal shape of lens

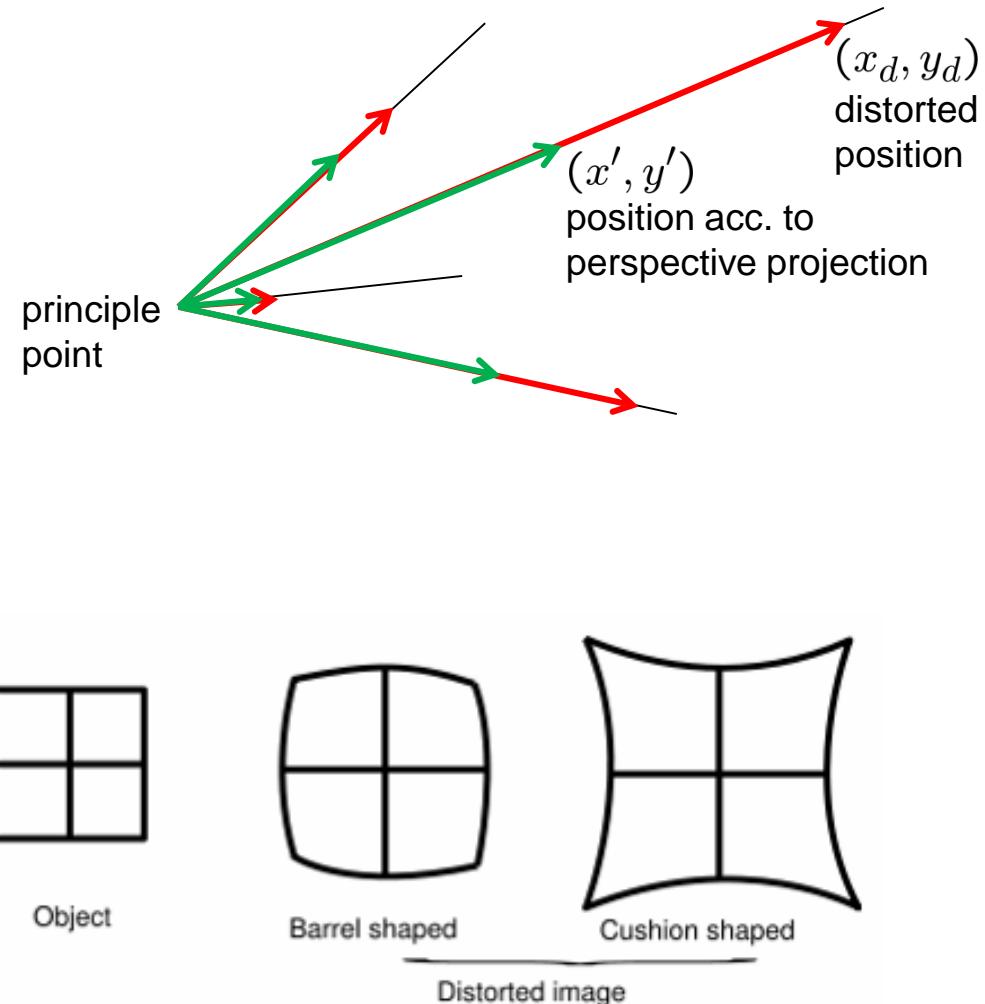
- tangential distortion

- suboptimal mounting of lens



Radial Distortion

- points are shifted away from principle point
- radial distortion is symmetric
- amount of shifting depends nonlinearly from the distance to the principle point
- rectangular objects appear barrel-shaped or pincushion-shaped in the image



Radial Distortion cont.

- mathematical modeling with even polynomials:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = (1 + k_1 r^2 + k_2 r^4) \begin{pmatrix} x_d \\ y_d \end{pmatrix} \quad \text{with } r^2 = x_d^2 + y_d^2$$

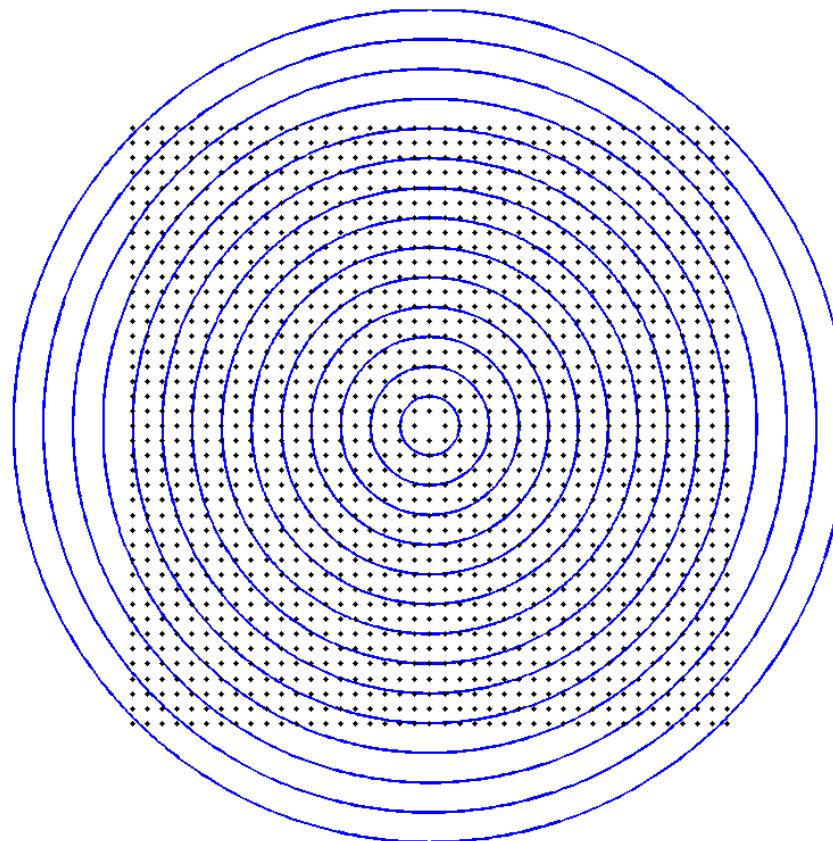
or in image coordinates:

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + (1 + k_1 r^2 + k_2 r^4) \begin{pmatrix} u_d - u_0 \\ v_d - v_0 \end{pmatrix}$$

with $r^2 = (u_d - u_0)^2 + (v_d - v_0)^2$

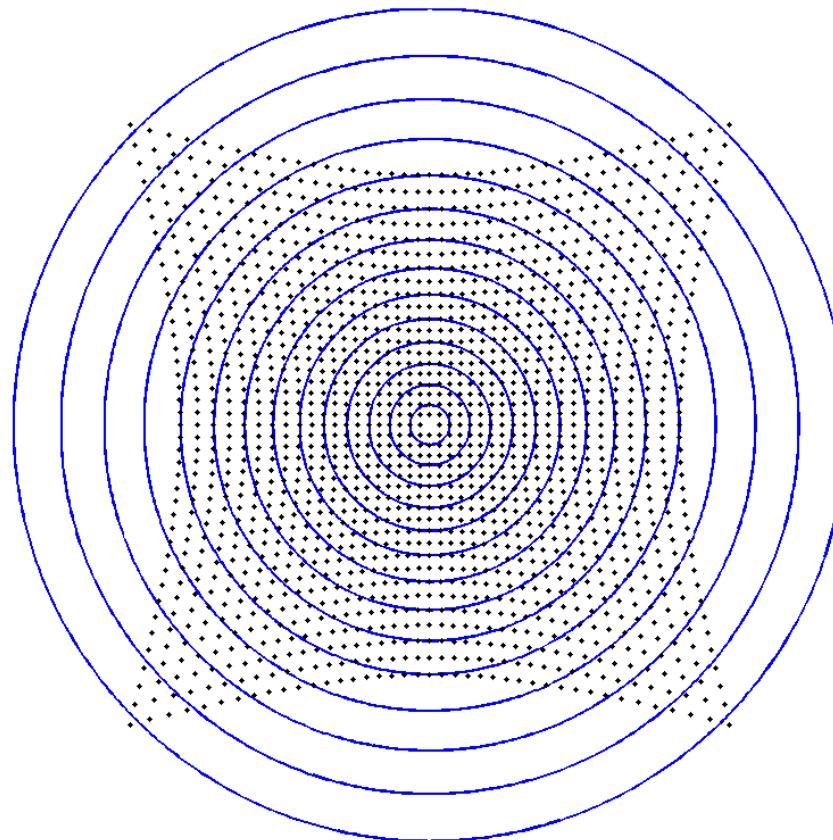
Radial Distortion cont.

$$\begin{aligned}k_1 &= 0 \\k_2 &= 0\end{aligned}$$



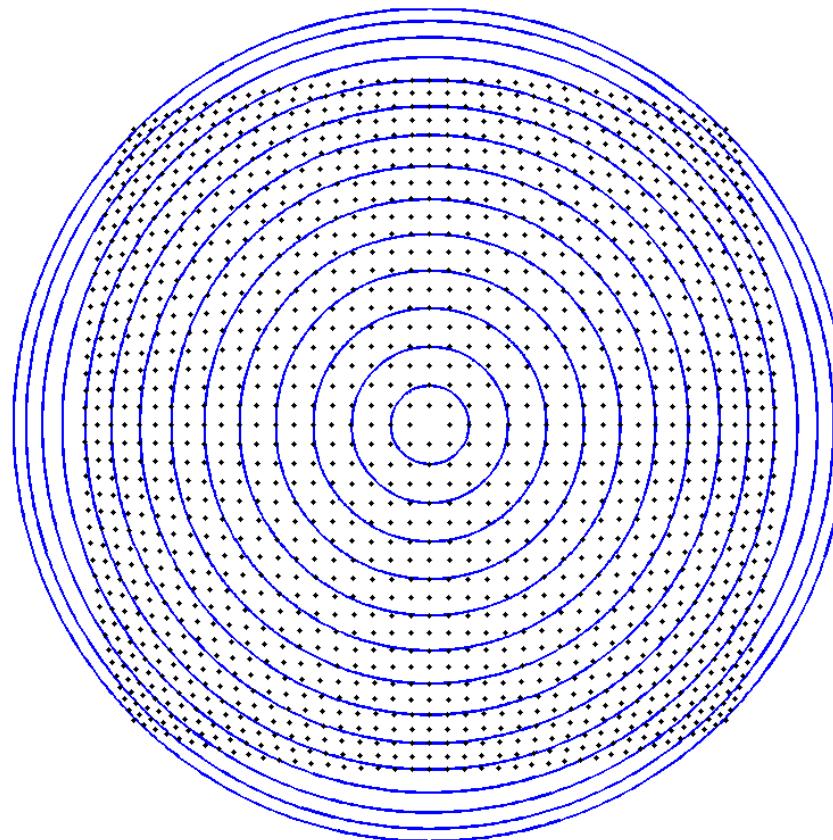
Radial Distortion cont.

$$\begin{aligned}k_1 &> 0 \\k_2 &\geq 0\end{aligned}$$



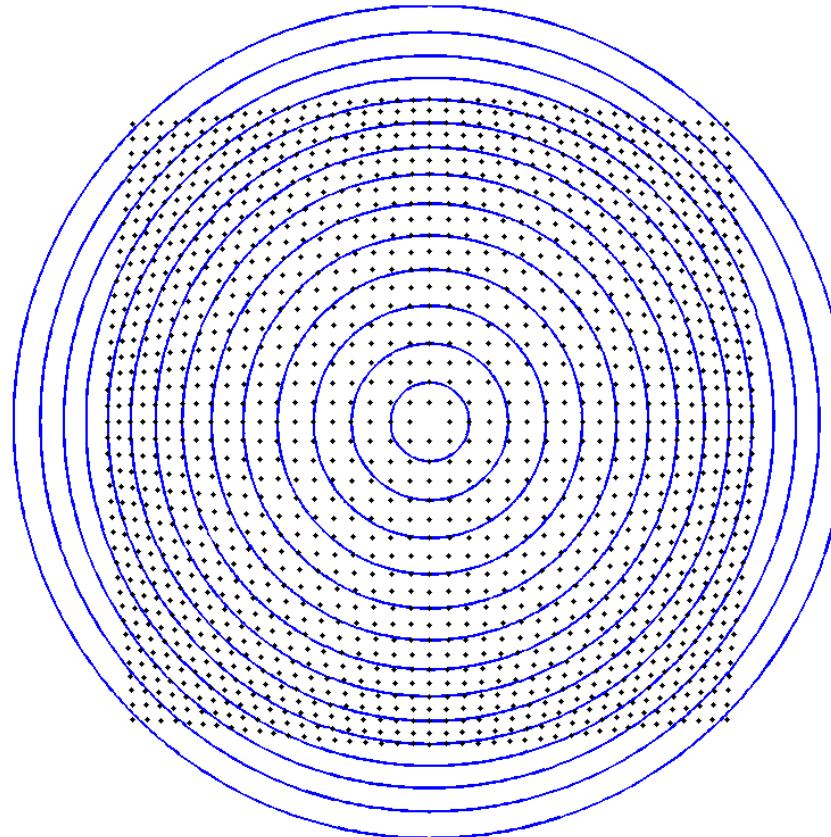
Radial Distortion cont.

$$\begin{aligned}k_1 &< 0 \\k_2 &\leq 0\end{aligned}$$



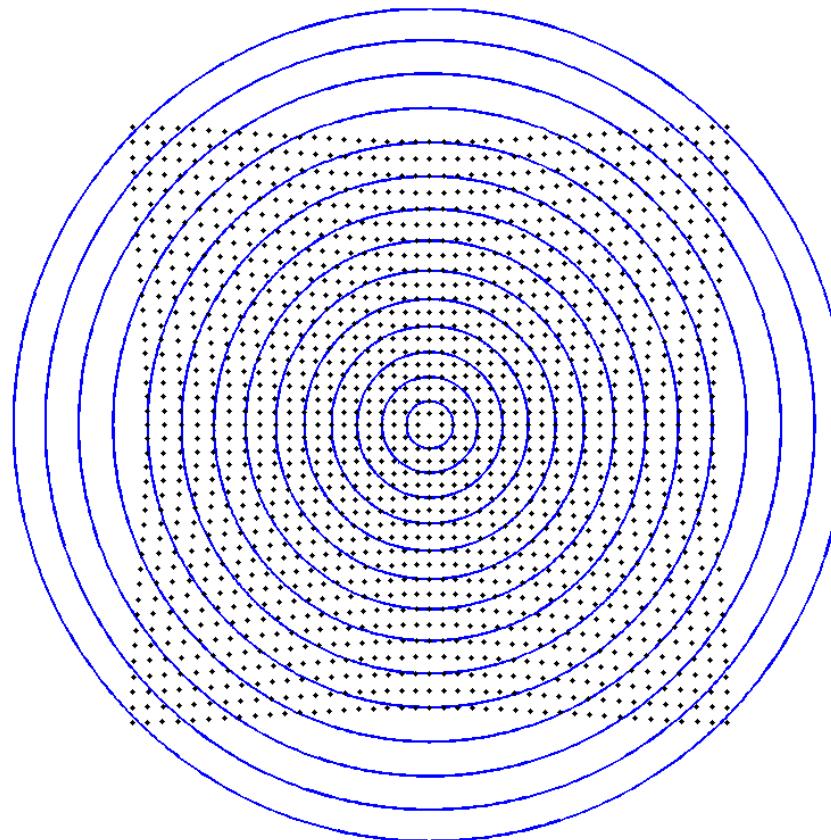
Radial Distortion cont.

$k_1 < 0$
 $k_2 > 0$



Radial Distortion cont.

$$\begin{aligned}k_1 &> 0 \\k_2 &< 0\end{aligned}$$



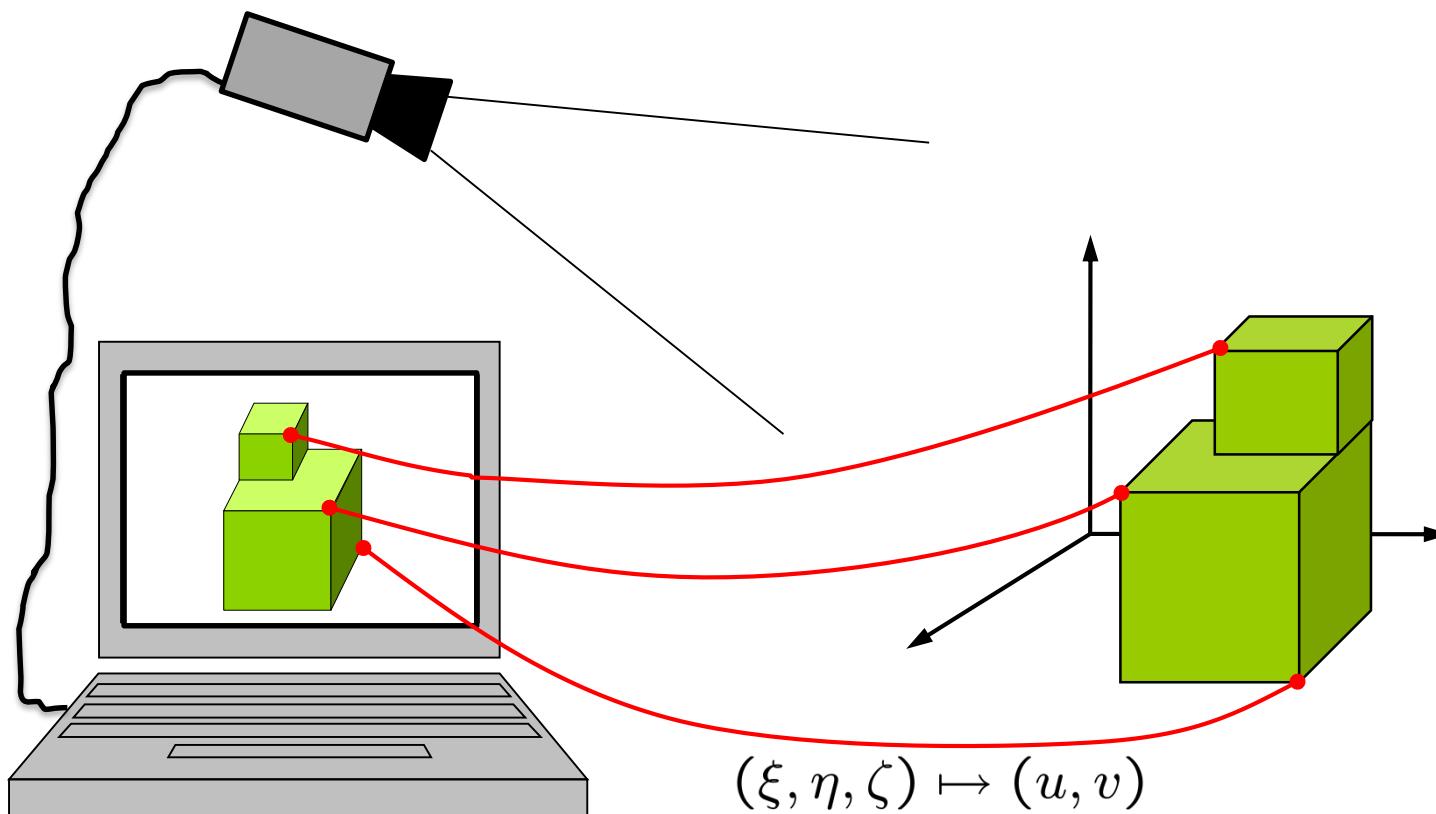
CAMERA CALIBRATION

Camera Calibration

- world-to-image mapping contains many parameters:
 - *intrinsic parameters*:
 $u_0, v_0, \alpha', \beta', \theta$
 - *extrinsic parameters*:
 R, \vec{t}
 - *distortion parameters*:
 k_1, k_2, \dots
- calibration = process to determine parameters

Camera Calibration cont.

- calibration: determine camera parameters from pairs of image points and world points



Camera Calibration cont.

- from one or several pictures we get corresponding points:
 $(\xi_i, \eta_i, \zeta_i) \mapsto (u_i, v_i)$
- find camera parameters A, R, \vec{t} that map (ξ_i, η_i, ζ_i) onto (u_i, v_i) as good as possible
- several approaches. Here:
 1. Tsai's approach
 2. Zhang's approach

Camera Calibration cont.

- world-to-image mapping:

$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \underbrace{A \cdot \begin{pmatrix} R & | & \vec{t} \end{pmatrix}}_{=:M} \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$

- M is 3x4 matrix

$$M = \begin{pmatrix} m_{1,1} & \dots & m_{1,4} \\ \vdots & \ddots & \vdots \\ m_{3,1} & \dots & m_{3,4} \end{pmatrix}$$

- we get:

$$\vec{m}_{1,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{1,4} - u(\vec{m}_{3,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{3,4}) = 0$$

$$\vec{m}_{2,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{2,4} - v(\vec{m}_{3,1:3} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + m_{3,4}) = 0$$

Camera Calibration cont.

- determining camera parameters by minimizing:

$$\begin{aligned} & \sum_i \left(\left(\vec{m}_{1,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{1,4} - u_i (\vec{m}_{3,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{3,4}) \right)^2 \right. \\ & \quad \left. + \left(\vec{m}_{2,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{2,4} - v_i (\vec{m}_{3,1:3} \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i \end{pmatrix} + m_{3,4}) \right)^2 \right) \end{aligned}$$

第一行和第二行：控制图像坐标 (u, v) 与世界坐标点 $(\text{\textit{xi}, \textit{eta}, \textit{zeta}})$ 的线性映射关系。

- zeroing partial derivatives:

$$\begin{pmatrix} \sum_i S_i & 0 & -\sum_i u_i S_i \\ 0 & \sum_i S_i & -\sum_i v_i S_i \\ -\sum_i u_i S_i & -\sum_i v_i S_i & \sum_i (u_i^2 + v_i^2) S_i \end{pmatrix} \cdot \begin{pmatrix} \vec{m}_{1,1:4}^T \\ \vec{m}_{2,1:4}^T \\ \vec{m}_{3,1:4}^T \end{pmatrix} = \vec{0}$$

第三行：深度信息 z 对映射的影响。

with

$$S_i = (\xi_i, \eta_i, \zeta_i, 1)^T$$

Camera Calibration cont.

- solution: Eigenvector with respect to smallest Eigenvalue
 - 1 degree of freedom: length of solution 最小特征值对应于最优解

- structure of the solution:

$$M = A \cdot (R | \vec{t}) = \begin{pmatrix} \vec{m}_{1,1:3} & m_{1,4} \\ \vec{m}_{2,1:3} & m_{2,4} \\ \vec{m}_{3,1:3} & m_{3,4} \end{pmatrix}$$

$$\text{with } \vec{m}_{1,1:3} = \alpha' \vec{r}_{1,1:3} - \beta' \cot \theta \vec{r}_{2,1:3} + u_0 \vec{r}_{3,1:3}$$

$$m_{1,4} = \alpha' t_1 - \beta' \cot \theta t_2 + u_0 t_3$$

$$\vec{m}_{2,1:3} = \frac{\beta'}{\sin \theta} \vec{r}_{2,1:3} + v_0 \vec{r}_{3,1:3}$$

$$m_{2,4} = \frac{\beta'}{\sin \theta} t_2 + v_0 t_3$$

$$\vec{m}_{3,1:3} = \vec{r}_{3,1:3}$$

$$m_{3,4} = t_3$$

Camera Calibration cont.

- R is a rotation matrix:

$$\|\vec{r}_{1,1:3}\| = 1$$

$$\|\vec{r}_{2,1:3}\| = 1$$

$$\|\vec{r}_{3,1:3}\| = 1$$

$$\langle \vec{r}_{1,1:3}, \vec{r}_{2,1:3} \rangle = 0$$

$$\langle \vec{r}_{2,1:3}, \vec{r}_{3,1:3} \rangle = 0$$

$$\langle \vec{r}_{3,1:3}, \vec{r}_{1,1:3} \rangle = 0$$

- Since $\vec{m}_{3,1:3} = \vec{r}_{3,1:3}$ choose solution with
 $\|\vec{m}_{3,1:3}\|^2 = 1$ (two possibilities, check $\det(R) = +1$)
- Given M , how can we derive camera parameters $R, t, \alpha', \beta', \theta, u_0, v_0$?

Camera Calibration cont.

$$\vec{r}_{3,1:3} = \vec{m}_{3,1:3}$$

$$t_3 = m_{3,4}$$

$$v_0 = \langle \vec{m}_{3,1:3}, \vec{m}_{2,1:3} \rangle$$

$$u_0 = \langle \vec{m}_{3,1:3}, \vec{m}_{1,1:3} \rangle$$

$$\frac{\beta'}{\sin \theta} = \sqrt{\|\vec{m}_{2,1:3}\|^2 - v_0^2}$$

$$t_2 = \left(\frac{\beta'}{\sin \theta} \right)^{-1} (m_{2,4} - v_0 t_3)$$

$$\vec{r}_{2,1:3} = \left(\frac{\beta'}{\sin \theta} \right)^{-1} (\vec{m}_{2,1:3} - v_0 \vec{r}_{3,1:3})$$

$$\beta' \cot \theta = \left(\frac{\beta'}{\sin \theta} \right)^{-1} (u_0 v_0 - \langle \vec{m}_{1,1:3}, \vec{m}_{2,1:3} \rangle)$$

$$\alpha' = \sqrt{\|\vec{m}_{1,1:3}\|^2 + (\beta' \cot \theta)^2 - u_0^2}$$

$$\vec{r}_{1,1:3} = (\alpha')^{-1} (\vec{m}_{1,1:3} + \beta' \cot \theta \vec{r}_{2,1:3} - u_0 \vec{r}_{3,1:3})$$

$$t_1 = (\alpha')^{-1} (m_{1,4} + \beta' \cot \theta t_2 - u_0 t_3)$$

$$\vec{m}_{1,1:3} = \alpha' \vec{r}_{1,1:3} - \beta' \cot \theta \vec{r}_{2,1:3} + u_0 \vec{r}_{3,1:3}$$

$$m_{1,4} = \alpha' t_1 - \beta' \cot \theta t_2 + u_0 t_3$$

$$\vec{m}_{2,1:3} = \frac{\beta'}{\sin \theta} \vec{r}_{2,1:3} + v_0 \vec{r}_{3,1:3}$$

$$m_{2,4} = \frac{\beta'}{\sin \theta} t_2 + v_0 t_3$$

$$\vec{m}_{3,1:3} = \vec{r}_{3,1:3}$$

$$m_{3,4} = t_3$$

$$\theta = \arccos \frac{\beta' \cot \theta}{\frac{\beta'}{\sin \theta}}$$

$$\beta' = \left(\frac{\beta'}{\sin \theta} \right) \sin \theta$$

Camera Calibration cont.

Summary: Tsai's approach

1. create an artificial scene with calibration markers (markers in general position)
2. measure 3d world position of calibration markers
3. make a picture
4. measure 2d image position of calibration markers
5. solve optimization problem to estimate matrix M
6. decompose M into A, R, t

R.Y. Tsai,

An Efficient and Accurate Camera Calibration Technique for 3D Machine Vision.
Proceedings of IEEE Conference on Computer Vision and Pattern Recognition,
Miami Beach, FL, pp. 364-374, 1986.

Camera Calibration: Zhang

Assume 3d-points on the plane $\zeta = 0$

These points are mapped by a camera to

$$\begin{aligned} z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} &= A \cdot (R \cdot \begin{pmatrix} \xi \\ \eta \\ 0 \end{pmatrix} + \vec{t}) \\ &= \underbrace{A \cdot (\vec{r}_{1:3,1}, \vec{r}_{1:3,2}, \vec{t})}_{=:H} \cdot \begin{pmatrix} \xi \\ \eta \\ 1 \end{pmatrix} \end{aligned}$$

H is called a *homography*

$$H = A \cdot (\vec{r}_{1:3,1}, \vec{r}_{1:3,2}, \vec{t})$$

Camera Calibration: Zhang cont.

If we know several homographies H_1, H_2, \dots, H_n , can we derive A ?

Let us first consider

$$B = A^{-T} A^{-1}$$

A is full rank, upper triangular matrix

→ A^{-1} exists and is also upper triangular matrix

→ B is symmetric, has 6 different entries

→ A^{-1} can be calculated from B via Cholesky decomposition

→ if we know B , we can derive A easily

$$B = \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{1,2} & b_{2,2} & b_{2,3} \\ b_{1,3} & b_{2,3} & b_{3,3} \end{pmatrix}$$

Camera Calibration: Zhang cont.

$$H = A \cdot \left(\vec{r}_{1:3,1}, \vec{r}_{1:3,2}, \vec{t} \right)$$

R is a rotation matrix, hence

$$\begin{aligned} 0 &= \langle \vec{r}_{1:3,1}, \vec{r}_{1:3,2} \rangle = \langle A^{-1} \vec{h}_{1:3,1}, A^{-1} \vec{h}_{1:3,2} \rangle \\ &= \vec{h}_{1:3,1}^T \cdot (A^{-T} A^{-1}) \cdot \vec{h}_{1:3,2} \\ &= \vec{h}_{1:3,1}^T \cdot B \cdot \vec{h}_{1:3,2} \end{aligned} \tag{1}$$

$$\begin{aligned} \langle \vec{r}_{1:3,1}, \vec{r}_{1:3,1} \rangle &= 1 = \langle \vec{r}_{1:3,2}, \vec{r}_{1:3,2} \rangle \\ \langle A^{-1} \vec{h}_{1:3,1}, A^{-1} \vec{h}_{1:3,1} \rangle &\quad \langle A^{-1} \vec{h}_{1:3,2}, A^{-1} \vec{h}_{1:3,2} \rangle \\ \vec{h}_{1:3,1}^T \cdot B \cdot \vec{h}_{1:3,1} &\quad \vec{h}_{1:3,2}^T \cdot B \cdot \vec{h}_{1:3,2} \\ \Rightarrow 0 &= \vec{h}_{1:3,1}^T \cdot B \cdot \vec{h}_{1:3,1} - \vec{h}_{1:3,2}^T \cdot B \cdot \vec{h}_{1:3,2} \end{aligned} \tag{2}$$

Hence, from one homography H we obtain two constraints (1), (2) for B

Camera Calibration: Zhang cont.

If we know several homographies H_1, H_2, \dots, H_n , can we derive A ?

- from each homography we obtain two constraints
- 3 homographies yield in total 6 constraints in order to estimate 6 parameters
- >3 homographies yield an overdetermined system of constraints
→ least squares method finds a matrix that minimizes the residuals

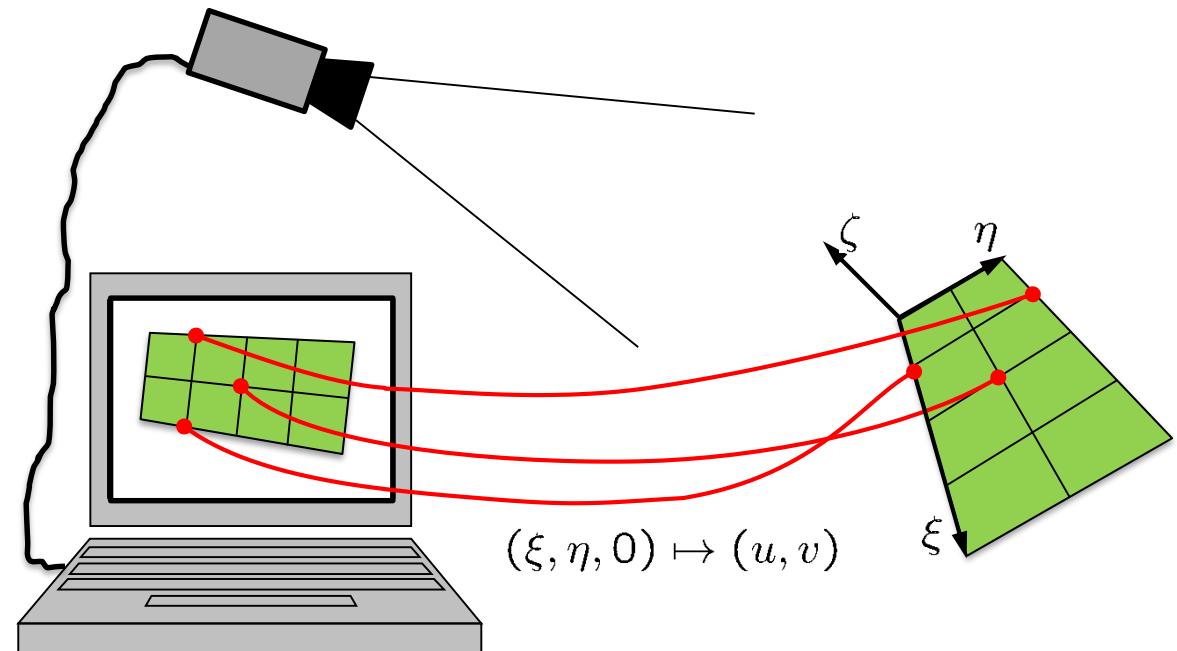
Camera Calibration: Zhang cont.

Sketch: Zhang's approach

1. ...
2. ...
3. ...
4. estimate homographies H
5. solve optimization problem to estimate matrix B
6. decompose B into A, R, t
7. ...

How do we get homographies?

Camera Calibration: Zhang cont.



- assume a set of point correspondences for points on a plane
- find homography H such that

$$z \cdot (u_i, v_i, 1)^T \approx H \cdot (\xi_i, \eta_i, 1)^T$$

- one correspondence yields two constraints

$$\vec{h}_{1,1:3} \cdot (\xi_i, \eta_i, 1)^T - u_i \cdot \vec{h}_{3,1:3} \cdot (\xi_i, \eta_i, 1)^T \approx 0$$

$$\vec{h}_{2,1:3} \cdot (\xi_i, \eta_i, 1)^T - v_i \cdot \vec{h}_{3,1:3} \cdot (\xi_i, \eta_i, 1)^T \approx 0$$

- least squares to minimize residuals and find best homography H

Camera Calibration: Zhang cont.

Sketch: Zhang's approach

1. create a plane with calibration markers at known positions
2. make several pictures of it with different position and orientation of plane
3. measure 2d image position of markers
4. estimate homographies H for each picture
5. solve optimization problem to estimate matrix B
6. decompose B into A , R , t
7. optimize all parameters using nonlinear least squares

Finally, we get

- intrinsic parameters A
- rotation R and translation t for each plane

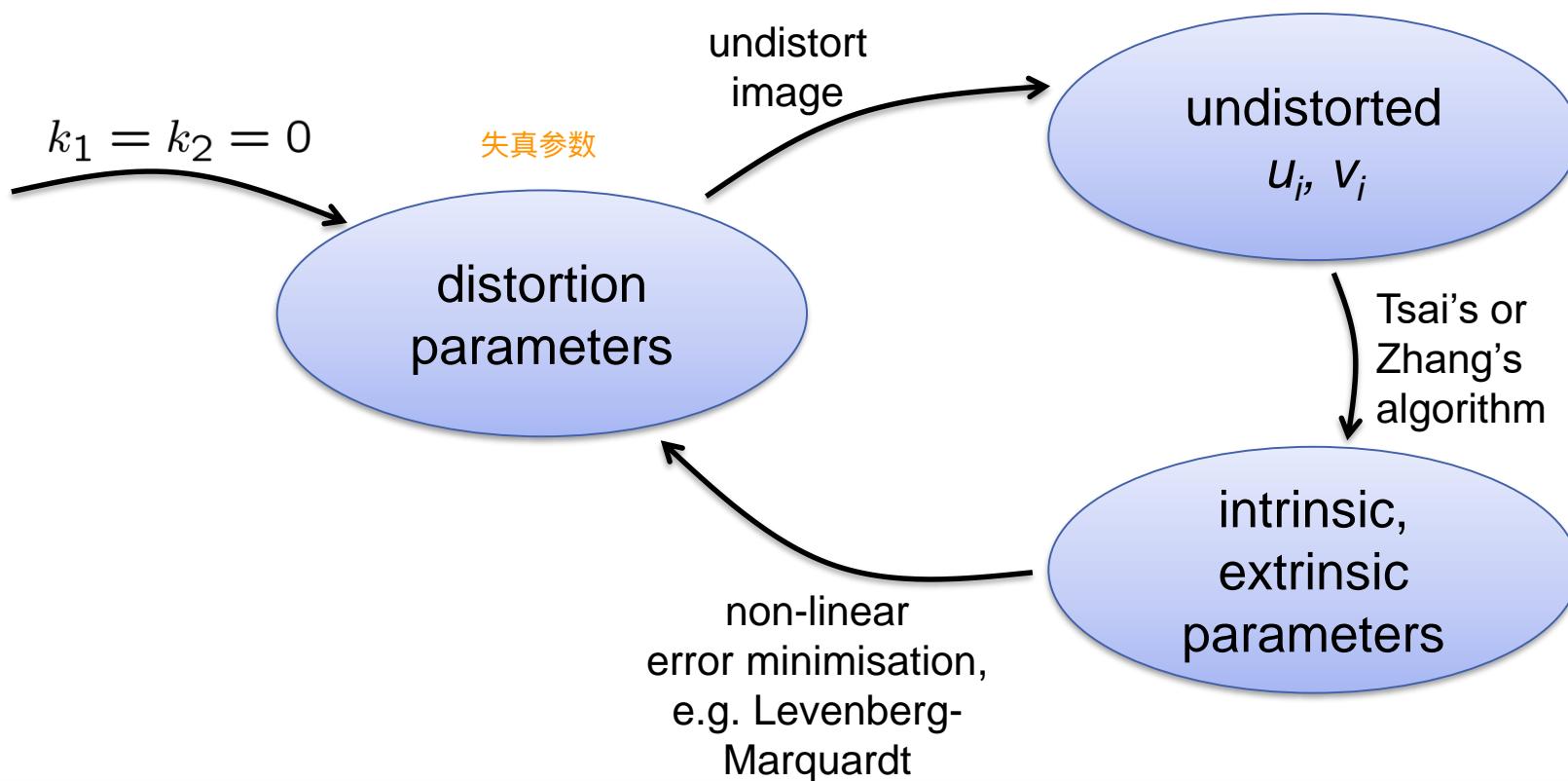
Z. Zhang,

A flexible new technique for camera calibration.

IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 22, no. 11,
pp. 1330-1334, 2000

Camera Calibration cont.

- calibrating distortion parameters k_1, k_2 :
 - non-linear optimization process
 - iterative estimation of intrinsic, extrinsic parameters and distortion parameters



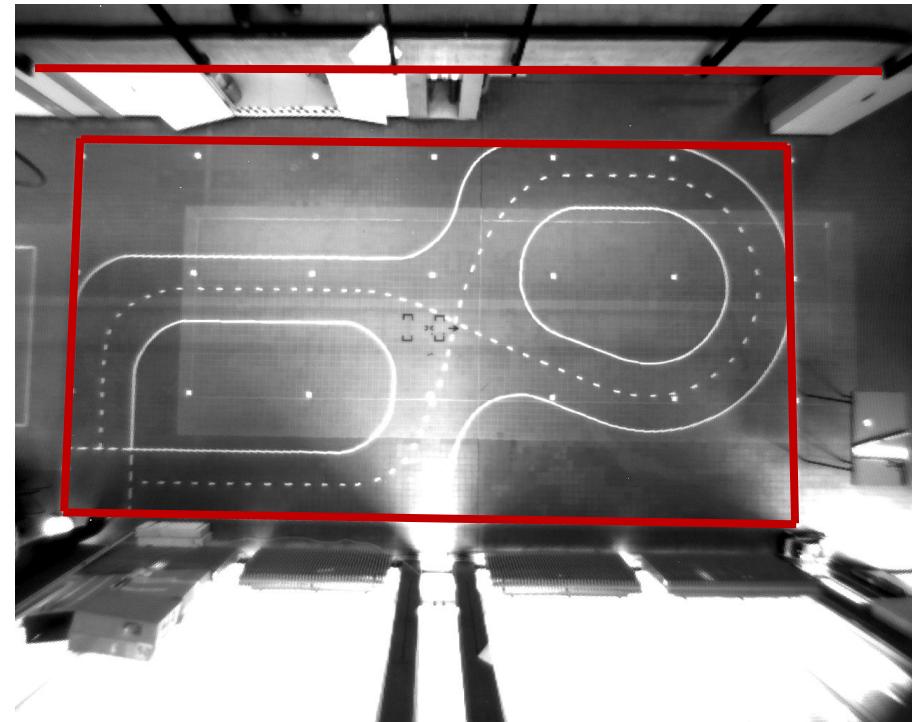
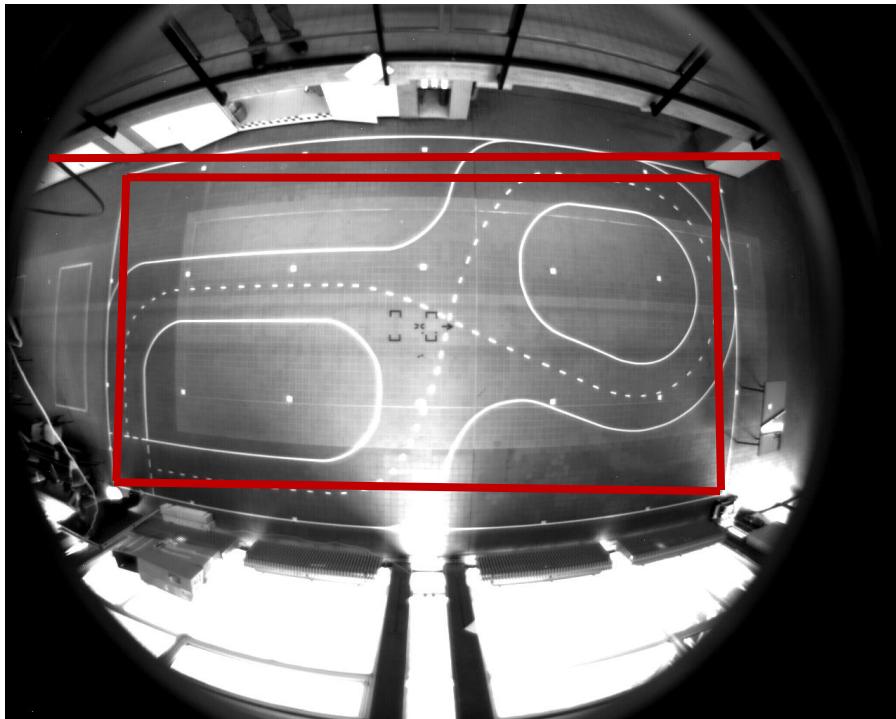
Camera Calibration cont.

- effect of undistortion:



Camera Calibration cont.

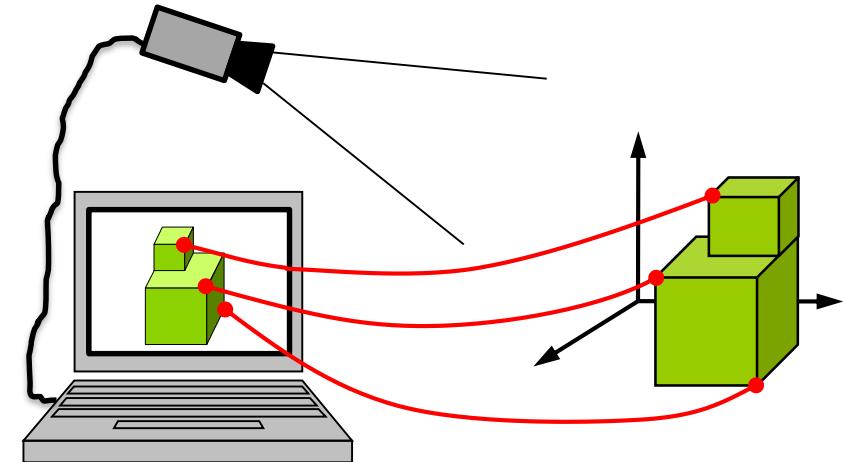
- example: camera with wide-angle lens



undistortion
(requires more complex
distortion model)

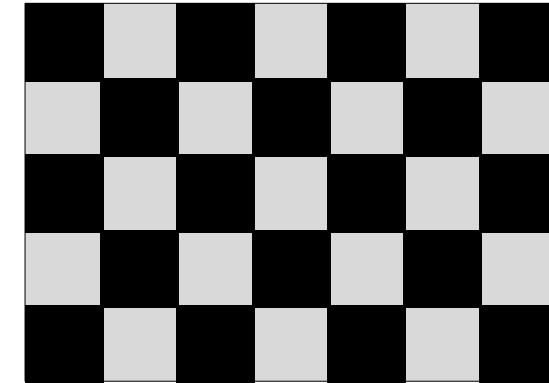
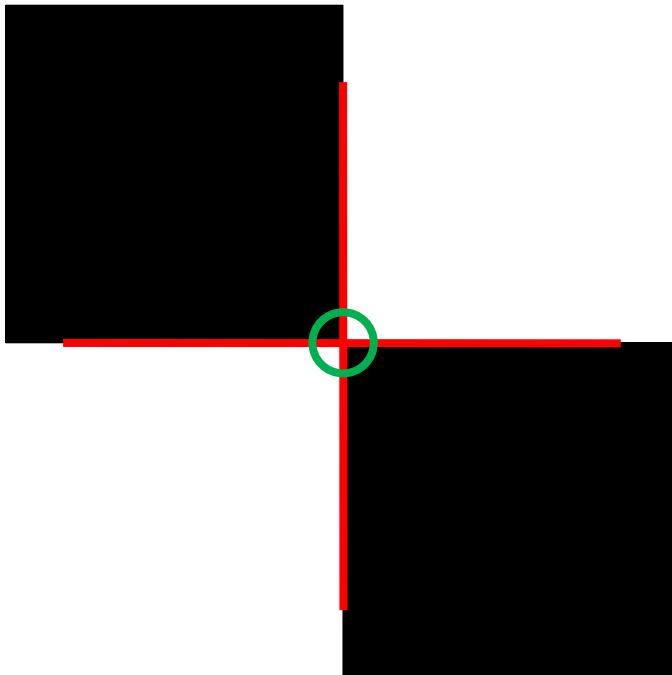
Camera Calibration cont.

- calibration markers
 - characteristic features
 - clearly recognizable
 - easy to determine position in world
 - easy to localize in image with high precision
 - features not coplanar in world (for Tsai's approach)
 - avoid occlusion
 - avoid shadow
 - as many as possible



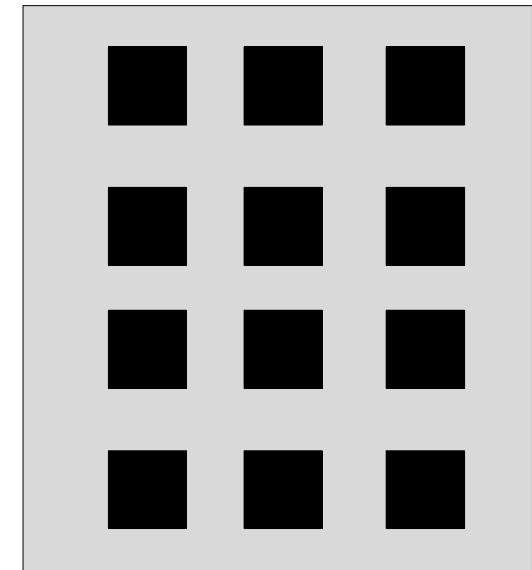
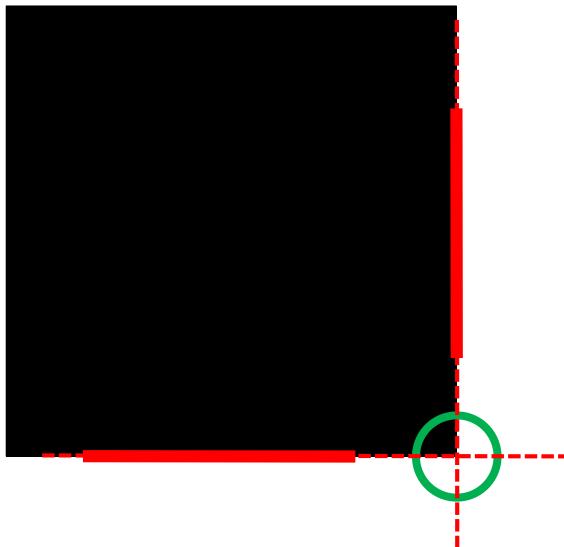
Camera Calibration cont.

- chessboard markers
 - determine image position calculating the point of intersection of horizontal and vertical edges



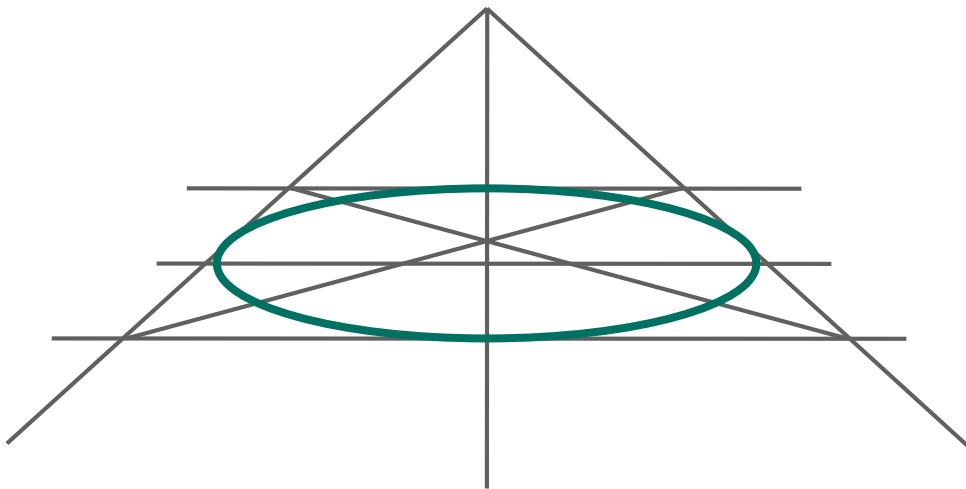
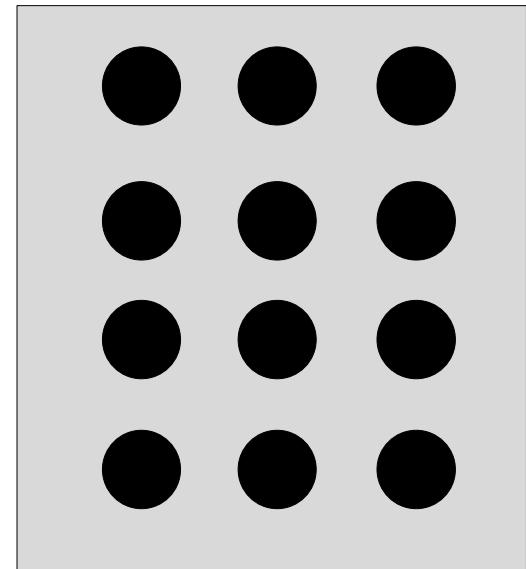
Camera Calibration cont.

- squares and rectangles
 - determine image position calculating the point of intersection of horizontal and vertical edges



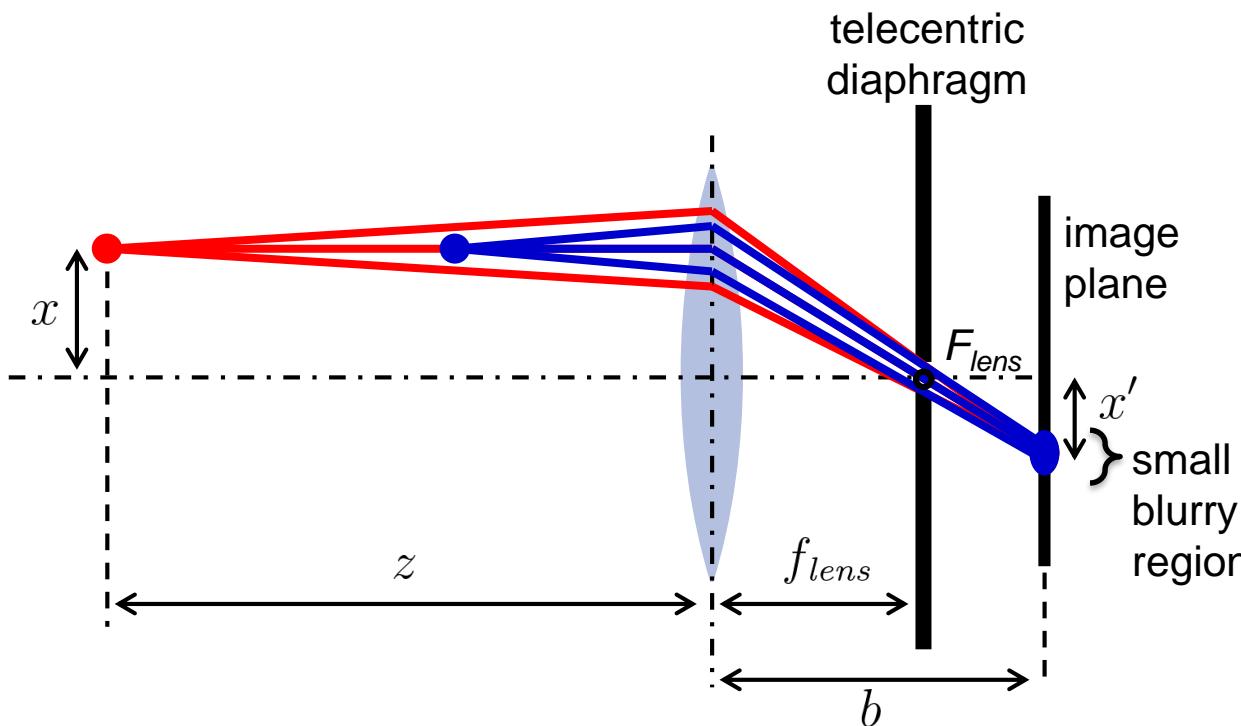
Camera Calibration cont.

- circles
 - determine center of resulting ellipse
 - iterative correction of mapping error of circle center



NON-STANDARD CAMERAS

Telecentric Lenses



magnification:

$$x' = \frac{b - f_{lens}}{f_{lens}} \cdot x$$

depth of field:

$$\Delta z = 2 \frac{\epsilon}{D} (z_0 - f_{lens})$$

pros: – magnification independent of object distance z

– improved depth of field

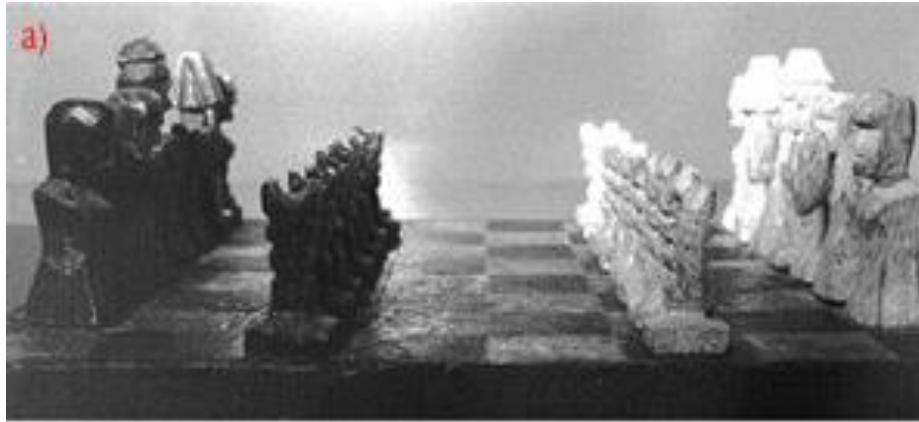
cons: – small aperture, lets poorly light through

– large, heavy, and expensive

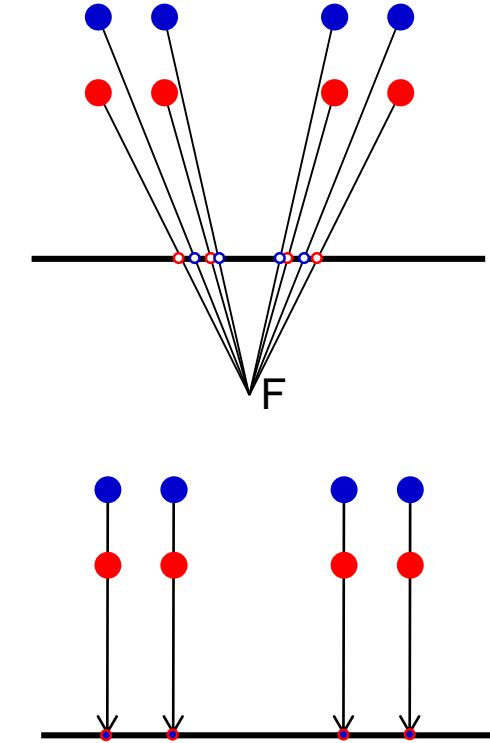
application area: – microscopy

Telecentric Lenses cont.

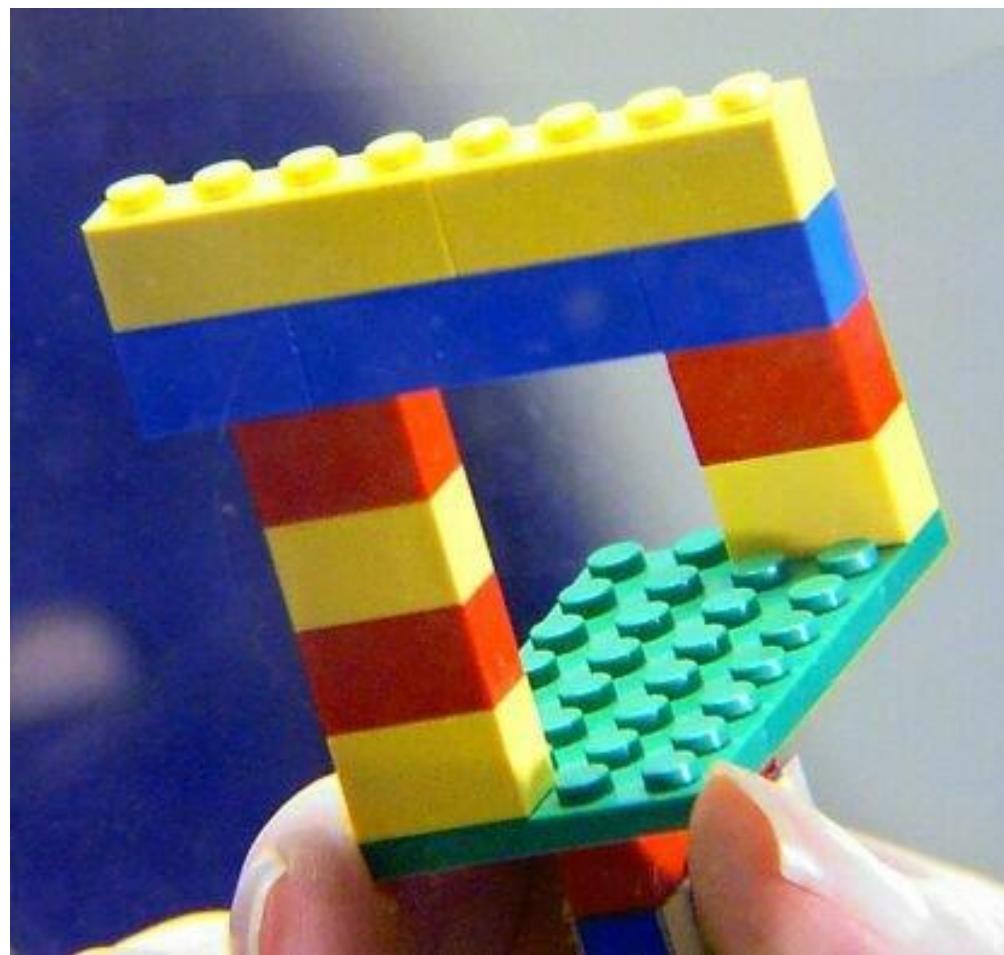
normal lens
(perspective projection)



telecentric lens
(orthographic projection)



Telecentric Lenses cont.

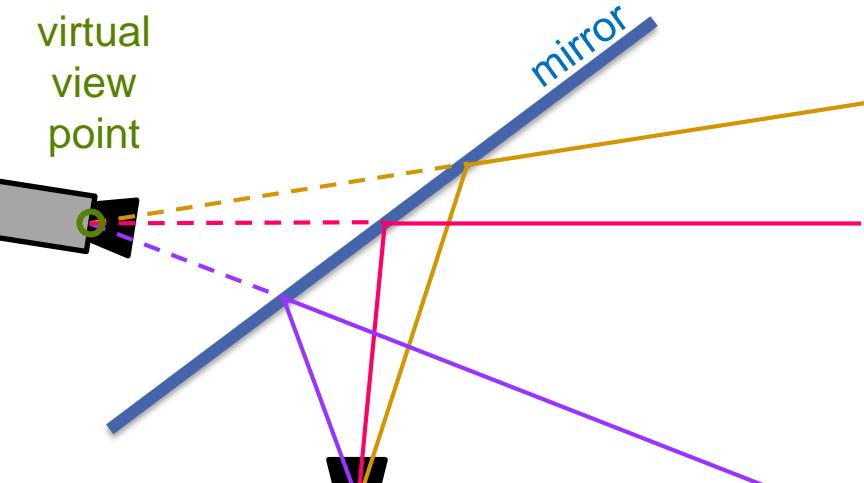


source: <http://www.lhup.edu/~dsimanek/3d/telecent.htm>

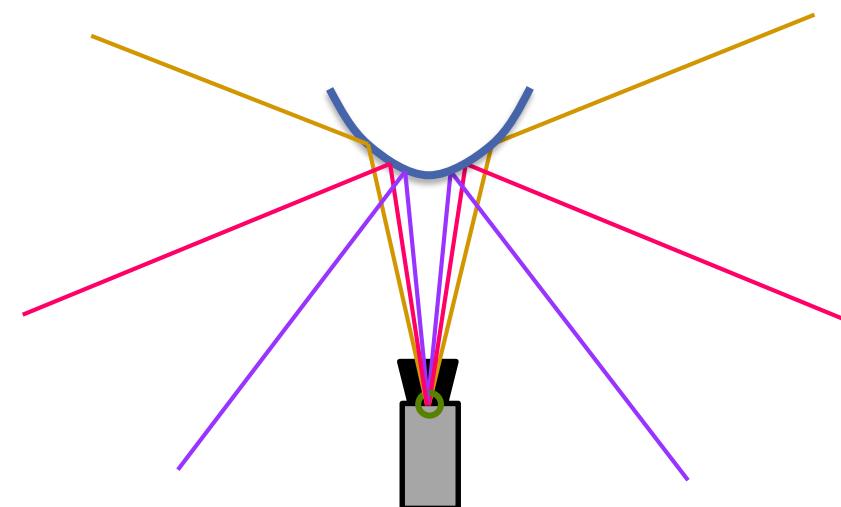
Catadioptric Cameras

- catadioptric cameras = cameras with mirrors
 - planar mirror

猫眼相机



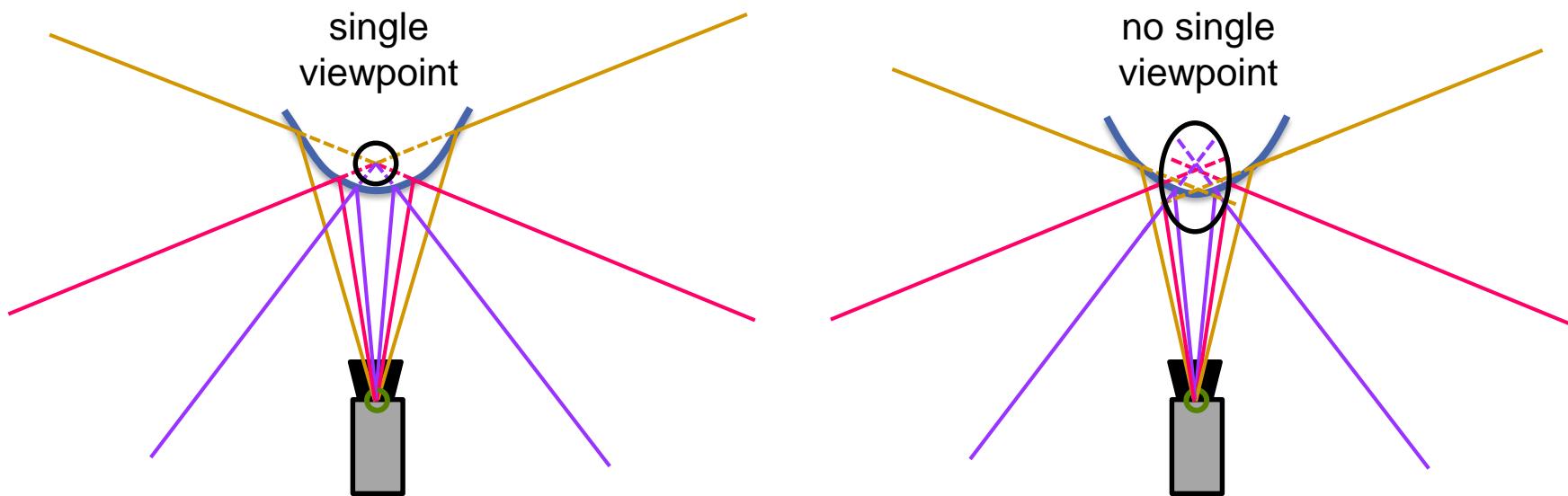
- curve-shaped mirrors



Catadioptric Cameras Cont.

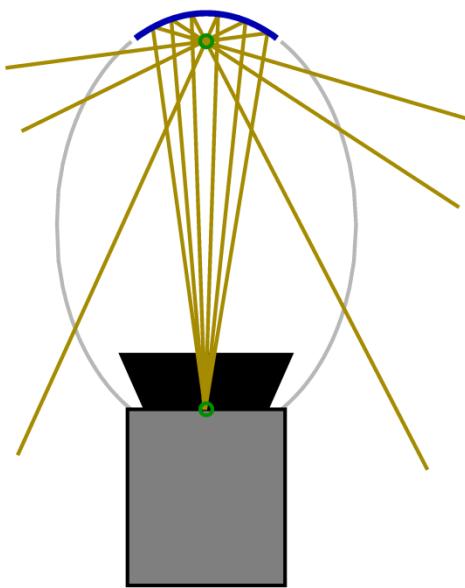
- *Single viewpoint*

A catadioptric camera has a single viewpoint if all object-mirror light rays intersect in a single point (e.g. if the mirror could be replaced by a pinhole camera)

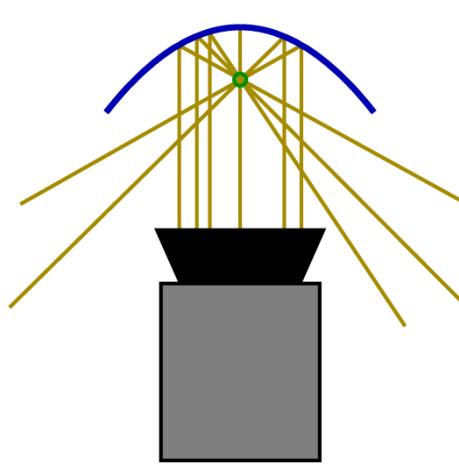


Catadioptric Cameras Cont.

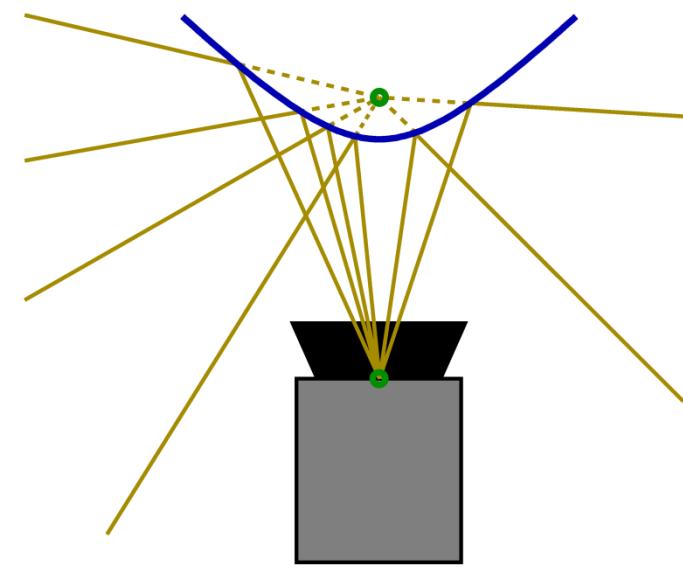
- camera setups with single viewpoint:



elliptic mirror+
camera with
standard lens

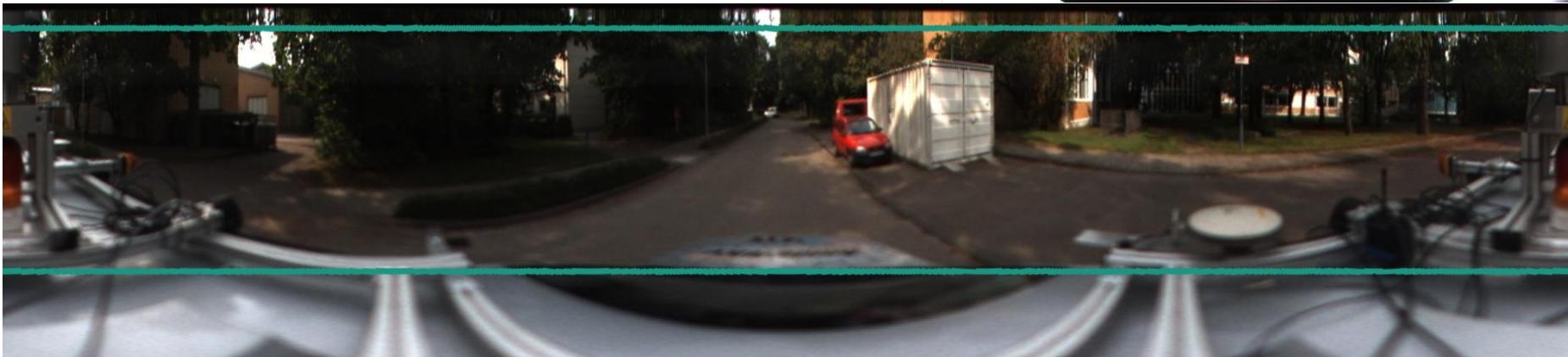
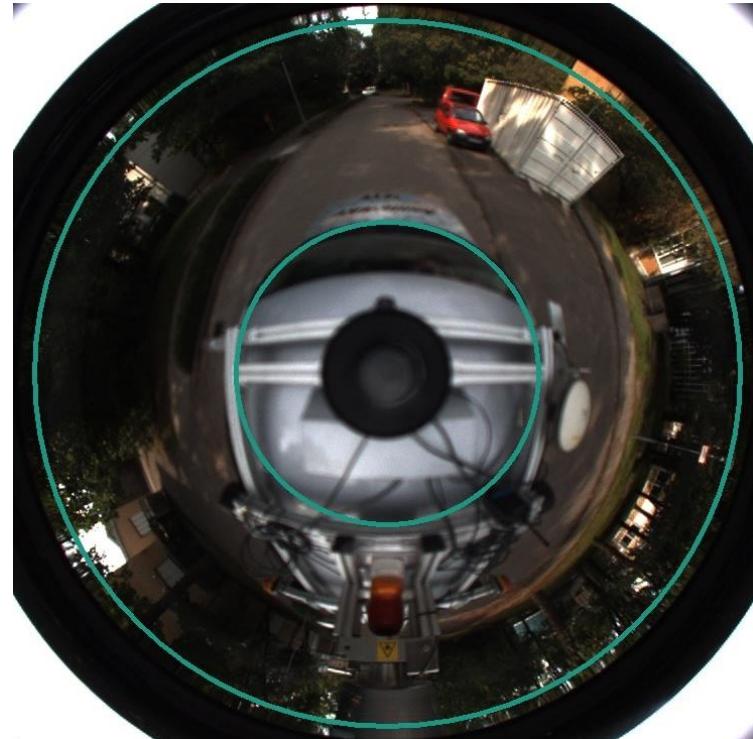
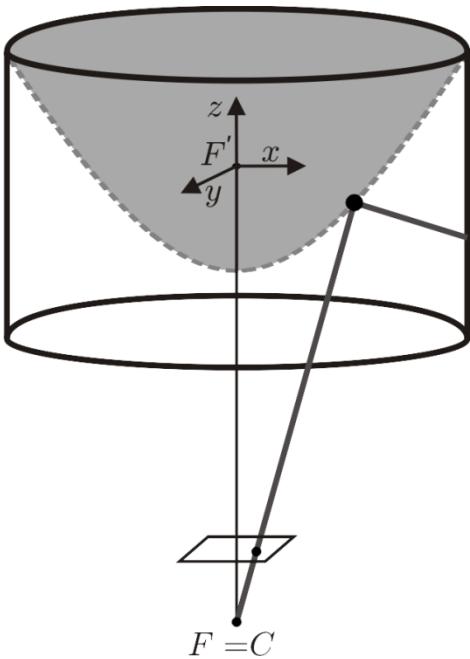


parabolic mirror+
camera with
telecentric lens



hyperbolic mirror+
camera with
standard lens

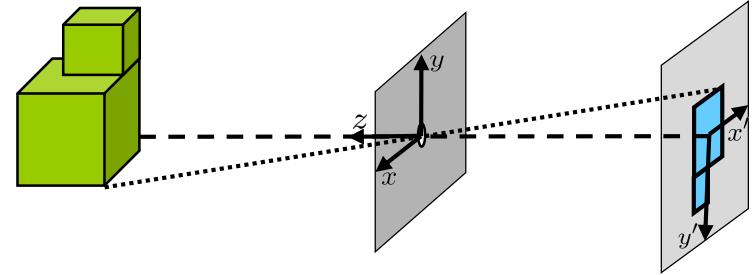
Catadioptric Cameras Cont.



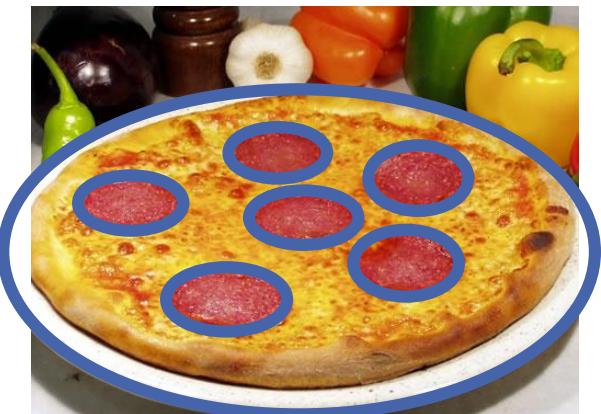
SUMMARY: CAMERA OPTICS

Summary

- **pinhole camera**
 - pinhole camera model
 - world-to-image-mapping
 - intrinsic/extrinsic camera parameters
 - properties of perspective projection
- **lenses**
- **camera calibration**
- **non-standard cameras**

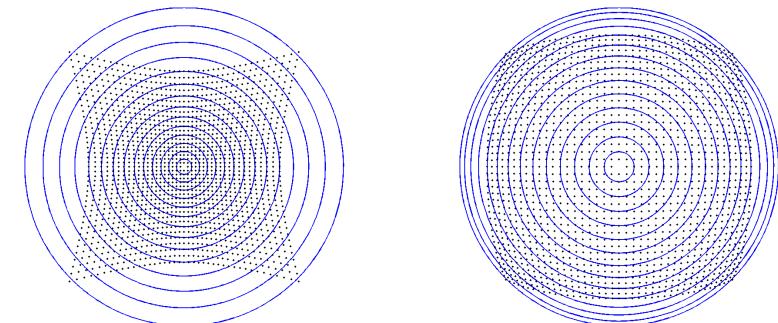
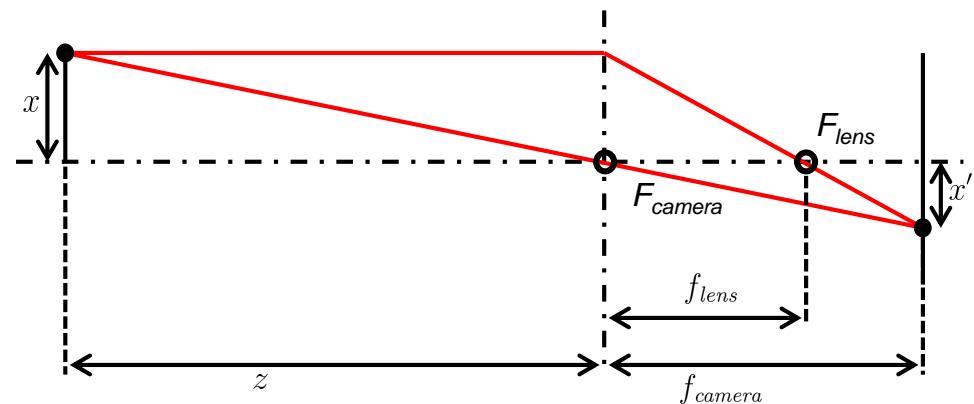


$$z \cdot \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = A \cdot \left(R \mid \vec{t} \right) \cdot \begin{pmatrix} \xi \\ \eta \\ \zeta \\ 1 \end{pmatrix}$$



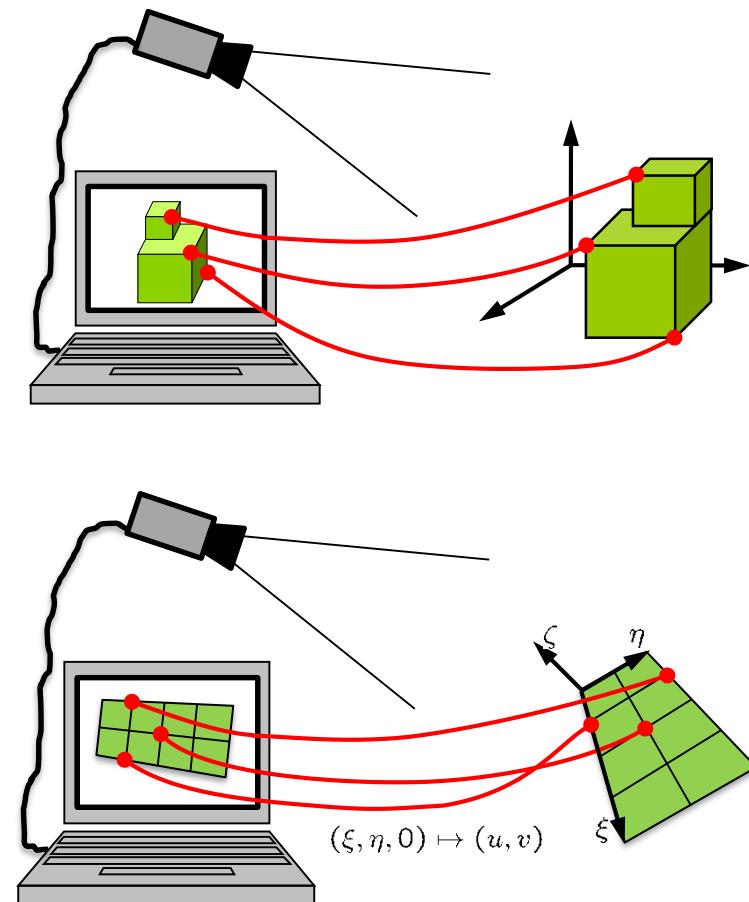
Summary cont.

- pinhole camera
- lenses
 - lens equation
 - depth of field
 - focus series
 - lens aberrations
 - radial distortion
- camera calibration
- non-standard cameras



Summary cont.

- pinhole camera
- lenses
- camera calibration
 - Tsai's camera calibration approach
 - Zhang's approach with homographies
 - calibration of distortion parameters
 - calibration markers and calibration objects
- non-standard cameras



Summary cont.

- pinhole camera
- lenses
- camera calibration
- non-standard cameras
 - telecentric lenses
 - catadioptric cameras

