Affix the authentification label over this field.

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Exam in "Machine Vision"

Date of exam: February 22, 2021
Time of exam: 14:00–15:00

Question 1 (6 points)

Let $f,g : \mathbb{R} \to \mathbb{R}$ be functions and $\alpha,\beta \in \mathbb{R}$. Prove the following property of the Fourier transform:

$$\mathcal{F}\{\alpha \cdot f(t) + \beta \cdot g(t)\} = \alpha \cdot \mathcal{F}\{f(t)\} + \beta \cdot \mathcal{F}\{g(t)\}$$

Note, that $\mathcal{F}\{h\}$ denotes the Fourier transform of a function h.

Question 2 (2+2 points)

Assume the two scenarios (a) and (b) described below. For each scenario name two possible techniques to improve the image quality. If you name more than two techniques, only the first two count for the answer.

(a) A photographer is standing at the coast and wants to make a picture of an island two kilometers away. However, on the pictures the island appears too small so that the details of the island cannot be recognized. What can the photographer do to get more details on the pictures?

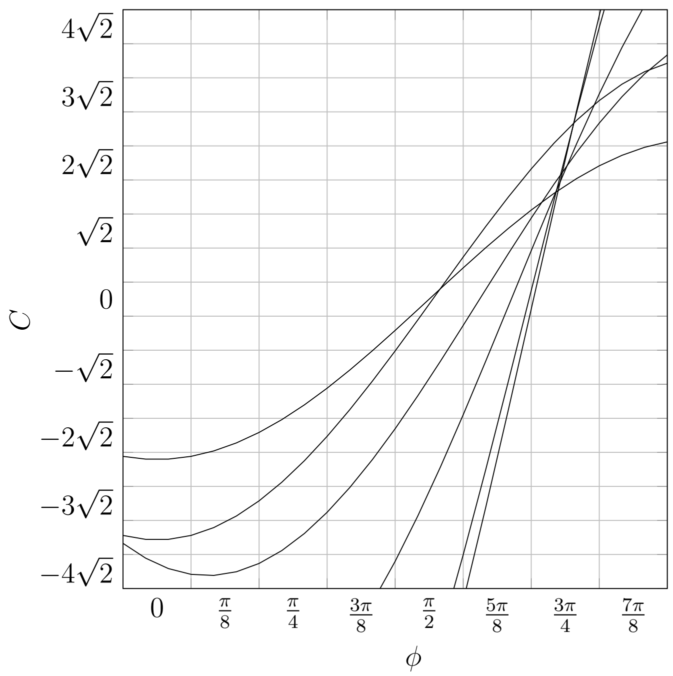
(b) In a machine that should sort letters according to their destination a camera makes pictures of the letters while they are passing by the camera with high speed. The distance between the letters and the camera is always the same and it is previously known. The problem is that the images are too dark. You are allowed to change the overall setup of the machine, to change the control parameters of the camera, and to change the lens of the camera. You are not allowed to replace the camera by another one. Mind that improving the image brightness should not lead to blurry images!

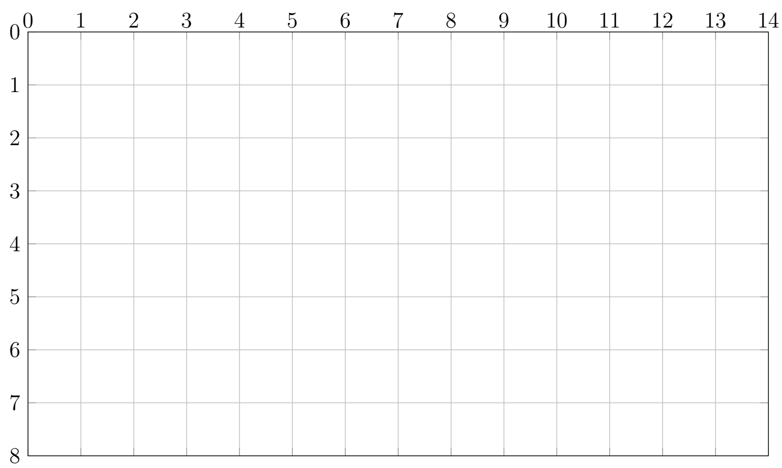
Question 3 (6 points)

Assume that six edge pixels have been extracted from a camera image. The Hough transform has been applied to each of the six pixels. The illustration below shows the Hough space with the resulting curves. The cells of the accumulator array are shown as gray boxes. Find the maximum in the accumulator array and mark it. Then, transform the maximum back into a line equations in normal form. Draw that line in the image below. Mind that the origin of the image is in its left upper corner, that an angle of 0 points to the right, and an angle of $\frac{\pi}{2}$ points downwards.

Remark:

θ	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$
$\sin(\theta)$	0	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	1	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$
$\cos(\theta)$	1	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	0	$-\frac{\sqrt{2-\sqrt{2}}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2+\sqrt{2}}}{2}$





Question 4 (4 points)

Assume we want to compare different colors which we represent in the HSV color space. Discuss briefly whether or not the Euclidean distance between two HSV triplets is a reasonable metric to measure the similarity of colors. The Euclidean distance between the two HSV triplets (h_1, s_1, v_1) and (h_2, s_2, v_2) is defined as

$$d_E((h_1, s_1, v_1), (h_2, s_2, v_2)) = \sqrt{(h_1 - h_2)^2 + (s_1 - s_2)^2 + (v_1 - v_2)^2}$$

Question 5 (8 points)

In this task we want to apply the k-means algorithm to segment a graylevel image into foreground and background by choosing $k = 2$. The algorithm is initialized with two prototype graylevels. Then, all pixels are assigned to the two segments and the new prototype graylevels are recalculated. These two steps are repeated until convergence.
The code below shows a Matlab implementation of the k-means algorithm with $k = 2$. The function takes as inputs a graylevel image I and two prototype graylevels inside the vector c. It returns the fully segmented image J in which each pixel takes the graylevel of the respective prototype.
Unfortunately, the code contains four logical errors. Find those errors and replace them by proper code.
You get one point for finding each error and one point for fixing it. Adding errors to the code or marking a proper part as erroneous will lead to minus points. The whole task will be awarded with at least zero points.

```
function [ J ] = k_means( I, c )
   while ( true )
     I_seg = ( I + c( 1 )) .^ 2 < ( I - c( 2 )) .^ 2;
     c_update( 1 ) = sum( sum( I .* I_seg )) / sum( sum( I_seg ));
     c_update( 2 ) = sum( sum( I * ( 1 - I_seg ))) / sum( sum( 1 - I_seg ));

     if ( c = c_update )
       J = c( 1 ) * I_seg + c( 2 ) * ( 1 - I_seg );
     else
       c = c_update;
     end
   end
end
```

Question 6 (7 points)

Assume a camera with thin lens which is described by its matrix of intrisic parameters

$$A = \begin{pmatrix} 500 & 0 & 400 \\ 0 & 500 & 300 \\ 0 & 0 & 1 \end{pmatrix}$$

We know that the optical focal length of the lens is $f_{lens} = 4.5$ mm and that the spacing of two neighboring pixels on the imager is 0.01 mm. Calculate the distance in front of the camera at which objects are mapped in a perfectly sharp way to the image plane.

Remark: Mind that the focal length of the lens f_{lens} is different from the focal length of the camera f_{camera}. Use millimeters as the unit for the focal lengths f_{lens} and f_{camera} and pixel per millimeter as the unit for the scaling factors α and β.

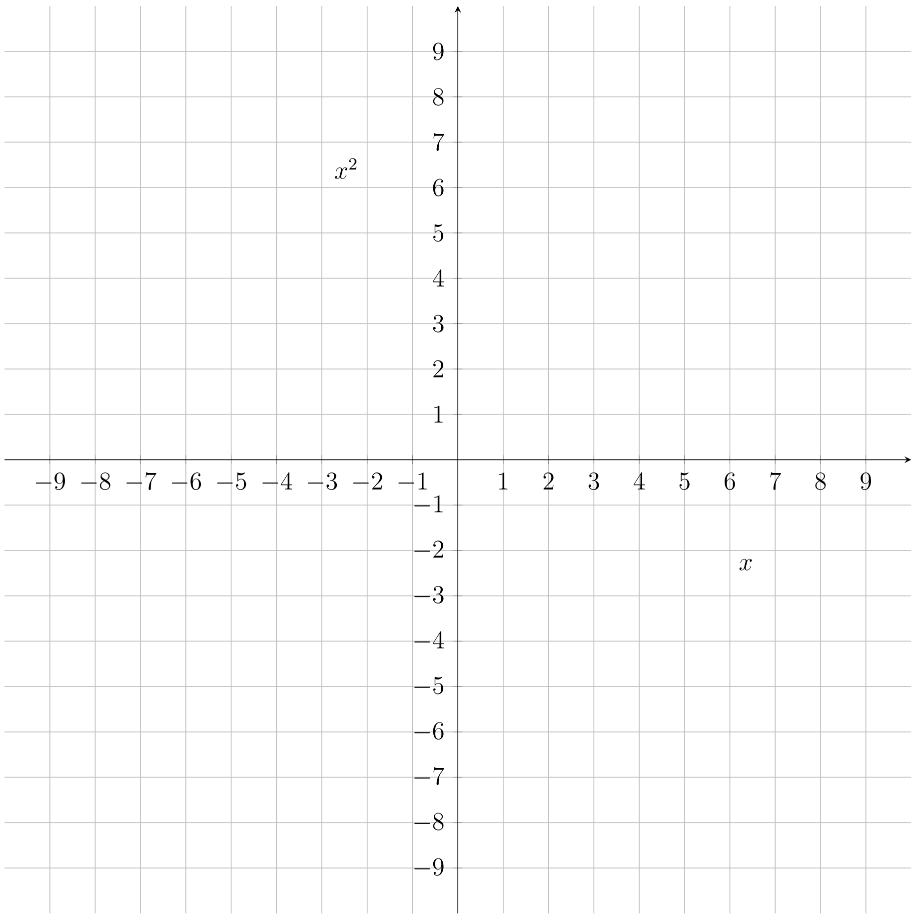
Question 7 (5 + 2 points)

Assume a binary classification problem with the six one-dimensional patterns shown in the table below.

(a) Apply the transform $\Phi_1(x) = \begin{pmatrix} x \\ x^2 \end{pmatrix}$ to each pattern and add the result in the diagram below. Are the patterns linearly separable after the non-linear transform Φ_1? Justify your answer briefly.

(b) Check whether the problem is linearly separable using the non-linear transformation $\Phi_2(x) = \begin{pmatrix} x^3 \\ x^2 \end{pmatrix}$ rather than Φ_1. Justify your answer briefly.

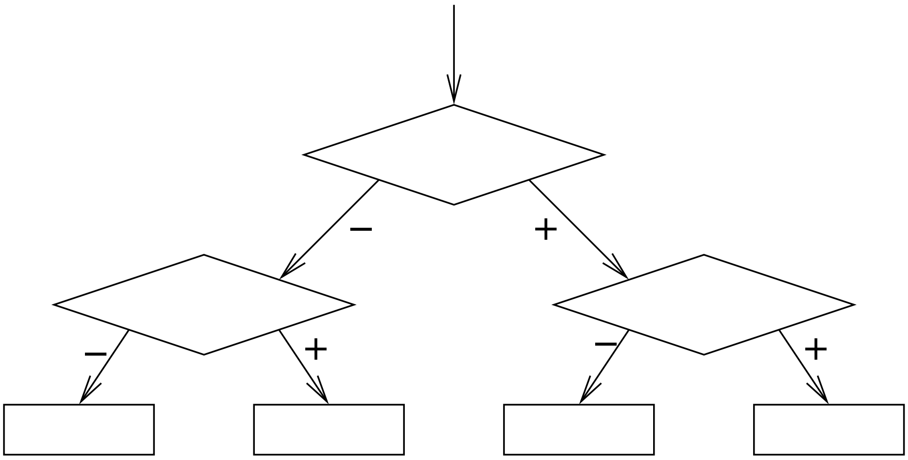
pattern No.	class label $d^{(i)}$	pattern $x^{(i)}$
1	$+1$	-3
2	-1	-1
3	-1	0
4	-1	1
5	$+1$	2
6	$+1$	3



Question 8 (10 points)

For a binary classification problem we want to train a decision tree with three internal nodes and four leaf nodes as depicted below. We have 16 training examples $(\vec{x}^{(i)}, d^{(i)})$ and can choose among five possible binary classifiers $c_1, \ldots, c_5$. The true class label $d^{(i)}$ of each training example $\vec{x}^{(i)}$ and the output of each of the five classifiers are listed in the table below.

Select the best classifier for each internal node and provide the class labels to the leaf nodes according to the training algorithm introduced in the lecture. You may enter your solution in the illustration of the decision tree.



example	label $d^{(i)}$	output of classifiers c_1	c_2	c_3	c_4	c_5
$\vec{x}^{(1)}$	+1	+1	−1	+1	−1	+1
$\vec{x}^{(2)}$	+1	+1	−1	+1	−1	−1
$\vec{x}^{(3)}$	+1	+1	+1	−1	−1	+1
$\vec{x}^{(4)}$	+1	+1	+1	−1	−1	+1
$\vec{x}^{(5)}$	+1	−1	+1	+1	−1	+1
$\vec{x}^{(6)}$	+1	−1	+1	−1	+1	+1
$\vec{x}^{(7)}$	+1	−1	+1	+1	+1	+1
$\vec{x}^{(8)}$	+1	−1	+1	+1	+1	+1
$\vec{x}^{(9)}$	−1	+1	−1	−1	+1	+1
$\vec{x}^{(10)}$	−1	+1	−1	−1	+1	+1
$\vec{x}^{(11)}$	−1	+1	−1	−1	+1	−1
$\vec{x}^{(12)}$	−1	+1	−1	−1	−1	−1
$\vec{x}^{(13)}$	−1	−1	+1	+1	−1	+1
$\vec{x}^{(14)}$	−1	−1	+1	+1	+1	−1
$\vec{x}^{(15)}$	−1	−1	−1	−1	+1	+1
$\vec{x}^{(16)}$	−1	−1	−1	−1	+1	+1

Question 9 (8 points)

Create a multi layer perceptron (MLP) that implements the function

$$x \mapsto \begin{cases} 0 & \text{if } x \leq -4 \\ x + 4 & \text{if } -4 < x \leq 0 \\ 2 \cdot x + 4 & \text{if } 0 < x \end{cases}$$

The MLP should be composed out of at most four perceptrons. For each perceptron provide the activation function and the weights. Don't forget the bias weight. Draw the structure of the MLP.