

Machine Vision

Chapter 2: Image Preprocessing

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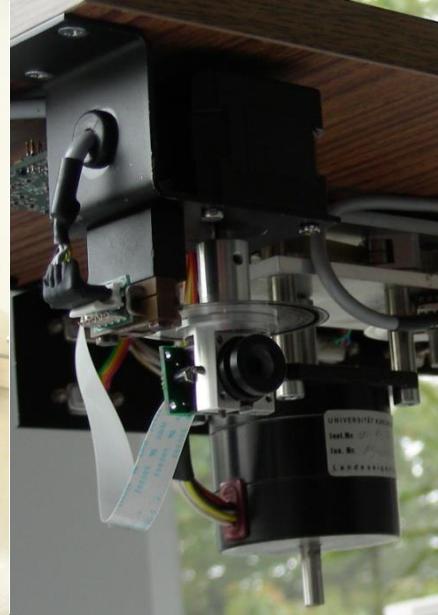
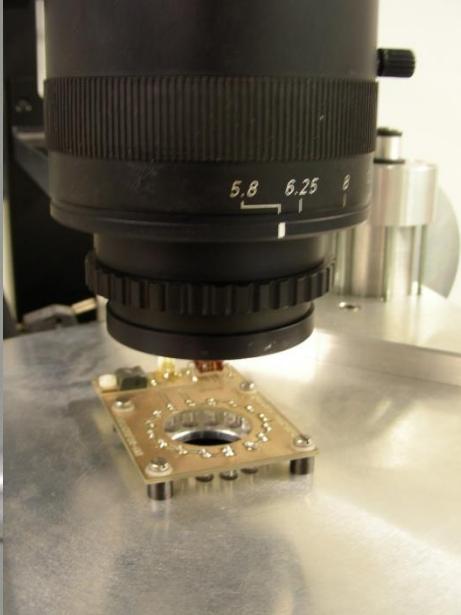
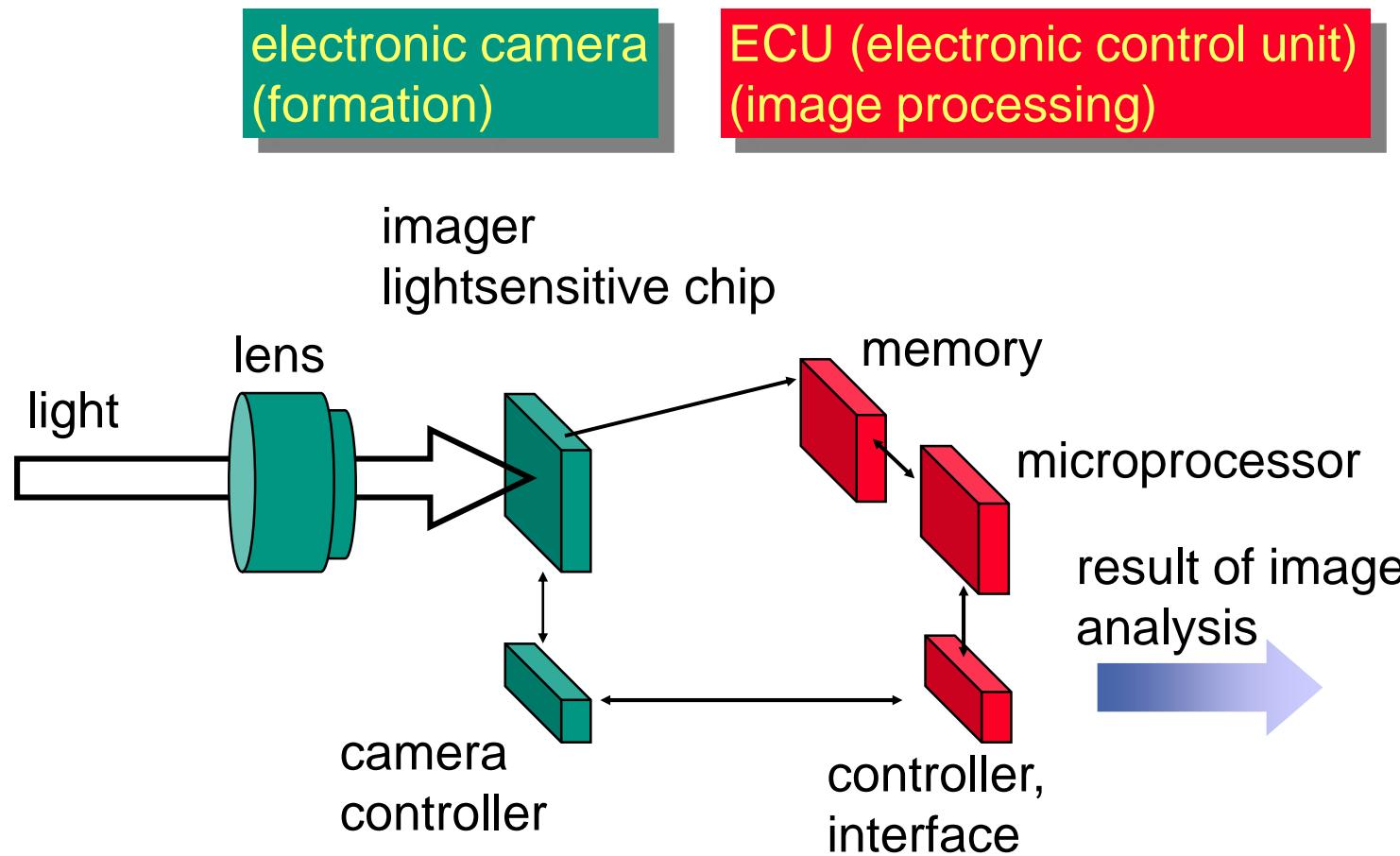


Image Formation and Analysis



Imager

- Process of image formation:

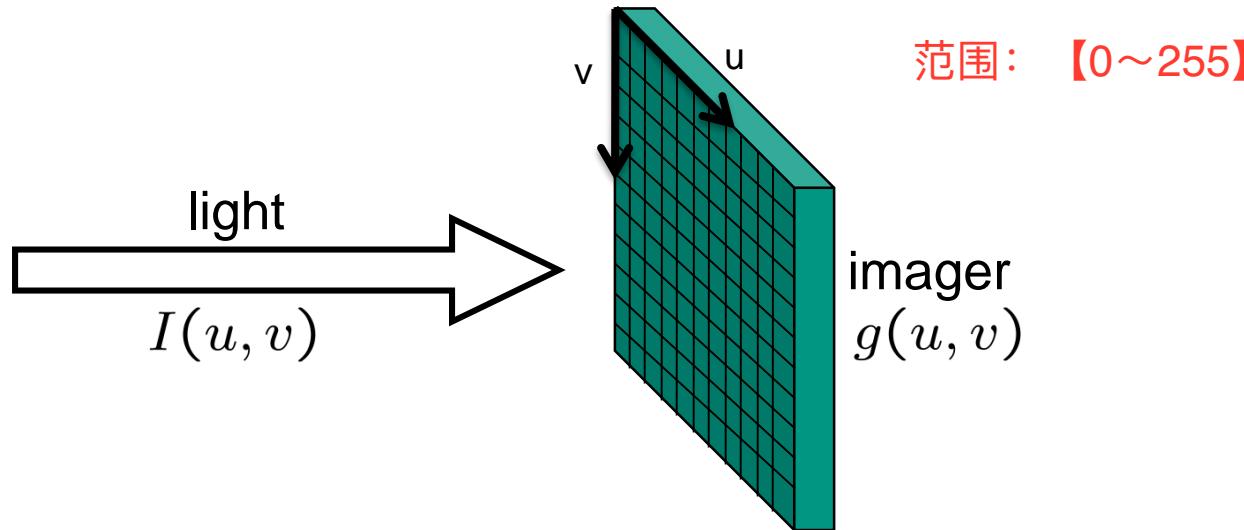
- incident light intensity:

$$I : \mathbb{R}^2 \rightarrow \mathbb{R}$$

- output of imager:

$$g : \{0, \dots, w - 1\} \times \{0, \dots, h - 1\} \rightarrow \{0, \dots, g_{max}\}$$

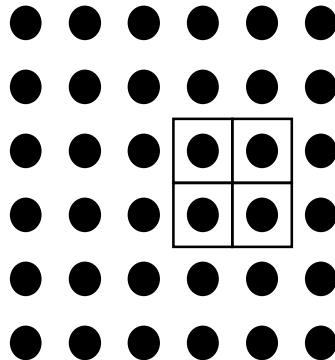
w, h : image width, height



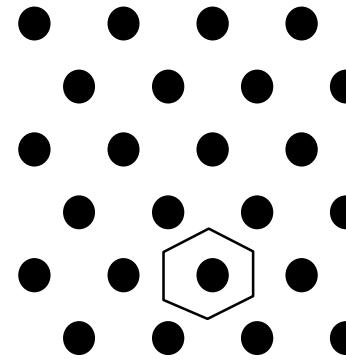
- Process of image formation:
 - **sampling**
evaluate light intensity on a regular grid of points
 - **quantization**
map continuous signals to discrete values (natural numbers)
 - **blur and noise**
 - **color**
will be discussed later. Here: only light intensity/grey level images

Sampling

- 2D grids used for sampling



rectangular



hexagonal

六边形取样hexagonal名字记住

- electronic cameras: rectangular, equidistant grids
- biology: hexagonal grids with varying resolution

Sampling: Moiré Patterns

- Moiré patterns
 - sampling might cause artifacts

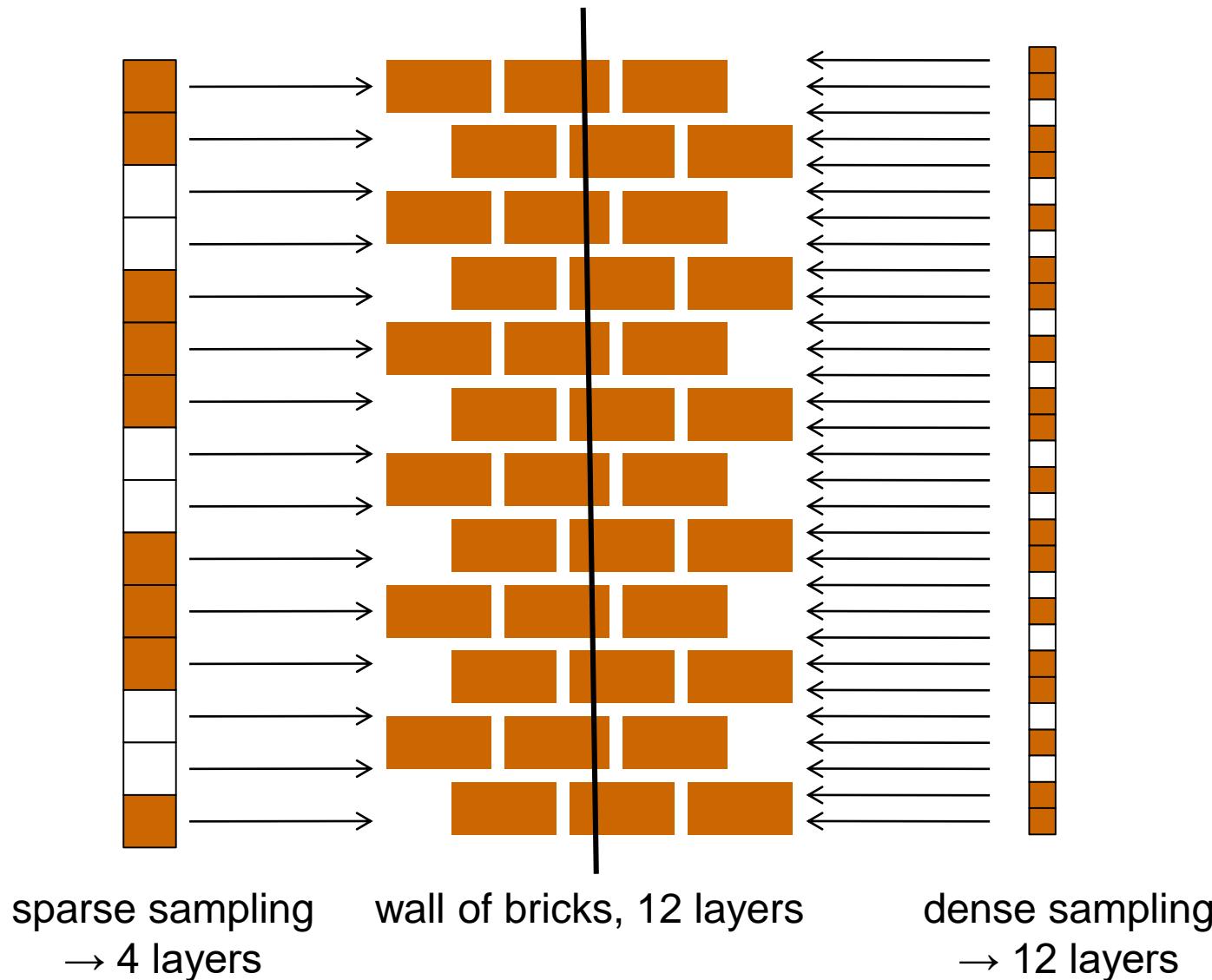


original picture



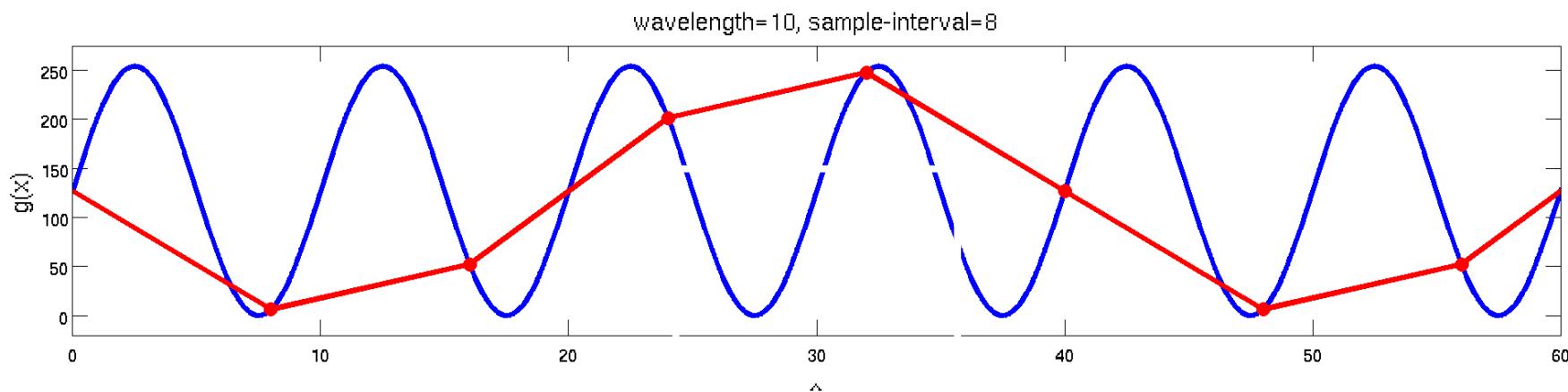
picture with Moiré pattern

Sampling: Moiré Patterns



Sampling: Moiré Patterns cont.

- 1D-example of Moiré patterns:



莫尔纹产生原因

The occurrence of Moiré patterns
depends on the sampling rate
compared to the maximal
frequency of the signal (image)

Nyquist-Shannon Sampling Theorem

经典的nyquist-shannon采样定理

If f is band bounded signal with cutoff frequency k_0
then it is completely determined by giving its ordinates at a series of
points spaced at most $\frac{1}{2k_0}$, i.e. the sample frequency must be
larger than $2k_0$

The cutoff frequency is the frequency at which a filter or system starts to significantly attenuate (reduce) the amplitude of the signal.

This term is typically used in the context of filters, such as low-pass, high-pass, band-pass, or band-stop filters.

- Questions:

- what is a band-bounded signal?
- what is a cutoff frequency?

A band-bounded signal (or bandlimited signal) is a signal whose frequency components

are confined within a specific finite range or band of frequencies.

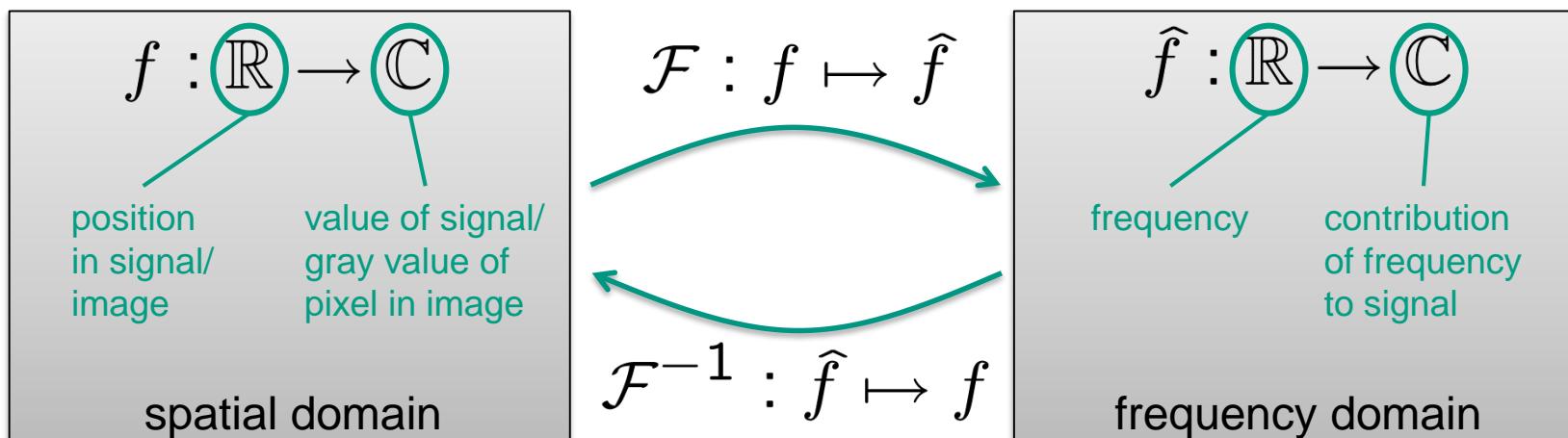
In other words, it has non-zero frequency components only within a certain range of frequencies,
while outside this range, the signal's frequency content is zero.

Fourier Transform

- Assume a periodic signal

$$f : \mathbb{R} \rightarrow \mathbb{C}$$

- Then, we can define the Fourier transform of f



$$\hat{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx$$

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(k) e^{2\pi i k x} dk$$

Fourier Transform

- Properties:

- the Fourier transform is **linear**

$$\mathcal{F}\{\alpha f(x) + \beta g(x)\}(k) = \alpha \hat{f}(k) + \beta \hat{g}(k)$$

- **shifting** a signal along the x-axis only changes the complex angles in frequency domain but not the amplitudes

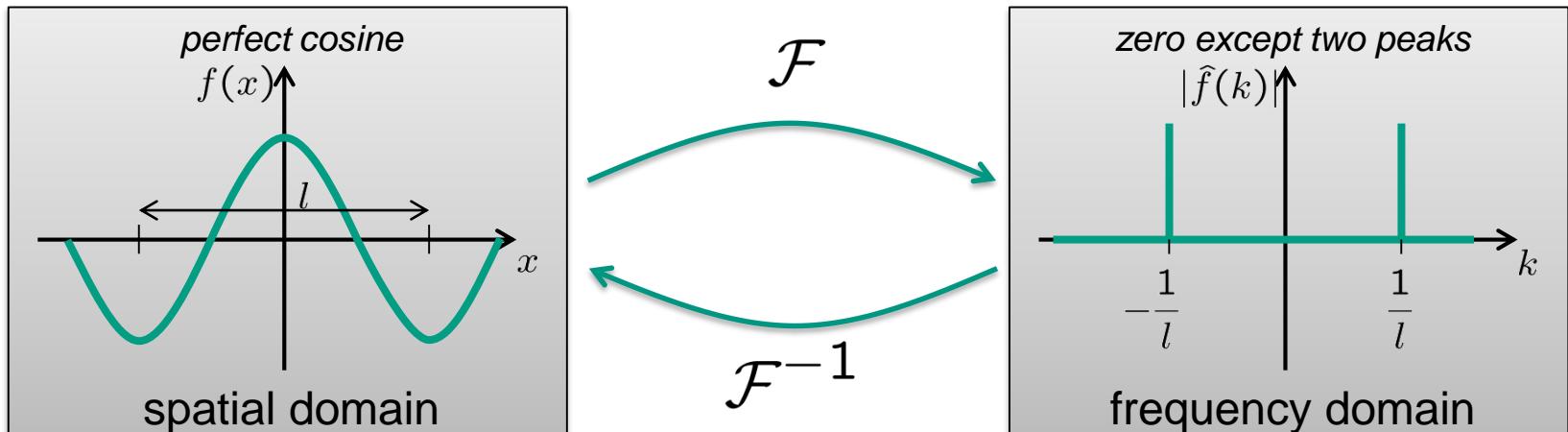
$$\mathcal{F}\{f(x - \xi)\}(k) = e^{-2\pi i \xi k} \hat{f}(k)$$

- **rescaling** the x-axis in the spatial domain rescales the frequency axis in a reciprocal way

$$\mathcal{F}\{f(\alpha x)\}(k) = \frac{1}{|\alpha|} \hat{f}\left(\frac{k}{\alpha}\right)$$

Fourier Transform

- Properties:
 - a cosine in spatial domain generates two peaks in frequency domain



- the peaks are located at position reciprocal to the period length
峰值的位置与周期长度成反比
- if the signal in spatial domain is a linear combination of cosines, the Fourier transform will be a set of peaks in frequency domain
如果空间域中的信号是余弦的线性组合, 傅立叶变换将在频域中呈现一组峰值
- intuitive interpretation: the Fourier transform decomposes a periodic signal into a (potentially infinite) linear combination of cosines
傅立叶变换将周期信号分解为 (可能是无限的) 余弦线性组合

Fourier Transform

- Observation
 - smooth periodic functions with small slope can be composed out of cosines with large period
 - periodic functions with large slope require cosines with small period
 - periodic functions that are discontinuous or have discontinuous derivatives require cosines with unbounded frequencies
- Definition

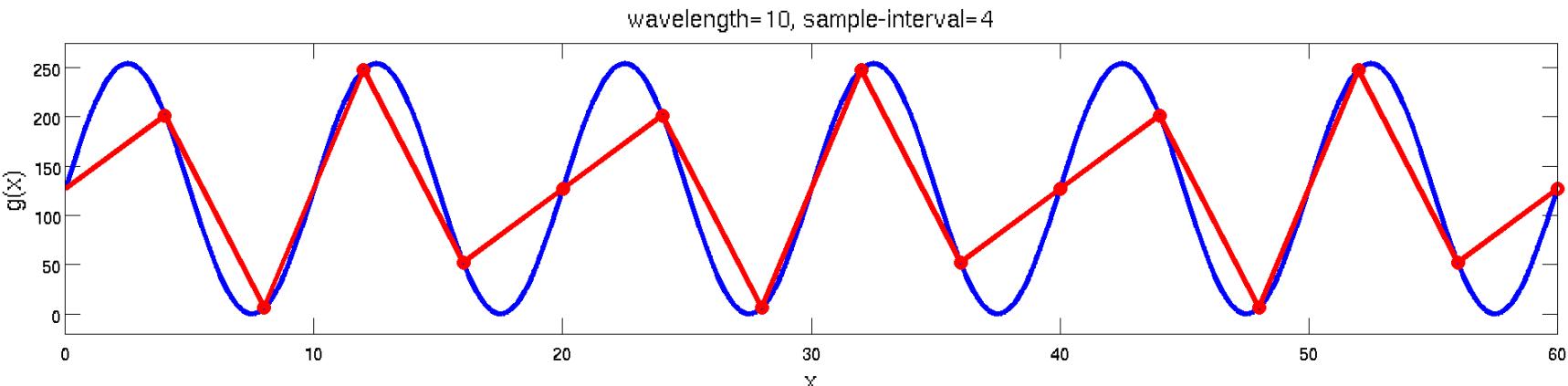
A signal f is band bounded with cutoff frequency k_0 if its Fourier transform is zero for all frequencies larger than the cutoff frequency, i.e.

$$\hat{f}(k) = 0 \text{ for all } k \text{ with } |k| \geq k_0$$

Nyquist-Shannon Sampling Theorem revisited

If f is band bounded signal with cutoff frequency k_0
then it is completely determined by giving its ordinates at a series of
points spaced at most $\frac{1}{2k_0}$, i.e. the sample frequency must be
larger than $2k_0$

Nyquist-Shannon Sampling Theorem



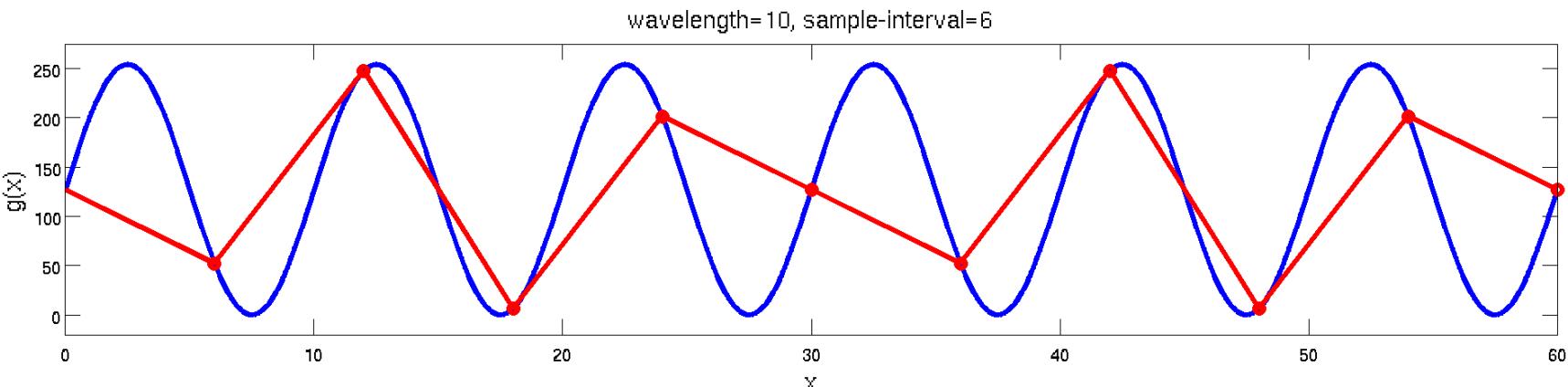
- signal is band bounded (sine function)
- sampling frequency high enough

$$f_{sample} = \frac{1}{4} > 2f_{signal} = \frac{2}{10}$$

- reconstruction of the signal possible

If f is band bounded signal with cutoff frequency k_0
then it is completely determined by giving its ordinates at a series of
points spaced at most $\frac{1}{2k_0}$, i.e. the sample frequency must be
larger than $2k_0$

Nyquist-Shannon Sampling Theorem



- signal is band bounded (sine function)
- but

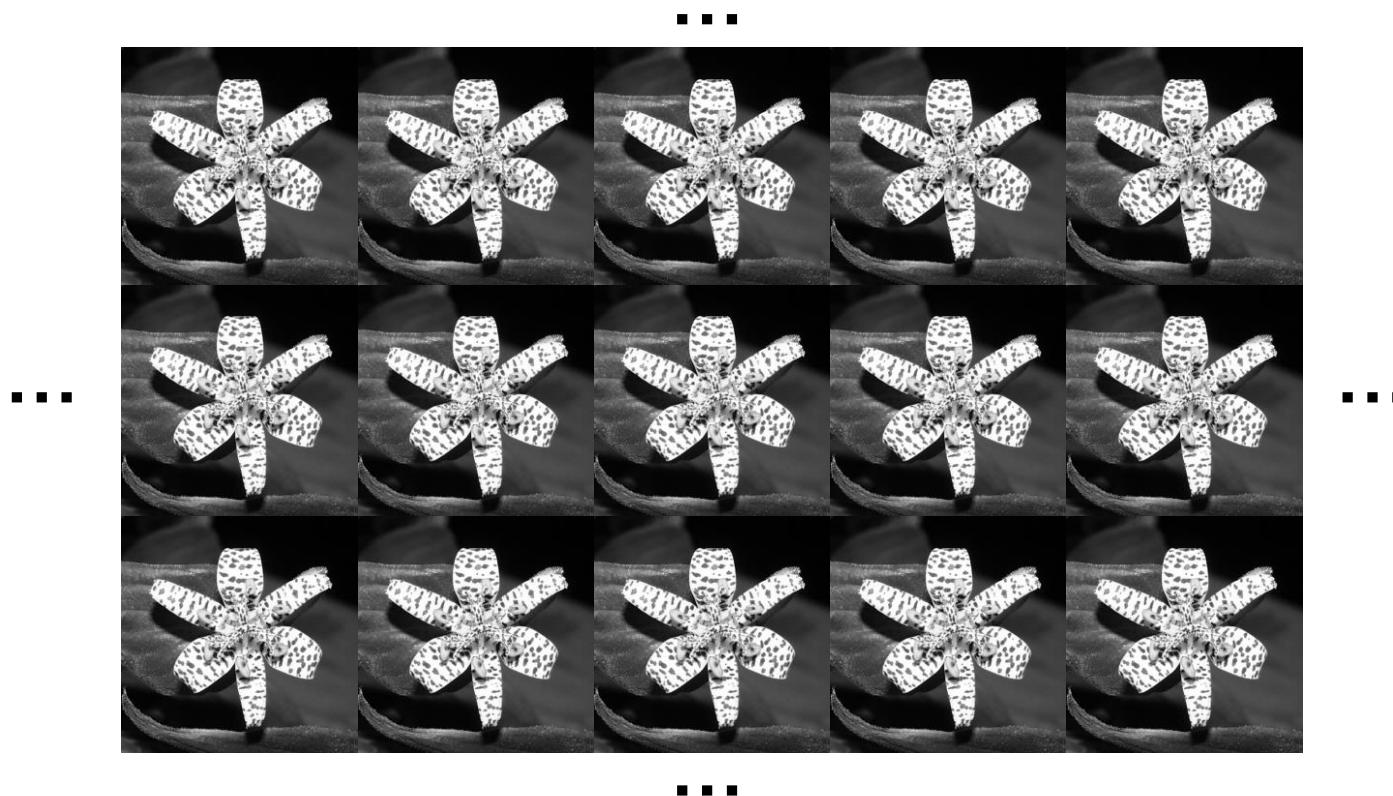
$$f_{sample} = \frac{1}{6} < 2f_{signal} = \frac{2}{10}$$

- reconstruction of the signal impossible

If f is band bounded signal with cutoff frequency k_0
then it is completely determined by giving its ordinates at a series of
points spaced at most $\frac{1}{2k_0}$, i.e. the sample frequency must be
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Sampling Theorem and Images

- Remarks:
 - analysis analogously possible for 2d signals
 - image is not periodic, but we can make it periodic by copying it repeatedly to the left, right, top, and bottom

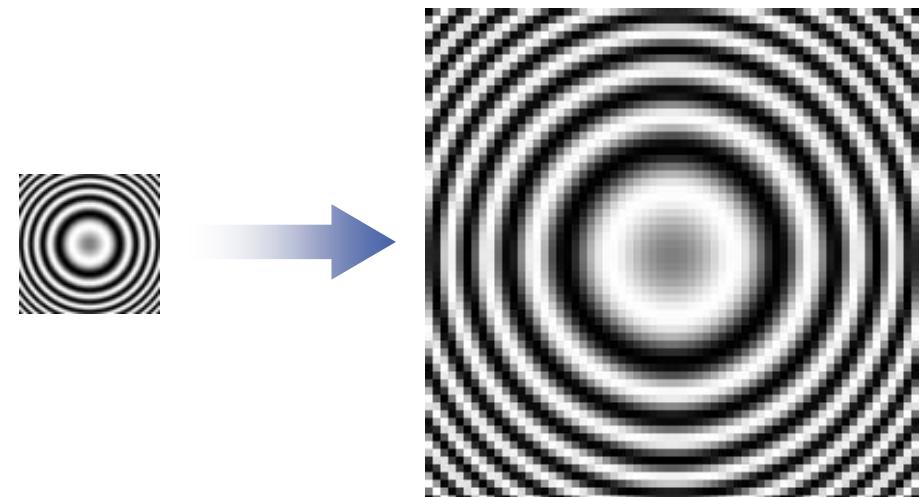
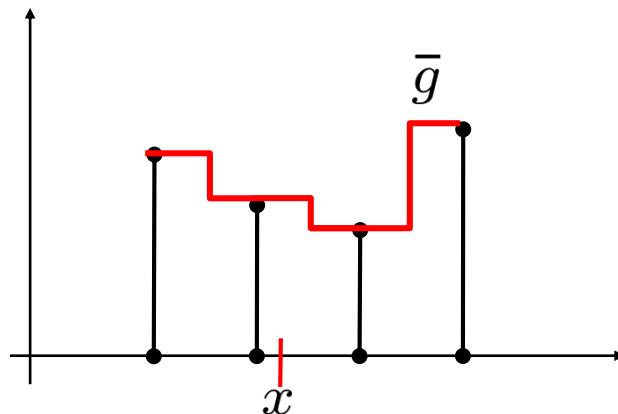


Sampling Theorem and Images

- Questions:
 - how can we determine the sampling frequency of a camera?
 - what can we do if we find that the sampling theorem is violated?

Image Scaling and Interpolation

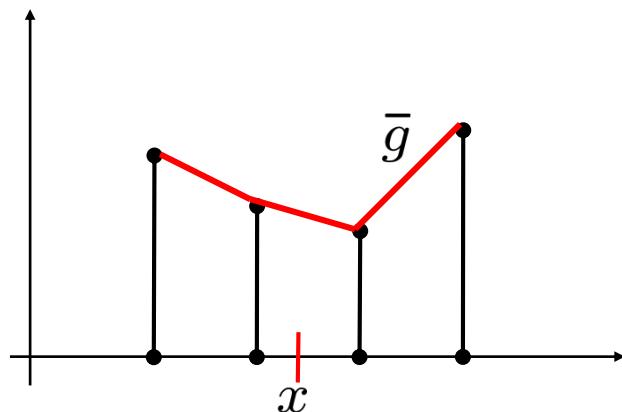
- changing the image size
- scaling needs evaluation of the image at non-integer positions → interpolation
- nearest neighbor interpolation:
 - approximating the grey level function with a step function
 - take the grey value of the nearest integer position
 - problem: aliasing 混叠



Interpolation cont.

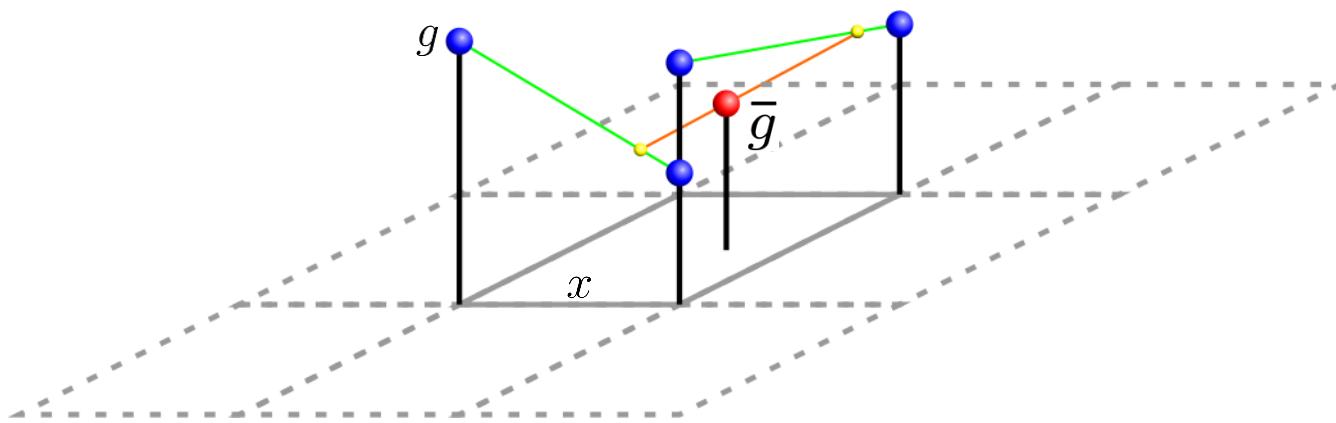
- linear interpolation in 1D
 - fit linear function locally around x

$$\bar{g}(x) = g(\lfloor x \rfloor) + (x - \lfloor x \rfloor)(g(\lfloor x \rfloor + 1) - g(\lfloor x \rfloor))$$



Interpolation cont.

- extension of linear interpolation to 2D:



– interpolate from 4 neighboring pixels

添加计算表达式

$$g = \alpha + \beta_1 x + \beta_2 y$$

Interpolation cont.

- cubic interpolation 三次方插值

- fit cubic polynomial to the grey level
 - solve

$$\bar{g}(x) = a \cdot (x - \lfloor x \rfloor)^3 + b \cdot (x - \lfloor x \rfloor)^2 + c \cdot (x - \lfloor x \rfloor) + d$$

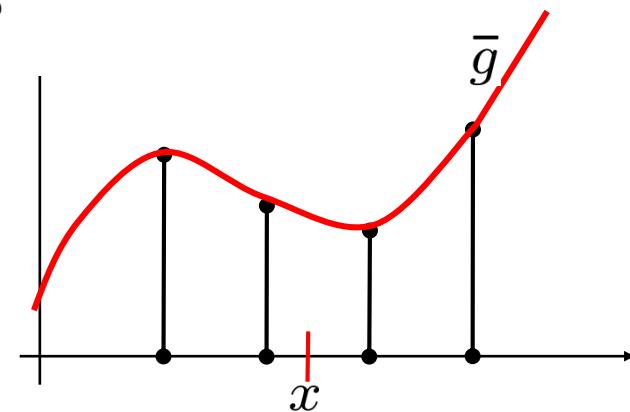
yields:

$$a = -\frac{1}{6}g(\lfloor x \rfloor - 1) + \frac{1}{2}g(\lfloor x \rfloor) - \frac{1}{2}g(\lfloor x \rfloor + 1) + \frac{1}{6}g(\lfloor x \rfloor + 2)$$

$$b = \frac{1}{2}g(\lfloor x \rfloor - 1) - g(\lfloor x \rfloor) + \frac{1}{2}g(\lfloor x \rfloor + 1)$$

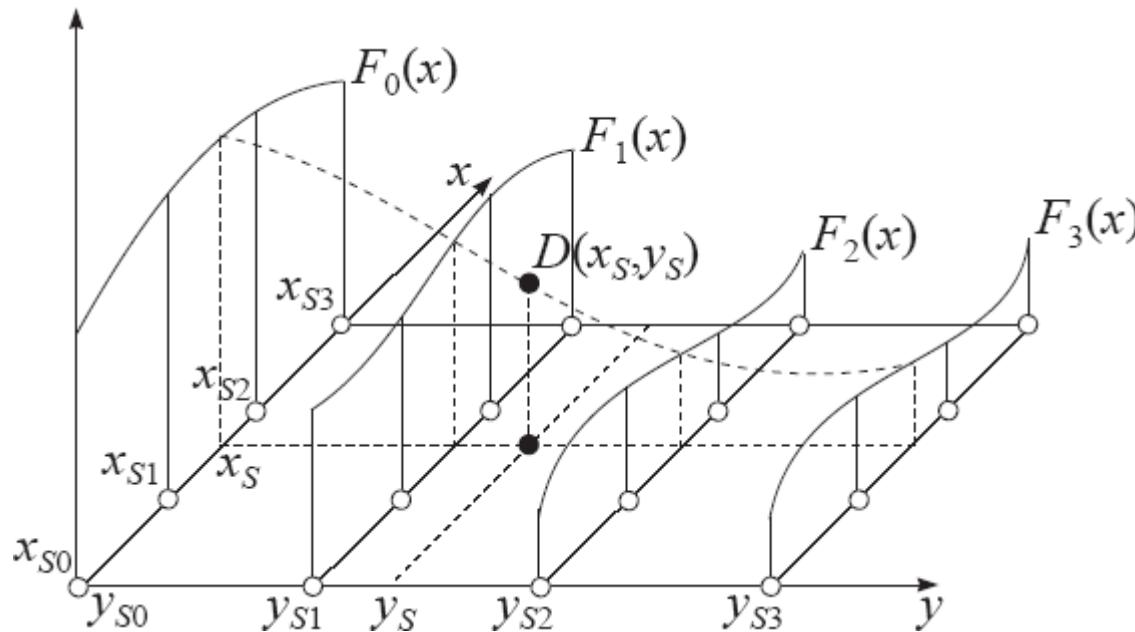
$$c = -\frac{1}{3}g(\lfloor x \rfloor - 1) - \frac{1}{2}g(\lfloor x \rfloor) + g(\lfloor x \rfloor + 1) - \frac{1}{6}g(\lfloor x \rfloor + 2)$$

$$d = g(\lfloor x \rfloor)$$



Interpolation cont.

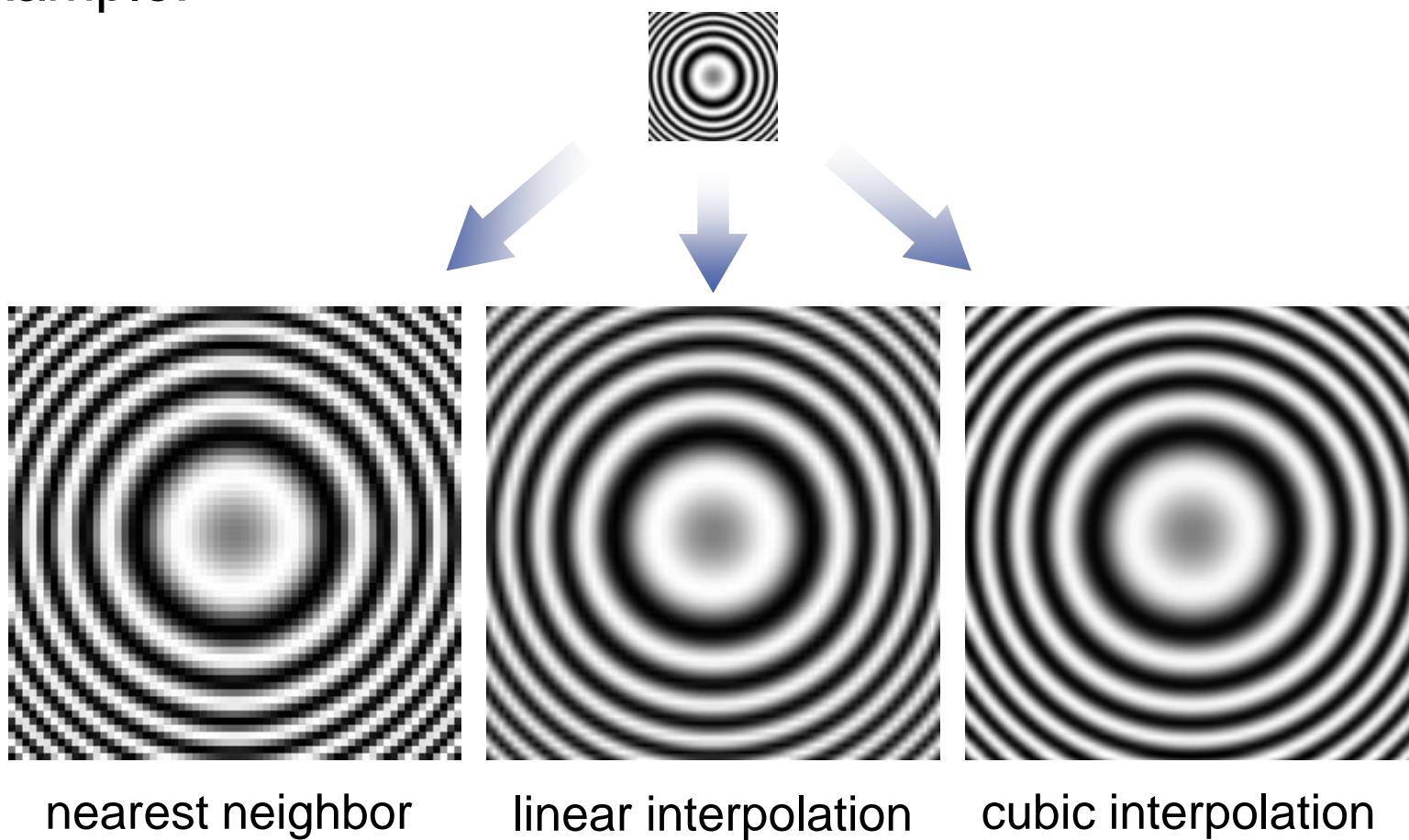
- extension of cubic interpolation to 2D:



- interpolation from 16 neighboring pixels

Interpolation cont.

- Example:

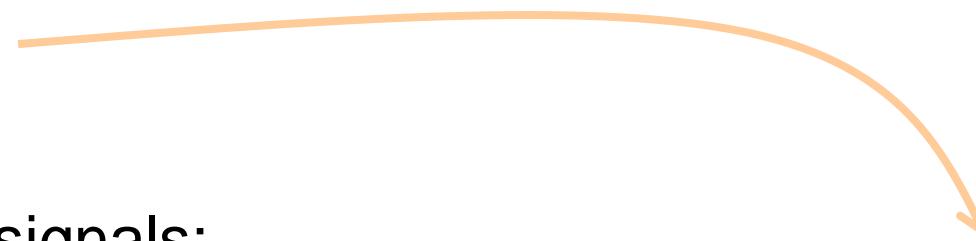


- Process of image formation:
 - **sampling**
evaluate light intensity on a regular grid of points
 - **quantization**
map continuous signals to discrete values (natural numbers)
 - **blur and noise**
 - **color**
will be discussed later. Here: only light intensity/grey level images

Quantization

- incident light:

$$I : \mathbb{R}^2 \rightarrow \mathbb{R}$$



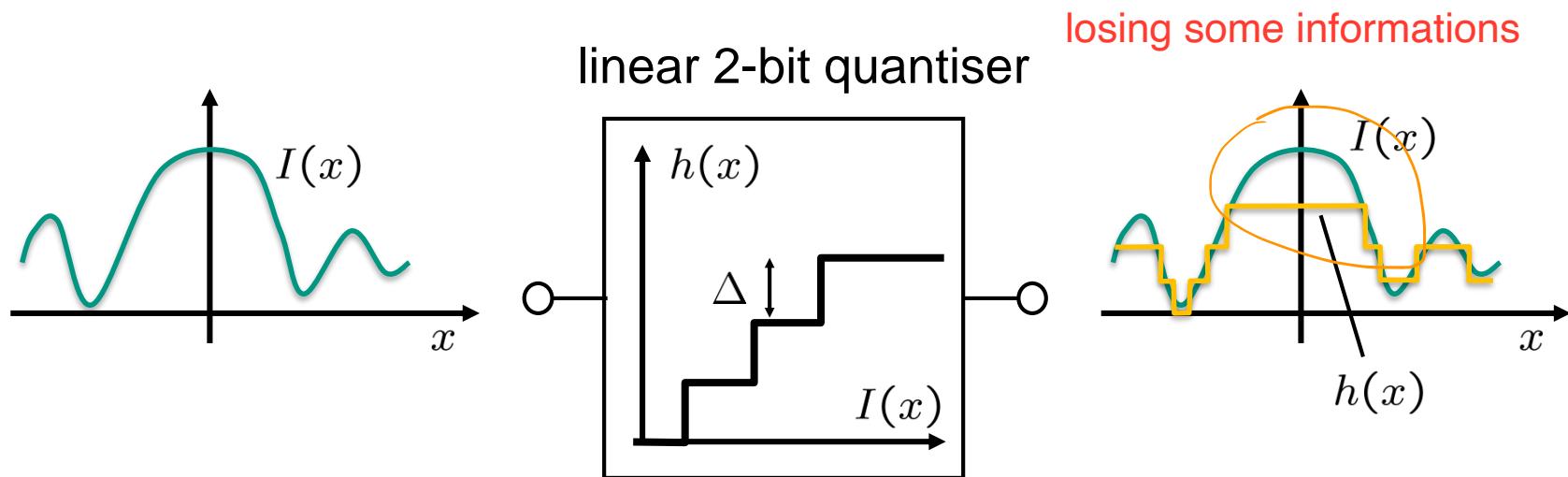
- digital camera signals:

$$g : \{0, \dots, w - 1\} \times \{0, \dots, h - 1\} \rightarrow \{0, \dots, g_{max}\}$$

w, h : image width, height

- need transformation from real valued light intensity to discrete digital signals (analog-to-digital converter)

Quantization cont.



- characteristic with equidistant steps (“linear”) of size Δ :

$$g(x) = \max\{0, \min\{g_{max}, \left\lfloor \frac{I(x)}{\Delta} + \frac{1}{2} \right\rfloor \}\}$$

$$h(x) = \Delta g(x)$$

- error of non-overdriven quantiser:

$$I(x) - h(x) \in [-\frac{\Delta}{2}, \frac{\Delta}{2}]$$

Quantization cont.

- characteristic of digital cameras:
 - linear
 - logarithmic
- grey level cutoff value
 - 1 (binary images, “bitmaps”) → 1 bit/pixel
 - 255 → 8 bit/pixel = 1 byte/pixel
 - 4095 → 12 bit/pixel = 1.5 byte/pixel
 - 65535 → 16 bit/pixel = 2 byte/pixel
- correction of grey level distribution
 - image too dark/too bright
 - low contrast 低对比度
 - non-linear camera characteristic

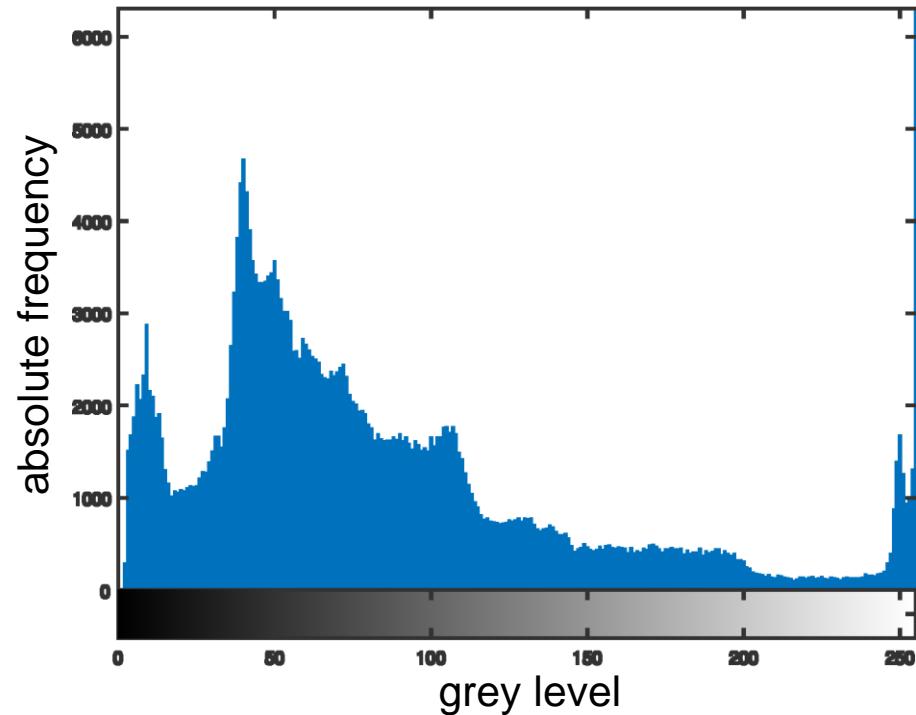


linear characteristic



logarithmic characteristic

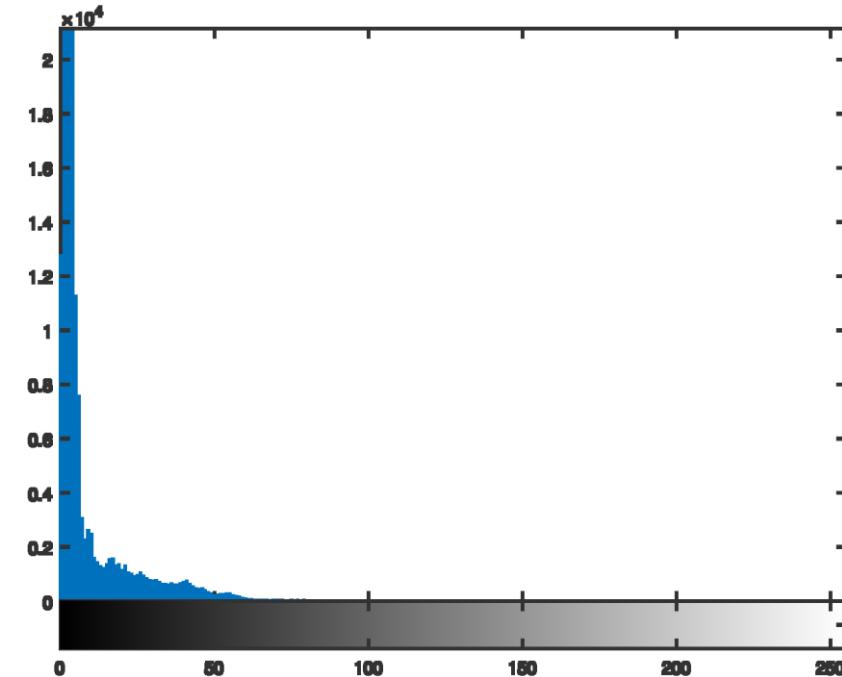
Grey Level Histogram



- grey level histograms display distribution of grey levels

灰度直方图显示灰度分布

Grey Level Histogram cont.



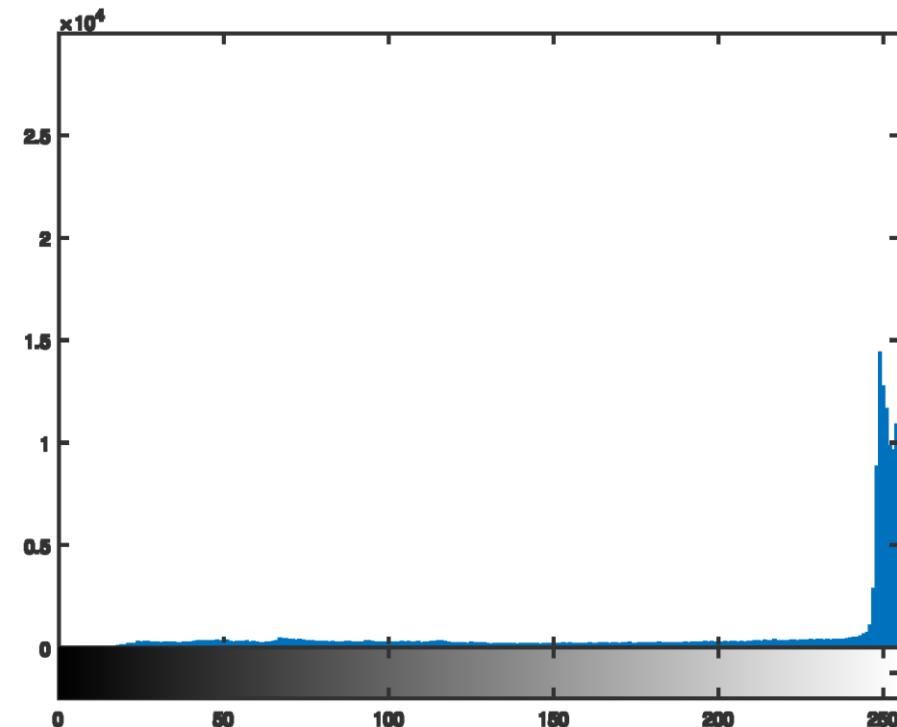
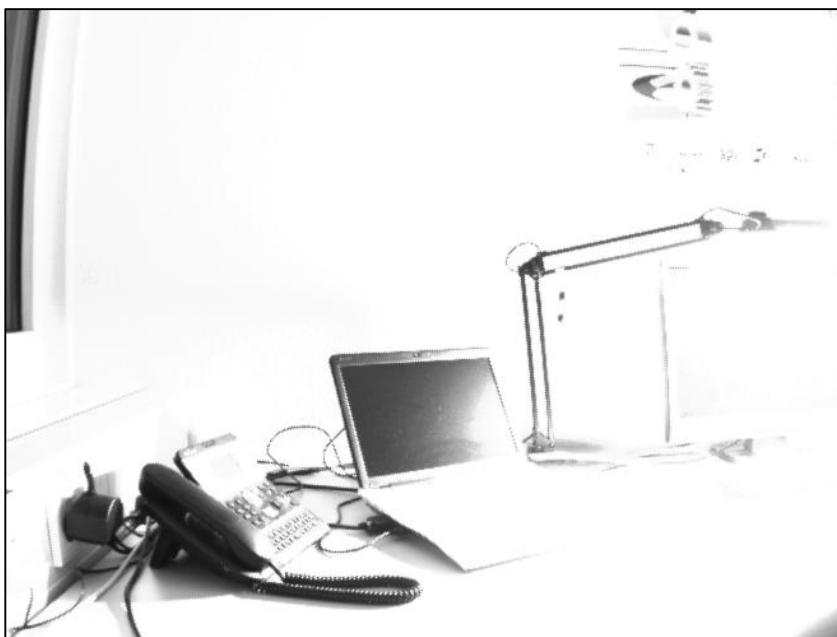
Underexposed images: 欠曝解决方法

- open aperture of camera
- increase exposure time of camera
- increase gain
- add additional light sources

- multiply grey values by a constant
- auto-exposure implemented in many digital cameras

normal will the noise also be amplified

Grey Level Histogram cont.



Overexposed images: 过度曝光不可弥补

- information loss due to cutoff value, no reconstruction possible
- close aperture of camera
- reduce exposure time of camera
- auto-exposure

Grey Level Histogram cont.



$$\gamma = 1$$

γ越大，曝光越低

$$\gamma = 0.5$$



$$\gamma = 2$$

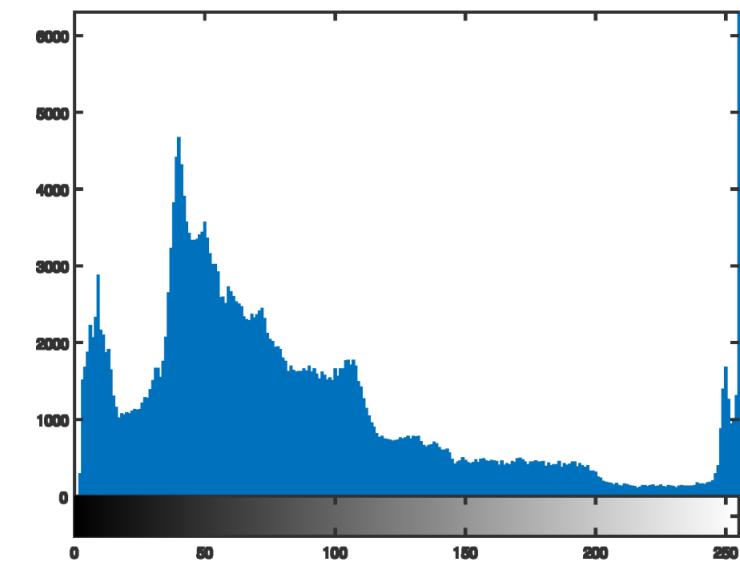


Gamma correction:

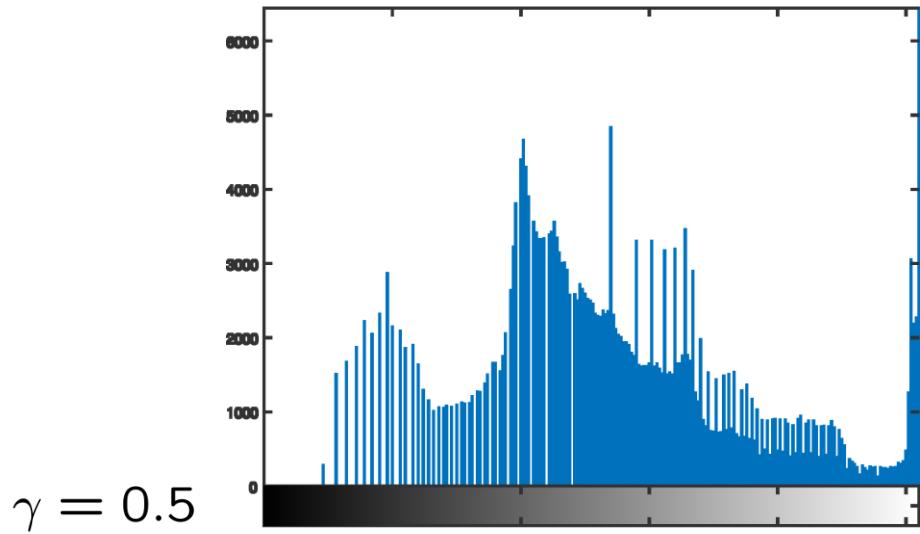
$$g_{out} = g_{max} \left(\frac{g_{in}}{g_{max}} \right)^\gamma$$

- keeps black and white
- nonlinear transformation

Grey Level Histogram cont.

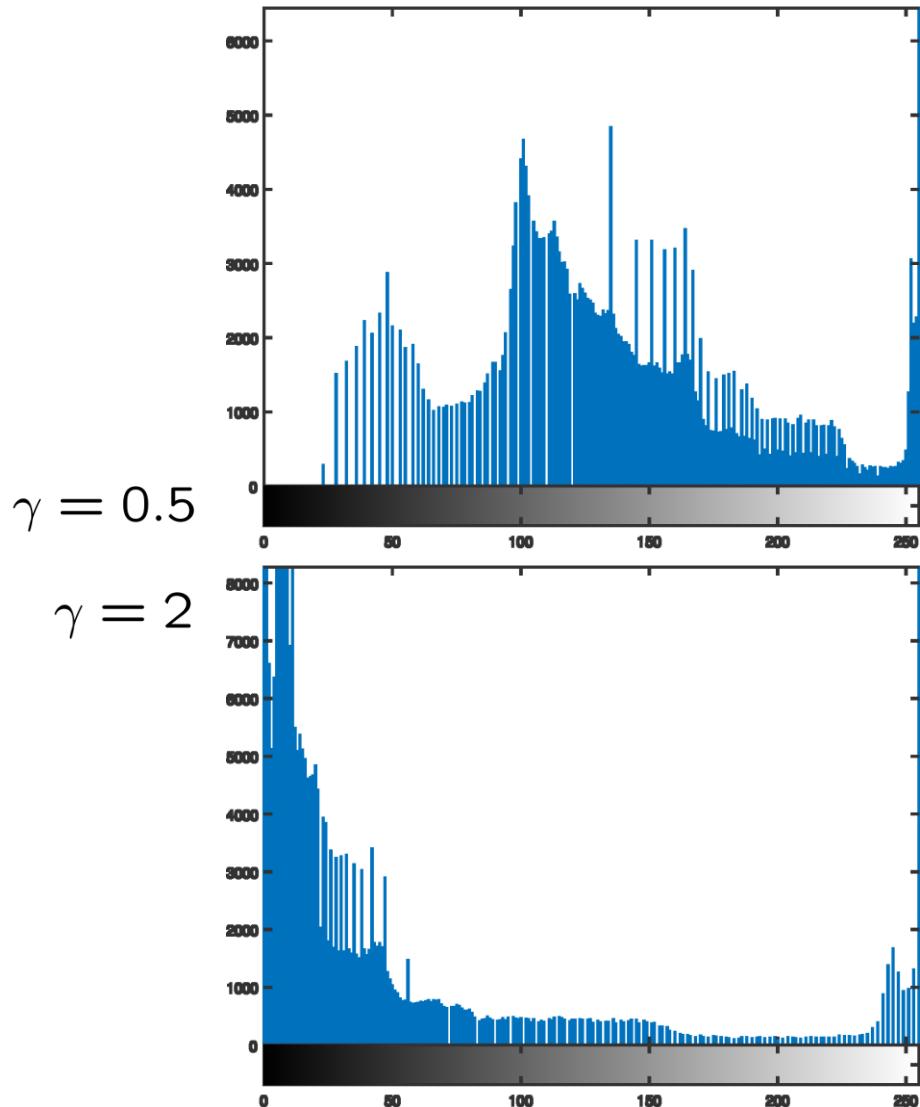


$\gamma = 1$



$\gamma = 0.5$

$\gamma = 2$

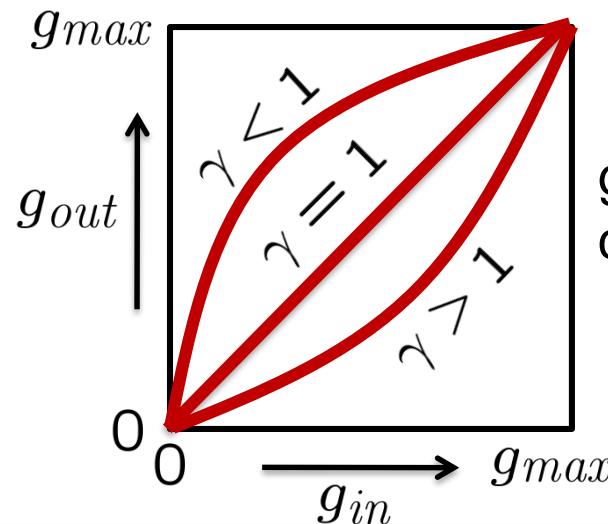
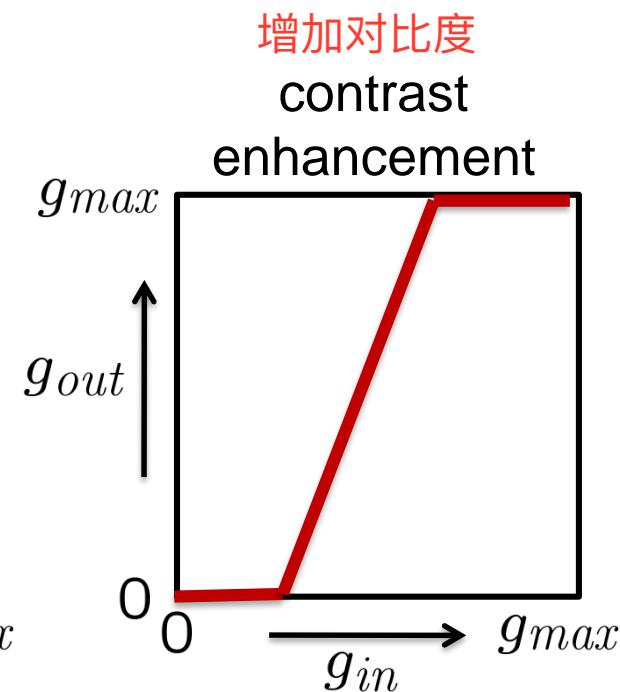
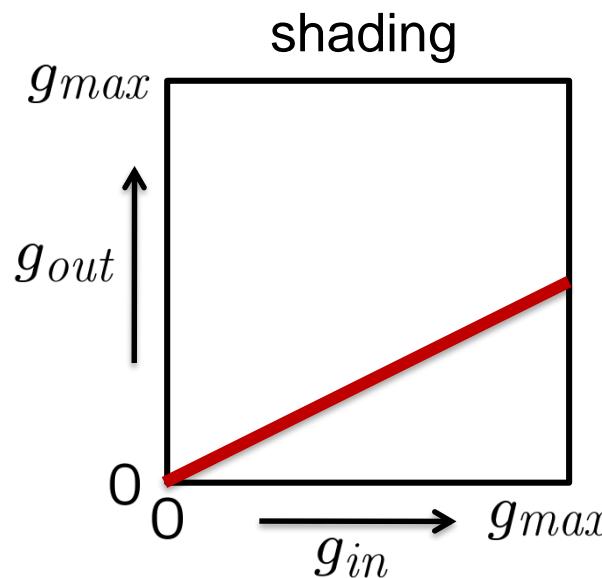
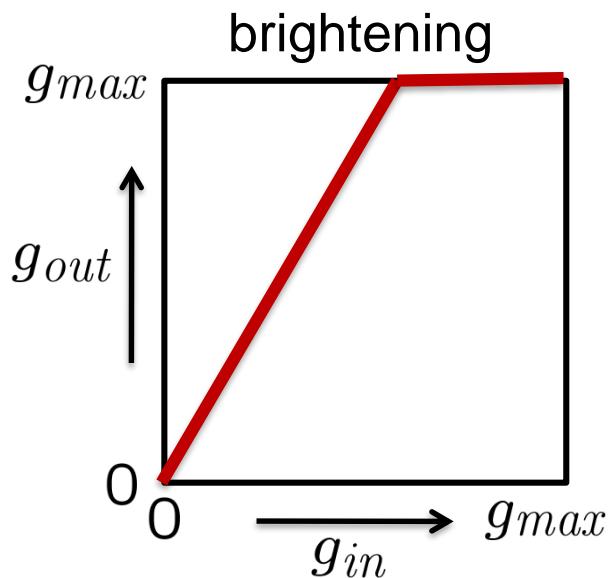


Gamma correction:

$$g_{out} = g_{max} \left(\frac{g_{in}}{g_{max}} \right)^\gamma$$

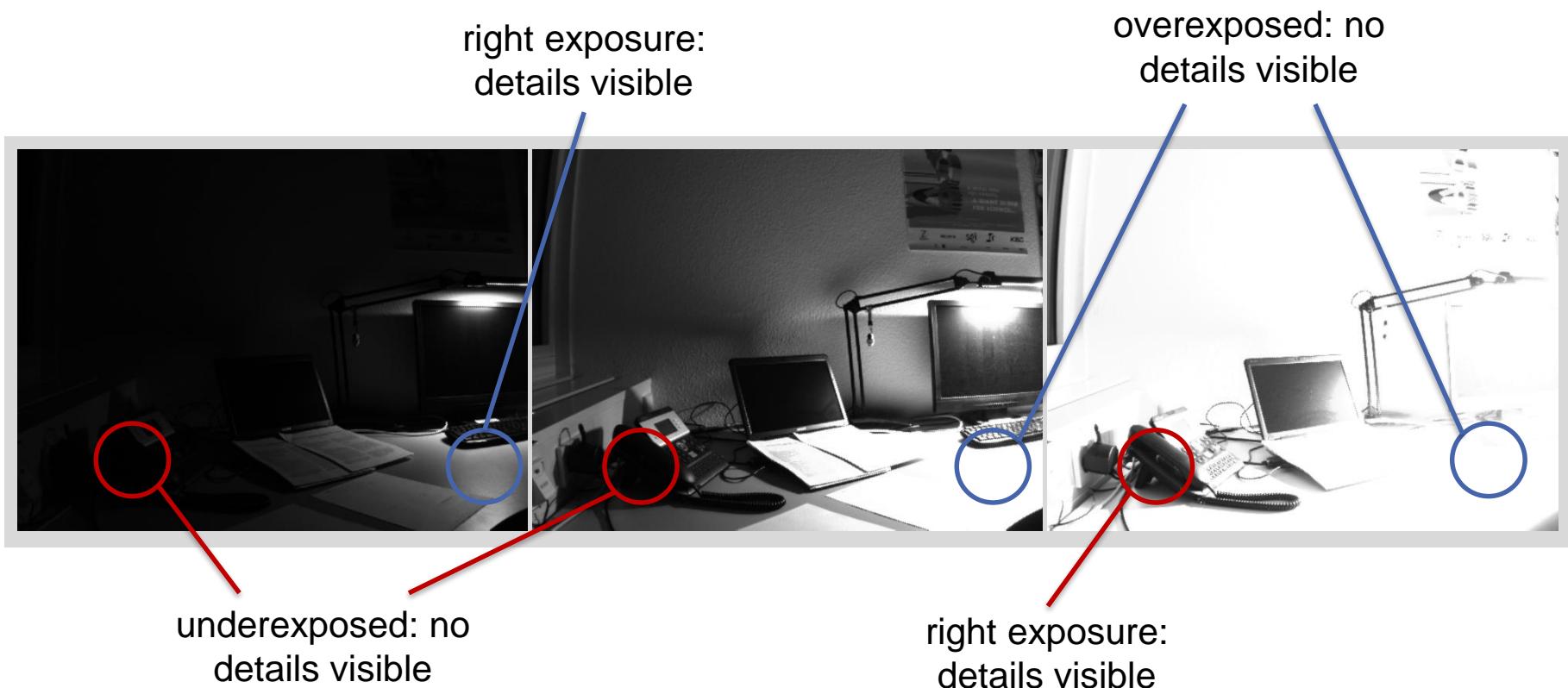
- keeps black and white
- nonlinear transformation

Grey Level Transformations

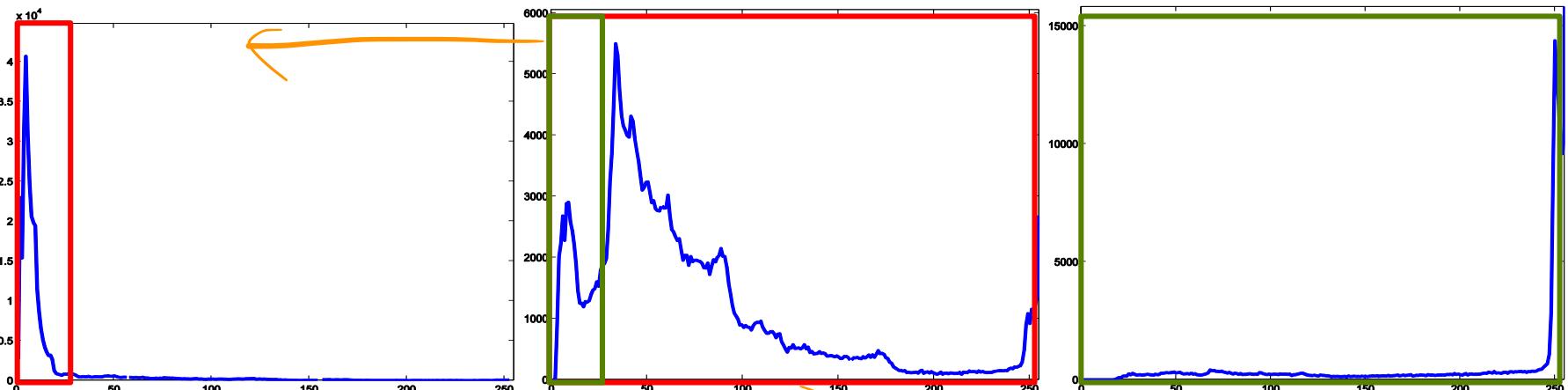


Exposure Series

- exposure bracketing, high dynamic range imaging (HDRI):
increase the grey value resolution combining over- and underexposed images



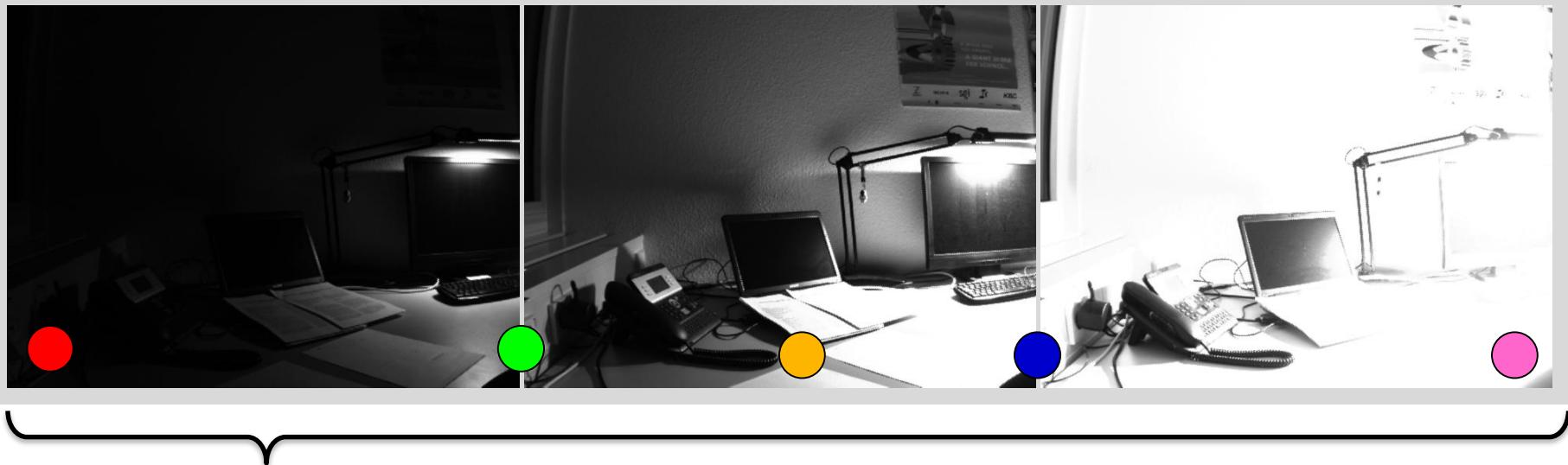
Exposure Series cont.



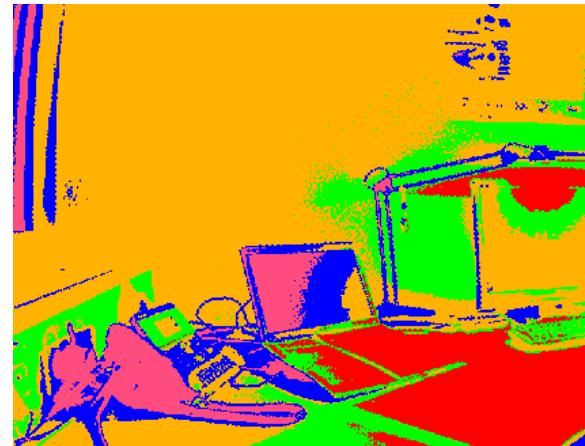
gray level histograms

gray scales differ by a constant factor

Exposure Series cont.

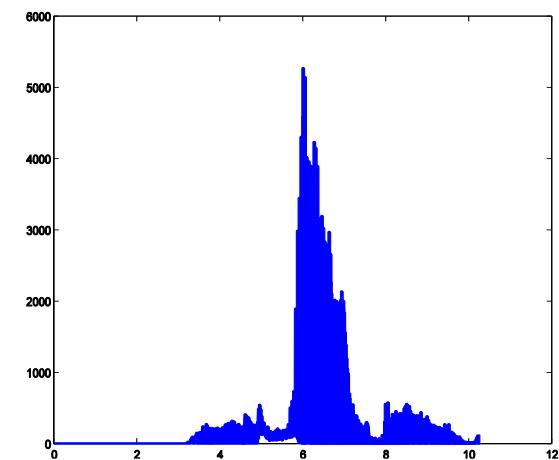


HDRI image (after log transform)



HDRI mixing

根据颜色判断出哪一部分取哪一种曝光程度的图片



HDRI histogram
(after log transform)

- Process of image formation:
 - **sampling**
evaluate light intensity on a regular grid of points
 - **quantization**
map continuous signals to discrete values (natural numbers)
 - **blur and noise** 对应书本中filtering / operator章节
模糊
 - **color**
will be discussed later. Here: only light intensity/grey level images

Convolution Operator

The convolution operator

- takes two functions f, g
- creates a new function $h = g * f$
- which is defined pointwise by

$$h(x) = \int_{-\infty}^{\infty} f(\tau)g(x - \tau)d\tau$$

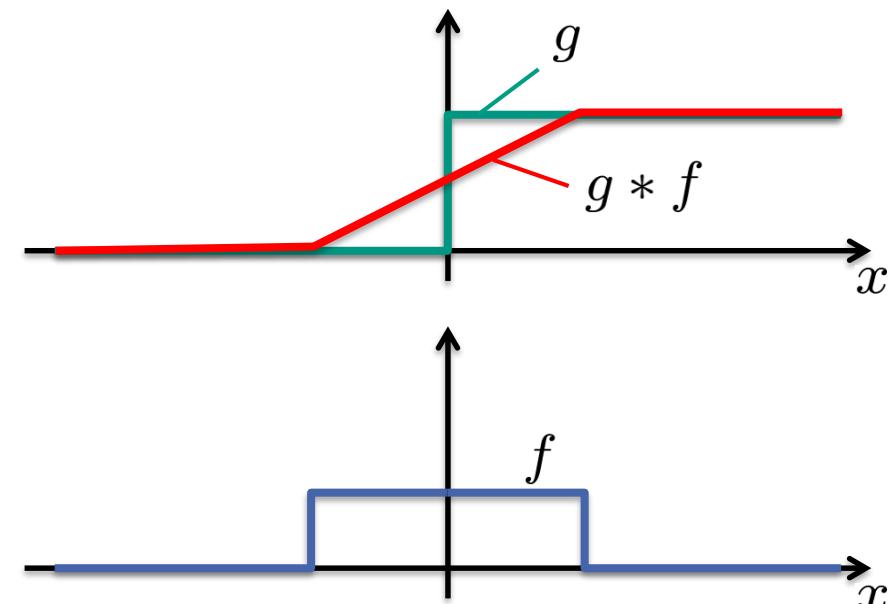
- we interpret
 - g is a gray level image
 - f is a filter function
 - h is a filtered image
- convolution implements a linear filter

Convolution Operator

- Example

$$g(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{2} & \text{if } -1 \leq x \leq +1 \\ 0 & \text{if } x > +1 \end{cases}$$



$$\begin{aligned}(g * f)(x) &= \int_{-\infty}^{\infty} f(\tau)g(x - \tau)d\tau = \int_{-\infty}^x f(\tau) \cdot 1 d\tau + \int_x^{\infty} f(\tau) \cdot 0 d\tau \\&= \int_{-\infty}^x f(\tau) d\tau = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{2}(x + 1) & \text{if } -1 \leq x \leq +1 \\ 1 & \text{if } x > +1 \end{cases}\end{aligned}$$

Convolution Operator

- Properties of convolution

- commutativity

$$f * g = g * f$$

- associativity

$$(f * g) * h = f * (g * h)$$

- linearity

$$f * (\alpha g + \beta h) = \alpha(f * g) + \beta(f * h)$$

- relationship with Fourier transform

$$\mathcal{F}(f * g) = \mathcal{F}(f) \cdot \mathcal{F}(g)$$

$$\mathcal{F}(f \cdot g) = \mathcal{F}(f) * \mathcal{F}(g)$$

Convolution of Images

- Convolution can be extended
 - to the 2d case

$$f, g : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$(g * f)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau, \rho) g(x - \tau, y - \rho) d\tau d\rho$$

- to the case of function which we can evaluate only at integer positions

$$f, g : \mathbb{Z} \rightarrow \mathbb{R}$$

$$(g * f)(u) = \sum_{k=-\infty}^{\infty} f(k) g(u - k)$$

$$f, g : \mathbb{Z}^2 \rightarrow \mathbb{R}$$

$$(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l) g(u - k, v - l)$$

Convolution of Images

$$(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l)g(u - k, v - l)$$

Diagram illustrating the convolution formula:

- (g) gray level image
- (f) filter
- ((u, v)) pixel to evaluate

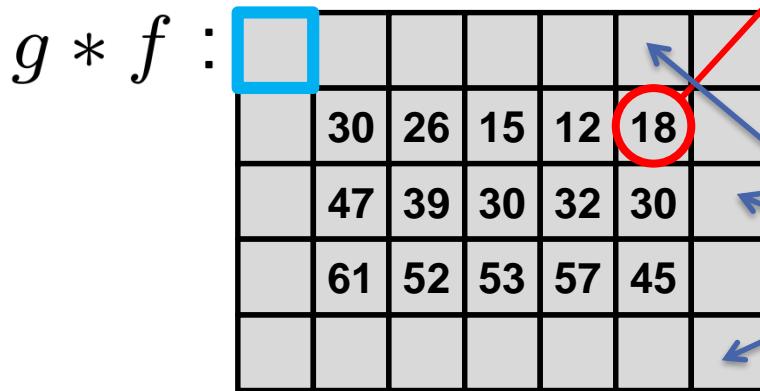
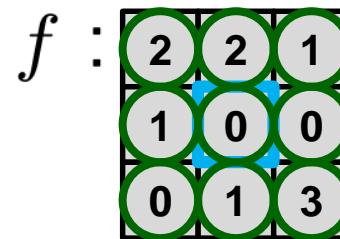
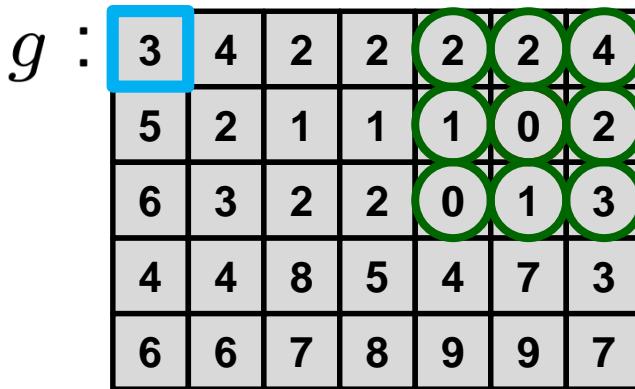
A red oval encloses the double summation. A red arrow points from the text "when k, l is outside the image area , $f(XX)$ is zero" to the summation area.

problem: sums run to infinity!

- in practice, filters and images have limited size.
We assume that all gray levels outside of filter size are 0

Convolution of Images

- Example



$$(g * f)(5, 1) =$$
$$+ f(-1, -1)g(6, 2) + f(0, -1)g(5, 2) + f(1, -1)g(4, 2)$$
$$+ f(-1, 0)g(6, 1) + f(0, 0)g(5, 1) + f(1, 0)g(4, 1)$$
$$+ f(-1, 1)g(6, 0) + f(0, 1)g(5, 0) + f(1, 1)g(4, 0)$$

boundary pixels are typically left free
since convolution requires evaluation
of pixels outside of image g

$$(g * f)(u, v) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f(k, l)g(u - k, v - l)$$

Blur and Noise

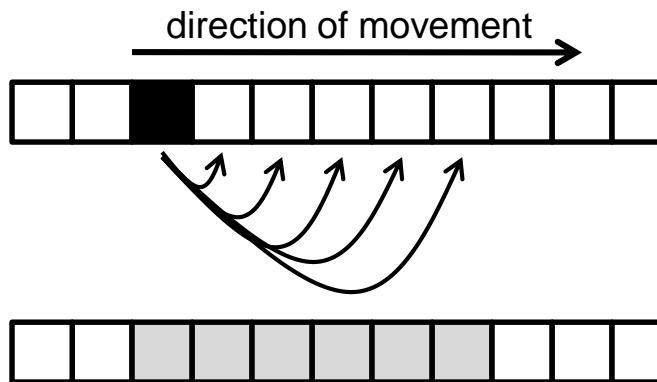
- types of blur and noise:
 - motion blur
 - defocus aberration
 - statistical noise of sensor cells and amplifiers
 - malfunctioning sensor cells



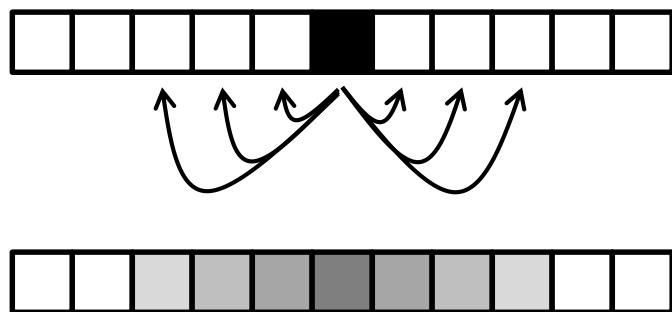
image source: wikipedia

Models of Blur

- Motion blur:



- Gaussian blur:



Models of Blur cont.

- blur can be modeled with convolution

$$g_{blurred} = g_{sharp} * p$$

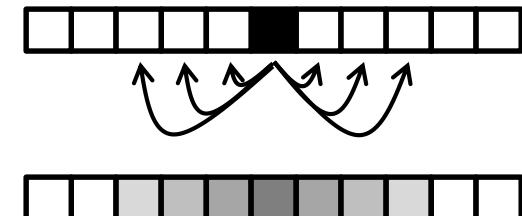
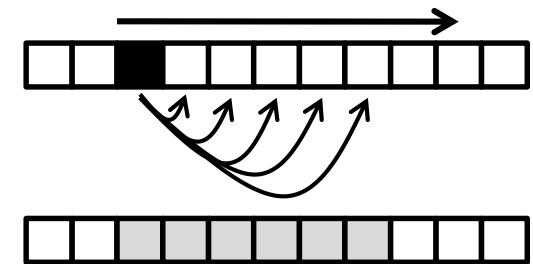
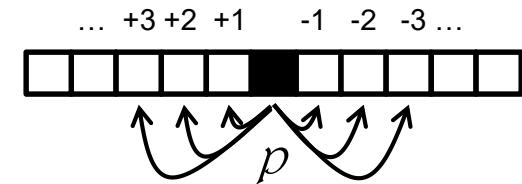
p : “point-spread-function” models blur

- motion blur (along x-axis by n pixels):

$$p_{motion}(x) = \begin{cases} \frac{1}{n} & \text{if } -n < x \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Gaussian blur (with variance σ^2):

$$p_{Gauss}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$



Wiener Deconvolution

- techniques to obtain sharp image from blurred image based on Wiener filter

$$g_{blurred} = g_{sharp} * p + v$$

p : point-spread-function

v : pixel noise

assume g_{sharp} and v be independent

$$g_{restored} = f * g_{blurred}$$

find optimal f that minimizes:

$$e(k) = \mathbb{E} [|\hat{g}_{sharp}(k) - \hat{g}_{restored}(k)|^2]$$

(\hat{g} denotes Fourier transform of g)

(\mathbb{E} denotes expectation value)

Wiener Deconvolution cont.

$$\begin{aligned} e(k) &= \mathbb{E} [|\hat{g}_{sharp}(k) - \hat{g}_{restored}(k)|^2] \\ &= \mathbb{E} [|\hat{g}_{sharp}(k) - \hat{f}(k)\hat{g}_{blurred}(k)|^2] \\ &= \mathbb{E} [|\hat{g}_{sharp}(k) - \hat{f}(k)(\hat{p}(k)\hat{g}_{sharp}(k) + \hat{v}(k))|^2] \\ &= \mathbb{E} [|((1 - \hat{f}(k)\hat{p}(k))\hat{g}_{sharp}(k) - \hat{f}(k)\hat{v}(k)|^2] \\ &= (1 - \hat{f}(k)\hat{p}(k))(1 - \hat{f}(k)\hat{p}(k))^* \mathbb{E} [\hat{g}_{sharp}(k)\hat{g}_{sharp}^*(k)] \\ &\quad - (1 - \hat{f}(k)\hat{p}(k))\hat{f}^*(k) \mathbb{E} [\hat{g}_{sharp}(k)\hat{v}^*(k)] \\ &\quad - \hat{f}(k)(1 - \hat{f}(k)\hat{p}(k))^* \mathbb{E} [\hat{v}(k)\hat{g}_{sharp}^*(k)] \\ &\quad + \hat{f}(k)\hat{f}^*(k) \mathbb{E} [\hat{v}(k)\hat{v}^*(k)] \end{aligned}$$

independence of signal and noise yields:

$$\mathbb{E} [\hat{g}_{sharp}(k)\hat{v}^*(k)] = \mathbb{E} [\hat{v}(k)\hat{g}_{sharp}^*(k)] = 0$$

denote:

$$S(k) = \mathbb{E} [\hat{g}_{sharp}(k)\hat{g}_{sharp}^*(k)], \quad N(k) = \mathbb{E} [\hat{v}(k)\hat{v}^*(k)]$$

$$e(k) = (1 - \hat{f}(k)\hat{p}(k))(1 - \hat{f}(k)\hat{p}(k))^* S(k) + \hat{f}(k)\hat{f}^*(k)N(k)$$

Wiener Deconvolution cont.

$$\begin{aligned}
 e(k) &= (1 - \hat{f}(k)\hat{p}(k))(1 - \hat{f}(k)\hat{p}(k))^* \cdot S(k) + \hat{f}(k)\hat{f}^*(k) \cdot N(k) \\
 &= |1 - \hat{f}(k)\hat{p}(k)|^2 \cdot S(k) + |\hat{f}(k)|^2 \cdot N(k)
 \end{aligned}$$

let $\hat{f}(k) = \alpha + i\beta$, $\hat{p}(k) = \gamma + i\delta$

$$e(k) = ((1 - \alpha\gamma + \beta\delta)^2 + (\alpha\delta + \beta\gamma)^2) \cdot S(k) + (\alpha^2 + \beta^2) \cdot N(k)$$

$$\begin{aligned}
 \frac{\partial e}{\partial \alpha} &= (-2\gamma(1 - \alpha\gamma + \beta\delta) + 2\delta(\alpha\delta + \beta\gamma)) \cdot S(k) + 2\alpha \cdot N(k) \\
 &= (2(\gamma^2 + \delta^2) \cdot S(k) + 2N(k)) \cdot \alpha - 2\gamma S(k)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial e}{\partial \beta} &= (2\delta(1 - \alpha\gamma + \beta\delta) + 2\gamma(\alpha\delta + \beta\gamma)) \cdot S(k) + 2\beta \cdot N(k) \\
 &= (2(\gamma^2 + \delta^2) \cdot S(k) + 2N(k)) \cdot \beta + 2\delta S(k)
 \end{aligned}$$

Wiener Deconvolution cont.

$$\frac{\partial e}{\partial \alpha} = (2(\gamma^2 + \delta^2) \cdot S(k) + 2N(k)) \cdot \alpha - 2\gamma S(k) = 0 !$$

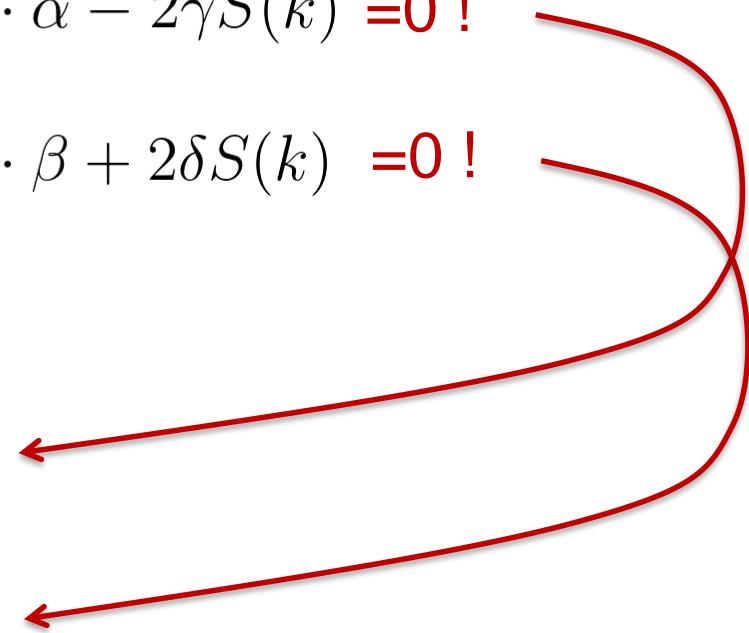
$$\frac{\partial e}{\partial \beta} = (2(\gamma^2 + \delta^2) \cdot S(k) + 2N(k)) \cdot \beta + 2\delta S(k) = 0 !$$

zeroing derivatives

$$\alpha = \frac{\gamma S(k)}{(\gamma^2 + \delta^2) \cdot S(k) + N(k)}$$

$$\beta = \frac{-\delta S(k)}{(\gamma^2 + \delta^2) \cdot S(k) + N(k)}$$

$$\hat{f}(k) = \alpha + i\beta = \frac{\hat{p}^*(k)S(k)}{|\hat{p}(k)|^2 \cdot S(k) + N(k)}$$



Wiener Deconvolution cont.

- zeroing the derivative of e to obtain the minimum yields:

$$\hat{f}(k) = \frac{\hat{p}^*(k)S(k)}{\hat{p}(k)\hat{p}^*(k)S(k) + N(k)} = \frac{\hat{p}^*(k)}{|\hat{p}(k)|^2 + (\frac{S(k)}{N(k)})^{-1}}$$

which defines the optimal linear filter (**Wiener filter**)

- $\frac{S(k)}{N(k)}$ is the signal-to-noise ratio

- in the noiseless case:

$$\hat{f}(k) = \frac{1}{\hat{p}(k)} \quad (\text{if } N(k) = 0)$$

- but: $\frac{S(k)}{N(k)}$ and $\hat{p}(k)$ must be known

Wiener Deconvolution cont.



original image

motion blur



restored image



Wiener Deconvolution cont.

Gaussian blur



original image

restored image



Models of Noise

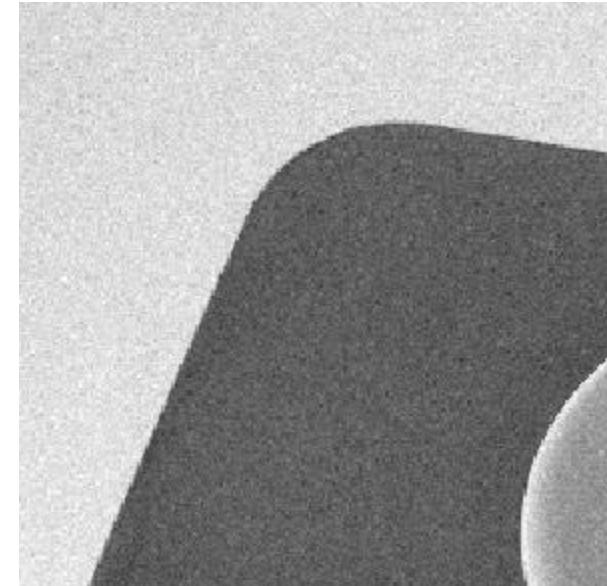
- statistical noise:

$$g_{noisy}(x, y) = g_{sharp}(x, y) + v(x, y)$$

$$v(x, y) \sim N(0, \sigma^2) \text{ i.i.d.}$$

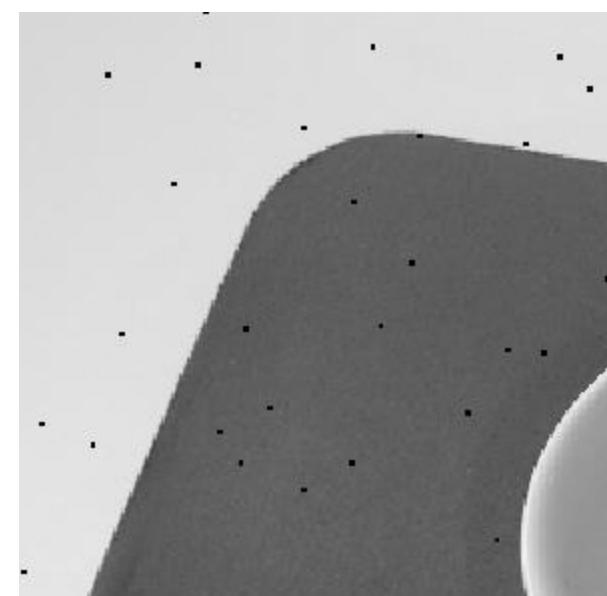
独立均匀分布

(i.i.d. = independent and identically distributed)



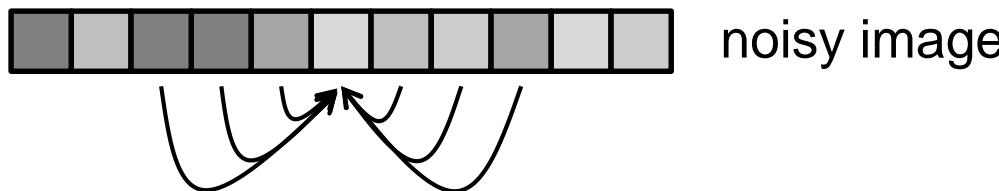
- malfunctioning sensors:

$$g_{noisy}(x, y) = \begin{cases} g_{sharp}(x, y) & \text{with probability } p \\ \text{arbitrary} & \text{otherwise} \end{cases}$$



Statistical Noise

- basic idea: averaging (smoothing)

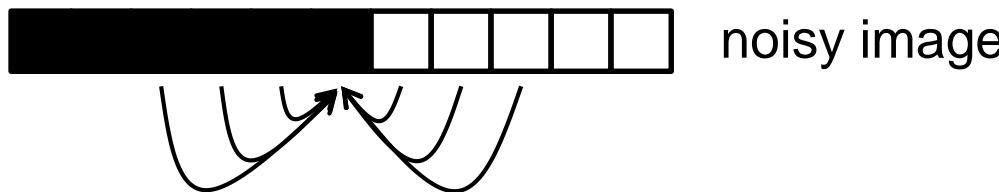


noisy image



image after smoothing

- works well in homogeneous areas, but fails at grey level edges



noisy image



image after smoothing

Smoothing Filters

- rectangular filter

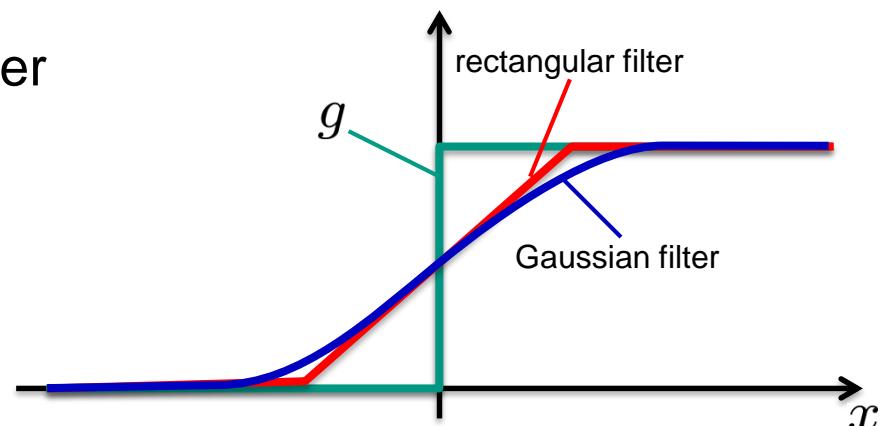
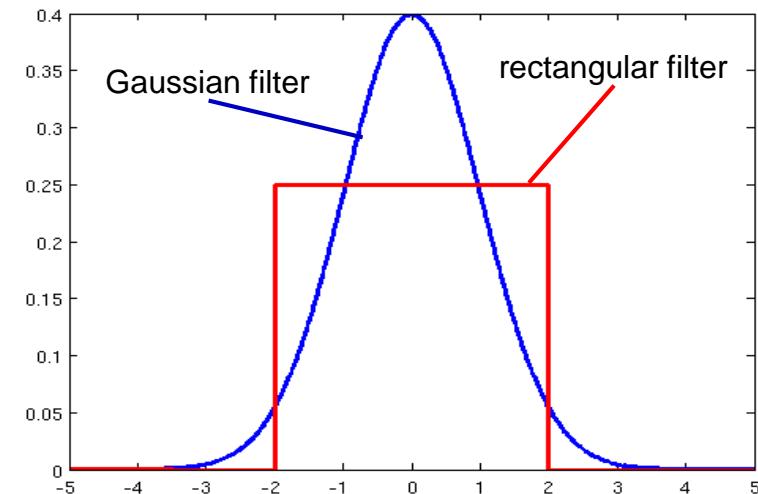
$$f(x) = \begin{cases} \frac{1}{a} & \text{if } |x| < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

the larger parameter a , the stronger smoothing

- Gaussian filter

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}$$

the larger parameter σ , the stronger smoothing



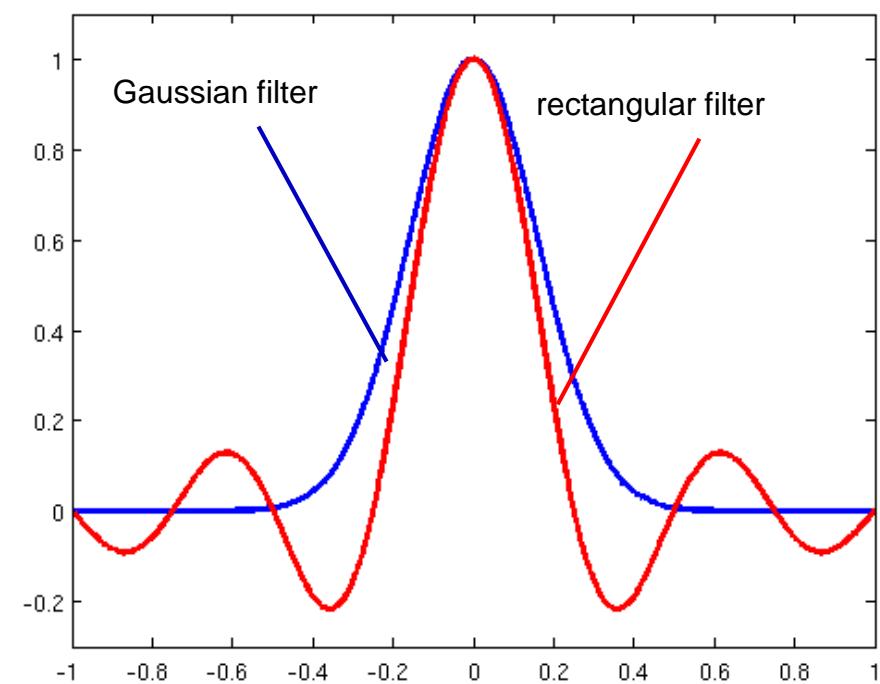
Smoothing Filters

- Fourier transform of smoothing filters

$$\mathcal{F}(\text{rect.filter})(k) = \text{sinc}(ak) = \begin{cases} \frac{\sin(\pi ak)}{\pi ak} & \text{if } k \neq 0 \\ 1 & \text{if } k = 0 \end{cases}$$

$$\mathcal{F}(\text{Gauss.filter})(k) = e^{-2\pi^2\sigma^2k^2}$$

Fourier Filter is better because of the high frequency



→ Smoothing: low pass filtering

Smoothing Filters for Images

- rectangular filter masks:

$$\frac{1}{9} \cdot \begin{matrix} 1 & 1 & 1 \\ 1 & \boxed{1} & 1 \\ 1 & 1 & 1 \end{matrix}$$

3x3 square mask

$$\frac{1}{25} \cdot \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & \boxed{1} & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}$$

5x5 square mask

$$\frac{1}{19.8} \cdot \begin{matrix} 0.1 & 0.8 & 1 & 0.8 & 0.1 \\ 0.8 & 1 & 1 & 1 & 0.8 \\ 1 & 1 & \boxed{1} & 1 & 1 \\ 0.8 & 1 & 1 & 1 & 0.8 \\ 0.1 & 0.8 & 1 & 0.8 & 0.1 \end{matrix}$$

5x5 disc mask

Discrete Convolution cont.

二项式滤波器是高斯滤波器的近似，可以通过二项式展开系数生成。随着滤波器尺寸的增加（如更大的矩阵），二项式滤波器会更接近于高斯滤波器。

- Gaussian filter masks:

$$\frac{1}{39} \cdot \begin{array}{|c|c|c|} \hline 1 & 4 & 1 \\ \hline 4 & 19 & 4 \\ \hline 1 & 4 & 1 \\ \hline \end{array}$$

3x3 mask, $\sigma^2=0.25$

$$\frac{1}{273} \cdot \begin{array}{|c|c|c|c|c|} \hline 1 & 4 & 7 & 4 & 1 \\ \hline 4 & 16 & 26 & 16 & 4 \\ \hline 7 & 26 & 41 & 26 & 7 \\ \hline 4 & 16 & 26 & 16 & 4 \\ \hline 1 & 4 & 7 & 4 & 1 \\ \hline \end{array}$$

5x5 mask, $\sigma^2=1$

二项式滤波器具有计算效率高的优点，并且对于许多应用来说，它提供了足够好的平滑效果

- binomial filter masks

approximations to Gaussian masks using binomial coefficients $\binom{n}{k}$

$$\binom{2}{0} \cdot \binom{2}{0} + \binom{2}{1} \cdot \binom{2}{1} + \binom{2}{2} \cdot \binom{2}{0}$$
$$\binom{2}{0} \cdot \binom{2}{1} + \binom{2}{1} \cdot \binom{2}{1}$$
$$\binom{2}{0} \cdot \binom{2}{2} + \binom{2}{1} \cdot \binom{2}{1}$$

$$\frac{1}{16} \cdot \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

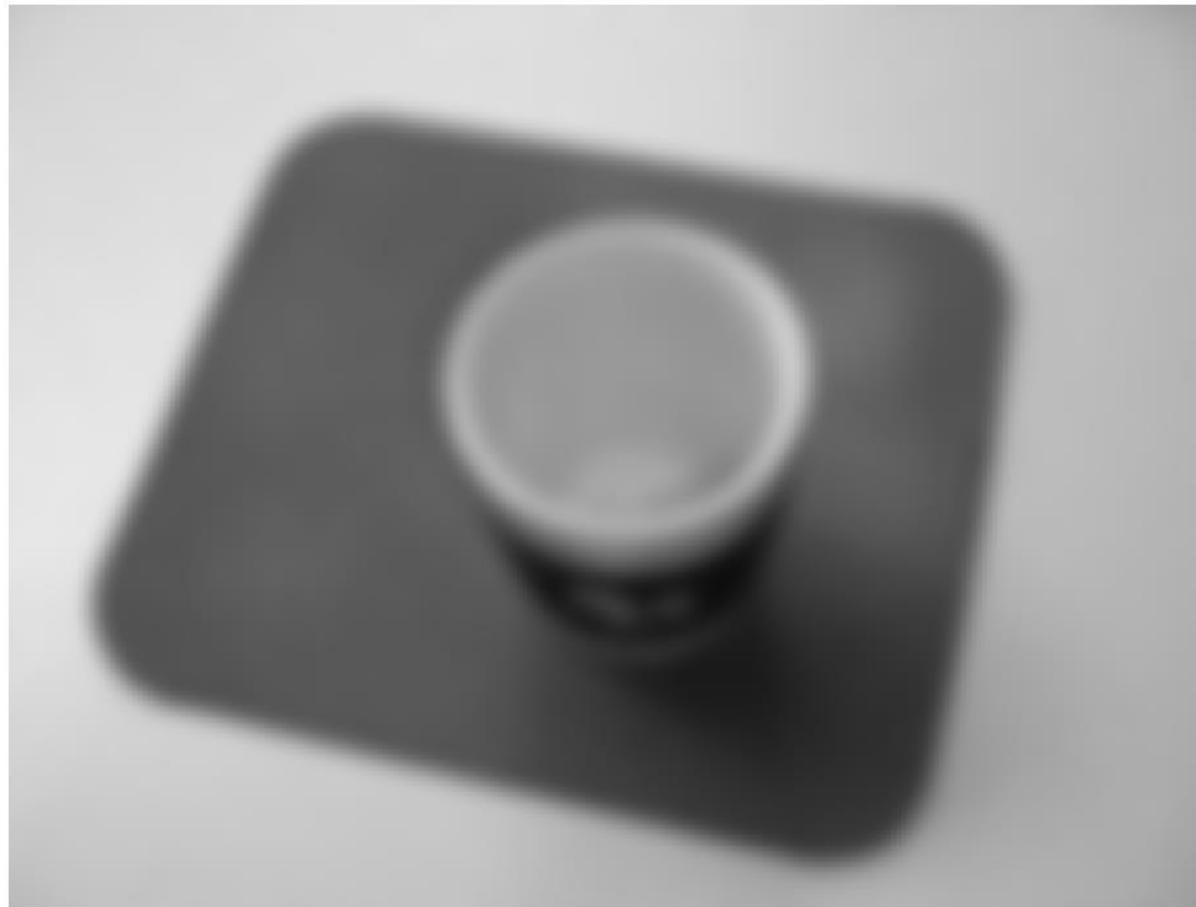
3x3 mask

$$\frac{1}{256} \cdot \begin{array}{|c|c|c|c|c|} \hline 1 & 4 & 6 & 4 & 1 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 6 & 24 & 36 & 24 & 6 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array}$$

5x5 mask

Smoothing Filters cont.

Gaussian, sigma = 7



Bilateral filter

Gaussian filter

- filter mask independent of image content

$$\tilde{g}(u, v) \propto \sum_{i,j} \left(e^{-\frac{1}{2} \frac{i^2 + j^2}{\sigma^2}} \cdot g(u+i, v+j) \right)$$

gray level after filteringdistance dependent weightgray level

Bilateral filter

- filter mask dependent on image content

$$\tilde{g}(u, v) \propto \sum_{i,j} \left(e^{-\frac{1}{2} \frac{\|g(u+i, v+j) - g(u, v)\|^2}{\rho^2}} \cdot e^{-\frac{1}{2} \frac{i^2 + j^2}{\sigma^2}} \cdot g(u+i, v+j) \right)$$

gray level after filteringdistance dependent weightgray level

content dependent weight

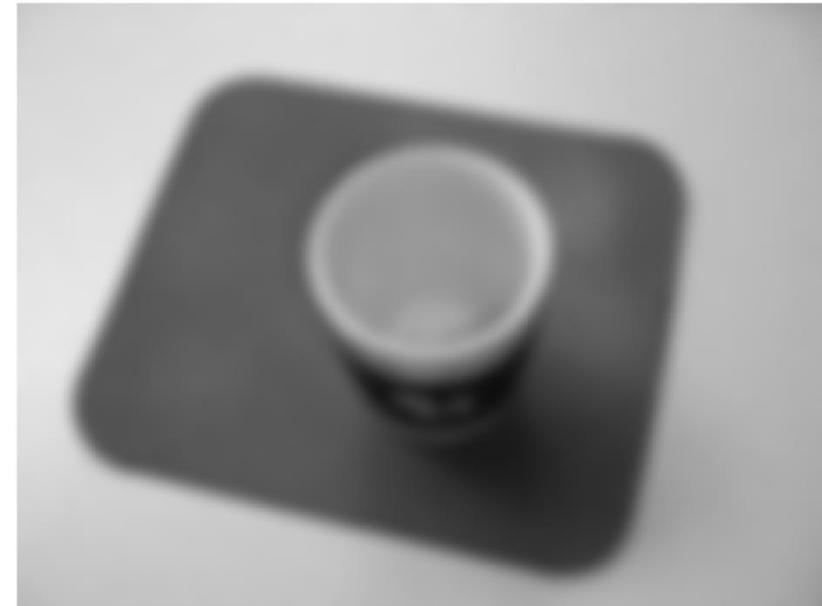
- smooth over edges and gross outliers

- reduces smoothing at edges and gross outliers

Bilateral Filters cont.



Gaussian filter
 $\sigma = 7$



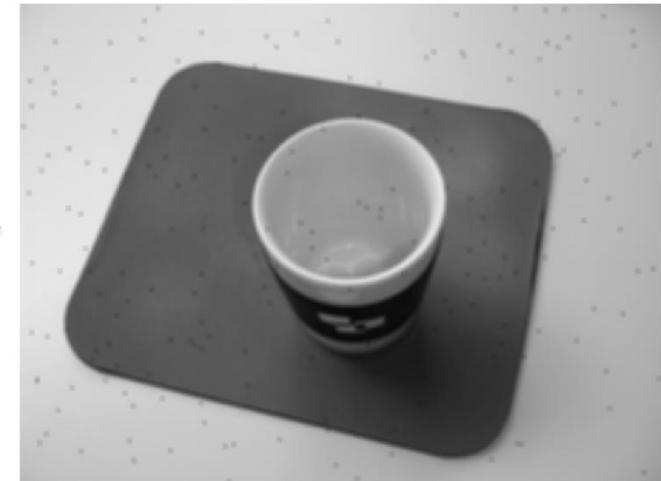
bilateral filter
 $\sigma = 7, \rho = 20$



Salt-and-Pepper Noise

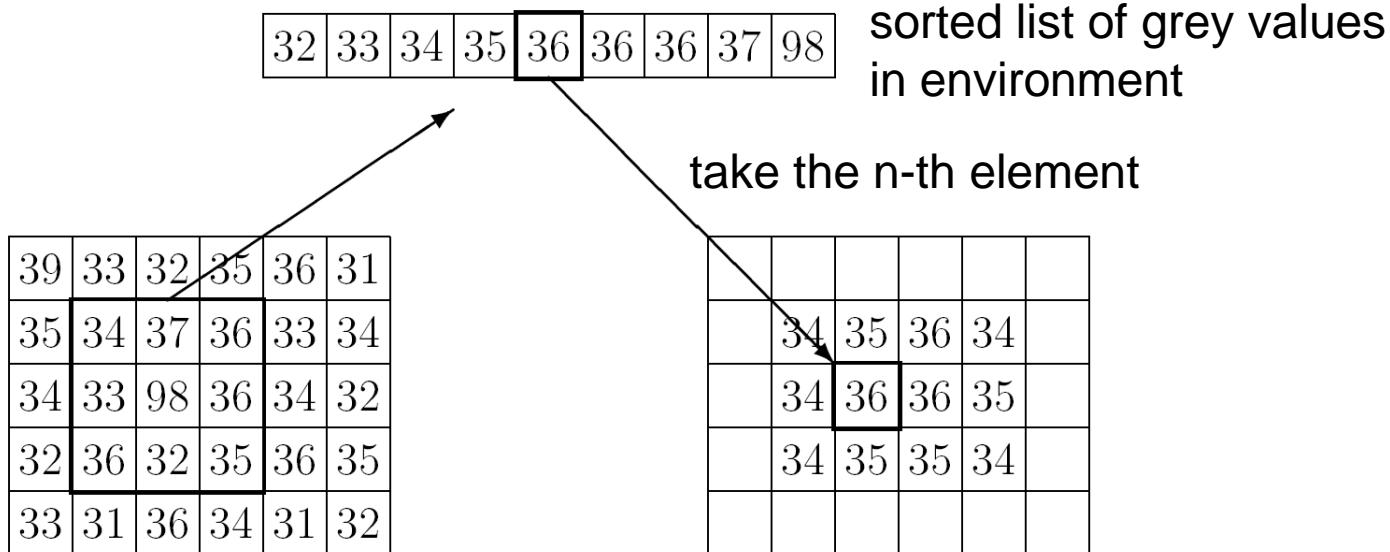


→ Gaussian
filter



→ smoothing not appropriate for salt-and-pepper noise

Median filter



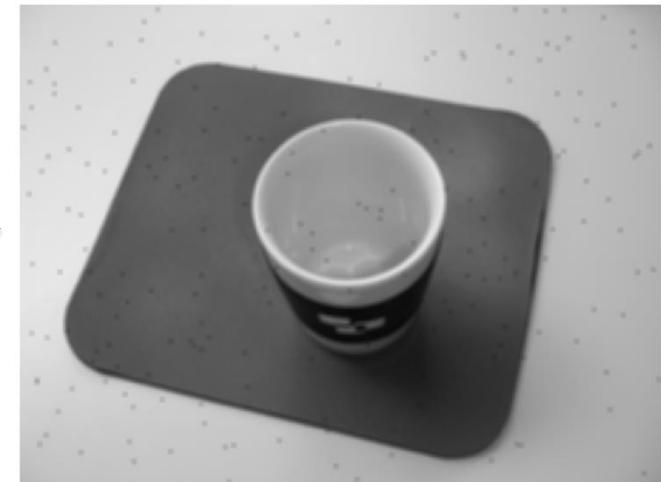
median filter:

- sort grey values in environment around reference pixel
- take the grey value in the middle of the sorted list

Median filter



Gaussian filter



median filter



SUMMARY: IMAGE PREPROCESSING

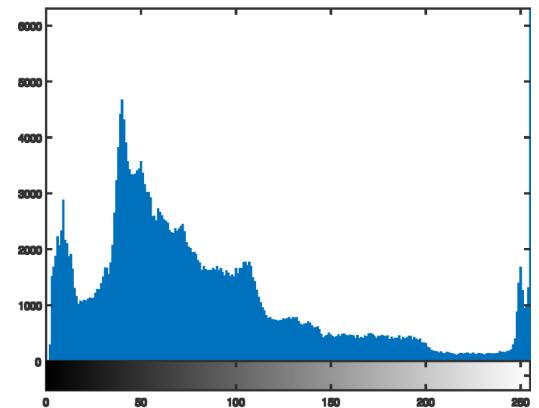
Summary

- **sampling**
 - Moiré patterns
 - sampling theorem
 - Fourier transform
- **quantization**
- **blur and noise**



Summary cont.

- sampling
- quantization
 - discrete grey values
 - histogram transformation
 - high dynamic range imaging
- blur and noise



Summary cont.

- **sampling**
- **quantization**
- **blur and noise**
 - convolution
 - models of blur and noise
 - optimal image restoration
(Wiener deconvolution)
 - smoothing filters
 - rectangular
 - Gaussian
 - bilateral
 - median filter

