5.37 $p(x_1,x_2) = P_{X_1}(x_1)\cdot P_{X_2}(x_2)$

$P(x)=\begin{cases} 0.2 & x=25 \\ 0.5 & x=40 \\ 0.3 & x=65 \end{cases}$

a)

$x_2 \backslash x_1$	25	40	65
25	0.04	0.1	0.06
40	0.1	0.25	0.15
65	0.06	0.15	0.09

$\bar{X} = \frac{x_1+x_2}{2}$ (average weight)

$$E(\bar{X}) = \sum_{x_1}\sum_{x_2} \bar{x}\cdot p(x_1,x_2)$$

$$= 25\times0.04 + 40\times0.25 + 65\times0.09 + 65\times0.1 + 90\times0.06$$

$$+ 105\times0.15$$

$$= 44.5 \text{ oz}$$

$$\mu = 25\times0.2 + 40\times0.5 + 65\times0.3 = 44.5$$
$$= E(\bar{X})$$

b) $\sigma = 25^2\times0.2 + 40^2\times0.5 + 65^2\times0.3 - 44.5^2 = 212.25$

s^2

$x_1 \backslash x_2$	25	40	65
25	0	112.5	800
40	112.5	0	312.5
65	800	312.5	0

$(40-32.5)^2+(25-32.5)^2 = 112.5$

$(65-45)^2+(25-45)^2 = 800$

$(65-52.5)^2+(40-52.5)^2 = 312.5$

$$E(S^2) = 0 + 0.1\times2\times112.5 + 0.06\times2\times800 + 0.15\times2\times312.5 = \boxed{212.25}$$

$$E(S^2) = \sigma^2$$

5.38

X	0	1	2
$P(X)$	.2	.5	.3

$\mu = 1.1$, $\sigma^2 = 0.49$

T_0	0	1	2	3	4
P_{T_0}	0.04	0.2	0.37	0.3	0.09

a) $P_{T_0}(0) = P(0) \cdot P(0) = 0.04$

$P_{T_0}(1) = 0.2 \times 0.5 \times 2 = 0.2$

$P_{T_0}(2) = 0.2 \times 0.3 \times 2 + 0.5 \times 0.5 = 0.37$

$P_{T_0}(3) = 0.5 \times 0.3 \times 2 = 0.3$

$P_{T_0}(4) = 0.3 \times 0.3 = 0.09$

b) $\mu_{T_0} = 0 + 0.2 + 0.74 + \cancel{0.9} + 0.36 = \boxed{2.2 = 2\mu}$

c) $\sigma^2_{T_0} = 0 + 0.2 + 4 \times 0.37 + 9 \times 0.3 + 16 \times 0.09 - 2.2^2 = \boxed{0.98 = 2\sigma^2}$

d) $E(T_0) = \boxed{4.4}$

$V(T_0) = \boxed{1.96}$

e) $P(T_0 = 8) = 0.09 \times 0.09 = \boxed{0.0081}$

$P(T_0 \geq 7) = P(T_0 = 8) + P(T_0 = 7)$

$= 0.0081 + 0.3 \times 0.09 \times 2$

$\boxed{= 0.0621}$

5.40 a)

$M: 0, 5, 10.$

$P_X(0) = \frac{5}{10} = 0.5$

$P_X(5) = \frac{3}{10} = 0.3$

$P_X(10) = \frac{2}{10} = 0.2$

$P_M(0) = 0.5^3 = 0.125$

$P_M(10) = 1 - P(\text{no } \$10) = 1 - (0.5 + 0.3)^3 = 0.488$

$P_M(5) = 1 - P_M(0) - P_M(10) = 0.3870$

5.46 $\mu = 12$ $\sigma = 0.04$

a) $E(\bar{x}) = \mu = \boxed{12 \text{ cm}}$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.04}{4} = \boxed{0.01 \text{ cm}}$

b) $E(\bar{x}) = 12$ cm

$\sigma_{\bar{x}} = \frac{0.04}{8} = 0.005$ cm

c) When $n = 64$, it is more likely that $\bar{x}$ is within 0.01 cm of 12 cm.

This is because the larger the sample size, the smaller the standard deviation.

5.50 $\mu = 10{,}000$ psi , $\sigma = 500$ psi

a) $n = 40 > 30$. (Approximate normal distribution)

$$P(9900 < X < 10200) = \Phi\left(\frac{10200 - 10000}{500}\right) - \Phi\left(\frac{9900 - 10000}{500}\right)$$

$$= 0.6554 - 0.4207$$

$$= 0.2347$$

b) We probably cannot as it would not be a good approximation according to central limit theorem.

5.73 $\mu_A = 105$ ksi $\sigma_A = 8$ ksi

$\mu_B = 100$ ksi $\sigma_B = 6$ ksi

a) $E(\bar{X}) = 105$ ksi $\sigma_{\bar{X}} = \frac{8}{\sqrt{40}} = 1.2649$ ksi

$E(\bar{Y}) = 100$ ksi $\sigma_{\bar{Y}} = \frac{6}{\sqrt{35}} = 1.0142$ ksi

b) $E(\bar{X}-\bar{Y}) = 105-100 = 5$ ksi

$\sigma_{\bar{X}-\bar{Y}} = \sqrt{\sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2} = 1.6213$ ksi

$\bar{X}$ & $\bar{Y}$ are independent random variables. the results are linear combinations.

c) $P(-1 \le \bar{X}-\bar{Y} \le 1) = \Phi\left(\frac{1-5}{1.6213}\right) - \Phi\left(\frac{-1-5}{1.6213}\right)$

$= 0.0067$

d) $P(\bar{X}-\bar{Y} \ge 10) = 1 - \Phi\left(\frac{10-5}{1.6213}\right) = 0.0010$

I would not doubt that, as it has such a low possibility to have $\bar{X}-\bar{Y} \ge 10$, inferring that $\mu_1 - \mu_2$ is much lower.

6.3

a) Using sample mean to estimate :

$\overline{X} = 1.3481 \approx \bar{x}$

b) sample median :

$\tilde{X} = 1.3950 \approx \tilde{x}$

c) $\Phi\left(\frac{x-\mu}{\sigma}\right) = 0.9$

Estimate using 90th percentile of the sample.

$X = 1.7550$

d) $P(X<1.5) = \Phi\left(\frac{1.5-\mu}{\sigma}\right)$

using sample mean & sample standard deviation: $\overline{X}$ & $\hat{\sigma}$

$\hat{\sigma} = 0.3385$

$P(X<1.5) \approx \Phi\left(\frac{1.5-1.3481}{0.3385}\right) = 0.6732$

e) $S_{\bar{X}} = \frac{0.3385}{\sqrt{16}} = 0.0846$

6.9

a) An unbiased estimator for poisson distribution would be the sample mean.

$$\bar{X} = \frac{37 + 2\times42 + 3\times30 + 4\times13 + 5\times7 + 6\times2 + 7}{150} = \boxed{2.1133}$$

b) $$S_{\bar{X}} = \frac{\sigma_x}{\sqrt{n}} = \sqrt{\frac{2.113}{150}} = \boxed{0.1187}$$