Q 1 $\quad f_X(x) = \frac{2x+4}{9} \quad -2 \le x \le 1$

$\quad = 0 \quad$ otherwise.

$Y = X^2$.

i). $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

$= \int_{-2}^{x} \frac{2x+4}{9}\,dx$

$= \frac{x^2+4x}{9} \Big|_{-2}^{x}$

$= \frac{x^2+4x}{9} - \frac{4-8}{9} = \frac{x^2+4x+4}{9}$

$\Rightarrow F_X(x) = \begin{cases} 0 & x < -2 \\ \frac{(x+2)^2}{9} & -2 \le x \le 1 \\ 1 & x > 1. \end{cases}$

For $y \in [0, 1]$.

$F_Y(y) = P(Y < y) = P(X^2 < y) = P(-\sqrt{y} \le X \le \sqrt{y})$

$= F_X(\sqrt{y}) - F_X(-\sqrt{y})$

$= \frac{(\sqrt{y}+2)^2}{9} - \frac{(-\sqrt{y}+2)^2}{9}$

$= \frac{8\sqrt{y}}{9}$

For $y \in [1, 4]$.

$$F_Y(y) = F_X(1) - F_X(-\sqrt{y})$$

$$= 1 - \frac{(-\sqrt{y}+2)^2}{9}.$$

$$\Rightarrow \boxed{F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{8\sqrt{y}}{9} & 0 \le y \le 1 \\ 1 - \frac{(-\sqrt{y}+2)^2}{9} & 1 \le y \le 4 \\ 1 & y > 4 \end{cases}}$$

ii) $f_Y(y) = \frac{d F_Y(y)}{dy}$

$$= \begin{cases} \frac{4}{9} y^{-\frac{1}{2}} \\ \frac{-2(-\sqrt{y}+2) \cdot \frac{1}{2} y^{-\frac{1}{2}}}{9} \\ 0 \end{cases} = \boxed{\begin{cases} \frac{4}{9} y^{-\frac{1}{2}} & 0 \le y \le 1 \\ \frac{1 - \frac{2}{\sqrt{y}}}{9} & 1 \le y \le 4 \\ 0 & \text{otherwise} \end{cases}}$$

iii) $P(Y > 1) = 1 - P(Y < 1)$.

$$= 1 - \frac{8}{9} = \boxed{\frac{1}{9}}$$

Q. 2. $\underline{X}$ is uniform over $[0, 2]$.

$$f_Y(y) = 2y \quad \text{if } 0 \le y \le 1$$
$$= 0 \quad \text{otherwise.}$$

x & y independent.

$$f_{\underline{X}}(x) = \frac{1}{2} \quad 0 \le x \le 2$$
$$0 \quad \text{otherwise.}$$

$r^2 = x^2 + y^2 \quad [0, 5]$.

i). $P(x^2 + y^2 \le 1)$.

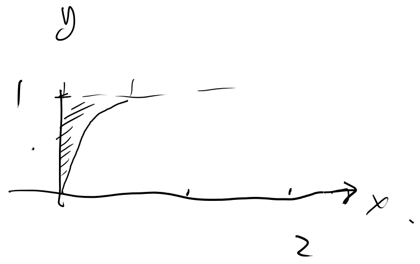
$f_{\underline{X}Y}(x, y) = f_{\underline{X}}(x) \cdot f_Y(y)$

$= y$.

$$= \iint_{x^2+y^2 \le 1} f_{\underline{X}Y}(x, y)\, dx\, dy$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 r \cdot \sin(\theta) \cdot r\, dr\, d\theta$$

$$= \boxed{\frac{1}{3}}$$



ii). $P(y^2 \le 2x) = P(\frac{y^2}{2} \le x)$.

$$= 1 - \int_0^1 \int_{\frac{y^2}{2}}^{\frac{1}{2}} y\, dx\, dy.$$

$$= \boxed{\frac{7}{8}}$$

iii) $Z = \max\{X, Y\}$. Find $f_Z(z)$.

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 < x < 2 \\ 1 & x > 2 \end{cases} \qquad F_Y(y) = \begin{cases} 0 & y < 0 \\ y^2 & 0 < y < 1 \\ 1 & \end{cases}$$

$$F_Z(z) = P(\max\{X, Y\} < z)$$

$$= P(X \leq z \cap Y \leq z).$$

$$= \frac{z \cdot z^2}{2}. \quad \text{for} \quad 0 < z < 1$$

$$\Rightarrow \boxed{F_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{z^3}{2} & 0 \leq z \leq 1 \\ \frac{z}{2} & 1 \leq z \leq 2 \\ 1 & 2 < z \end{cases}}$$

Q3. $f_{XY}(x,y) = a x^2 y^2$ for $0 \le x \le y \le 1$

$= 0$ otherwise.

i)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a\, x^2 y^2 \, dxy = 1.$$

$$a \int_0^1 \int_0^y x^2 y^2 \, dx\, dy = 1$$

$$\frac{a}{18} = 1$$

$$\boxed{a = 18}$$

ii)

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dy$$

$$= 18 \cdot \int_x^1 x^2 y^2 \, dy$$

$$= 18 \cdot x^2 \cdot \frac{y^3}{3} \Big|_x^1$$

$$= \boxed{6\,(x^2 - x^5)}.$$

Q.4. Binomial R.V. $n=5$.

$P(X=3) = 0.2637$.

$P(X=4) = 0.3955$.

Find p.

$\binom{n}{k} p^k (1-p)^{n-k}$.

10, −20, 10. 0 0 −0.2637

$0.2637 = 10 \cdot p^3 \cdot (1-p)^2 = 10(p^5 - 2p^4 + p^3)$ (1)

$0.3955 = 5 \cdot p^4 (1-p) = 5(p^4 - p^5)$. (2)

−5, 5, 0, 0. 0, −0.3955

roots for the 2 equations are.

(1) $\begin{cases} 1.1344 \\ 0.75 \\ 0.4361 \\ -0.1602 \pm 0.2131i \end{cases}$, (2) $\begin{cases} 0.8444 \\ 0.75 \\ -0.0568 \pm 0.5065i \\ -0.4808 \end{cases}$

The common root is $\boxed{p = 0.75}$