1. $R_{♂} = 3397$, $\mu_{♂} = 42828.314258067$

$r_p = 1.1 R_{♂}$, $r_a = 6.0 R_{♂}$. $\theta_c^* = 90°$ @ t_c .

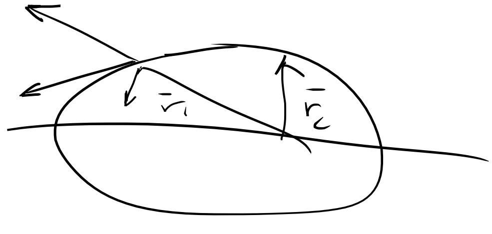
single maneuver to circularize the orbit.

$$a = \frac{r_p + r_a}{2} = 3.55 R_{♂} = 1.2059 \times 10^4 \text{ km}$$

$$e = 1 - \frac{r_p}{a} = 0.6901$$

$$p = a(1-e^2) = 6.3155 \times 10^3 \text{ k} \qquad r_c = p$$

a) $$4.5 R_{♂} = r_1 = \frac{p}{1 + e\cos\theta_1^*}$$



$$\theta_1^* = \cos^{-1}\left[\left(\frac{p}{r_1} - 1\right) \cdot \frac{1}{e}\right]$$

$$\theta_1^* = \pm 148.2487°$$

The earliest opportunity after t_c to reach r_1 is at $\underline{\theta_1^* = 148.2487°}$ ascending

$$v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a}} = \boxed{1.4325 \text{ km/s}}$$

$$\gamma_1 = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_1 v_1}\right) = \boxed{+41.3171°} \text{ ascending}$$

$$E_1 = 2 \cdot \tan^{-1}\left(\tan\left(\frac{\theta_1^*}{2}\right) \cdot \sqrt{\frac{1-e}{1+e}}\right)$$

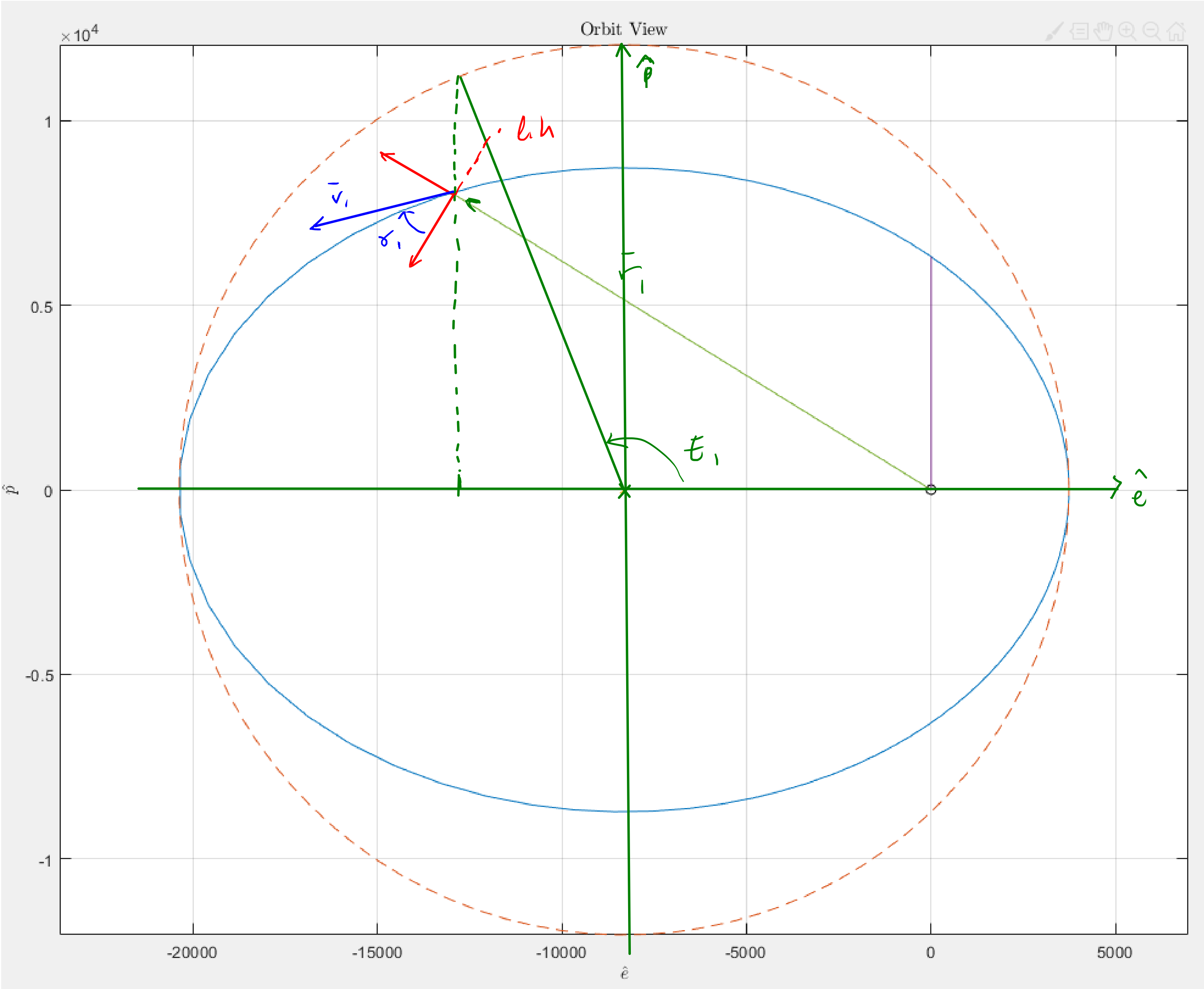
$$= 1.9690 \text{ rad}$$

$$\boxed{= 112.8149°}$$

$$t_1 - t_p = \sqrt{\frac{a^3}{\mu}}\,(E_1 - e\sin E_1)$$

$$= \boxed{8.5290 \times 10^3 \text{ sec}}$$

b)



c) $E_c = 2 \cdot \tan^{-1}\left(\tan 45^\circ \cdot \sqrt{\frac{1-e}{1+e}}\right)$

$= 0.8091 \text{ rad} = 46.3587^\circ$

$t_c - t_p = \sqrt{\frac{a^3}{\mu}} \cdot (E_c - e \sin E_c) = 1.9816 \times 10^3 \text{ sec}$

$t_1 - t_c = \boxed{6.5474 \times 10^3 \text{ sec}}$

d) $r_1^+ = r_1$, on circular orbit, $e = 0$. $r_1 = a = p$

$= \boxed{1.5287 \times 10^4 \text{ km}}$

$v_1 = \sqrt{\frac{\mu}{r_1}}$

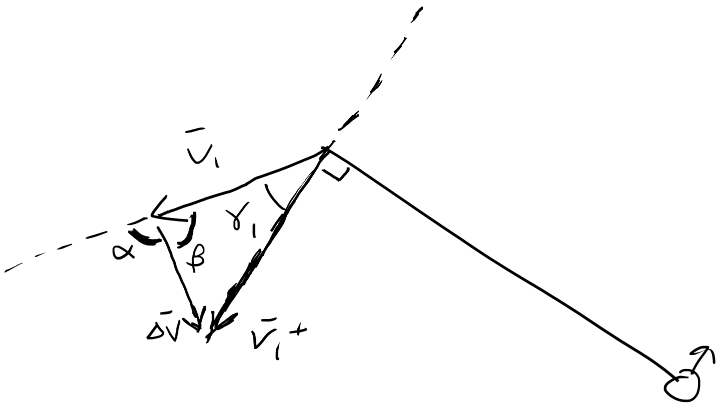
$\boxed{= 1.6738 \text{ km/s}}$

$\gamma_1^+ = \cos^{-1} \frac{\sqrt{\mu p}}{r_1^+ v_1^+} = \cos^{-1} \frac{\sqrt{\mu / r_1^+}}{v_1^+} = \boxed{0^\circ}$ circular orbit.

$\Delta\gamma_1 = \gamma_1$

$\Delta V = \sqrt{v_1^{+2} + v_1^2 - 2 v_1 v_1^+ \cos\Delta\gamma_1}$

$\boxed{= 1.1189 \text{ km/s}}$



$\frac{\Delta V}{\sin\Delta\gamma_1} = \frac{v_1^+}{\sin\beta}$

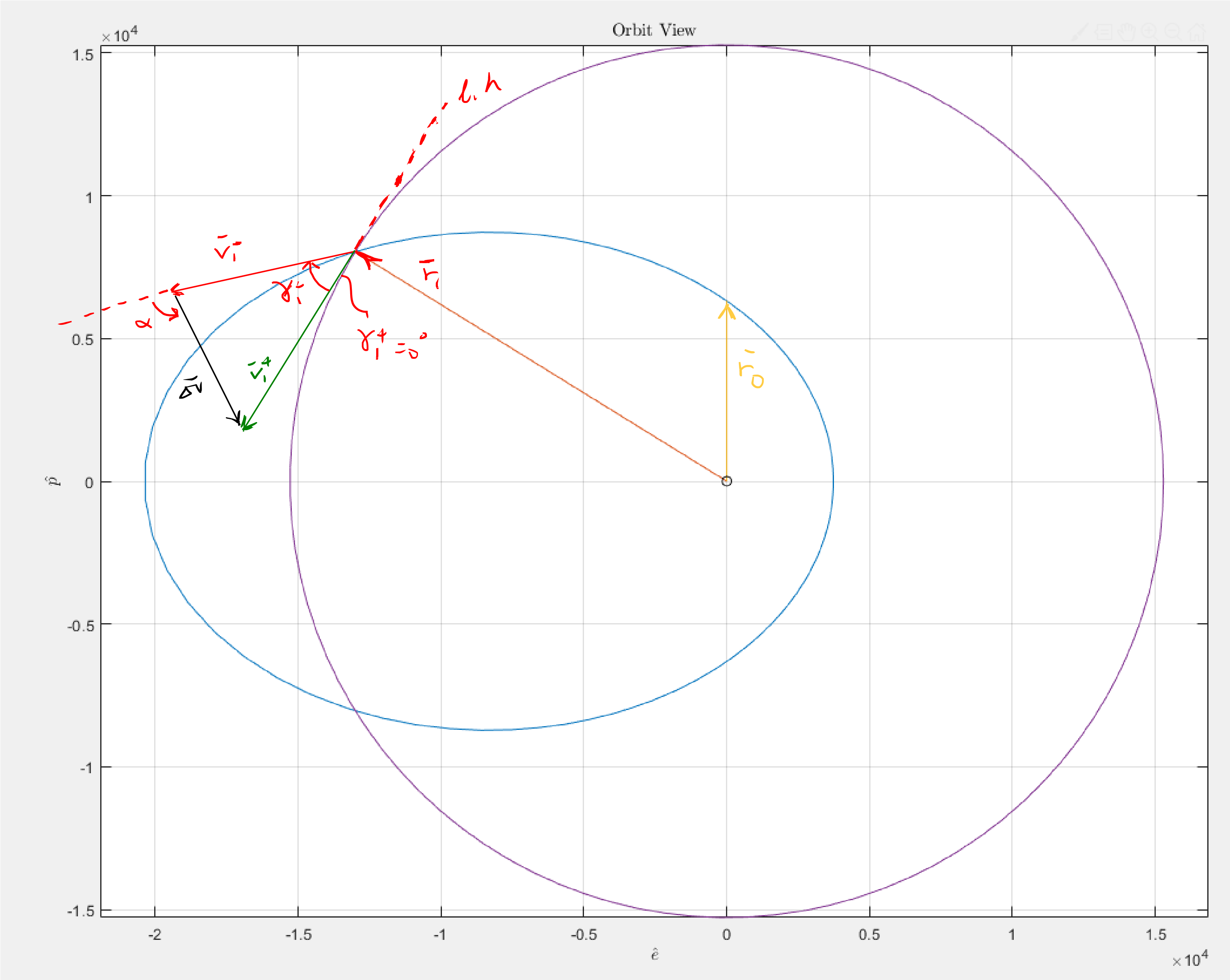
$\beta = \sin^{-1}\left(\frac{v_1^+}{\Delta V} \cdot \sin\Delta\gamma_1\right)$

$\beta = 80.9862^\circ$

$\alpha = 180 - \beta$

$= \boxed{99.0138^\circ}$

e)



f)

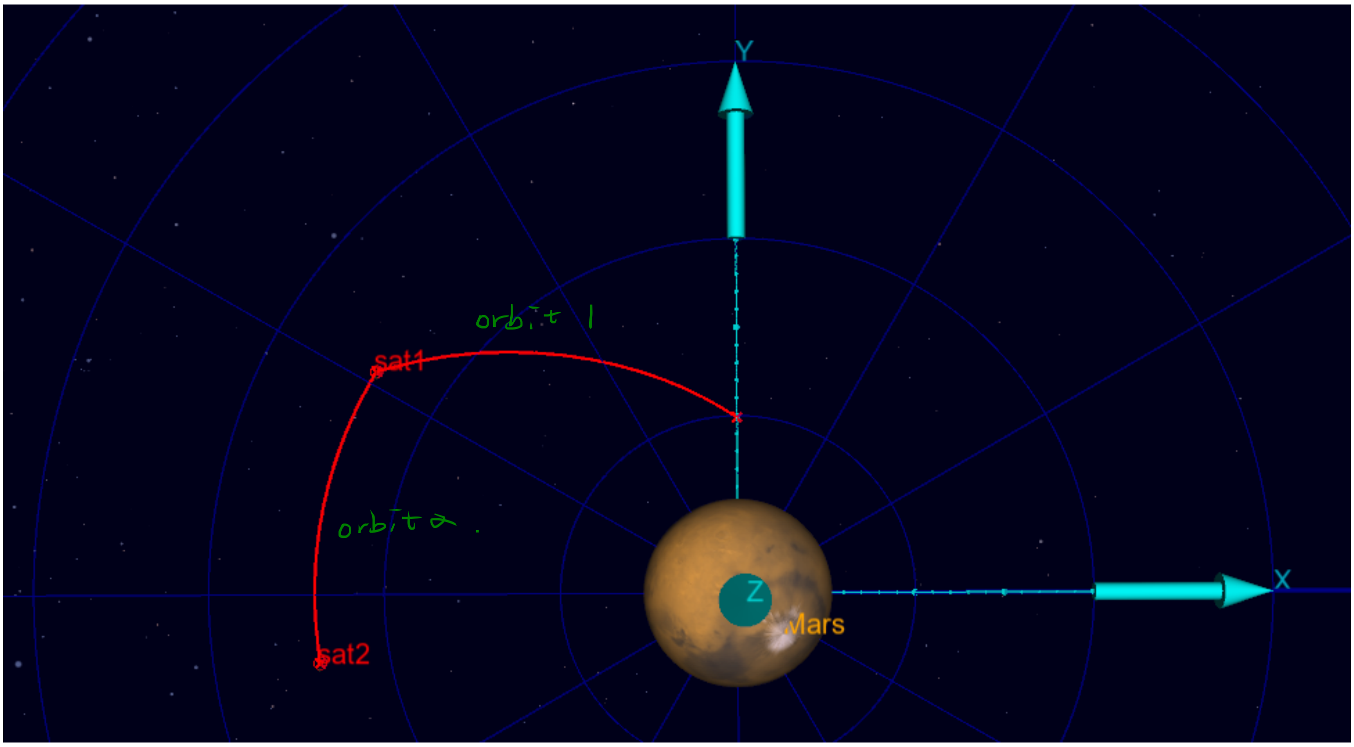
$\bar{v}_1^-$			$\bar{v}_1^+$		
v_x	v_y	v_z	sat2.MarsMJ2000Eq.VX	sat2.MarsMJ2000Eq.VY	sat2.MarsMJ2000Eq.VZ
-1.370216313540919	-0.4173318414722971	0	-0.8808107697051142	-1.423302171925606	0

$$\Delta \bar{v} = \bar{v}_1^+ - \bar{v}_1^- = \underline{0.4894\,\hat{x} - 1.0060\,\hat{y} \quad km/s}$$

$$|\Delta \bar{v}| = \underline{1.1187 \text{ km/s} \simeq 1.1189 \text{ km/s}}$$

The ΔV calculated using GMAT is very close to the one calculated in (d). This could be because of numerical error, or slight difference in certain parameter values.

orbit 2 started at where orbit 1 ended



2. $e = 0.4$ $\quad a = 4R_{\oplus}$ $\quad$ single in-plane maneuver at $\underline{\theta^* = 135°}$ ascending

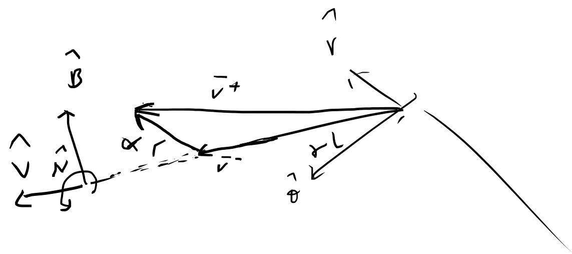
$|\Delta\bar{v}| = 0.90$ km/s, $\alpha = +45°$

a) $p = a(1-e^2) = 2.1431 \times 10^4$ km

$$r = \frac{p}{1 + e\cos\theta^*} = 2.9883 \times 10^4 \text{ km}$$

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = 3.3248 \text{ km/s}$$

$$\gamma = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r \cdot v}\right) = \boxed{21.5240°}$$



$\bar{r} = r\hat{r}$

$$\boxed{= 2.9883\ \hat{r} \text{ km}}$$

$\bar{v} = v \cdot (\sin\gamma\ \hat{r} + \cos\gamma\ \hat{\theta})$

$$\boxed{= 1.2198\ \hat{r} + 3.0929\ \hat{\theta} \text{ km/s}}$$

b)

$$\Delta\bar{v}^{r\theta h} = |\Delta\bar{v}| \cdot (-\sin(\alpha+\gamma)\ \hat{r} + \cos(\alpha+\gamma)\ \hat{\theta})$$

$$\boxed{= 0.8255\ \hat{r} + 0.3585\ \hat{\theta} \text{ km/s}}$$

$$R = \begin{bmatrix} c\theta^* & -s\theta^* & 0 \\ s\theta^* & c\theta^* & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.7071 & -0.7071 & 0 \\ 0.7071 & -0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

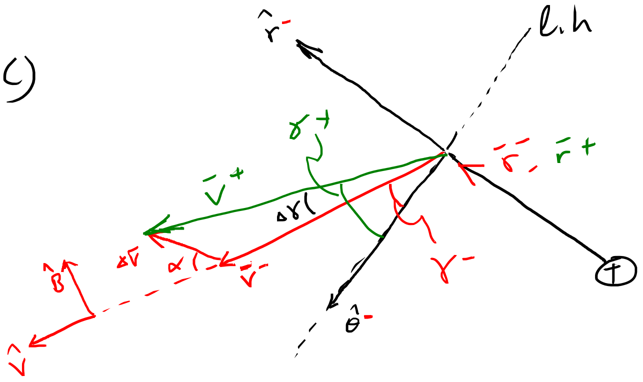
$\Delta\bar{v}^{eph} = R \cdot \Delta\bar{v}^{r\theta h}$

$$\boxed{= -0.8372\ \hat{e} + 0.3302\ \hat{p} \text{ km/s}}$$

$\Delta\bar{v}^{VNB} = |\Delta\bar{v}| \cdot (\cos\alpha\ \hat{v} + \sin\alpha\ \hat{B})$

$$= \boxed{0.6364\ \hat{v} + 0.6364\ \hat{B} \text{ km/s}}$$

c)



$$r^+ = r^- = \boxed{2.9883 \times 10^4 \text{ km}}$$

$$\bar{r}^+ = \bar{r}^- = 2.9883 \times 10^4 \ \hat{r} \text{ km}$$

$$\bar{v}^+ = \bar{v}^- + \Delta\bar{v}$$

$$= 2.0453 \ \hat{r} + 3.4514 \ \hat{\theta} \text{ km/s}$$

$$v^+ = |\bar{v}^+| = \boxed{4.0120 \text{ km/s}}$$

$$\beta = 135^\circ$$

$$\sin\Delta\gamma = \frac{\Delta v}{v^+} \cdot \sin\beta$$

$$\Delta\gamma = 9.1271^\circ$$

$$\gamma^+ = \gamma^- + \Delta\gamma$$

$$\boxed{= 30.6511^\circ}$$

d)

$$\gamma^+ = \cos^{-1}\left(\frac{\sqrt{\mu p^+}}{r^+ v^+}\right)$$

$$p^+ = (\cos\gamma^+ \cdot r^+ v^+)^2 / \mu$$

$$= 2.6687 \times 10^4 \text{ km}$$

$$a^+ = -\frac{\mu}{2\left(\frac{v^{+2}}{2} - \frac{\mu}{r^+}\right)}$$

$$\boxed{= 3.7668 \times 10^4 \text{ km}}$$

$$e^+ = \sqrt{1 - \frac{p^+}{a^+}} = \boxed{0.5399}$$

$$h^+ = |\bar{r}^+ \times \bar{v}^+| = r^+ v^+ \cos\gamma^+ = \boxed{1.0314 \times 10^5} \text{ km}^2/\text{s}$$

$$\mathbb{P}^+ = 2\pi\sqrt{\frac{a^{+3}}{\mu}}$$

$$\boxed{= 7.2756 \times 10^4 \text{ sec}}$$

$$\mathcal{E} = -\frac{\mu}{2a}$$

$$\boxed{= -5.2910 \text{ km}^2/\text{s}^2}$$

$$\theta^{*+} = \cos^{-1}\left(\left(\frac{p^+}{r^+} - 1\right) \cdot \frac{1}{e^+}\right)$$

$$\boxed{= 101.4239^\circ}$$

$$E^+ = 2 \cdot \tan^{-1}\left(\tan\left(\frac{\theta^{*+}}{2}\right) \cdot \sqrt{\frac{1-e}{1+e}}\right)$$

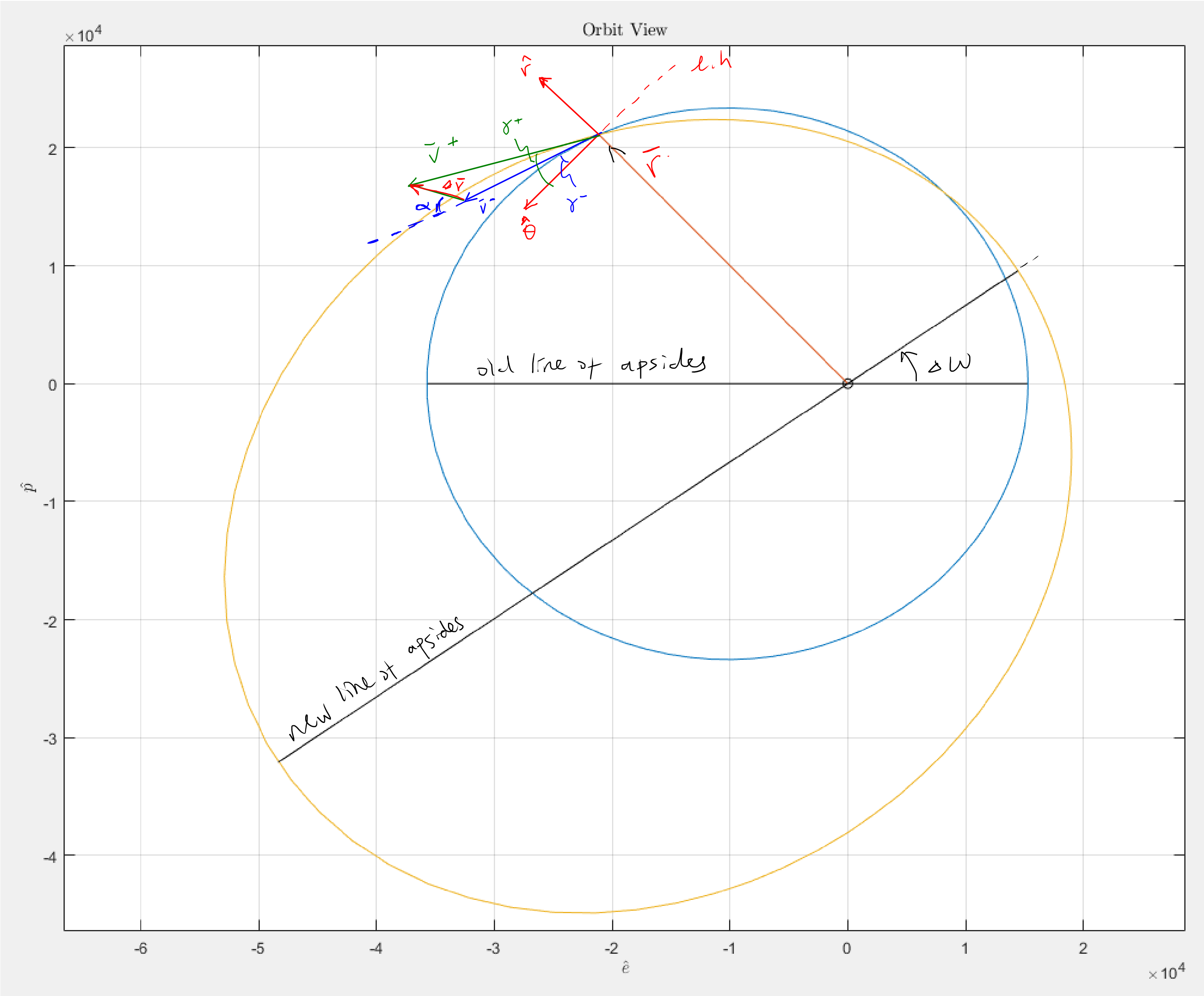
$$\boxed{= 67.4930^\circ}$$

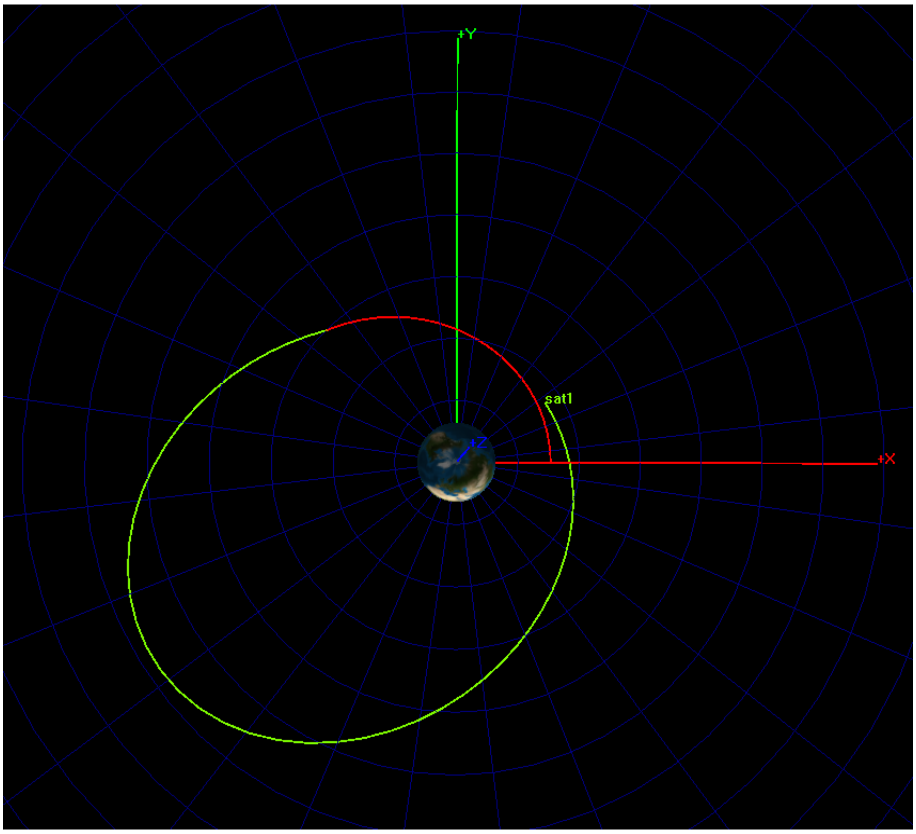
$$t - t_p = \sqrt{\frac{a^{+3}}{\mu}} \cdot (E^+ - e^+ \cdot \sin E^+) = \boxed{7.8645 \times 10^3 \text{ sec}}$$

$$r_p = a^+(1 - e^+) = \boxed{1.7330 \times 10^4 \text{ km}}$$

$$\Delta\omega = \theta^{*-} - \theta^{*+} = \boxed{33.5761^\circ}$$

e) $\delta\omega > 0$

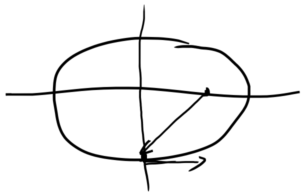




GMAT analysis checks out.

3、 $a = 3R_\oplus$, $e = 0.6$, raise perigee, $r_p^+ = 2R_\oplus$, $e^+ = 0.4$

$E = -90^\circ$



, $r = a$

a) $r^- = a$

$p^- = a^-(1-e^{-2}) = 2.1431\times10^4$ km

$\bar{r}^- = a\,\hat{r}$

$= \boxed{2.5513\times10^4\,\hat{r}\ \text{km}}$

$\theta^{*-} = 2\cdot\tan^{-1}\left(\tan\frac{E}{2}\cdot\sqrt{\frac{1+e}{1-e}}\right)$

$= -126.8699^\circ$

$v^- = \sqrt{\frac{2\mu}{r^-} - \frac{\mu}{a}} = 3.9527$ km/s

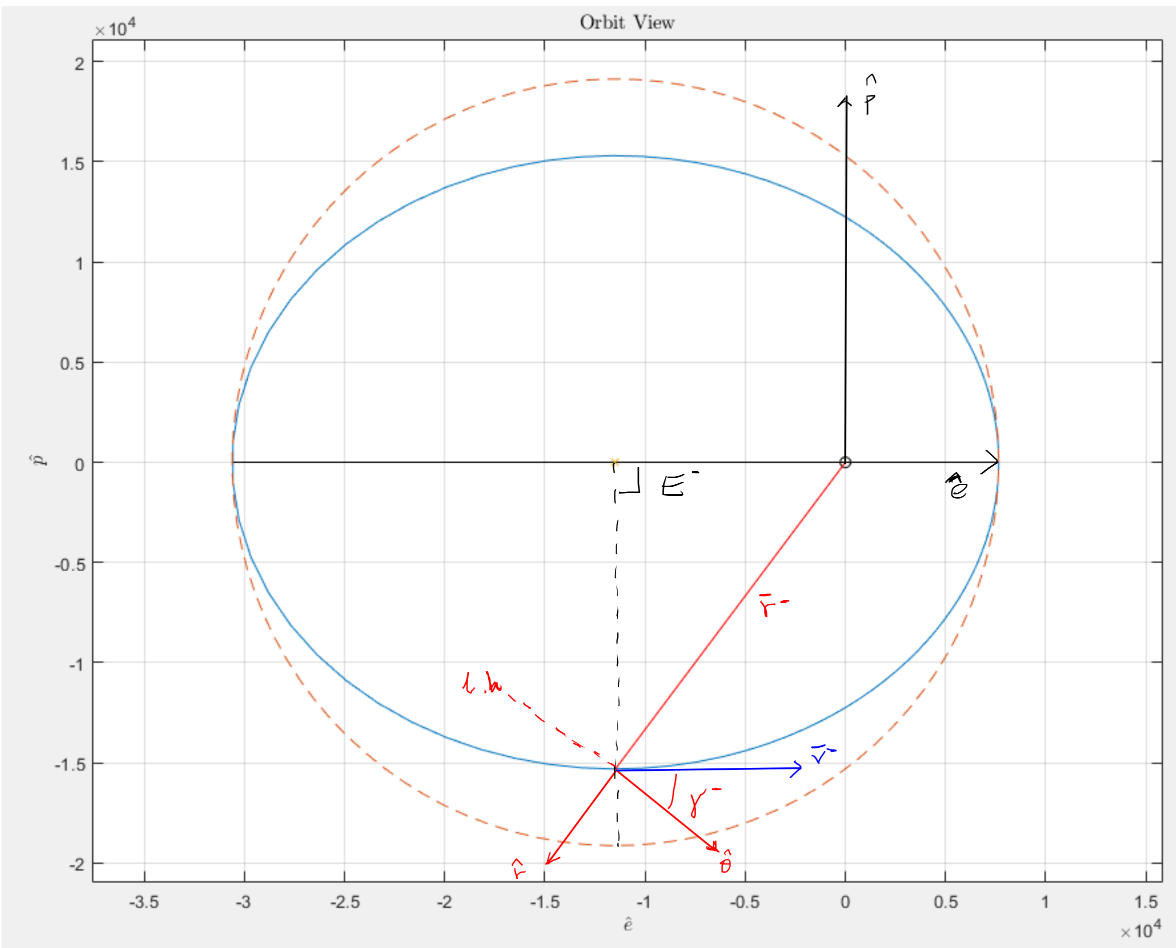
$\gamma^- = \cos^{-1}\left(\frac{\sqrt{\mu p^-}}{r^- v^-}\right) = \boxed{-36.8699^\circ}$

$\bar{v}^- = v^-(\sin\gamma^-\,\hat{r} + \cos\gamma^-\,\hat{\theta})$

$\boxed{= -1.5811\,\hat{r} + 3.6227\,\hat{\theta}\ \text{km/s}}$

$t - t_p = \sqrt{\frac{a^3}{\mu}}\,(E^- - e\sin E^-)$

$= \boxed{-4.0659\times10^3\ \text{sec}}$



b) $e^+ = 0.4$, $r_p^+ = 2R_\oplus$

$$a^+ = \frac{r_p^+}{1-e^+}$$

$$= 2.1260\times10^4 \text{ km}$$

$$p^+ = r_p^+(1+e^+)$$

$$= 1.7859\times10^4 \text{ km}$$

$$\theta^{*+} = \cos^{-1}\left(\frac{p^+ - r^+}{r^+ e^+}\right)$$

$$= \pm 99.5941^\circ$$

$$v^+ = \sqrt{\frac{2\mu}{r^+} - \frac{\mu}{a^+}}$$

$$= 4.7869 \text{ km/s}$$

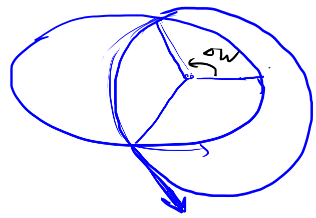
$$\gamma^+ = \cos^{-1}\left(\frac{\sqrt{\mu p^+}}{r^+ v^+}\right)$$

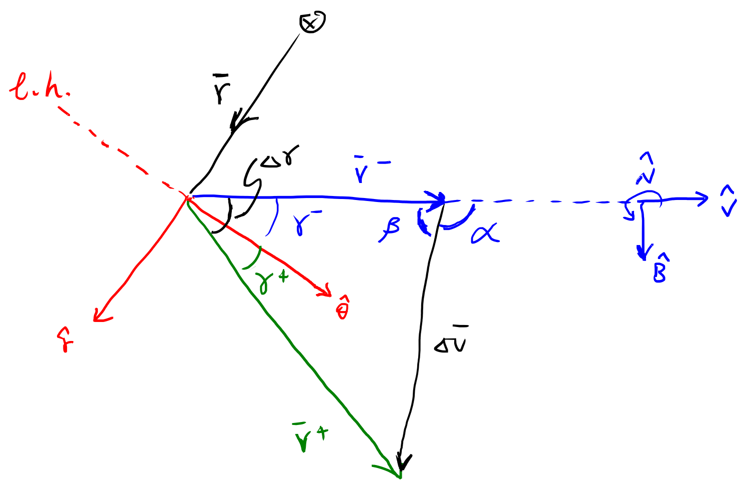
$$= \pm 22.9078^\circ$$

To raise perigee, $\Delta\omega > 0$

$\Rightarrow \theta^* = +99.5941^\circ$

$$\boxed{\gamma^+ = +22.9078^\circ}$$





$$\Delta\gamma = \gamma^+ - \gamma^- = 59.7777^\circ$$

$$\bar{r}^+ = \bar{r}^- = \boxed{1.9134\times10^4\ \hat{r} \text{ km}}$$

$$\bar{v}^+ = v^*\cdot(\sin\gamma^+\ \hat{r} + \cos\gamma^+\ \hat{\theta})$$

$$= \boxed{1.8633\ \hat{r} + 4.4094\ \hat{\theta} \quad \text{km/s}}$$

$$\Delta\bar{v}^{r\theta h} = \bar{v}^+ - \bar{v}^-$$

$$= \boxed{4.6018\ \hat{r} + 0.7518\ \hat{\theta} \quad \text{km/s}}$$

$$\Delta v = \|\Delta\bar{v}\| = \boxed{4.6638 \text{ km}}$$

$$\beta = \sin^{-1}\left(\frac{v_n}{\Delta v}\cdot\sin\Delta\gamma\right)$$

$$= 62.4886^\circ$$

$$\alpha = 180^\circ - \beta$$

$$= 117.5154^\circ$$

$$\Delta\bar{v}^{VNB} = \Delta v\cdot(\cos\alpha\ \hat{V} + \sin\alpha\ \hat{B})$$

$$= \boxed{-2.1546\ \hat{V} + 4.1363\ \hat{B} \quad \text{km/s}}$$

c)

$\boxed{a^{+} = 2.2160 \times 10^{4} \text{ km}}$

$\boxed{e^{+} = 0.4}$

$\boxed{r_p^{+} = 1.2756 \times 10^{4} \text{ km}}$

$r_a^{+} = 2a^{+} - r_p^{+}$

$\boxed{= 2.9765 \times 10^{4} \text{ km}}$

$E^{+} = 2 \cdot \tan^{-1}\left(\tan\left(\frac{\theta^{*+}}{2}\right) \cdot \sqrt{\frac{1-e^{+}}{1+e^{+}}}\right)$

$= \boxed{75.5225^\circ}$

$\boxed{\gamma^{+} = 22.9078^\circ}$

$\mathbb{P} = 2\pi \cdot \sqrt{\frac{a_n^3}{\mu}}$

$\boxed{= 3.0851 \times 10^{4} \text{ sec}}$

$\mathcal{E} = -\frac{\mu}{2a_n}$

$= \boxed{-9.3742 \text{ km}^2/\text{s}^2}$

$\boxed{t - t_p = 3.6196 \times 10^{3} \text{ sec}}$

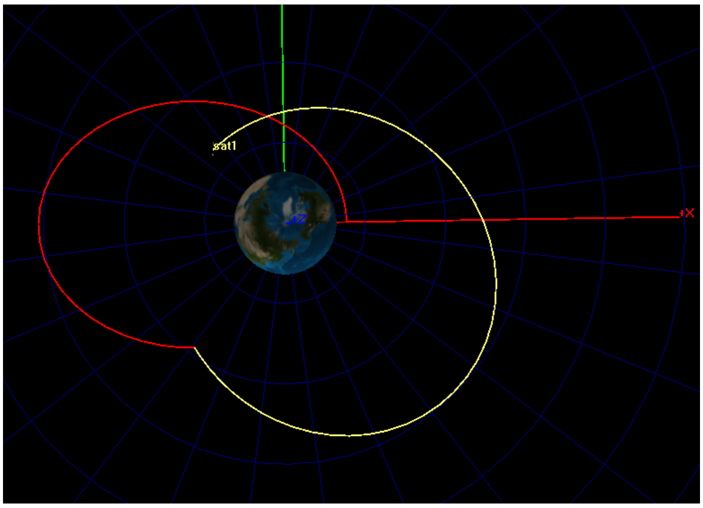
$\Delta\omega = \theta^{*-} - \theta^{*+}$

$= -226.4640^\circ$

$= \boxed{133.5360^\circ}$

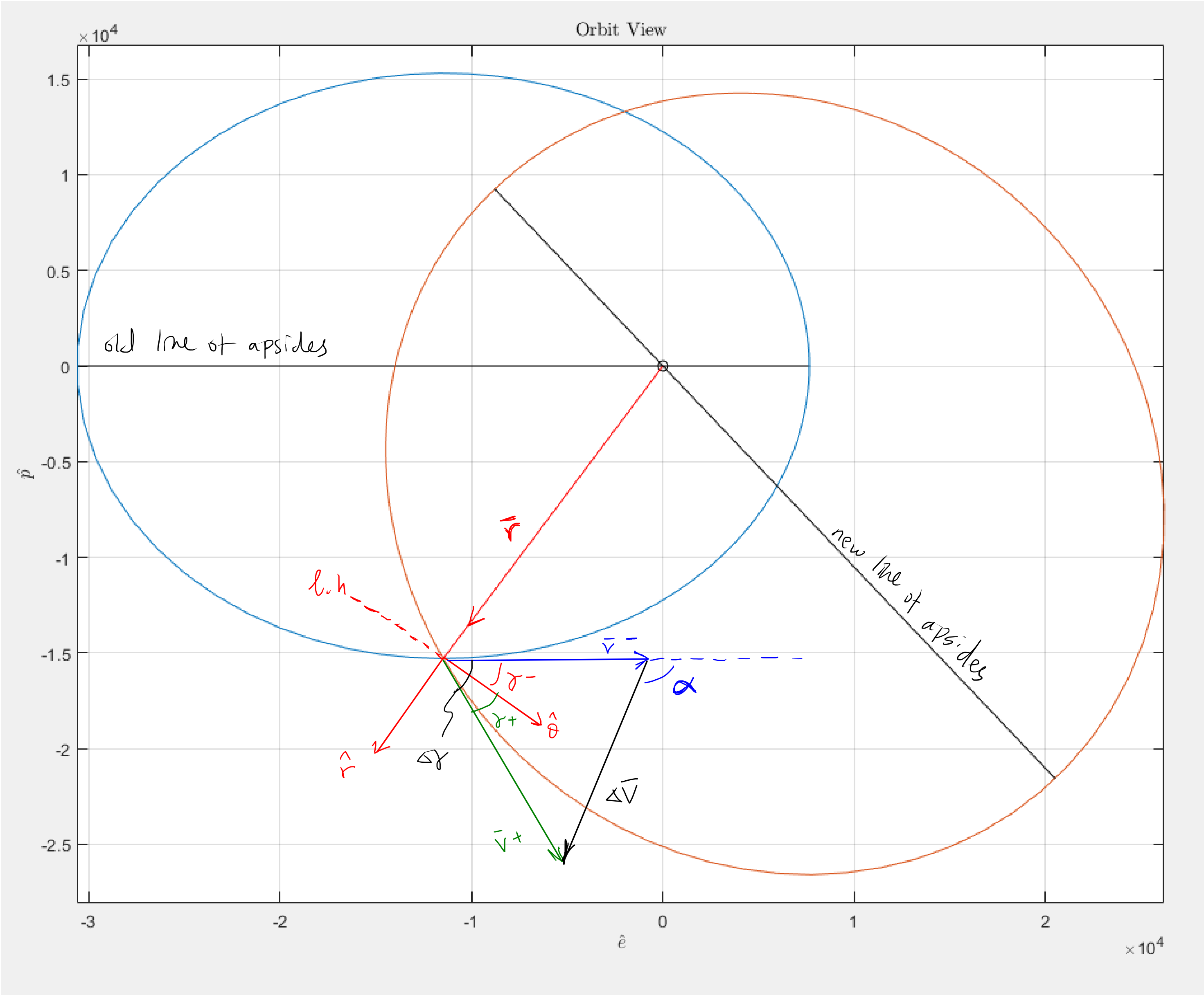
$\mathbb{P} - (t - t_p) = \boxed{2.7231 \times 10^{4} \text{ sec}}$
$= \boxed{0.3152 \text{ days}}$

time to reach perigee in new orbit.



GMAT results check out

d)



Orbit View
old line of apsides
new line of apsides
$\bar{r}$
$\hat{r}$
$\hat{\theta}$
l.h
$\bar{v}^-$
α
γ^-
γ^+
$\Delta\gamma$
$\Delta\bar{V}$
$\bar{v}^+$
$\hat{p}$
$\hat{e}$
$\times 10^4$
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-2
-1
0
1
2
$\times 10^4$