Ex. 1

$$2\ddot{q}_1 + \ddot{q}_2 + \sin q_1 = 0$$
$$\ddot{q}_1 + 2\ddot{q}_2 + \sin q_2 = 0.$$

$$\begin{cases} 3\ddot{q}_1 + 2\sin q_1 - \sin q_2 = 0 \\ 3\ddot{q}_2 + 2\sin q_2 - \sin q_1 = 0 \end{cases}$$

$$\begin{cases} \ddot{q}_1 = \frac{1}{3}\sin q_2 - \frac{2}{3}\sin q_1 \\ \ddot{q}_2 = \frac{1}{3}\sin q_1 - \frac{2}{3}\sin q_2 \end{cases}$$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$ $\boxed{\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{1}{3}\sin x_2 - \frac{2}{3}\sin x_1 \\ \frac{1}{3}\sin x_1 - \frac{2}{3}\sin x_2 \end{bmatrix}}$

Ex. 2.

$$\ddot{q}_1 + \dot{q}_2 + q_1^3 = 0$$
$$\dot{q}_1 + \dot{q}_2 + q_2^3 = 0$$

$$\begin{cases} \ddot{q}_1 = -\dot{q}_2 - q_1^3 \\ \dot{q}_2 = -\dot{q}_1 - q_2^3 \end{cases} \qquad \begin{cases} \ddot{q}_1 = q_2^3 + \dot{q}_1 - q_1^3 \\ \dot{q}_2 = -\dot{q}_1 - q_2^3 \end{cases}$$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \end{bmatrix}$

$$\boxed{\dot{x} = \begin{bmatrix} x_3 \\ -x_3 - x_2^3 \\ x_2^3 + x_3 - x_1^3 \end{bmatrix}}$$

Ex. 3. $\ddot{q}_1 + q_1 + 2\dot{q}_2 = 0$

$\ddot{q}_1 + \dot{q}_2 + q_2 = 0$

$$\begin{cases} \dot{q}_2 + q_1 - q_2 = 0 \\ \frac{1}{2}\ddot{q}_1 + q_2 - \frac{1}{2}q_1 = 0. \end{cases} \qquad \begin{cases} \dot{q}_2 = q_2 - q_1 \\ \ddot{q}_1 = q_1 - 2q_2 \end{cases}$$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \end{bmatrix}$

$$\boxed{\dot{x} = \begin{bmatrix} x_3 \\ x_2 - x_1 \\ x_1 - 2x_2 \end{bmatrix}}$$

Ex. 4

$$\begin{cases} q_1(k+2) + q_1(k) + 2q_2(k+1) = 0 \\ q_1(k+2) + q_1(k+1) + q_2(k) = 0. \end{cases}$$

$$\begin{cases} q_2(k+1) = \frac{1}{2}\left(q_1(k+1) + q_2(k) - q_1(k)\right) \\ q_1(k+2) = -q_1(k+1) - q_2(k) \end{cases}$$

$q_1(k) + 2q_2(k+1) - q_1(k+1) - q_2(k) = 0$

$2q_2(k+1) = q_1(k+1) + q_2(k) - q_1(k)$

let $x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} q_1(k) \\ q_2(k) \\ q_1(k+1) \end{bmatrix}$

$$x(k+1) = \begin{bmatrix} x_3(k) \\ \frac{1}{2}x_3(k) + \frac{1}{2}x_2(k) - \frac{1}{2}x_1(k) \\ -x_3(k) - x_2(k) \end{bmatrix}$$

Ex. 5 $\quad x(k+1) = x(k) - \dfrac{g(x(k))}{g'(x(k))}$ . show $x^e$ is an equilibrium state of the system. iff $g(x^e)=0$.

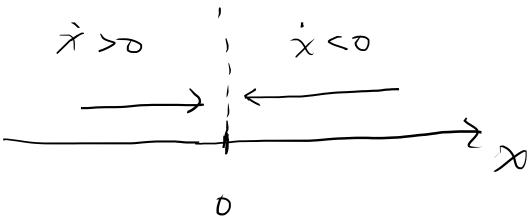
$$x^e = x^e - \frac{g(x^e)}{g'(x^e)}$$

$$0 = \frac{g(x^e)}{g'(x^e)}$$

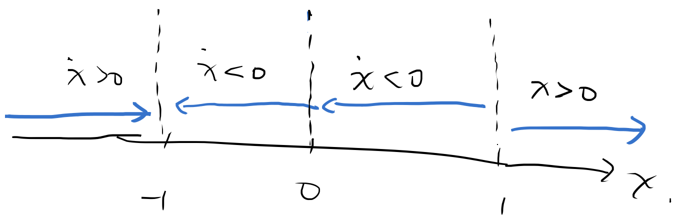
$\Rightarrow g(x^e) = 0$. (shown)

Ex 6.

$$\dot{x} = -\alpha \operatorname{sgn}(x)$$




Ex. 7 $\quad \dot{x} = x^4 - x^2 = x^2(x^2-1)$



Ex 8.

$$\dot{x} = -x^3$$

$$\frac{dx}{dt} = -x^3$$

$$\frac{dx}{-x^3} = dt$$

$$\int \frac{1}{-x^3}\,dx = \int dt$$

$$\frac{x^{-2}}{2} = t + c$$

$$\boxed{x = \pm\sqrt{\frac{1}{2(t+c)}}}$$

$$x_0 = \sqrt{\frac{1}{2c}}$$

$$c = \frac{1}{2x_0^2}$$

$$x = \pm\sqrt{\frac{1}{2(t+\frac{1}{2x_0^2})}}$$

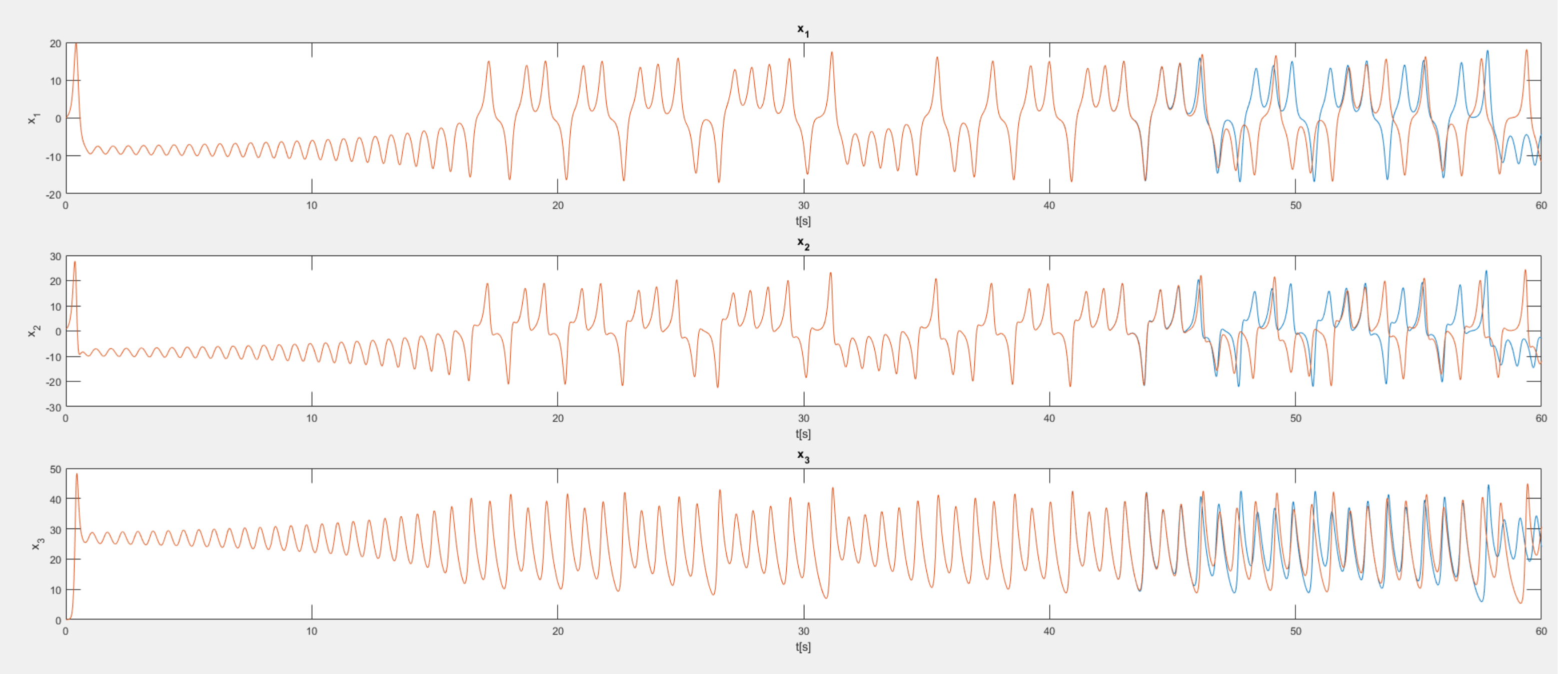
Ex. 9

$$\dot{x}_1 = \sigma(x_2 - x_1)$$

$$\dot{x}_2 = r x_1 - x_2 - x_1 x_3$$

$$\dot{x}_3 = -b x_3 + x_1 x_2$$

$\sigma = 10$, $b = \frac{8}{3}$, $r = 28$. simulate with $\bar{x}_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 1+eps \\ 0 \end{bmatrix}$




It's quite evident that small deviation in initial conditions can cause big difference in system behavior as time progresses, even though the system behaved similarly in the first 45 seconds.