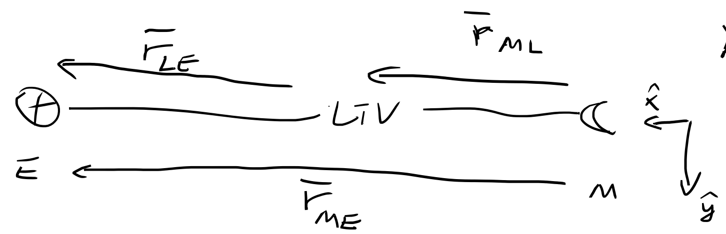
Exam 2. Zhanpeng Yang.

☾ − M(oon)
⊕ − E(arth)
LTV − L(TV)

1. $\mu_{LTV} <<< \mu_{☾}$

a) $r_{ML} = 50\, r_{☾}$



using EOM of m_i relative to m_q.

$$\ddot{\bar{r}}_{qi} + \frac{G(m_i + m_q)}{r_{qi}^3}\bar{r}_{qi} = G\sum_{j=1}^{n} m_j\left(\frac{\bar{r}_{ij}}{r_{ij}^3} - \frac{\bar{r}_{qj}}{r_{qj}^3}\right)$$

Analyze the dominant acceleration from the moon, and perturbation cause by the Earth, we can determine if 2BD model is suitable for this problem.

$$\ddot{\bar{r}}_{ML} + \underbrace{\frac{\cancel{\mu_L} + \mu_M}{r_{ML}^3}\cdot\bar{r}_{ML}}_{\text{dominant term}} = \mu_E\cdot\underbrace{\left(\frac{\bar{r}_{LE}}{r_{LE}^3} - \frac{\bar{r}_{ME}}{r_{ME}^3}\right)}_{\text{perturbing term}}$$

$$\ddot{\bar{r}}_{ML} = \mu_E\left(\frac{1}{r_{LE}^2} - \frac{1}{r_{ME}^2}\right) - \mu_M\cdot\frac{1}{r_{ML}^2}$$

$\mu_E = 4\times10^5\ km^3/sec^2$

$\mu_M = 5000\ km^3/sec^2$

$r_{ML} = 50\cdot r_{☾} = 50\cdot1500 = 7.5\times10^4\ km$

$r_{ME} = 4\times10^5\ km$

$r_{LE} = r_{ME} - r_{ML} = 3.25\times10^5\ km$

$$\ddot{\bar{r}}_{ML} = \overset{\text{direct}}{3.7870\times10^{-6}} - \overset{\text{indirect}}{2.5\times10^{-6}} - 8.8889\times10^{-7}$$

$$= \underbrace{1.2870\times10^{-6}}_{\text{From Earth}} - \underbrace{8.8889\times10^{-7}}_{\text{From Moon.}}$$

As the EOM shows, the Earth's perturbing acceleration is significantly larger than dominant acceleration from the Moon, thus a 2BD-model is <u>NOT</u> suitable for this problem.

b) $p = 4R_{\zeta} = 6000$ km. $\qquad \mu = \mu_M = 5000\ km^3/s^2$

$e = \frac{1}{\sqrt{3}} \qquad a = \frac{p}{1-e^2} = 1.5p = 9000$ km

After maneuver.

$\mathcal{E}_N = +\frac{1}{6}\ km^2/s^2 \qquad$ Hyperbolic orbit.

$h_N = 8200\ km^2/s$

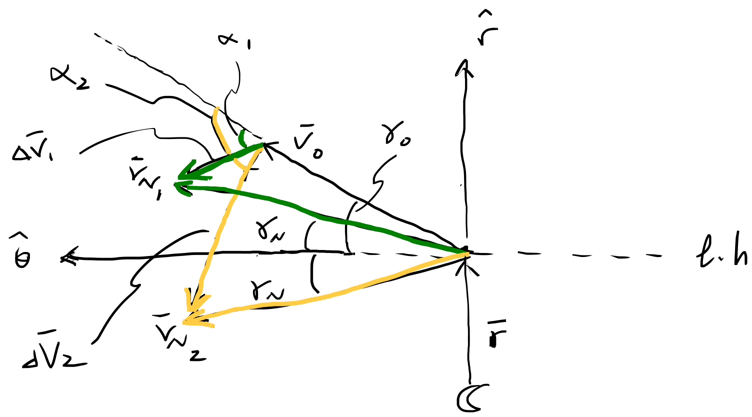
i) At the end of latus rectum, $\theta_0^* = 90°$

$$r_0 = \frac{p}{1+e\cos(\theta_0^*)} = 6000\ km$$

$$V_c = \sqrt{\frac{\mu}{r_0}} = \sqrt{5000/6000} = 0.9129\ km/s$$

$$V_e = \sqrt{2}\,V_c = \boxed{1.2910\ km/s}$$

ii)



There are two options for the new $\bar{V}_N$, one is ascending ($\bar{V}_{N_1}$), and the other one is descending ($\bar{V}_{N_2}$)

It is clear that the ascending $\bar{V}_{N_1}$ requires the smallest $|\Delta\bar{V}|$ from the vector diagram.

γ_N is positive.

iii)

$$v_0 = \sqrt{\frac{2\mu}{r_0} - \frac{\mu}{a}} = \sqrt{\frac{2\times 5000}{6000} - \frac{5000}{9000}} = \frac{\sqrt{10}}{3} = \underline{1.0541 \text{ km/s}}$$

$$\gamma_0 = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_0 \cdot v_0}\right) = \cos^{-1}\left(\frac{\sqrt{5\times 6}}{6\times \sqrt{10}/3}\right) = \cos^{-1}\left(\sqrt{\frac{30}{40}}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= +30^\circ$$

$r_n = r_0 = 6000 \text{ km}$

$$v_n = \sqrt{2\left(\varepsilon_n + \frac{\mu}{r_n}\right)} = \sqrt{2\cdot\left(\frac{1}{6} + \frac{5}{6}\right)} = \sqrt{2} = \underline{1.4142 \text{ km/s}}$$

$$h_n = r_n \cdot v_n \cdot \cos\gamma_n$$

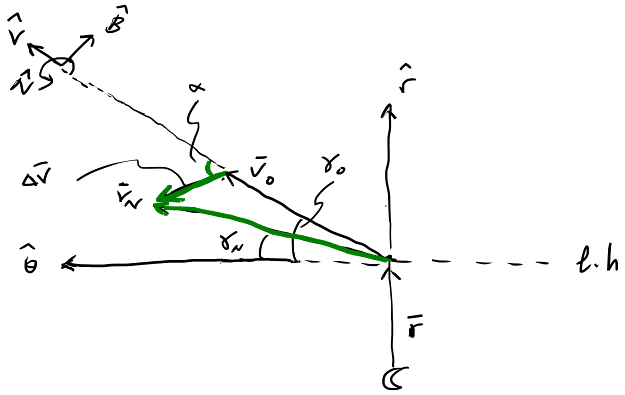
$$\gamma_n = \cos^{-1}\left(\frac{8200}{6000\cdot\sqrt{2}}\right) = +14.8993 \quad \text{(ascending)}$$

$$\Delta\gamma = \gamma_0 - \gamma_N = 15.1007^\circ$$

$$\Delta V^2 = V_n^2 + V_0^2 - 2V_n V_0 \cos(\Delta\gamma)$$

$$\Delta V = \sqrt{2 + \frac{10}{9} - 2\cdot\frac{\sqrt{20}}{3}\cdot\cos(15.1007)}$$

$$\boxed{= 0.4823 \text{ km/s}}$$



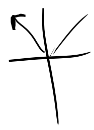
$$\frac{\Delta V}{\sin\Delta\gamma} = \frac{V_n}{\sin\beta}$$

$$\beta = \sin^{-1}\left(\frac{V_n}{\Delta V}\cdot\sin\Delta\gamma\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{2}}{0.4823}\cdot\sin(15.1007)\right)$$

$$= \sin^{-1}(0.7639)$$

$$= 49.8086^\circ,\ \underline{130.1914^\circ}$$



$$= 130.1914^\circ$$

$$\boxed{\alpha = -49.8086^\circ}$$

$$\Delta\bar{V}^{VNB} = \Delta V\cdot(\cos\alpha\,\hat{V} + \sin\alpha\,\hat{B})$$

$$= 0.4823\cdot\left[\cos(-49.8086^\circ)\,\hat{V} + \sin(-49.8086^\circ)\,\hat{B}\right]$$

$$\boxed{= 0.3112\,\hat{V} - 0.3684\,\hat{B} \text{ km/s}}$$

iv) $e_n = \sqrt{\left(\frac{r_n v_n^2}{\mu} - 1\right)^2 \cdot \cos^2\gamma_n + \sin^2\gamma_n}$

$= \sqrt{\left(\frac{6000 \cdot 2}{5000} - 1\right)^2 \cdot \cos^2(14.8993) + \sin^2(14.8993)}$

$\boxed{= 1.3771}$

$\theta_N^* = \tan^{-1}\left(\frac{\frac{r_n v_n^2}{\mu} \cos\gamma_n \cdot \sin\gamma}{\frac{r_n v_n^2}{\mu} \cos^2\gamma_n - 1}\right)$

$= \tan^{-1}\left(\frac{\frac{12}{5} \cdot \cos(14.8993) \cdot \sin(14.8993)}{\frac{12}{5} \cdot \cos^2(14.8993) - 1}\right)$

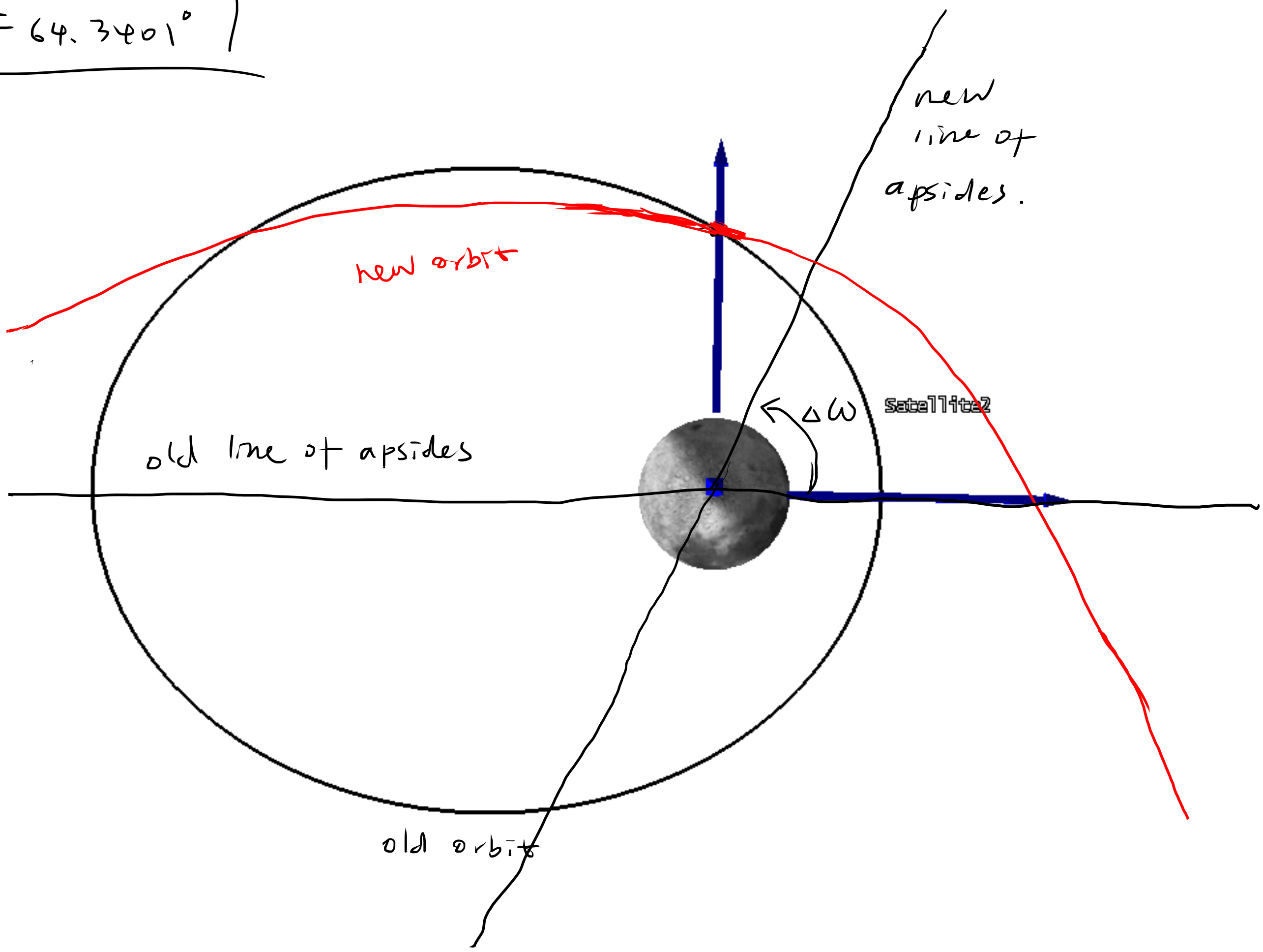
$= 25.6599°, -154.3401°$

$\boxed{= 25.6599°}$ ascending

$\Delta\omega = \theta_0^* - \theta_N^*$

$= 90° - 25.6599°$

$\boxed{= 64.3401°}$



c) orbital maneuver calculated in b) can be used to send LTVs from lunar orbits to earth orbits,

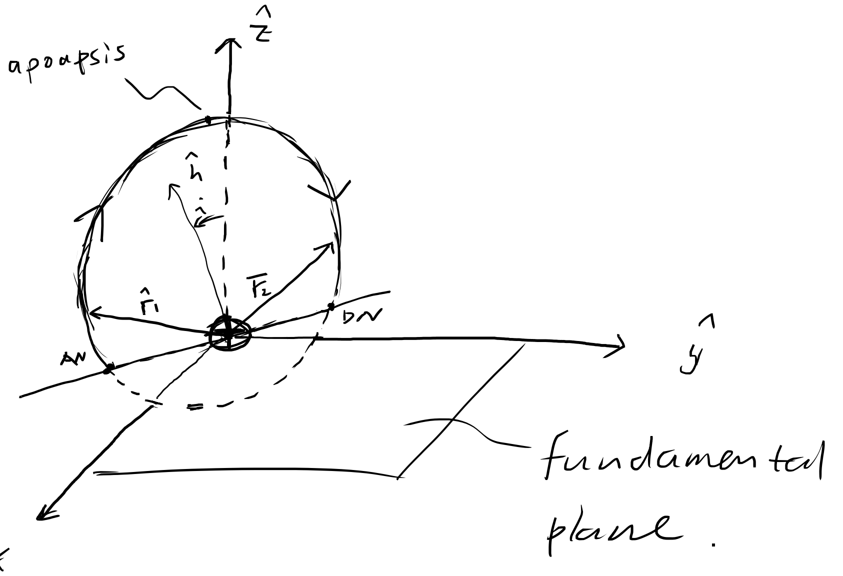
Results from a) allows us to analysis whether the 2BD-model is valid, which affects the accuracy of the maneuver calculated in b).

With a large perturbation from the Earth as calculated in a), it is unlikely that this LTV will be sent to the desired orbit using maneuver derived from 2BD model. More factors needs to be considered to perform this transfer.

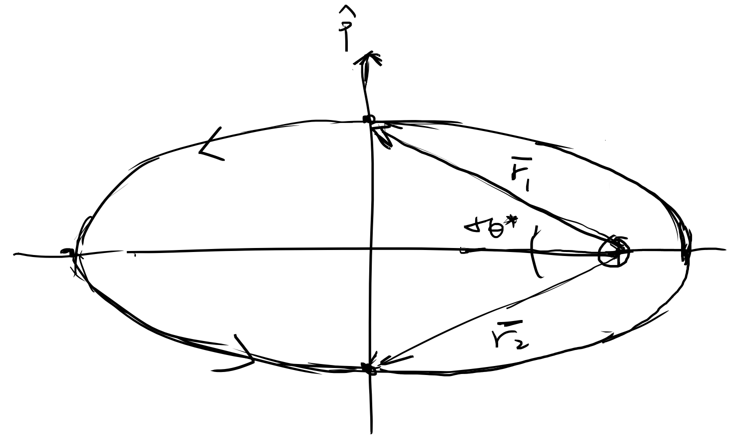
2. $\bar{r}_1 = a_n(\cos 45^\circ \hat{x} + \sin 45^\circ \hat{z})$ Ends of semi-minor axis. $r = a$.

$\bar{r}_2 = a_n(\cos 45^\circ \hat{y} + \sin 45^\circ \hat{z})$

a) (i)



(ii)



b) $\Delta\theta^* = \cos^{-1} \dfrac{\bar{r}_1 \cdot \bar{r}_2}{|\bar{r}_1| \cdot |\bar{r}_2|}$

$= \cos^{-1} \dfrac{\sin(45^\circ)^2}{1 \cdot 1}$

$= \cos^{-1}(\frac{1}{2})$

$= \pm 60^\circ$

$= \boxed{+60^\circ}$ by inspection.

$\hat{h} = \dfrac{\bar{r}_1 \times \bar{r}_2}{|\bar{r}_1 \times \bar{r}_2|}$

$$\begin{matrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{matrix}$$

$= \dfrac{-\frac{1}{2}\hat{x} - \frac{1}{2}\hat{y} + \frac{1}{2}\hat{z}}{\sqrt{3}/2}$

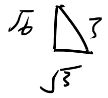
$= \boxed{-\frac{\sqrt{3}}{3}\hat{x} - \frac{\sqrt{3}}{3}\hat{y} + \frac{\sqrt{3}}{3}\hat{z}}$

$\hat{z}$ is positive,

$\hat{h}$ is $\boxed{\text{above}}$ the fundamental plane.

c) Ω, i, ω, e, p

$\cos i = \frac{\sqrt{3}}{3}$,



$\sin i = \frac{\sqrt{6}}{3}$

$i = \pm 54.7356^\circ$, inspect a(i), perigee below f-p.

$\boxed{i = 54.7356^\circ}$ $0^\circ < i < 180^\circ$

$$\begin{cases} \sin\Omega \cdot \sin i = \sqrt{3}/3 \\ -\cos\Omega \cdot \sin i = \sqrt{3}/3 \end{cases}$$

$$\begin{cases} \sin\Omega = \frac{\sqrt{2}}{2} & \Omega = 45^\circ, 135^\circ \\ \cos\Omega = -\frac{\sqrt{2}}{2} & \Omega = \pm 135^\circ \end{cases}$$

$\boxed{\Omega = 135^\circ}$

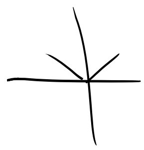
$\hat{r}_1 = \bar{r}_1 = \frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{z}$

$$\begin{matrix} -\frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{3}}{3} & 0 \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{2}}{2} \end{matrix}$$

$\hat{\theta}_1 = \frac{\hat{h}_1 \times \hat{r}_1}{\| \hat{h}_1 \times \hat{r}_1 \|}$

$= \frac{-\frac{\sqrt{6}}{6}\hat{x} + \frac{\sqrt{6}}{3}\hat{y} + \frac{\sqrt{6}}{6}\hat{z}}{\sqrt{\frac{1}{6} + \frac{6}{9} + \frac{1}{6}}}$

$= -\frac{\sqrt{6}}{6}\hat{x} + \frac{\sqrt{6}}{3}\hat{y} + \frac{\sqrt{6}}{6}\hat{z}$



$\frac{\sqrt{3}}{2}$

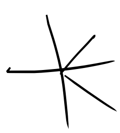
$$\begin{cases} \sin\theta_1 \sin i = \frac{\sqrt{2}}{2} \\ \cos\theta_1 \sin i = \frac{\sqrt{6}}{6} \end{cases} \Rightarrow \begin{cases} \theta_1 = \sin^{-1}\left(\frac{\sqrt{2}}{2} \cdot \frac{3}{\sqrt{6}}\right) \\ \theta_1 = \cos^{-1}\left(\frac{\sqrt{6}}{6} \cdot \frac{3}{\sqrt{6}}\right) \end{cases} \Rightarrow \begin{matrix} \theta_1 = 60^\circ, 120^\circ \\ \theta_1 = \pm 60^\circ \end{matrix}$$

$\frac{1}{2}$

$\omega = \theta_1 - \theta_1^*$

$= 60^\circ - 150^\circ$

$\boxed{\theta_1 = 60^\circ}$



$\boxed{\omega = -90^\circ}$ checks out with the sketch a)(i)

Inspect a ii) . $\bar{r}_1$ & $\bar{r}_2$ are symmetrical about SMA

$$\theta_1^* = 180° - \frac{\Delta\theta^*}{2} = 150° \quad . \quad \theta_2^* = -150°$$

$$\begin{cases} r_1 = \dfrac{p}{1+e\cos\theta_1^*} \\ p = a_n(1-e^2) \end{cases} \Rightarrow \begin{cases} a_n = \dfrac{p}{1-\frac{\sqrt{3}}{2}e} \\ p = a_n(1-e^2) \end{cases} \Rightarrow \begin{cases} p = a_n(1-\frac{\sqrt{3}}{2}e) \\ p = a_n(1-e^2) \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} e = \frac{\sqrt{3}}{2} \\ p = \frac{1}{4}a_n \end{cases}}$$

d) $\boxed{\begin{matrix} \theta_1^* = 150° \\ \theta_2^* = -150° \end{matrix}}$

$$v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_n}} = \sqrt{\frac{\mu}{a_n}}$$

$$v_2 = v_1$$

$$\gamma_1 = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_1 v_1}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{\mu \cdot a_n/4}}{a_n \cdot \sqrt{\mu/a_n}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \pm 60°$$

$\boxed{\gamma_1 = +60°}$ ascending $\boxed{\gamma_2 = -60°}$ descending

e) $v_C = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{\mu}{a_n}}$

$$\bar{r}_2 = f\bar{r}_1 + g\bar{v}_1$$

$$\bar{v}_1 = \frac{\bar{r}_2 - f\bar{r}_1}{g}$$

$$v_1 = \sqrt{\frac{\mu}{a_n}} = v_c$$

$$\bar{v}_1 = k_1 v_c \hat{v}_1 = v_c \hat{v}_1 \qquad \boxed{k_1 = 1}$$

$$f = 1 - \frac{r_2}{p}\left[1 - \cos(\Delta\theta^*)\right]$$

$$= 1 - \frac{a_n}{a_n/4}\left[1 - \cos(60^\circ)\right]$$

$$= 1 - 4 \times \left(1 - \frac{1}{2}\right)$$

$$= -1$$

$$g = \frac{r_1 r_2}{\sqrt{\mu p}} \cdot \sin(\Delta\theta^*)$$

$$= \frac{a_n^2}{\sqrt{\mu \cdot a_n/4}} \cdot \sin(60^\circ)$$

$$= \cancel{2}\sqrt{\frac{a_n^3}{\mu}} \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$$= \sqrt{\frac{3a_n^3}{\mu}} = a_n\sqrt{\frac{3a_n}{\mu}}$$

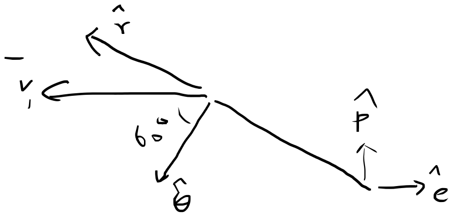
$$\bar{v}_1 = \frac{\bar{r}_2 + \bar{r}_1}{\sqrt{\frac{3a_n^3}{\mu}}}$$

$$= \frac{a_n\left(\frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y} + \sqrt{2}\hat{z}\right)}{\cancel{a_n}\sqrt{\frac{3a_n}{\mu}}}$$

$$\frac{6\times 2 + 4\times 6}{36} = \frac{12+24}{36} = 1$$

$$= \boxed{\frac{\sqrt{6}}{6}\cdot\sqrt{\frac{\mu}{a_n}}\,\hat{x} + \frac{\sqrt{6}}{6}\sqrt{\frac{\mu}{a_n}}\,\hat{y} + \frac{\sqrt{6}}{3}\sqrt{\frac{\mu}{a_n}}\,\hat{z}} \text{ km/s}$$

$|\bar{v}_1| = \sqrt{\frac{\mu}{a_n}}$, checks out

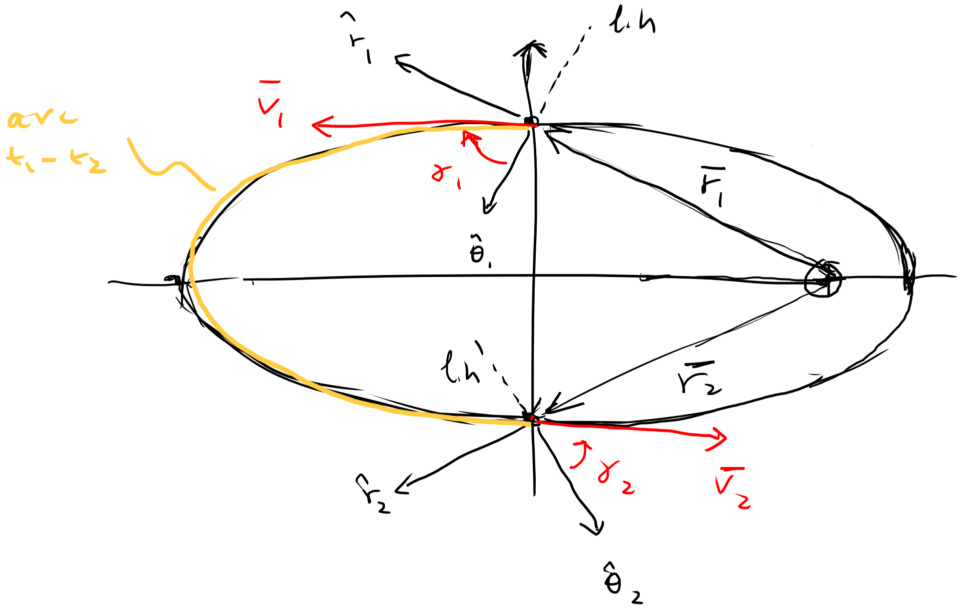


$$\boxed{\bar{v}_1^{eph} = -\sqrt{\frac{\mu}{a_n}}\,\hat{e}} \text{ km/s}$$

$\mu \approx \mu_{Earth}$

$$\boxed{\bar{v}_1^{roh} = \frac{\sqrt{3}}{2}\sqrt{\frac{\mu}{a_n}}\,\hat{r} + \frac{1}{2}\sqrt{\frac{\mu}{a_n}}\,\hat{\theta}} \text{ km/s}$$

f)



3、 $\theta_1^* = 90°$, $\theta_2^* = 225°$, $(t_2 - t_1)$?

$r_p = 2R_\oplus$, $r_a = 10 R_\oplus$.

$a = 6 R_\oplus$ $\quad e = 1 - \frac{r_p}{a} = \frac{2}{3}$

$$E_1 = 2 \cdot \tan^{-1}\left(\tan\frac{\theta_1^*}{2} \cdot \sqrt{\frac{1-e}{1+e}}\right)$$

$$= 2 \cdot \tan^{-1}\left(\sqrt{\frac{1}{5}}\right)$$

$$= 48.1897° = 0.8411 \text{ rad}$$

$$E_2 = 2 \cdot \tan^{-1}\left(\tan\frac{\theta_2^*}{2} \sqrt{\frac{1}{5}}\right)$$

$$= 2 \cdot \tan^{-1}\left(\tan(112.5°)\sqrt{\frac{1}{5}}\right)$$

$$= -94.3877° = -1.6474 \text{ rad}$$

$$t - t_p = \sqrt{\frac{a^3}{\mu}}(E - e\sin E)$$

$$t_2 - t_1 = \left[(E_2 - E_1) - e(\sin E_2 - \sin E_1)\right]\sqrt{\frac{a^3}{\mu}}$$

$$= \left[(-1.6474 - 0.8411) - \frac{2}{3}\cdot\left(\sin(-94.3877°) - \sin(48.1897°)\right)\right] \cdot \sqrt{\frac{(6R_\oplus)^3}{\mu}}$$

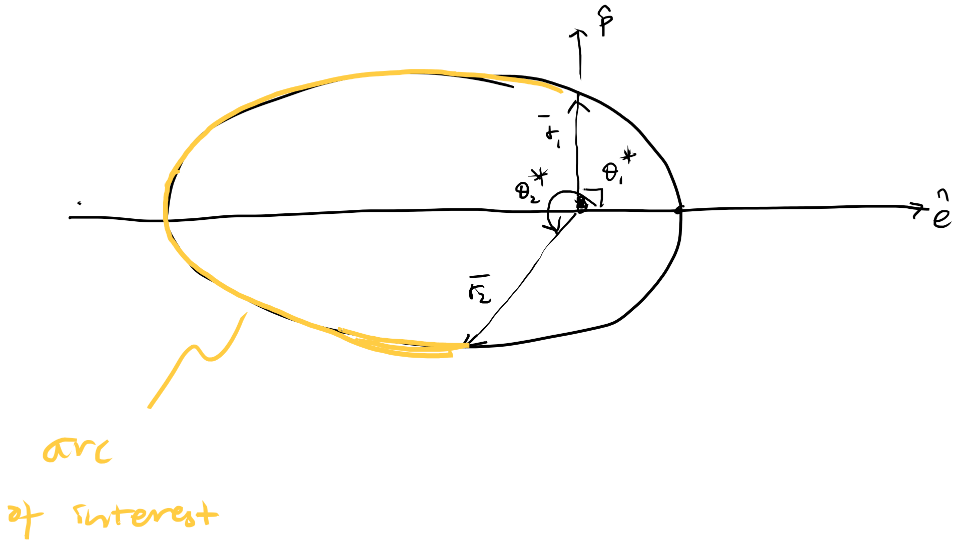
$$= \left[-2.4885 - (-1.1616)\right] \cdot \sqrt{\frac{(6 \cdot 6400)^3}{4 \times 10^5}}$$

$$= -15786.99605 \text{ sec}$$

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} = 74756.11256 \text{ sec}$$

$\therefore$ the actual $t_2 - t_1$ is

$$74756.11256 - 15786.99605 = \boxed{58969.11651 \text{ sec}}$$


$\hat{p}$
$\bar{r}_1$
θ_2^*
θ_1^*
$\hat{e}$
$\bar{r}_2$
arc
of interest