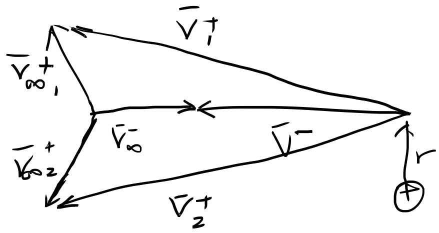
1. a) In order to return to Earth, the spacecraft need a darkside pass.

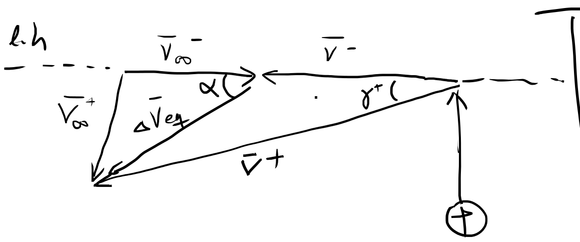
The passage in PS8 was a ligh-side pass.
In order to return to the Earth, a dark-side pass is better, because the resulting velocity vector would be pointing closer to the Earth.




As shown in this vector diagram, $\bar{v}_{\infty 1}^+$ is the result of a lighside pass, which causes $\bar{v}_1^+$ to point away from the Earth. Whereas $\bar{v}_{\infty 2}^+$ is a darkside pass, causing $\bar{v}_2^+$ to point towards the Earth.

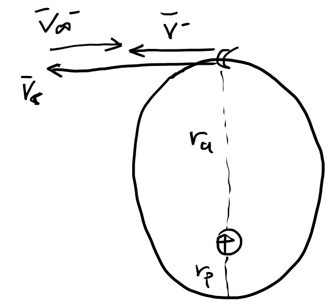
---

From PS8.



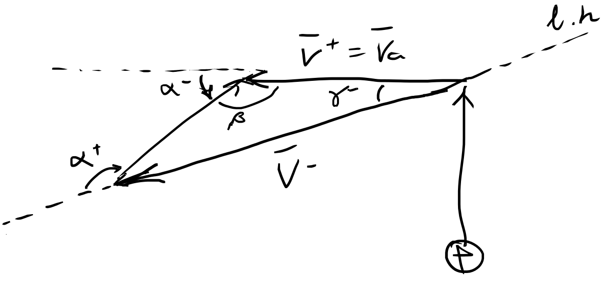

$r^+ = r_a = 384400$ km
$v^+ = 1.4573$ km/s
$\gamma^+ = 33.7046°$
$\theta^{*+} = 66.18°$

conditions immediately after encounter.




To offset effect of lunar gravity, a maneuver of $-\Delta \bar{V}_{eq}$ is required to put the spacecraft back on the second half of the Hohmann transfer path.

$\Delta \bar{v} = -\Delta \bar{V}_{eq} \Rightarrow |\Delta \bar{v}| = \boxed{1.3062 \text{ km/s}}$ from PS8



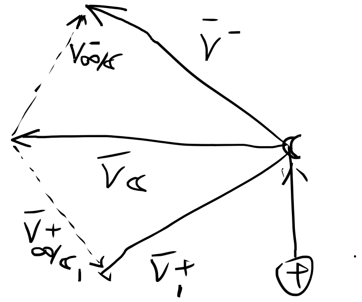

$\alpha = \alpha^+ = \beta + \gamma = 180° - \alpha^- + \gamma^-$

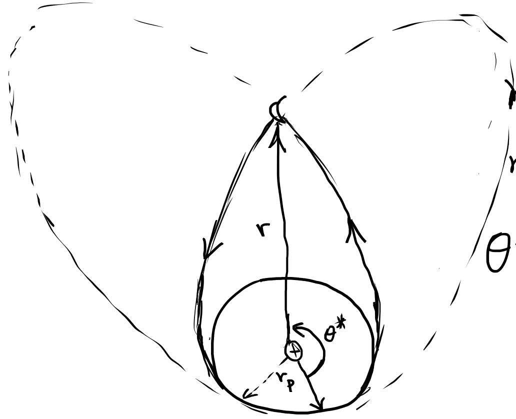
$= 180° - 38.2529 + 33.7046°$

$\boxed{\alpha = 175.4517°}$

This maneuver seems reasonable, as the $\Delta V$ cost is small. However, if the maneuver is missed, the spacecraft will be slung on a hyperbolic orbit which will probably miss the Earth, and the crew cannot return home.

b)




$r_p = 190 + R_\oplus$

$r = a_\zeta$

$\theta^* = 173.8°$

$$\left.\begin{aligned} r &= \frac{p}{1+e\cos\theta^*} \\ r_p &= \frac{p}{1+e} \end{aligned}\right\} \quad \begin{aligned} & r\cdot(1+e\cos\theta^*) = r_p\cdot(1+e). \\ & r - r_p = e(r_p - r\cdot\cos\theta^*) \\ & e = \frac{r-r_p}{r_p - r\cos\theta^*} = \boxed{0.9720.} \end{aligned}$$

$p = r_p\cdot(1+e) = 1.2952\times10^4$ km

$a = p/(1-e^2) = \boxed{2.3450\times10^5 \text{ km}}$

$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu_\oplus}} = \boxed{1.1301\times10^6 \text{ sec}}$

$\mathcal{E}_1 = -\frac{\mu}{2a} = \boxed{-0.8499 \text{ km}^2/\text{s}^2}$

$r_p = 6568.1363$ km

$r_a = \frac{p}{1-e} = \boxed{4.6242\times10^5 \text{ km}}$

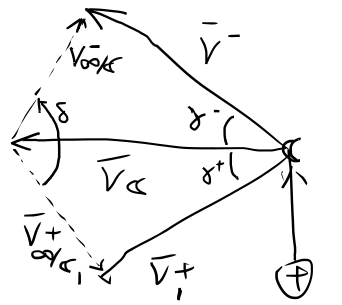
$E = \cos^{-1}\left(\frac{1}{e} - \frac{r}{ae}\right) = \boxed{2.2885 \text{ rad.}}$

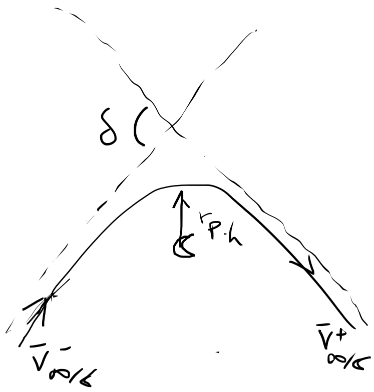
$t - t_p = \sqrt{\frac{a^3}{\mu_\oplus}}\cdot(E - e\sin E) = \boxed{2.7992\times10^5 \text{ sec}} = \text{TOF}$ from perigee.

$n_\zeta = \sqrt{\frac{\mu_\oplus}{a_\zeta^3}} = 2.6491\times10^{-6}$ rad/s.

$\phi = \pi - n_\zeta\cdot\text{TOF} = \boxed{137.5131°}$

c)





$r^- = 384400$ km

$v^- = \sqrt{\frac{2\mu_\oplus}{r^-} - \frac{\mu_\oplus}{a}} = 0.6116$ km/s

$V_{☾} = \sqrt{\frac{\mu_\oplus}{a_L}} = 1.0183$ km/s

$\gamma^- = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r^- v^-}\right) = 72.2043^\circ$ (ascending)

$V_{\infty/☾} = \sqrt{v^{-2} + V_☾^2 - 2v^- V_☾ \cdot \cos\gamma^-} = 1.0150$ km/s

$V_{\infty/☾}^2 = \frac{\mu_☾}{|a_h|} \Rightarrow |a_h| = \frac{\mu_☾}{V_{\infty/☾}^2} = 4.7585 \times 10^3$ km

$\sin\frac{\delta}{2} = \frac{v^-}{V_{\infty/☾}} \cdot \sin\gamma^- = \frac{1}{e_h} \Rightarrow \begin{cases} e_h = 1.7430 \\ \delta = 70.0187 \end{cases}$

$R_☾ = 1738.2$ km

$r_{p.h} = |a_h| \cdot (e_h - 1)$

$\boxed{= 3.5358 \times 10^3 \text{ km}} > R_☾$

This is a reasonable $r_p$.

Altitude is

$r_{p.h} - R_☾ = \boxed{1.7976 \times 10^3 \text{ km}}$

The return orbit is mirrored.

$r^+ = r^- = 384400$ km

$v^+ = v^- = 0.6116$ km/s

$\gamma^+ = -\gamma^- = -72.2043^\circ$

$\theta^{*+} = -\theta^{*-} = -173.8^\circ$

$a^+ = a^- = 2.3450 \times 10^5$ km

$e^+ = e^- = 0.9720$

$r_p^+ = r_p^- = 6568.1363$ km

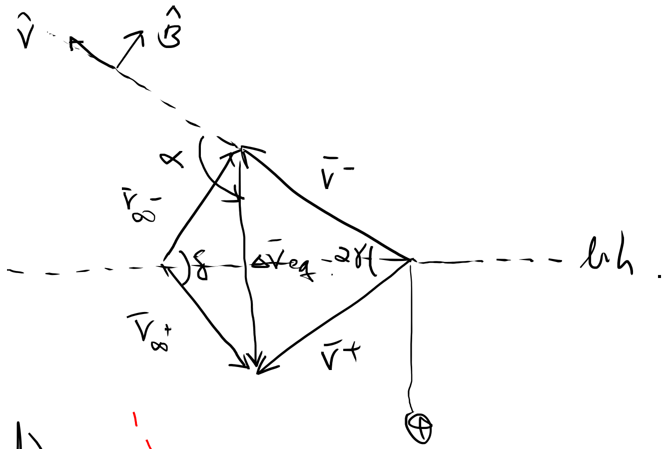
$r_a^+ = r_a^- = 4.6242 \times 10^5$ km

$\mathbb{P}^+ = \mathbb{P}^- = 1.1301 \times 10^6$ sec

$\mathcal{E}^+ = \mathcal{E}^- = -0.8499$ km²/s²

$\Delta\omega = \theta^{*-} - \theta^{*+} = 347.6^\circ$

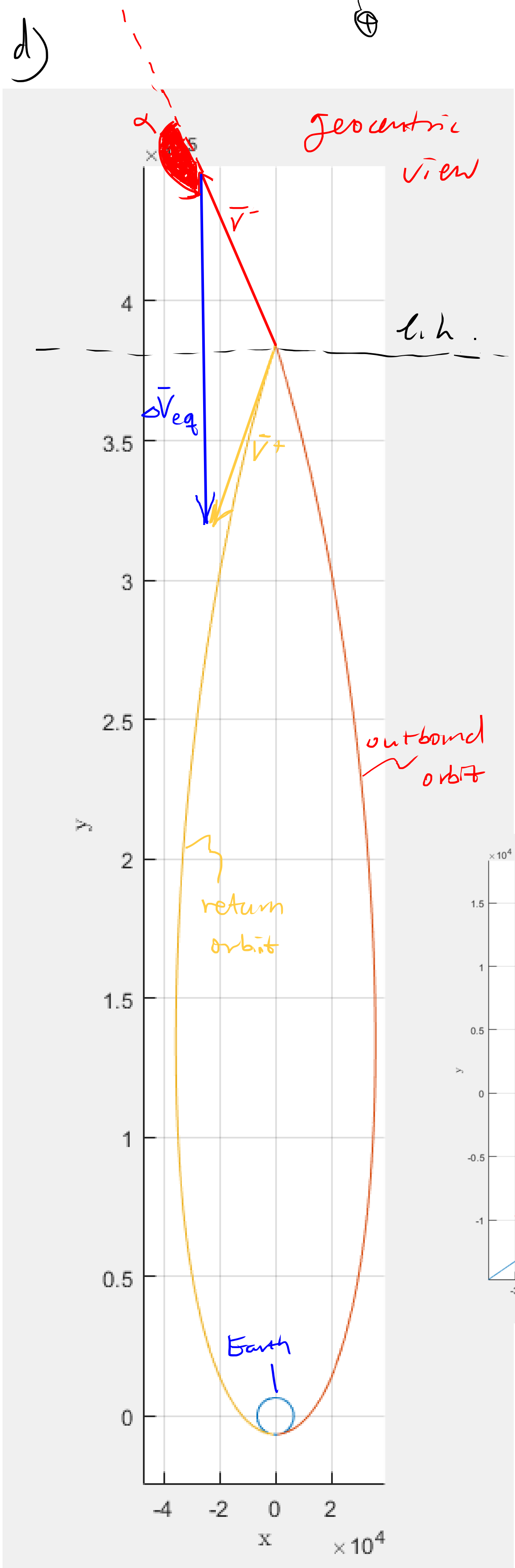
$\boxed{\Delta\omega = -12.4^\circ}$

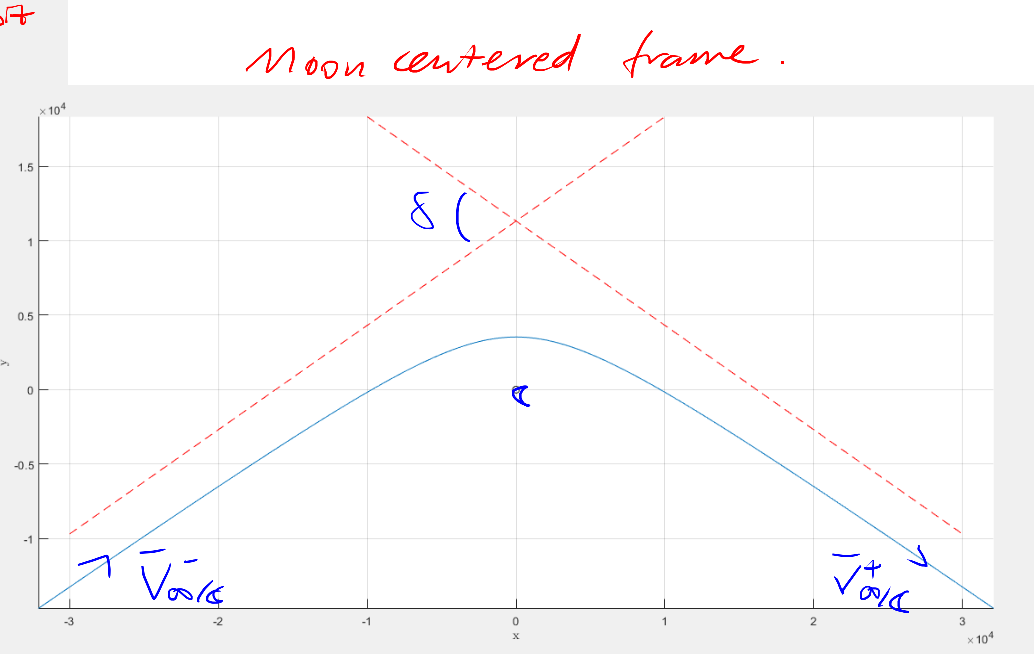



$$|\Delta \bar{V}_{eq}| = \sqrt{2v_\infty^2 - 2v_\infty^2 \cdot \cos\delta} = \boxed{1.1647 \text{ km/s}}$$

$$\alpha = 180^\circ - \frac{180^\circ - 2\delta}{2} = \boxed{162.2043^\circ}$$

d)



2. a) Earth Launch: 8/03/2004 = 2453221 units in Julian dates

First Venus Flyby: 10/24/2006 = 2454003

First Mercury Flyby: 1/14/2008 = 2454480

MOI : 3/18/2011 = 2455639

(i) $TOF_1 = 782$ days

(ii) $TOF_2 = 1259$ days

(iii) $TOF_3 = 2418$ days

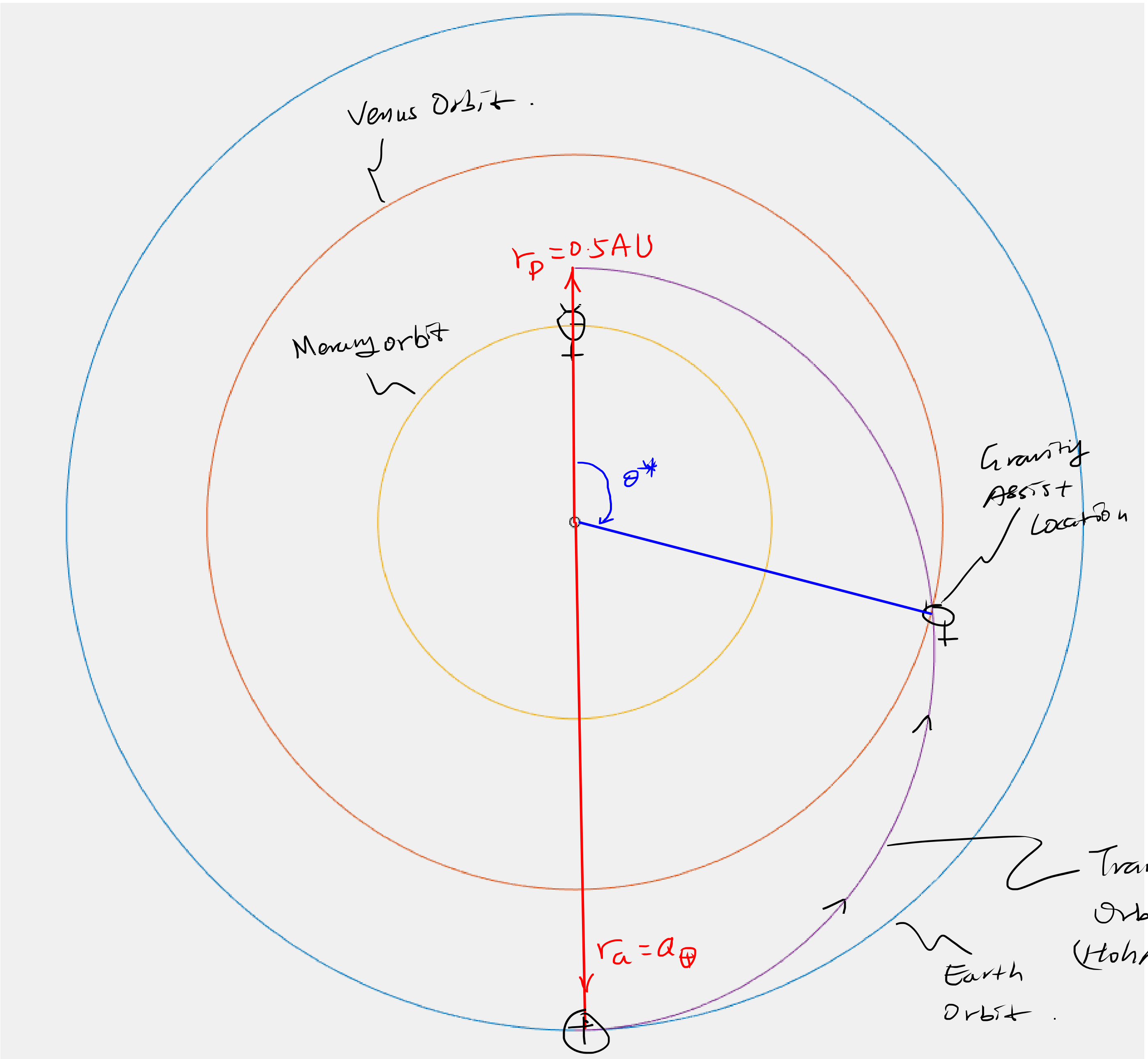
$r_p = 0.5\,AU$

$r_a = a_{\oplus} = 1\,AU$

$r_p = a(1-e)$ $\quad e = 1 - \frac{r_p}{a}$

$r_a = a(1+e)$ $\quad = \frac{1}{3}$

b)



$e_T = \frac{1}{3}$. $a_T = 0.75$ AU. $p_T = a_T \cdot (1 - e_T^2) = \frac{2}{3}$ AU

$$r_{enc} = a_{\text{♀}} = \frac{p_T}{1 + e_T \cos\theta^*} \Rightarrow \theta^*_{enc} = \cos^{-1}\left(\frac{p_T}{a_{\text{♀}} e_T} - \frac{1}{e_T}\right)$$

$$= \boxed{-103.5902^\circ}$$ (descending)

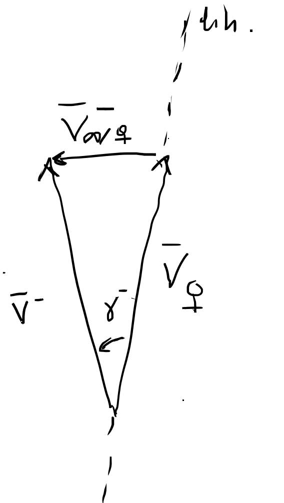
$$\mathbb{P}_T = 2\pi\sqrt{\frac{a_T^3}{\mu_\odot}} = 2.0498 \times 10^7 \text{ sec}.$$

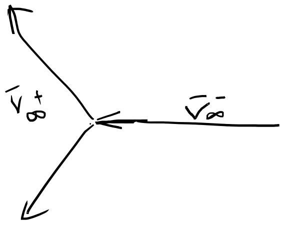
$$E_{enc} = 2 \cdot \tan^{-1}\left(\tan\frac{\theta^*_{enc}}{2} \cdot \sqrt{\frac{1-e_T}{1+e_T}}\right) = -1.4639 \text{ rad}.$$

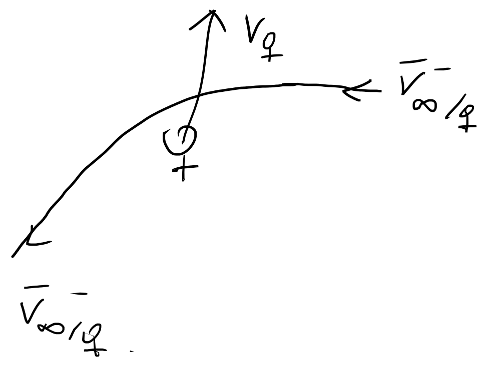
$$t_{enc} - t_p = \sqrt{\frac{a_T^3}{\mu_\odot}} \cdot (E_{enc} - e_T \cdot \sin E_{enc}) = -1.6228 \times 10^6 \text{ sec}.$$

$$TOF = \frac{1}{2}\mathbb{P}_T - |t_{enc} - t_p| = \boxed{8.6260 \times 10^6 \text{ sec}}$$ apoapsis to encounter.

c) $\frac{2990}{R_{\text{♀}}} = 0.4941$. The actual encounter is $\boxed{1.4941\, R_{\text{♀}}} = r_{ph}$.



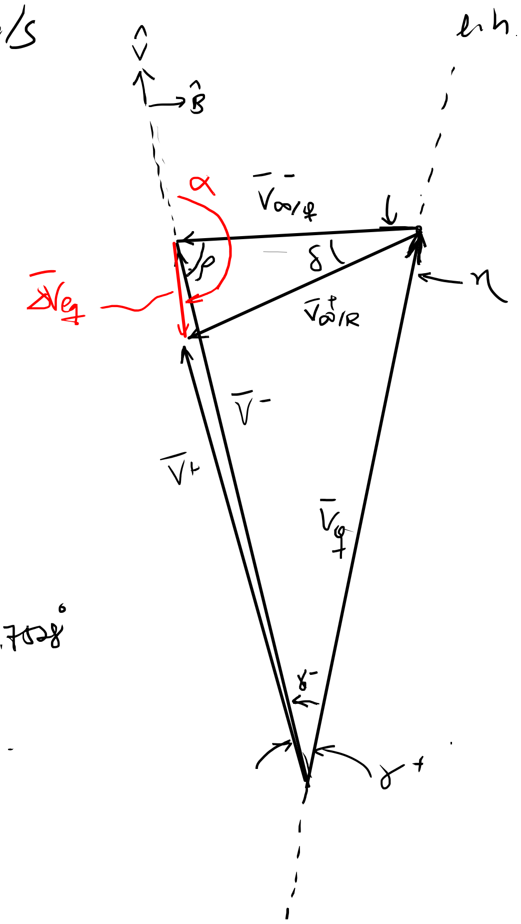

To reach Mercury, the spacecraft should loose energy. by choosing a flyby that results in a smaller $|\bar v^+|$. As shown in the diagrams above, it should lead ahead of Venus.

$$v^- = \sqrt{\frac{2\mu_\odot}{r^-} - \frac{\mu_\odot}{a_T}} = 35.6384 \text{ km/s}$$

$$v_{\text{♀}} = \sqrt{\frac{\mu_\odot}{a_{\text{♀}}}} = 35.0209 \text{ km/s}$$

$$\gamma^- = \cos^{-1}\left(\frac{\sqrt{\mu_\odot \cdot p_T}}{r^- \cdot v^-}\right) = -19.3683^\circ \text{ (descending)}$$

$$v_{\infty/\text{♀}}^- = \sqrt{v_{\text{♀}}^2 + v^{-2} - 2 v_{\text{♀}} v^- \cos\gamma^-} = 11.9017 \text{ km/s}$$




$$|a_h| = \frac{\mu_{\text{♀}}}{v_{\infty/\text{♀}}^{-2}} = 2.2934 \times 10^3 \text{ km}$$

$$e_h = \frac{r_{p,h}}{|a_h|} + 1 = 4.9420$$

$$\delta = 2 \cdot \sin^{-1}\frac{1}{e_h} = 23.3457^\circ$$

$$\frac{\sin\eta}{v^-} = \frac{\sin|\gamma^-|}{v_{\infty/\text{♀}}^-} \Rightarrow \eta = \sin^{-1}\left(\frac{v^-}{v_{\infty/\text{♀}}^-} \cdot \sin|\gamma^-|\right) = \underline{83.2472^\circ}, \ 96.7528^\circ$$

$$\rho = \sin^{-1}\left(\frac{v_{\text{♀}}}{v_{\infty/\text{♀}}^-} \cdot \sin|\gamma^-|\right) = \underline{77.3845^\circ}, \ 102.6155^\circ$$

$$\rho + \eta + |\gamma^-| = 180^\circ \text{ (check)}$$

$$\Delta V_{eq} = \sqrt{2V_{\infty/\text{♀}}^2 - 2V_{\infty/\text{♀}}^2 \cos\delta} = \boxed{4.8160 \text{ km/s}}$$

$$\alpha = 180^\circ - \rho + \frac{180 - \delta}{2} = \boxed{180.9427^\circ}$$

$$v^+ = \sqrt{v_{\text{♀}}^2 + V_{\infty/\text{♀}}^2 - 2v_{\text{♀}} \cdot V_{\infty/\text{♀}} \cos(\eta - \delta)} = \boxed{30.8231 \text{ km/s}}$$

$$\gamma^+ = \sin^{-1}\left(\frac{V_{\infty/\text{♀}}}{V^+} \cdot \sin(\eta - \delta)\right) = \boxed{-19.5156^\circ} \text{ descending}$$

$$\sqrt{v^{-2} + v^{+2} - 2v^- v^+ \cos(|\gamma^+| - |\gamma^-|)} = 4.8160 = \Delta V_{eq} \text{ (check).}$$

$$r^+ = r^- = a_{\text{♀}} = \boxed{108207284 \text{ km}}$$

$$e^+ = \sqrt{\left(\frac{r^+ v^{+2}}{\mu_\odot} - 1\right)^2 \cdot \cos^2\gamma^+ + \sin^2\gamma^+} = \boxed{0.3959}$$

$$\theta^{*+} = \tan^{-1}\left(\frac{\frac{r^+ V^{+2}}{\mu_\odot} \cdot \cos\gamma^+ \cdot \sin\gamma^+}{\left(\frac{r^+ v^{+2}}{\mu_\odot}\right) \cdot \cos^2\gamma^+ - 1}\right) = \boxed{-141.9659^\circ}$$

$$p^+ = r^+ \cdot (1 + e^+ \cos\theta^{*+}) = 7.4467 \times 10^7 \text{ km}$$

$$a^+ = p^+/(1 - e^{+2}) = \boxed{8.8306 \times 10^7 \text{ km}}$$

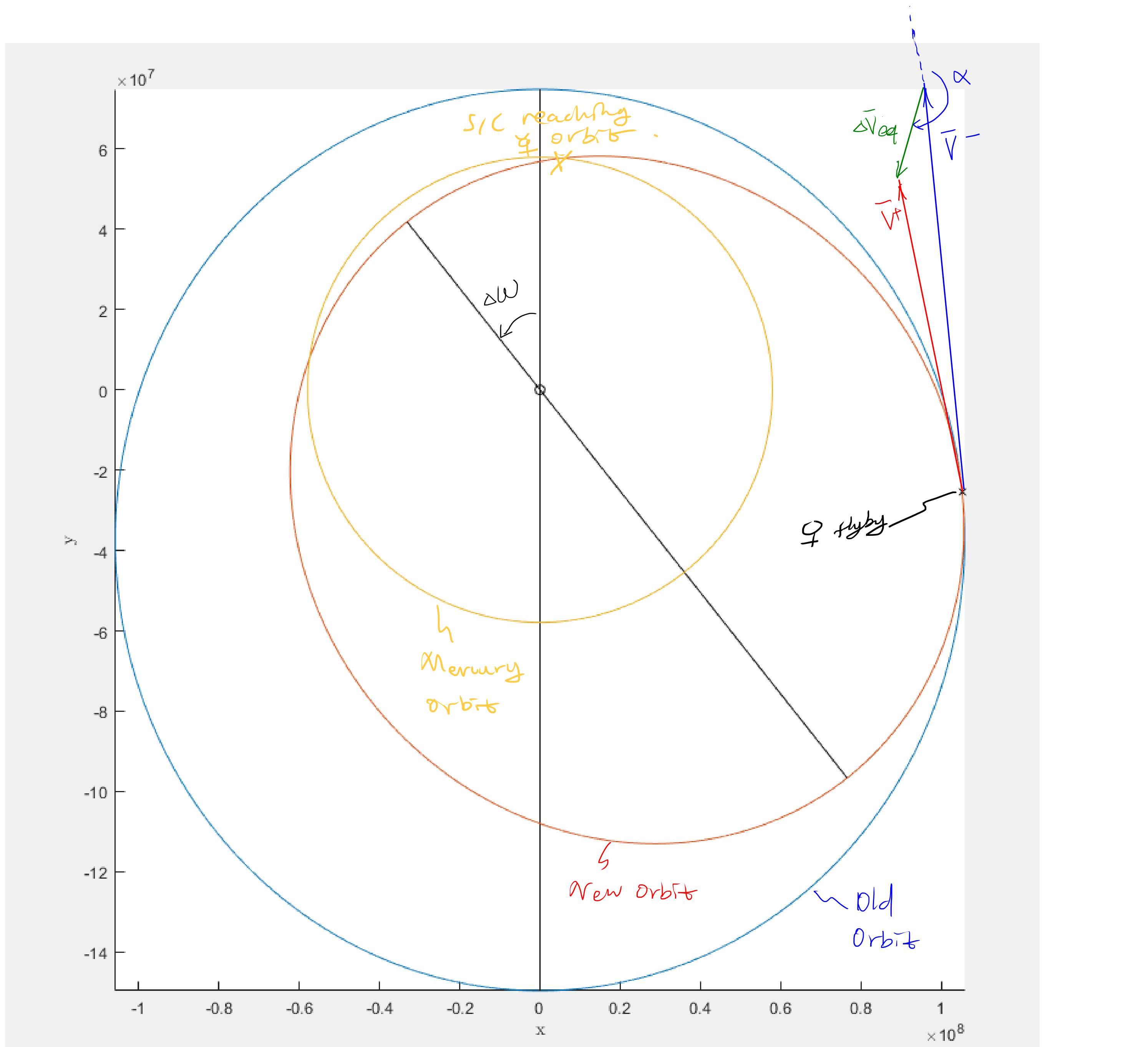
$$r_p^+ = a^+(1 - e^+) = \boxed{5.3348 \times 10^7 \text{ km}}$$

$$r_a^+ = a^+(1 + e^+) = \boxed{1.2326 \times 10^8 \text{ km}}$$

$$\mathbb{P}^+ = 2\pi\sqrt{\frac{a^{+3}}{\mu_\odot}} = \boxed{1.4312 \times 10^7 \text{ sec}}$$

$$\Delta\omega = \theta^{*-} - \theta^{*+} = \boxed{38.3758^\circ}$$

d)



I will probably try a slightly larger perihelion value than 0.5AU, such that after venus flyby the new orbit will intercept Mercury's orbit tangentially, to reduce the cost of final maneuver into Mercury parking orbit.

3, $a_R = 15\, R_{Jup}$. $R_R = 3000$ km.

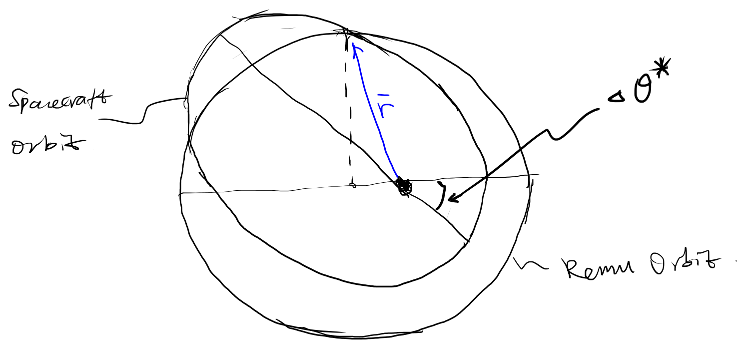
$e_R = 0.25$ $\mu_R = 1\times10^5\ km^3/s^2$

$r_p = 7.5\, R_{Jup}$

$e = 0.5$

$a = r_p(1-e^2) = 15 \cdot R_{Jup}$.

a)




$\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\right) - \tan^{-1}\left(\sqrt{\frac{1-e_R}{1+e_R}}\right)$

In Remus' orbit:

$r = a_R$

$\theta^* = \cos^{-1}\left(\frac{p_R}{r e_R} - \frac{1}{e_R}\right) = 104.4775°$

In s/c's orbit

$r = r$

$\theta^* = \cos^{-1}\left(\frac{p}{re} - \frac{1}{e}\right) = 120°$

$$\boxed{\Delta\theta^* = 15.5225°}$$

b) $\boxed{r^- = 1072380 \text{ km}}$

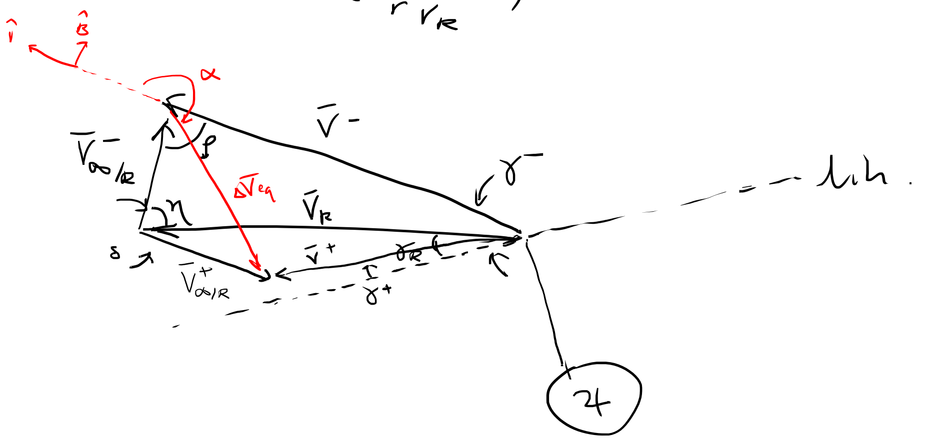
$v^- = \sqrt{\frac{2\mu_J}{r^-} - \frac{\mu_J}{a}} = \boxed{10.8702 \text{ km/s}}$

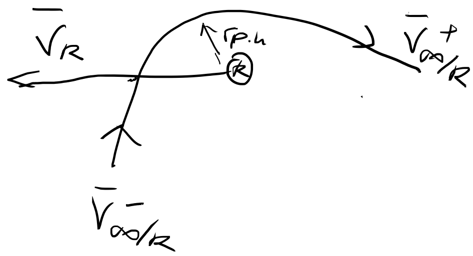
$\gamma^- = \cos^{-1}\left(\frac{\sqrt{\mu_J p}}{r^- v^-}\right) = \boxed{30°}$ (ascending)

$\boxed{\theta^{*-} = 120°}$

c) $V_R = \sqrt{\frac{2\mu_{24}}{r} - \frac{\mu_{24}}{a_R}} = 10.8702 \text{ km/s} = v^-$

$\gamma_R = \cos^{-1}\left(\frac{\sqrt{\mu_{24} p_R}}{r v_R}\right) = 14.4775°$ (ascending)




The spacecraft should pass ahead of Remus to decrease its orbit energy. by reducing $|\bar{v}^+|$

d). $r_{p.h} = R_R + 1500 = 4500$ km.

$V^-_{\infty/R} = \sqrt{V_R^2 + V^{-2} - 2V_R V^- \cos(\gamma^- - \gamma_R)} = 2.9359$ km/s

$|a_h| = \frac{\mu_R}{V^{-2}_{\infty/R}} = 1.1601 \times 10^4$ km

$e_h = \frac{r_{ph}}{|a_h|} + 1 = 1.3879$

$\delta = 2 \cdot \sin^{-1}(\frac{1}{e_h}) = 92.1949°$

$\eta = \sin^{-1}\left(\frac{V^-}{V^-_{\infty/R}} \cdot \sin(\gamma^- - \gamma_R)\right) = 82.2388°$ $\quad \rho + \eta + \gamma^- - \gamma_R = 180°$

$\rho = \sin^{-1}\left(\frac{V_R}{V_{\infty/R}} \cdot \sin(\gamma^- - \gamma_R)\right) = 82.2388°$

$V^+ = \sqrt{V^{+2}_{\infty/R} + V_R^2 - 2V^+_{\infty/R} V_R \cos(\delta - \eta)} = \boxed{7.9946 \text{ km/s}}$

$\gamma^+ = \gamma_R - \sin^{-1}\left(\frac{V^+_{\infty/R}}{V^+} \cdot \sin(\delta - \eta)\right) = \boxed{10.8371°}$ ascending.

$r^+ = r^- = \boxed{1072380 \text{ km}}$

$$\theta^{*+} = \tan^{-1}\left( \frac{\frac{r^+ v^{+2}}{\mu_4} \cdot \cos\gamma^+ \cdot \sin\gamma^+}{\frac{r^+ v^{+2}}{\mu_4} \cdot \cos^2\gamma^+ - 1} \right) = \boxed{168.2022^\circ}$$

$$e^+ = \sqrt{\left(\frac{r^+ v^{+2}}{\mu_4} - 1\right)^2 \cdot \cos^2\gamma^+ + \sin^2\gamma^+} = \boxed{0.4885}$$

$$p^+ = r \cdot (1 + e^+ \cos\theta^{*+}) = 5.5955 \times 10^5 \text{ km}$$

$$a^+ = p^+ / (1 - e^{+2}) = \boxed{7.3496 \times 10^5 \text{ km}}$$

$$r_p^+ = a^+ (1 - e^+) = \boxed{3.7590 \times 10^5 \text{ km}}$$

$$r_a^+ = a^+ (1 + e^+) = \boxed{1.0940 \times 10^6 \text{ km}}$$

$$P^+ = 2\pi \sqrt{\frac{a^{+3}}{\mu_4}} = \boxed{3.5170 \times 10^5 \text{ sec}}$$

$$\Delta\omega = \theta^{*-} - \theta^{*+} = \boxed{-48.2022^\circ}$$

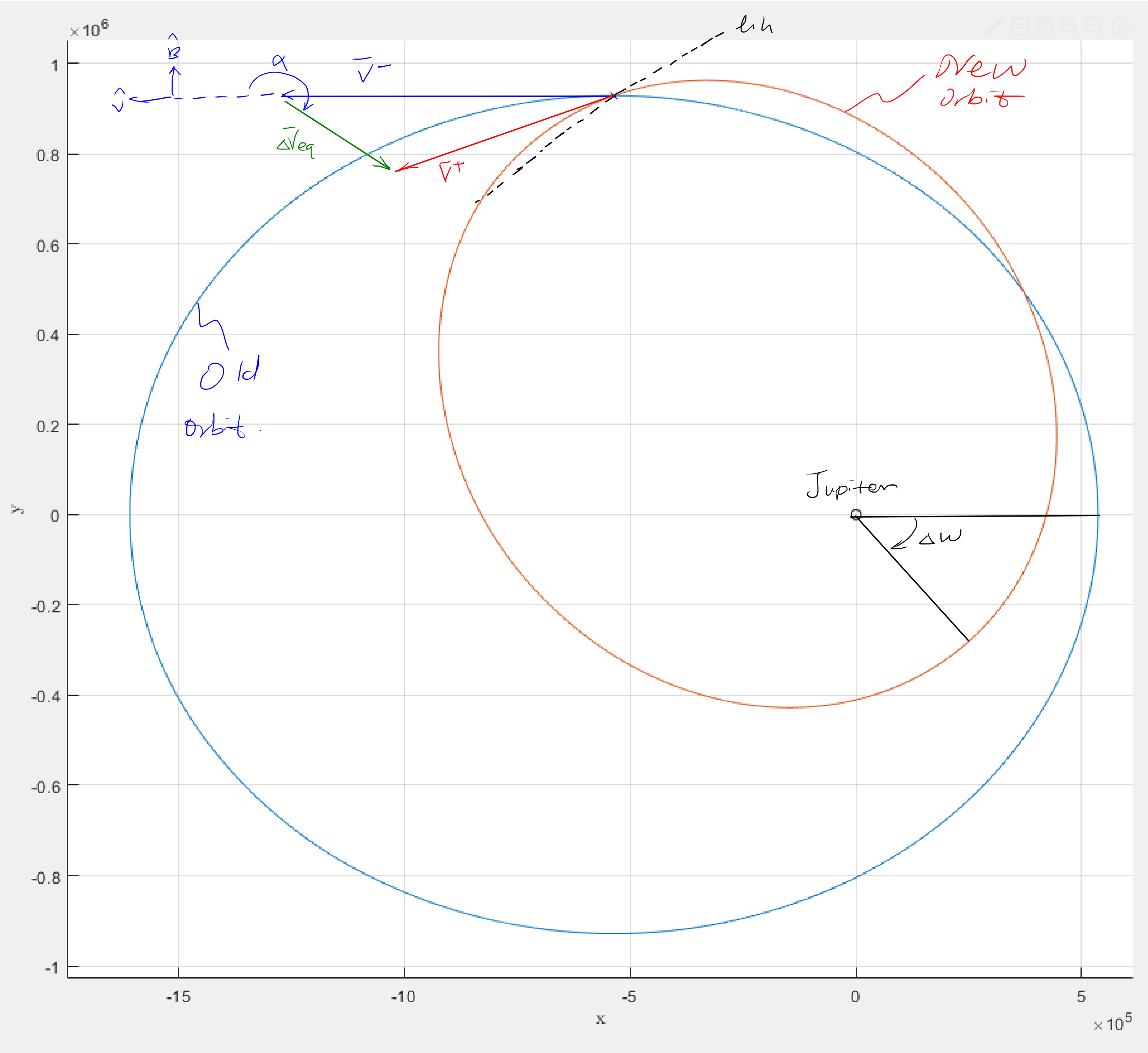
$$\mathcal{E} = \frac{\mu_4}{2a^+} = \boxed{-86.2037 \text{ km}^2/\text{s}^2}$$

$$\Delta V_{eq} = \sqrt{2 \cdot V_{\infty/4}^2 \cdot (1 - \cos\delta)} = \boxed{4.2308 \text{ km/s}}$$

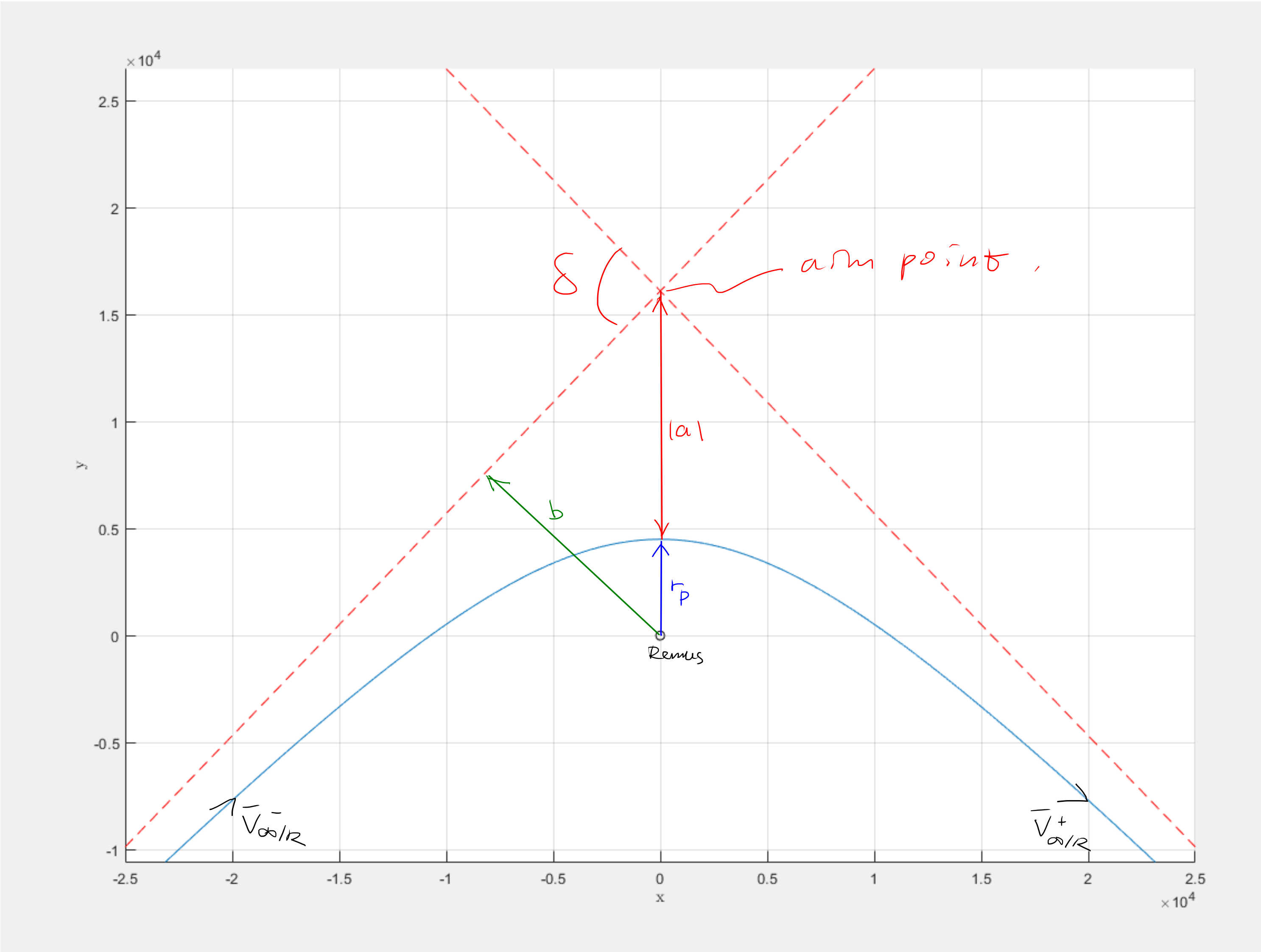
$$\alpha = 180^\circ + \left(\beta - \frac{180^\circ - \delta}{2}\right) = \boxed{218.3362^\circ}$$

$$\overline{\Delta V_{eq}} = \Delta V_{eq} \cdot (\cos\alpha\, \hat{V} + \sin\alpha\, \hat{B}) = \boxed{-3.3186\, \hat{V} - 2.6243\, \hat{B} \text{ km/s}}$$

e)



$\times 10^6$
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
y
-15
-10
-5
0
5
x
$\times 10^5$
$\hat{B}$
$\hat{J}$
$\alpha$
$\bar{V}^-$
$\Delta \bar{V}_{eq}$
$\bar{V}^+$
$\ell.h$
new orbit
Old orbit.
Jupiter
$\Delta\omega$



×10^4
2.5
2
1.5
1
0.5
0
-0.5
-1
y
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
x
×10^4
δ
aim point.
|a|
b
r_P
Remus
V∞/R -
V∞/R +