1. Obtain the describing function of the nonlinear function

$$\phi(y) = y^5 \quad (\text{odd})$$

$$\phi(a\sin\theta) = a^5 \sin^5\theta$$

$$N = \frac{2}{\pi a} \cdot \int_0^{\pi} a^5 \sin^6\theta \, d\theta = \boxed{\frac{5a^4}{8}}$$

2. Determine whether or not the following Duffing system has a periodic solution. Determine the approximate amplitude & periode of all periodic solutions,

$$\ddot{y} - y + y^3 = 0$$

$$\begin{cases} \ddot{y} - y = u \\ u = -\phi(y), \text{ where } \phi(y) = y^3 \end{cases}$$

$$\hat{G}(s) = \frac{1}{s^2 - 1} \qquad N(a) = \frac{3a^2}{4}$$

$$1 + \frac{1}{-w^2 - 1} \cdot \frac{3a^2}{4} = 0$$

$$1 - \frac{3a^2}{4w^2 + 4} = 0 \quad \rightarrow \text{there is a periodic solution.}$$

$$4w^2 + 4 = 3a^2$$

$$w = \sqrt{\frac{3a^2 + 4}{4}} = \frac{\sqrt{3a^2 + 4}}{2}, \quad T = \frac{2\pi}{w} = \frac{4\pi}{\sqrt{3a^2 + 4}}$$

The system has periodic solution of all amplitudes a & period

$$T = \frac{4\pi}{\sqrt{3a^2 + 4}}$$

3. · · · · periodic solution., approximate amplitude & period

$$\ddot{y} + \mu\left(\frac{\dot{y}^3}{3} - \dot{y}\right) + y = 0. \quad , \mu \in \mathbb{R}, \ \mu > 0.$$

$$\begin{cases} \ddot{y} - \mu\dot{y} + y = \frac{\mu}{3} u \\ u = -\dot{y}^3 \end{cases}$$

$$\hat{G}(s) = \frac{\mu s/3}{s^2 - \mu s + 1}, \quad \phi(\dot{y}) = \dot{y}^3$$

$$N(a) = \frac{3a^2}{4} \in \mathbb{R}.$$

$$1 + N(a)G(j\omega) = 0$$

$$1 + \frac{\mu(j\omega)/3}{-\omega^2 - \mu(j\omega) + 1} N(a) = 0.$$

$$-\omega^2 - \mu(j\omega) + 1 + \frac{\mu(j\omega)}{3} N(a) = 0.$$

$$\begin{cases} 1 - \omega^2 = 0 \\ \mu\omega\left(\frac{a^2}{4} + 1\right) = 0. \end{cases} \Rightarrow \begin{cases} \omega = \pm 1 \\ a = \pm 2 \end{cases}. \Rightarrow \boxed{\begin{cases} \omega = 1 \\ a = 2 \end{cases}}$$

4. Use the describing method to predict periodic solution to

$$\dot{x}(t) = -x(t) - 2\,\mathrm{sgn}[x(t-h)]$$

$$\dot{x} + x = 2u.$$

$$u = \phi(y), \quad \phi(y) = \mathrm{sgn}(y)$$

$$y = x(t-h)$$

$$\hat{G}(s) = \frac{2}{s+1} e^{-hs}$$

$$N(a) = \frac{2}{\pi a} \int_{0-h}^{\frac{\pi}{\omega}-h} \phi(a\sin(\omega t)) \cdot \sin(\omega t) \cdot dt.$$

$$= \frac{2}{\pi a} \left[\int_{-h}^{0} -\sin(\omega t)\,\omega\,dt + \int_{0}^{\frac{\pi}{\omega}-h} \sin(\omega t)\,\omega\,dt \right]$$

$$= \frac{2}{\pi a} \left[1 - \cos(h\omega) + \cos(h\omega) + 1 \right] = \frac{4}{\pi a}.$$

$$1 + \hat{G}(j\omega) N(a) = 0.$$

$$1 + e^{hj\omega} \frac{2}{j\omega + 1} \cdot N(a) = 0$$

$$1 + j\omega + 2(\cos\omega h - j\sin\omega h)\, N(a) = 0.$$

$$\omega - 2\sin(\omega h) \cdot N(a) = 0 \quad ①$$

$$1 + 2\cos(\omega h) \cdot N(a) = 0 \quad ②$$

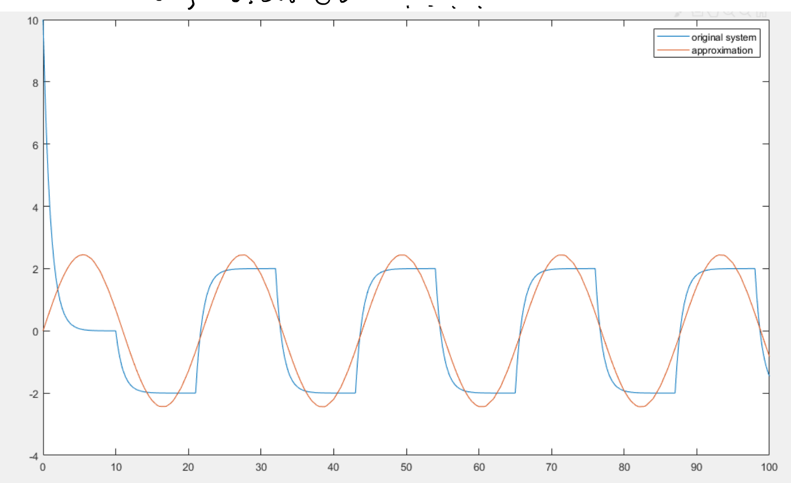
$$\Rightarrow \quad \omega = -\tan(\omega h).$$

use $h = 10$

$$\omega = 0,\ 0.2862 \ \ldots$$

$$N(a) = -\frac{1}{2\cos(\omega h)} = 0.5202$$

$$\underline{a = 2.4476.}$$



5. $\ddot{q} = u$.

$u = -K_p q - K_D \dot{q} - \text{sat}(\tilde{u})$, where $\tilde{u} = K_I \int q$

a). For $K_p = 1$, $K_D = 2$. determine the largest of $K_I \geq 0$ for which the closed loop system is asymptotically stable about $q(t) \equiv 0$.

$\hat{G}(s) = \frac{1}{s^2}$. $\hat{Y}(s) = \frac{K_d s^2 + k_p s + k_i}{s^3 + K_d s^2 + k_p s + k_i} = \frac{2s^2 + s + k_i}{s^3 + 2s^2 + s + k_i}$

$s^3 + 2s^2 + s + k_i = 0$.

$(jw)^3 + 2(jw)^2 + jw + k_i = 0$.

$-jw^3 - 2w^2 + jw + k_i = 0$.

$\begin{cases} -w^3 + w = 0 \\ k_i - 2w^2 = 0 \end{cases}$ $w = \pm 1, 0$ $k_i = 2, 0$. $\boxed{k_i < 2}$

b). . . . . . use describing function to determine the smallest value $k_i \geq 0$ for which the closed loop system has a periodic solution.

$\ddot{q} + q + 2\dot{q} = -\text{sat}(\tilde{u})$

$\hat{G}(s) = \frac{1}{s^2 + 2s + 1}$ $u = -\text{sat}(\tilde{u})$. suppose the saturation is at e