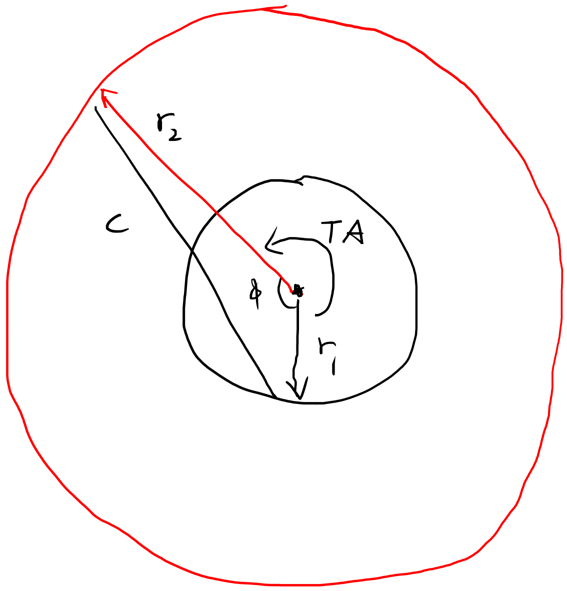
1. TOF = 15 hr. TA = 240° $\phi = 120°$.



TA = 240° > 180°

$\boxed{\text{Type 2}}$

From the example:

$r_1 = a_{phobos} = 9376$ km $\mu_♂ = 42828.314258067$ km³/s²

$r_2 = a_{Deimos} = 23458$ km $R_♂ = 3397$ km

$c = 2.9294 \times 10^4$ km $= 8.6 R_♂$

$s = 3.1039 \times 10^4$ km $= 9.144 R_♂$ $a_{min} = 15532$ km.

$$TOF_{par} = \frac{1}{3}\sqrt{\frac{2}{\mu_♂}}\left[s^{3/2} + (s-c)^{3/2}\right] = 3.5114 \text{ hr} < TOF$$

checks elliptical transfer

$\alpha_0 = 180°$ $\beta_0 = 27.62°$

$$TOF_{min} = \sqrt{\frac{a_{min}^3}{\mu_♂}}\left[(\alpha_0 - S\alpha_0) - (\beta_0 - S\beta_0)\right] = 8.1145 \text{ hr} < TOF$$

$\boxed{\text{Type 2B}}$

Solve a in Lambert's Equation using secant method.

$$\boxed{a = 1.8040 \times 10^4 \text{ km}}$$

$$\alpha = 223.7811^\circ$$

$$\beta = -25.5940^\circ$$

$$p = \frac{4a(s-r_1)(s-r_2)}{c^2} \sin^2\left(\frac{\alpha \pm \beta}{2}\right)$$

$$= \boxed{1.3524 \times 10^4 \text{ km}} \quad \text{or} \quad 0.9378 \times 10^4 \text{ km}$$

choose large p for small e ⇒ small Δae

$$e = \sqrt{1 - \frac{p}{a}} = \boxed{0.5003}$$

$\boxed{r_p = 9014.5 \text{ km} > R_\delta}$ check

$\boxed{r_a = 2706.5 \text{ km}}$

$$\xi = -\frac{\mu_\delta}{2a} = \boxed{-1.1871 \text{ km}^2/s^2}$$

$$V_{dep} = \sqrt{\frac{2\mu_\delta}{r_1} - \frac{\mu_\delta}{a}} = \boxed{2.6003 \text{ km/s}}$$

$$V_{arr} = \sqrt{\frac{2\mu_\delta}{r_2} - \frac{\mu_\delta}{a}} = \boxed{1.1302 \text{ km/s}}$$

$$\theta^*_{dep} = \cos^{-1}\left(\frac{p}{r_1 e} - \frac{1}{e}\right) = \boxed{-27.8257^\circ}$$

$$\theta^*_{arr} = \cos^{-1}\left(\frac{p}{r_2 e} - \frac{1}{e}\right) = \boxed{-147.8257^\circ}$$

$\theta^*_A - \theta^*_D = -120^\circ$

$TA = -120^\circ = 240^\circ$

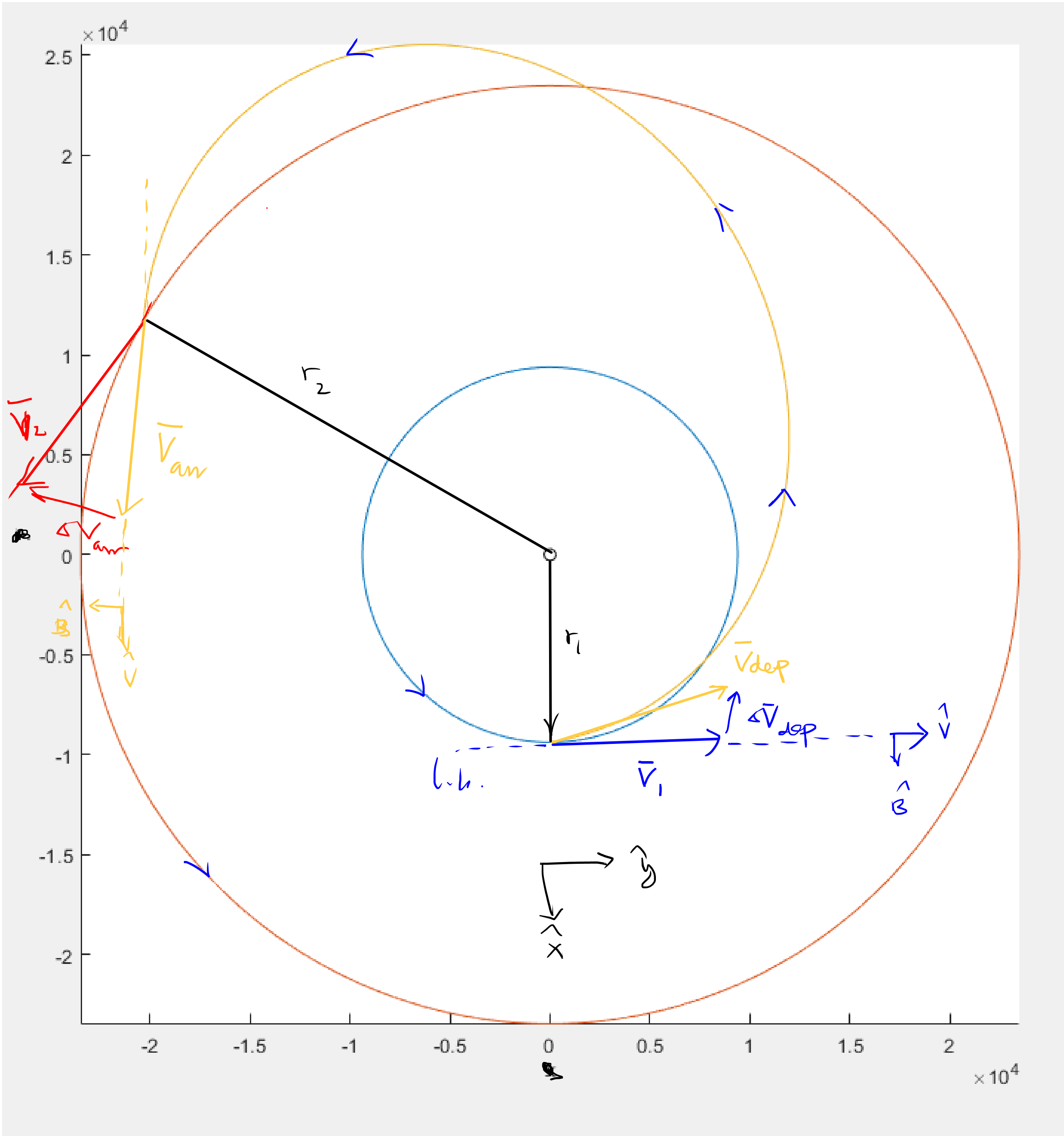
check.

$$\gamma_{dep} = \cos^{-1}\left(\frac{\sqrt{\mu_\delta p}}{r_1 V_{dep}}\right) = \boxed{-9.1963^\circ}$$

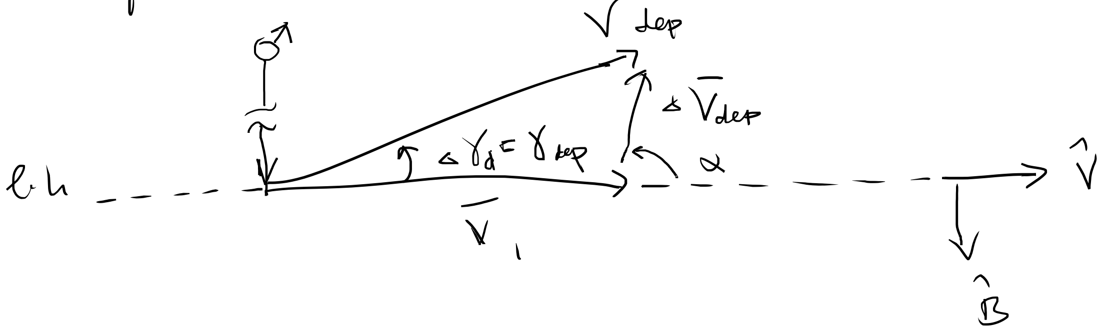
descending at both locations

$$\gamma_{arr} = \cos^{-1}\left(\frac{\sqrt{\mu_\delta p}}{r_2 V_{arr}}\right) = \boxed{-24.8007^\circ}$$

b)



i) Departure:



$$V_1 = \sqrt{\frac{\mu_{♂}}{a_{ph}}} = 2.1373 \text{ km/s}$$

$$\Delta V_{dep} = \sqrt{v_1^2 + V_{dep}^2 - 2 v_1 V_{dep} \cos|\gamma_{dep}|} = 0.5977 \text{ km/s}$$

$$\bar{V}_1 = 2.1373 \, \hat{v} \text{ km/s}$$

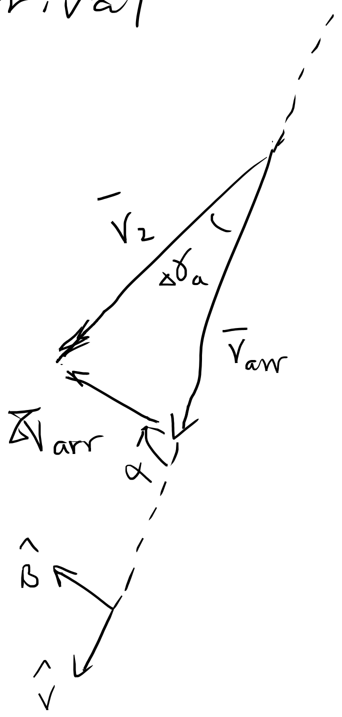
$$\bar{V}_{dep} = 2.6003 \cdot (\sin(90° - \Delta\gamma_d)\, \hat{v} + \cos(90° - \Delta\gamma_d)\, \hat{B})$$

$$= 2.5669 \, \hat{v} - 0.4156 \, \hat{B} \text{ km/s.}$$

$$\Delta\bar{V}_{dep} = \bar{V}_{dep} - \bar{V}_1 = \boxed{0.4296 \, \hat{v} - 0.4156 \, \hat{B} \text{ km/s}}$$

$$\alpha_d = \cos^{-1}\left(\frac{\Delta\bar{V}_{dep} \cdot \bar{V}_1}{|\Delta\bar{V}_{dep}| \cdot |V_1|}\right) = \boxed{-44.0472°}$$

ii) Arrival



$$V_2 = \sqrt{\frac{\mu_g}{r_2}} = 1.3512 \text{ km/s}$$

$$\Delta\gamma_a = 0 - \gamma_{arr} = -\gamma_{arr}$$

$$\Delta V_{arr} = 0.5749 \text{ km/s}$$

$$\bar{V}_{arr} = 1.1302 \, \hat{v} \text{ km/s}$$

$$\bar{V}_2 = 1.3512 \cdot (\sin(90° - \Delta\gamma_a)\, \hat{v} + \cos(90° - \Delta\gamma_a)\, \hat{B})$$

$$= 1.2266 \, \hat{v} + 0.5668 \, \hat{B} \text{ km/s}$$

$$\Delta\bar{V}_{arr} = \bar{V}_2 - \bar{V}_{arr}$$

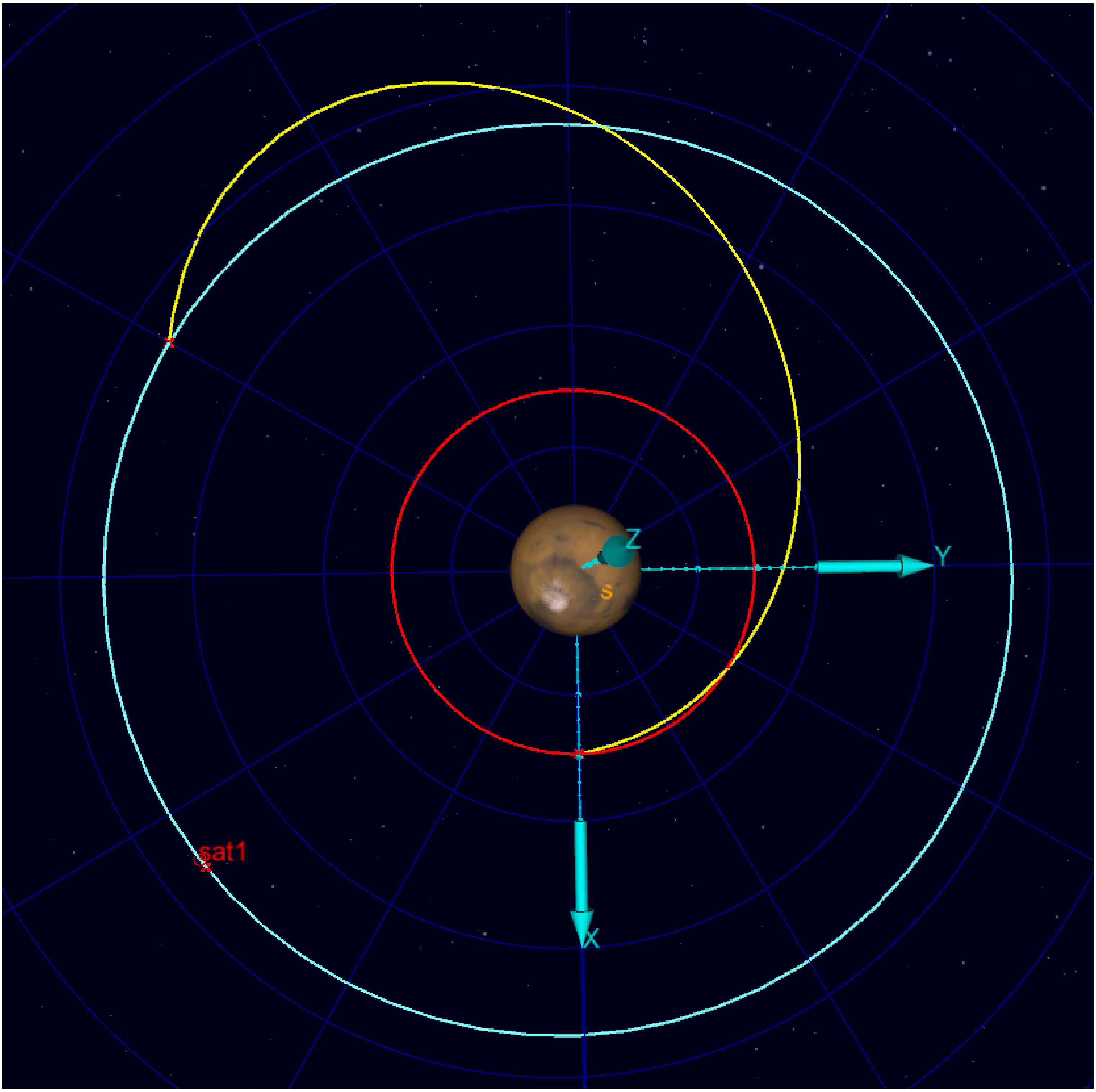
$$= \boxed{0.0964 \, \hat{v} + 0.5668 \, \hat{B}} \text{ km/s}$$

$$\alpha_a = \cos^{-1}\left(\frac{\Delta\bar{V}_{arr} \cdot \bar{V}_{arr}}{|\Delta\bar{V}_{arr}| \cdot |\bar{V}_{arr}|}\right) = \boxed{80.3491°}$$

$$\Delta V_{total} = \Delta V_{dep} + \Delta V_{arr} = \boxed{1.1726 \text{ km/s}} < 1.402 \text{ km/s}$$

This cost is smaller than the cost of minimal energy transfer

c)



The transfer passes through both periapsis & apoapsis

d) $n_{ph} = \sqrt{\frac{\mu_g}{a_{ph}^3}} = 2.2795 \times 10^{-4} \ s^{-1}$

$n_{de} = \sqrt{\frac{\mu_{de}}{a_{de}^3}} = 5.7601 \times 10^{-5} \ s^{-1}$

$P_{ph} = 2\pi / n_{ph} = 2.7564 \times 10^4 \text{ sec} = \underline{7.6566 \text{ hr}}$

$P_{De} = 2\pi / n_{de} = 1.0908 \times 10^4 \text{ sec} = \underline{30.3004 \text{ hr}}$

$\phi = 240° - n_{de} \cdot TOF = \boxed{61.7845°}$

$t_s = \frac{2\pi}{n_{Ph} - n_{De}} = 3.6884 \times 10^4 \text{ sec} = \boxed{10.2456 \text{ hr}}$

The synodic period is slightly longer than the period of Phobos but smaller than the period of Deimos, which is quite frequent in this problem.

2 .a) $TA = 120°$, $TOF = 160$ days

$r_1 = a_{\oplus}$ $\quad r_2 = a_{♂}$, $\Phi = 120°$ Type 1

$$c = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\Phi)} = 3.2930 \times 10^8 \text{ km}$$

$$s = \frac{1}{2}(r_1 + r_2 + c) = 3.5342 \times 10^8 \text{ km}$$

$$a_{min} = \frac{1}{2} s = 1.7671 \times 10^8 \text{ km}$$

$$TOF_{par} = \frac{1}{3}\sqrt{\frac{2}{\mu_s}}\left[s^{3/2} - (s-c)^{3/2}\right] = 97.7346 \text{ day} < 160 \text{ days } (TOF)$$

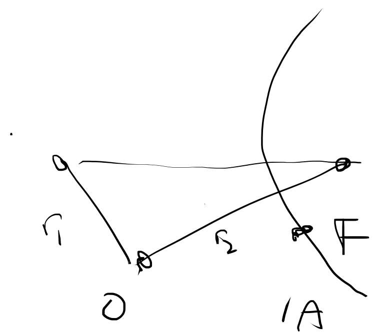
the transfer is elliptical

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a_m}} = \pi$$

$$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a_m}} = 30.2884°$$

$$TOF_{min} = \sqrt{\frac{a_m^3}{\mu_s}}\left[(\alpha_0 - \sin\alpha_0) - (\beta_0 - \sin\beta_0)\right] = 232.6504 \text{ days} > 160 \ (TOF)$$

the transfer type is $\boxed{1A}$



b) Solve of a iteratively:

$$\boxed{a = 2.0028 \times 10^8 \text{ km}}$$

$\alpha = 139.8778^\circ$

$\beta = 28.4108^\circ$

$\boxed{\text{type 1A}}$

$$p = 4a \cdot (s - r_1)(s - r_2) \cdot \sin^2\left(\frac{\alpha \pm \beta}{2}\right)$$

$$p = \boxed{1.8697 \times 10^8 \text{ km}} \quad \text{or} \quad 1.2904 \times 10^8 \text{ km}$$

choose larger P for smaller e

$$e = \sqrt{1 - \frac{p}{a}} = \boxed{0.2577}$$

$$\boxed{\begin{aligned} r_p &= 1.4866 \times 10^8 \text{ km} \\ r_a &= 2.5189 \times 10^8 \text{ km} \end{aligned}}$$

$$\mathcal{E} = -\frac{\mu_s}{2a} = \boxed{-331.3247 \text{ km}^2/\text{s}^2}$$

$$v_{dep} = \sqrt{\frac{2\mu_s}{r_1} - \frac{\mu_s}{a}} = \boxed{33.3408 \text{ km/s}}$$

$$v_{arr} = \boxed{22.4005 \text{ km/s}}$$

$$\theta^*_{dep} = \cos^{-1}\left(\frac{p}{r_1 e} - \frac{1}{e}\right) = \boxed{14.2201^\circ}$$

$$\theta^*_{arr} = \cos^{-1}\left(\frac{p}{r_2 e} - \frac{1}{e}\right) = \boxed{134.2201^\circ}$$

$\theta^*_A - \theta^*_D = 120^\circ = TA$

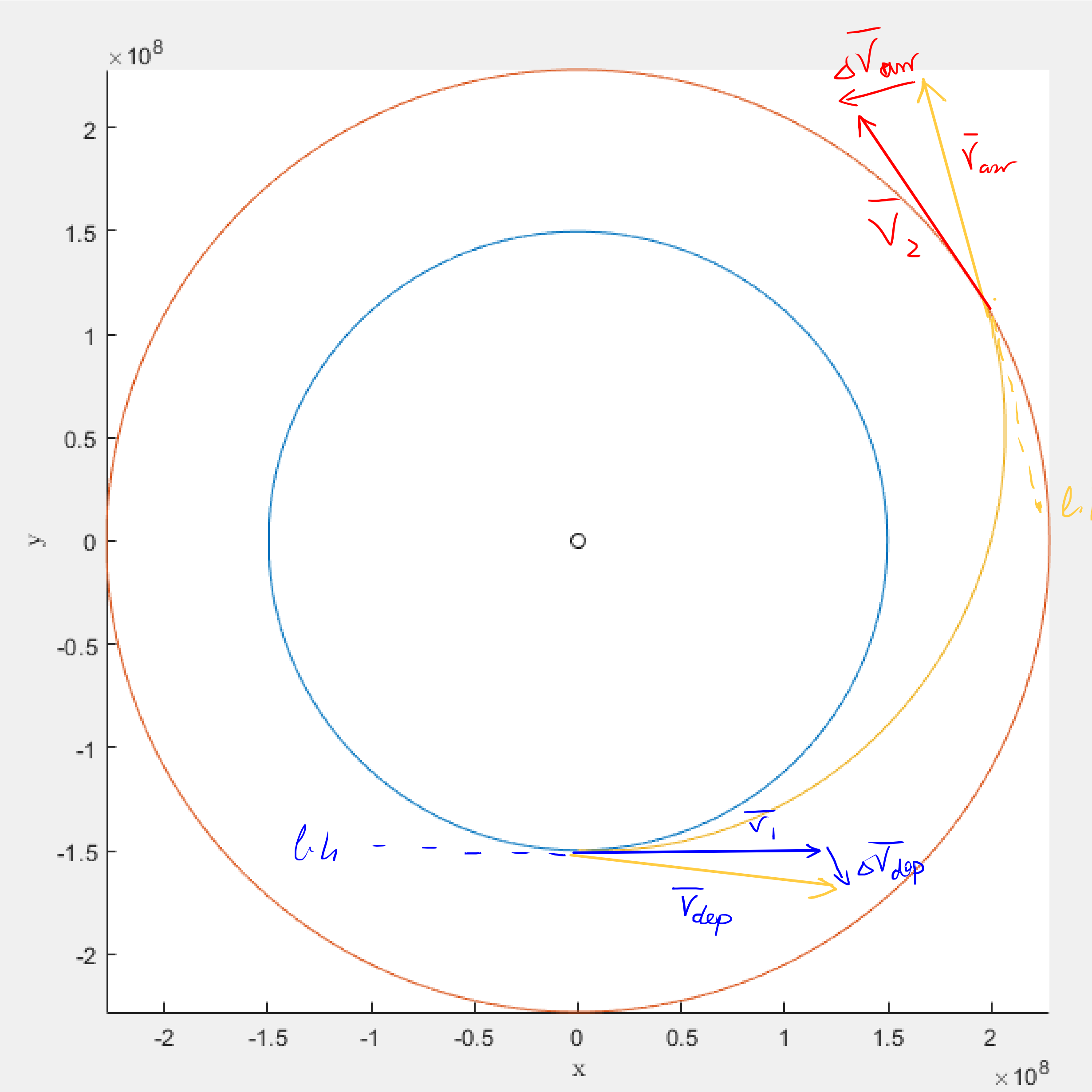
check.

$$\gamma_{dep} = \cos^{-1}\left(\frac{\sqrt{\mu_s p}}{r_1 v_{dep}}\right) = \boxed{2.8999^\circ}$$

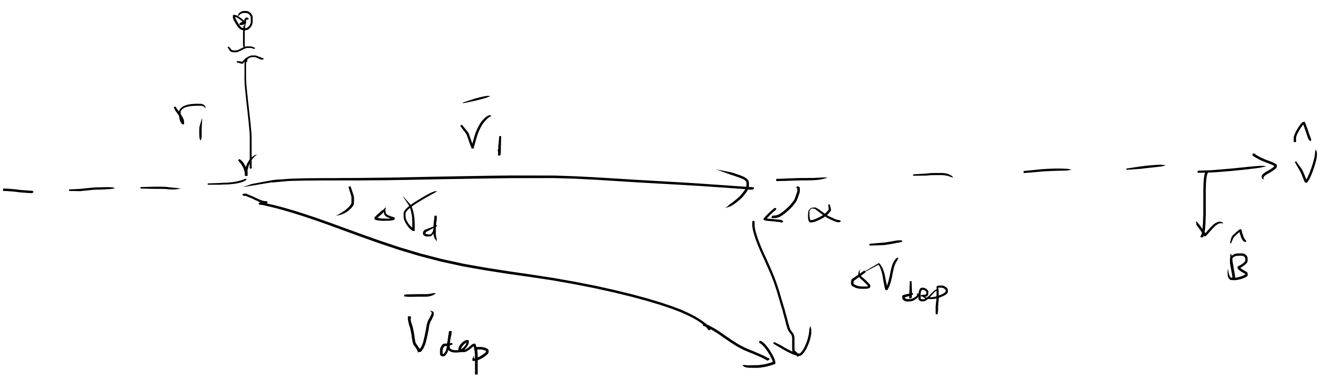
(Ascending at both locations)

$$\gamma_{arr} = \cos^{-1}\left(\frac{\sqrt{\mu_s p}}{r_2 v_{arr}}\right) = \boxed{12.6903^\circ}$$

c)



i) Departure :



$$\Delta\gamma_d = \gamma_{dep}$$

$$V_1 = \sqrt{\frac{\mu_s}{r_1}} = 29.7847 \text{ km/s} \quad , \quad V_{dep} = 33.3408 \text{ km/s}$$

$$\Delta V_{dep} = \sqrt{V_1^2 + V_{dep}^2 - 2V_1V_d\cos|\Delta\gamma|} = 3.8973 \text{ km/s}$$

$$\bar{V}_1 = 29.7847\,\hat{v} \text{ km/s}$$

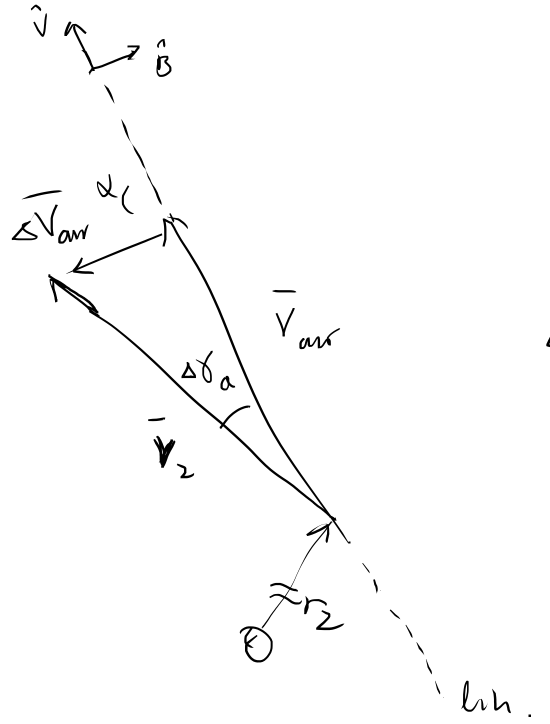
$$\bar{V}_{dep} = 33.3408 \cdot \left(\sin(90° - \Delta\gamma)\,\hat{v} + \cos(90° - \Delta\gamma)\,\hat{B}\right)$$

$$= 33.2981\,\hat{v} + 1.6867\,\hat{B} \text{ km/s}$$

$$\Delta\bar{V}_{dep} = \bar{V}_{dep} - \bar{V}_1 = \boxed{3.5134\,\hat{v} + 1.6867\,\hat{B} \text{ km/s}}$$

$$\alpha_{dep} = \cos^{-1}\left(\frac{\Delta\bar{V}_d \cdot \bar{V}_1}{|\Delta\bar{V}_d|\,|\bar{V}_1|}\right) = \boxed{25.6451°}$$

ii) Arrival



$$V_2 = \sqrt{\frac{\mu_s}{r_2}} = 24.1291 \text{ km/s}$$

$$V_{arr} = 22.4005 \text{ km/s}$$

$$\Delta\gamma_a = -\gamma_{arr}$$

$$\Delta V_{arr} = \sqrt{V_2^2 + V_{arr}^2 - 2V_2V_a\cos|\Delta\gamma_a|}$$

$$= 5.4217 \text{ km/s}$$

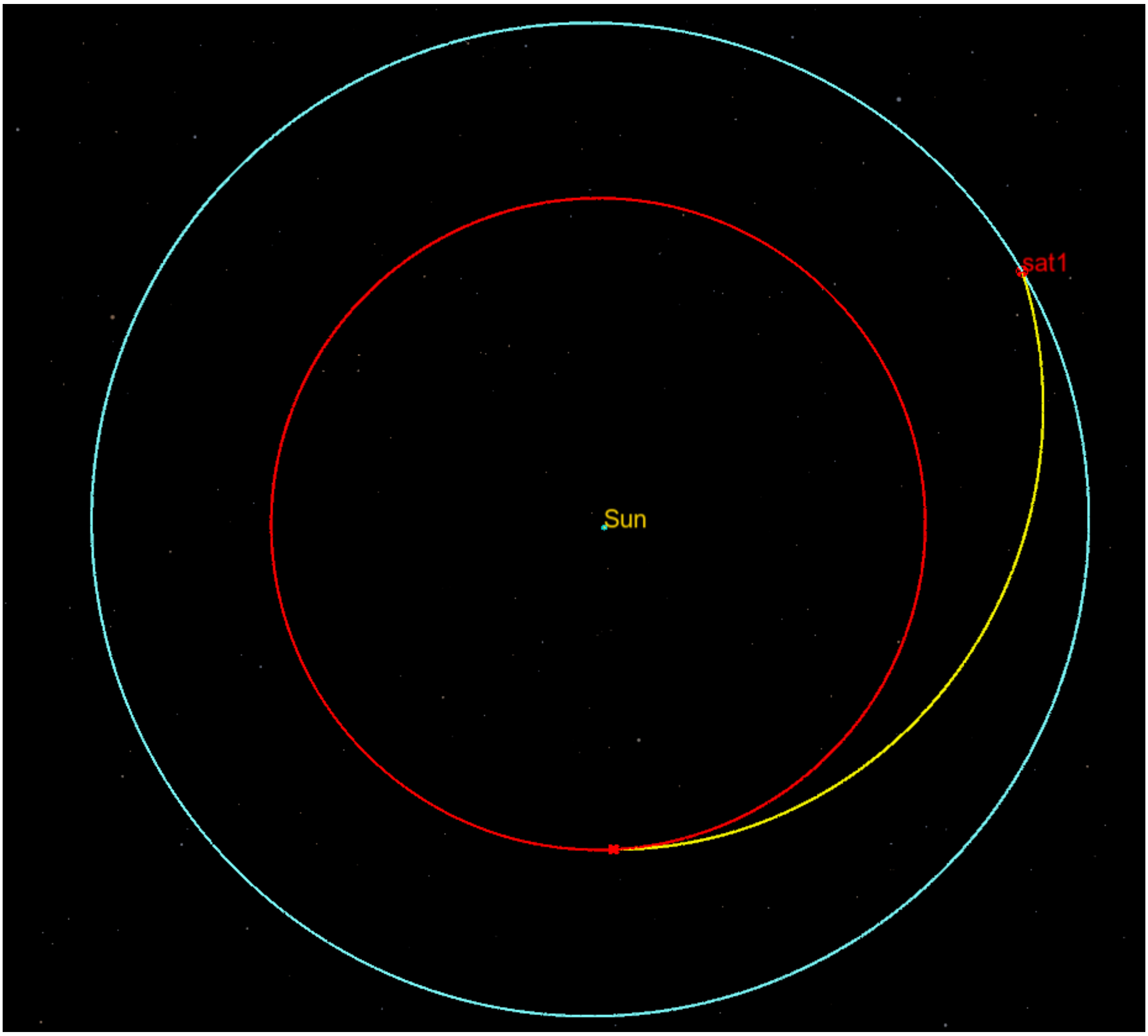
$$\bar{V}_{arr} = 22.4005\,\hat{v} \text{ km/s}$$

$$\bar{V}_2 = 24.1291 \cdot \left[\sin(90° - \Delta\gamma_a)\,\hat{v} + \cos(90° - \Delta\gamma_a)\,\hat{B}\right] \qquad \Delta V_{total} = 9.3190 \text{ km/s}$$

$$= 23.5397\,\hat{v} - 5.3007\,\hat{B} \text{ km/s}$$

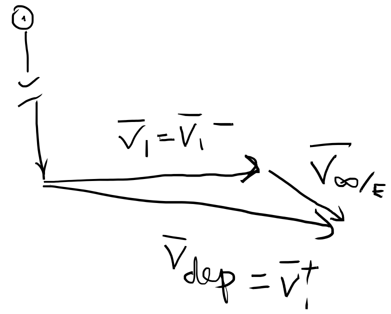
$$\Delta\bar{V}_{arr} = \bar{V}_2 - \bar{V}_{arr} = \boxed{1.1392\,\hat{v} - 5.3007\,\hat{B} \text{ km/s}} \quad \alpha = \cos^{-1}\left(\frac{\Delta\bar{V}_a \cdot \bar{V}_a}{|\Delta\bar{V}_a|\,|\bar{V}_a|}\right) = \boxed{-77.8703°}$$

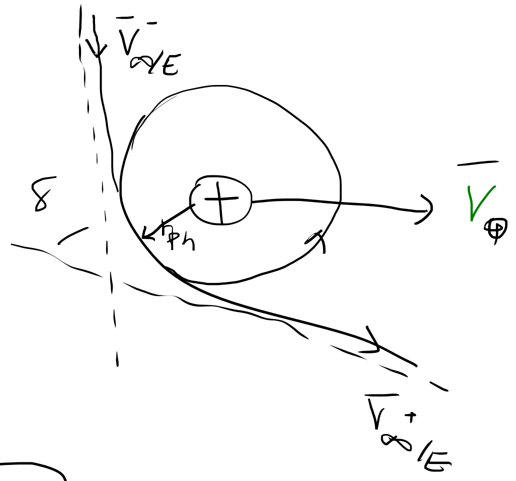
d)



See matlab plot above in c). GMAT result checks out.

f)





pass behind,

$\boxed{|\bar{V}_{\infty/E}| = 3.8973 \text{ km/s}}$, $r_{p.h} = 1000 + R_\oplus$

$$|a_h| = \frac{\mu_E}{V_{\infty/E}^2} = 2.6243 \times 10^4 \text{ km}$$

$$e_h = \frac{r_{p.h}}{|a_h|} + 1 = 1.0381 \quad , \quad \delta = 148.8857^\circ$$

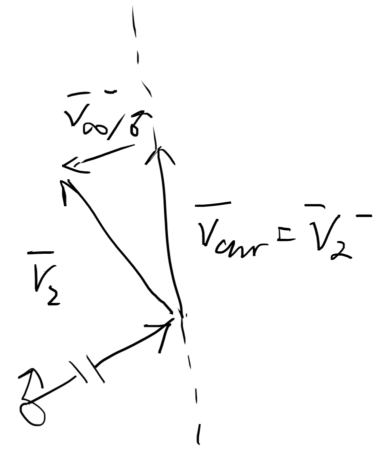
$$V_{P.h} = \sqrt{\frac{2\mu_E}{r_{ph}} + \frac{\mu_E}{a_h}} = \boxed{11.1013 \quad \text{km/s}}$$

V_{ph}
$\vec{v_c} \quad \vec{\Delta V}$

$$V_c = \sqrt{\frac{\mu_E}{r_{ph}}} = 7.3501 \text{ km/s.} \quad |\bar{\Delta V}| = 3.7511 \text{ km/s}$$

If the maneuver is tangential, it is quite reasonable for the Earth launched vehicle to reach this speed with the associated ΔV.

g)



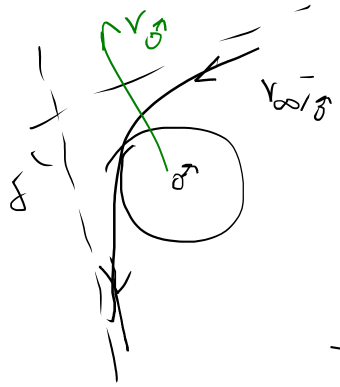
$|\bar{V}_{\infty/♂}| = 5.4217$ km/s

$|a_h| = 1.4570 \times 10^3$ km

$e_h = 1.3432$

$\delta = 96.2331^\circ$

$\boxed{v_{p,h} = 7.1677 \text{ km/s}}$



pass ahead,

It's unlikely that Mark can reach this speed directly from the surface of the Mars

3. a) $TA = 120°$, $TOF = 92$ days

$r_1 = a_\oplus$ $r_2 = a_♂$, $\Phi = 120°$ Type 1

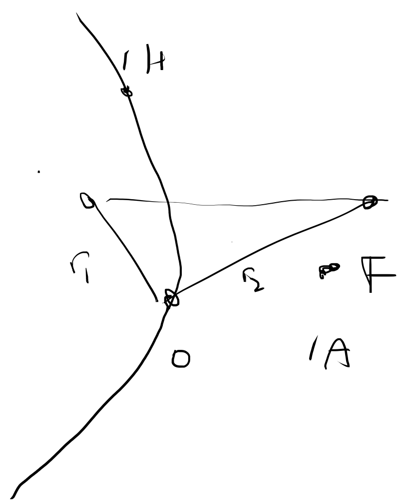
$c = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\Phi)} = 3.2930 \times 10^8$ km

$S = \frac{1}{2}(r_1 + r_2 + c) = 3.5342 \times 10^8$ km

$a_{min} = \frac{1}{2} S = 1.7671 \times 10^8$ km

$TOF_{par} = \frac{1}{3}\sqrt{\frac{2}{\mu_S}}\left[S^{3/2} - (S-c)^{3/2}\right] = 97.7346$ day > 92 days (TOF)

the transfer is hyperbolic $\boxed{\text{Type 1H}}$



b). Solve for $|a|$ iteratively.

$\boxed{\text{Type 1H}}$

$$\boxed{|a| = 8.2606 \times 10^{8} \text{ km}}$$

$\alpha' = 51.2724°$

$\beta' = 13.8128°$

$$p = \frac{4|a|(s-r_1)(s-r_2)}{c^2} \sinh^2\left(\frac{\alpha' \pm \beta'}{2}\right)$$

$$= \boxed{2.7962 \times 10^{8} \text{ km}} \text{ or } 0.8629 \times 10^{8} \text{ km}.$$

$$\boxed{\begin{aligned} r_p &= 1.2964 \times 10^{8} \text{ km} \\ r_a &= \infty \end{aligned}}$$

choose larger p

$\boxed{e = 1.1569}$ $\boxed{\mathcal{E}_e = -80.3289 \text{ km}^2/\text{s}^2}$

$$v_{dep} = \sqrt{\frac{2\mu_s}{r_1} + \frac{\mu_s}{|a|}} = \boxed{43.9876 \text{ km/s}}$$

$$v_{arr} = \sqrt{\frac{2\mu_s}{r_2} + \frac{\mu_s}{|a|}} = \boxed{36.4017 \text{ km/s}}$$

$$\theta^*_{dep} = \cos^{-1}\left(\frac{p}{r_1 e} - \frac{1}{e}\right) = \boxed{-41.3057°}$$

$$\theta^*_{arr} = \cos^{-1}\left(\frac{p}{r_2 e} - \frac{1}{e}\right) = \boxed{78.6993°}$$

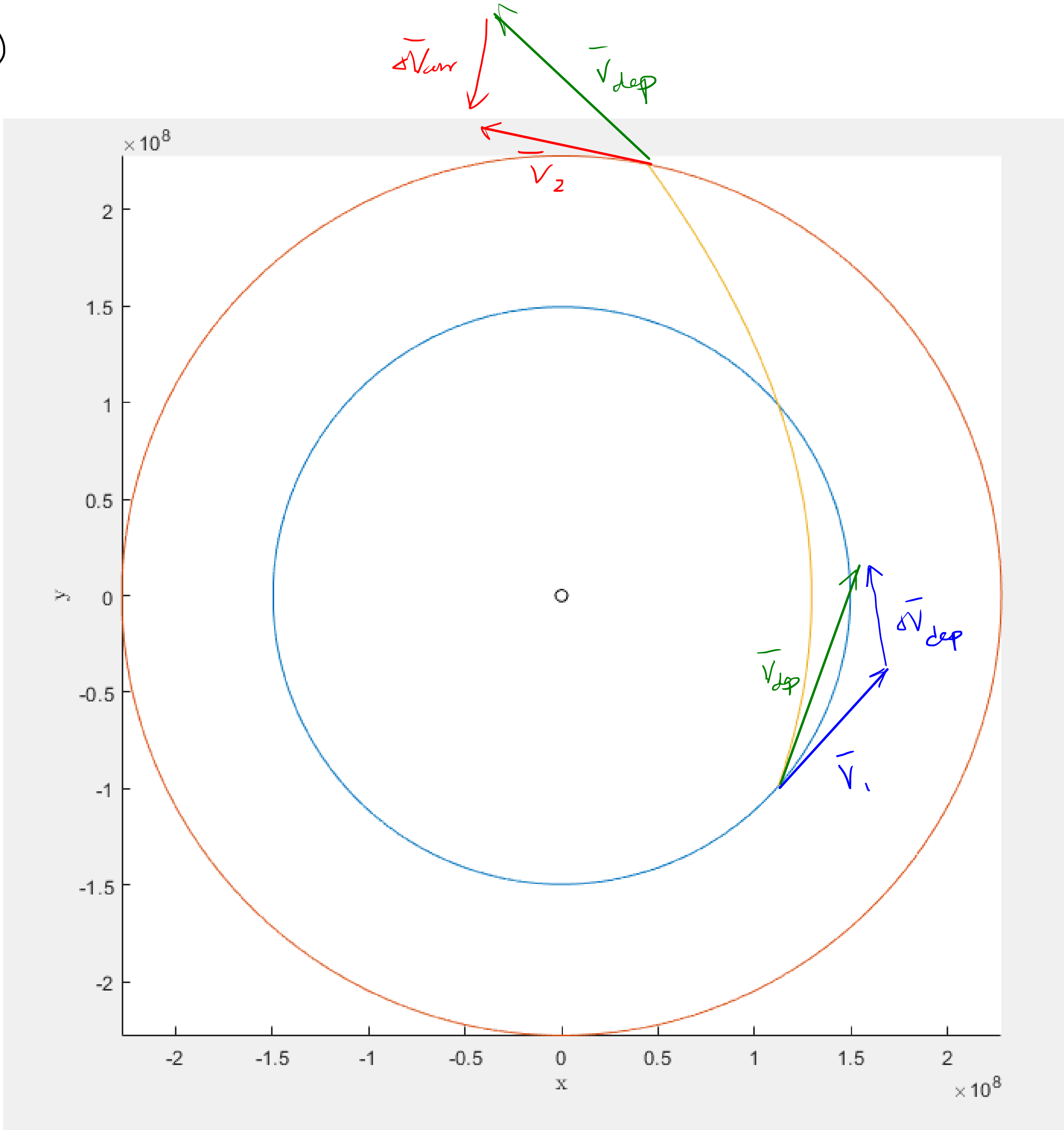
$\theta^*_A - \theta^*_D = 120° = TA$

check.

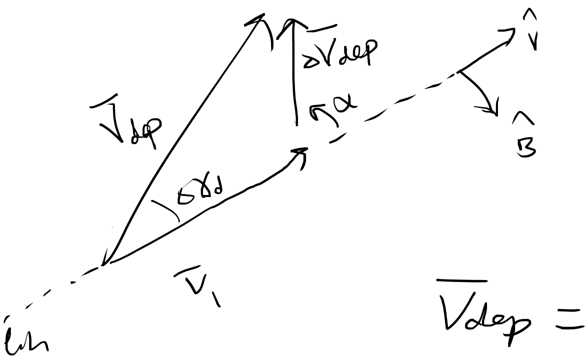
$$\gamma_{dep} = \cos^{-1}\left(\frac{\sqrt{\mu_s p}}{r_1 v_{dep}}\right) = \boxed{-22.2211°} \text{ (descending)}$$

$$\gamma_{arr} = \cos^{-1}\left(\frac{\sqrt{\mu_s p}}{r_2 v_{arr}}\right) = \boxed{42.7637°} \text{ (ascending)}$$

c)



i) Departure:



$\bar{V}_1 = 29.7847$ km/s $\bar{V}_{dep} = 43.9876$ km/s

$\Delta\gamma_d = \gamma_d$

$\bar{V}_1 = 29.7847\,\hat{V}$ km

$\bar{V}_{dep} = 43.9876\left(\sin(90^\circ - \delta\gamma)\,\hat{V} + \cos(90^\circ - \delta\gamma)\,\vec{B}\right)$

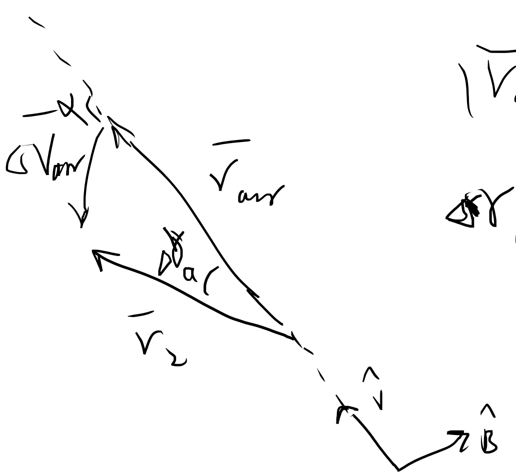
$= 40.7207\,\hat{V} - 16.6353\,\hat{B}$ km/s

$$\Delta\bar{V}_{dep} = \bar{V}_{dep} - \bar{V}_1 = \boxed{10.9361\,\hat{V} - 16.6353\,\hat{B}\ \text{km/s}}$$

$$|\Delta\bar{V}_{dep}| = 19.9081\ \text{km/s}$$

$$\alpha = -56.6791^\circ$$

ii) Arrival.



$$V_2 = 24.1291\ \text{km/s}$$

$$|\bar{V}_{arr}| = 36.4017\,\hat{V}\ \text{km/s}$$

$$\delta\gamma_a = -\gamma_a$$

$$|\bar{V}_2| = 24.1291\left[\sin(90^\circ - \delta\gamma_a)\,\hat{V} + \cos(90^\circ - \delta\gamma_a)\,\hat{B}\right]$$

$$= 17.7146\,\hat{V} - 16.3831\,\hat{B}\ \text{km/s}$$

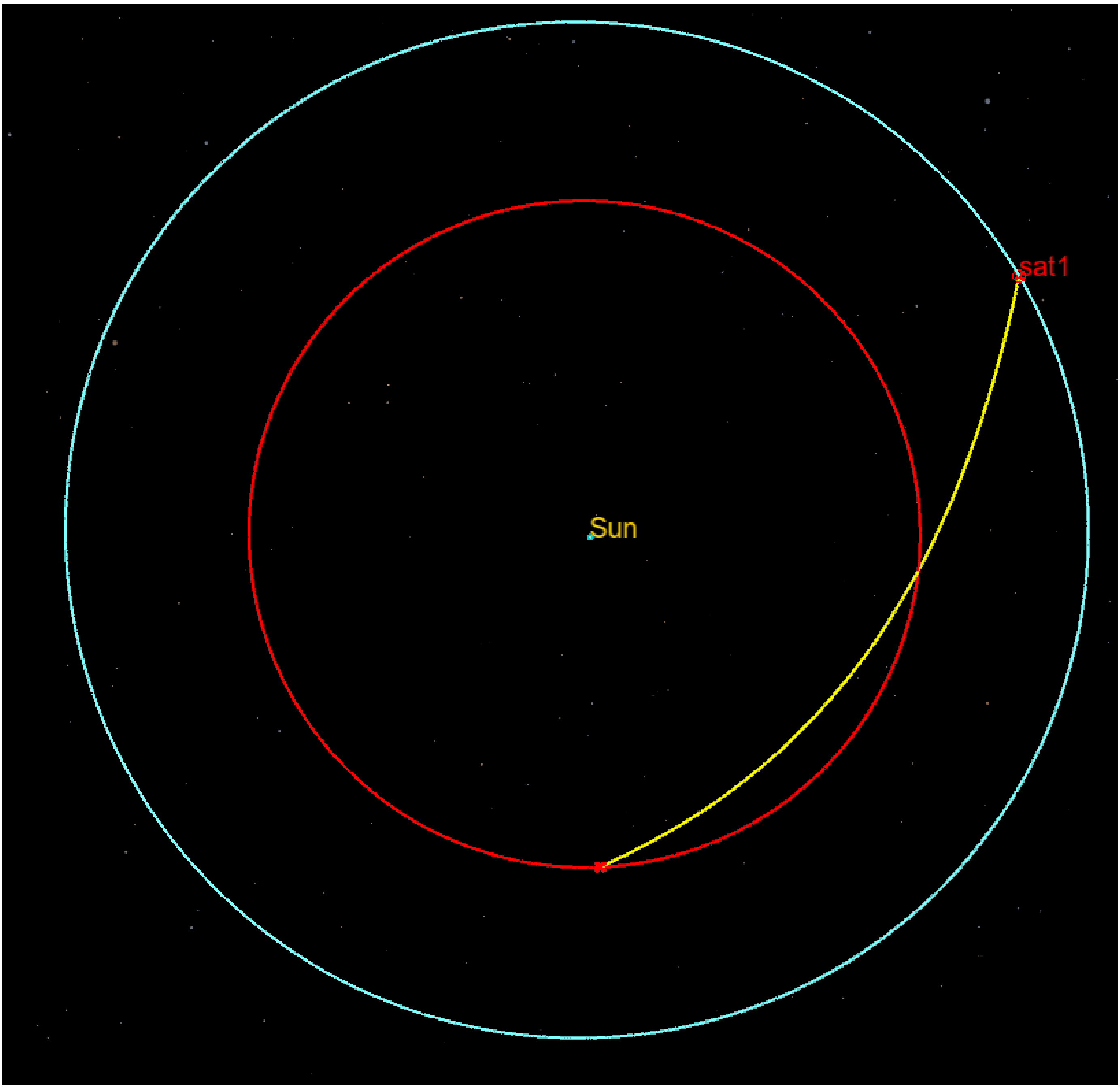
$$\Delta\bar{V}_{arr} = \bar{V}_2 - \bar{V}_{arr} = \boxed{-18.6871\,\hat{V} - 16.3831\,\hat{B}\ \text{km/s}}$$

$$|\Delta\bar{V}_{arr}| = 24.8519\ \text{km/s}$$

$$\alpha = -138.7587^\circ$$

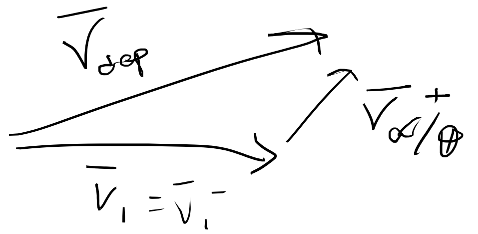
$$\Delta V_{total} = 44.7599\ \text{km/s}$$

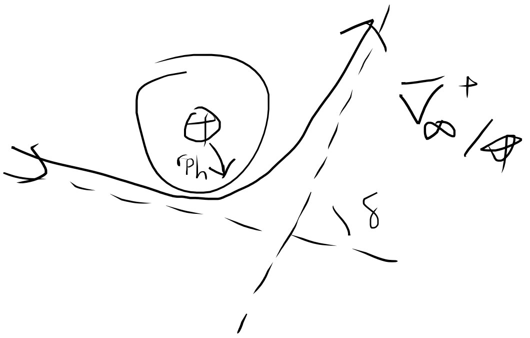
d) See c) for matlab plot



GMAT result checks out

f)





$$\boxed{|\bar{V}_{\infty/\oplus}| = 19.9081 \text{ km/s}}$$

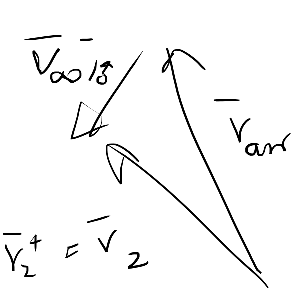
$|a_h| = 1.0057 \times 10^3$ km

$e_h = 1.9943$. $\delta = 60.1889°$

$$\boxed{V_{P,h} = 22.4584 \text{ km/s}}$$

this transfer seems unreasonable

g)



$|\bar{V}_{\infty/♂}^{-}| = 24.8519$ km/s

$|a_h| = 69.3047$ km

$e_h = 8.2104$

$\delta = 13.9917°$

$V_{ph} = 25.2902$ km/s $\gg V_c = 9.2551$ km/s

It's unlikely that Mark can reach this speed from the surface of the Mars.

It's also quite unreasonable to reach Mars in 92 days.

4. a) Choose TOF of 175 days, with the same space triangle. The resulting transfer is 1A

We know that shortening the TOF will cost more ΔV thus we can try increasing the TOF by a few days and calculate ΔV_{total}.

b). For the new TOF

$a = 1.5857 \times 10^8$ km $\qquad \alpha = 150.9563^\circ$

$\qquad \beta = 29.0987^\circ$

$p = 1.7789 \times 10^8$ km

$e = 0.2379$ $\qquad r_p = 1.4370 \times 10^8$ km

$\mathcal{E} = -351.8983$ km²/s² $\qquad r_a = 2.3343 \times 10^8$ km

$V_{dep} = 32.7179$ km $\qquad V_{arr} = 21.6624$ km

$\theta^*_{dep} = -37.3526^\circ$ $\qquad \theta^*_{arr} = -157.3526^\circ$

$\gamma_{dep} = -6.5216^\circ$ $\qquad \gamma_{arr} = -6.6955^\circ$

$\bar{V}_1 = 29.7847$ km/s $\qquad \Delta\bar{V}_{dep} = 4.7757$ km/s

$\bar{V}_2 = 24.1291$ km/s $\qquad \Delta\bar{V}_{arr} = 3.7650$ km/s

$\Delta\bar{V}_{total} = 8.5408$ km/s

This ΔV_{total} is smaller than the ΔV_{total} in the case of TOF = 160 days. (9.3 km/s)

ΔV_{dep} is slightly larger than before but ΔV_{arr} is slightly smaller than before.

I think the reason why the ΔV_{total} cost improved is that the slight longer TOF resulted in a comparatively lower energy orbit, and at the same time, the resulting departure γ and arrival γ are smaller, making the maneuvers more tangent, and the v^- & v^+ have closer values.

c) The next step would be iteratively solve for best TOF at this space triangle.

Once we have a best TOF for a variety of space triangles (different TA), then we can compare the results and choose the best pair.