STAT511 HW-3 Zhanpeng Yang

2.12

A: student have VISA card

B: student have MasterCard.

$P(A) = 0.5$ $P(A \cap B) = 0.25$

$P(B) = 0.4$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.4 - 0.25 = \boxed{0.65}$

b) $1 - P(A \cup B) = 1 - 0.65 = \boxed{0.35}$

c) $P(A \cap B') = P(A \cup B) - P(B)$

$= 0.65 - 0.4 \boxed{= 0.25}$

2.26

$P(A_1) = 0.12 \quad P(A_2) = 0.07 \quad P(A_3) = 0.05$

$P(A_1 \cup A_2) = 0.13 \quad P(A_1 \cup A_3) = 0.14$

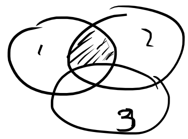
$P(A_2 \cup A_3) = 0.10 \quad P(A_1 \cap A_2 \cap A_3) = 0.01$

a) $P(A_1') = 1 - 0.12 = \boxed{0.88}$

b) $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$

$= 0.12 + 0.07 - 0.13$

$= \boxed{0.06}$



c) $P(A_1 \cap A_2 \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3)$

$= 0.06 - 0.01$

$= 0.05$

d) P(at most 2 defect) $= 1 -$ P(3 defects)

$= 1 - P(A_1 \cap A_2 \cup A_3)$

$= 1 - 0.01$

$= 0.99$

2.32

Receiver : 5

Disc player : 4

Speaker : 3

Turntable : 4

a) $5 \times 4 \times 3 \times 4 = 240$

b) $1 \times 1 \times 3 \times 4 = 12$

c) $4 \times 3 \times 3 \times 3 = 108$

d) $240 - 108 = 132$

e) $P(\text{at least } 1 \text{ sony}) = \frac{132}{240} = \boxed{0.55}$

$P(\text{exactly } 1 \text{ sony})$

$= P(\text{Receiver sony}) + P(\text{player sony}) + P(\text{speaker sony}) + P(\text{turntable sony})$

$= \frac{3 \times 3 \times 3}{240} + \frac{4 \times 3 \times 3}{240} + 0 + \frac{4 \times 3 \times 3}{240}$

$\boxed{= 0.4125}$

2.38)

4 – 40W
5 – 60W   select 3 bulbs at random
6 – 75W

a) $P(2\text{-}75W) = \frac{6}{15} \times \frac{5}{14} \times \frac{9}{13} \times 3 = \boxed{\frac{27}{91}}$

b) $P(\text{same rating}) = \frac{6\times5\times4 + 5\times4\times3 + 4\times3\times2}{15\times14\times13} = \boxed{\frac{34}{455}}$

c) $P(\text{one of each}) = \frac{6\times5\times4}{15\times14\times13} \times 3! = \boxed{\frac{24}{91}}$

d) $P(\text{at least six}) = \frac{\binom{9}{5}}{\binom{15}{5}} = \boxed{\frac{6}{143}}$

2.47) $P(A) = 0.5 \quad P(B) = 0.4 \quad P(A \cap B) = 0.25$

a) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.25}{0.5} = \boxed{\dfrac{1}{2}}$

b) $P(B'|A) = \dfrac{P(B' \cap A)}{P(A)} = \dfrac{0.25}{0.5} = \boxed{\dfrac{1}{2}}$

c) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.25}{0.4} = \boxed{\dfrac{5}{8}}$

c) $P(A'|B) = \dfrac{P(A' \cap B)}{P(B)}$

$= \dfrac{P(A \cup B) - P(A)}{P(B)}$

$= \dfrac{0.65 - 0.5}{0.4} = \boxed{\dfrac{3}{8}}$

e) $P(A | A \cup B) = \dfrac{P(A)}{P(A \cup B)} = \dfrac{0.5}{0.65} = \boxed{\dfrac{10}{13}}$

2.48)

a) $P(A_2 | A_1) = \dfrac{P(A_2 \cap A_1)}{P(A_1)}$

$= \dfrac{0.06}{0.12} = \boxed{\dfrac{1}{2}}$

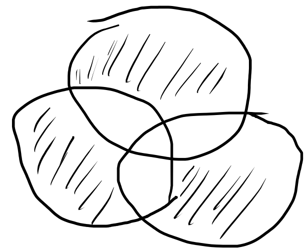
b) $P(A_1 \cap A_2 \cap A_3 | A_1) = \dfrac{0.01}{0.12} = \boxed{\dfrac{1}{12}}$

c) P(exactly 1 defect | at least 1 defect)

$= \dfrac{P(\text{exactly 1 defect})}{P(\text{at least 1 defect})}$

P(exactly 1 defect) $= P(A_1 \cap A_2' \cap A_3') + P(A_1' \cap A_2 \cap A_3') + P(A_1' \cap A_2' \cap A_3)$

$= P(A_1 \cup A_2 \cup A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_3 \cap A_1)$
$+ 2P(A_1 \cap A_2 \cap A_3)$



$P(A_1 \cap A_2) = 0.06$

$P(A_1 \cap A_3) = 0.12 + 0.05 - 0.14 = 0.03$

$P(A_2 \cap A_3) = 0.07 + 0.05 - 0.1 = 0.02$

$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3)$
$- P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

$= 0.12 + 0.07 + 0.05 - 0.06 - 0.03 - 0.02 + 0.01$

$= 0.14$

P (exactly 1 defect) = $0.14 - 0.06 - 0.03 - 0.02 + 0.02 = 0.05$

$$\Rightarrow \frac{0.05}{0.14} = \boxed{\frac{5}{14}}$$

d) $$P(A_3' \mid A_1 \cap A_2) = \frac{P(A_3' \cap A_1 \cap A_2)}{P(A_1 \cap A_2)}$$

$$= \frac{0.05}{0.06} = \boxed{\frac{5}{6}}$$

2.68) $P(A_1) : 0.5$ $P(D|A_1) = 0.3$, $P(L|A_1) = 0.1$

$P(A_2) : 0.3$ $P(D|A_2) = 0.25$, $P(L|A_2) = 0.2$

$P(A_3) : 0.2$ $P(D|A_3) = 0.4$, $P(L|A_3) = 0.25$

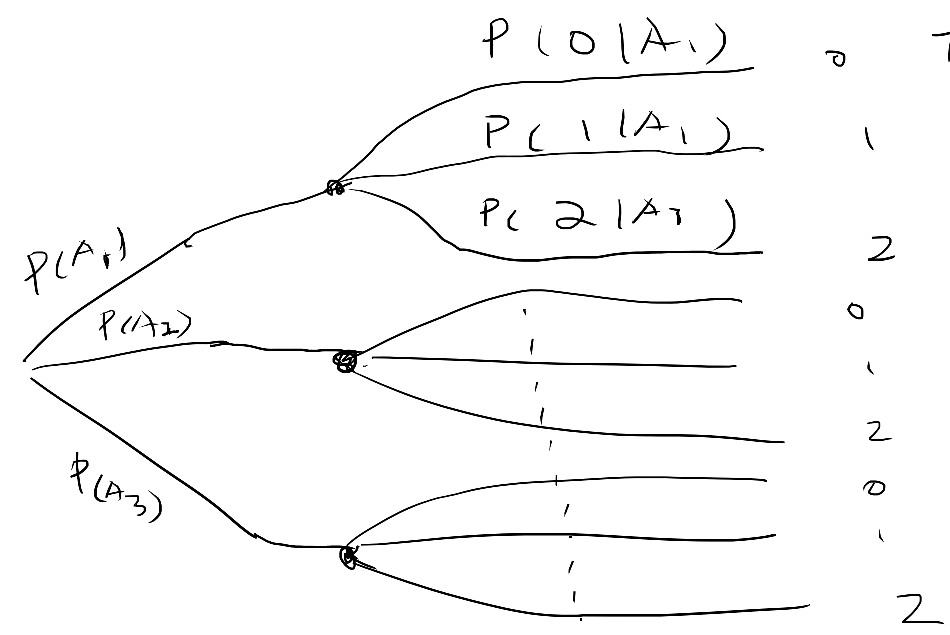
If she's late at exactly one location,

find probability she flew on #1, #2 & #3

$P(A_n | 1)_{n=1,2,3}$?

(Late at L.A is independent of being late at D.C)

$P(A_n | 1) = \frac{P(A_n \cap 1)}{P(1)}$



$P(0|A_1) = P(D'|A_1) \times P(L'|A_1)$

$= 0.7 \times 0.9 = 0.63$

$P(1|A_1) = P(D'|A_1) \times P(L|A_1) + P(D|A_1) \times P(L'|A_1)$

$= 0.7 \times 0.1 + 0.3 \times 0.9 = 0.34$

$P(2|A_1) = P(D|A_1) \times P(L|A_1) = 0.03$

$P(A_1 \cap 1) = P(A_1) \times P(1 | A_1) = 0.5 \times 0.34 = 0.17$

$P(1 | A_2) = 0.25 \times 0.8 + 0.75 \times 0.2 = 0.35$

$P(A_2 \cap 1) = 0.35 \times 0.3 = \underline{0.105}$

$P(1 | A_3) = 0.4 \times 0.75 + 0.6 \times 0.25 = 0.45$

$P(A_3 \cap 1) = 0.2 \times 0.45 = 0.09$

$P(1) = 0.17 + 0.105 + 0.09 = 0.365$

$P(A_1 | 1) = \dfrac{0.17}{0.365} = \boxed{0.4658}$

$P(A_2 | 1) = \dfrac{0.105}{0.365} = \boxed{0.2877}$

$P(A_3 | 1) = \dfrac{0.09}{0.365} = \boxed{0.2466}$

2.80)

P(system works)

$= 1 - P(\text{not work})$

$P(\text{not work}) = P(1' \cap 2') \times (1 - P(3 \cap 4))$

$= 0.1 \times 0.1 \times (1 - 0.1 \times 0.1)$

$= 0.0099$

$P(\text{system work}) = 1 - 0.0099 = \boxed{0.9981}$