P130. 1. $f_X(x) = c e^{-|x|}$

1). $\int_{-\infty}^{\infty} c e^{-|x|} dx = 1.$

$c\left[\int_{-\infty}^{0} e^{x} dx + \int_{0}^{\infty} e^{-x} dx\right] = 1.$

$c \quad (1 + 1) = 1$

$\boxed{c = \frac{1}{2}}$

2) $\mu_x = \int_{-\infty}^{\infty} x \cdot \frac{e^{-|x|}}{2} dx.$

$= \frac{1}{2}\left[\int_{-\infty}^{0} x e^{x} dx + \int_{0}^{\infty} x e^{-x} dx\right]$

$= \frac{1}{2} \cdot [-1 + 1]$

$\mu_x = Ex = \boxed{0}$

3) $\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) \, dx$

$= \frac{1}{2} \cdot \left[\int_{-\infty}^{0} x^2 e^{x} dx + \int_{0}^{\infty} x^2 e^{-x} dx\right]$

$= \frac{1}{2} \cdot (2+2)$

$E(x-\mu_x)^2 = \boxed{2}$

P130. 2. $F_X(x) = \begin{cases} 0 & x < -1 \\ a + b\arcsin(x) & -1 \le x \le 1 \\ 1 & 1 < x \end{cases}$

$-\frac{\pi}{2}$

1). 2). $\begin{cases} a + b\sin^{-1}(-1) = 0 \\ a + b\sin^{-1}(1) = 1 \end{cases}$ $\Rightarrow \begin{cases} \boxed{a = \frac{1}{2}} \\ \boxed{b = \frac{1}{\pi}} \end{cases}$

$\frac{\pi}{2}$

3) $f_X(x) = \frac{d}{dx} F_X(x)$

$= \frac{1}{\pi\sqrt{1-x^2}}$

$$\boxed{f_X = \begin{cases} \frac{1}{\pi\sqrt{1-x^2}} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}}$$

4). $\mu_X = \int_{-\infty}^{\infty} x f_X(x)\,dx$

$= \int_{-1}^{1} \frac{x}{\pi\sqrt{1-x^2}}\,dx$

$= \boxed{0}$

5). $\sigma_X^2 = \int_{-\infty}^{\infty} (x-\mu_X)^2 f_X(x)\,dx$

$= \int_{-1}^{1} \frac{x^2}{\pi\sqrt{1-x^2}}\,dx$

$= \boxed{\frac{1}{2}}$

PB1. 4.

$$f_{X_n} = \frac{1}{2} \quad \text{if} \quad n \le |x| \le n+1$$

$$= 0 \quad \text{otherwise.}$$

$$\mu_{X_n} = \int_{-\infty}^{\infty} x f_{X_n} \, dx$$

$$= \int_{n}^{n+1} \frac{x}{2} \, dx + \int_{-n-1}^{-n} \frac{x}{2} \, dx.$$

$$= \frac{x^2}{4} \Big|_{n}^{n+1} + \frac{x^2}{4} \Big|_{-n-1}^{-n}$$

$$= 0.$$

$$\sigma_{X_n}^2 = \int_{-\infty}^{\infty} \frac{(x - \mu_{X_n})^2}{2} \, dx.$$

$$= \int_{n}^{n+1} \frac{x^2}{2} \, dx + \int_{-n-1}^{-n} \frac{x^2}{2} \, dx$$

$$= \frac{1}{6} \left[x^3 \Big|_{n}^{n+1} + x^3 \Big|_{-n-1}^{-n} \right]$$

$$= n^2 + n + \frac{1}{3}$$

$$\lim_{n \to \infty} \sigma_{X_n}^2 \to \infty$$

P140. 4. $f_X = \frac{1}{3}$ $\quad -1 \le x \le 2$. $\quad Y = X^2$. $\quad Y \in [0, 4]$.

$= 0$ $\quad$ otherwise.

$$F_X = \begin{cases} 0 & x < -1 \\ \frac{x+1}{3} & -1 \le x \le 2 \\ 1 & 2 < x \end{cases}$$

$F_Y = P(Y < y) = P(X^2 < y) = P(\sqrt{y} \le X \le \sqrt{y})$.

For $y \in [0, 1]$.

$F_Y = F_X(\sqrt{y}) - F_X(-\sqrt{y}) = \frac{2\sqrt{y}}{3}$

For $y \in [1, 4]$.

$F_Y = F_X(\sqrt{y}) - F_X(-1) = \frac{\sqrt{y}+1}{3}$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{2\sqrt{y}}{3} & 0 \le y \le 1 \\ \frac{\sqrt{y}+1}{3} & 1 \le y \le 4. \\ 1 & 4 < y \end{cases}$$

$$\boxed{f_Y(y) = \begin{cases} \frac{1}{3\sqrt{y}} & 0 \le y \le 1 \\ \frac{1}{6\sqrt{y}} & 1 \le y \le 4 \\ 0 & \text{otherwise} \end{cases}}$$

(iii)

$$\mu_Y = \int_{-\infty}^{\infty} y f_Y(y)\, dy$$

$$= \int_0^1 \frac{\sqrt{y}}{3}\, dy + \int_1^4 \frac{\sqrt{y}}{6}\, dy$$

$$= \frac{2}{9} + \frac{7}{9} = \boxed{1} \quad (i)$$

$$E_Y = E_{X^2} = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$= \int_{-1}^{2} \frac{x^2}{3}\, dx$$

$$= \frac{x^3}{9}\Big|_{-1}^{2} = \frac{8+1}{9} = \underline{1}$$

$$\sigma_Y^2 = \int_{-\infty}^{\infty} (y-1)^2 f_Y(y)\, dy$$

$$= \int_0^1 \frac{(y-1)^2}{3\sqrt{y}}\, dy + \int_1^4 \frac{(y-1)^2}{6\sqrt{y}}\, dy$$

$$= \frac{16}{45} + \frac{38}{45}$$

$$= \boxed{\frac{6}{5}} \quad (ii)$$

$$\sigma_Y^2 = E(Y-\mu_Y)^2 = E(X^2-1)^2$$

$$= \int_{-1}^{2} \frac{(x^2-1)^2}{3}\, dx$$

$$= \underline{\frac{6}{5}}$$

$$\frac{1}{2} = \int_{-\infty}^{v} f_Y(y)\, dy$$

$$= F_Y(v)$$

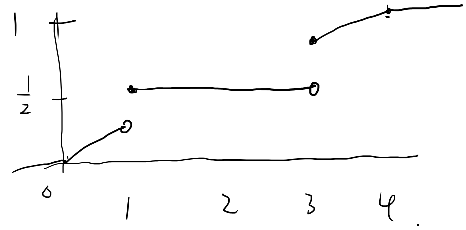
$$\frac{1}{2} = \frac{2\sqrt{v}}{3}$$

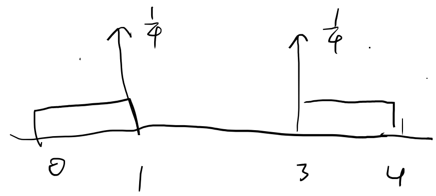
$$\boxed{v = \frac{9}{16}} \quad (iii)$$

$$\int_{-\infty}^{\infty} |y - v| f_Y(y)\, dy.$$

iv) It is unique.

P141 5. $F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ \frac{x}{4} & [0, 1) \\ \frac{1}{2} & [1, 3) \\ \frac{x}{4} & [3, 4) \\ 1 & [4, \infty) \end{cases}$





i) $f_X(x) = \begin{cases} \frac{1}{4} & 0 \le x < 1 \\ \frac{1}{4}\delta(x-1) & x = 1 \\ \frac{1}{4}\delta(x-3) & x = 3 \\ \frac{1}{4} & 3 \le x < 4 \\ 0 & \text{otherwise}. \end{cases}$

$$\int_{-\infty}^{\infty} g(x)\,\delta(x - x_0)\,dx = g(x_0).$$

$$E X = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$= \int_0^1 \frac{x}{4}\,dx + 1 \cdot P(X=1) + 3 \cdot P(X=3) + \int_3^4 \frac{x}{4}\,dx.$$

$$= \frac{1}{8} + \frac{1}{4} + \frac{3}{4} + 2 - \frac{9}{8} = \boxed{2}.$$

$$\sigma_X^2 = E X^2 - 2^2 = \frac{1}{4}\int_0^1 x^2\,dx + \frac{1}{4}\int_3^4 x^2\,dx + 1^2 P(X=1) + 3^2 P(X=3) - 4$$

$$= \frac{1}{12} + \frac{37}{12} + \frac{1}{4} + \frac{9}{4} - 4$$

$$= \boxed{\frac{5}{3}}.$$

$$\boxed{\sigma_X = \sqrt{\frac{5}{3}}}$$

ii). smallest $v_X = 1$,

iii) All medians lies between $\underline{[1, 3]}$

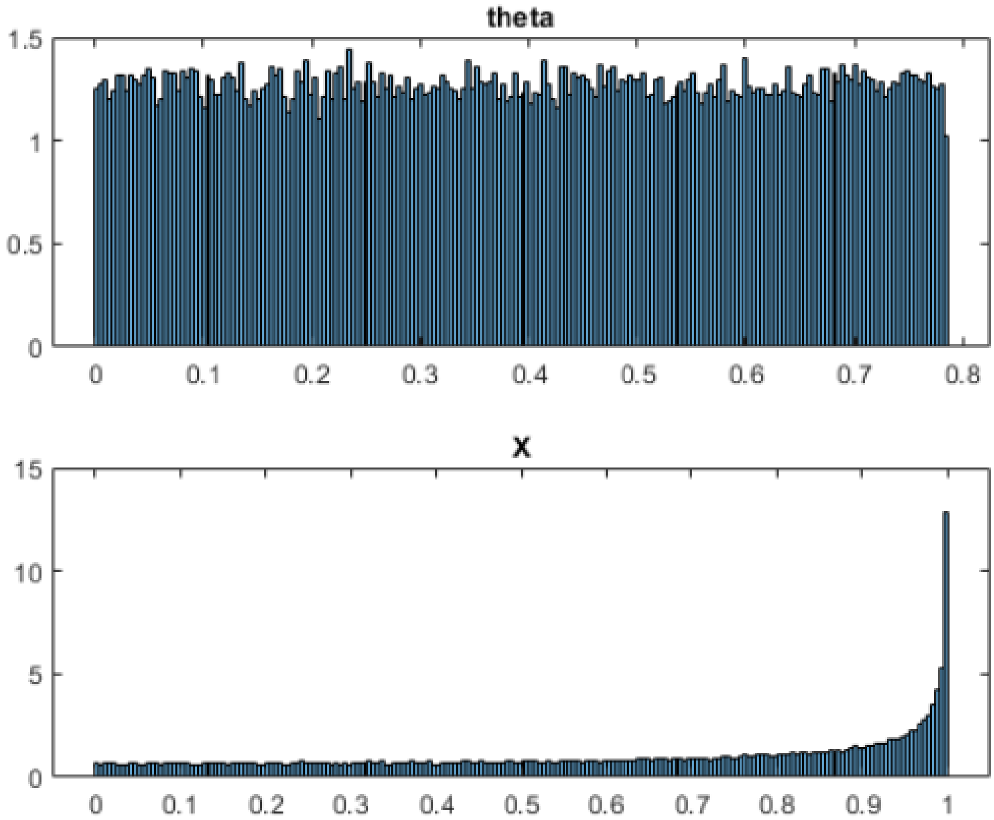
$P(X \leq 1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. $P(X \geq 1) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \geq \frac{1}{2}$

$P(X \leq 3) = \frac{3}{4}$ $P(X \geq 3) = \frac{1}{2}$.

iv). $\frac{1}{2} = F_X(V)$. , $\underline{V \in [1, 3)}$

v) . $P(X \leq 3) = \frac{3}{4}$. $\boxed{v = 3}$

P145 P1. $\Theta \in U[0, \frac{\pi}{4}]$. $X = \sin(2\Theta)$. $\alpha = 1$. $0 < X < \alpha$.




```
mu_X = 0.6365
std_X = 0.3082
med_X = 0.7076
```

The numerical results are close to the analytical results.

P 16 1. $f_{XY}(x,y) = k(x^2+y^2)$ if $x^2+y^2 \le 1$ $\quad r = \sqrt{X^2+Y^2}$

$= 0$ otherwise.

i). $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k(x^2+y^2)\,dx\,dy = 1.$

$k\int_0^{2\pi}\int_0^1 r^2 \cdot r\,dr\,d\theta = 1.$ $\quad \boxed{k = \frac{2}{\pi}}$

$\frac{\pi}{2}k = 1$

ii). $F_r(r) = \frac{2}{\pi}\int_0^{2\pi}\int_0^r r^3\,dr\,d\theta = r^4 \Rightarrow \begin{cases} 0 & r<0 \\ r^4 & 0\le r\le 1 \\ 1 & 1<r \end{cases}$

$f_r(r) = \frac{dF_r}{dr} = \boxed{\begin{cases} 4r^3 & 0\le r\le 1 \\ 0 & \text{otherwise.} \end{cases}}$

iii) $\mu_r = E_r = \int_{-\infty}^{\infty} r\cdot f_r(r)\,dr = \int_0^1 r\cdot 4r^3\,dr = \boxed{\frac{4}{5}}$

$\sigma_r^2 = Er^2 - \mu_r^2 = \int_0^1 r^2\,4r^3\,dr - \frac{16}{25} = \frac{2}{3} - \frac{16}{25} = \boxed{\frac{2}{75}}$

$\nu = \min\{r : F_r(r) \ge \frac{1}{2}\}$.

$r^4 = \frac{1}{2}$ $\quad \boxed{\nu = \left(\frac{1}{2}\right)^{\frac{1}{4}}}$

iv) $E|r-\nu| = \int_{-\infty}^{\infty} |r-\nu|\, f_r(r)\,dr.$

$= \int_0^1 |r-\nu|\,4r^3\,dr.$

$= \int_0^{\nu} (\nu-r)4r^3\,dr + \int_{\nu}^1 (r-\nu)4r^3\,dr$

$\boxed{= 0.1273}$

P162 - 3.

$f_{XY}(x,y) = k\,xy \quad$ if $0 \le y \le x \le 1$.

$= 0 \quad$ otherwise.

i). $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx\,dy = 1$.

$\int_0^1 \int_0^x k\,xy\,dy\,dx = 1$. $\quad \boxed{k = 8}$

$k \cdot \frac{1}{8} = 1$.

ii) $\mu_X = E_X = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_{-\infty}^{\infty} x \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy\,dx$.

$= \int_{-\infty}^{\infty} x \cdot \int_{-\infty}^{\infty} 8xy\,dy \cdot dx$.

$= \int_0^1 x \cdot \int_0^x 8xy\,dy\,dx$.

$= \int_0^1 x \cdot 4x^3\,dx$

$= \frac{4}{5}x^5 \Big|_0^1$

$= \boxed{\frac{4}{5}}$

$E_Y = \frac{8}{15}$

$\sigma_X^2 = E_X^2 - \mu_X^2$

$= \int_0^1 x^2 \int_0^x 8xy\,dy\,dx - \frac{16}{25}$

$= \frac{2}{3} - \frac{16}{25}$

$= \boxed{\frac{2}{75}}$

iv). $E(Y-X)^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (y-x)^2 \cdot f_{XY}(x,y)\,dx\,dy$

$= \int_0^1 \int_0^x (y-x)^2 \cdot 8xy\,dy\,dx$

$= \boxed{\frac{1}{9}}$

$E|Y-X| = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |y-x| f_{XY}(x,y)\,dx\,dy$

$= \int_0^1 \int_0^x (x-y) \cdot 8xy\,dx\,dy$

$= \boxed{\frac{4}{15}} \quad = E_X - E_Y$

P175 2. $\{V_n\}_0^\infty$ iid descrete r.v.

$P(V_n=1)=p \quad P(V_n=-1)=q \quad . \quad P(V_n=-2)=e.$

$Y_{n+1}=Y_n+V_n$, where $Y_0=y_0$ is a constant integer.

$$Y_n = Y_0+\sum_{j=0}^{n-1} V_j$$

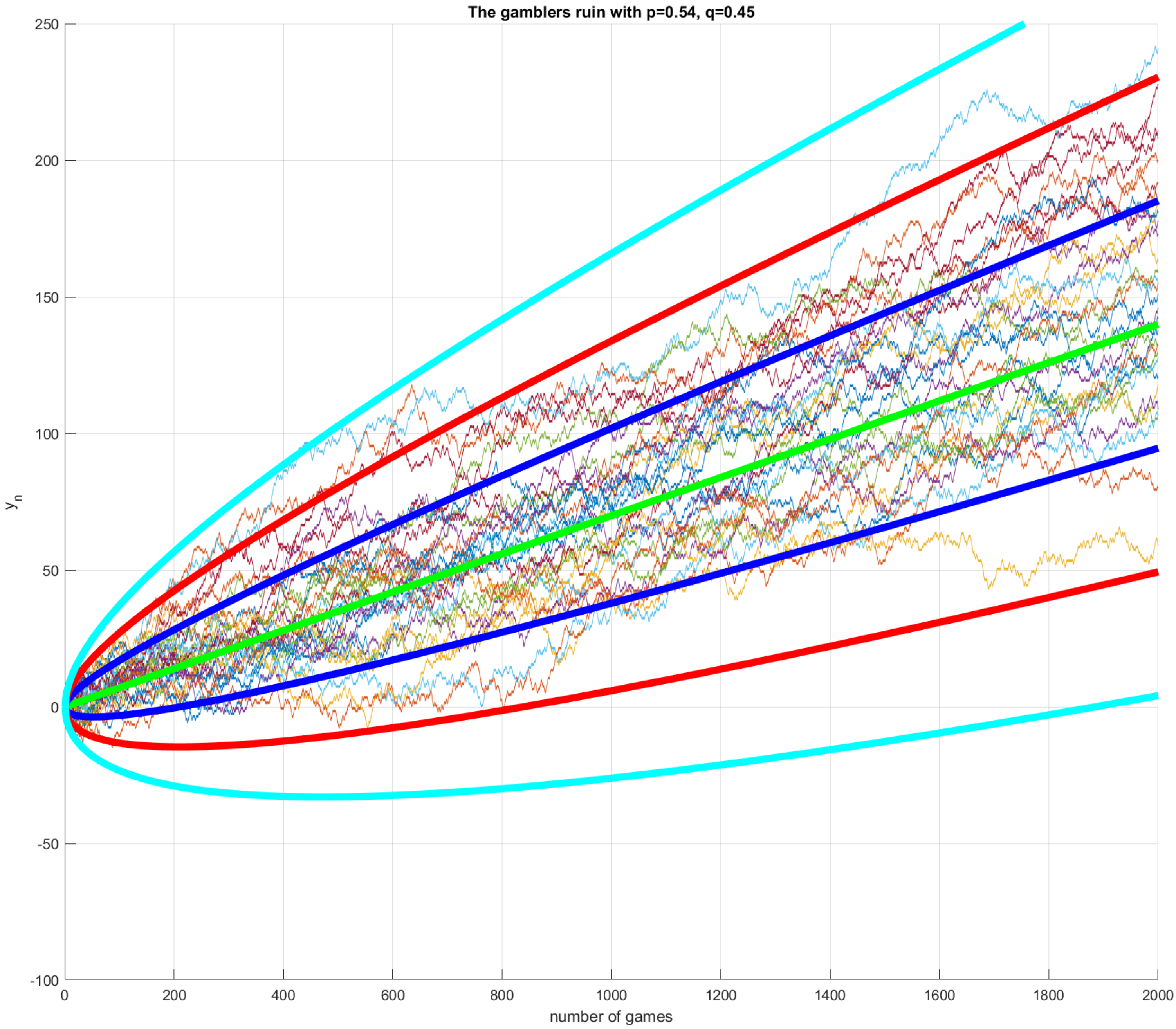
$$\mu_{V_j} = E_{V_j} = 1\cdot P(V_j=1) - 1\, P(V_j=-1) - 2\cdot P(V_j=-2)$$
$$= p-q-2e.$$

$$\sigma_{V_j}^2 = E_{V_j^2} - \mu_{V_j}^2$$
$$= 1\cdot p + 1\cdot q + 4e - (p-q-2e)^2$$
$$= p+q+4e-p^2+2pq-q^2+4e(p-q)-4e^2$$
$$= p+q-p^2+2pq-q^2 + 4e(1+p-q-e).$$
$$= p+q-(p-q)^2+8ep$$
$$= p+q-p(1-q-e)+2pq-q(1-\cancel{p}-e)$$
$$= \cancel{p}+\cancel{q}-\cancel{p}+pq+pe+2pq-\cancel{q}+pq+qe.$$
$$= 4pq+9pe+qe.$$

$$\mu_{Y_n} = E_{Y_n} = E(Y_0+\sum_0^{n-1} V_j) = y_0+n(p-q-2e).$$

$$\sigma_{Y_n}^2 = n(4pq+9pe+qe)$$

$$\sigma_{Y_n} = \sqrt{n(4pq+9pe+qe)}$$



The gamblers ruin with p=0.54, q=0.45
250
200
150
100
50
0
-50
-100
y_n
0
200
400
600
800
1000
1200
1400
1600
1800
2000
number of games