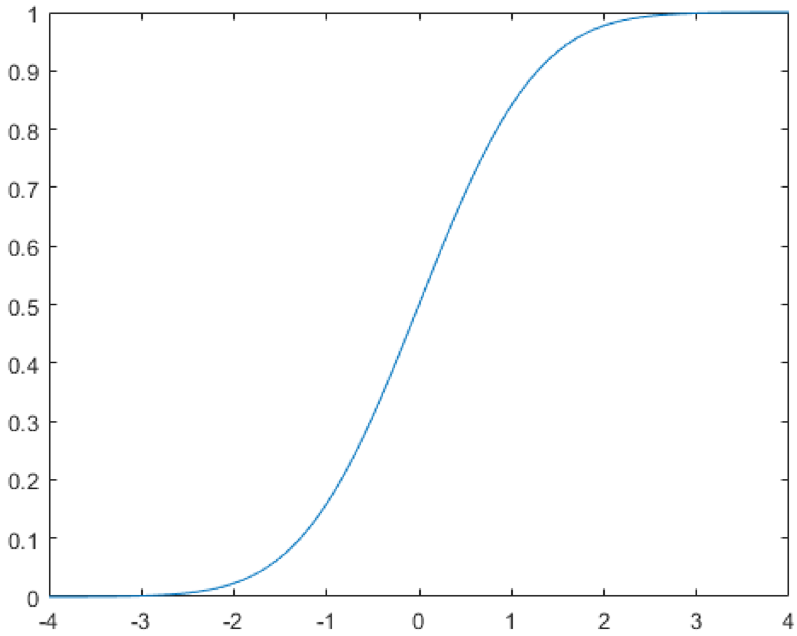
P 70 1, 2, 3.

P 82 1, 2, 4 part (i) only

P 88 3, 5, 7 (i) to (iii)

P70. 1.

i)



ii) $P(X < 1.5) = \underline{0.9332}$

iii) $P(-0.5 < X \leq 1.5) = \underline{0.6247}$

P70.

2. $X \in N(\mu_1, \sigma_1)$.

$Y = \alpha X + \beta$ s.t. $y \in N(\mu_2, \sigma_2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right)$$

$$f_Y(y) = \frac{1}{|\alpha|} f_X\left(\frac{y-\beta}{\alpha}\right)$$

$$= \frac{1}{|\alpha|} \cdot \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(y-\beta-\alpha\mu_1)^2}{2\alpha^2\sigma_1^2}\right)$$

$$= \frac{1}{\sqrt{2\pi(\alpha\sigma_1)^2}} \exp\left(-\frac{[y-(\beta+\alpha\mu_1)]^2}{2(\alpha\sigma_1)^2}\right)$$

If Y is Gaussian:

$$\begin{cases} |\alpha\sigma_1| = \sigma_2 \Rightarrow \alpha = \pm\frac{\sigma_2}{\sigma_1} \\ \beta + \alpha\mu_1 = \mu_2 \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} \alpha = \frac{\sigma_2}{\sigma_1} \\ \beta = \mu_2 - \frac{\sigma_2}{\sigma_1}\mu_1 \end{cases} \text{ or } \begin{cases} \alpha = -\frac{\sigma_2}{\sigma_1} \\ \beta = \mu_2 + \frac{\sigma_2}{\sigma_1}\mu_1 \end{cases}}$$

p 70. 3. $\underline{X}$ is Gaussian. $\mathcal{N}(0,1)$

$\underline{Y} = \underline{X}^2$.

$$F_{\underline{Y}}(y) = P(\underline{Y} \le y) = P(X^2 \le y)$$

$$= P(\sqrt{y} \le X \le \sqrt{y})$$

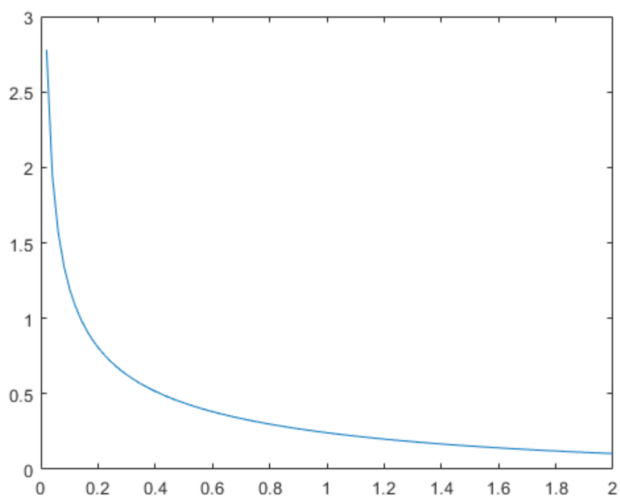
$$= P(X \le \sqrt{y}) - P(X \le -\sqrt{y})$$

$$= \Phi(\sqrt{y}) - \Phi(-\sqrt{y})$$

$$f_{\underline{Y}}(y) = \frac{d}{dy} F_{\underline{Y}}(y) = \frac{1}{2\sqrt{y}} \Phi(\sqrt{y}) + \frac{1}{2\sqrt{y}} \Phi(-\sqrt{y})$$

$$= \frac{1}{\sqrt{y}} \cdot \left(\frac{1}{\sqrt{2\pi}} e^{\frac{-y}{2}} \right)$$

$$\boxed{= \frac{1}{\sqrt{2\pi y}} e^{-\frac{y}{2}}} \quad . \text{ for } y \in (0, \infty).$$



P82. 1. $\overset{\text{Head}}{p} = \frac{3}{5}$. $\overset{\text{Tail}}{q} = \frac{2}{5}$.

$$P(5H \cup 6H \cup 7H) = ?$$

$$= P(5H) + P(6H) + P(7H) - \cancel{P(5H \cap 6H)} - \cancel{P(5H \cap 7H)} - \cancel{P(6H \cap 7H)}$$
$$- \cancel{P(5H \cap 6H \cap 7H)}$$

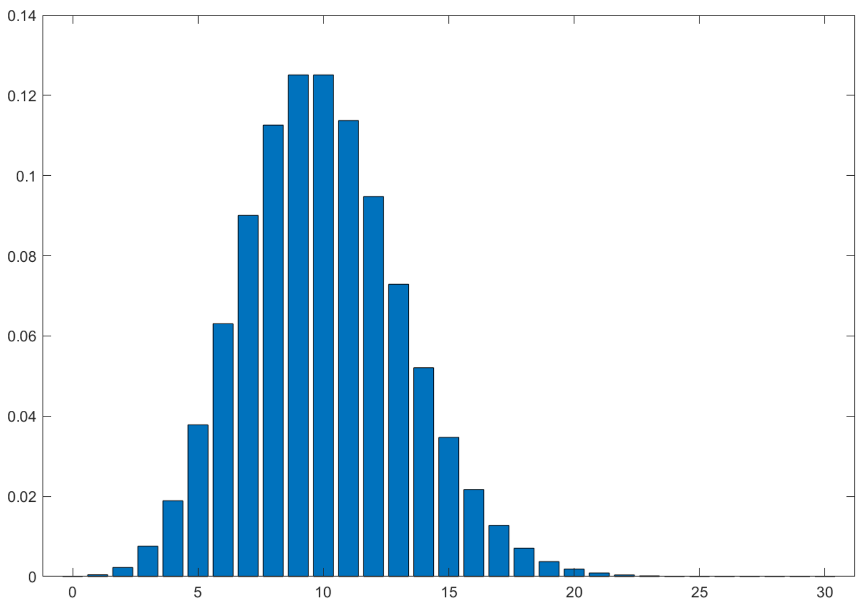
$$= \binom{10}{5}\left(\frac{3}{5}\right)^5 \cdot \left(\frac{2}{5}\right)^5 + \binom{10}{6}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4 + \binom{10}{7}\left(\frac{3}{5}\right)^7 \cdot \left(\frac{2}{5}\right)^3.$$

$$= 0.2007 + 0.2508 + 0.2150.$$

$$\boxed{\approx 0.6665}.$$

2. (i) $P(8 \le X \le 12.5) = \sum_{n=8}^{12} P(X=k) \approx \boxed{0.5713}$

(ii).



4. (i)

For the stationary solution:

$$\dot{p} = Ap = 0.$$

$$P_k(t) = \pi_k$$

$$A\{\pi_k\}_0^\infty = 0.$$

$$\begin{cases} -\lambda\pi_0 + \mu\pi_1 = 0 \\ \lambda\pi_0 - (\lambda+\mu)\pi_1 + \mu\pi_2 = 0 \\ \lambda\pi_1 - (\lambda+\mu)\pi_2 + \mu\pi_3 = 0 \\ \vdots \end{cases}$$

$$\pi_1 = \frac{\lambda}{\mu}\pi_0.$$

$$\pi_2 = \frac{1}{\mu}\left(\frac{\lambda^2}{\mu} + \lambda - \lambda\right)\pi_0 = \frac{\lambda^2}{\mu^2}\pi_0.$$

$$\pi_3 = \frac{1}{\mu}\left(\frac{\lambda^3}{\mu^2} + \frac{\lambda^2}{\mu} - \frac{\lambda^2}{\mu}\right)\pi_0 = \frac{\lambda^3}{\mu^3}\pi_0$$

$$\vdots$$

$$\pi_k = \left(\frac{\lambda}{\mu}\right)^k \pi_0.$$

$\sum_{k=0}^{\infty} \pi_k = 1$ since it is probability.

$$\sum_0^\infty \left(\frac{\lambda}{\mu}\right)^k \pi_0 = 1.$$

$\pi_0 \left(\frac{1}{1-\frac{\lambda}{\mu}}\right) = 1.$ since $\frac{\lambda}{\mu} < 1$;

$\pi_0 = 1 - \frac{\lambda}{\mu}.$ $\Rightarrow$ $\boxed{\pi_k = \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^k}$

P58. 3. $f_Y(y) = \frac{3}{2} y^2$ if $|y| \le 1$

$= 0$ otherwise.

i). $F_Y(y) = \int_{-\infty}^{y} \frac{3}{2} y^2 \, dy$

$= \int_{-1}^{y} \frac{3}{2} y^2 \, dy$

$= \frac{y^3}{2} \Big|_{-1}^{y}$

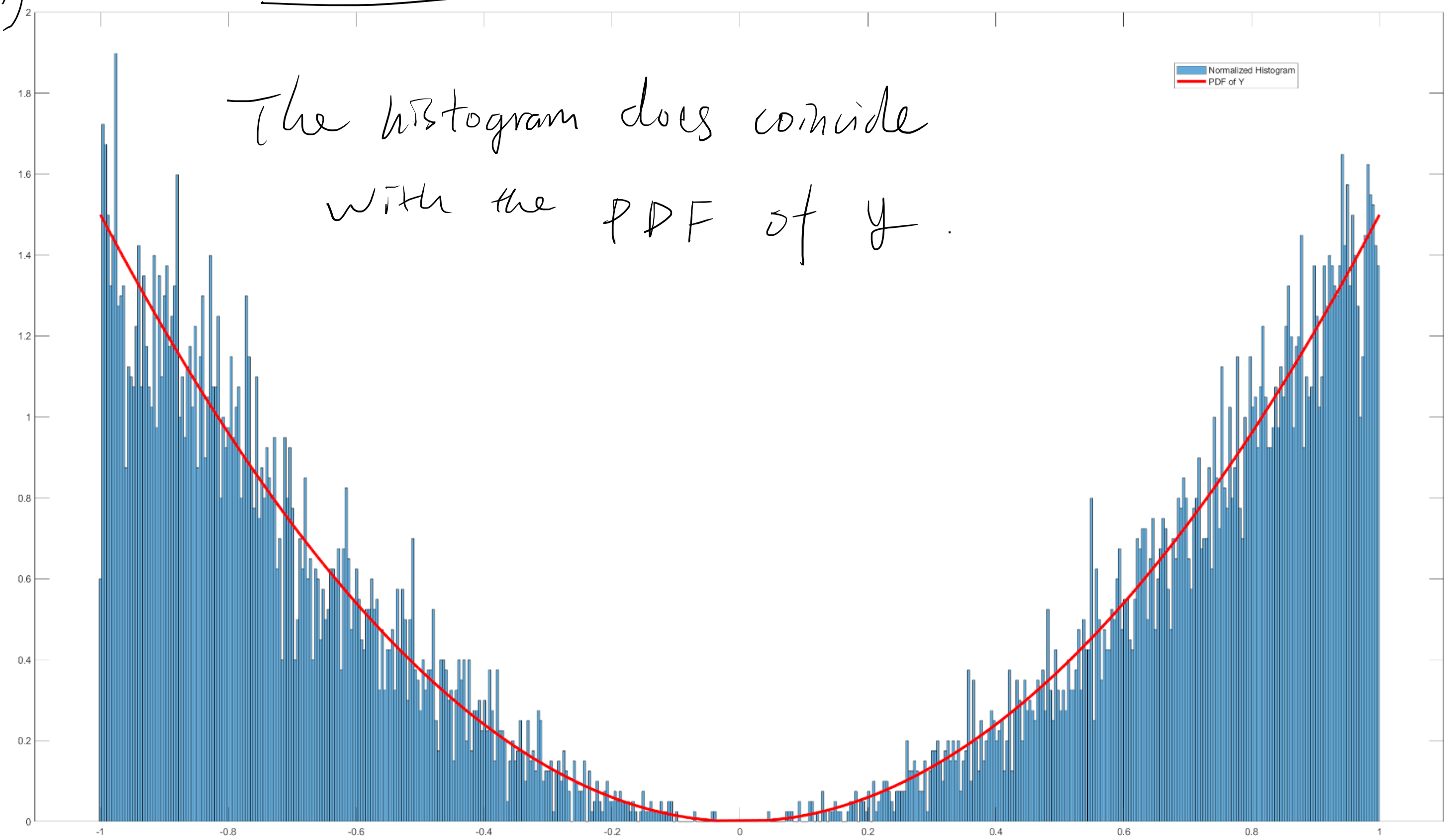
$= \frac{y^3+1}{2} \Rightarrow \boxed{F_Y(y) = \begin{cases} 0 & y < -1 \\ \frac{y^3+1}{2} & -1 \le y < 1 \\ 1 & y \ge 1. \end{cases}}$

ii) $x = F(y) = \frac{y^3+1}{2}$.

$y^3 = 2x-1$

$\underline{y = \text{sign}(2x-1)\,|2x-1|^{\frac{1}{3}}}$

iii)



The histogram does coincide with the PDF of y.

5. $f_Y(y) = b e^{-4|y|}$ for $-\infty < y < \infty$

i). $\int_{-\infty}^{\infty} b e^{-4|y|} dy = 1$.

$$b \cdot 2 \int_0^{\infty} e^{-4y} dy = 1.$$

$$b \cdot 2 \cdot \frac{1}{4} = 1$$

$$\boxed{b = 2.}$$

ii) $F_Y(y) = \int_{-\infty}^{y} f_Y(y)\, dy$

For $y < 0$.

$$F_Y(y) = \int_{-\infty}^{y} 2e^{4y} dy = \frac{1}{2} e^{4y} \Big|_{-\infty}^{y} = \frac{e^{4y}}{2}.$$

For $y \geq 0$.

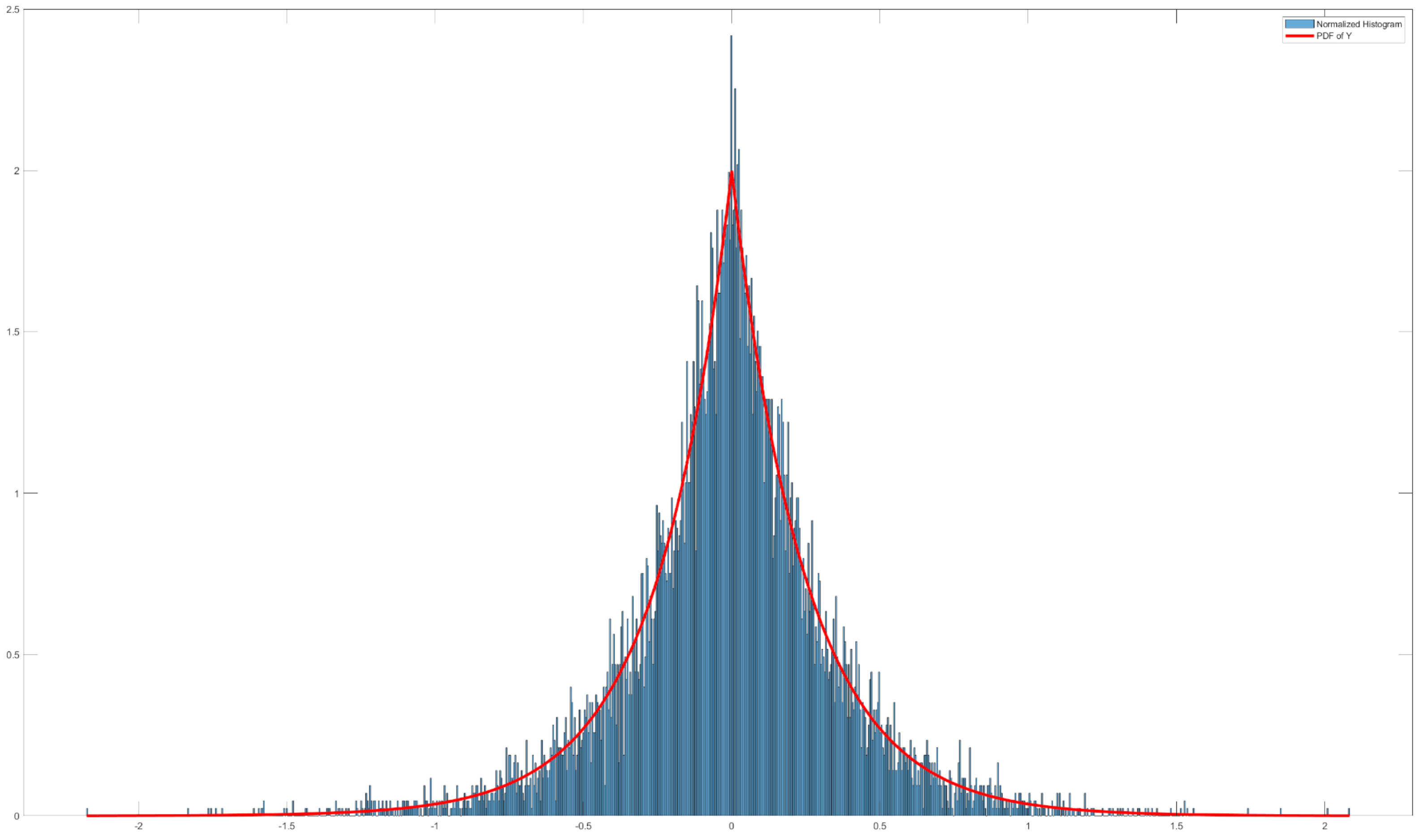
$$F_Y(y) = 1 - \int_y^{\infty} 2e^{-4y} dy = 1 - \frac{1}{2} e^{-4y} \Big|_y^{\infty} = 1 - \frac{e^{-4y}}{2}.$$

$$\Rightarrow \boxed{F_Y(y) = \begin{cases} \dfrac{e^{4y}}{2} & y < 0 \\ 1 - \dfrac{e^{-4y}}{2} & y \geq 0. \end{cases}}$$

iii) $x = F_Y(y) = \begin{cases} \frac{e^{4y}}{2} & y < 0 \Rightarrow x < \frac{1}{2} \\ 1 - \frac{e^{-4y}}{2} & y \geq 0 \Rightarrow x \geq \frac{1}{2} \end{cases}$

$$\boxed{F_Y^{-1}(x) = \begin{cases} \frac{\ln(2x)}{4} & 0 < x < \frac{1}{2} \\ -\frac{\ln(2-2x)}{4} & \frac{1}{2} \leq x < 1 \end{cases}}$$

iv)



The PDF does coincide with the histogram.

7. $P(H) = \frac{51}{100} = p$. $P(T) = \frac{48}{100} = q$. $P(E) = \frac{1}{100} = e$.

i). Let W_n be winning \$200 starting with n dollars.

$$P(W_n) = P(W_n | H) P(H) + P(W_n | T) P(T) + P(W_n | E) P(E)$$

$$P(n) = P(n+1) p + P(n-1) q + P(n-2) e .$$

$$P(n+1) p - P(n) + P(n-1) q + P(n-2) e = 0 .$$

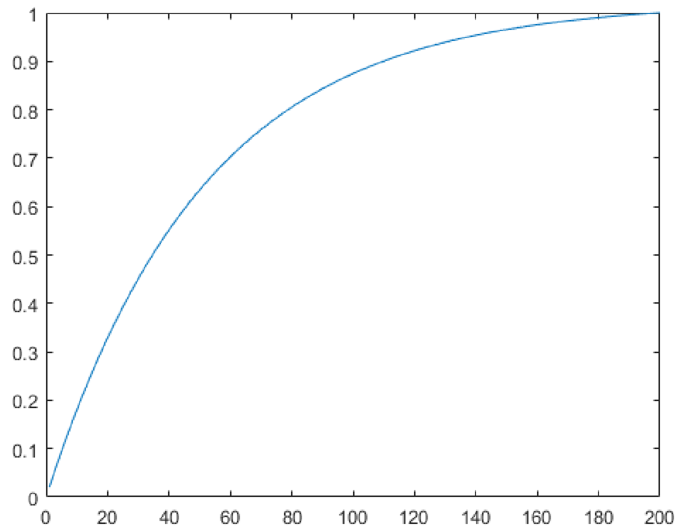
$$p\lambda^3 - \lambda^2 + q\lambda + e = 0 .$$

$$\lambda = \underset{\lambda_1}{1}, \underset{\lambda_2}{0.9808}, \underset{a_3}{-0.02} .$$

$$\begin{cases} P(0) = 0 \\ P(-1) = 0 \\ P(200) = 1 \end{cases} \quad P(n) = a_1 + a_2 \cdot 0.9808^n + a_3 (-0.02)^n$$

$$\begin{cases} 0 = a_1 + a_2 + a_3 \\ 0 = a_1 + a_2 \cdot 0.9808^{-1} + a_3 (-0.02)^{-1} \\ 1 = a_1 + a_2 \cdot 0.9808^{200} + a_3 (-0.02)^{200} \end{cases} \qquad \begin{cases} a_1 = 1.021 \\ a_2 \doteq -1.0206 \\ a_3 \doteq -0.0004 . \end{cases}$$

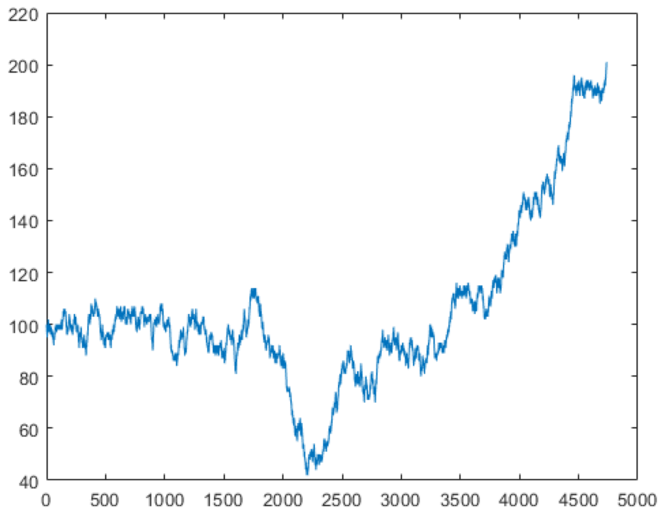
$$\boxed{P(n) = 1.021 - 1.0206 (0.9808)^n - 0.0004 (-0.02)^n .}$$



ii) . At $n = 100$, $P(W_{100}) = \underline{0.8745}$.

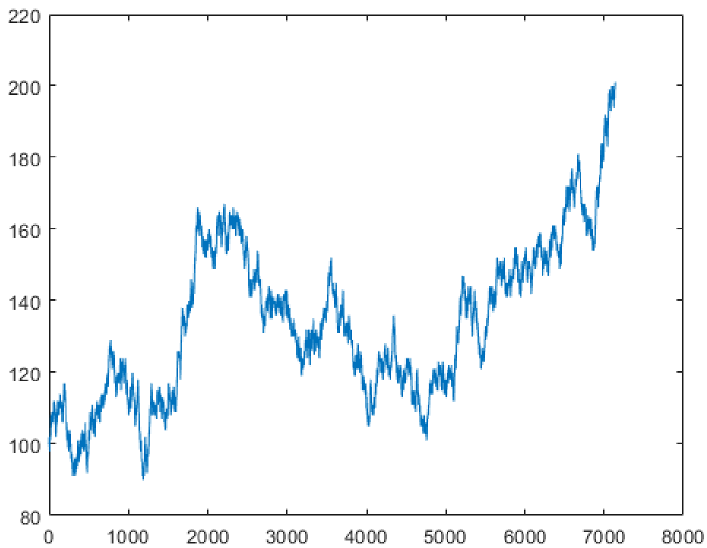
sim 1:

Gambler achieves \$200 after $\underline{4736}$ games.



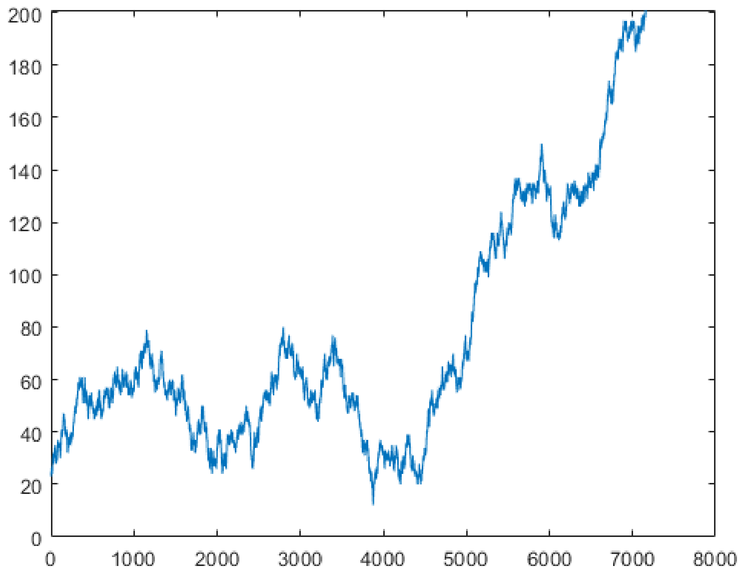
sim 2:

Gambler achieves \$200 after 7136 games.

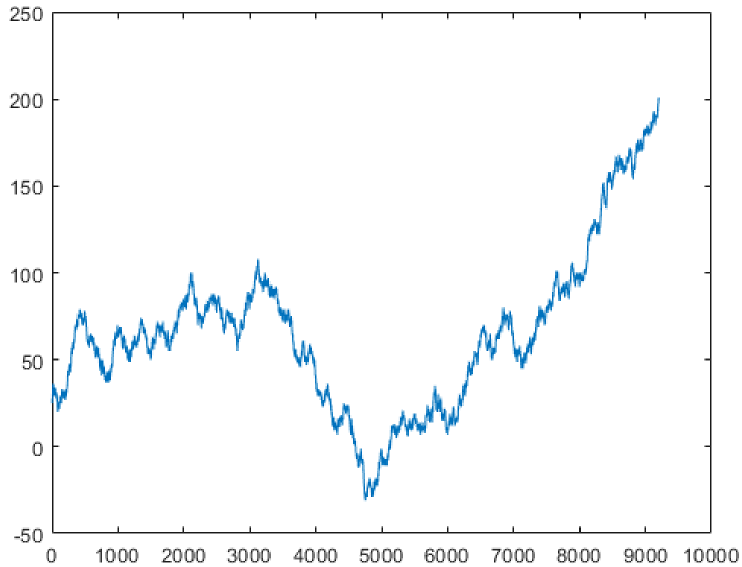


iii) $\boxed{\text{At } n = 35}$. $P(n) = 0.5036$.

iv).



wins after 7155 games.



Briefly goes broke. but eventually wins after 9197 games