3.1 2)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ linearly independent, but not orthogonal

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. linearly independent & orthogonal.

3.1 6)

$$\begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}1\\-1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\-2\end{pmatrix}$$ orthogonal to both.

$$\left\|\begin{pmatrix}1\\1\\1\end{pmatrix}\right\| = \sqrt{3}, \quad \left\|\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\| = \sqrt{2}, \quad \left\|\begin{pmatrix}1\\1\\-2\end{pmatrix}\right\| = \sqrt{6}$$

The orthonormal basis are

$$\begin{pmatrix}\frac{\sqrt{3}}{3}\\ \frac{\sqrt{3}}{3}\\ \frac{\sqrt{3}}{3}\end{pmatrix}, \begin{pmatrix}\frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2}\\ 0\end{pmatrix}, \begin{pmatrix}\frac{\sqrt{6}}{6}\\ \frac{\sqrt{6}}{6}\\ \sqrt{6}\end{pmatrix}$$

3.1 12)

Orthogonal component of $R(A)$;

$$\begin{pmatrix}1\\0\\2\end{pmatrix} \times \begin{pmatrix}1\\1\\4\end{pmatrix} = \begin{pmatrix}-2\\-2\\1\end{pmatrix}$$

$$\begin{pmatrix}3\\3\\3\end{pmatrix} + \begin{pmatrix}-2\\-2\\1\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix}$$

$$\begin{pmatrix}3\\3\\3\end{pmatrix} = \begin{pmatrix}1\\1\\4\end{pmatrix} - \begin{pmatrix}-2\\-2\\1\end{pmatrix}$$

$$x_r \quad - \quad x_n$$

3.2 4)

When a & b are on the same line, the angle θ between both vectors is zero. The Schwarz Inequality describes the ratio of $a^T b$ & $\|a\| \cdot \|b\|$, which is also the definition of $|\cos\theta| \leq 1$. At $\theta = 0$. $\cos\theta = 1$.

$$|a^T b| = \|a\| \cdot \|b\|$$

If a & b are on opposite sides of the origin,

$$|a^T b| = \|a\| \cdot \|b\|, \text{ but } a^T b = -\|a\| \cdot \|b\|$$

3.2 b) Triangle inequality.

$$2\sqrt{xy} \le x+y$$

$$\sqrt{xy} \le \frac{x+y}{2}.$$

3.2 12)

$x + 2y = 0.$ $\qquad a = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$x = -2y.$

$$P = \frac{a \cdot a^T}{a^T a} = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix} \cdot \frac{1}{4+1} = \begin{bmatrix} \frac{4}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

3.3 4) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$, $x = \begin{bmatrix} u \\ v \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$$Ax - b = \begin{bmatrix} u-1 \\ v-3 \\ u+v-4 \end{bmatrix},$$

$$E^2 = (u-1)^2 + (v-3)^2 + (u+v-4)^2$$

$$= 2u^2 + 2uv - 10u + 2v^2 - 14v + 26$$

$$\left.\begin{aligned} \frac{dE^2}{du} &= 4u + 2v - 10 = 0 \\ \frac{dE^2}{dv} &= 2u + 4v - 14 = 0 \end{aligned}\right\} \Rightarrow \left\{ \begin{aligned} 2u + v &= 5 \\ u + 2v &= 7 \end{aligned}\right.$$

$$A^TA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad A^Tb = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$A^TA\hat{x} = A^Tb$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \hat{x} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$\left.\begin{aligned} 2x_1 + x_2 &= 5 \\ x_1 + 2x_2 &= 7 \end{aligned}\right\}$ this is the same as $\begin{aligned} 2u + v &= 5 \\ u + 2v &= 7 \end{aligned}$

$$\hat{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = b$$

$p = b$ is because $Ax = b$ is consistent

A minimal error solution is found.

3.3 24)

$t = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ $y = \begin{bmatrix} 2 \\ 0 \\ -3 \\ -5 \end{bmatrix}$, $t = A$

$A^T A \hat{x} = A^T y$

$6 \hat{x} = -15$

$\hat{x} = -2.5$

$\boxed{y = -2.5x}$