3.4 6)

$$Q = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{14}} & \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{14}} & \bar{c}_3 \\ \frac{1}{\sqrt{3}} & \frac{-3}{\sqrt{14}} & \end{bmatrix}$$
$$\begin{array}{ccc} \bar{c}_1 & \bar{c}_2 & \end{array}$$

$$\bar{c}_3 = \bar{c}_1 \times \bar{c}_2 = \frac{1}{\sqrt{42}} \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix}$$

$$\|\bar{c}_3\| = \frac{1}{\sqrt{42}} \cdot \sqrt{25+16+1} = 1$$

$\therefore$ $\bar{c}_3$ is a unit vector and orthogonal to $\bar{c}_1$ & $\bar{c}_2$

$$Q = \begin{array}{c} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{14}} & \boxed{\frac{-5}{\sqrt{42}}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{42}} \\ \frac{1}{\sqrt{3}} & \frac{-3}{\sqrt{14}} & \frac{1}{\sqrt{42}} \end{bmatrix} \begin{array}{c} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \end{array} \\ \bar{c}_3 \end{array}$$

$$\left.\begin{aligned} \bar{r}_1 \cdot \bar{r}_1 &= \frac{1}{3} + \frac{1}{14} + \frac{25}{42} = \frac{14+3+25}{42} = 1 \\ \bar{r}_2 \cdot \bar{r}_2 &= \frac{1}{3} + \frac{4}{14} + \frac{16}{42} = \frac{14+12+16}{42} = 1 \\ \bar{r}_3 \cdot \bar{r}_3 &= \frac{1}{3} + \frac{9}{14} + \frac{1}{42} = \frac{14+27+1}{42} = 1. \end{aligned}\right\} \|\bar{r}_1\| = \|\bar{r}_2\| = \|\bar{r}_3\| = 1.$$

$$\left.\begin{aligned} \bar{r}_1 \cdot \bar{r}_2 &= \frac{1}{3} + \frac{2}{14} - \frac{20}{42} = 0 \qquad \bar{r}_1 \perp \bar{r}_2 \\ \bar{r}_1 \cdot \bar{r}_3 &= \frac{1}{3} - \frac{3}{14} + \frac{-5}{42} = 0 \qquad \bar{r}_1 \perp \bar{r}_3 \\ \bar{r}_2 \cdot \bar{r}_3 &= \frac{1}{3} - \frac{6}{14} + \frac{4}{42} = 0 \qquad \bar{r}_2 \perp \bar{r}_3 \end{aligned}\right\} \bar{r}_1 \perp \bar{r}_2 \perp \bar{r}_3$$

$\therefore$ All row vectors are unit vectors and orthognal to each other,

<u>checked</u>

3.4 10) q_1 & q_2 are outputs from Graham - Schmidt.

what are the inputs. a & b

a & b can be any independent vector.

3.4 (14) $a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $c = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, find orthonormal vectors, q_1, q_2, q_3

$$q_1 = \frac{a}{\|a\|} = \boxed{\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}}$$

$$B = b - (q_1^T b) \cdot q_1$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$$

$\frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$

$$q_2 = \frac{B}{\|B\|} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}}$$

$$C = c - (q_1^T c) q_1 - (q_2^T c) \cdot q_2$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{6} \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{6} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} \qquad q_3 = \frac{C}{\|C\|} = \boxed{\begin{bmatrix} -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}}$$

3.4 22) $\|b_1 \sin x - \cos x\|^2 = \int_0^{2\pi} (b_1 \sin x - \cos x)^2 \, dx$

$$\int_0^{2\pi} (b_1 \sin x - \cos x)^2 dx = \int_0^{2\pi} b_1^2 \sin^2 x - 2 b_1 \sin x \cos x + \cos^2 x \, dx$$

$$= b_1^2 \int_0^{2\pi} \sin^2 x + \int_0^{2\pi} \cos^2 x \, dx - \cancel{2b_1 \int_0^{2\pi} \sin x \cdot \cos x \, dx} \; 0$$

$$= \pi b_1^2 + \pi$$

$$\frac{\partial \|b_1 \sin x - \cos x\|^2}{\partial b_1} = 2\pi b_1 = 0.$$

$$\boxed{b_1 = 0}$$

If $b_1 = 0$. $f(x) = \cos x$

$$b_1 = \frac{\int_0^{2\pi} f(x) \sin x \, dx}{\int_0^{2\pi} \sin^2 x \, dx} = \frac{\overset{0}{\cancel{\int_0^{2\pi} \cos x \sin x \, dx}}}{\int_0^{2\pi} \sin^2 x \, dx} = 0$$

3.4 24) $x^3 + ax^2 + bx + c \perp 1, x, x^2 - \frac{1}{3}$, $-1 \le x \le 1$

$$\int_{-1}^{1} (x^3 + ax^2 + bx + c) \cdot 1 \, dx = 0.$$

$$\frac{x^4}{4} + \frac{a}{3}x^3 + \frac{b}{2}x^2 + cx \Big|_{-1}^{1} = 0$$

$$0 + \frac{2}{3}a + 0 + 2c = 0$$

$$\underline{\frac{2}{3}a + 2c = 0}$$

$$\int_{-1}^{1} (x^3 + ax^2 + bx + c) \cdot x \, dx = 0.$$

$$\int_{-1}^{1} x^4 + ax^3 + bx^2 + cx \, dx = 0$$

$$\frac{x^5}{5} + \frac{a}{4}x^4 + \frac{b}{3}x^3 + \frac{c}{2}x^2 \Big|_{-1}^{1} = 0.$$

$$\frac{2}{5} + 0 + \frac{2}{3}b + 0 = 0$$

$$\boxed{b = -\frac{3}{5}}$$

$$\int_{-1}^{1} (x^3 + ax^2 + bx + c) \cdot (x^2 - \frac{1}{3}) \, dx = 0$$

$$\int_{-1}^{1} (x^5 + ax^4 + bx^3 + cx^2 - \frac{x^3}{3} - \frac{a}{3}x^2 - \frac{b}{3}x - \frac{c}{3}) \, dx = 0$$

$$\cancel{\frac{x^6}{6}} + \frac{a}{5}x^5 + \cancel{\frac{b}{4}x^4} + \frac{c}{3}x^3 - \cancel{\frac{x^4}{12}} - \frac{a}{9}x^3 - \cancel{\frac{b}{6}x^2} - \frac{c}{3}x \Big|_{-1}^{1} = 0.$$

$$\frac{2}{5}a + \cancel{\frac{2}{3}c} - \frac{2}{9}a - \cancel{\frac{2}{3}c} = 0.$$

$$(\frac{2}{5} - \frac{2}{9})a = 0$$

$$\underline{a = 0}$$

$$\underline{c = 0}$$

The polynomial is $\boxed{x^3 - \frac{3}{5}x}$

3.5 2)

$$F P = \bar{F}$$

$$P = F^{-1} \bar{F}$$

$$P = \frac{1}{n} \bar{F} \cdot \bar{F}$$

$$F = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & & w^{n-1} \\ 1 & w^2 & w^4 & \ddots & w^{2(n-1)} \\ \vdots & \vdots & & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \cdots & w^{(n-1)^2} \end{bmatrix}$$

$$\bar{F} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w^{-1} & w^{-2} & \cdots & w^{1-n} \\ 1 & w^{-2} & w^{-4} & \ddots & w^{2(1-n)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{1-n} & w^{2(1-n)} & \cdots & w^{(1-n)^2} \end{bmatrix}$$

$F(0,:) \cdot P = \bar{F}(0,:)$, $[1\ 1\ 1 \cdots 1] \cdot P = [1\ 1\ 1 \cdots 1]$,

$F(1,:) \cdot P = \bar{F}(1,:)$

$$[1\ \ w\ \ w^2 \cdots w^{n-1}]\, P = [1\ \ w^{-1}\ \ w^{-2} \cdots w^{1-n}]$$

Since w is a root of unity, $w^n = 1$.

$$w^{-1} = (1) \cdot w^{-1} = w^{n-1}$$

$$w^{-2} = w^{n-2}$$

$$\cdots\cdots$$

$$w^{1-n} = w^1$$

$$[1\ \ w\ \ w^2 \cdots w^{n-1}]\, P = [1\ \ w^{n-1}\ \ w^{n-2} \cdots w]$$

It is clear that the 0th column stayed same, 1st column swapped with $(n-1)$th column, 2nd $\longleftrightarrow$ $(n-2)$th, so on and so forth.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & \cdots 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots 1 & 0\ 0 \\ \vdots & \vdots & \vdots & \vdots & \iddots & \vdots \\ & & 1 & 0 & & \\ 0 & 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

3.5 b) $\sqrt{\overline{w_{128}}} = e^{i \cdot \frac{2\pi}{128} \cdot \frac{1}{2}} = e^{i\frac{2\pi}{256}} = w_{256}$

$w_{128}^2 = e^{i\frac{2\pi}{64}} = w_{64}$