1.

$a = 6 R_♃$ $\quad \Omega = 45^\circ$ $\quad p = a(1-e^2) = 4.5 R_♃$

$e = 0.5$ $\quad \omega = 30^\circ$

$i = 30^\circ$ $\quad \theta^* = -165^\circ$

At $\theta^* = 150^\circ$, $\Delta \bar{V} = 0.1\hat{x} + 0.25\hat{y} + 0.35\hat{z}$ km/s
(Ascending)

a)

$$\theta = \theta^* + \omega = 150^\circ + 30^\circ = 180^\circ$$

$$C_{313} = \begin{bmatrix} C_\Omega C_\theta - S_\Omega C_i \cancel{S_\theta} & -C_\Omega \cancel{S_\theta} - S_\Omega C_i C_\theta & S_\Omega S_i \\ S_\Omega C_\theta + C_\Omega \cancel{C_i S_\theta} & -S_\Omega \cancel{S_\theta} + C_\Omega C_i C_\theta & -C_\Omega S_i \\ S_i \cancel{S_\theta} & S_i C_\theta & C_i \end{bmatrix}$$

$$= \begin{bmatrix} -0.7071 & 0.6124 & 0.3536 \\ -0.7071 & -0.6124 & -0.3536 \\ 0 & -0.5 & 0.8660 \end{bmatrix}$$

$$\Delta \bar{V}^{r\theta h} = C_{313}^T \cdot \Delta \bar{V}^{xyz}$$

$$= \boxed{-0.2475\,\hat{r} - 0.2669\,\hat{\theta} + 0.1250\,\hat{h} \text{ km/s}}$$

@ $\theta^* = 180^\circ$

$r = \dfrac{p}{1+e\cos 180^\circ} = 2.6461 \times 10^4$ km $\quad V = \sqrt{\dfrac{2\mu}{r} - \dfrac{\mu}{a}} = 1.0372$ km/s

$\gamma = \cos^{-1}\left(\dfrac{\sqrt{\mu p}}{r V}\right) = +23.7940^\circ$ (ascending)

VNB is $r\theta h$ rotated around $\hat{h}$ by $-\gamma$.
(BVN)

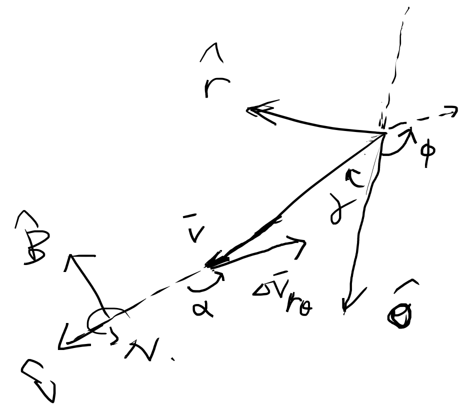
$$\Delta \bar{V}^{BVN} = \begin{bmatrix} C_\gamma & S_\gamma & 0 \\ -S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \Delta \bar{V}^{r\theta h}$$

$$= \begin{bmatrix} 0.9150 & -0.4034 & 0 \\ 0.4034 & 0.9150 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -0.2475 \\ -0.2669 \\ 0.2501 \end{bmatrix} = \underbrace{\boxed{-0.3440\,\hat{V} + 0.2501\,\hat{N} - 0.1188\,\hat{B} \text{ km/s}}}_{\Delta \bar{V}^{VNB}}$$

$$\Delta V_{r\theta} = \sqrt{0.2475^2 + 0.2669^2} = 0.3640 \text{ km/s}$$

$$\beta = \tan^{-1}\left(\frac{\Delta V_h}{\Delta V_{r\theta}}\right) = \tan^{-1}\left(\frac{0.2501}{0.3640}\right) = 34.4933^\circ$$

$$\phi = \tan^{-1}\left(\frac{\Delta V_r}{\Delta V_\theta}\right) = \tan^{-1}\left(\frac{-0.2475}{-0.2669}\right) = -137.1566^\circ$$



$$\Delta V_{BV} = \sqrt{0.3288^2 + 0.2348^2} = \boxed{0.3640 \text{ km/s} = \Delta V_{r\theta}}$$

They are equal

$$\alpha = \phi - \gamma = -137.1566^\circ - 23.7940^\circ = \boxed{-160.9505^\circ}$$

Out of plane $\Delta \bar{V}$ component is $\Delta \bar{V}_h$ ($\Delta \bar{V}_N$) $= 0.2501$ km/s

$$\boxed{\frac{0.2501}{|\Delta \bar{V}|} = 56.63\% \text{ out of plane}}$$

b) $\bar{V}_0^{\,roh} = V_0 \cdot (\sin\gamma_0 \, \hat{r} + \cos\gamma_0 \, \hat{\theta}) = 0.4185 \, \hat{r} + 0.9490 \, \hat{\theta}$ km/s

$$\bar{V}_0^{\,XYZ} = C_{313} \cdot \bar{V}_0^{\,roh}$$

$$= 0.2853 \, \hat{x} - 0.8771 \, \hat{y} - 0.4745 \, \hat{z} \text{ km/s}$$

$$\bar{V}^+ = \bar{V}_n^{\,XYZ} = \Delta \bar{V}^{XYZ} + \bar{V}_0^{\,XYZ}$$

$$\boxed{= 0.3853 \, \hat{x} - 0.6271 \, \hat{y} - 0.1245 \, \hat{z}} \text{ km/s}$$

$\bar{r}_0^{\,roh} = 2.6961 \times 10^4 \, \hat{r}$ km

$\bar{r}^+ = \bar{r}^-$

$= C_{313} \cdot r_0^{\,roh}$

$$\bar{r}^+ = \boxed{-1.9064 \times 10^4 \, \hat{x} - 1.9064 \times 10^4 \, \hat{y} \text{ km}}$$

c) $V_n = |\bar{v}^+| = 0.7464$ km/s.

$$a^+ = \frac{-\mu}{2\cdot\left(\frac{v^{+2}}{2} - \frac{\mu}{r^+}\right)} = \boxed{16307 \text{ km}}$$

$$\bar{h}^+ = r^+ \times v^+ = (0.2374\hat{x} - 0.2374\hat{y} + 1.5299\hat{z}) \times 10^4 \text{ km}^2/\text{s}$$

$$|\bar{h}^+| = 1.9589 \times 10^4 \text{ km}^2/\text{s} = \sqrt{\mu p^+}$$

$$p^+ = \frac{h^{+2}}{\mu} = a(1-e^{+2})$$

$$e^+ = \sqrt{1 - \frac{h^2}{\mu a^+}} = \boxed{0.6722}$$

$$\hat{h}^+ = \frac{\bar{h}^+}{|\bar{h}^+|} = 0.1212\hat{x} - 0.1212\hat{y} + 0.9852\hat{z}$$

$\cos i^+ = 0.9852$ $\qquad$ $\sin i^+ = 0.1714$.

$$\boxed{i^+ = 9.8680^\circ}$$

$$\left.\begin{array}{l} \sin\Omega^+ \sin i^+ = 0.1212 \\ -\cos\Omega^+ \sin i^+ = -0.1212 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \Omega^+ = 45^\circ, 135^\circ \\ \Omega^+ = \pm 45 \end{array}\right\} \Rightarrow \boxed{\Omega = 45^\circ}$$

$$\hat{r} = -0.7071\hat{x} - 0.7071\hat{y}$$

$$\hat{v} = 0.5162\hat{x} - 0.8401\hat{y} - 0.1668\hat{z}$$

$\hat{r} \cdot \bar{v}^+ = 0.1710 > 0$ ascending.

$$\left.\begin{array}{l} \sin i^+ \sin\theta^+ = 0 \\ \sin i^+ \cos\theta^+ = -0.1668 \end{array}\right\} \theta^+ = -180^\circ$$

$$\theta^{*+} = \cos^{-1}\left(\frac{1}{e^+}\left(\frac{p^+}{r^+} - 1\right)\right)$$

$$= \pm 173.3198^\circ = \boxed{173.3198^\circ} \text{ ascending}$$

$$\omega^+ = \theta^+ - \theta^{*+} = -353.3198^\circ$$

$$\boxed{\omega^+ = 6.6802^\circ}$$

d) In local frame, Mars centers,

Element 1 : $\hat{V}$

Element 2 : $\hat{N}$

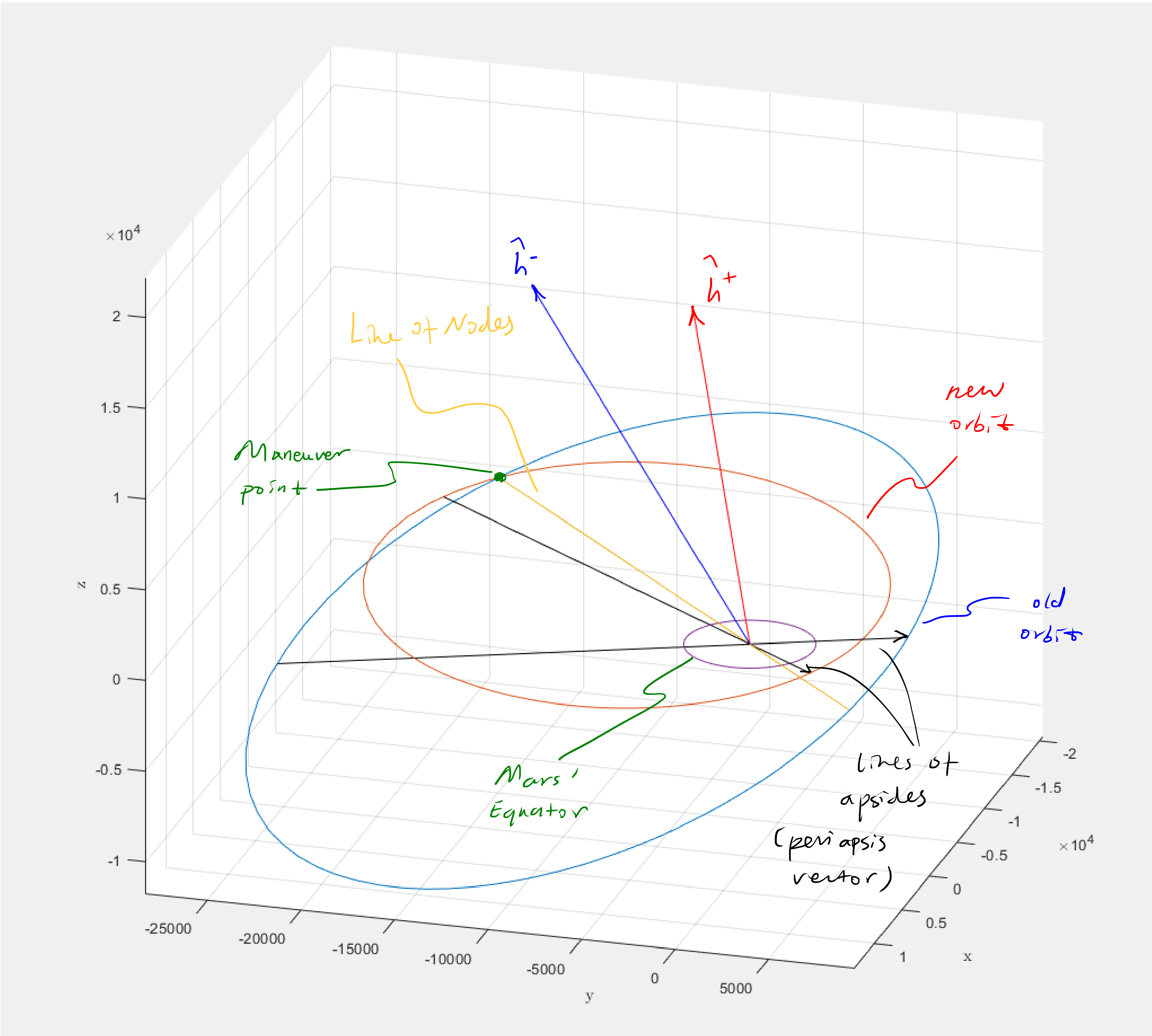
Element 3 : $\hat{B}$.

In MarsMJ2000Eq frame

E_1 : $\hat{x}$

E_2 : $\hat{y}$

E_3 : $\hat{z}$



2. Departure : Aug 05, 2011

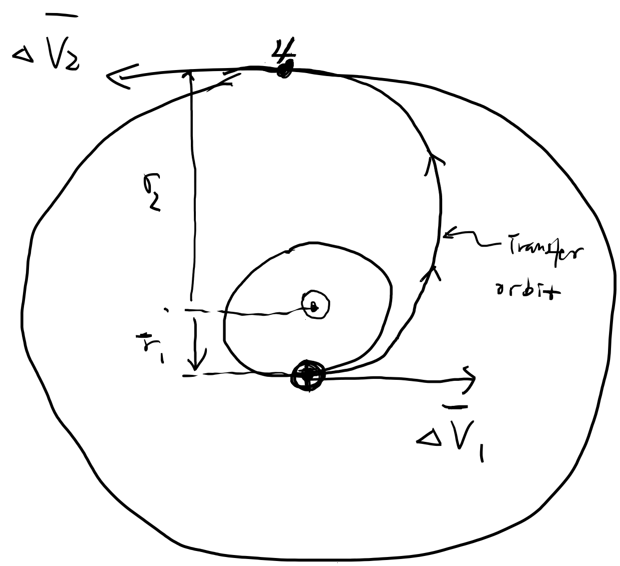
Arrival : Jul 05, 2016

a) Julian Days

$\boxed{\begin{array}{l} \text{Dep : } 2455779 \text{ days} \\ \text{Arr : } 2457575 \text{ days} \end{array}}$

$\boxed{\begin{array}{l} \text{TOF : } 1796 \text{ days} \\ \quad 4.9205 \text{ years} \end{array}}$

b)



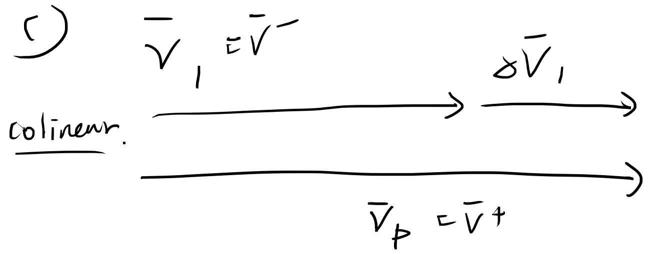
$r_1 = a_{Earth}$ ($a_\oplus$)

$r_2 = a_{Jupiter}$ ($a_{♃}$)

$a_T = \frac{1}{2}(r_1 + r_2)$

$e_T = 1 - \frac{r_1}{a_T}$

c)

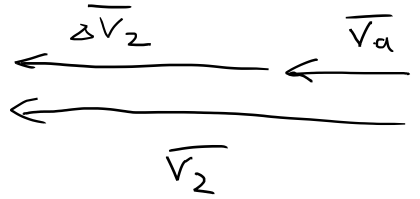


$\alpha = 0°$

$$V_1 = \sqrt{\frac{\mu}{r_1}} = 29.7847$$

$$V_p = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_T}} = 38.5772$$

$$\Delta V_1 = V_p - V_1 = 8.7925$$



$\alpha = 0°$

$$V_2 = \sqrt{\frac{\mu}{r_2}} = 13.0583$$

$$V_a = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_T}} = 7.4152$$

$$\Delta V_2 = V_2 - V_a = 5.6432$$

$$|\Delta \bar{V}|_{total} = \Delta V_1 + \Delta V_2 = \boxed{14.4357 \text{ km/s}}$$

$$\mathbb{P}_T = 2\pi \cdot \sqrt{\frac{a_T^3}{\mu}} = 1.7235 \times 10^8 \text{ sec}$$

$$TOF = \frac{1}{2} \mathbb{P}_T = 8.6176 \times 10^8 \text{ sec} = \boxed{2.7326 \text{ years}}$$

The $|\Delta \bar{V}|_{total}$ is quite large that would require a significant amount of fuel. This is not a easily achievable maneuver by the spacecraft.

However, the TOF is certainly shorter in the Hohmann transfer than in the actual flight.

d) $n_1 = \frac{2\pi}{\mathbb{P}_\oplus}$

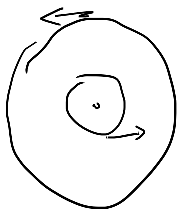
$\Phi = \pi - n_2 \cdot TOF$

$n_2 = \frac{2\pi}{\mathbb{P}_4}$

$\boxed{= 97.1562^\circ}$

$t_s = \frac{2\pi}{n_1 - n_2} = \boxed{3.4462 \times 10^7 \text{ sec}}$
$= \boxed{1.0928 \text{ years}}$

e) $e_2' = 0.0485359$



$r_2' = r_{p,4} = a_4(1-e_4) = 7.4051 \times 10^8$ km

$V_2' = \sqrt{\frac{2\mu}{r_2'} - \frac{\mu}{a_J}} = 13.7083$ km/s

$a_T' = \frac{1}{2}(r_1 + r_2') = 4.4505 \times 10^8$ km

$V_a' = \sqrt{\frac{2\mu}{r_2'} - \frac{\mu}{a_T'}} = 7.7616$ km/s

$\Delta V_2' = V_2' - V_a' = 5.9467$ km/s

$V_p' = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_T'}} = 38.4195$ km/s

$\Delta V_1' = V_p' - V_1 = 8.6348$ km/s

$|\Delta \bar{V}|_{total} = 8.6348 + 5.9467 = \boxed{14.5816 \text{ km/s}}$

$\mathbb{P}_T = 2\pi\sqrt{\frac{a_T'^3}{\mu}} = 1.6193 \times 10^8$ sec

$TOF = \frac{1}{2}\mathbb{P}_T = \boxed{8.0967 \times 10^7 \text{ sec} = 2.5675 \text{ years}}$

The perihelion arrival time is different

f) Synodic period is unaffected as period of Jupiter & Earth are unchanged.

$t_s = \frac{2\pi}{n_1 - n_2} = \frac{2\pi}{2\pi/\mathbb{P}_1 - 2\pi/\mathbb{P}_2} = \frac{1}{\frac{1}{\mathbb{P}_1} - \frac{1}{\mathbb{P}_2}} = \frac{\mathbb{P}_1 \cdot \mathbb{P}_2}{\mathbb{P}_2 - \mathbb{P}_1} = f(\mathbb{P}_1, \mathbb{P}_2)$

3.

a) $a = R_E + 200$

$r_1 = r_2 = a$

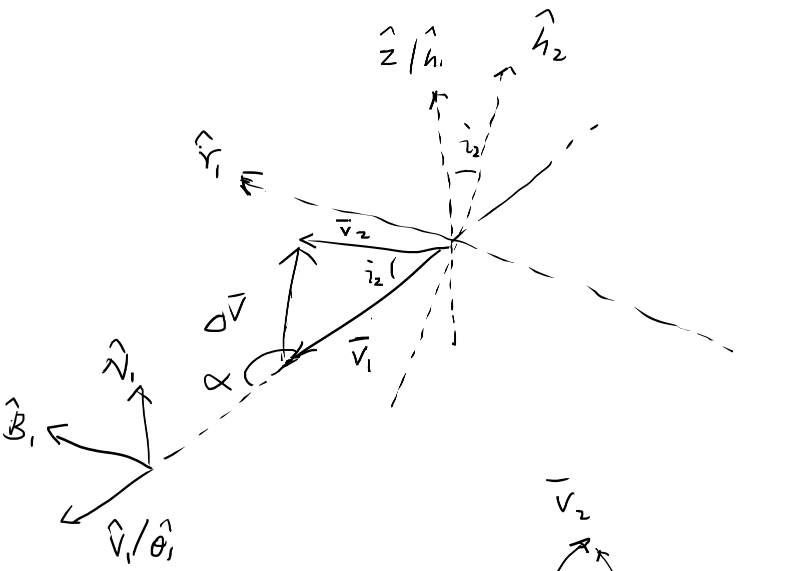
$V_1 = \sqrt{\frac{\mu}{r_1}}$

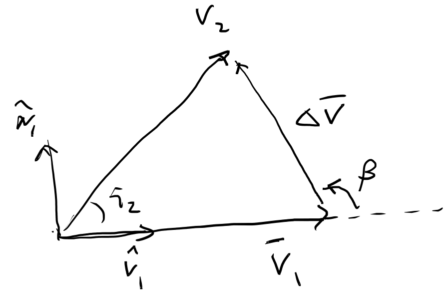
$V_2 = V_1 = 7.7843$ km/s

$\bar{i}_1 = 0$

$\bar{i}_2 = 57°$

$\boxed{\alpha = 180°}$





$\Delta v = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos(i_2)}$

$\boxed{= 7.4287 \text{ km/s}}$

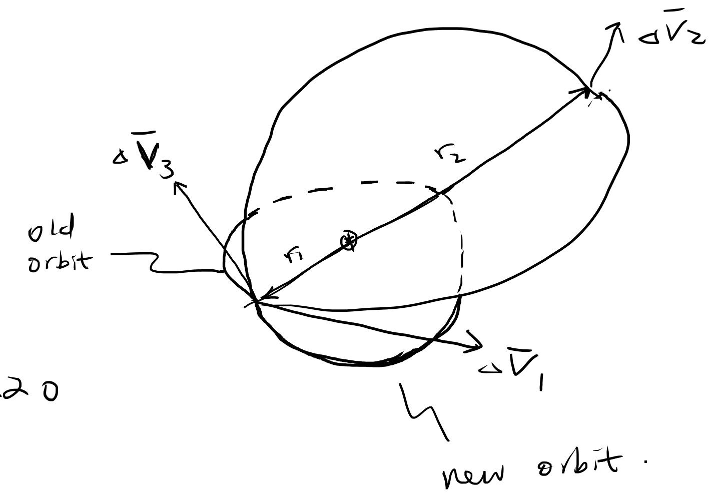
$\bar{\Delta V} = \Delta V \cdot (\cos(\beta)\, \hat{V}_1 + \sin(\beta)\, \hat{N}_1)$

$= -3.5446\, \hat{V}_1 + 6.5284\, \hat{N}_1$ km/s

$\beta = 180 - (180 - i_2)/2$

$\boxed{\beta = 118.5°}$

b)



$r_1 = R_\oplus + 220$

$r_2 = 55 R_\oplus$

$r_0 = r_1$ $\quad r_{p,T} = r_1$

$r_n = r_1$ $\quad r_{a,T} = r_2$.

$a_T = \frac{1}{2}(r_{a_T} + r_{p_T}) = 1.7869 \times 10^5$ km

$e_T = 1 - \frac{r_{pT}}{a_T} = 0.9632$

Maneuver 1.

Tangential.

$\bar{v}_1 = \bar{v}^-$ $\quad \Delta\bar{v}_1$

$\bar{v}_p = \bar{v}^+$

$$v_1 = \sqrt{\frac{\mu}{r_1}} = 7.7843 \text{ km/s}$$

$$v_p = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_T}} = 10.9068 \text{ km/s}$$

$$\Delta v_1 = v_p - v_1 = \boxed{3.1226 \text{ km/s}}$$

$$\boxed{\alpha_1 = 0^\circ, \beta_1 = 0^\circ}$$

Maneuver 3

Tangential.

$\bar{v}_p = \bar{v}^-$

$\bar{v}^+ = \bar{v}_1$ $\quad \Delta\bar{v}_3$

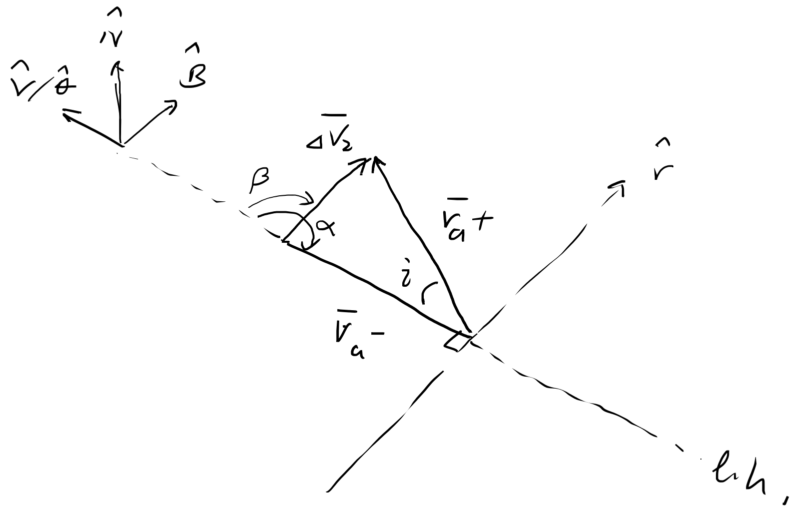
$$\Delta\bar{v}_3 = -\Delta\bar{v}_1 \qquad \Delta v_3 = \Delta v_1$$

$$\boxed{= 3.1226 \text{ km/s}}$$

$$\boxed{\alpha_3 = 180^\circ, \beta_3 = 180^\circ}$$

Maneuver 2.

Out of plane.



$$v_a = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_T}} = 0.2045 \text{ km/s}$$

$$\Delta v_2 = \sqrt{2v_a^2 - 2v_a^- v_a^+ \cos i}$$

$$= \boxed{0.1952 \text{ km/s}}$$

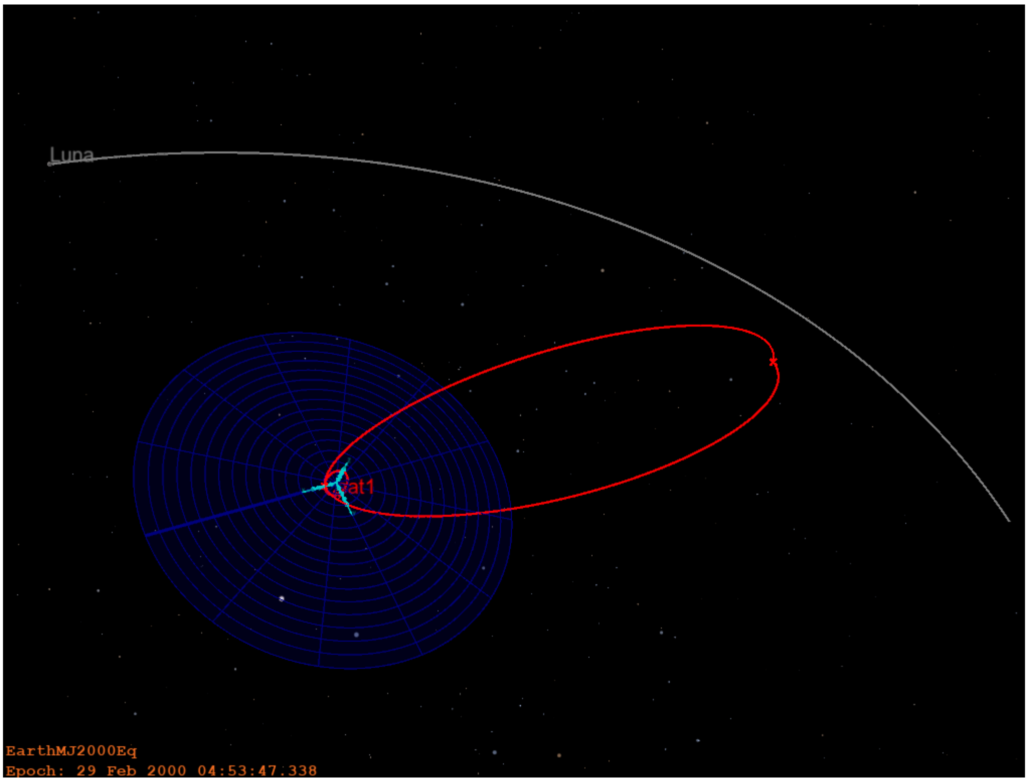
$$\boxed{\alpha_2 = 180^\circ, \beta_2 = 118.5^\circ}$$

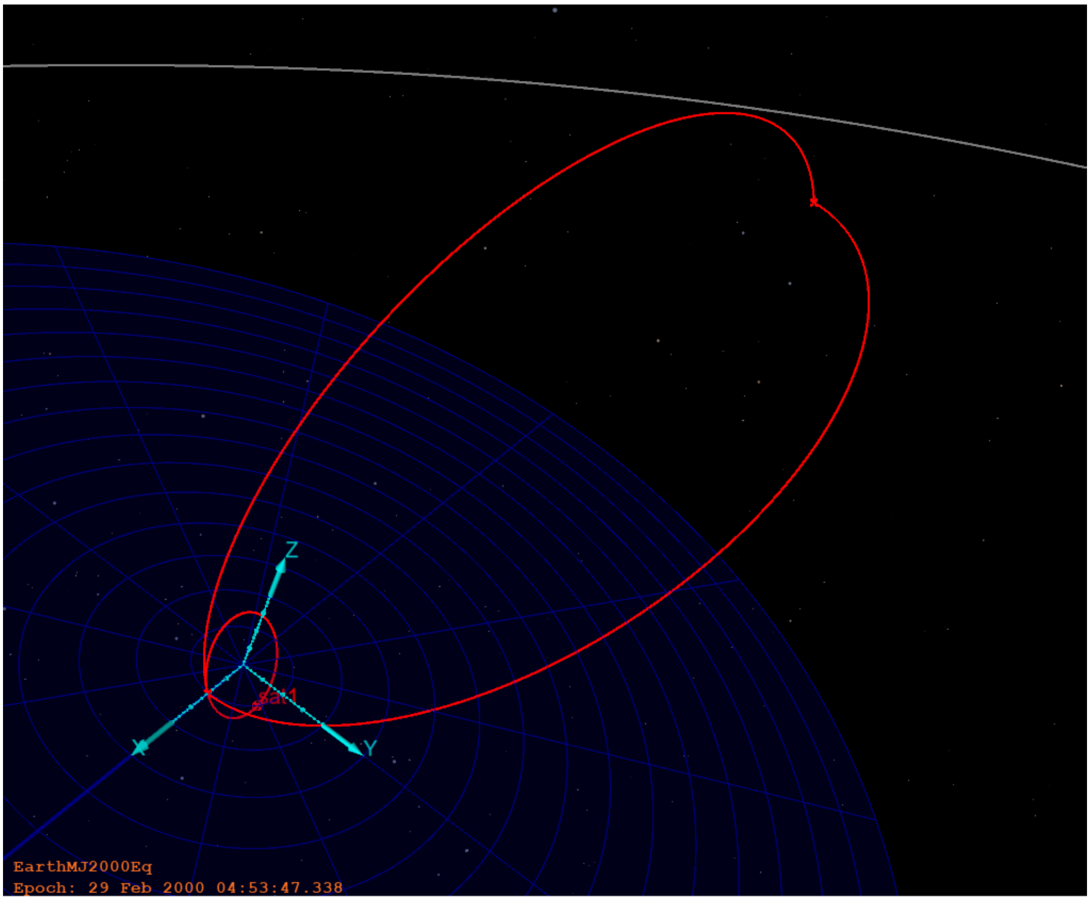
c) $|\Delta\bar{v}|$ total $= 3.1226 \times 2 + 0.1952$

$$= \boxed{6.4403 \text{ km/s}}$$

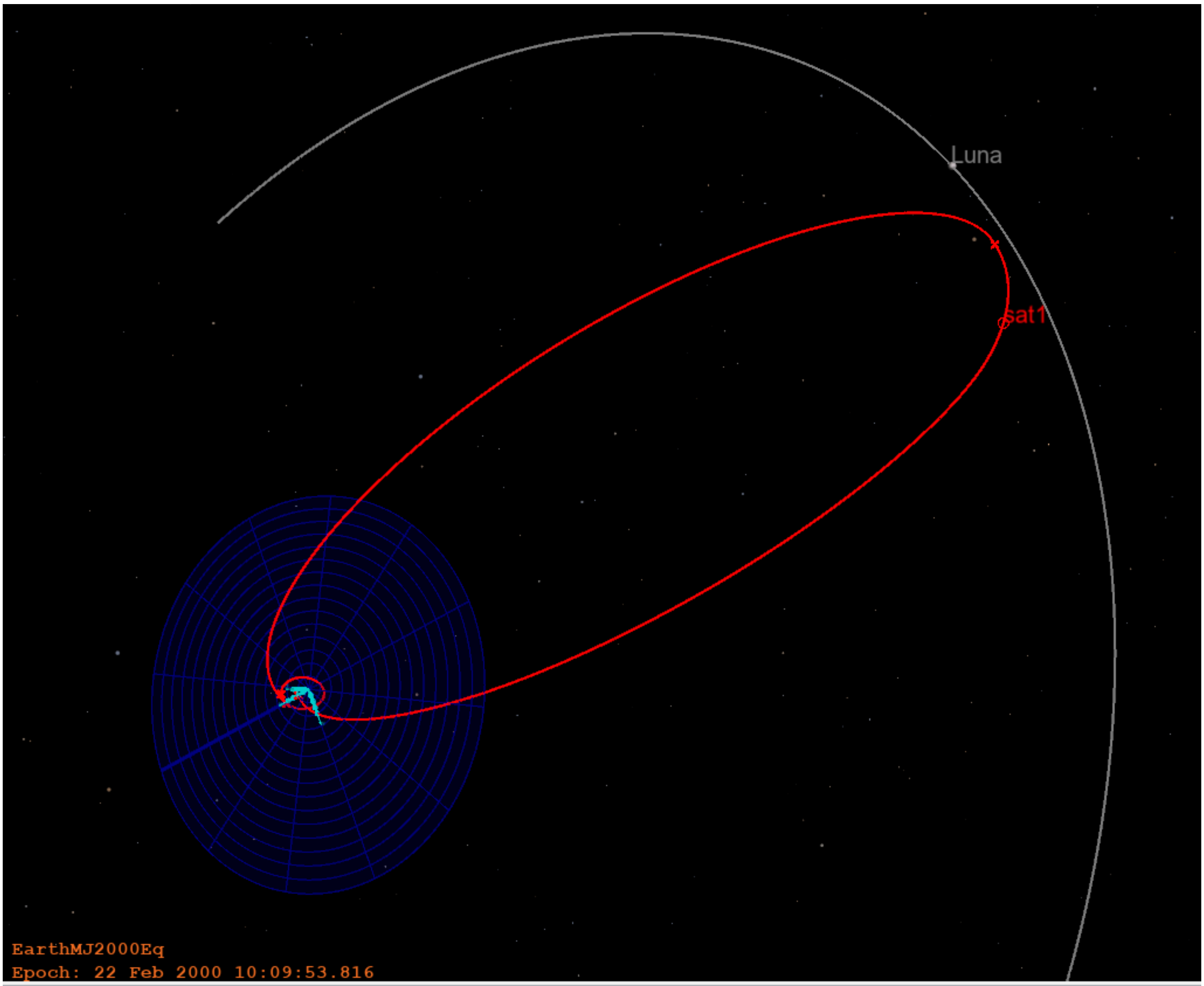
$$ToF = P_T = 2\pi\sqrt{\frac{a_T^3}{\mu}} = \boxed{7.5172 \times 10^5 \text{ sec}}$$

Total cost for bi-elliptic maneuver is smaller, the time penalty is the TOF d)





The intermediate orbit does get a little close to lunar orbit, and will be affected by lunar gravity e)



By choosing a starting date that will result in the S/C close to lunar at apogee of intermediate orbit, we can see the intermediate orbit being significantly stretched by lunar gravity. The final orbit is no longer circular.