hw4

March 19, 2021

[1]:
```
%config IPCompleter.use_jedi = False
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import cvxpy as cp
import pandas as pd
np.set_printoptions(precision=4)

from pathlib import Path
fig_path = str(Path().absolute())+'/figures/hw4/'
print(fig_path)
data_path = str(Path().absolute())+'/hw4/data/'
print(data_path)
```

```
/home/zpyang/grad_courses/2021_spring/ece595_ml/figures/hw4/
/home/zpyang/grad_courses/2021_spring/ece595_ml/hw4/data/
```

1 Exercise 3

3 a)

$$J(\theta) = -\frac{1}{N}\sum_{n=1}^{N}[y_n \cdot \log h_\theta(x_n) + (1-y_n)\cdot \log(1-h_\theta(x_n))]$$

$$= -\frac{1}{N}\sum_{n=1}^{N}[y_n \cdot \log(\frac{h_\theta(x_n)}{1-h_\theta(x_n)}) + \log(1-h_\theta(x_n))]$$

$$= -\frac{1}{N}\{\sum_{n=1}^{N}[y_n \cdot \log(\frac{h_\theta(x_n)}{1-h_\theta(x_n)})] + \sum_{n=1}^{N}\log(1-h_\theta(x_n))\}$$

where $h_\theta(x) = \frac{1}{1+\exp\{-\theta^T x_n\}}$

Subbing $h_\theta(x)$ back into the loss function, we obtain

$$J(\theta) = -\frac{1}{N}\{\sum_{n=1}^{N}[y_n \cdot \log(\frac{\frac{1}{1+\exp\{-\theta^T x_n\}}}{1-\frac{1}{1+\exp\{-\theta^T x_n\}}})] + \sum_{n=1}^{N}\log(1-\frac{1}{1+\exp\{-\theta^T x_n\}})\}$$

$$= -\frac{1}{N}\{\sum_{n=1}^{N} y_n \log(e^{-\theta^T x_n}) + \sum_{n=1}^{N}\log(1-\frac{e^{\theta^T x_n}}{1+e^{\theta^T x_n})}\}$$

$$= -\frac{1}{N}\{\sum_{n=1}^{N} -y_n \theta^T x_n - \sum_{n=1}^{N} \log{(1 + e^{\theta^T x_n})}\}$$

$$= -\frac{1}{N}\{(\sum_{n=1}^{N} -y_n x_n)^T \theta - \sum_{n=1}^{N} \log{(1 + e^{\theta^T x_n})}\}$$

[2]:

```
# 3 b)

c0 = pd.read_csv(data_path+"class0.txt", delimiter='\s+', header=None).
 ↪to_numpy()
c1 = pd.read_csv(data_path+"class1.txt", delimiter='\s+', header=None).
 ↪to_numpy()

n0 = c0.shape[0]
n1 = c1.shape[0]
N = n0 + n1
x = np.vstack([c0,c1])
y = np.hstack([np.zeros(n0), np.ones(n1)]).reshape(-1,1)
X = np.hstack([x, np.ones((N,1))])

lambd = 0.0001

theta = cp.Variable((3,1))
loss = -cp.sum(cp.multiply(y, X @ theta)) \
        + cp.sum(cp.log_sum_exp(cp.hstack([np.zeros((N,1)), X @ theta]),␣
 ↪axis=1))
reg = cp.sum_squares(theta)
prob = cp.Problem(cp.Minimize(loss/N + lambd*reg))
prob.solve()
omega = theta.value

omega
```

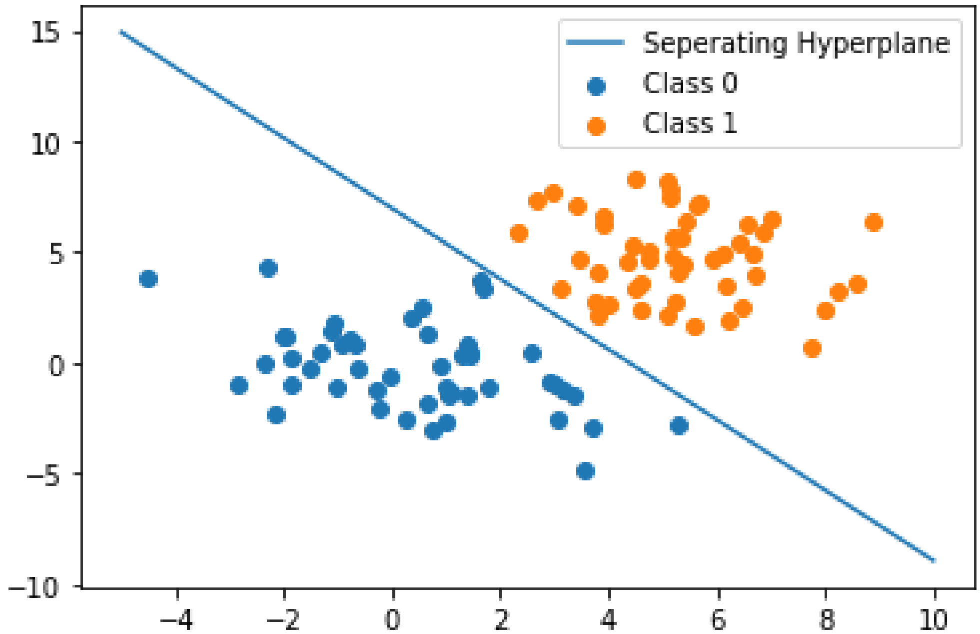
[2]:

```
array([[  2.3786],
       [  1.4975],
       [-10.4365]])
```

[3]:

```
# 3 c)

x1 = np.linspace(-5,10,2)
x2 = -omega[0]/omega[1] * x1 - omega[2]/omega[1]

plt.figure
plt.scatter(c0[:,0], c0[:,1], label='Class 0')
plt.scatter(c1[:,0], c1[:,1], label='Class 1')
plt.plot(x1,x2, label='Seperating Hyperplane')
plt.legend()
```

[3]: <matplotlib.legend.Legend at 0x7f262525cf10>

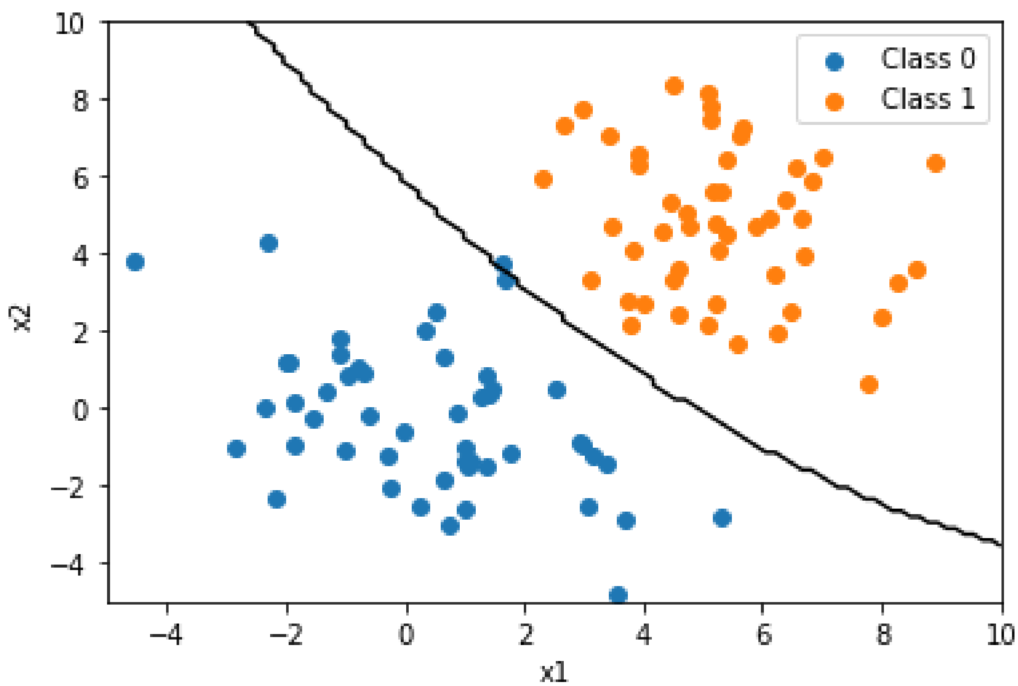


[4]:
```
# 3 d)
mu0 = c0.mean(0)
mu1 = c1.mean(0)

SIG0 = np.cov(c0.T)
SIG1 = np.cov(c1.T)

invSIG1 = np.linalg.inv(SIG1)
invSIG0 = np.linalg.inv(SIG0)
detSIG1 = np.linalg.det(SIG1)
detSIG0 = np.linalg.det(SIG0)

LOG0 = -0.5*np.log(detSIG0)
LOG1 = -0.5*np.log(detSIG1)
```

[5]:
```
from tqdm.notebook import tqdm

x1 = np.linspace(-5,10,N)
x2 = np.linspace(-5,10,N)
X1,X2 = np.meshgrid(x1,x2)

grid = np.zeros((N,N))

for i in range(N):
```

```
    for j in range(N):
        xi = np.array([X1[i,j], X2[i,j]]).T
        k0 = -0.5*(xi - mu0).T @ invSIG0 @ (xi - mu0) + LOG0
        k1 = -0.5*(xi - mu1).T @ invSIG1 @ (xi - mu1) + LOG1
        if k1 > k0:
            grid[i,j] = 1
plt.figure()
plt.contour(X1,X2,grid, levels=[0.5], cmap='gray')
plt.scatter(c0[:,0], c0[:,1], label='Class 0')
plt.scatter(c1[:,0], c1[:,1], label='Class 1')
plt.xlabel('x1')
plt.ylabel('x2')
plt.legend()
```

[5]: <matplotlib.legend.Legend at 0x7f262304c8b0>



2 Exercise 4

[6]:
```
# 4 a)

h = 1
K = np.zeros((N,N))
for i in range(N):
```

```python
    for j in range(N):
        K[i,j] = np.exp(-np.sum((X[i,:]-X[j,:])**2)/h)

print('The 47th to 52th elements of the Kernel Matrix: \n', K[47:52, 47:52])
```

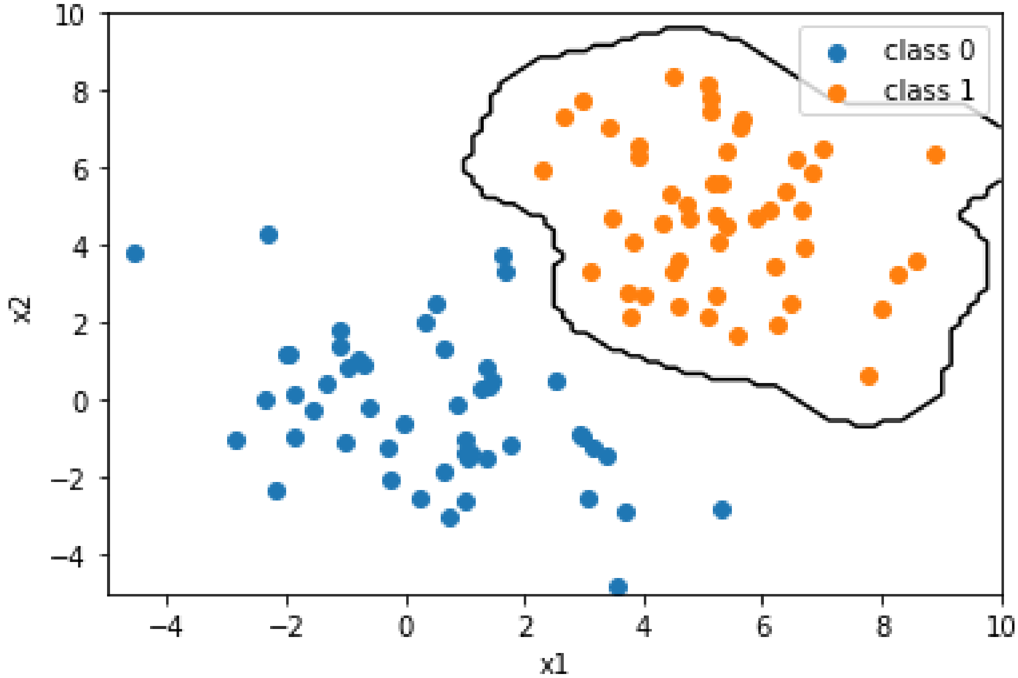
```
The 47th to 52th elements of the Kernel Matrix:
 [[1.0000e+00 5.0531e-25 6.0654e-20 4.6547e-29 4.0689e-17]
 [5.0531e-25 1.0000e+00 3.9593e-13 2.6936e-33 5.3878e-12]
 [6.0654e-20 3.9593e-13 1.0000e+00 2.3035e-65 3.7842e-34]
 [4.6547e-29 2.6936e-33 2.3035e-65 1.0000e+00 2.1628e-06]
 [4.0689e-17 5.3878e-12 3.7842e-34 2.1628e-06 1.0000e+00]]
```

[7]:
```python
# 4 b)

alpha = cp.Variable((N,1))
loss = -cp.sum(cp.multiply(y, K @ alpha)) \
        + cp.sum(cp.log_sum_exp(cp.hstack([np.zeros((N,1)), K @ alpha]),␣
 ↪axis=1))
reg = cp.quad_form(alpha, K)
prob = cp.Problem(cp.Minimize(loss/N + lambd*reg))
prob.solve()
a = alpha.value
print('The first 2 elements of alpha: ', a[0:2].T)
```

```
The first 2 elements of alpha:  [[-0.9525 -1.2105]]
```

[8]:
```python
from numpy.matlib import repmat
out = np.zeros((N,N))
for i in range(N):
    for j in range(N):
        data = repmat(np.array([x1[j], x2[i], 1]).reshape((1,3)), N, 1)
        phi = np.exp(-np.sum((X-data)**2, axis=1)/h)
        out[i,j] = np.dot(phi.T, a)

plt.figure()
plt.scatter(c0[:,0], c0[:,1], label='class 0')
plt.scatter(c1[:,0], c1[:,1], label='class 1')
plt.contour(x1, x2, out>0.5, levels=[0.5], cmap='gray')
plt.xlabel('x1')
plt.ylabel('x2')
plt.legend()
```

[8]:
```
<matplotlib.legend.Legend at 0x7f2620d97130>
```


class 0
class 1
x1
x2
−4
−2
0
2
4
6
8
10