2.2.1 Exercise

Problem 1. Find a constant a such that

$$\begin{aligned} f_{\mathbf{x}}(x) &= a\sin(x) && \text{if } 0 \le x \le \pi \\ &= 0 && \text{otherwise} \end{aligned}$$

is a density function. For this constant a find the distribution function $F_{\mathbf{x}}(x)$.

Problem 2. Find a constant a such that

$$f_{\mathbf{x}}(x) = ae^{-|x|} \qquad \text{for } -\infty < x < \infty$$

is a density function.

- For this constant a find the distribution function $F_{\mathbf{x}}(x)$.
- Find $P(1 \le \mathbf{x})$.

Problem 3. Find a constant c such that

$$\begin{aligned} f_{\mathbf{x}}(x) &= \frac{c}{\sqrt{x}} && \text{if } 0 < x \le 4 \\ &= 0 && \text{otherwise} \end{aligned}$$

is a density function.

- For this constant c find the distribution function $F_{\mathbf{x}}(x)$.
- Find $P(|\mathbf{x}| \le 1)$.

Problem 4. Consider the uniform random variable $\mathbf{x}$ over $[0, 1]$ whose density function is given by

$$\begin{aligned} f_{\mathbf{x}}(x) &= 1 && \text{if } 0 \le x \le 1 \\ &= 0 && \text{otherwise.} \end{aligned}$$

Let $\mathbf{y}$ be the random variable defined by $\mathbf{y} = \mathbf{x}^2$.

- Find the distribution function $F_{\mathbf{y}}(y)$ for $\mathbf{y}$.
- Find the density function $f_{\mathbf{y}}(y)$ for $\mathbf{y}$.

HINT: $F_{\mathbf{y}}(y) = P(\mathbf{y} \le y) = P(\mathbf{x}^2 \le y)$.