4.5

$$f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a) $\int_0^2 kx^2\,dx = 1$

$\frac{k}{3}x^3 \Big|_0^2 = 1$

$\frac{8}{3}k = 1$

$k = \boxed{\frac{3}{8}}$

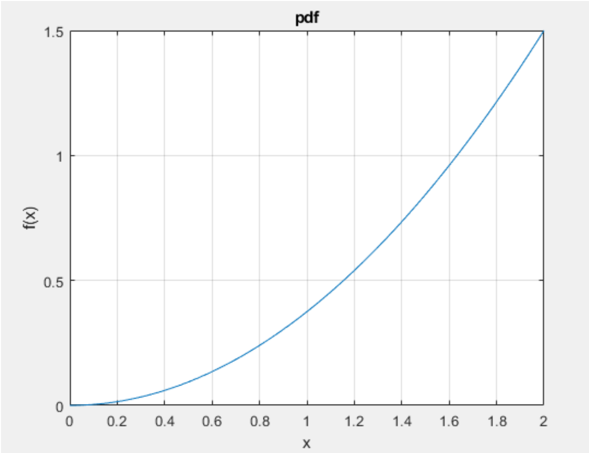
b) $f(x \le 1) = \int_0^1 \frac{3}{8}x^2\,dx = \frac{1}{8}x^3\Big|_0^1 = \boxed{\frac{1}{8}}$

c) $f(1 \le x \le 1.5) = \int_1^{1.5} \frac{3}{8}x^2\,dx = \frac{1}{8}x^3\Big|_1^{1.5} = \boxed{0.2969}$

d) $f(x \ge 1.5) = \int_{1.5}^{2} \frac{3}{8}x^2\,dx = \frac{1}{8}x^3\Big|_{1.5}^{2} = \boxed{0.5781}$

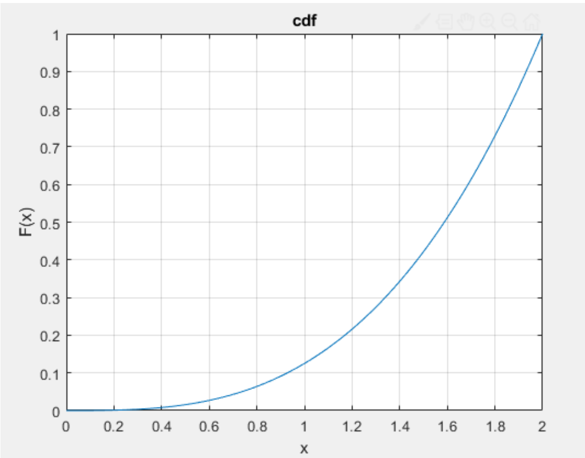
4.1b $f(x) = \begin{cases} \frac{3}{8}x^2 & 0<x< \\ 0 & \text{otherwise} \end{cases}$ X : lecture time past the hour in minutes.

a)



$F(x) = \int_0^x \frac{3}{8} y^2 dy = \frac{1}{8} y^3 \Big|_0^x = \frac{x^3}{8}$

$$\boxed{F(x) = \begin{cases} \frac{x^3}{8} & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}}$$



b) $P(X \le 0.5) = F(0.5) = \boxed{\frac{1}{64}}$

c) $P(0.25 < X \le 0.5) = F(0.5) - F(0.25) = \frac{1}{64} - \frac{1}{8 \times 64} = \boxed{0.0137}$

d) $P(X \le x) = 0.75$

$\frac{x^3}{8} = 0.75$

$\boxed{x = 1.8171}$

e) $E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_0^2 x \cdot \frac{3}{8} x^2 dx = \frac{3}{32} x^4 \Big|_0^2 = \boxed{1.5 \text{ min}}$

$V(x) = \int_0^2 (x-1.5)^2 \cdot \frac{3}{8} x^2 dx = \int_0^2 \frac{3}{8} x^4 dx - 1.5^2 = 2.4 - 1.5^2 = 0.15$

$\sigma(x) = \sqrt{0.15} = \boxed{0.3873 \text{ min}}$

f) $1 - P(1.5 - 0.3873 < X < 1.5 + 0.3873) = 1 - [F(1.8873) - F(1.1127)]$

$= 1 - 0.6681 = \boxed{0.3319}$

4.33 Normal distribution. $\mu = 46.8$ $\sigma = 1.75$

a) $P(X \le 50) = \Phi\left(\frac{50-46.8}{1.75}\right) = \boxed{0.9663}$

b) $P(X \ge 48) = 1 - P(X \le 48) = 1 - \Phi\left(\frac{48-46.8}{1.75}\right) = \boxed{0.2464}$

c) $P(46.8 - 1.75 \times 1.5 \le X \le 46.8 + 1.75 \times 1.5)$

$= \Phi(1.5) - \Phi(-1.5) = \boxed{0.8664}$

4.35 Norm dist. $\mu = 8.8$. $\sigma = 2.8$

a) $P(X \geq 10) = 1 - \Phi\left(\frac{10-8.8}{2.8}\right) = \boxed{0.3341}$

$P(X > 10) = P(X \geq 10) = \boxed{0.3341}$

b) $P(X > 20) = 1 - \Phi\left(\frac{20-8.8}{2.8}\right) = 3.1671 \times 10^{-5} \approx \boxed{0}$

c) $P(5 < X < 10) = \Phi\left(\frac{10-8.8}{2.8}\right) - \Phi\left(\frac{5-8.8}{2.8}\right) = \boxed{0.5785}$

d) $P(8.8 - c < X < 8.8 + c) = 0.98$

$\Phi\left(\frac{c}{2.8}\right) - \Phi\left(\frac{-c}{2.8}\right) = 0.98 \Rightarrow \Phi\left(\frac{c}{2.8}\right) = 0.99$

$\frac{c}{2.8} = 2.33$

$\boxed{c = 6.524}$

e) $1 - P(X < 10)^4 = 1 - 0.6659^4 = \boxed{0.8034}$

4.46 $\mu = 70, \quad \sigma = 3$

a) $P(67 < X < 75) = \Phi\left(\frac{75-70}{3}\right) - \Phi\left(\frac{67-70}{3}\right)$

$= 0.9522 - 0.1587 = \boxed{0.7936}$

b) $P(70-c < X < 70+c) = 0.95$

$\Phi\left(\frac{c}{3}\right) - \Phi\left(\frac{-c}{3}\right) = 0.95 \Rightarrow \Phi\left(\frac{c}{3}\right) = 0.975$

$\frac{c}{3} = 1.96$

$c = 5.88$

c) $0.7936 \times 10 = \boxed{7.936}$

d) $P(X < 73.48) = \underline{0.8997}$

$P(Y \leq 8) = B(8, 10; 0.8997)$

$= \sum_{y=0}^{8} \binom{10}{y} \cdot 0.8997^{y} \cdot 0.1003^{1-y}$

$\boxed{= 0.2651}$

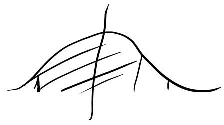
4.48

a) $P(-1.72 \le Z \le -0.55) = P(0.55 \le Z \le 1.72)$

$= \Phi(1.72) - \Phi(0.55)$

$= 0.2484$

b) $P(-1.72 \le Z \le 0.55)$



$= \Phi(0.55) - (1 - P(1.72))$

$= 0.7088 - 0.0427$

$\boxed{= 0.6661}$

It is not necessary to tabulate negative z values, because the curve is symmetric

4.54 $n = 200$, $p = 0.1$ $np = 20 > 10$

a) $P(X \le 30) = B(30, 200, 0.1) = \Phi\left(\frac{30+0.5-20}{\sqrt{200\times0.1\times0.9}}\right) = \boxed{0.9933}$

b) $P(X < 30) = B(29, 200, 0.1) = \Phi\left(\frac{9.5}{\sqrt{18}}\right) = \boxed{0.9874}$

c) $P(15 \le X \le 25) = B(25, 200, 0.1) - B(15, 200, 0.1)$

$= \Phi\left(\frac{5.5}{\sqrt{18}}\right) - \Phi\left(\frac{-4.5}{\sqrt{18}}\right)$

$= 0.9026 - 0.1444$

$\boxed{= 0.7582}$