STAT 511 HW-4

3.10

a) $T \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

b) $X \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

c) $U \in \{0, 1, 2, 3, 4, 5, 6\}$

d) $Z \in \{0, 1, 2\}$

3.12

a) $P(y \le 50) = 0.05 + 0.10 + 0.12 + 0.14 + 0.25 + 0.17$

$= 0.83$

b) $P(y > 50) = 1 - 0.83 = 0.17$

c) $P(y \le 49) = 0.66$

$P(y \le 47) = 0.27$

3.31

$\mu_Y = 48.84$

$48.84 + 2.12 = 50.96$

$48.84 - 2.12 = 46.72$

$V(Y) = E(X^2) - (E(X))^2$

$= 2389.84 - 48.84^2$

$\boxed{= 4.4944}$

$\sigma_Y = \sqrt{V(Y)}$ $\boxed{= 2.12}$

$P(46.72 < Y < 50.96) = 0.12 + 0.14 + 0.25 + 0.17$

$\boxed{= 0.68}$

3.35

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

X: demand   cost: \$2   price: \$4

3 copies: cost $= 2\times 3 = \$6$.

Expected demand: $\frac{1}{15} + \frac{2\times 2}{15} + \frac{3\times 3}{15} + \frac{3\times 4}{15} + \frac{3\times 3}{15} + \frac{3\times 2}{15}$

$= \frac{41}{15} = 2.7333$

Expected revenue: $4\times\frac{41}{15} = 10.9333$

Expected profit: $10.9333 - 6 = \underline{\$4.9333}$

4 copies: cost $= 2\times 4 = \$8$

Expected demand: $\frac{1}{15} + \frac{2\times 2}{15} + \frac{3\times 3}{15} + \frac{4\times 4}{15} + \frac{4\times 3}{15} + \frac{4\times 2}{15} = \frac{50}{15}$

Expected revenue: $4\times\frac{50}{15} = 13.3333$

Expected profit: $13.3333 - 8 = \underline{\$5.3333}$

$5.3333 > 4.9333$

Buying 4 copies is better.

3.49 (-c) Let X be number of "seconds"

a) $P(X=1) = \frac{9}{10}^5 \times \frac{1}{10} \times \binom{6}{1} = \boxed{0.3543}$

b) $P(X \geq 2) = 1 - P(X \leq 1)$

$P(X \leq 1) = P(0) + P(1)$

$P(0) = \frac{9}{10}^6 = 0.5314$

$P(1) = 0.3543$

$P(X \geq 2) = 1 - 0.5314 - 0.3543 = \boxed{0.1143}$

c) _ _ _ _ _

$\frac{9}{10}$ $\frac{1}{10}$ $\frac{9}{10}$ $\frac{9}{10}$ $\frac{9}{10}$

Case 1: First 4 are not seconds.

0.9^4

case 2: 1 second in first 4.

$0.1 \times 0.9^4 \times 4$

Total probability: $0.9^4 + 0.1 \times 0.9^4 \times 4 = \boxed{0.9185}$

3.56 Let X be number of students that got special treatment.

a) $P(X=1) = \binom{25}{1} \cdot 0.02 \cdot 0.98^{24} = \boxed{0.3079}$

b) $P(X \geq 1) = 1 - P(X<1) = 1 - P(X=0) = 1 - 0.98^{25} = \boxed{0.3965}$

c) $P(X \geq 2) = 1 - P(X=0) - P(X=1) = 1 - 0.98^{25} - 0.3079 = \boxed{0.0886}$

d) $E(X) = 0.02 \times 25 = 0.5$

$V(X) = 0.02 \times 25 \times (1 - 0.02) = 0.49$

$\sigma_X = \sqrt{0.49} = 0.7$

$P(0.5 - 0.7 \times 2 < X < 0.5 + 0.7 \times 2)$

$= P(-0.9 < X < 1.9)$

$= P(0 \leq X \leq 1)$

$= P(X=0) + P(X=1)$

$= 0.98^{25} + 0.3079$

$\boxed{= 0.9114}$

e) $\dfrac{0.5 \times 4.5 + 24.5 \times 3}{25} = \boxed{3.03 \text{ hours}}$

3.71

a) $n = 15, \quad M = 10, \quad N = 20$

$$P(X=x) = h(x; 15, 10, 20)$$

$$= \frac{\binom{10}{x}\binom{10}{15-x}}{\binom{20}{15}}, \quad x = 5, 6, 7, 8, 9, 10$$

b) $$P(X=10) = \frac{\binom{10}{10}\binom{10}{5}}{\binom{20}{15}} = 0.0163$$

$$P(X=5) = \frac{\binom{10}{5}\binom{10}{10}}{\binom{20}{15}} = 0.0163$$

$$P = 0.0163 + 0.0163 = \boxed{0.0326}$$

c) $$E(X) = n \cdot \frac{M}{N} = 15 \times \frac{10}{20} = 7.5$$

$$V(X) = \left(\frac{N-n}{N-1}\right) \cdot n \cdot \frac{M}{N} \cdot \left(1 - \frac{M}{N}\right) = \frac{5}{19} \cdot 15 \cdot \frac{10}{20} \cdot \left(1 - \frac{10}{20}\right) = 0.9868$$

$\sigma_X = 0.9934$

$$P(7.5 - 0.9934 < X < 7.5 + 0.9934) = P(6.5066 < X < 8.4934)$$

$$= P(7 \le X \le 8)$$

$$= P(X=7) + P(X=8)$$

$$\boxed{= 0.6966}$$

3.87 $\alpha = 4$ /hr

a) $t = 2$ hr

$$P(X=10) = \frac{e^{-8} \cdot (8)^{10}}{10!} = \boxed{0.0993}$$

b) $t = 0.5$ hr

$$P(X=0) = \frac{e^{-2} \cdot (2)^{0}}{1} = \boxed{0.1353}$$

c) $\mu = 4 \times 0.5 = \boxed{2 \text{ calls}}$