

7.1

$$a) \bar{x} \pm 2.81 \frac{\sigma}{\sqrt{n}}$$

$$\Phi(2.81) = 0.9975$$

$$\Phi(-2.81) = 0.0025$$

$$CL = 0.9975 - 0.0025 = \boxed{0.9950}$$

$$b) \bar{x} \pm 1.44 \frac{\sigma}{\sqrt{n}}$$

$$\Phi(1.44) = 0.9251$$

$$CL = 0.9251 - 0.0749 = \boxed{0.8502}$$

$$\Phi(-1.44) = 0.0749$$

$$c) CL = 0.9970$$

$$\Phi(z_{\alpha/2}) - [1 - \Phi(z_{\alpha/2})] = 0.9970$$

$$\Phi(z_{\alpha/2}) = 0.9985$$

$$\boxed{z_{\alpha/2} = 2.96}$$

$$d) CL = 0.75$$

$$\Phi(z_{\alpha/2}) = 0.875$$

$$\boxed{z_{\alpha/2} = 1.15}$$

7.2 (114.4, 115.6) (114.1, 115.9)

a) $z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = \frac{115.6 - 114.4}{2} = 0.6$, $z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = \frac{115.9 - 114.1}{2} = 0.6$

$\bar{x} = 115.0$

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b) (114.4, 115.6) has CL of 90%

(114.1, 115.9) has CL of 99%.

This because the later has a longer interval, hence a larger critical value, and inferring a bigger confidence interval.

$$7.5 \quad \sigma = 0.75$$

a) If $\bar{x} = 4.85$ $n = 20$. $CL = 95\%$.

$$\Phi(z_{\alpha/2}) = \frac{1.95}{2} = 0.975$$

$$z_{\alpha/2} = 1.96$$

$$CI = \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 4.85 \pm 1.96 \cdot \frac{0.75}{\sqrt{20}}$$

$$= 4.85 \pm 0.3287 \quad | \quad (4.5213, 5.1787)$$

b) If $\bar{x} = 4.56$, $n = 16$, $CL = 98\%$

$$\Phi(z_{\alpha/2}) = \frac{1.98}{2} = 0.99$$

$$z_{\alpha/2} = 2.33$$

$$CI = 4.56 \pm 2.33 \frac{0.75}{\sqrt{16}}$$

$$= 4.56 \pm 0.4369 \quad (4.1231, 4.9969)$$

c) $CL = 0.95$, $w = 0.4$

$$\Phi(z_{\alpha/2}) = 0.975$$

$$z_{\alpha/2} = 1.96$$

$$n = \left(2 \frac{\sigma}{w} \cdot z_{\alpha/2} \right)^2$$

$$n = \left(2 \cdot \frac{0.75}{0.4} \cdot 1.96 \right)^2$$

$$n = 54.0225$$

$$n \geq 55$$

d) $CL = 0.99$, $w = 0.2 \cdot 2 = 0.4$

$$\Phi(z_{\alpha/2}) = 0.995$$

$$z_{\alpha/2} = 2.58$$

$$n = \left(2 \cdot \frac{0.75}{0.4} \cdot 2.58 \right)^2$$

$$n = 93.6058$$

$$n \geq 94$$

$$7.20 \quad n = 4722, \quad \hat{p} = 15\%, \quad CL = 0.99$$

$$\Phi(z_{\alpha/2}) = 0.995$$

$$z_{\alpha/2} = 2.58$$

$$\tilde{p} = \frac{\hat{p} + z_{\alpha/2}^2 / 2n}{1 + z_{\alpha/2}^2 / n} = 0.1505$$

$$z_{\alpha/2} \frac{\sqrt{\hat{p}\hat{q}(n + z_{\alpha/2}^2 / 4n^2)}}{1 + z_{\alpha/2}^2 / n} = 0.0134$$

$$CI: (0.1371, 0.1639)$$

$$7.49(c) \quad n = 18 \quad df = 17$$

$$\bar{x} = 38.6611$$

$$s = 8.4732$$

$$98\% \text{ CI}, \alpha = 0.02$$

$$t_{0.01, 17} = 2.567$$

The interval is

$$38.6611 \pm 2.567 \cdot \frac{8.47}{\sqrt{18}} = \boxed{(33.5363, 43.7859)}$$

7.50 $n=5$, 95% CI (229.764, 233.504).

$$\bar{X} = \frac{229.764 + 233.504}{2} = 231.634$$

$$\frac{W}{2} = 233.504 - 229.764 = 1.87$$

$$t_{0.25, 4} = 2.776$$

$$1.87 = 2.776 \cdot \frac{S}{\sqrt{5}}$$

$$S = 1.5063$$

For 99% CI, $\alpha = 0.01$

$$t_{0.005, 4} = 4.604$$

the interval is $231.634 \pm 4.604 \frac{1.5063}{\sqrt{5}} = (228.533, 234.735)$