8.39 $n = 150$, $x = 82$ $p_0 = 40\%$. $\alpha = 0.01$

$H_0 : P = 40\%$.
$H_a : P \neq 40\%$.

$z_{\alpha/2} = 2.5758$

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{82/150 - 0.4}{\sqrt{0.4 \cdot 0.6 / 150}} = 3.6667 > z_{\alpha/2}.$$

$\boxed{\text{reject } H_0.}$

If $\alpha = 0.05$

$z_{\alpha/2} = 1.96 < z$. still reject H_0.

14.7 $H_0: P_1 = 0.25, P_2 = 0.25, P_3 = 0.25, P_4 = 0.25$

H_a: otherwise. $\alpha = 0.01$

$1361/4 = 340.25$

$$\chi^2 = \frac{(328-340.25)^2 + (334-340.25)^2 + (372-340.25)^2 + (327-340.25)^2}{340.25}$$

$= 4.0345 \approx 4.03$

$\chi^2_{0.01,3} = 11.344 > 4.03$. Do not reject Ho.

P-value $= P(\chi^2_3 > 4.03) > 0.1 > \alpha$.

Do not reject Ho

14.17

$H_0: \hat{P}_i = \dfrac{\hat{\lambda}^{k_i} e^{-\hat{\lambda}}}{k_i!}$ Assume a test at $\alpha = 0.01$

H_a: Otherwise.

$16 + 2 \cdot 16 + 3 \cdot 18 + 4 \cdot 15 + 5 \cdot 9 + 6 \cdot 6 + 7 \cdot 5 + 8 \cdot 3 + 9 \cdot 4 + 10 \cdot 3 + 11 \cdot 0 + 12$

$= 380$.

$\hat{\lambda} = \hat{\mu} = \dfrac{380}{120} = 3.167$

$\dfrac{3.167^0 \cdot e^{-3.167}}{0!}$

i	0	1	2	3	4	5	6	≥ 7
O	24	16	16	18	15	9	6	16
$\hat{p}$	0.0421	0.1335	0.2113	0.2230	0.1766	0.1118	0.0590	0.0426
$\hat{E}$	5.0573	16.0147	25.3565	26.7652	21.1892	13.4198	7.0827	5.1006

$\chi^2 = \sum_{i=0}^{7} \dfrac{(O_i - E_i)^2}{E_i} = 103.9959$. $k = 8,\ d = 1$

$\chi^2_{0.01, 6} = 16.812 < \chi^2$

Reject H_0

The distribution is not poisson

14.19 $\quad P(H)=\theta_1 \quad P(L)=\theta_2 \quad P(N)=1-\theta_1-\theta_2$

$P_1=\theta_1^2 \quad P_2=\theta_2^2 \quad P_3=(1-\theta_1-\theta_2)^2 \quad P_4=2\theta_1\theta_2$

$P_5=2\theta_1(1-\theta_1-\theta_2) \quad P_6=2\theta_2(1-\theta_1-\theta_2)$

$n_1=49. \quad n_2=26 \quad n_3=14 \quad n_4=20 \quad n_5=53 \quad n_6=38 \quad n=200$

$$f=[\pi_1(\theta_1,\theta_2)]^{n_1}\cdots[\pi_6(\theta_1,\theta_2)]^{n_6}$$

$$=\theta_1^{2n_1}\cdot\theta_2^{2n_2}\cdot(1-\theta_1-\theta_2)^{2n_3}\cdot(2\theta_1\theta_2)^{n_4}\cdot[2\theta_1(1-\theta_1-\theta_2)]^{n_5}\cdot[2\theta_2(1-\theta_1-\theta_2)]^{n_6}$$

$$\ln(f)=2n_1\ln(\theta_1)+2n_2\ln(\theta_2)+2n_3\ln(1-\theta_1-\theta_2)+n_4\ln(2\theta_1\theta_2)+$$

$$+n_5\ln(2\theta_1(1-\theta_1-\theta_2))+n_6\ln(2\theta_2(1-\theta_1-\theta_2))$$

$1-\theta_1-\theta_2-\theta_1$

$$\frac{\partial}{\partial\theta_1}\ln(f)=\frac{2n_1}{\theta_1}+\frac{2n_3}{1-\theta_1-\theta_2}\cdot(-1)+\frac{n_4}{\cancel{2\theta_1\theta_2}}\cdot\cancel{2\theta_2}+\frac{n_5}{\cancel{2}\theta_1(1-\theta_1-\theta_2)}\cdot(\cancel{2}-\cancel{4}\theta_1-\cancel{2}\theta_2)$$

$$+\frac{n_6}{\cancel{2\theta_2}(1-\theta_1-\theta_2)}\cdot(-2\cancel{\theta_2})$$

$n_5\cdot\frac{1}{\theta_1}+\frac{-1}{1-\theta_1-\theta_2}$

$$=\frac{2n_1+n_4+n_5}{\theta_1}+\frac{-2n_3-n_5-n_6}{1-\theta_1-\theta_2}=0.$$

$$\underbrace{\left(\frac{2n_1+n_4+n_5}{\theta_1}=\frac{2n_3+n_5+n_6}{1-\theta_1-\theta_2}\right)} \qquad \frac{171}{\theta_1}=\frac{119}{1-\theta_1-\theta_2}$$

$$\frac{\partial}{\partial\theta_2}\ln(f)=\frac{2n_2}{\theta_2}+\frac{2n_3}{1-\theta_1-\theta_2}\cdot(-1)+\frac{n_4}{\cancel{2\theta_1}\theta_2}\cdot\cancel{2\theta_1}+\frac{n_5}{\cancel{2\theta_1}(1-\theta_1-\theta_2)}\cdot(-\cancel{2\theta_1})$$

$$+\frac{n_6}{\cancel{2}\theta_2(1-\theta_1-\theta_2)}\cdot\cancel{2}(1-\theta_1-2\theta_2)$$

$$=\frac{2n_2+n_4+n_6}{\theta_2}+\frac{-2n_3-n_5-n_6}{1-\theta_1-\theta_2}=0$$

$$\underbrace{\left(\frac{2n_2+n_4+n_6}{\theta_2}=\frac{2n_3+n_5+n_6}{1-\theta_1-\theta_2}\right)} \qquad \frac{110}{\theta_2}=\frac{119}{1-\theta_1-\theta_2}$$

$$\frac{\theta_1}{k_1} = \frac{1-\theta_1-\theta_2}{k_3} \qquad k_1 = 2n_1+n_4+n_5 = 171$$

$$\frac{\theta_2}{k_2} = \frac{1-\theta_1-\theta_2}{k_3} \qquad k_2 = 2n_2+n_4+n_6 = 110$$

$$k_3 = 2n_3+n_5+n_6 = 119$$

$$\hat{\theta}_1 = \frac{171}{2n} = \frac{171}{400} = 0.4275$$

$$\hat{\theta}_2 = \frac{110}{400} = 0.2750$$

$\hat{p}$	0.1828	0.0756	0.0885	0.2351	0.2544	0.1636
E	36.5512	15.1250	17.7012	47.0250	50.8725	32.7250

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 29.3033$$

Assume $\alpha = 0.01$ $\quad k = 6,\ d = 2 \quad df = 6-1-2 = 3$

$\chi^2_{0.01,3} = 11.3449 < \chi^2$ reject H_0

9.50 a) $p_1 = \frac{35}{80}$. $p_2 = \frac{66}{80}$ $p = \frac{35+66}{160}$

$H_0 : p_1 = p_2$

$H_a = p_1 \neq p_2$ $\alpha = 0.01$

a)

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) \cdot \left(\frac{2}{80}\right)}} = -5.0797 \qquad |z| > z_{\alpha/2}$$

P-value = $P(|z| > 5.0797) \approx 0 < \alpha$. <u>reject H_0</u>

14.26

Observed

Treatment	S Number of Fruits Matured	f Number of Fruits Aborted	
Control	141	206	347
Two leaves removed	28	69	97
Four leaves removed	25	73	98
Six leaves removed	24	78	102
Eight leaves removed	20	82	102
	238	508	746.

$H_0 : p_1 = p_2 = \cdots = p_5$

H_a: otherwise.

$\alpha = 0.01$

$$\hat{p} = \frac{238}{238+508} = 0.319 \qquad 1-\hat{p} = 0.681$$

Expected table:

S	f
110.7051	236.2949
30.9464	66.0536
31.2654	66.7346
32.5416	69.4584
32.5416	69.4584.

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 24.8206$$

$$\chi_{0.01,4} = 13.2767 \quad < \chi^2$$

reject H_0.

14.32

		Usage Level			
	$i \backslash j$	Never (1)	Rarely (2)	Frequently (3)	$n_{i\cdot}$
Political Views	Liberal (1)	479	173	119	771
	Conservative (2)	214	47	15	276
	Other (3)	172	45	85	302
	$n_{\cdot j}$	865	265	219	

$n = 1349.$

$H_0: p_{ij} = p_{i\cdot}\, p_{\cdot j}$, $i = 1,2,3$. $j = 1,2,3$. $\alpha = 0.01$ $df = 2 \cdot 2 = 4$

H_a: Otherwise.

$i /$

$\hat{P}_{1\cdot} = \frac{771}{1349} = 0.5715$

$\hat{P}_{2\cdot} = \frac{276}{1349} = 0.2046$

$\hat{P}_{3\cdot} = \frac{302}{1349} = 0.2239$

$j /$

$\hat{P}_{\cdot 1} = \frac{865}{1349} = 0.6412$

$\hat{P}_{\cdot 2} = \frac{265}{1349} = 0.1964$

$\hat{P}_{\cdot 3} = \frac{85}{1349} = 0.1623$

Expected table:

E = usage level.

$$\begin{array}{cc} & \\ \text{Political view} & \begin{pmatrix} 494.3773 & 151.4566 & 125.1660 \\ 176.9755 & 54.2179 & 44.8065 \\ 193.6471 & 59.3254 & 49.0274 \end{pmatrix} \end{array}$$

$$\chi^2 = \sum \frac{(n_{ij} - \hat{e}_{ij})^2}{\hat{e}_{ij}}$$

$= 64.6542$

$\chi^2_{0.01,4} = 13.2767 < 64.5642$

Reject H_0.

They are not independent.

14.36

	1	2	3	$\hat{P}_{i\cdot}$
1	0.13	0.19	0.28	0.6
2	0.07	0.11	0.22	0.4
$\hat{P}_{\cdot j}$	0.2	0.3	0.5	1

$n = 100$. $\alpha = 0.1$.

$df = 1 \cdot 2 = 2$.

a)

H_0: $P_{ij} = P_{i\cdot} \cdot P_{\cdot j}$

H_a: otherwise.

$n \cdot P_{ij} - n \cdot \hat{P}_{ij}$

Observed table:

n_{ij}	1	2	3
1	13	19	28
2	7	11	22.

$$\chi^2 = \sum \frac{(n_{ij} - \hat{e}_{ij})^2}{\hat{e}_{ij}}$$

$= 0.6806$.

Expected table:

$\hat{e}_{ij}$	1	2	3
1	12	18	30
2	8	12	20.

$\chi^2_{0.1, 2} = 4.6052 > 0.6806$

$\boxed{\text{Do not reject } H_0}$

b) $n = 1000 = 100 \cdot 10$.

$\chi^2 = 10 \cdot 0.6806 = 6.806 > 4.6052$. $\boxed{\text{Reject } H_0}$

c) $\frac{4.6052}{0.6806} = 6.7664$

$n_{min} = 6.7664 \cdot 100 = 676.64 \approx \boxed{677}$