8.32 $n = 115$, $\bar{x} = 11.3$, $s = 6.43$ $\mu_0 = 15$

$H_0 : \mu = 15$?

$H_a : \mu < \mu_0$

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{11.3 - 15}{6.43/\sqrt{115}}$$

$$= -6.1708$$

using a test with significance level of $\alpha = 0.05$, $z_\alpha = 1.645$

$z < -z_\alpha$

$-6.1708 < -1.645$

Thus we reject Ho, and Ha can be assumed.

The average daily zinc intake falls below recommended allowance.

8.51 Use table A-8

a) Upper-tailed test, $df = 8$, $t = 2.0$

P-value = $\boxed{0.040}$

b) Lower-tailed test, $df = 11$, $t = -2.4$

P-value = $\boxed{0.018}$

c) Two tailed test, $df = 15$. $t = -1.6$

P value = $2 * 0.065 = \boxed{0.13}$

d) Upper-tailed test, $df = 19$, $t = -0.4$

P-value = $1 - 0.347 = \boxed{0.653}$

e) Upper-tailed test, $df = 5$, $t = 5.0$

P-value = $1 - 0.9979 = \boxed{0.0021}$

f) Two-taild test, $df = 40$, $t = -4.8$

P-value = $2 \times 1.1173 \times 10^{-5} = \boxed{2.2316 \times 10^{-5}}$

8.9 $H_0: p=0.5$, $H_a: p \neq 0.5$, $n=25$

a). $R_1 = \{x: x \le 7 \text{ or } x \ge 18\}$, $R_2 = \{x: x \le 8\}$

$R_3 = \{x: x \ge 17\}$

R_1 is more suitable as a rejection region, because the Ha indicated $p \neq 0.5$, which means we need to consider both end of the distribution for testing.

b) Type I: The proportion of people favors company 1 is 0.5, but we rejected it, saying that one company is favored over the other.

Type II: The proportion of people favors company 1 is not 0.5, but we rejected it. Saying that both companies are equally favored.

c) X has a binomial distribution with $p=0.5$, $n=25$.

$$P(\text{type I error}) = P(X \le 7) + P(X \ge 18)$$

$$= B(7, 25, 0.5) + 1 - B(17, 25, 0.5)$$

$$= 0.0216 + 1 - 0.9784$$

$$= 0.0432$$

d) For $p=0.3$, $n=25$

$$P(\text{type II error}) = P(8 \le X \le 17)$$

$$= B(17, 25, 0.3) - B(7, 25, 0.3)$$

$$= \underline{0.4881}$$

For $p = 0.4$

$P(\text{type II error}) = B(17, 25, 0.4) - B(7, 25, 0.4)$

$= 0.8452$

For $p = 0.6$

$P(\text{type II error}) = B(17, 25, 0.6) - B(7, 25, 0.6)$

$= 0.8452$

For $p = 0.7$

$P(\text{type II error}) = B(17, 25, 0.7) - B(7, 25, 0.7)$

$= 0.4881$

e) $x = 6 < 7$,

H_0 is rejected, H_a is favored.

8.11) $n=25$, $\sigma=0.2$ kg.

a). $H_0: \bar{x}=\mu_0=10\text{kg}$.

$H_a: \mu \neq 10\text{ kg}$

b) $R=\{\bar{x}\geq 10.1032 \text{ or } \bar{x}\leq 9.8968\}$. Find type I error.

$\alpha = P(\text{type I error}) = P(\bar{X}\geq 10.1032)+P(\bar{X}\leq 9.8968)$

$$= 1-\Phi\left(\frac{10.1032-10}{0.2/\sqrt{25}}\right)+\Phi\left(\frac{9.8968-10}{0.2/\sqrt{25}}\right)$$

$$= 1-0.9951+0.0049$$

$$= \underline{0.0098}$$

c) $P(\text{type II error}) = P(9.8968\leq \bar{X}\leq 10.1032)$

$$\beta(10.1) = \Phi\left(\frac{10.1032-10.1}{0.2/5}\right)-\Phi\left(\frac{9.8968-10.1}{0.2/5}\right)$$

$$= \underline{0.5319}$$

$$\beta(9.8) = \Phi\left(\frac{10.1032-9.8}{0.04}\right)-\Phi\left(\frac{9.8968-9.8}{0.04}\right)$$

$$= \underline{0.0078}$$

d) $Z=(\bar{X}-10)/(\sigma/\sqrt{n})$, $R=\{\bar{x}\geq 10.1032 \text{ or } \bar{x}<9.8968\}$.

Find c, where $Z\geq c$ or $Z\leq -c$

$$\frac{10.1032-10}{0.04}=2.58, \quad \frac{9.8968-10}{0.04}=-2.58$$

$$c=\underline{2.58}$$

e) $n=10$. $\alpha=0.05$

$0.05 = 1-\Phi(z)+\Phi(z)$

$0.05 = 2\Phi(-Z_{\alpha=0.05})$

$-Z_{\alpha=0.05} = -1.960$

$Z_{\alpha=0.05} = 1.96$

$\frac{\bar{x}-10}{0.2/\sqrt{10}} = 1.96$ $\quad$ $\frac{\bar{x}-10}{0.2/\sqrt{10}} = -1.96$

$\bar{x} = 10.1240$ $\quad$ $\bar{x} = 9.8760$

$R_{\alpha=0.05} = \{\bar{x} \geq 10.1240 \text{ or } \bar{x} \leq 9.8760\}$

f) $\bar{x} = 10.0203 < 10.1240$, $\bar{x} > 9.8760$. out of rejection region

H_0 can not be rejected.

g) $z \geq 2.58$ or $z \leq -2.58$

8.18 $\sigma = 9$, $H_0: \mu_0 = 75$, $H_a: \mu < 75$. $n=25$

a) $\dfrac{72.3-75}{9/\sqrt{25}} = \underline{-1.5}$

b) $\alpha = 0.01$

$0.01 = \Phi(-z_\alpha)$

$-z_\alpha = -2.33$

$z = -1.5 > -2.33 = -z_\alpha$.

<u>H_0 cannot be rejected</u>, test value is outside rejection region.

c) $\Phi(-2.88) = 0.002$.

d) $\beta(70) = 1 - \Phi(-z_\alpha + \dfrac{\mu_0 -}{\sigma/\sqrt{n}})$

$= 1 - \Phi(-2.88 + \dfrac{75-70}{9/5})$

$= 1 - \Phi(-0.1022)$

$\approx 1 - \Phi(-0.10)$

$= 0.5398$

e) $\beta(70) = 0.01$

$0.01 = 1 - \Phi(z_\beta)$

$z_\beta = 2.33$

$n = \left[\dfrac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}\right]^2 = \left[\dfrac{9}{5}(2.33+2.33)\right]^2$

$= 70.36$

$\approx \underline{71}$

f) $z_\alpha = -2.33$. $\mu = 76$., $n=100$

$p(\text{type I error}) = \alpha(76)$

$= (z < -2.33)$

$= (z' < -2.33 + \dfrac{75-76}{9/10})$

$= (z' < -3.44) = \underline{2.8968 \times 10^{-4}} \approx 0$

$\dfrac{x-\mu_0}{\sigma/\sqrt{n}} < -2.33$

$\dfrac{x-\mu}{\sigma/\sqrt{n}} < -2.33 + \dfrac{\mu_0-\mu}{\sigma/\sqrt{n}}$.

f) alternatively,

$$Z_\alpha = -2.33 . \quad \mu = 76 , \quad n' = 100 .$$

$$\bar{x} = \mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} = 75 + (-2.33) \cdot 1.8 = 70.806 .$$

$$Z' = \frac{\bar{x} - \mu'}{\sigma / \sqrt{n'}} = \frac{70.806 - 76}{0.9} = -5.771$$

$\alpha(76) = P(\text{type 1 error})$

$= P(H_0 \text{ is rejected})$

$= P(Z < -2.33 \mid \mu = 75, n = 25)$

$= P(\bar{X} < \bar{x} \mid \mu = 75, n = 25)$

$= P(Z' < -5.771 \mid \mu = 76, n = 100)$

$= \Phi(-5.771)$

$\approx \underline{0}$

I don't know which way is right, but both methods yield close to zero probability.

8.25 $\mu_0 = 5.5$, $n = 16$, $\sigma = 0.3$. $\bar{x} = 5.25$

a). $H_0: \mu_0 = 5.5$, $H_a: \mu_0 \neq 5.5$

$$z = \frac{5.25 - 5.5}{0.3/4} = -3.33$$

using $\alpha = 0.05$ two-tailed test,

$-z_{0.025} = -1.96$

$-3.33 < -1.96$

$z < -z_{\alpha/2}$

In the rejection region.

Thus Ho is rejected, the true average differs from 5.5.

b) $\alpha = 0.01$ $n = 16$, $\mu = 5.6$.

$z_{0.005} = 2.58$.

$\alpha(5.6) = P(\text{Reject } H_0)$

$= P(z \leq -2.58 \cup z \geq 2.58)$

$= P(z \leq -2.58) + 1 - z(\leq 2.58)$

$= P\left(z' \leq -2.58 + \frac{5.5-5.6}{0.3/4}\right) + 1 - P\left(z \leq 2.58 + \frac{5.5-5.6}{0.3/4}\right)$

$= \Phi(-3.9133) + 1 - \Phi(1.2467)$ | $= \Phi(-3.91) + 1 - \Phi(1.25)$

$= 4.5522 \times 10^{-5} + 1 - 0.8937$ | $= 0 + 1 - 0.8944$

$= \underline{0.1063}$ (matlab) | $= \underline{0.1056}$ (table)

c) $\beta = 0.01$, $z_\beta = 2.33$.

$$n = \left[\sigma \frac{(z_{\alpha/2} + z_\beta)}{\mu_0 - \mu'}\right]^2 = \left[0.3 \frac{(2.58 + 2.33)}{-0.1}\right]^2 = 216.97 \approx \underline{217}$$