Appendix A 2)

Intersection:

a) y axis , (0,1,0)

b) line through (1,1,1) , the line is on the plane.

c) zero vector

d) zero vector

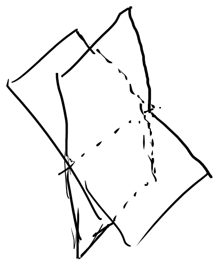
Sum:

a) $\mathbb{R}^3$

b) plane through (1,2,0) & (0,1,1)

c) $\mathbb{R}^3$

d) $\mathbb{R}^3$



2.5 2)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Let b be the linear combination of columns of A

$Ax = b$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} x_3 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$x_1 - x_2 = b_1$
$x_2 - x_3 = b_2$
$x_1 - x_3 = b_3$

$$\underline{(x_1 - x_2) + (x_2 - x_3) - (x_1 - x_3) = 0 = b_1 + b_2 - b_3}$$

shown from columns

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & b_1 \\ 0 & 1 & -1 & b_2 \\ 1 & 0 & -1 & b_3 \end{array}\right] \xRightarrow{r_1 + r_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & b_1 + b_2 \\ 0 & 1 & -1 & b_2 \\ 1 & 0 & -1 & b_3 \end{array}\right] \xRightarrow{r_1 - r_3} \left[\begin{array}{ccc|c} 0 & 0 & 0 & b_1 + b_2 - b_3 \\ 0 & 1 & -1 & b_2 \\ 1 & 0 & -1 & b_3 \end{array}\right]$$

$$\Rightarrow \underline{b_1 + b_2 - b_3 = 0}$$

shown from rows

This means the potential difference in the loop is zero

25 4)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

It's quite evident that A^TA is symmetric. $A_{ji} = A_{ij}$

$$\text{reduce}\left(\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\right) \Rightarrow \begin{bmatrix} 0 & 3 & -3 \\ -1 & 2 & -1 \\ 0 & -3 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 2 & -1 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{bmatrix},$$ rank $= 2 < 3$ singular.

$$\Rightarrow \begin{bmatrix} -1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_3 = 0$
$x_2 - x_3 = 0$ $\quad \bar{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} x_3$

$N(A^TA) \in \left\{ c\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \middle| c \in \mathbb{R} \right\}$, null space of A^TA span all multiples of $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$ $2 \times 2 - 1 \times 1 = 3 \neq 0$, not singular

2.6 2)

$\rightarrow \leftarrow \rightarrow \leftarrow \rightarrow \leftarrow \downarrow \rightarrow \uparrow \leftarrow \downarrow \rightarrow \uparrow \leftarrow$

It is a reflection

2.6 8) $a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$a_0 (2+3t) + a_1 t (2+3t) \cdots\cdots$

$2a_0 + 3a_0 t + 2a_1 t + 3a_1 t^2 + \cdots\cdots + 3a_3 t^4$

$2a_0 + (3a_0 + 2a_1)t + \cdots\cdots + 3a_3 t^4$

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

2.6 (10) $\ddot{u} = u$ $\quad s^2 - 1 = 0$, $s = \pm 1$

$u = c_1 e^{t} + c_2 e^{-t}$

$u(0) = c_1 + c_2 = x$

$u'(0) = c_1 - c_2 = y$

$$\boxed{\begin{cases} c_1 = \dfrac{x+y}{2} \\ c_2 = \dfrac{x-y}{2} \end{cases}}$$

$u = \frac{x+y}{2} e^{t} + \frac{x-y}{2} e^{-t}$

$u = \frac{e^{t}+e^{-t}}{2} x + \frac{e^{t}-e^{-t}}{2} y$

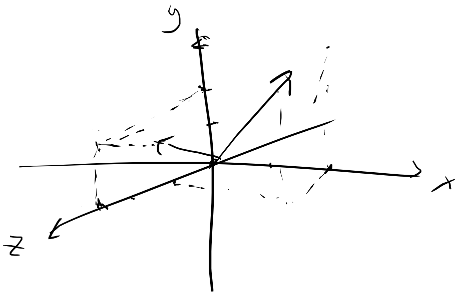
$$\begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix}$$

The matrix is $\boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$

2b 20)

$$\underset{X}{(x_1, x_2, x_3)} \xrightarrow{R} \underset{X'}{(x_2, x_3, x_1)}$$

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



Let the axis of rotation be a,

$$Ra = a$$

$$\begin{bmatrix} a_2 \\ a_3 \\ a_1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \Rightarrow \begin{cases} a_2 = a_1 \\ a_3 = a_2 \\ a_1 = a_3 \end{cases} \quad a_1 = a_2 = a_3, \quad \boxed{a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

$$tr(R) = 1 + 2\cos(\theta) = 0$$

$$\cos(\theta) = -\frac{1}{2}$$

$$\boxed{\theta = \frac{2}{3}\pi} = 120^\circ$$