hw3

March 5, 2021

[1]:
```
%config IPCompleter.use_jedi = False
import numpy as np
from scipy import stats
import matplotlib.pyplot as plt
import cvxpy as cp
import pandas as pd
np.set_printoptions(precision=4)

from pathlib import Path
fig_path = str(Path().absolute())+'/figures/hw3/'
print(fig_path)
data_path = str(Path().absolute())+'/hw3/data/'
print(data_path)
```

```
/home/zpyang/grad_courses/2021_spring/ece595_ml/figures/hw3/
/home/zpyang/grad_courses/2021_spring/ece595_ml/hw3/data/
```

1 Exercise 2

[2]:
```
train_cat = pd.read_csv(data_path+"train_cat.txt", delimiter=',', header=None)
train_cat = np.matrix(train_cat)
train_grass = pd.read_csv(data_path+"train_grass.txt", delimiter=',',␣
 ↪header=None)
train_grass = np.matrix(train_grass)
n_cat = train_cat.shape[1]
n_grass = train_grass.shape[1]
```

1.1 2 a)

$$P_{Y|X}(C_1|x) \gtrless_{C_0}^{C_1} P_{Y|X}(C_0|x)$$

$$P_{X|Y}(x|C_1) \cdot P_Y(C_1) \gtrless_{C_0}^{C_1} P_{X|Y}(x|C_0) \cdot P_Y(C_1)$$

$$\frac{1}{(2\pi)^{d/2}|\Sigma_1|^{1/2}} \exp\{-1/2(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)\}\cdot\pi_1 \gtrless_{C_0}^{C_1} \frac{1}{(2\pi)^{d/2}|\Sigma_0|^{1/2}} \exp\{-1/2(x-\mu_0)^T\Sigma_1^{-1}(x-\mu_0)\}\cdot\pi_0$$

taking $\log(\cdot)$ on both sides

$$-\frac{d}{2}\log(2\pi)-\frac{1}{2}\log|\Sigma_1|-\frac{1}{2}(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)+\log(\pi_1) \gtrless_{C_0}^{C_1} -\frac{d}{2}\log(2\pi)-\frac{1}{2}\log|\Sigma_0|-\frac{1}{2}(x-\mu_0)^T\Sigma_0^{-1}(x-\mu_0)+\log(\pi$$

$\frac{d}{2}\log{(2\pi)}$ terms cancel out on both sides

$$\boxed{-\frac{1}{2}(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)+\log{(\pi_1)}-\frac{1}{2}\log{|\Sigma_1|}\gtrless_{C_0}^{C_1}-\frac{1}{2}(x-\mu_0)^T\Sigma_0^{-1}(x-\mu_0)+\log{(\pi_0)}-\frac{1}{2}\log{|\Sigma_0|}}$$

[3]:
```
K1 = train_cat.shape[1]
K0 = train_grass.shape[1]

pi1 = K1/(K1+K0)
pi0 = K0/(K1+K0)

mu1 = np.mean(train_cat,axis=1)
mu0 = np.mean(train_grass, axis=1)

SIG1 = 1/n_cat*sum([(train_cat[:,i]-mu1)*(train_cat[:,i]-mu1).T for i in␣
 ↪range(n_cat)])
SIG0 = 1/n_grass*sum([(train_grass[:,i]-mu0)*(train_grass[:,i]-mu0).T for i in␣
 ↪range(n_grass)])

print('mu1[0:2]:',mu1[0:2].T)
print('mu0[0:2]:',mu0[0:2].T)
print('____________________')
print('SIG1[0:2,0:2]:\n',SIG1[0:2,0:2])
print('SIG0[0:2,0:2]:\n',SIG0[0:2,0:2])
print('____________________')
print('pi_1:', pi1)
print('pi_0:', pi0)
```

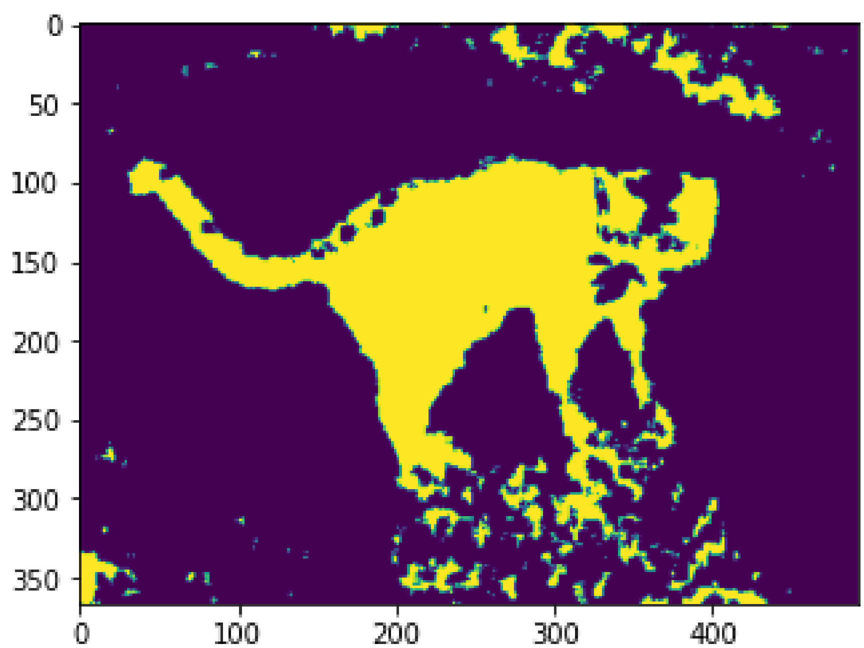
```
mu1[0:2]: [[0.4408 0.4387]]
mu0[0:2]: [[0.4825 0.4864]]
____________________
SIG1[0:2,0:2]:
 [[0.0431 0.0353]
 [0.0353 0.0425]]
SIG0[0:2,0:2]:
 [[0.0645 0.0369]
 [0.0369 0.0662]]
____________________
pi_1: 0.171349288935137
pi_0: 0.828650711064863
```

[4]:
```
invSIG1 = np.linalg.inv(SIG1)
invSIG0 = np.linalg.inv(SIG0)
detSIG1 = np.linalg.det(SIG1)
detSIG0 = np.linalg.det(SIG0)

k1 = np.log(pi1) - 1/2*np.log(detSIG1)
```

```
k0 = np.log(pi0) - 1/2*np.log(detSIG0)
```

[5]:
```
def decision(mu, invSIG, k, x):
    return -1/2*(x-mu).T @ invSIG @ (x-mu) + k

Y = plt.imread(data_path+"cat_grass.jpg")/255
M,N = Y.shape
pred = np.zeros((M-8,N-8))
tau_vec = []
for i in range(M-8):
    for j in range(N-8):
        block = Y[i:i+8, j:j+8]
        x = block.reshape(1,-1).T
        d1 = decision(mu1, invSIG1, k1, x)
        d0 = decision(mu0, invSIG0, k0, x)
        if d1 > d0 :
            pred[i,j] = 1
pred=pred.astype('int32')
plt.imshow(pred)
```

[5]: <matplotlib.image.AxesImage at 0x7f8d9ead5340>



1.2 2 d)

[6]:
```
truth = plt.imread(data_path+"truth.png")
print(np.sum(truth))
truth = truth.astype('int32')
print(np.sum(truth))
truth = truth[0:M-8,0:N-8]

MAE = 1/((M-8)*(N-8))*np.sum(np.abs(pred-truth))
print('MAE',MAE)
```

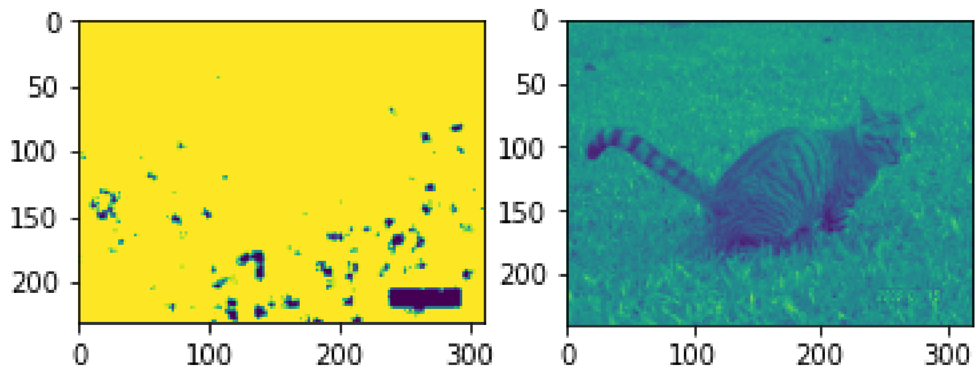
```
31598.99
30073
MAE 0.0897521100551605
```

[7]:
```
TP = np.sum(np.floor(((truth + pred)/2)))
PD_bd = TP/np.sum(truth)

FP = np.sum(pred)-TP
PF_bd = FP/((M-8)*(N-8)-np.sum(truth))
print('PD_bd', PD_bd)
print('PF_bd', PF_bd)
```

```
PD_bd 0.8346357197486117
PF_bd 0.0746423374155265
```

1.3 2 e)

[8]:
```
Y = plt.imread(data_path+"cat_grass_2.jpeg")/255
Y = Y[:,:,2]
M,N = Y.shape
pred_1 = np.zeros((M-8,N-8))
for i in range(M-8):
    for j in range(N-8):
        block = Y[i:i+8, j:j+8]
        x = block.reshape(1,-1).T
        if decision(mu1, invSIG1, k1, x) > decision(mu0, invSIG0, k0, x):
            pred_1[i,j] = 1
plt.subplot(1,2,1)
plt.imshow(pred_1)
plt.subplot(1,2,2)
plt.imshow(Y[:,:])
plt.show()
```



2

3 Exercise 3

3.1 3 a)

$$P_{Y|X}(C_1|x) \gtrless P_{Y|X}(C_0|x)$$

$$P_{X|Y}(x|C_1)\pi_1 \gtrless P_{X|Y}(x|C_0)\pi_0$$

$$\frac{P_{X|Y}(x|C_1)}{P_{X|Y}(x|C_0)} \gtrless \frac{\pi_0}{\pi_1} = \tau$$

[9]:
```
tau = pi0/pi1
print('tau:',tau)
np.log(tau)
```

```
tau: 4.836032388663967
```

[9]:
```
1.5760946301378869
```

3.2 3 b)

ass

[23]:
```
from tqdm.notebook import tqdm

Y = plt.imread(data_path+"cat_grass.jpg")/255
M,N = Y.shape

c = 1/2*(np.log(detSIG0)-np.log(detSIG1))
```

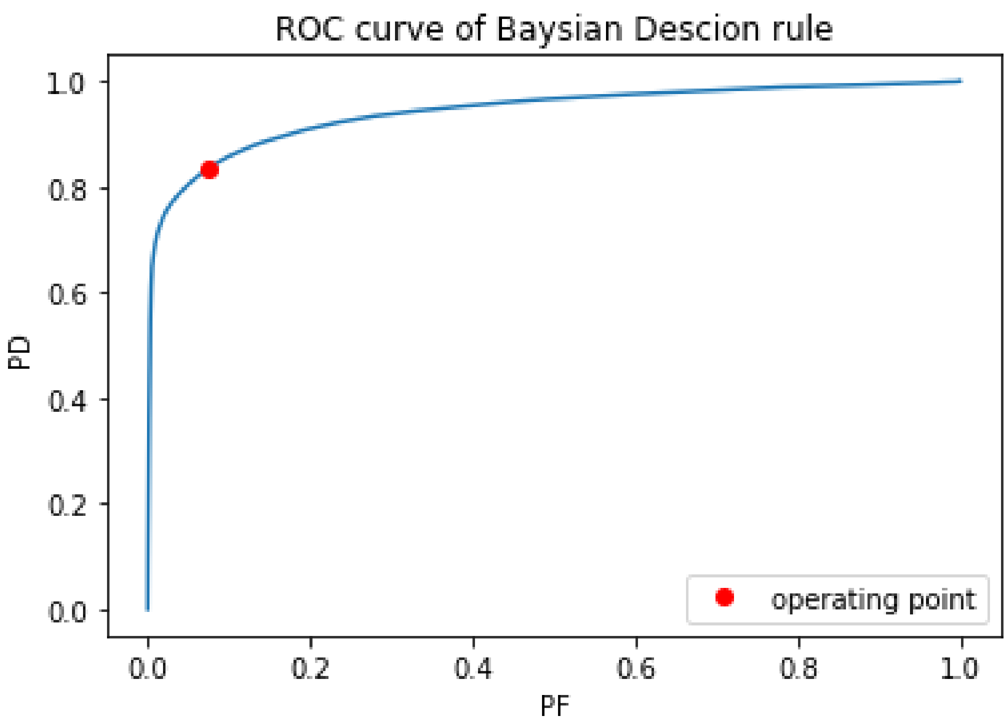
```
def likelihood_ratio(x, mu0, mu1, invSIG0, invSIG1, c):
    return (-0.5*(x-mu1).T @ invSIG1 @ (x-mu1) + 0.5*(x-mu0).T @ invSIG0 @␣
 ↪(x-mu0)) + c

tau_calc_mat = np.zeros((M-8,N-8))
for i in range(M-8):
    for j in range(N-8):
        block = Y[i:i+8, j:j+8]
        x = block.reshape(1,-1).T
        tau_calc_mat[i,j] = likelihood_ratio(x, mu0, mu1, invSIG0, invSIG1, c)
print("tau_calc_mat done")
```

```
tau_calc_mat done
```

[27]:
```
# 3 c)
n = 1000
tau_vec = np.linspace(-400,50,n)

PD = np.zeros(n)
PF = np.zeros(n)

truth = plt.imread(data_path+"truth.png")
truth = truth[0:M-8,0:N-8]
truth_int = truth.astype('int32')
truth = truth_int.astype('bool')

for k in tqdm(range(n)):
    pred = tau_calc_mat > tau_vec[k]

    TP = np.sum((pred & truth))
    PD[k] = TP/np.sum(truth)

    FP = np.sum(pred)-TP
    PF[k] = FP/((M-8)*(N-8)-np.sum(truth))

plt.plot(PF, PD)
plt.plot(PF_bd, PD_bd, 'ro', label='operating point')
plt.xlabel('PF')
plt.ylabel('PD')
plt.title('ROC curve of Baysian Descion rule')
plt.legend()
```

```
  0%|          | 0/1000 [00:00<?, ?it/s]
```

[27]:
```
<matplotlib.legend.Legend at 0x7f8d9adbd280>
```



[26]:
```
print(np.sum(truth))
print(np.sum(truth_int))
```

```
30073
30073
```

3.3 3 d)

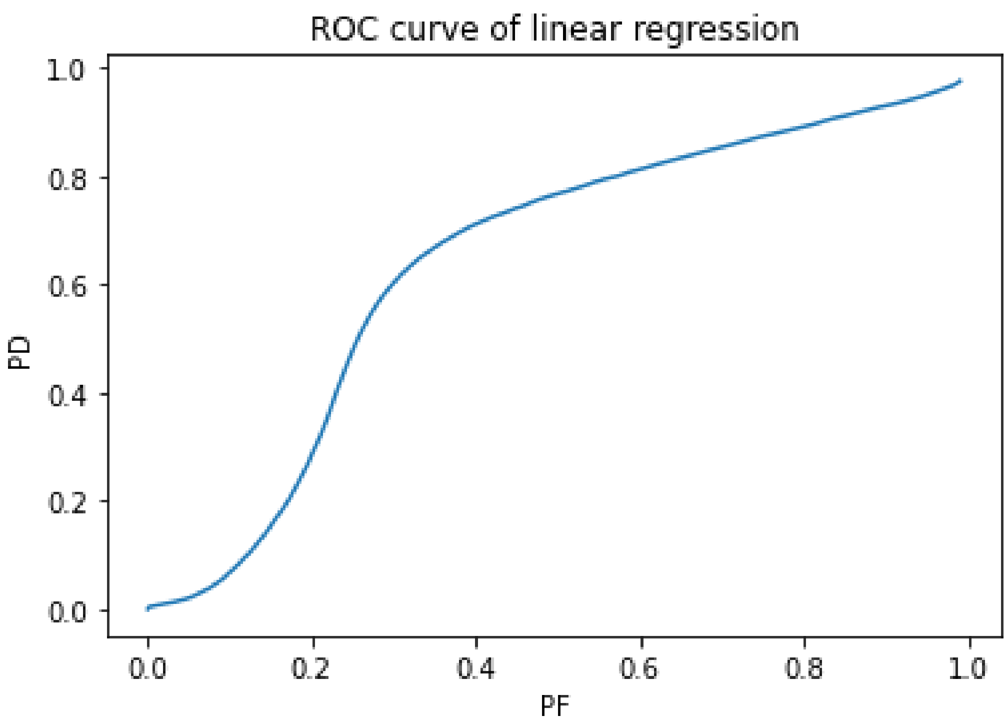
[32]:
```
X1 = train_cat.T
X0 = train_grass.T
A = np.vstack((X1,X0))
b = np.hstack((np.ones(K1),-np.ones(K0)))
theta = np.linalg.lstsq(A,b, rcond=None)[0]
```

[41]:
```
tau_calc_mat = np.zeros((M-8,N-8))
for i in range(M-8):
    for j in range(N-8):
        block = Y[i:i+8, j:j+8]
        x = block.reshape(1,-1).T
        d = theta.T @ x
        tau_calc_mat[i,j] = theta.T @ x

n = 1000
```

```
tau_vec = np.linspace(-1,0,n)
PD_lr = np.zeros(n)
PF_lr = np.zeros(n)
for k in tqdm(range(n)):
    pred = tau_calc_mat > tau_vec[k]

    TP = np.sum((pred & truth))
    PD_lr[k] = TP/np.sum(truth)

    FP = np.sum(pred)-TP
    PF_lr[k] = FP/((M-8)*(N-8)-np.sum(truth))

plt.plot(PF_lr, PD_lr)
plt.xlabel('PF')
plt.ylabel('PD')
plt.title('ROC curve of linear regression')
```

```
  0%|          | 0/1000 [00:00<?, ?it/s]
```

[41]: Text(0.5, 1.0, 'ROC curve of linear regression')



[ ]: