

Final Exam

```
set(groot, 'defaultAxesTickLabelInterpreter','latex');  
set(groot, 'defaultLegendInterpreter','latex');
```

P1

```
e1 = [1 0 0]';  
e2 = [0 1 0]';  
e3 = [0 0 1]';  
T = [3 2 1; 2 3 2; 1 2 3];
```

```
e1'*T*e2
```

```
ans = 2
```

```
e2'*T*e2
```

```
ans = 3
```

```
e3'*T*e2
```

```
ans = 2
```

P2

```
rng(1000);  
u = randn(1,20)
```

```
u = 1×20  
    0.3106   -1.3391    0.3024    1.7215   -0.2158   -1.1695   -0.1961   -1.2406 ...
```

```
rng(2000);  
v = randn(1,20)
```

```
v = 1×20  
    0.3170    0.3342   -0.0107   -0.5568   -0.5770    0.1019   -1.0280    0.2335 ...
```

```
A = @ (n) [exp(-n/50), 1; 2, cos(n/50)];  
B = @ (n) [exp(-n/50)*cos(n/50); 1];  
C = @ (n) [1+exp(-n/50) 2+sin(n/50)];  
D = @ (n) [1+1/2*sin(n/50)];
```

```
x0 = [0;0];  
xh0 = [0;0];
```

```

Q0 = eye(2);

tf = 15;
x = zeros(2,tf);
y = zeros(1,tf);

xh = zeros(2,tf);
Qn = Q0;

xhh = zeros(2,tf);

for n = 1:tf
    An = A(n-1);
    Bn = B(n-1);
    Cn = C(n-1);
    Dn = D(n-1);

    del_n = An*Qn*Cn'*inv(Cn*Qn*Cn' + Dn*Dn');

    x(:,n+1) = An*x(:,n) + Bn*u(n);
    y(:,n) = Cn*x(:,n) + Dn*v(n);

    xh(:,n+1) = An*xh(:,n) + del_n*(y(n) - Cn*xh(:,n));
    xhh(:,n) = xh(:,n) + Qn*Cn'*inv(Cn*Qn*Cn' + Dn*Dn')*(y(n)-Cn*xh(:,n));

    Qn1 = An*Qn*An' + Bn*Bn' - An*Qn*Cn'*inv(Cn*Qn*Cn' + Dn*Dn')*Cn*Qn*An';
    Qn = Qn1;
end

xh(1,9:11)

```

```

ans = 1×3
    -31.5387    -75.9469   -174.4413

```

```
xhh(2,9:11)
```

```

ans = 1×3
    -48.0377   -111.7605   -260.9927

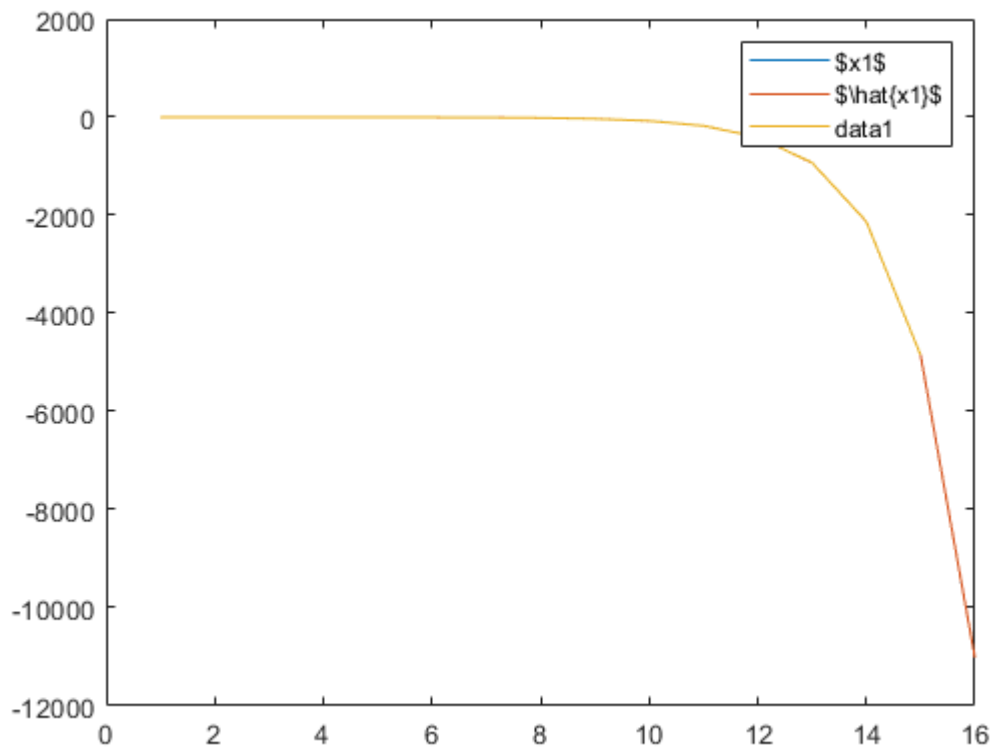
```

```

n_vec = [1:1:tf+1];
figure()
plot(n_vec,x(1,:))
hold on
plot(n_vec, xh(1,:))
legend('$x1$', '$\hat{x1}$')

plot(n_vec(1:tf), xhh(1,:))

```



Warning: Error updating Legend.

String scalar or character vector must have valid interpreter syntax:
 \hat{x}_1

P3

```
clear;
rng(69)
x = randn(1)
```

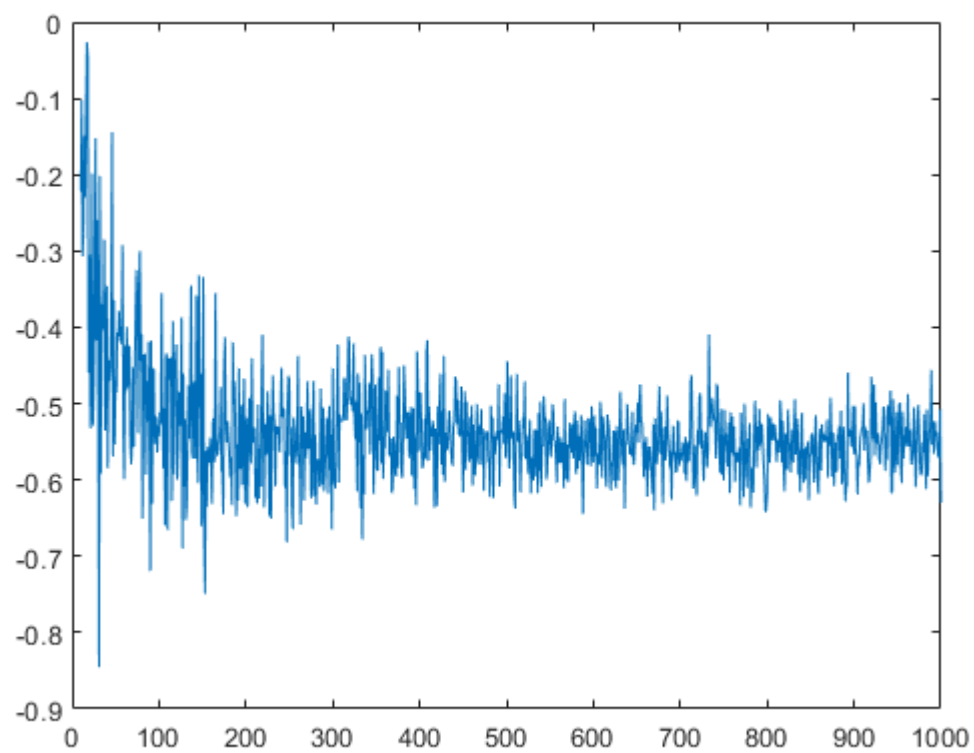
x = -0.5573

```
n = [10:1:1e3];
xh_vec = zeros(1,length(n));
error_vec = zeros(1,length(n));
rng('shuffle')
for i = 1:length(n)
    v = randn(1,i);
    y = x + v;

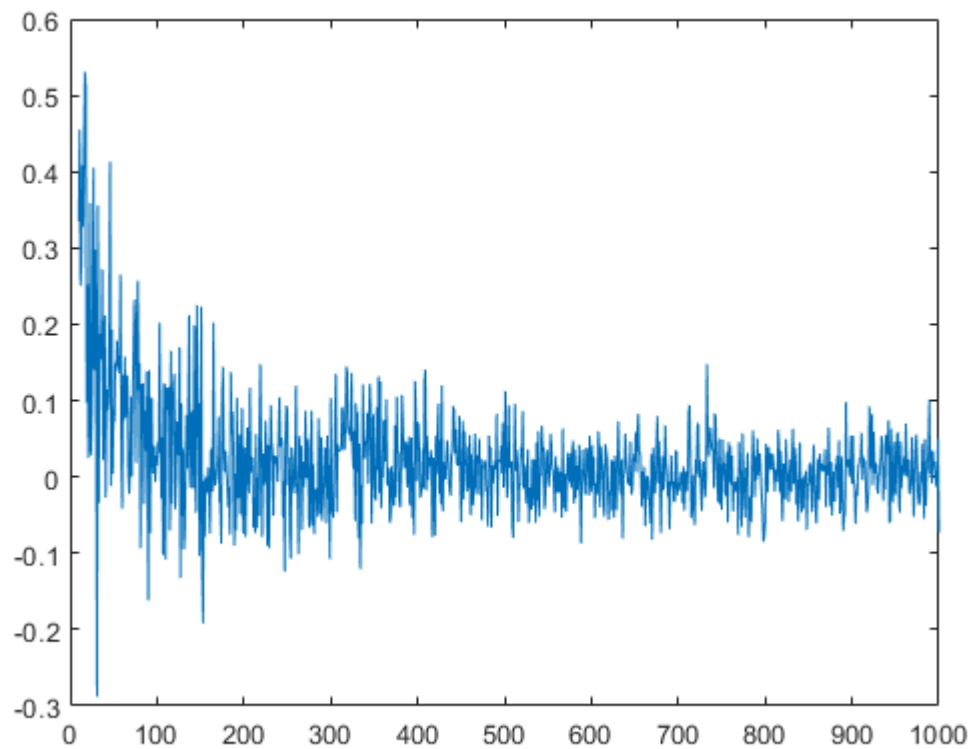
    xh_vec(i) = (1-(n(i))/(1+n(i)))*sum(y);
    error_vec(i) = xh_vec(i)-x;
end

figure()
```

```
plot(n, xh_vec)
```



```
plot(n, error_vec)
```



```
syms n
simplify(1-(n-n^2/(n+1)))
```

ans =

$$\frac{1}{n+1}$$

P4

```
A = [0 1 0 0; 0 -0.2 3 0; 0 0 0 1; 0 -0.5 30 0]
```

A = 4×4

0	1.0000	0	0
0	-0.2000	3.0000	0
0	0	0	1.0000
0	-0.5000	30.0000	0

```
B1 = [0;1;0;5]
```

B1 = 4×1

0
1
0
5

```
B = [0 0;1/4 0;0 0;0 1/10]
```

```
B = 4x2
      0      0
    0.2500    0
      0      0
      0    0.1000
```

```
C = [1 0 0 0;0 0 1 0]
```

```
C = 2x4
      1      0      0      0
      0      0      1      0
```

```
D = [1/4 0;0 1/20]
```

```
D = 2x2
    0.2500    0
      0    0.0500
```

```
invD2 = inv(D*D');
```

```
P = are(A',C'*invD2*C,B*B')
```

```
P = 4x4
    0.0792    0.0505    0.0013    0.0025
    0.0505    0.0771    0.0133    0.0668
    0.0013    0.0133    0.0273    0.1490
    0.0025    0.0668    0.1490    0.8152
```

```
L = P*C'*invD2
```

```
L = 4x2
    1.2671    0.5179
    0.8081    5.3104
    0.0207   10.9191
    0.0400   59.6184
```

```
q = [1, 5, 1, 5]
```

```
q = 1x4
      1      5      1      5
```

```
K = lqr(A,B1,diag(q), 0.5)
```

```
K = 1x4
   -1.4142   -4.4316   18.8017    5.0905
```

```
sim('sys_noisy.slx', 30)
```

```
set(0, 'defaultTextInterpreter', 'latex');
x1 = x.Data(1,:);
x2 = x.Data(2,:);
x3 = x.Data(3,:);
```

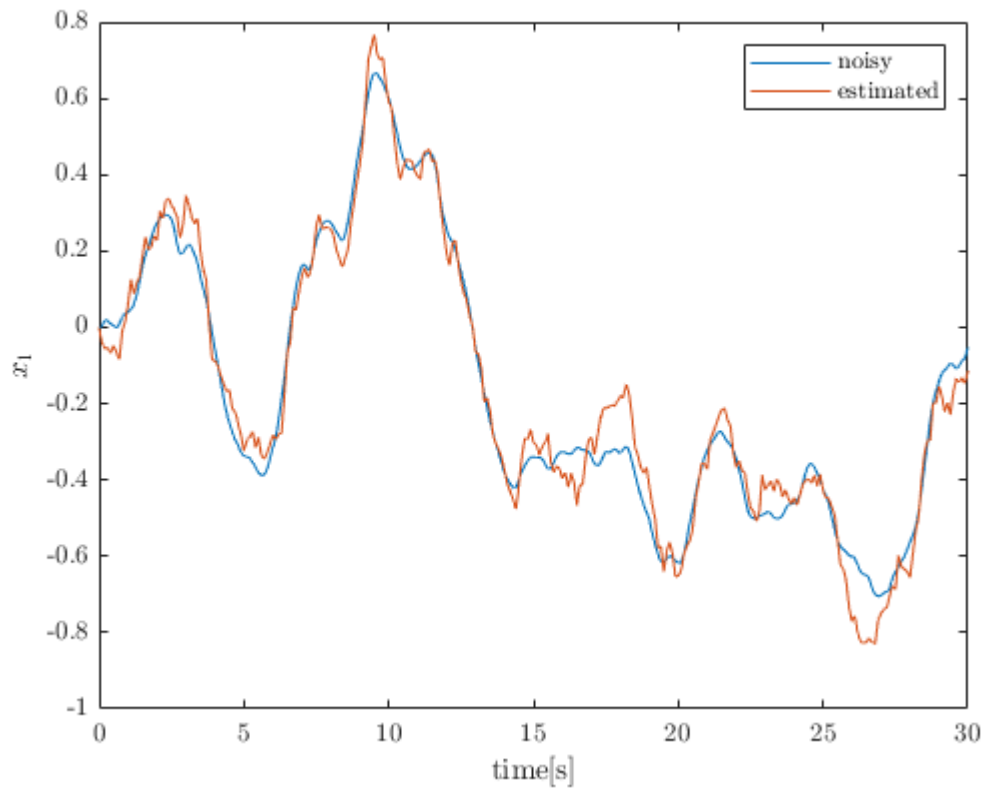
```

x4 = x.Data(4,:);
t = x.Time;

xh1 = xh.Data(1,:);
xh2 = xh.Data(2,:);
xh3 = xh.Data(3,:);
xh4 = xh.Data(4,:);

figure
plot(t,x1)
hold on
plot(t,xh1)
xlabel('time[s]')
ylabel('$x_1$')
legend('noisy','estimated')

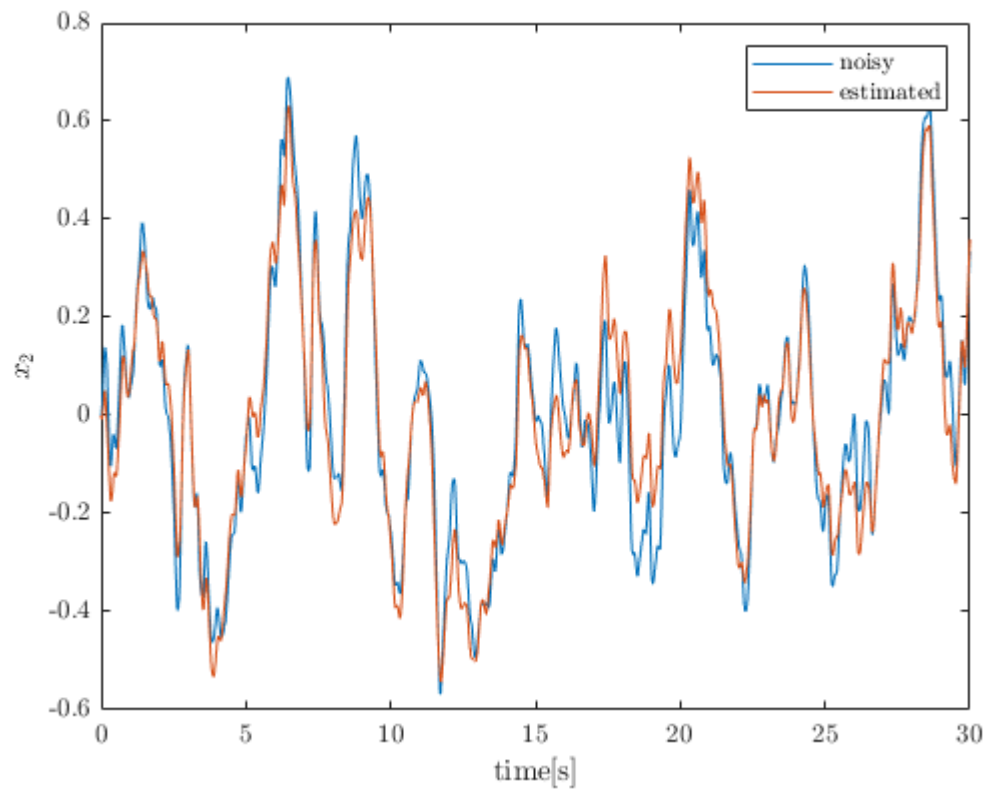
```



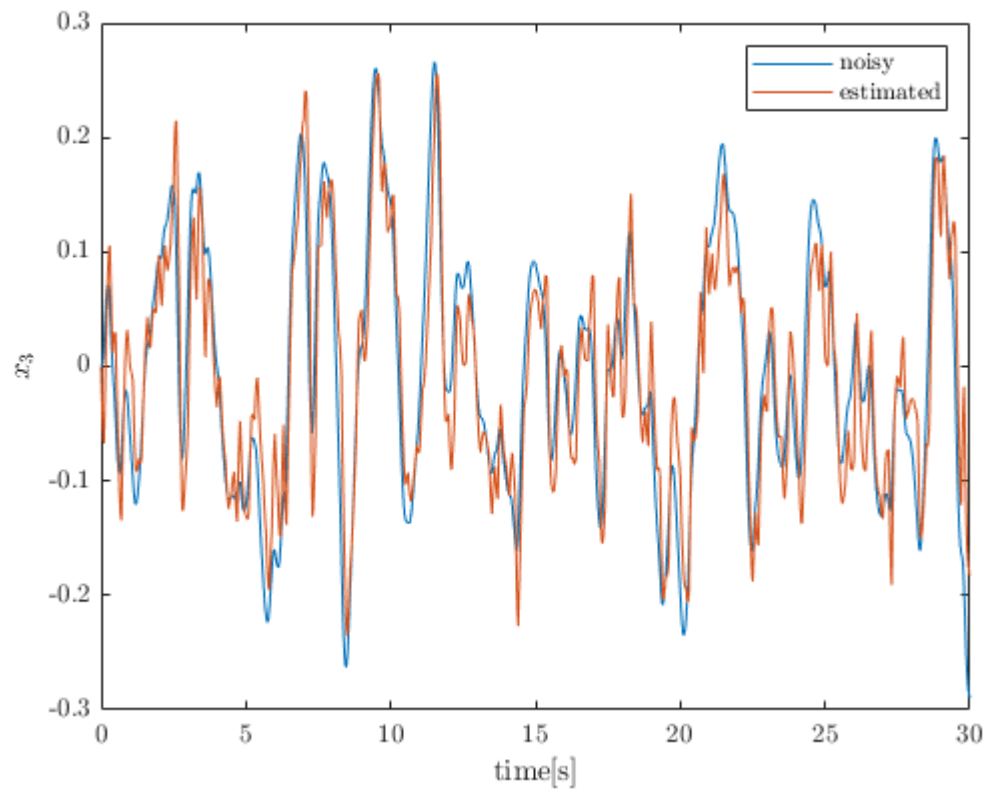
```

figure
plot(t,x2)
hold on
plot(t,xh2)
xlabel('time[s]')
ylabel('$x_2$')
legend('noisy','estimated')

```



```
figure
plot(t,x3)
hold on
plot(t,xh3)
xlabel('time[s]')
ylabel('$x_3$')
legend('noisy','estimated')
```

```
figure
plot(t,x4)
hold on
plot(t,xh4)
xlabel('time[s]')
ylabel('$x_4$')
legend('noisy','estimated')
```

