hw1

February 5, 2021

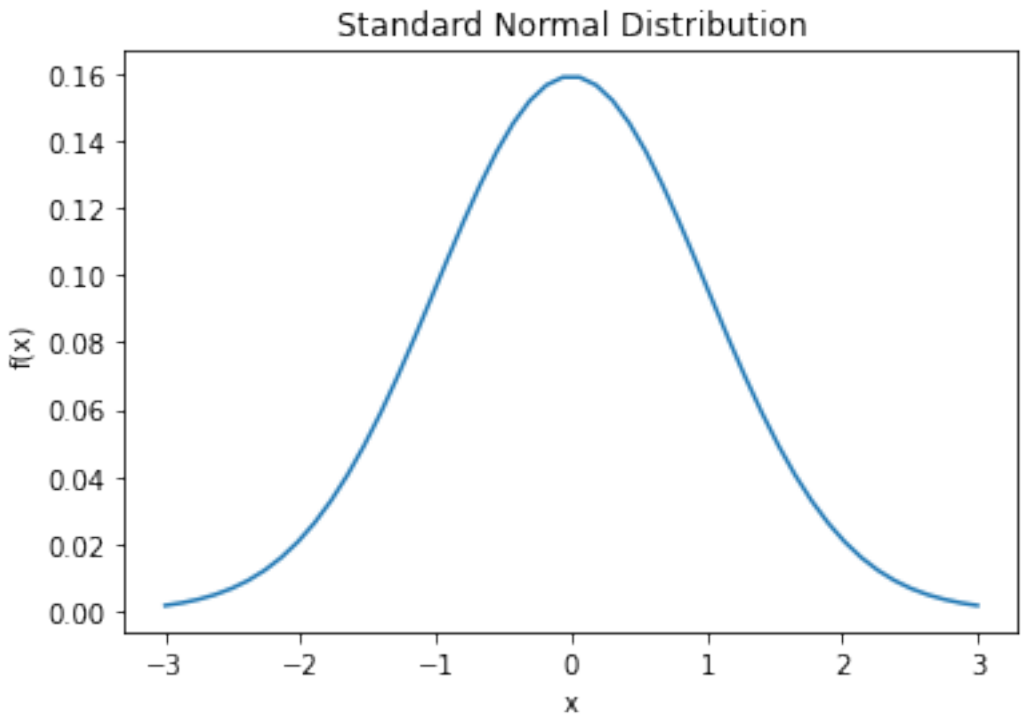
[1]:
```
%config IPCompleter.use_jedi = False
import numpy as np
np.set_printoptions(precision=4)
from scipy import stats
import matplotlib.pyplot as plt
from pathlib import Path

fig_path = str(Path().absolute())+'/figures/hw1/'
print(fig_path)
```

```
/home/zpyang/grad_courses/2021_spring/ece595_ml/figures/hw1/
```

1 Exercise 1

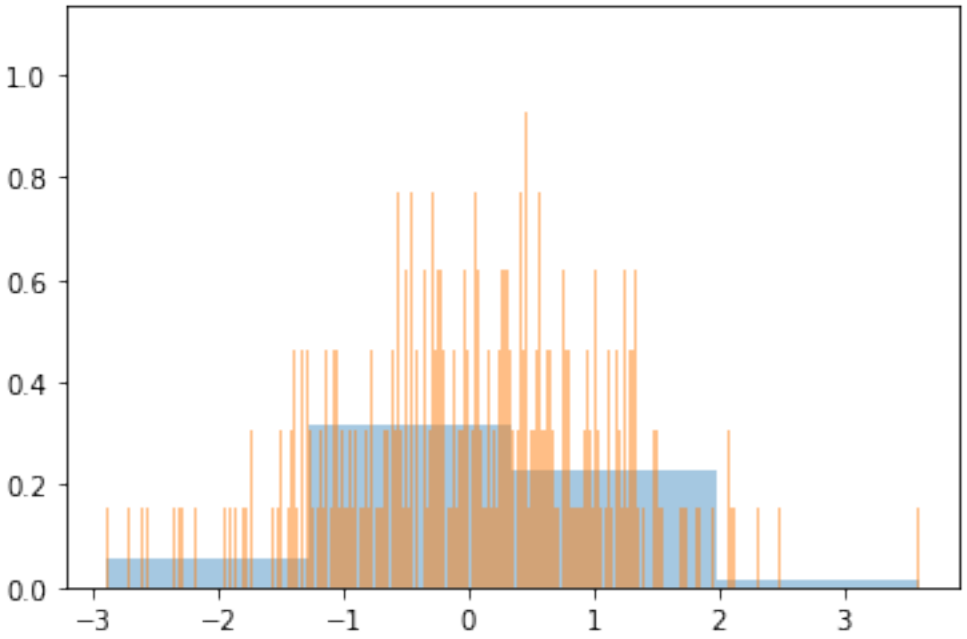
1.1 1 a)

[2]:
```
mu = 0
sigma = 1
f = lambda x:1/(2*np.pi*sigma**2)*np.exp(-(x-mu)**2/(2*sigma**2))
x_vec = np.linspace(-3,3)

plt.figure()
plt.plot(x_vec, f(x_vec))
plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('Standard Normal Distribution')
plt.savefig(fig_path+'hw1_ex1_a')
```



1.2 1 b)

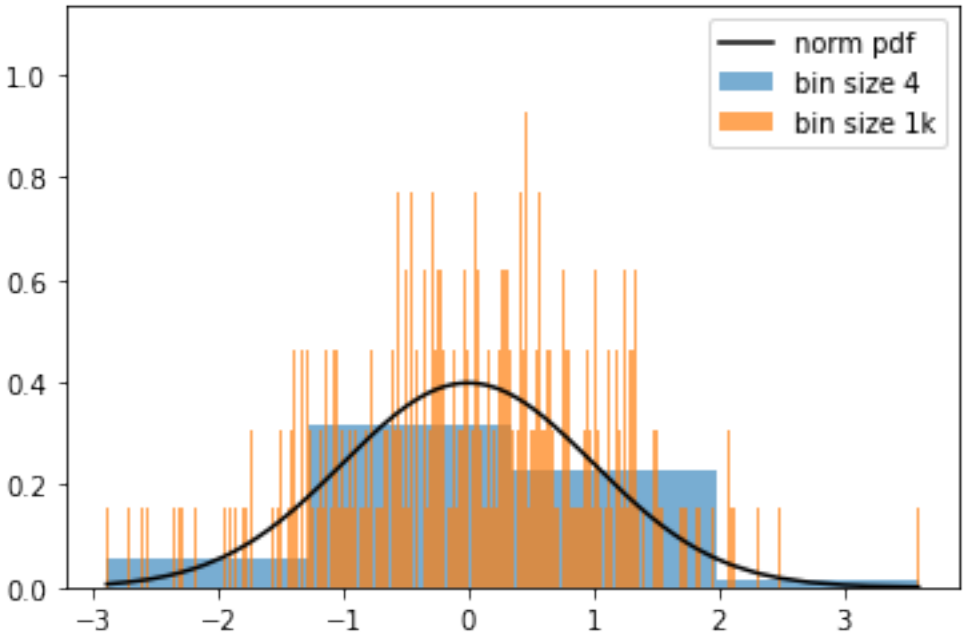
[3]:
```
# i)
samples = np.random.normal(loc=0,scale=1,size=1000)

# ii)
plt.figure()
plt.hist(samples, density=True, bins=4, alpha=0.4);
plt.hist(samples, density=True, bins=1000, alpha=0.5);
```



[4]:
```
# iii)
[mu, sigma] = stats.norm.fit(samples)
print("mu: "+str(mu)+"\nsigma: "+str(sigma))
```

```
mu: 0.07577138510009958
sigma: 1.000040657053501
```

[5]:
```
# iv)
samples = np.sort(samples)
p = stats.norm.pdf(samples)
plt.figure()
plt.hist(samples,density=True, bins=4, alpha=0.6, label='bin size 4')
plt.hist(samples,density=True, bins=1000, alpha=0.7, label='bin size 1k')
plt.plot(samples, p, color='k', label='norm pdf')
plt.legend(loc='best')
plt.savefig(fig_path+'hw1_ex1_b')
```

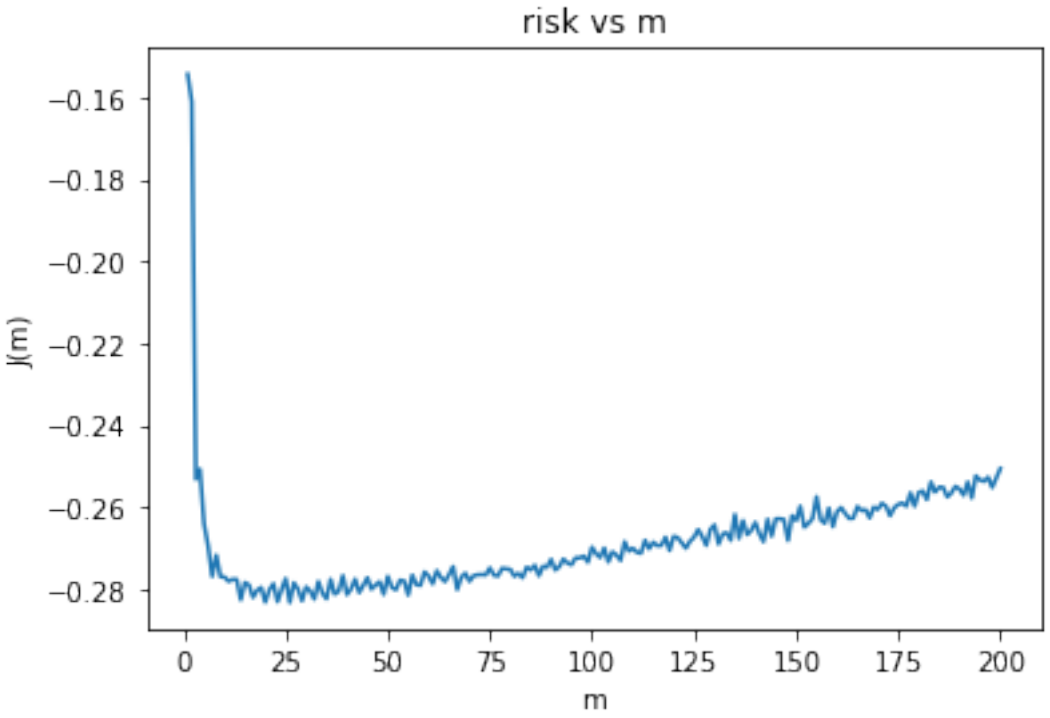


1.3 1 c)

[6]:
```
# i)
n = 1000
s_range = np.array([min(samples),max(samples)])
m_vec = np.linspace(1,200,200)
h = (s_range[1]-s_range[0])/m_vec

# def sum_implicit_prob(m_vec):
#     p_sum_vec = []
#     for mj in m_vec:
#         interval = [s_range[0]+(s_range[1]-s_range[0])/mj*j for j in␣
 ↪range(int(mj)+1)]
#         cdf_vec = stats.norm.cdf(interval)
#         pj_vec = np.array([cdf_vec[i+1]-cdf_vec[i] for i in␣
 ↪range(len(cdf_vec)-1)])
#         pj_sum = sum(pj_vec**2)
#         p_sum_vec.append(pj_sum)
#     return np.array(p_sum_vec)

def sum_implicit_prob(m_vec):
    p_sum_vec = []
    for mj in m_vec:
        hist, bins = np.histogram(samples, bins=int(mj))
        p_sum_vec.append(sum((hist/n)**2))
```

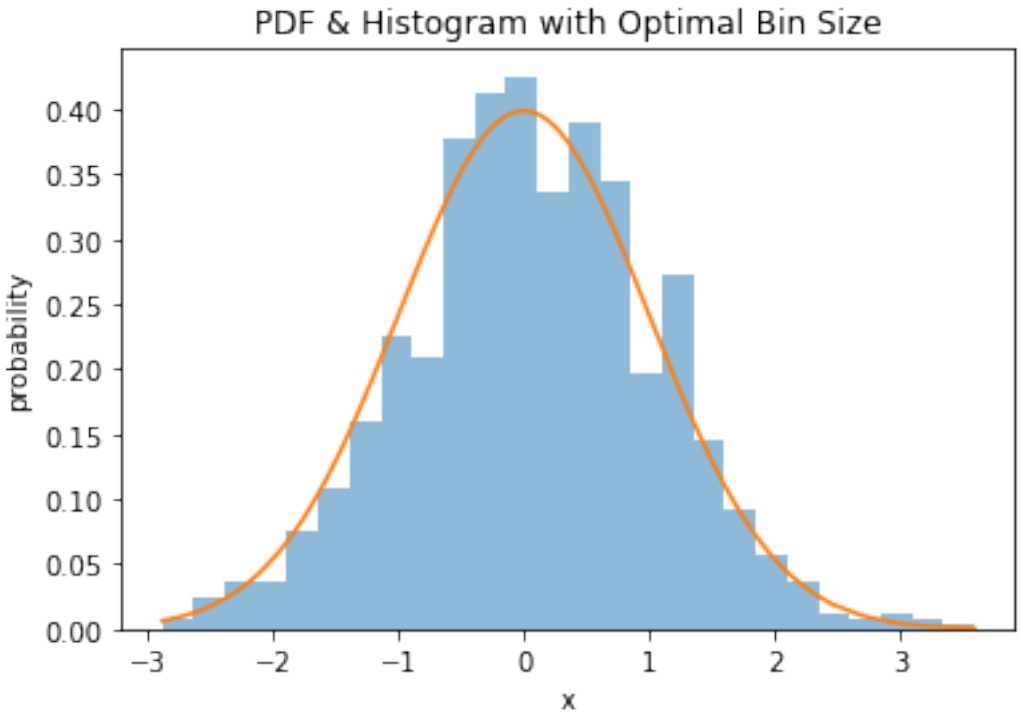
```
    return np.array(p_sum_vec)

J = lambda m: 2/(h*(n-1))-(n+1)/(h*(n-1))*sum_implicit_prob(m)
J_vec = J(m_vec)

m_star = m_vec[np.argmin(J_vec)]

print("m* and J*: ",m_star,min(J_vec))

plt.figure()
plt.plot(m_vec, J_vec)
plt.xlabel('m')
plt.ylabel('J(m)')
plt.title('risk vs m')
plt.savefig(fig_path+'hw1_ex1_ci')
```

```
m* and J*:  26.0 -0.2832622935732081
```



[7]:
```
plt.figure()
plt.hist(samples, density=True, bins=int(m_star), alpha= 0.5)
plt.plot(samples, p)
plt.title("PDF & Histogram with Optimal Bin Size")
plt.xlabel('x')
```

```
plt.ylabel('probability')
plt.savefig(fig_path+'hw1_ex1_ciii')
```

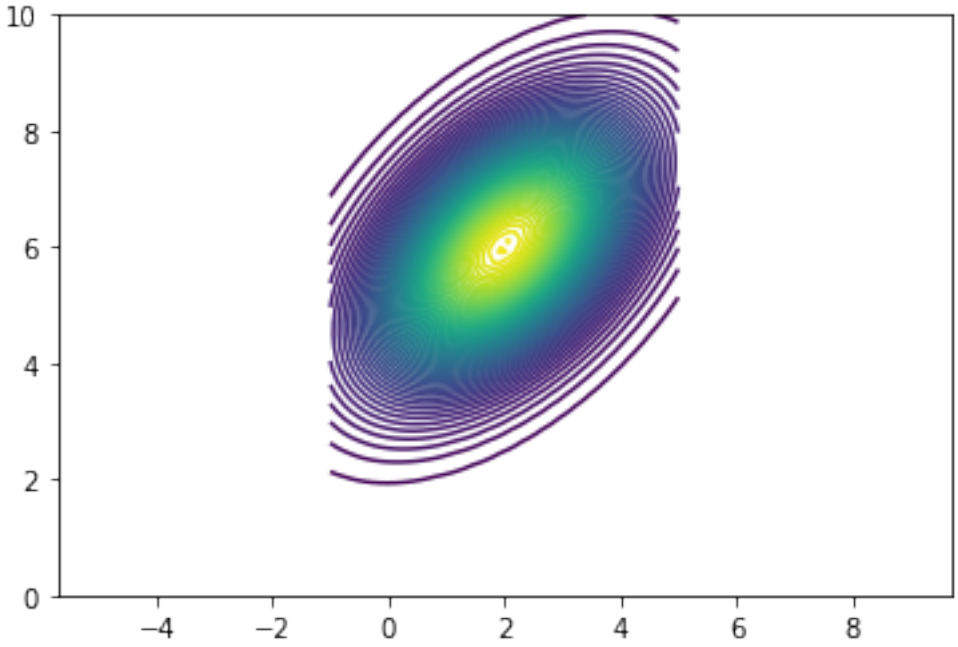


2 Exercise 2

2.1 2 a)

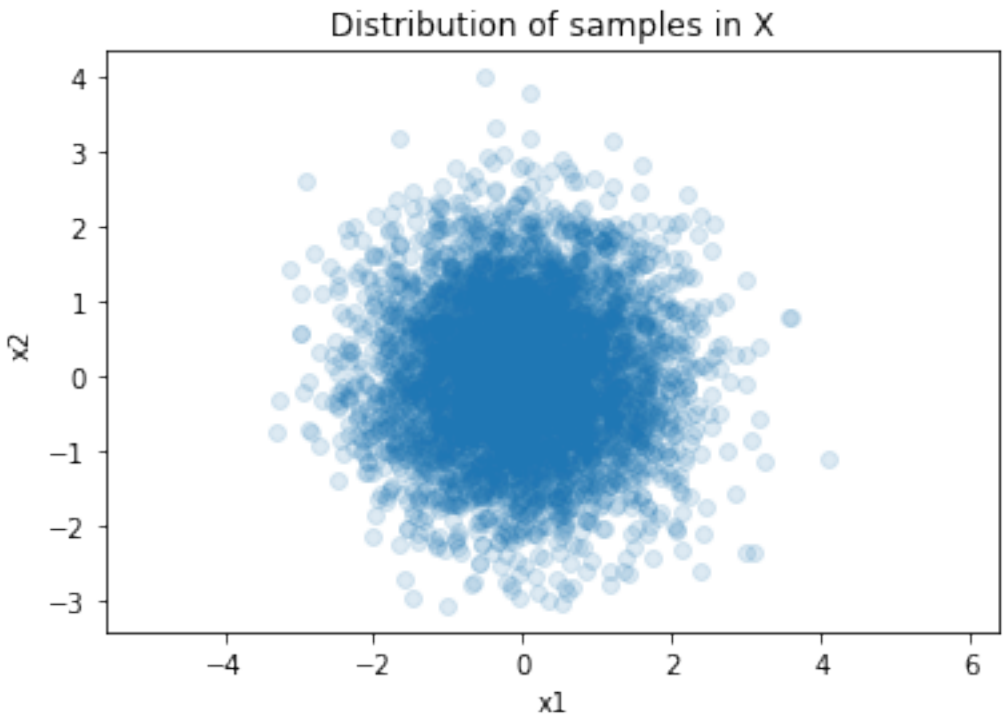
[8]:
```
# ii)
mu = np.array([[2],
               [6]])
sigma = np.array([[2,1],[1,2]])
sigma_inv = np.linalg.inv(sigma)

fx = lambda x: 1/np.sqrt((2*np.pi)**2*np.linalg.det(sigma)) \
        *np.exp(-0.5*(x-mu).T @ sigma_inv @ (x-mu))

x1 = np.linspace(-1,5)
x2 = np.linspace(0,10)
X1, X2 = np.meshgrid(x1,x2)

FX = 1/(2*np.pi*np.sqrt(3))*np.exp(-1/6*(2*X1**2+4*X1-2*X1*X2+2*X2**2-20*X2+56))

plt.figure()
```

```
plt.contour(X1, X2, FX , levels=69)
plt.axis('equal')
plt.savefig(fig_path+'hw1_ex2_aii')
```



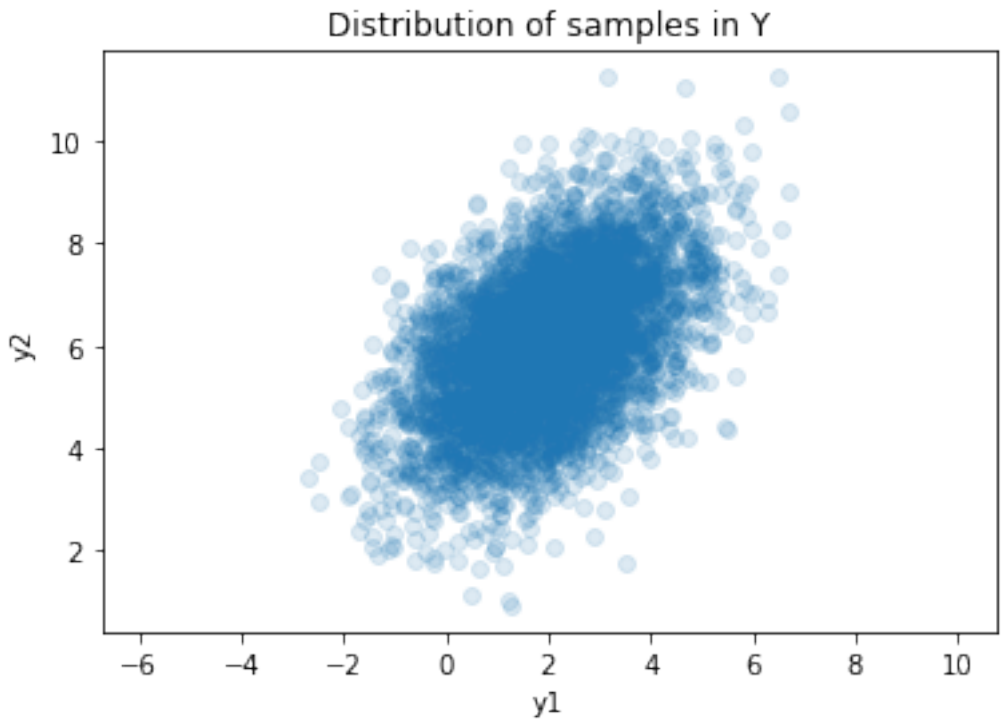
2.2 2 c)

[9]:
```
# i)
samples_std = np.random.multivariate_normal(np.array([0,0]),np.
 ↪array([[1,0],[0,1]]),size=5000)
plt.figure()
plt.scatter(samples_std[:,0],samples_std[:,1], alpha=0.15)
plt.xlabel('x1')
plt.ylabel('x2')
plt.title('Distribution of samples in X')
plt.axis('equal')
plt.savefig(fig_path+'hw1_ex2_ci')
```



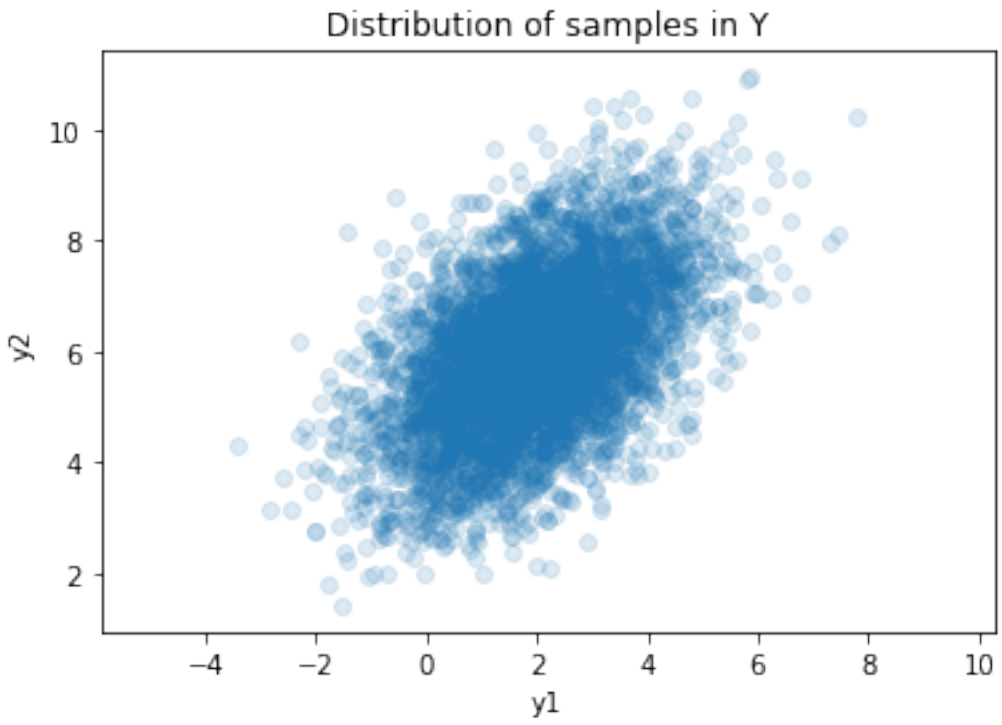
[10]:
```
# ii)
A = np.array([[1/np.sqrt(2),np.sqrt(3/2)],[-1/np.sqrt(2),np.sqrt(3/2)]])
b = mu
samples_tf = A @ samples_std.T + b
print(A)
plt.figure()
plt.scatter(samples_tf[0,:],samples_tf[1,:], alpha=0.15)
plt.xlabel('y1')
plt.ylabel('y2')
plt.title('Distribution of samples in Y')
plt.axis('equal')
plt.savefig(fig_path+'hw1_ex2_cii1')
```

```
[[ 0.7071  1.2247]
 [-0.7071  1.2247]]
```

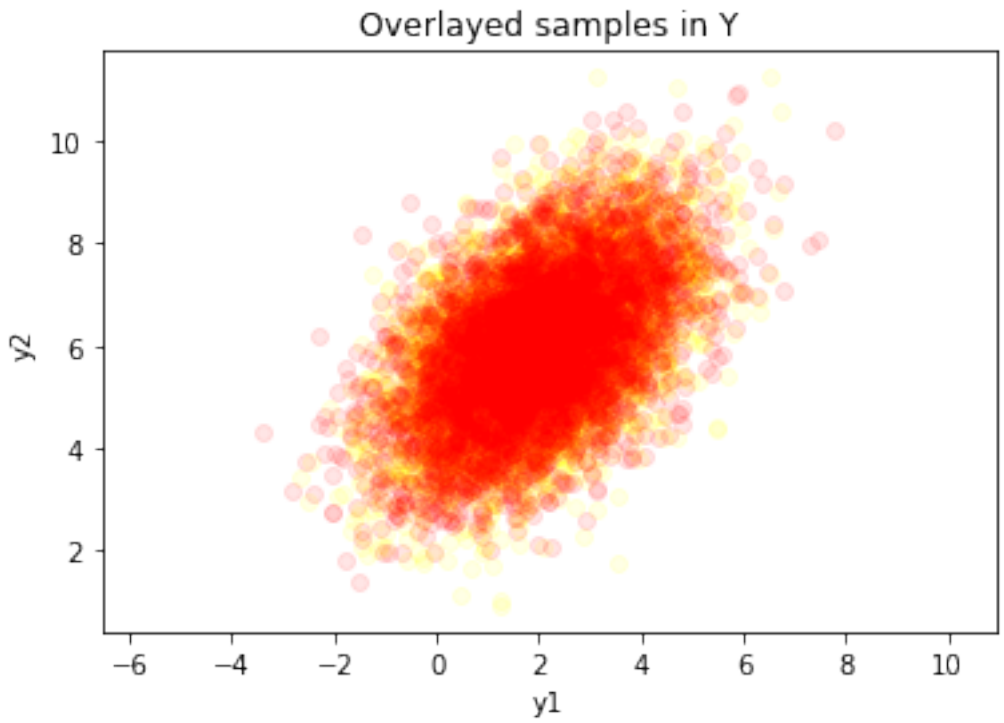


[11]:
```
w,v = np.linalg.eig(sigma)
w = np.diag(w)
A_n = v @ np.sqrt(w)
print(w,v)
print(A_n)
samples_tf = A_n @ samples_std.T + b

plt.figure()
plt.scatter(samples_tf[0,:],samples_tf[1,:], alpha=0.15)
plt.xlabel('y1')
plt.ylabel('y2')
plt.title('Distribution of samples in Y')
plt.axis('equal')
plt.savefig(fig_path+'hw1_ex2_cii2')
```

```
[[3. 0.]
 [0. 1.]] [[ 0.7071 -0.7071]
 [ 0.7071  0.7071]]
[[ 1.2247 -0.7071]
 [ 1.2247  0.7071]]
```



[12]:
```
plt.figure()

samples_tf = A @ samples_std.T + b
plt.scatter(samples_tf[0,:],samples_tf[1,:], alpha=0.1, color='yellow')

samples_tf = A_n @ samples_std.T + b
plt.scatter(samples_tf[0,:],samples_tf[1,:], alpha=0.1, color='red')
plt.xlabel('y1')
plt.ylabel('y2')
plt.title('Overlayed samples in Y')
plt.axis('equal')
plt.savefig(fig_path+'hw1_ex2_ciii')
```



Exercise 3

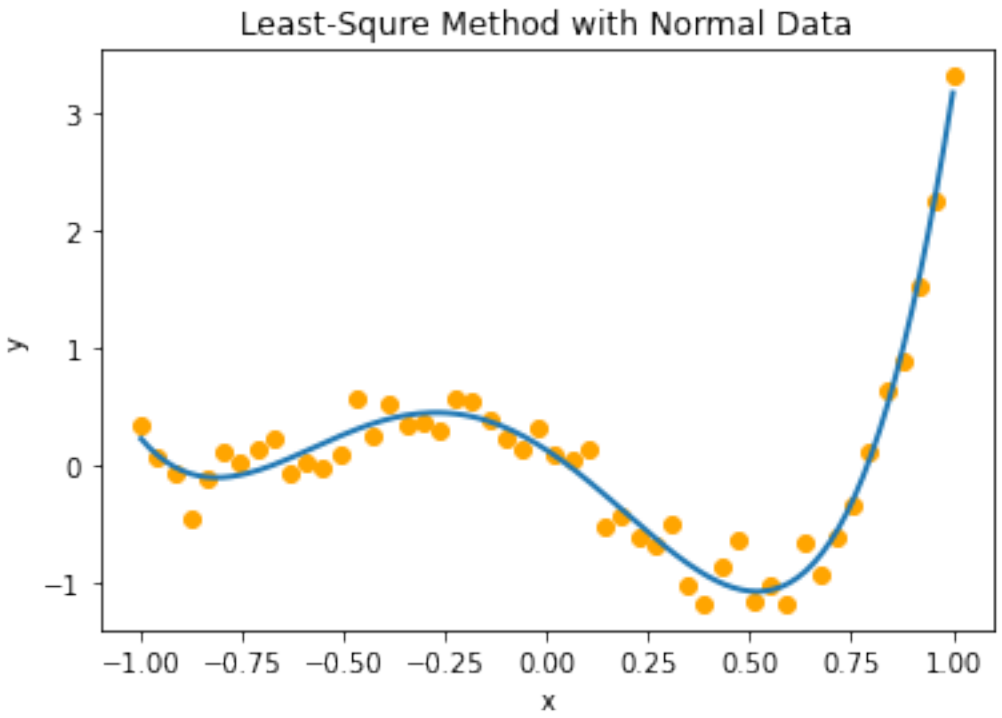
[13]:
```
from scipy.special import eval_legendre
N = 50
x = np.linspace(-1,1,N)
L0 = eval_legendre(0,x)
L1 = eval_legendre(1,x)
L2 = eval_legendre(2,x)
L3 = eval_legendre(3,x)
L4 = eval_legendre(4,x)
```

2.3 3 a) b) c)

[14]:
```
beta = np.array([-0.001, 0.01, 0.55, 1.5, 1.2])
y = beta[0]*L0 + beta[1]*L1 + beta[2]*L2 + beta[3]*L3 + beta[4]*L4 + 0.2*np.
 ↪random.randn(N)

# X = np.column_stack((L0,L1,L2,L3,L4))
X = np.column_stack((x**0,x**1,x**2,x**3,x**4))

theta = np.linalg.lstsq(X, y, rcond=None)[0]
t     = np.linspace(-1, 1, 200);
```

```
T = np.column_stack((t**0,t**1,t**2,t**3,t**4))
yhat = theta @ T.T

print(theta)
plt.scatter(x,y, color='orange')
plt.plot(t,yhat, linewidth=2)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Least-Squre Method with Normal Data')
plt.savefig(fig_path+'hw1_ex3_lstsqr_normal')
```

```
[ 0.1268 -2.2668 -3.3552  3.7368  4.9232]
```

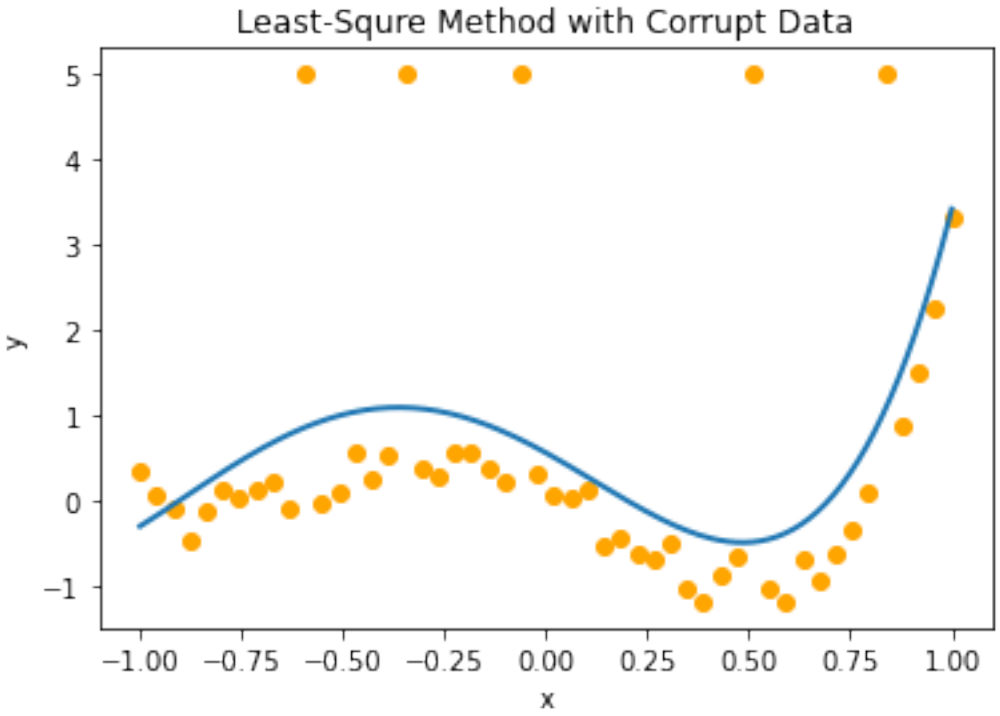


2.4 3 d)

[15]:
```
idx = [10,16,23,37,45]
y[idx] = 5

# X = np.column_stack((L0,L1,L2,L3,L4))
X = np.column_stack((x**0,x**1,x**2,x**3,x**4))

theta = np.linalg.lstsq(X, y, rcond=None)[0]
t     = np.linspace(-1, 1, 200);
```

```
T = np.column_stack((t**0,t**1,t**2,t**3,t**4))
yhat = theta @ T.T

print(theta)
plt.scatter(x,y,color='orange')
plt.plot(t,yhat, linewidth=2)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Least-Squre Method with Corrupt Data')
plt.savefig(fig_path+'hw1_ex3_lstsqr_corrupt')
```

```
[ 0.5658 -2.6131 -1.9346  4.4699  2.9324]
```



2.5 3 e) f)

[16]:
```
from scipy.optimize import linprog

c = np.hstack([np.zeros((1,5)), np.ones((1,N))])
X = np.column_stack((x**0,x**1,x**2,x**3,x**4))
A = np.block([
    [X, -np.eye(N)],
    [-X, -np.eye(N)],
```

```
])
b = np.hstack([y,-y]).reshape(N*2,1)
result = linprog(A_ub=A, b_ub=b, c=c, bounds=(None,None))
beta_h = result.x[0:5]
beta_h
```

[16]:
```
array([ 0.135 , -2.4814, -3.1449,  3.9349,  4.6272])
```

[17]:
```
T = np.column_stack((t**0,t**1,t**2,t**3,t**4))
yhat = beta_h @ T.T

plt.scatter(x,y,color='orange')
plt.plot(t,yhat, linewidth=2)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Linear Programming with Corrupt Data')
plt.savefig(fig_path+'hw1_ex3_linprog_corrupt')
```