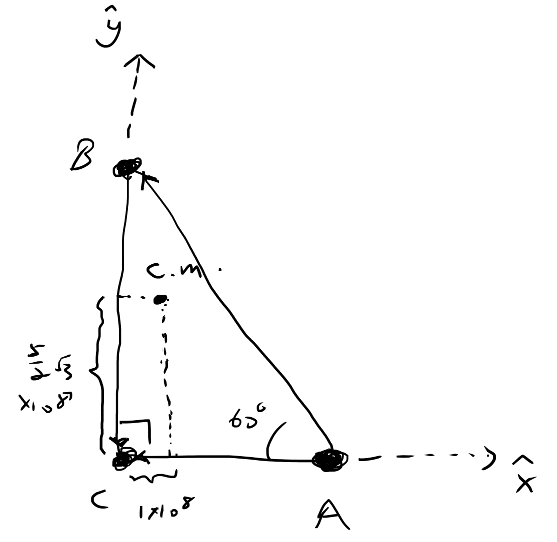
AAE 532 Exam 1 Zhanpeng Yang

1. a) It is to infer uniformly distributed gravity field around each body, thus allowing us to use the gravity equations.



$16+48=64$

$\bar{r}_{AC} = -4\times10^8\,\hat{x}$

$\bar{r}_{BC} = 4\sqrt{3}\times10^8\,\hat{y}$

$\bar{r}_{AB} = \bar{r}_{AC} - \bar{r}_{BC} = -4\times10^8\,\hat{x} + 4\sqrt{3}\times10^8\,\hat{y}$

Let $\bar{r}_A = -\bar{r}_{AC}$, $\bar{r}_B = -\bar{r}_{BC}$, $\bar{r}_C = 0$.

(C coincides the origin at this moment)

$M\bar{r}_{cm} = m_A\bar{r}_A + m_B\bar{r}_B + m_C\bar{r}_C$

$$\bar{r}_{cm} = \frac{2\times4\times10^{16}\,\hat{x} + 5\times4\sqrt{3}\times10^{16}\,\hat{y}}{8\times10^8}$$

$$\boxed{\bar{r}_{cm} = 1\times10^8\,\hat{x} + \frac{5}{2}\sqrt{3}\times10^8\,\hat{y}\ \text{km}}$$

b) i) motion of C relative to A perturbed by B.

$$\ddot{\bar{r}}_{AC} + \frac{G(m_A+m_C)}{r_{AC}^3}\bar{r}_{AC} = Gm_B\left(\frac{\bar{r}_{CB}}{r_{CB}^3} - \frac{\bar{r}_{AB}}{r_{AB}^3}\right)$$

Both formulations are correct, because they are based on different reference point, which will give us different information on the interested body.

ii)
$$m_C\ddot{\bar{r}}_C = -G\frac{m_A m_C}{r_{AC}^3}\bar{r}_{AC} - G\frac{m_B m_C}{r_{BC}^3}\bar{r}_{BC}$$

iii) Independent variable: t, (time)

Dependent variables: $\bar{r}_C$, $\dot{\bar{r}}_C$, $\bar{r}_A$, $\bar{r}_B$ (each with $\hat{x}$, $\hat{y}$ components, assuming planar motion)

c) Dominant Acceleration;

i)
$$-\frac{G(m_A+m_C)}{r_{AC}^3}\bar{r}_{AC} = -\frac{3\times10^8}{(4\times10^8)^3}\times(-4\times10^8\,\hat{x}) = \boxed{1.875\times10^{-9}\,\hat{x}\ km/s^2}$$

mag: 1.875×10^{-9} km/s²

Direct Perturbation:

$$\frac{Gm_B\,\bar{r}_{CB}}{r_{CB}^3} = 5\times10^8\times\frac{4\sqrt{3}\times10^8\,\hat{y}}{(4\sqrt{3}\times10^8)^3} = \boxed{1.0417\times10^{-9}\,\hat{y}\ km/s^2}$$

mag: 1.0417×10^{-9} km/s²

Indirect Perturbation:

$$Gm_B\frac{\bar{r}_{AB}}{r_{AB}^3} = 5\times10^8\times\frac{(-4\times10^8\,\hat{x}+4\sqrt{3}\times10^8\,\hat{y})}{(8\times10^8)^3}$$

$$= \boxed{-3.90625\times10^{-10}\,\hat{x}+6.7658\times10^{-10}\,\hat{y}\ km/s^2}$$

mag: 7.8125×10^{-10} km/s²

Net Perturbation:

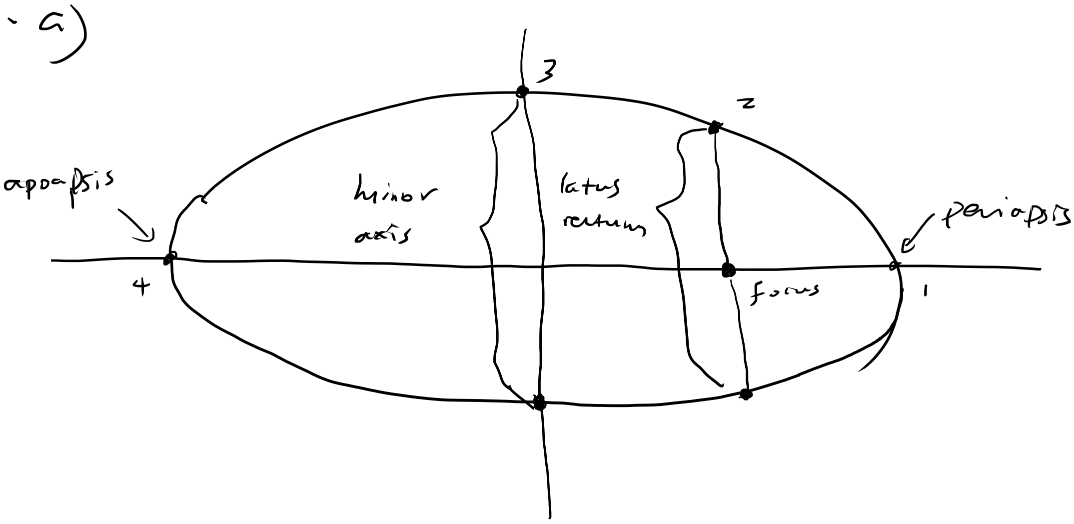
$$Gm_B\frac{\bar{r}_{CB}}{r_{CB}^3} - Gm_B\frac{\bar{r}_{AB}}{r_{AB}^3} = \boxed{3.90625\times10^{-10}\,\hat{x}+3.6512\times10^{-10}\,\hat{y}\ km/s^2}$$

mag: 5.3470×10^{-10} km/s²

(ii) Dominant term has the largest magnitude, followed by direct perturbation and then indirect perturbation. The net perturbation has the smallest magnitude.

(iii) The dominant term is positive in $\hat{x}$, and the net perturbing term is also positive in both $\hat{x}$ & $\hat{y}$. The net acceleration in $\hat{x}$ is greater than that in $\hat{y}$. Since A is on the $\hat{x}$ axis at this instance, the distance between A & C will tend to decrease.

2. a)



b) · Periapsis : $\theta^* = 0°$

$$r_1 = \frac{p}{1+e} = \boxed{a(1-e)}$$

Apoapsis : $\theta^* = 180$

$$r_4 = \frac{p}{1-e} = \boxed{a(1+e)}$$

Latus rectum.

$$r_2 = p = \boxed{a(1-e^2)}$$

Minor axis

$$r_3 = a(1 - e\cos 90°) = \boxed{a}$$

c) $\xi = \frac{v^2}{2} - \frac{\mu}{r}$

$\xi = -\frac{\mu}{2a}$. $\mu = -2a\xi$ $v^2 = 2v_c^2 - \frac{\mu}{a}$

$= 2v_c^2 + 2\xi$

$$v^2 = 2\xi + 2\frac{r}{\mu}$$

$$v^2 = 2\xi + \frac{2r}{-2a\xi}$$

$$v^2 = 2\xi - \frac{r}{a\xi}$$

Periapsis:

$$v_1 = \sqrt{2\xi - \frac{r_1}{a\xi}}$$

$$= \sqrt{2\xi - \frac{a(1-e)}{a\xi}}$$

$$\boxed{v_1 = \sqrt{2\xi - \frac{1-e}{\xi}}}$$

apoapsis

$$v_4 = \sqrt{2\xi - \frac{r_4}{a\xi}}$$

$$= \sqrt{2\xi - \frac{a(1+e)}{a\xi}}$$

$$\boxed{v_4 = \sqrt{2\xi - \frac{1+e}{\xi}}}$$

latus rectum:

$$v_2 = \sqrt{2\xi - \frac{r_2}{a\xi}} = \boxed{\sqrt{2\xi - \frac{1-e^2}{\xi}}}$$

minor axis:

$$v_3 = \sqrt{2\xi - \frac{r_3}{a\xi}} = \boxed{\sqrt{2\xi - \frac{1}{\xi}}}$$

d) $r_1 = 0.5a$

$r_4 = 1.5a$

$r_2 = 0.75a$

$r_3 = a$

$v_1 = \sqrt{2\xi - \frac{1}{2\xi}}$

$v_3 = \sqrt{2\xi - \frac{1}{\xi}}$

$v_4 = \sqrt{2\xi - \frac{3}{2\xi}}$

$v_2 = \sqrt{2\xi - \frac{3}{4\xi}}$

e)

3. $m = 8000\,kg$. $\bar{c}_1 = \bar{0}$, $|\bar{h}| = 2.5\times10^4\ km^2/s$, $|\xi| = 1.2351\ km^2/s$

$R_♂ = 3400\,km$ $\mu_♂ = 4.2\times10^4\ km^3/s^2$

a) $|\bar{c}_1| = 0$ tells us that the center of the mass of the system is not moving in this reference frame, thus we only need to consider rotational motions.

b)
$$\xi = -\frac{\mu}{2a}$$

$$1.2351 = \frac{4.2\times10^4}{2a}$$

$$\boxed{a = 1.7002\times10^4\ km} = 5.0006 R_♂$$

$$p = \frac{h^2}{\mu} = \frac{(2.5\times10^4)^2}{(4.2\times10^4)} = \boxed{1.4881\times10^4\ km}$$

$= 4.3768\,R_♂$

$$a = \frac{p}{1-e^2}$$

$$e = \sqrt{1-\frac{p}{a}} = \sqrt{1-\frac{1.4881}{1.7002}} = \boxed{0.3532}$$

e

$$b = a\sqrt{(1-e^2)}$$
$$= 1.7002\times10^4\sqrt{(1-0.3532^2)}$$
$$\boxed{b = 1.5906\times10^4\ km}$$
$= 4.6782\,R_♂$

$$r_p = a(1-e) = \boxed{1.0997\times10^4\ km} = 3.2344R_♂$$

$$r_a = a(1+e) = \boxed{2.3007\times10^4\ km} = 6.7668\,R_♂$$

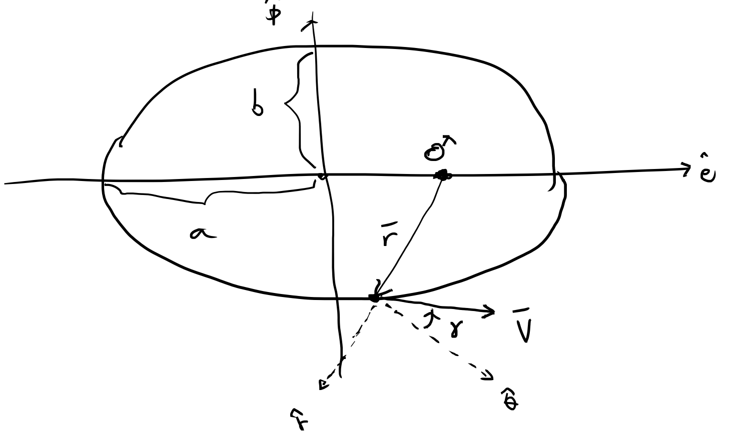
$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}}$$
$$= 2\pi\sqrt{\frac{(1.7002\times10^4)^3}{4.2\times10^4}}$$
$$\boxed{= 6.7968\times10^4\ s}$$

c)
$$r = \frac{p}{1+e\cos(\theta^*)} = \frac{1.4881\times10^4}{1+0.3532\cdot\cos(-110^\circ)} = \boxed{1.6926\times10^4\ km}$$

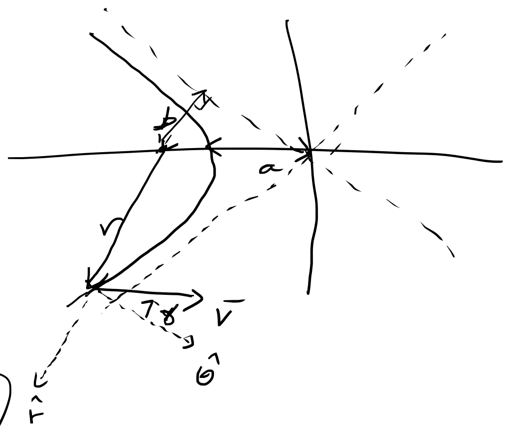
$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \rightarrow \frac{\xi}{2}.$$

$$= \sqrt{\frac{2\times4.2\times10^4}{1.6926\times10^4} - \frac{1.2351}{2}}$$

$$= 2.1845\ km/s$$



d). If $\xi > 0$. it is a hyperbolic orbit.



$a = -\frac{\mu}{2\xi} = \boxed{-1.7002\times10^4 \text{ km}}$

$p = \boxed{1.4881\times10^4 \text{ km}}$

$e = \sqrt{1+\frac{2\xi h^2}{\mu^2}} = \sqrt{1+\frac{2\times1.2351\times(2.5\times10^4)^2}{(4.2\times10^4)^2}} = \boxed{1.3694}$

$b = |a|\sqrt{e^2-1} = \boxed{1.5906\times10^4 \text{ km}}$

$r_p = |a|\sqrt{e-1} = \boxed{1.0334\times10^4 \text{ km}}$

$r_a = \infty$ p, & b remained the same for both orbit.

$P = \infty$

$\boxed{r = 1.6926\times10^4 \text{ km}}$

$v = \sqrt{\frac{2\times4.2\times10^4}{1.6926\times10^4}+\frac{1.2351}{2}} = \boxed{2.3623 \text{ km/s}}$ The velocity is way faster.

$\sqrt{\frac{2\times4.2\times10^4}{1.6926\times10^4}+\frac{1.2351}{2}}$

e) For the elliptic orbit:

For the hyperbolic orbit: $\theta_\infty = \sin^{-1}\frac{1}{e} = 46.91^\circ$ This the angle of two asymptotes.