1. a) $r_p = 1.5 R_\oplus$ $\quad \Omega = 0°$ $\quad R_\oplus = 6378.1363$ km

$r_a = 200 R_\oplus$ $\quad \theta^* = 0°$

$\omega = 0°$

$i = 30°$

$$\frac{1-e}{1+e} = \frac{1.5}{200}$$

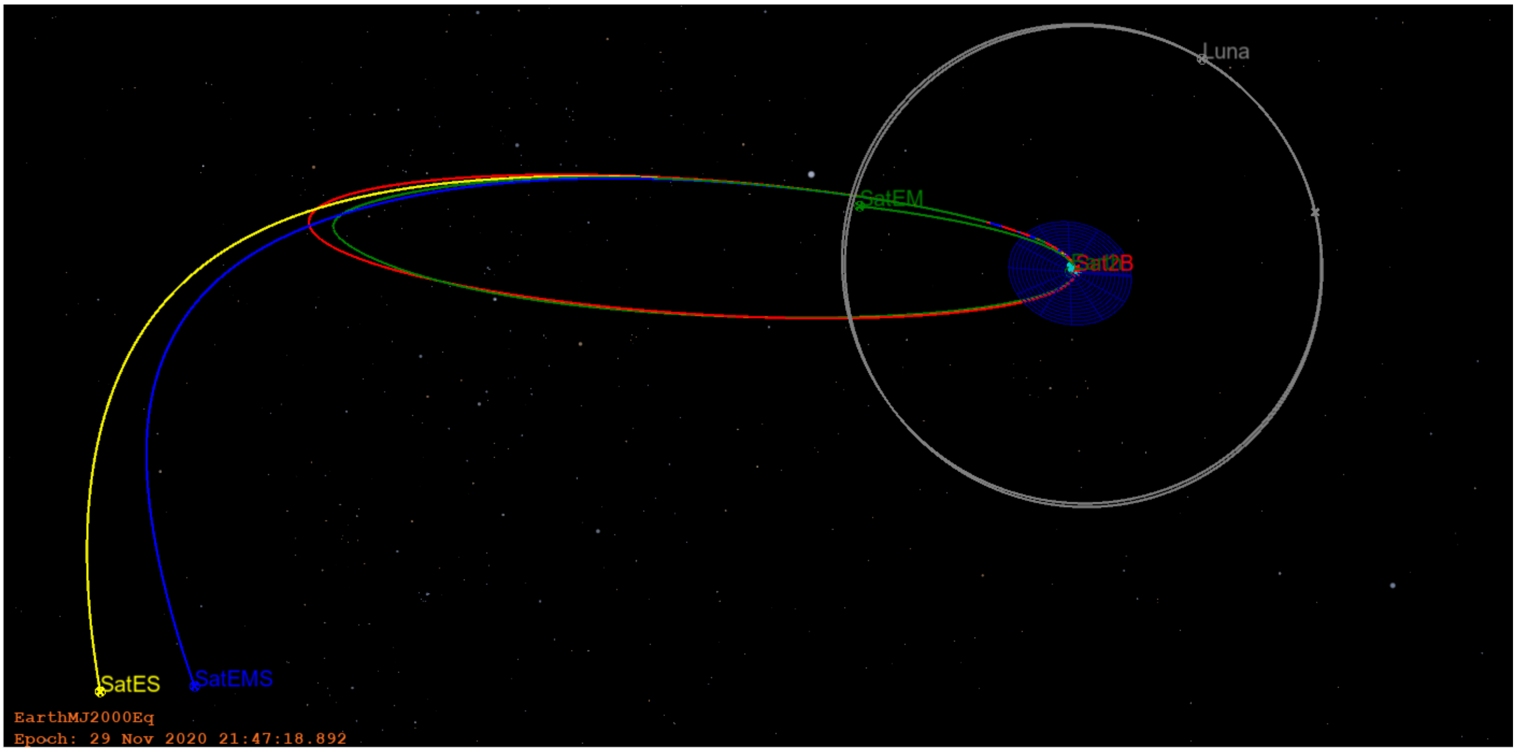
$$1-e = \frac{3}{400} + \frac{3}{400}e$$

$$\frac{397}{400} = \frac{403}{400}e$$

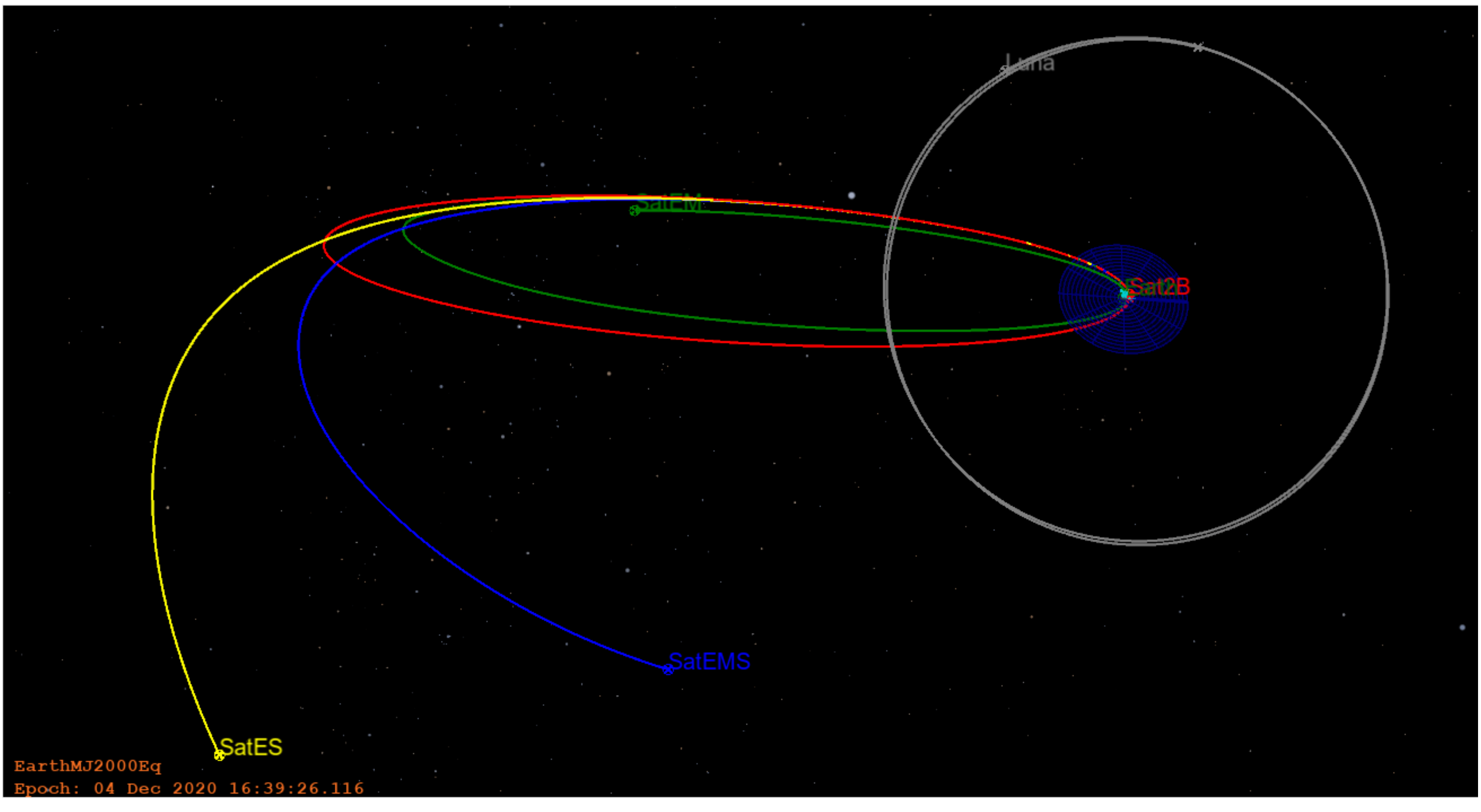
$$e = \frac{397}{403}$$

$$\left.\begin{array}{l} r_p = a(1-e) \\ r_a = a(1+e) \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a = 642597.2322 \text{ km} \\ e = \frac{397}{403} \approx 0.98511166253 \end{array}\right.$$

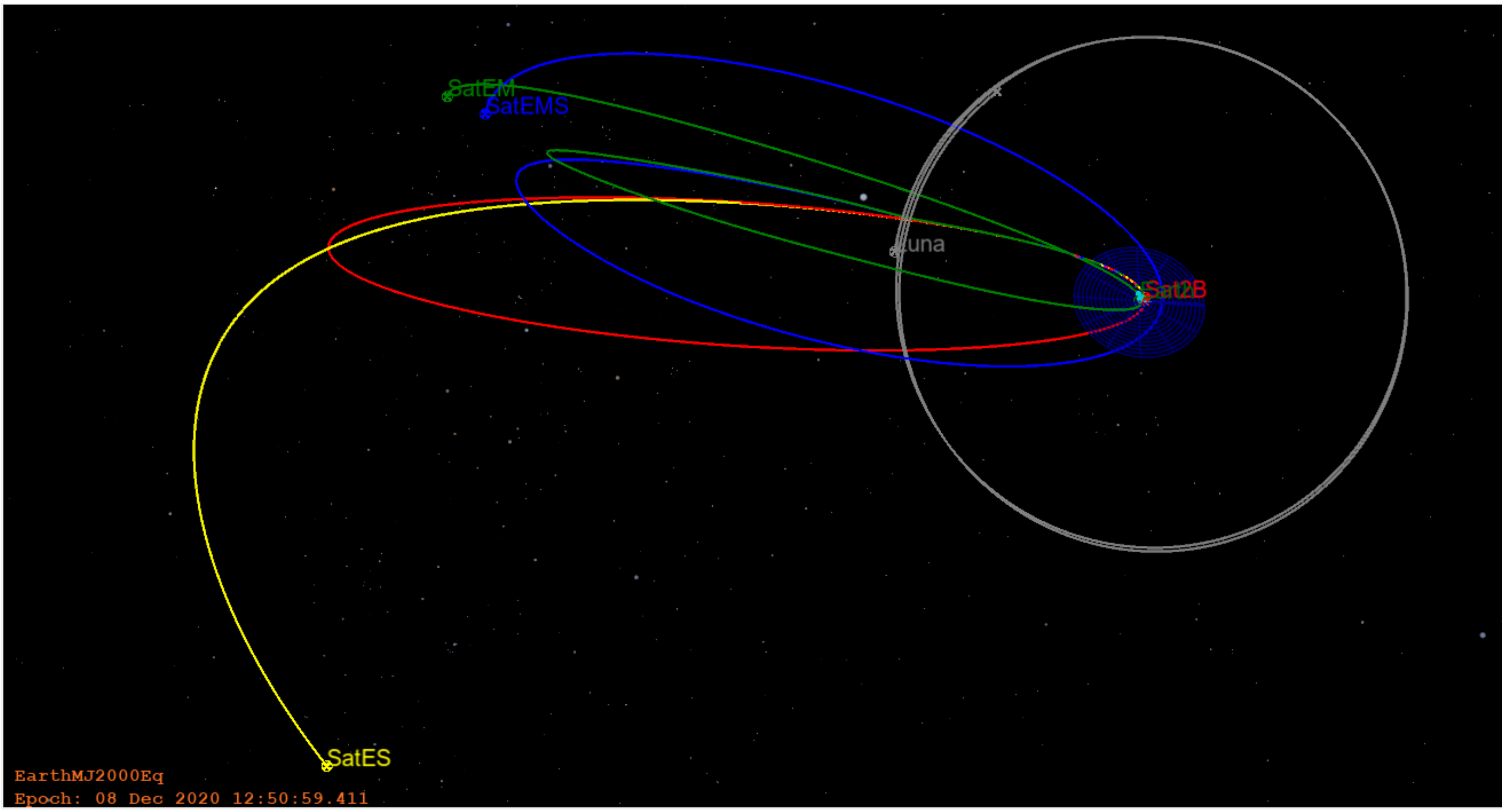
Start Epoch: Oct 02



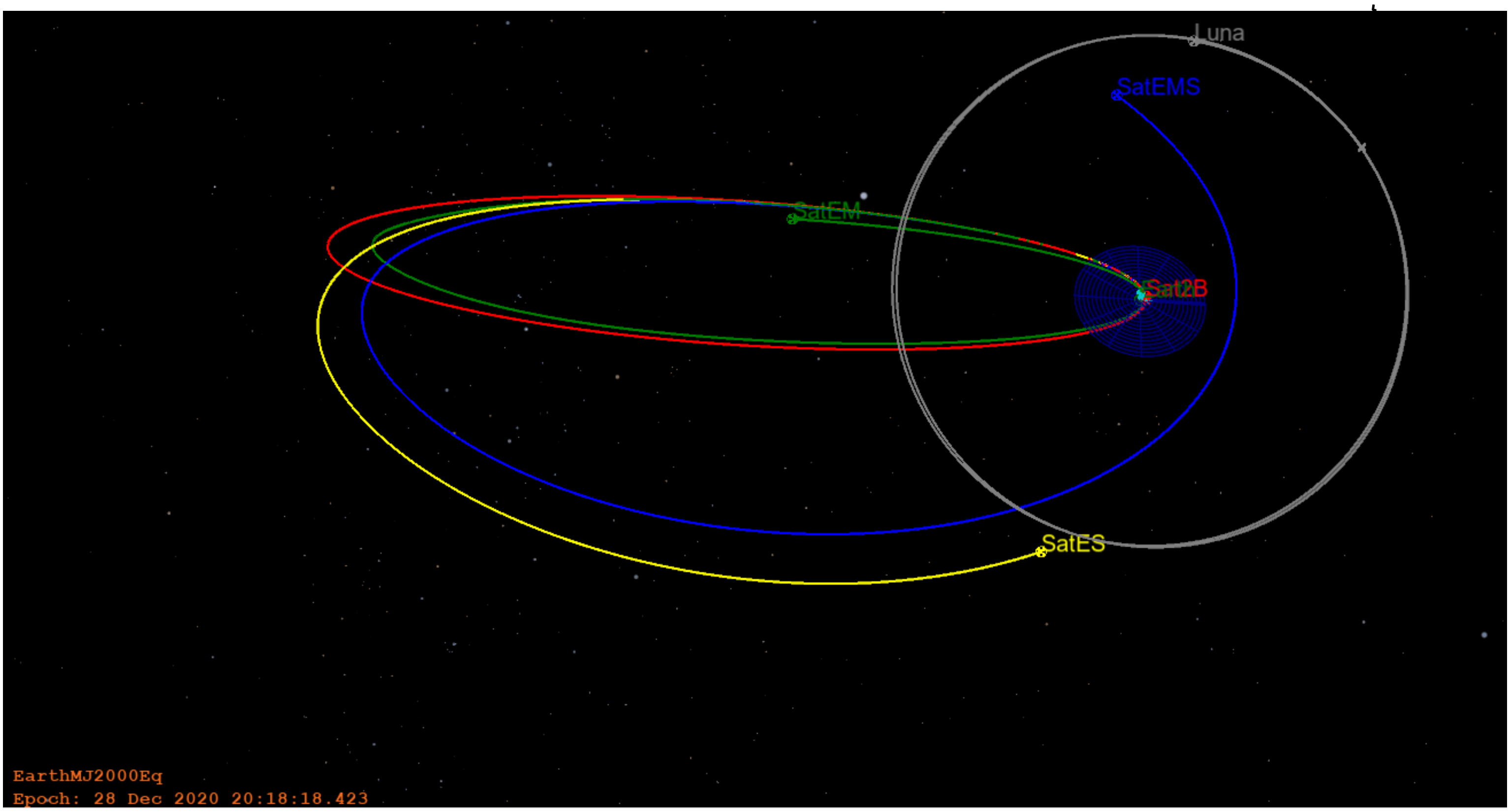
Start Epoch: Oct 07



Start Epoch: Oct 11



Start Epoch: Oct 31



Different models yield different orbits.

The 2BD model is no longer adequate, as perturbing forces from other celestial bodies are significant.

I would include both the Sun & the Moon to formulate a 4BD model. As shown on the orbit plot, 4BD model yields a very different trajectory than the 3BD / 2BP models, which includes more information and the most precise model.

Different epoch dates result in different orbits. This is because the initial perturbing accelerations are different on different dates, as the Sun & the Moon travel to a new location on each starting epoch.

b)

2-Body Model				
Epoch started at Oct 02				
a	e	r_p	Energy	h
642597.2321436529	0.9851116625286942	9567.204450281739	-0.3101479601540618	87006.93811984058
r_f	v_f	θ_f^*	FPA	γ_f
10269.96775330297	8.775202664023691	30.44591518638943	74.89397904730147	90-FPA
Epoch started at Oct 07				
a	e	r_p	Energy	h
642597.2321435203	0.9851116625286909	9567.204450281906	-0.3101479601541257	87006.93811984187
r_f	v_f	θ_f^*	FPA	γ_f
10265.85447641848	8.776974651037763	30.3607605394309	74.93621346465579	90-FPA
Epoch started at Oct 11				
a	e	r_p	Energy	h
642597.2321431228	0.9851116625286817	9567.20445028191	-0.3101479601543176	87006.93811984058
r_f	v_f	θ_f^*	FPA	γ_f
10320.54251699518	8.753501640830207	31.47053627960411	74.38580551918359	90-FPA
Epoch started at Oct 31				
a	e	r_p	Energy	h
642597.2321433731	0.9851116625286874	9567.204450281926	-0.3101479601541968	87006.93811984095
r_f	v_f	θ_f^*	FPA	γ_f
10296.27563262954	8.763894368715579	30.98396494457327	74.62712404704764	90-FPA

Earth-Moon Model				
Epoch started at Oct 02				
a	e	r_p	Energy	h
611081.9368294963	0.9864681475638223	8269.070595590416	-0.3261432039442014	80916.67719289198
r_f	v_f	θ_f^*	FPA	γ_f
351854.764881928	1.270206148368623	165.103625238773	10.43097057102369	90-FPA
Epoch started at Oct 07				
a	e	r_p	Energy	h
568615.5303199108	0.9862777820948763	7802.666211287308	-0.350500839535408	78597.80079343601
r_f	v_f	θ_f^*	FPA	γ_f
767270.9877892353	0.5813833215959732	173.4300446756124	10.14833636275611	90-FPA
Epoch started at Oct 11				
a	e	r_p	Energy	h
534046.8754593359	0.9838860081436503	8605.627002060715	-0.3731886280180576	82493.2671290264
r_f	v_f	θ_f^*	FPA	γ_f
1058758.731050387	0.08112193837241011	180.2721757758503	106.1640354628631	90-FPA
Epoch started at Oct 31				
a	e	r_p	Energy	h
598379.0985296473	0.9866057097736284	8014.863311100716	-0.3330668154013496	79665.95881541379
r_f	v_f	θ_f^*	FPA	γ_f
536833.927752162	0.9049149278620259	169.5816777286275	9.438735725806545	90-FPA

Earth-Sun Model				
Epoch started at Oct 02				
a	e	r_p	Energy	h
1374260.227141098	0.3629441741642292	875480.4839146261	-0.1450236402203158	689653.835945731
r_f	v_f	θ_f^*	FPA	γ_f
1794644.783314038	0.3926368297625418	157.4172570571872	78.1607626550163	90-FPA
Epoch started at Oct 07				
a	e	r_p	Energy	h
1221145.815373773	0.3281311006054918	820449.8949753863	-0.1632075533002563	659045.4907352207
r_f	v_f	θ_f^*	FPA	γ_f
1621761.562295125	0.4063861269034955	180.8152329472982	90.3981085160806	90-FPA
Epoch started at Oct 11				
a	e	r_p	Energy	h
1119682.777263637	0.3554269666259336	721717.3241575218	-0.1779970405877497	624439.7105371114
r_f	v_f	θ_f^*	FPA	γ_f
1465821.609704632	0.4334343476506163	200.6292171111442	100.6273753547192	90-FPA
Epoch started at Oct 31				
a	e	r_p	Energy	h
810221.2538031581	0.7506193882429439	202053.4719320005	-0.2459824644373248	375489.6005505422
r_f	v_f	θ_f^*	FPA	γ_f
405205.1976832372	1.214675062777516	260.2542060283752	130.2802242493301	90-FPA

Earth-Moon-Sun Model				
Epoch started at Oct 02				
a	e	r_p	Energy	h
1159599.715702167	0.4116794226768316	682216.3742056809	-0.1718698427149222	619580.8687841462
r_f	v_f	θ_f^*	FPA	γ_f
1632524.793409209	0.3802423176146617	174.9363998882631	86.47542833862009	90-FPA
Epoch started at Oct 07				
a	e	r_p	Energy	h
800569.8795779545	0.5860720757080043	331378.2285043956	-0.2489479380051962	457712.0364187732
r_f	v_f	θ_f^*	FPA	γ_f
910015.4162193869	0.6149261822566867	223.8797304936007	125.1210507729914	90-FPA
Epoch started at Oct 11				
a	e	r_p	Energy	h
524418.1654380752	0.9141641594817735	45013.87401340352	-0.3800406505436623	185324.0034256037
r_f	v_f	θ_f^*	FPA	γ_f
1002965.08547173	0.1864478511880883	180.7259460699941	97.67832268082154	90-FPA
Epoch started at Oct 31				
a	e	r_p	Energy	h
814842.7805605406	0.8278984313496098	140235.7207379147	-0.2445873308381021	319649.6573430056
r_f	v_f	θ_f^*	FPA	γ_f
308746.8247208884	1.446678625116222	101.8317693704417	45.69635960605956	90-FPA

Examining r_p's, there is no scenario where collision occurs.

Perturbation from the Moon reduces r_p.

It does not occur at the start of epoch.

2 $r_p = 1.5\ R_♂$ $r_a = 6.5\ R_♂$ $M = -90°$, $R_♂ = 3397$ km

$\mu \approx \mu_♂$

a) Find $a, e, p, h, \mathbb{P}, \xi, r, v, \theta^*, E, \gamma, (t-t_p)$

$r_p = a(1-e)$

$r_a = a(1+e)$

$\frac{1.5}{6.5} = \frac{1-e}{1+e}$

$\frac{3}{13} + \frac{3}{13}e = 1-e$

$\frac{16}{13}e = \frac{10}{13}$

$\boxed{e = \frac{10}{16} = \frac{5}{8} = 0.625}$

$\boxed{a = \frac{r_p}{1-e} = 13588\ \text{km}}$

$r_p = \frac{p}{1+e}$

$\boxed{p = r_p(1+e) = 8280.2\ \text{km}}$

$\boxed{h = \sqrt{p\mu} = 1.8832 \times 10^4\ \text{km}^2/\text{s}}$

$\boxed{\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = 4.8089 \times 10^4\ \text{sec}}$

$\boxed{\xi = \frac{-\mu}{2a} = -1.576\ \text{km}^2/\text{s}^2}$

$\tan\frac{\theta^*}{2} = \left(\frac{1+e}{1-e}\right)^{\frac{1}{2}} \tan\frac{E}{2}$

$\boxed{\theta^* = -149.4535°}$

descending orbit

$E - e\sin E = M$

$\boxed{E = -2.1078\ \text{rad} = -120.7691°}$

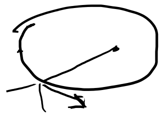
$\boxed{r = a(1 - e\cos E) = 1.7933 \times 10^4\ \text{km}}$

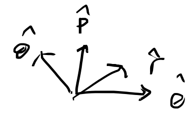
$\boxed{v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = 1.2746\ \text{km/s}}$

$\boxed{\gamma = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r \cdot v}\right) = -34.5256°}$

$\boxed{t - t_p = \frac{M}{n} = \frac{M}{\sqrt{\frac{\mu}{a^3}}} = -1.2022 \times 10^4\ \text{sec}}$

b) $\bar{r}_0 = 1.7932\times10^4\ \hat{r}$ km





$\bar{v}_0 = 1.2746\,(\cos\gamma\,\hat{\theta} + \sin\gamma\,\hat{r}) = -0.7224\,\hat{r} + 1.0501\,\hat{\theta}$ km/s

$$R_{ep\to r\theta} = \begin{bmatrix} c\theta^* & s\theta^* & 0 \\ -s\theta^* & c\theta^* & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\bar{r}_0^{\,ep} = R^T\cdot\bar{r}_0 = -1.5443\times10^4\,\hat{e} - 0.9114\times10^4\,\hat{p} \text{ km}}$$

$$\boxed{\bar{v}_0^{\,ep} = R^T\cdot\bar{v}_0 = 1.1559\,\hat{e} - 0.5372\,\hat{p} \text{ km}}$$

c) $t_f - t_p = t_0 - t_p + \frac{1}{2}\mathbb{P}$

$M_f = \sqrt{\frac{\mu}{a^3}}\,(t_f - t_p) = 1.5708$ rad $= 90°$

$E_f = 2.1078$ rad $= 120.7691°$

$$\theta^*_f = 2\cdot\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\cdot\tan\frac{E_f}{2}\right) = \boxed{149.4530°}$$

$$\bar{r} = \left\{1 - \frac{a}{r_0}\left[1-\cos(E-E_0)\right]\right\}\bar{r}_0 + \left\{(t-t_0) + \left[\frac{\sin(E-E_0)-(E-E_0)}{n}\right]\right\}\bar{v}_0$$

$$= \boxed{-0.1188\,\bar{r}_0 - 1.4949\times10^4\,\bar{v}_0 \text{ km}}$$

$r = \frac{p}{1+e\cos\theta^*} = 1.7933\times10^4$ km

$$\bar{v} = \frac{na^2}{r r_0}\cdot\sin(E-E_0)\,\bar{r}_0 + \left\{1 - \frac{a}{r}\left[1-\cos(E-E_0)\right]\right\}\bar{v}_0$$

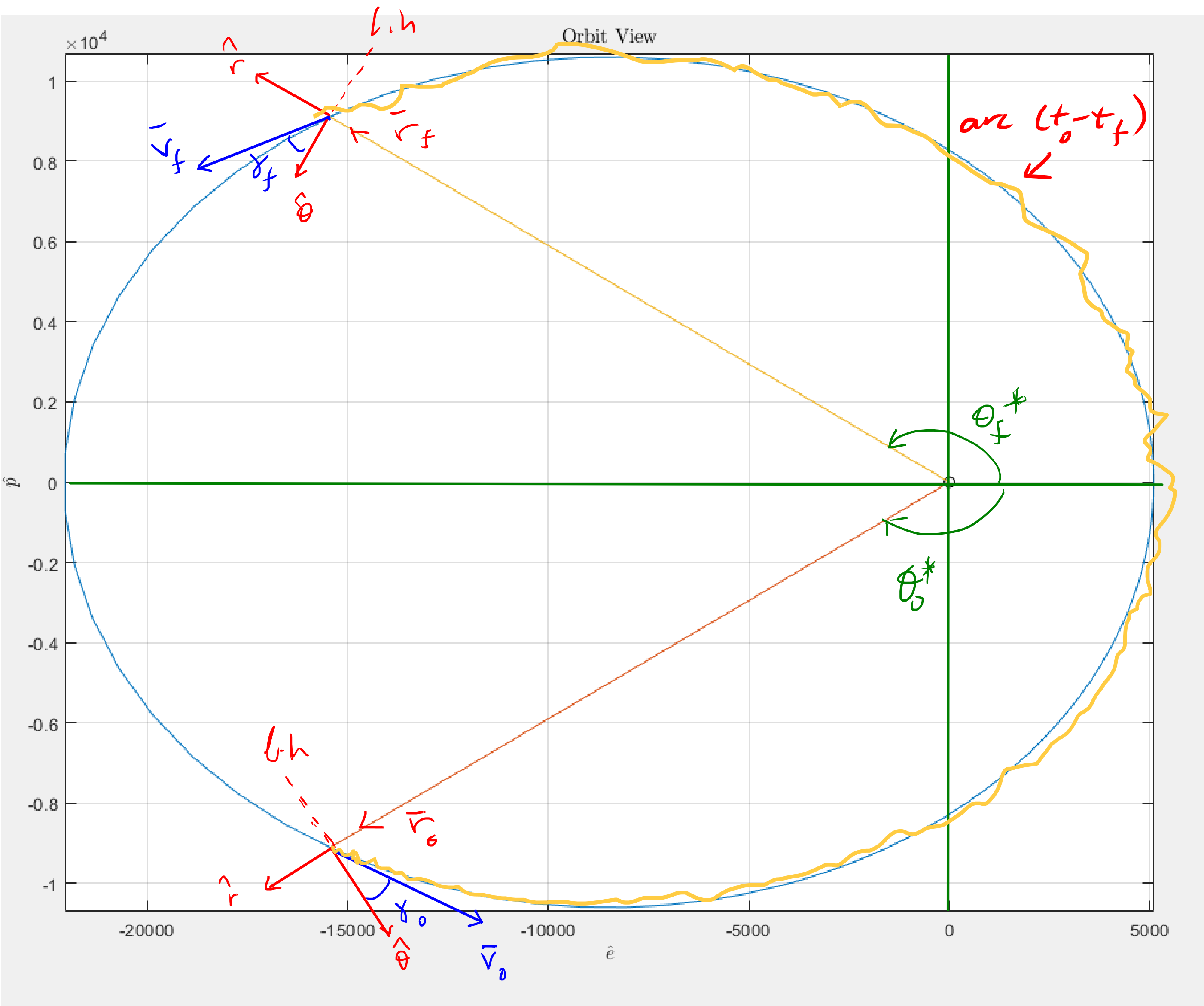
$$= \boxed{-6.5950\times10^{-5}\,\bar{r}_0 - 0.1188\,\bar{v}_0 \text{ km/s}}$$

$$\bar{r} = \left\{1 - \frac{r}{p}\left[1-\cos(\theta^*-\theta^*_0)\right]\right\}\bar{r}_0 + \frac{r_0 r}{\sqrt{\mu p}}\sin(\theta^*-\theta^*_0)\,\bar{v}_0$$

$$= \underbrace{-0.1188}_{f(\theta^*-\theta^*_0)}\bar{r}_0 \underbrace{-1.4949\times10^4}_{g(\theta^*-\theta^*_0)}\bar{v}_0$$

This is the same as the $\bar{r}$ calculated using $f(E-E_0)$ & $g(E-E_0)$

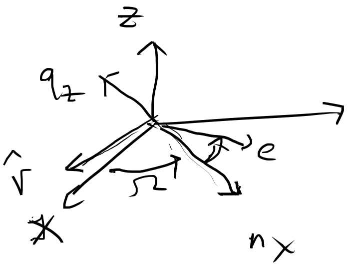
d)

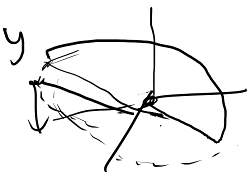

Orbit View
×10^4
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-20000
-15000
-10000
-5000
0
5000
ê
p̂
l.h
r̂
$\bar{r}_f$
$\bar{v}_f$
γ_f
$\hat{\theta}$
arc $(t_o - t_f)$
θ_f^*
θ_o^*
l.h
$\bar{r}_o$
r̂
γ_o
$\hat{\theta}$
$\bar{v}_o$

3, $a = 20 R_{\oplus}$ $\quad \Omega = 45^\circ$

$e = 0.6$ $\quad \omega = 30^\circ$

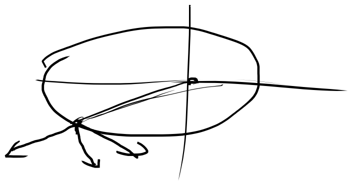
$i = 34^\circ$ $\quad \theta = 235^\circ$





a) Find $\bar{r}, \bar{v}, r, v, \gamma, \theta^*, \mathbb{P}, M, E, (t-t_p)$.

$\theta^* = \theta - \omega = \boxed{205^\circ}$ $>180^\circ$ descending orbit



$p = a(1-e^2) = 8.1640 \times 10^4$ km

$E = \tan^{-1}\left(\tan\left(\frac{\theta^*}{2}\right) \cdot \sqrt{\frac{1-e}{1+e}}\right) \cdot 2 = \boxed{-132.1760^\circ}$

$r = \frac{p}{1 + e \cdot \cos\theta^*} = \boxed{1.7895 \times 10^4 \text{ km}}$

$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} = \boxed{1.1533 \text{ km/s}}$

$\gamma = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r \cdot v}\right) = \boxed{-29.0659^\circ}$ descending orbit.

$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = \boxed{4.5342 \times 10^5 \text{ sec}}$

$M = E - e \cdot \sin(E) = \boxed{-106.6994^\circ}$

$t - t_p = M / \sqrt{\mu / a^3} = \boxed{-1.3439 \times 10^5 \text{ sec}}$

$$\bar{r}^{r\theta h} = \boxed{1.7895 \times 10^5 \, \hat{r} \text{ km}}$$

$$\bar{v}^{r\theta h} = v \cdot (\sin\gamma \, \hat{r} + \cos\gamma \cdot \hat{\theta}) = \boxed{-.5603 \, \hat{r} + 1.0081 \, \hat{\theta} \text{ km/s}}$$

$$C_{313} = \begin{bmatrix} c_\Omega c_\theta - s_\Omega c_i s_\theta & -c_\Omega s_\theta - s_\Omega c_i c_\theta & s_\Omega s_i \\ s_\Omega c_\theta + c_\Omega c_i s_\theta & -s_\Omega s_\theta + c_\Omega c_i c_\theta & -c_\Omega s_i \\ s_i s_\theta & s_i c_\theta & c_i \end{bmatrix}$$

$$\bar{r}^{xyz} = C_{313} \, \bar{r}^{r\theta h}$$

$$= \begin{bmatrix} 0.0746 & 0.9155 & 0.3954 \\ -0.8858 & 0.2430 & -0.3954 \\ -0.4881 & -0.3207 & 0.8290 \end{bmatrix}$$

$$\bar{r}^{xyz} = C_{313} \, \bar{r}^{r\theta h}$$

$$\boxed{= 0.1335 \times 10^5 \, \hat{x} - 1.5851 \times 10^5 \, \hat{y} - 0.8197 \times 10^5 \, \hat{z} \text{ km}}$$

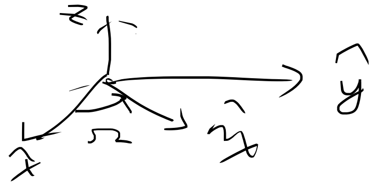
$$\bar{v}^{xyz} = C_{313} \, \bar{v}^{r\theta h}$$

$$\boxed{= 0.8810 \, \hat{x} + 0.7412 \, \hat{y} - 0.0667 \, \hat{z} \text{ km/s}}$$

$xyz \xrightarrow{\Omega} n_x, n_y, n_z$

$$C_3 = \begin{bmatrix} c\Omega & s\Omega & 0 \\ -s\Omega & c\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\hat{n}_x = \cos\Omega\,\hat{x} + \sin\Omega\,\hat{y}$



$$\bar{r}^n = C_3 \cdot \bar{r}^{xyz}$$

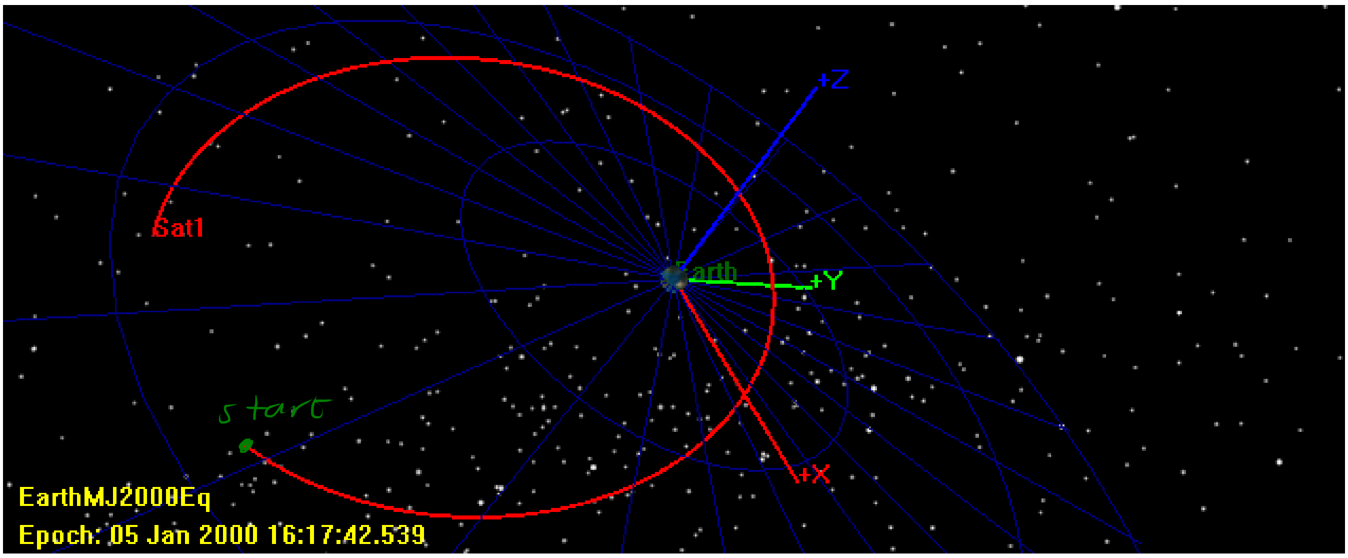
$$\boxed{= -1.0264\times10^5\,\hat{n}_x - 1.2153\times10^5\,\hat{n}_y - 0.8197\times10^5\,\hat{n}_z \text{ km}}$$

$$\bar{v}^n = C_3 \cdot \bar{v}^{xyz}$$

$$\boxed{= 1.1471\,\hat{n}_x - 0.0988\,\hat{n}_y - 0.0667\,\hat{n}_z \text{ km/s}}$$

---

b)



check output at $t = t_0$

Sat1.EarthMJ2000Eq.X	Sat1.EarthMJ2000Eq.Y	Sat1.EarthMJ2000Eq.Z
13353.66685000691	-158511.404929191	-81970.96799742116

Sat1.EarthMJ2000Eq.VX	Sat1.EarthMJ2000Eq.VY	Sat1.EarthMJ2000Eq.VZ
0.8810382912635342	0.7412445372022063	-0.06667456756496955

These values matches the calculation above.

c) $t_2 - t_p = t - t_p + 3 \times 24 \times 3600 = 1.2481 \times 10^5$ sec

$$M_2 = \sqrt{\frac{\mu}{a^3}} \cdot (t_2 - t_p) = 1.7296 \text{ rad} = 99.0985^\circ$$

$$E_2 = 2.2108 \text{ rad} = 126.6716^\circ$$

$$\theta_2^* = 2 \cdot \tan^{-1}\left(\tan\frac{E_2}{2} \cdot \sqrt{\frac{1+e}{1-e}}\right) = 2.6496 \text{ rad} = 151.8108^\circ$$

ascending orbit

$$r_2 = a(1 - e\cos E_2) = 1.7327 \times 10^5 \text{ km}$$

$$v_2 = \sqrt{2\frac{\mu}{r_2} - \frac{\mu}{a}} = 1.2149 \text{ km/s}$$

$$\gamma_2 = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_2 v_2}\right) = 0.5416 \text{ rad} = \boxed{31.0292^\circ}$$ ascending +ve

$$\bar{r}_2^{\,r\theta h} = 1.7327 \times 10^5 \, \hat{r} \text{ km}$$

$$\bar{v}_2^{\,r\theta h} = v_2 \cdot (\sin\gamma \, \hat{r} + \cos\gamma \, \hat{\theta}) = 0.6263 \, \hat{r} + 1.0411 \, \hat{\theta} \text{ km/s}$$

$$\theta_2 = \theta_2^* + \omega = 181.8108^\circ$$

$$C_{313}' = \begin{bmatrix} -0.6882 & 0.6083 & 0.3954 \\ -0.7253 & 0.5636 & -0.3954 \\ -0.0177 & 0.5589 & 0.8290 \end{bmatrix}$$

$$\bar{r}_2^{\,xyz} = C_{313}' \cdot \bar{r}_2^{\,r\theta h}$$

$$\boxed{= -1.1925 \times 10^5 \, \hat{x} - 1.2567 \times 10^5 \, \hat{y} - 0.0306 \times 10^5 \, \hat{z} \text{ km}}$$

$$\bar{v}_2^{\,xyz} = C_{313}' \cdot \bar{v}_2^{\,r\theta h}$$

$$\boxed{= 0.2022 \, \hat{x} - 1.0410 \, \hat{y} - 0.5929 \, \hat{z} \text{ km/s}}$$

$$\theta_2^* - \theta_1^* = 151.8108^\circ - 205 = \boxed{-53.1892^\circ}$$

$$E_2 - E_1 = 126.6716^\circ - (-132.1760^\circ) = \boxed{258.8477^\circ}$$

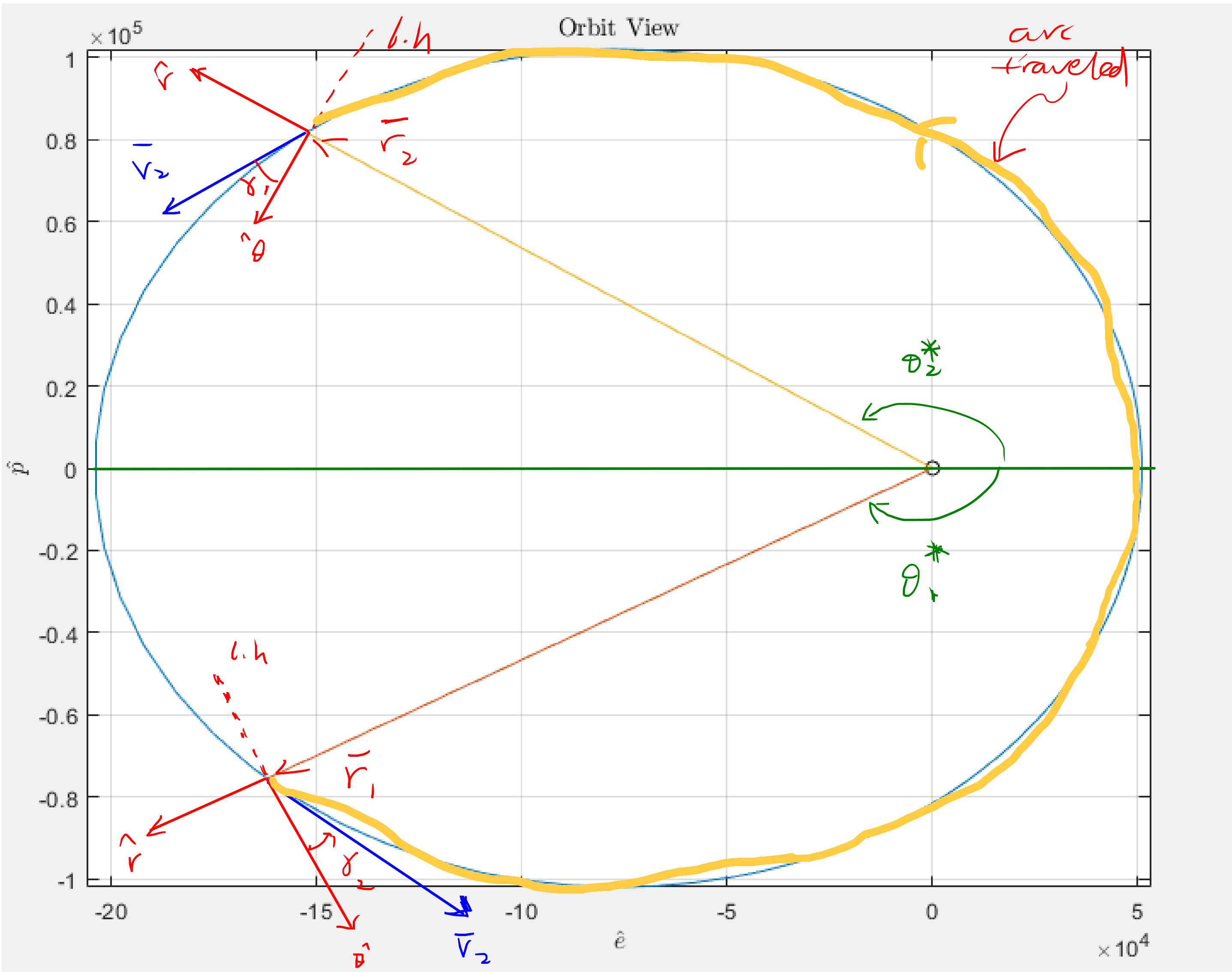
X	Y	Z
-119251.6914772449	-125671.2022335755	-3061.782742233393

v_x	v_y	v_z
0.2022456483708943	-1.040962141783962	-0.5929473913806762

EA	TA	MA	FPA
126.6716481422899	151.8108243820068	99.09846892850742	58.97084250302141

Results from GMAT matches the calculation above.

d)



4. $\bar{r}_1 = 0.15\, R_\oplus \hat{x} - 1.44\, R_\oplus \hat{y} - 0.65\, R_\oplus \hat{z}$

$\bar{v}_1 = 6.62 \hat{x} + 2.7 \hat{y} - 1.56 \hat{z}$ km/s

a) Find a, e, i, ω, Ω, γ, θ^*, M, E, $(t - t_p)$

$r_1 = \|\bar{r}_1\| = 1.0122 \times 10^4$ km

$v_1 = \|\bar{v}_1\| = 7.3176$ km/s

$$-\frac{\mu}{2a} = \frac{v_1^2}{2} - \frac{\mu}{r_1}$$

$$a = -\frac{\mu}{2} / \left(\frac{v_1^2}{2} - \frac{\mu}{r_1}\right) = \boxed{1.5811 \times 10^4 \text{ km}}$$

$h = \|\bar{r}_1 \times \bar{v}_1\| = 7.3092 \times 10^4$ km²/s

$= \sqrt{\mu p}$

$p = h^2/\mu = 1.3403 \times 10^4$ km

$= a(1-e^2)$

$e = \sqrt{1 - p/a} = \boxed{0.3903} < 1$, this is an elliptic orbit

$$\hat{h} = \frac{r_1 \times v_1}{h} = 0.3492 \hat{x} - 0.3551 \hat{y} + 0.8672 \hat{z}$$

$\cos i = 0.8672$

$\boxed{i = 29.8668^\circ}$

$\hat{r}_1 = \frac{\bar{r}_1}{r_1} = 0.0945 \hat{x} - 0.9074 \hat{y} - 0.4096 \hat{z}$

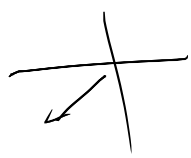
$\hat{\theta}_1 = \hat{h} \times \hat{r}_1 = 0.9323 \hat{x} + 0.2250 \hat{y} - 0.2833 \hat{z}$

$$\begin{cases} \sin\Omega \sin i = 0.3491 \\ -\cos\Omega \sin i = -0.3551 \end{cases}$$

$-\tan\Omega = -\frac{0.3491}{0.3551}$

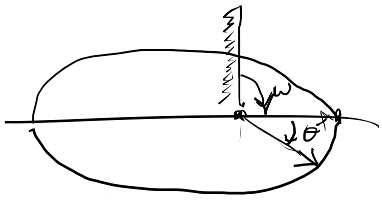
$\boxed{\Omega = 44.5201^\circ}$

$\tan\theta_1 = \frac{-0.4096}{-0.2833}$



$\theta_1 = -124.6678^\circ$

$\dot{r}_1 = \bar{v}_1 \cdot \hat{r}_1 = -1.1852$ km/s < 0, descending orbit



$$\theta_1^* = \pm\cos^{-1}\left(\left(\frac{p}{r_1}-1\right)\cdot\frac{1}{e}\right)$$

$$= \pm 33.8428$$

$$\boxed{= -33.8428^\circ}$$

$$\omega = \theta_1 - \theta_1^* = -124.6678^\circ + 33.8428^\circ = \boxed{-90.8250^\circ}$$

$$\gamma_1 = -\cos^{-1}\left(\frac{\sqrt{\mu p}}{r_1 v_1}\right) = \boxed{-9.3213^\circ} \text{ descending}$$

$$E_1 = 2\cdot\tan^{-1}\left(\tan\frac{\theta_1^*}{2}\cdot\sqrt{\frac{1-e}{1+e}}\right)$$

$$= \boxed{-22.7827^\circ}$$

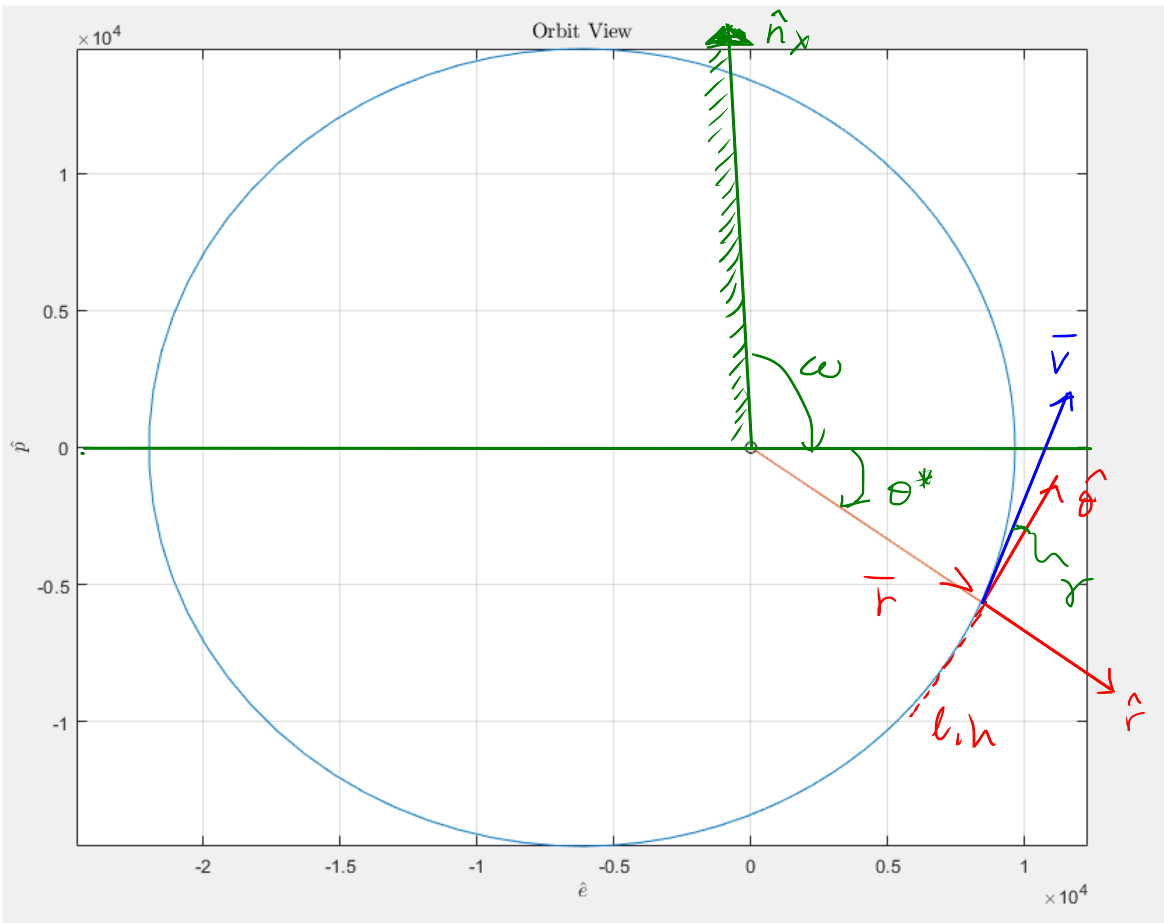
$$M_1 = E_1 - e\sin E_1$$

$$= \boxed{-14.1240^\circ}$$

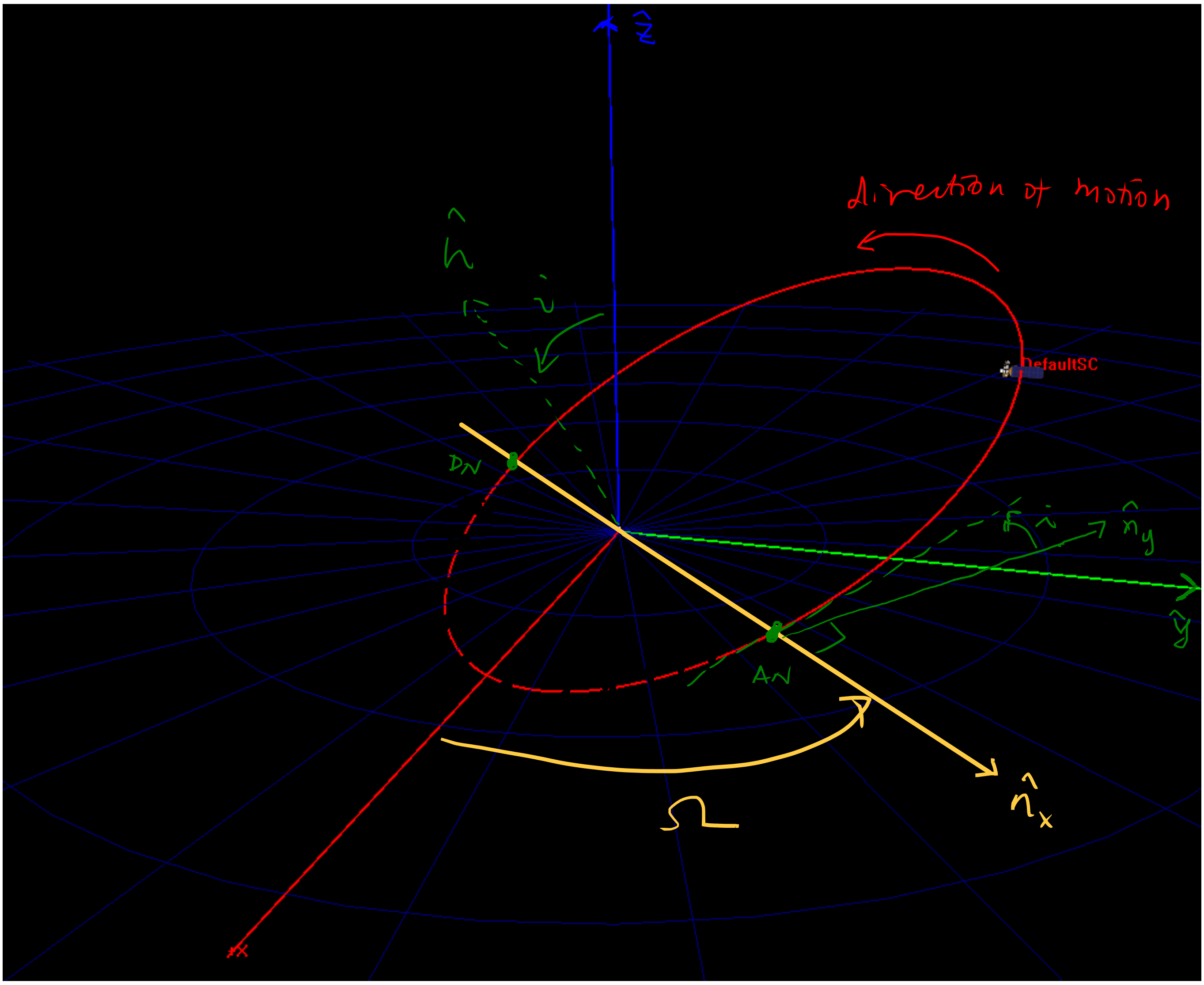
$$t_1 - t_p = M_1 / \sqrt{\frac{\mu}{a^3}}$$

$$\boxed{= -776.2720 \text{ sec}}$$

b)



c)



Periapsis is below the fundamental plane

because AOP is negative, (thus descends below fund. plane)

$\theta^*_{AN} = -\omega = \underline{90.8250^\circ}$

$\theta^*_{DN} = 180^\circ + 90^\circ.8250^\circ = \underline{270.8250^\circ}$