1. $e = 0.6$, $p = 6R_{\oplus}$, $\theta_0^* = 90°$

a) Determine $a, r_p, r_a, \mathbb{P}, \xi, r_0, v_0, E_0^*, \gamma_0$

$\mu = G(M_E + M_{S/C})$ since $M_E >>> m_{S/C}$

$\mu \approx GM_E = 398600.4415$

$$a = \frac{p}{(1-e^2)} = \frac{6R_{\oplus}}{1-0.36} = \boxed{5.9795 \times 10^4 \text{ km}}$$

$$r_p = \frac{p}{1+e} = \boxed{2.3918 \times 10^4 \text{ km}}$$

$$r_a = \frac{p}{1-e} = \boxed{9.5672 \times 10^4 \text{ km.}}$$

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = \boxed{1.4552 \times 10^5 \text{ sec}} = 40.4209 \text{ hours}$$

$$\xi = -\frac{\mu}{2a} = \boxed{-3.3333 \text{ km}^2/s^2}$$

$$r_0 = \frac{p}{1+e\cos\theta_0^*} = \frac{p}{1+e \cdot 0} = p = \boxed{3.8269 \times 10^4 \text{ km}}$$

$$\xi = \frac{v_0^2}{2} - \frac{\mu}{r_0} \Rightarrow v_0 = \sqrt{2(\xi + \frac{\mu}{r_0})} = \boxed{3.7637 \text{ km/s}}$$

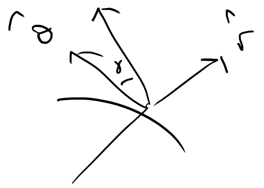
$$\cos E_0^* = \frac{a-r_0}{ae} \Rightarrow E_0^* = \cos^{-1}\left(\frac{a-r_0}{ae}\right) = \boxed{53.1301°}$$

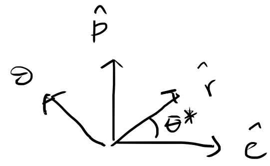
$$h = \sqrt{\mu p} , = r_0 v_0 \cos\gamma_0 \Rightarrow \gamma_0 = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_0 v_0}\right) = \boxed{30.9638°}$$

$$v_c = \sqrt{\frac{\mu}{r}} = 3.2274 \text{ km/s} , \sqrt{2}V_c = 4.5642$$

$\boxed{V_0 < \sqrt{2}V_c}$ This correct as the spacecraft is on an elliptic orbit.

1 b) $\gamma = 30.9638^\circ$, $r_0 = 3.8269\times10^4$ km
$\theta_0^* < 90^\circ$





$\bar{r}_0 = 3.8269\times10^4\ \hat{r}$ km

$\bar{v}_0 = v_0(\sin\gamma_0\,\hat{r} + \cos\gamma_0\,\hat{\theta})$

$\bar{v}_0 = 1.9364\,\hat{r} + 3.2274\,\hat{\theta}$

$\hat{r} = C_{\theta^*}\,\hat{e} + S_{\theta^*}\,\hat{p}$

$\hat{\theta} = -S_{\theta^*}\,\hat{e} + C_{\theta^*}\,\hat{p}$

At this instant. $\hat{r} = \hat{p}$, $\hat{\theta} = -\hat{e}$

$\bar{r}_0 = \boxed{3.8269\times10^4\ \hat{p}\ \text{km}}$

$\bar{v}_0 = \boxed{-3.2274\,\hat{e} + 1.9364\,\hat{p}}$ km/s

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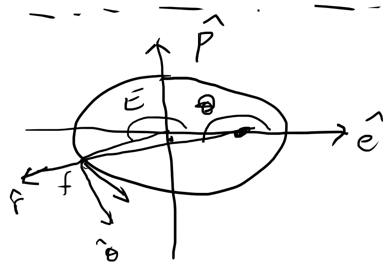
$E_f^* = 225^\circ$



$r_f = a(1 - e\cos E_f^*) = \boxed{8.5164\times10^4\ \text{km}}$

$v_f = \sqrt{2\left(\xi + \frac{\mu}{r_f}\right)} = \boxed{1.6415\ \text{km/s}}$

$r_f = \dfrac{p}{1+e\cos\theta_f^*} \Rightarrow \theta_f^* = \cos^{-1}\left(\dfrac{p}{e r_f} - \dfrac{1}{e}\right) = \boxed{203.4018^\circ}$

$\gamma_f = \cos^{-1}\left(\dfrac{\sqrt{\mu p}}{r_f v_f}\right) = \boxed{-27.9384^\circ}$

$\bar{r}_f = r_f(\cos\theta_f^*\,\hat{e} + \sin\theta_f^*\,\hat{p}) = \boxed{-7.8158\times10^4\,\hat{e} - 3.3825\times10^4\,\hat{p}\ \text{km}}$

$\bar{v}_f = v_f\left[\cos(\theta_f^* + 90^\circ + |\gamma_f|)\,\hat{e} + \sin(\theta_f^* + 90^\circ + |\gamma_f|)\,\hat{\theta}\right] = \boxed{1.2818\,\hat{e} - 1.0255\,\hat{p}\ \text{km/s}}$

$\gamma = \cos^{-1}\left(\dfrac{\sqrt{\mu p}}{r\sqrt{\frac{2\mu}{r}}}\right) = \cos^{-1}\left(\dfrac{r_p}{\sqrt{r}}\right)$

c) Find $t_0 - t_p$, $t_f - t_p$, $t_f - t_0$. $\Delta\theta^*$, ΔE

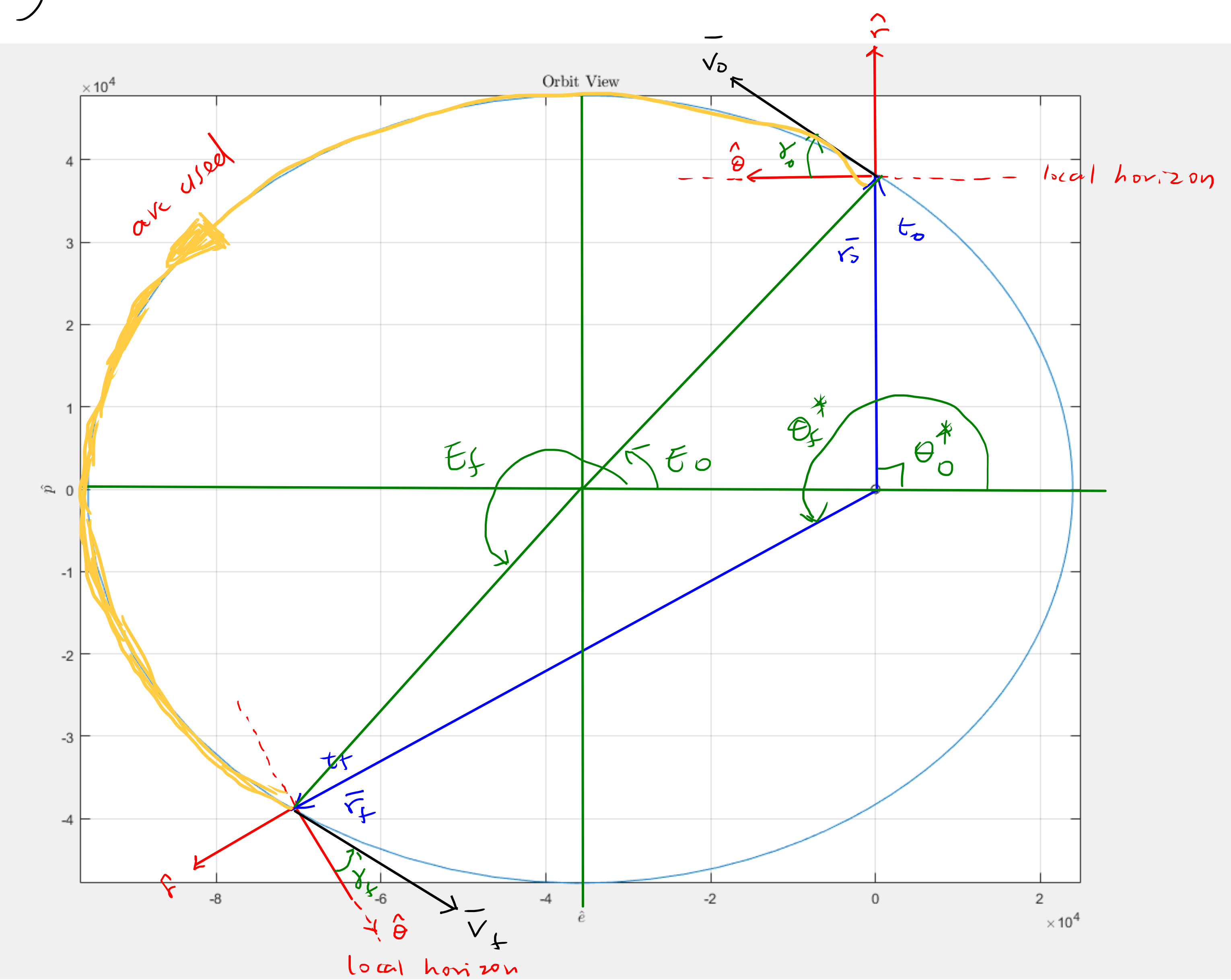
$$\sqrt{\frac{\mu}{a^3}}\,(t - t_p) = E - e\sin E$$

$$t - t_p = \frac{E - e\sin E}{\sqrt{\mu/a^3}}$$

$$\boxed{\begin{aligned} t_0 - t_p &= 1.0359\times10^4 \text{ sec} \\ t_f - t_p &= 1.0077\times10^5 \text{ sec} \\ t_f - t_0 &= 9.0414\times10^4 \text{ sec} \end{aligned}}$$

$$\boxed{\begin{aligned} \Delta\theta^* &= \theta^*_f - \theta^*_0 = 113.4018^\circ \\ \Delta E &= E_f - E_0 = 171.8699^\circ \end{aligned}}$$

d)

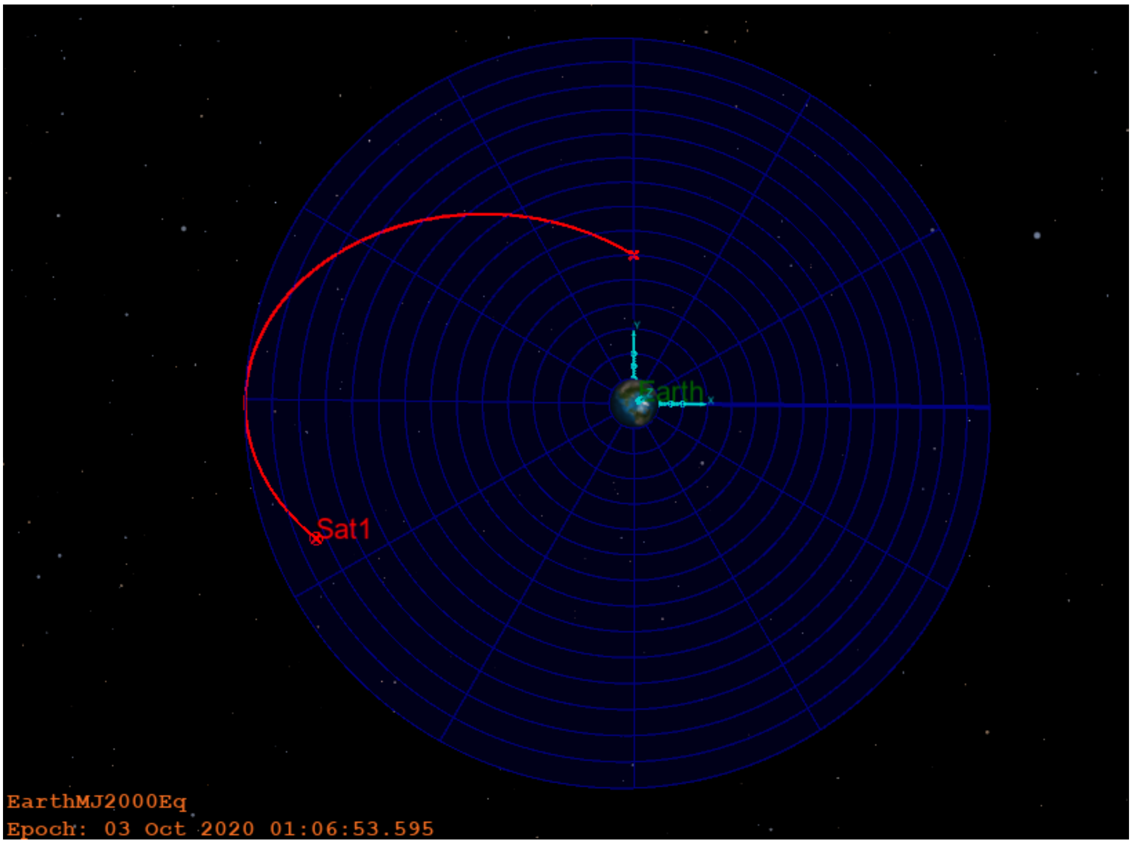


2. a) GMAT takes in semi-major axis (a), eccentricity (e), inclination (INC), right ascension of the ascending node (RAAN), argument of periapsis (AOP) and true anomaly (TA/θ^*) as initial state.

The rest of quantities can be found in the report file parameter list. $r = \sqrt{x^2+y^2}$, $\gamma = FPA$, $E = EA$, $v = \sqrt{vx^2+vy^2}$.

From GMAT, start time is 02 Oct, 2020, 00:00:00.000, end time is 03 Oct 2020, 01:06:53.595. The time difference is 25 hours, 6 minutes, 53.595 seconds. which is 9.0414×10^4 seconds.

This is the same as the calculation in problem 1.
Every other calculation also matches the result from GMAT.

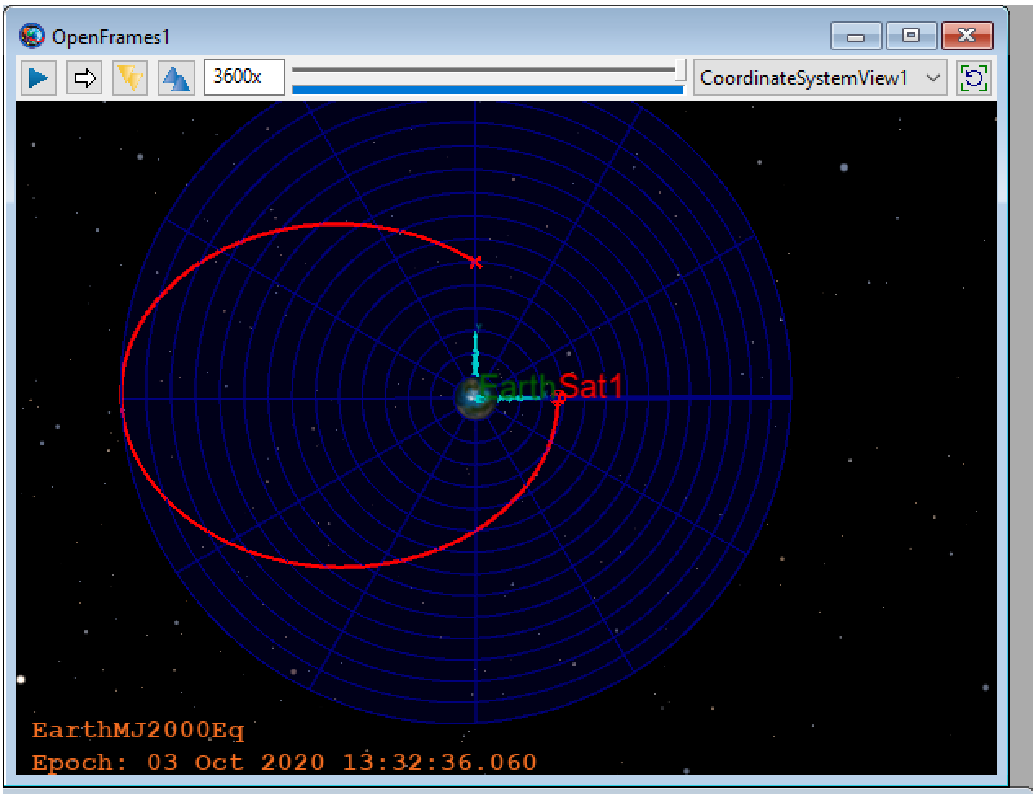


This orbit is also the same as the one plotted in matlab.

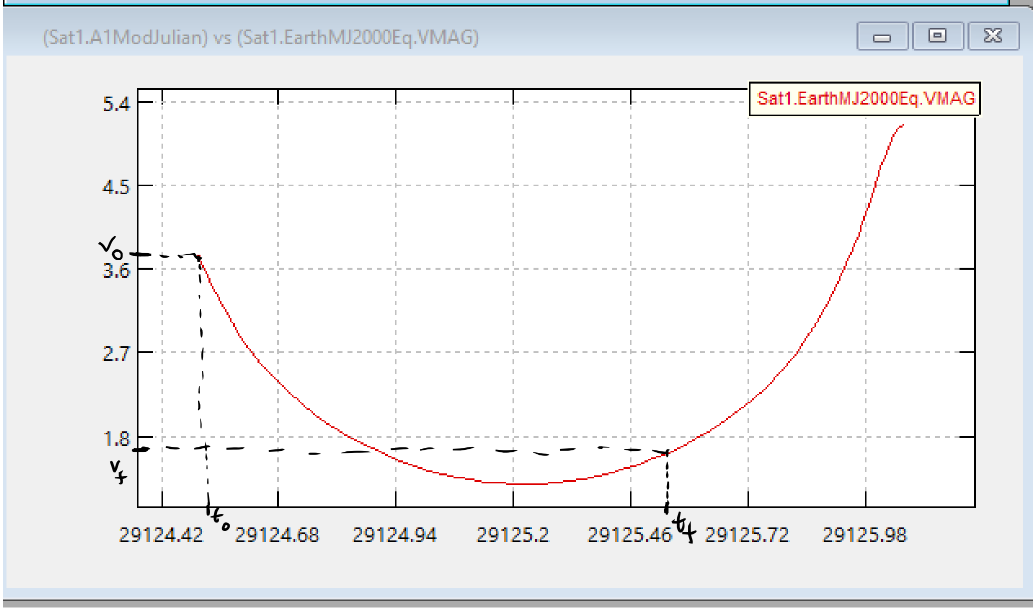
b) Report parameters at t_0 & t_f

Time	γ(deg)	θ^*(deg)	E(deg)	$r\ \hat{e}$(km)	$r\ \hat{p}$ (km)	$v\ \hat{e}$(km/s)	$v\ \hat{p}$ (km/s)
02 Oct 2020 00:00:00.000	59.03624346792647	90	53.13010235415596	-6.15409240559453e-12	38268.80000000002	-3.227352966736025	1.936411780041615
03 Oct 2020 01:06:53.595	117.9383527621815	203.4018390604809	225.0000000786629	-78158.44992306236	-33825.16003131537	1.281831493055231	-1.025465191627488

c)



Propagate from $\theta_0^* = 90°$ till spacecraft reaches periapsis



Yes, the max velocity location is at the periapsis.

3. a) $r_p = R_\oplus + 225$

$$V_c = \sqrt{\frac{\mu}{r_p}} = \boxed{7.7695 \text{ km/s}}$$

$$V_{escape} = \sqrt{2}\, V_c = \boxed{10.9877 \text{ km/s}}$$

$$\frac{V_{escape} - V_c}{V_c} \times 100\% = \boxed{41.4214\%}$$

b) $\frac{v^2}{2} - \frac{\mu}{r} = 0$ $\qquad p = r_p \cdot (1+e) = 2r_p$

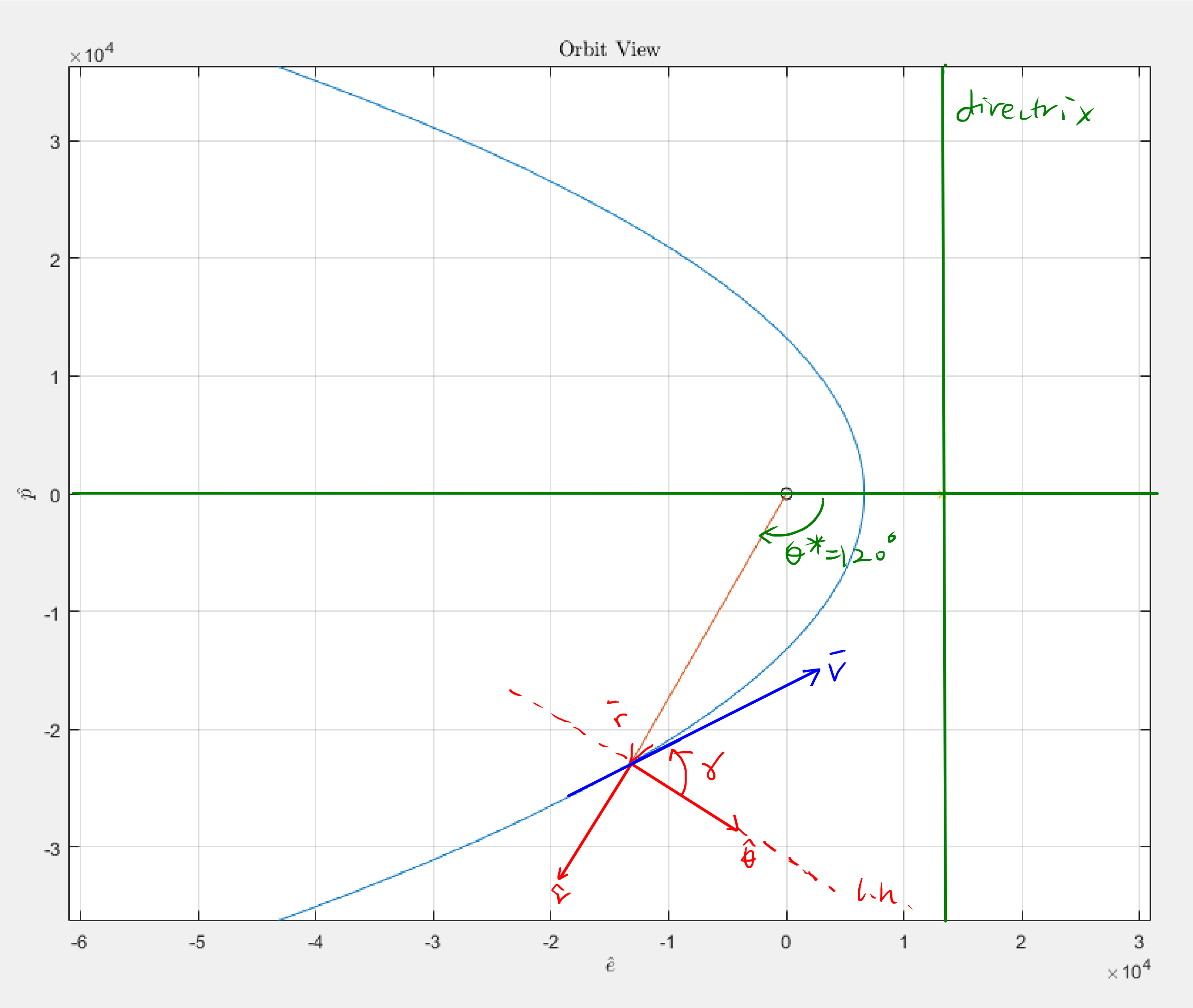
$$\underline{v = \sqrt{\frac{2\mu}{r}}} \qquad r = \frac{p}{1+\cos\theta^*} \Rightarrow \underline{\theta^* = \cos^{-1}\left(\frac{p}{r} - 1\right)}$$

$$6\sqrt{\frac{\mu}{p^3}}\,(t-t_p) = \tan^3\frac{\theta^*}{2} + 3\tan\frac{\theta^*}{2} \Rightarrow \underline{(t-t_p) = \frac{1}{6}\sqrt{\frac{p^3}{\mu}}\left(\tan^3\frac{\theta^*}{2} + 3\tan\frac{\theta^*}{2}\right)}$$

r	v (km/s)	θ^* (deg)	$t - t_p$ (days)
$2R_\oplus$	7.9054	87.9784	0.0176
$10R_\oplus$	3.5354	142.4616	0.1591
$75R_\oplus$	1.2909	166.5056	2.9178
$200R_\oplus$	0.7905	171.7483	12.5471
$800R_\oplus$	0.3953	175.8768	99.7995
$420R_\oplus$	0.5455	174.3084	38.0299

← extra distance

c)



$$\boxed{\theta^* + \gamma^* = 180° \quad \text{at } \theta^* = 120°}$$

$$\left.\begin{aligned} r_{\theta^*=120} &= 2.6413 \times 10^4 \text{ km} \\ V_{\theta^*=120} &= 5.4939 \text{ km/s} \end{aligned}\right\} \quad \gamma = 60°$$

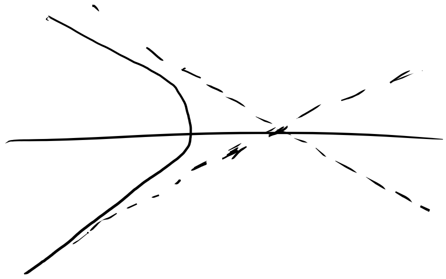
d) At $r = 75 R_\oplus$, the acceleration due to the Earth is

$$\frac{\mu_\oplus}{r^2} = 1.7419 \times 10^{-6} \text{ km/s}^2 .$$

Even though it is small, but over course of the flight in terms of days, its affect on the spacecraft is still significant. ($V_{75 R_\oplus} \rightarrow V_{200 R_\oplus}$)

Thus we should still model it as a two body problem at this point.

4.



$|a| = 7050$ $\qquad \theta^*_0 = -60^\circ$

$$\boxed{r_p = R_⊕ + 800 = 2.5382 \times 10^3 \text{ km}}$$

$$v_p = \sqrt{\frac{2\mu}{r_p} + \frac{\mu}{|a|}} \doteq \boxed{2.1351 \text{ km/s}}$$

$$e = 1 + \frac{r_p}{|a|} = 1.3600$$

$$b = |a| \cdot \sqrt{e^2 - 1} = \boxed{6.495 \times 10^3 \text{ km}}$$

$$h = \sqrt{p \cdot \mu} = \boxed{5.4193 \times 10^3 \text{ km}^2/\text{s}}$$

$$\delta = 2\sin^{-1}\left(\frac{1}{e}\right) = \boxed{94.6616^\circ}$$

$$v_\infty = \sqrt{\frac{\mu}{|a|}} = \boxed{0.8339 \text{ km/s}}$$

$$\xi = \frac{\mu}{2|a|} = \boxed{0.3477 \text{ km}^2/\text{s}^2}$$

At t_0, $\theta^*_0 = -60^\circ$

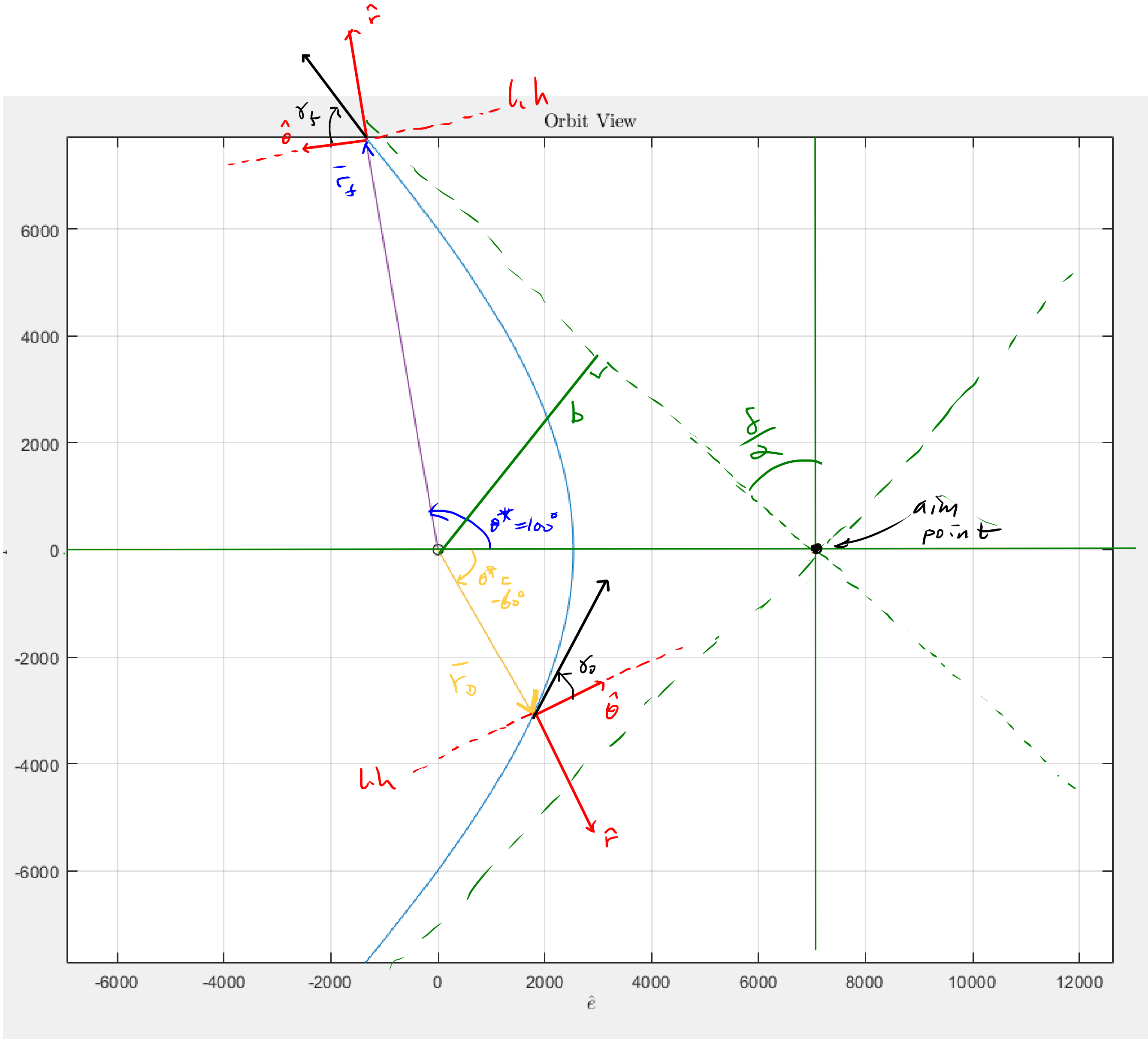
$$\boxed{r_0 = 3.5656 \times 10^3 \text{ km}} \qquad \boxed{v_0 = 1.8562 \text{ km/s}}$$

$$\gamma_0 = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_0 v_0}\right) = \boxed{35.0334^\circ}$$

$$H = \cosh^{-1}\left(\frac{r_0}{|a|e} + \frac{1}{e}\right) = \boxed{26.2924^\circ}$$

$$t_0 - t_p = \frac{e \sinh H_0 - H_0}{\sqrt{\mu / |a|^3}} = \boxed{1.5838 \times 10^3 \text{ sec}}$$

b). c)



At t_f, $\theta^*_f = 100°$

$r_f = 7.8423 \times 10^3$ km

$v_f = 1.3949$ km/s

$\gamma_f = 60.3041°$