1. $a_{comet} = 8AU = 8\times1.5\times10^8 = 1.2\times10^9$ km. $\mu_s = 1.3\times10^{11}$ km³/s²

$e_{comet} = 0.8$.

$v_J = 13$ $\theta^*_J = 90°$

$r_J = r_{Jup} = 5.2\text{ AU} = 7.8\times10^8$ km $\gamma_J = 5°$

$$e_J = \sqrt{\left(\frac{r_J v_J^2}{\mu_s} - 1\right)^2 \cdot \cos^2\gamma_J + \sin^2\gamma_J} = \sqrt{\left(\frac{7.8\times10^8 \times 13^2}{1.3\times10^{11}} - 1\right)^2 \cdot \cos^2(5) + \sin^2(5)} = 0.0883$$

$$a_J = -\mu_s \Big/ 2\left(\frac{V_J^2}{2} - \frac{\mu_s}{r_J}\right) = -1.3\times10^{11}/2\left(\frac{13^2}{2} - \frac{1.3\times10^{11}}{7.8\times10^8}\right) = 7.9108\times10^8 \text{ km}.$$

$$25R_J = 25\times7\times10^4 = 1.75\times10^6 \text{ km} = r_{p.h.}$$

a) $\boxed{r_c^- = r_J = 7.8\times10^8 \text{ km}}$

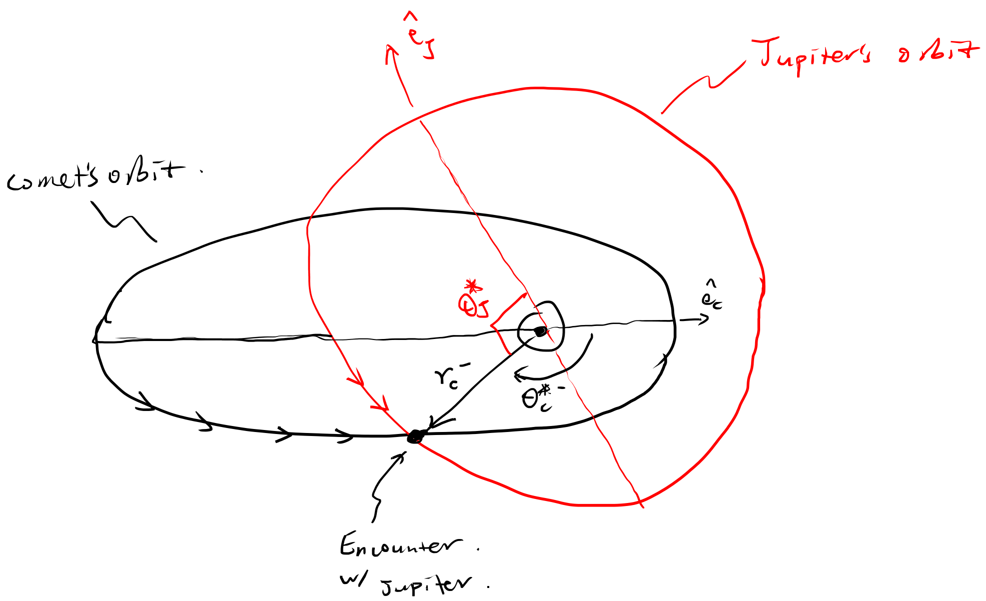
$$P_c = a_c\cdot(1-e_c^2) = 1.2\times10^9\times(1-0.8^2) = 4.32\times10^8 \text{ km}.$$

$$\theta_c^{*-} = \cos^{-1}\left(\frac{P_c}{r_c\cdot e_c} - \frac{1}{e_c}\right) = \cos^{-1}\left(\frac{4.32\times10^8}{7.8\times10^8\times0.8} - \frac{1}{0.8}\right) = \boxed{-123.8964°}$$ inbound/descend.

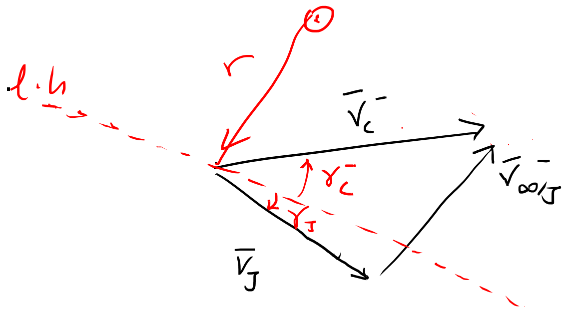
$$V_{comet}^- = V_c^- = \sqrt{\frac{2\mu_s}{r_c} - \frac{\mu_s}{a_c}} = \sqrt{\frac{2\times1.3\times10^{11}}{7.8\times10^8} - \frac{1.3\times10^{11}}{1.2\times10^9}} = \boxed{15 \text{ km/s}}$$

$$\gamma_c^- = \cos^{-1}\left(\frac{\sqrt{\mu_s P_c}}{r_c v_c}\right) = \cos^{-1}\left(\frac{\sqrt{1.3\times10^{11}\times4.32\times10^8}}{7.8\times10^8\times15}\right) = \boxed{-50.1699°}$$ inbound/descend

$$E_c^- = 2\tan^{-1}\left(\tan\frac{\theta_c^{*-}}{2}\cdot\sqrt{\frac{1-e_c}{1+e_c}}\right) = -64.0556°$$



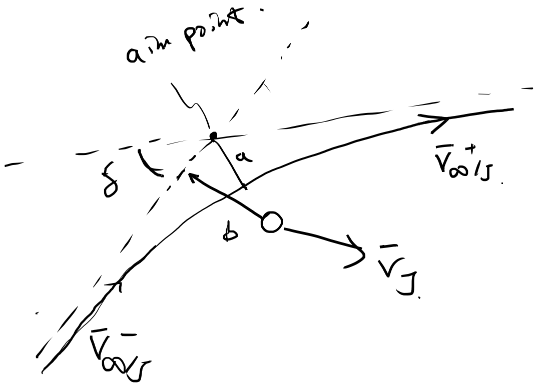
1 b)



$$V_{\infty/J} = \sqrt{V_c^{-2} + V_J^2 - 2V_c^- V_J \cos(|\gamma_J| + |\gamma_c^-|)}$$

$$= \sqrt{15^2 + 13^2 - 2 \times 15 \times 13 \times \cos(5 + 50.1699)}$$

$$= 13.0864 \text{ km/s}.$$



$$r_{p.h} = 25 R_J = 1.75 \times 10^6 \text{ km}.$$

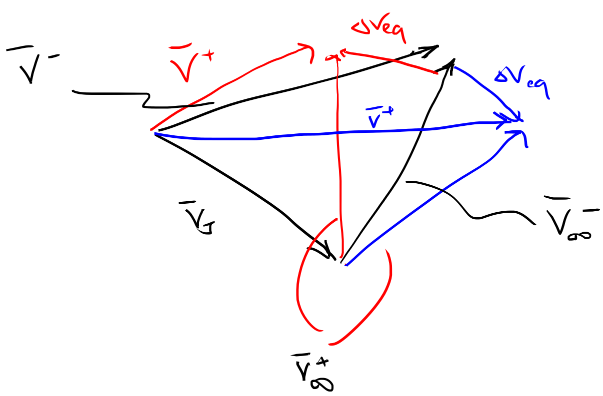
$$|a_h| = \frac{\mu_J}{V_{\infty/J}^2} = \frac{1.3 \times 10^8}{13.0864^2} = \boxed{7.5911 \times 10^5 \text{ km} = 10.8444 R_J}$$

$$e_h = \frac{r_{p.h}}{|a_h|} + 1 = \frac{1.75 \times 10^6}{7.5911 \times 10^5} + 1 = 3.3053$$

$$\delta = 2 \cdot \sin^{-1} \frac{1}{e_h} = \boxed{35.2206^\circ}$$

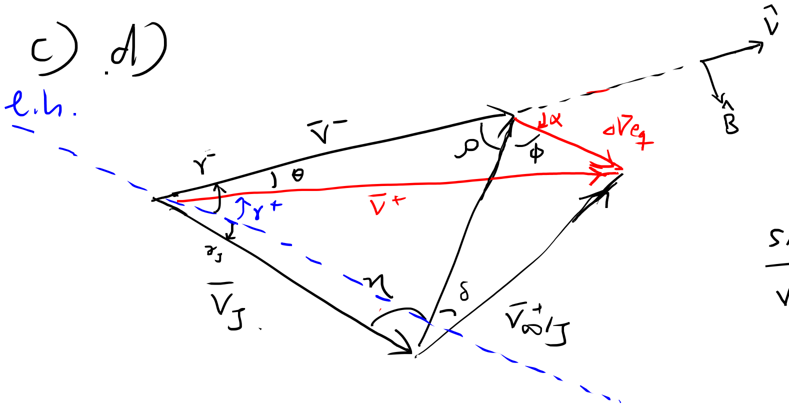
$$|b_h| = |a_h| \cdot \sqrt{e_h^2 - 1} = \boxed{2.3915 \times 10^6 \text{ km} = 34.1643 R_J}$$

The comet will pass behind Jupiter, because we know the energy of the comet's orbit increases.



As shown here, $\bar{V}^+$ with the blue ΔV_{eq} has a larger magnitude, indicating increased energy. The blue one corresponds to passing behind Jupiter.

c) d)



$$\phi = \frac{180° - \delta}{2} = \frac{180° - 35.2206°}{2} = 72.3897°$$

$$\frac{\sin\rho}{V_J} = \frac{\sin(|\gamma_J| + |\gamma^-|)}{V_{\infty/J}}$$

$$\rho = \sin^{-1}\left(\frac{13}{13.0864} \cdot \sin(55.1699)\right) = 54.6299°$$

$$|\Delta V_{eq}| = \sqrt{2V_{\infty/J}^2(1-\cos\delta)} = \sqrt{2 \times 13.0864^2 \times (1-35.2206)}$$

$$= \boxed{7.9184 \text{ km/s}}$$

$$\alpha = 180° - \phi - \rho = \boxed{52.9804°}$$

$$\bar{\Delta V}_{eq} = 7.9184 \cdot (\sin\alpha\,\hat{V} + \cos\alpha\,\hat{B})$$

$$= 6.3222\,\hat{V} + 4.7675\,\hat{B} \text{ km/s}$$

$$V^+ = \sqrt{V^{-2} + \Delta V_{eq}^2 - 2V^-\Delta V_{eq}\cdot\cos(\rho+\phi)}$$

$$= \sqrt{15^2 + 7.9184^2 - 2\times15\times7.9184\cdot\cos(127.0196)}$$

$$= \boxed{20.7540 \text{ km/s}}$$

$$\theta = \sin^{-1}\left(\frac{\Delta V_{eq}}{V^+}\sin(\rho+\phi)\right)$$

$$= 17.7359°.$$

$$|\gamma^+| = |\gamma^-| - \theta = 32.434°$$

$$\boxed{\gamma^+ = -32.434°}$$

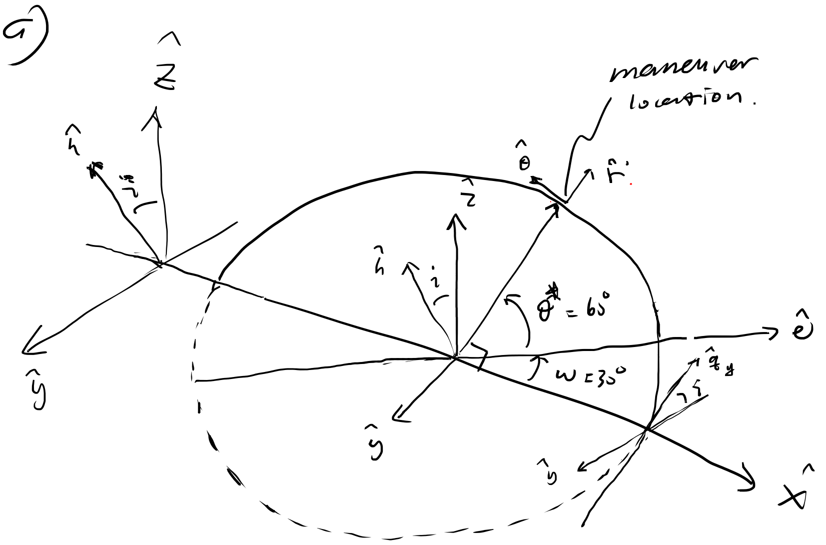
2. $a = 5R_\oplus = 32000$ km

$e = 0.4$ $\quad \Omega = 0°$ (RAAN)

$i = 30°$ $\quad \omega = 30°$ (AOP)

Maneuver at $\theta^* = 60°$

a)



$\hat{h}$ is in $\hat{y}$-$\hat{z}$ plane.

because $\Omega = 0°$, $\hat{h}$ is only rotated by $i = 30°$ about $\hat{x}$-axis.

b) $\Delta \bar{V} = -\frac{1}{2}\hat{y}$ km/s $\qquad \theta = 60° + 30° = 90°$

$$C_{313} = \begin{bmatrix} \cancel{C_\Omega C_\theta} - \cancel{S_\Omega} C_i S_\theta & -C_\Omega S_\theta - \cancel{S_\Omega C_i C_\theta} & \cancel{S_\Omega S_i} \\ \cancel{S_\Omega C_\theta} + C_\Omega C_i S_\theta & -\cancel{S_\Omega} S_\theta + C_\Omega C_i \cancel{C_\theta} & -C_\Omega S_i \\ S_i S_\theta & S_i \cancel{C_\theta} & C_i \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ C_i & 0 & -S_i \\ S_i & 0 & C_i \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0.866 & 0 & -0.5 \\ 0.5 & 0 & 0.866 \end{bmatrix}$$

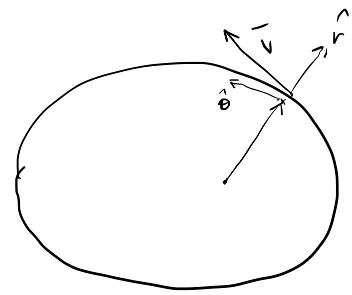
$$\Delta \bar{V}_{xyz} = C_{313} \cdot \Delta \bar{V}_{r\theta h}$$

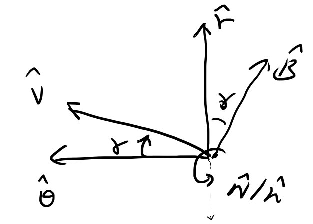
$$\Delta \bar{V}_{r\theta h} = \begin{bmatrix} 0 & 0.866 & 0.5 \\ -1 & 0 & 0 \\ 0 & -0.5 & 0.866 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.433 \\ 0 \\ 0.25 \end{bmatrix} = -0.433\,\hat{r} + 0.25\,\hat{h} \text{ km/s}$$

$$p = a(1-e^2) = 26880 \text{ km.} \qquad r = \frac{p}{1+e\cos\theta^*} = \frac{26800}{1+0.4\cos(60)} = 22400 \text{ km}$$

$$v = \sqrt{\frac{2\mu_E}{r} - \frac{\mu}{a}} = \sqrt{\frac{2\times 4\times10^5}{22400} - \frac{4\times10^5}{32000}} = 4.8181 \text{ km/s}$$

$$\gamma = \cos^{-1}\left(\frac{\sqrt{\mu_E p}}{rv}\right) = \cos^{-1}\left(\frac{\sqrt{4\times10^5\times 26880}}{22400\times 4.8181}\right) = 16.1013° \text{ (ascend)}$$





$\hat{r} = \sin(\gamma)\hat{V} + \cos(\gamma)\hat{B}$

$= 0.2773\hat{V} + 0.9608\hat{B}$

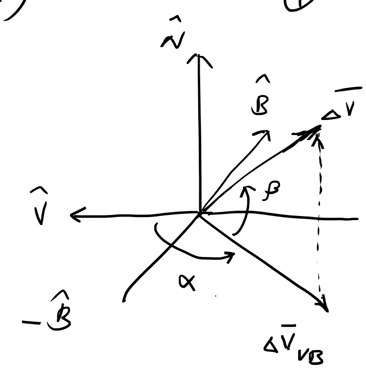
$\hat{h} = \hat{N}$

$$\Delta\bar{V}_{VNB} = -0.433 \times 0.2773\hat{V} - 0.433 \times 0.9608\hat{B} + 0.25\hat{N}$$

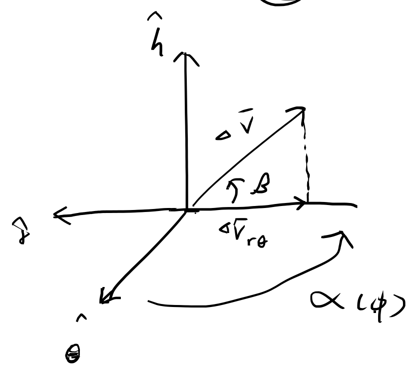
$$= \boxed{-0.1201\hat{V} + 0.25\hat{N} - 0.4160\hat{B} \text{ km/s}}$$

c) d)

①



②



① $\alpha = -90° - \gamma = \boxed{-106.1013°}$ $\qquad \beta = \tan^{-1}\left(\frac{0.25}{0.433}\right) = \boxed{30°}$

$|\Delta\bar{V}_{VB}| = \sqrt{0.1201^2 + 0.4160^2} = 0.433$ km/s

② $\alpha = \boxed{-90°}$ $\quad \beta = \boxed{30°}$ $\qquad |\Delta\bar{V}_N| = 0.25$ km/s

$|\Delta\bar{V}_{r\theta}| = 0.433$ km/s.

Since VNB is rθh rotated by γ about $\hat{h}$, there are now two ways of representing $\Delta\bar{V}$.

On the right, ① shows that α is the angle between $+\hat{V}$ and $\Delta\bar{V}_{VB}$, the in-plane component of $\Delta\bar{V}$. β is the angle between the in-plane component $\Delta\bar{V}_{VB}$ & actual $\Delta\bar{V}$ vector.

On the left, ② shows α is the angle between $+\hat{\theta}$ & $\Delta\bar{V}_{r\theta}$, the in-plane component.. β is the angle between the in-plane component $\Delta\bar{V}_{r\theta}$ and $\Delta\bar{V}$.

3. a). $c = \sqrt{(3R_\oplus)^2 + (4R_\oplus)^2} = \boxed{5R_\oplus = 32000 \text{ km}}$

b). $r_1 = 3R_\oplus = 19200$ km

$r_2 = 4R_\oplus = 25600$ km

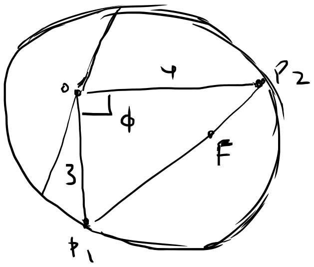
$s = 6R_\oplus$

3 4 5

$a_{min} = \frac{1}{4}(r_1 + r_2 + c) = \boxed{3R_\oplus = 19200 \text{ km}}$

$\varepsilon = -\frac{\mu_E}{2a_m} = \boxed{-10.4167 \text{ km}^2/\text{s}^2}$

c)



$\cos\phi = 0.$

$a=3,\ c=5,\ r_1=3,\ r_2=4.$

$$a c^2 p^2 + r_1 r_2 (1-0)\left[-2a(r_1+r_2) + r_1 r_2 (1+0)\right] p + a r_1^2 r_2^2 (-1)^2 = 0$$

$$(3\times25)R_\oplus^3 p^2 + 12R_\oplus^2\left[(-2\times3\times7)R_\oplus^2 + 12R_\oplus^2\right] p + 3\times9\times16\, R_\oplus^5 = 0.$$

$$75p^2 - 360R_\oplus p + 432R_\oplus^2 = 0.$$

$$25p^2 - 120R_\oplus R + 144R_\oplus^2 = 0.$$

$$(5p - 12R_\oplus)^2 = 0.$$

$$5p = 12R_\oplus$$

$$p = 2.4R_\oplus \quad \text{(Proven)}.$$

$$e = \sqrt{1 - \frac{p}{a}} = \sqrt{1 - \frac{2.4}{3}} = \sqrt{0.2} = \boxed{0.4472}$$

$$b = a\cdot\sqrt{1-e^2} = 3R_\oplus\cdot\sqrt{1-1+\frac{p}{a}} = 2.6833R_\oplus = \boxed{17173.\text{ km}}$$

d). $r_p = a(1-e) = \boxed{10613.76 \text{ km}}$

$r_a = a(1+e) = \boxed{27786.24 \text{ km}}$

$r_{dep} = r_1 = \boxed{19200 \text{ km}}$

$v_{dep} = \sqrt{\frac{2\mu_E}{r_{dep}} - \frac{\mu_E}{a}} = \boxed{4.5644 \text{ km/s}}$

$\gamma_{dep} = \cos^{-1}\left(\frac{\sqrt{\mu p}}{r_{dep} v_{dep}}\right) = \boxed{26.5662^\circ}$ ascending.

$E_{dep} = \boxed{90^\circ}$ $(r=a)$

$\theta^*_{dep} = \cos^{-1}\left(\frac{p}{r_{dep} e} - \frac{1}{e}\right) = \boxed{116.5659^\circ}$ ascending.

$\theta^*_{arr} = \cos^{-1}\left(\frac{p}{r_{arr} e} - \frac{1}{e}\right) = \boxed{-153.4384^\circ}$ descending

e)