HW-2 Zhanpeng Yang

1.6.6.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\boxed{A_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}$$

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$$A_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{array}\right]$$

$$\boxed{A_2^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{bmatrix}}$$

1.6 6

$$A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\boxed{A_3^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}$$

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B: square matrix

show $A = B + B^T$ is always symmetric

& $K = B - B^T$ is always skew symmetric

Find A & K when $B = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$

$$A^T = (B + B^T)^T = B^T + B = A$$

Thus A is symmetric

$$K^T = (B - B^T)^T = B^T - B = -(B - B^T) = -K$$

$K^T = -K$. thus K is skew-symmetric

For $B = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$. $A = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ $K = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$$B = \frac{1}{2}(A + K)$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

2! 2.

a) Let $v_1 = (0, a_1, b_1)$, $v_2 = (0, a_2, b_2)$

$v_1 + v_2 = (0, a_1 + a_2, b_1 + b_2)$ (In subspace)

$c \cdot v_1 = (0, ca_1, cb_1)$ (In subspace)

Thus this is a subspace

b) Let $v_1 = (1, a_1, b_1)$, $v_2 = (1, a_2, b_2)$

$v_1 + v_2 = (2, a_1 + a_2, b_1 + b_2)$ (not in subspace)

Thus this is not a subspace

c) Let v_1 be $(a_1, 0, b_1)$
v_2 be $(a_2, b_2, 0)$

$v_1 + v_2 = (a_1 + a_2, b_2, b_1)$
which no longer in either b_2 or b_3 plane

Thus ~~it~~ is not a subspace

d) $a(1,1,0) + b(2,0,1)$
(a & b can be any scalar),

Let $v_1 = a_1(1,1,0) + b_1(2,0,1)$
$v_2 = a_2(1,1,0) + b_2(2,0,1)$

$v_1 + v_2 = (a_1 + a_2)(1,1,0) + (b_1 + b_2)(2,0,1)$
$a_1 + a_2 \in \mathbb{R}$ $\quad$ $b_1 + b_2 \in \mathbb{R}$ (In subspace)

$c \cdot v_1 = ca_1(1,1,0) + cb_1(2,0,1)$ (In subspace)

This is a subspace

e) Let $v_1 = (a_1, b_1, c_1)$

$v_2 = (a_2, b_2, c_2)$

$v_1 + v_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

$c_1 + c_2 - (b_1 + b_2) + 3(a_1 + a_2)$

$= c_1 - b_1 + 3a_1 + c_2 - b_2 + 3a_2$

$= 0 + 0 = 0$. (In subspace)

$kv_1 = (ka_1, kb_1, kc_1)$

$kc_1 - kb_1 + 3ka_1 = k(c_1 - b_1 + 3a_1) = 0$ (In subspace)

Thus it is a subspace

2.1 4.

Let A be a symmetric matrix, $A_{ij} = A_{ji}$, $A_{ij} \in \mathbb{R}$

B be a lower triangular matrix. $B_{ij} = 0$, if $i < j$

The smallest subspace S that contains all As & Bs

is $S = \{ \alpha A + \beta B \mid \alpha, \beta \in \mathbb{R} \}$

A matrix $C \in S$ has entries that $\begin{cases} C_{ij} \in \mathbb{R} & i < j \\ C_{ij} \in \mathbb{R} & i \geq j \end{cases}$

$S = \mathbb{R}^{3 \times 3}$

The largest subspace contained in both subspaces

is the subspace of a diagonal matrix

$S_A \cap S_B = D$

$$\begin{bmatrix} d_1 & & 0 \\ & d_2 & \\ 0 & & d_3 \end{bmatrix}$$

2.2 4.

$$Mx = b$$

$$M = \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 0 & 0 \\ 3 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$[M \,\vdots\, b] = \left[\begin{array}{cc|c} 1 & 2 & b_2 \\ 0 & 0 & b_1 \\ 0 & 0 & b_3 \\ 0 & 0 & b_4 - 2b_2 \end{array}\right] \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 = b_2$$

$$x_1 = b_2 - 2x_2$$

$$\boxed{x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_2 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix} x_2}$$

complete solution

2.2 6.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 0 & b_1 \\ 0 & 1 & b_2 \\ 2 & 3 & b_3 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & b_1 \\ 0 & 1 & b_2 \\ 0 & 0 & b_3 - 2b_1 - 3b_2 \end{array}\right]$$

The constraint on b is

$$\boxed{b_3 - 2b_1 - 3b_2 = 0}$$