

$$I = 4, \quad J = 10.$$

	$x_{i.}$	$\bar{x}_{i.}$
1	260.8	26.08
2	246.9	24.69
3	299.5	29.95
4	338.4	28.64
	$x_{..}$	$\bar{x}_{..}$
	1145.6	28.64

$$SST_r = \frac{1}{J} \sum_{i=1}^I x_{i.}^2 - \frac{1}{IJ} x_{..}^2$$

$$= \frac{1}{10} \cdot (260.8^2 + 246.9^2 + 299.5^2 + 338.4^2) - \frac{1}{4 \cdot 10} \cdot (1145.6^2)$$

$$= 509.122$$

$$SSE = \sum_{i=1}^I \sum_{j=1}^J (x_{ij} - \bar{x}_{i.})^2$$

$$= 563.134$$

$$SST = \sum_{i=1}^I \sum_{j=1}^J x_{ij}^2 - \frac{1}{IJ} x_{..}^2$$

$$= 1072.256 = SST_r + SSE$$

$$MST_r = \frac{SST_r}{I-1} = 169.7073$$

$$MSE = \frac{SSE}{I(J-1)} = 15.6426$$

$$F = \frac{MST_r}{MSE} = 10.8490$$

$$df_r = I-1 = 3$$

$$df_E = I(J-1) = 36$$

$$f_{0.01, 3, 36} \approx 4.51$$

$10.85 > 4.51$, reject H_0 .

Source of variation	df	Sum of Squares	Mean Square	f
Treatment	3	509.122	169.7073	10.8490
Error	36	563.134	15.6426	
Total	39	1072.256		

10.26 J_i $X_{i.}$

1	4	56.4
2	5	64.0
3	4	55.3
4	4	52.4
5	5	85.7
6	4	72.4
		<hr/>
	n	$X_{..}$
	26	386.2

a)

$$SST = \sum_{i=1}^I \sum_{j=1}^{J_i} X_{ij}^2 - \frac{1}{n} X_{..}^2 = 113.6446$$

$$SST_r = \sum_{i=1}^I \frac{1}{J_i} X_{i.}^2 - \frac{1}{n} X_{..}^2 = 108.1851$$

$$SSE = SST - SST_r = 5.4595$$

$$MST_r = \frac{SST_r}{I-1} = 21.6370$$

$$df_r = 5$$

$$MSE = \frac{SSE}{n-I} = 0.2730$$

$$df_E = 20$$

$$f = \frac{MST_r}{MSE} = 79.2638$$

Assume test at 0.01 level

$$f_{0.01, 5, 20} = 4.10 \quad 79.2638 > 4.10 \quad \underline{\text{reject } H_0.}$$

Source of variance	df	Sum of squares	Mean square	f
Treatment	5	108.1851	21.6370	79.2638
Error	20	5.4595	0.2730	
Total	25	113.6446		

$$b) w_{ij} = Q_{\alpha, I, n-1} \cdot \sqrt{\frac{MSE}{2} \left(\frac{1}{J_i} + \frac{1}{J_j} \right)}$$

$$\text{at } \alpha = 0.01, I = 6, n-1 = 20,$$

$$Q = Q_{0.01, 6, 20} = 5.51$$

$$CI : (\mu_i - \mu_j) \pm w_{ij}$$

i	j	$\mu_i - \mu_j$	w_{ij}	CI	
1.0000	2.0000	1.3000	1.3655	-0.0655	2.6655
1.0000	3.0000	0.2750	1.4394	-1.1644	1.7144
1.0000	4.0000	1.0000	1.4394	-0.4394	2.4394
1.0000	5.0000	-3.0400	1.3655	-4.4055	-1.6745
1.0000	6.0000	-4.0000	1.4394	-5.4394	-2.5606
2.0000	3.0000	-1.0250	1.3655	-2.3905	0.3405
2.0000	4.0000	-0.3000	1.3655	-1.6655	1.0655
2.0000	5.0000	-4.3400	1.2874	-5.6274	-3.0526
2.0000	6.0000	-5.3000	1.3655	-6.6655	-3.9345
3.0000	4.0000	0.7250	1.4394	-0.7144	2.1644
3.0000	5.0000	-3.3150	1.3655	-4.6805	-1.9495
3.0000	6.0000	-4.2750	1.4394	-5.7144	-2.8356
4.0000	5.0000	-4.0400	1.3655	-5.4055	-2.6745
4.0000	6.0000	-5.0000	1.4394	-6.4394	-3.5606
5.0000	6.0000	-0.9600	1.3655	-2.3255	0.4055

$$c) \frac{(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)}{4} - \frac{(\bar{x}_5 + \bar{x}_6)}{2} = \hat{\theta} = -4.1638$$

$$\sum \frac{C_i^2}{J_i} = \left(\frac{1}{4}\right)^2 \cdot \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{5} + \frac{1}{4}\right) = 0.1719$$

for a 0.01 level, (99% CL)

$$t_{\alpha/2, n-1} = t_{0.005, 20} = 2.8453$$

$$CI: -4.1638 \pm 2.8453 \cdot \sqrt{0.273 \cdot 0.1719}$$

$$= \underline{(-4.7801, -3.5474)}$$

10.38

$$\bar{x}_{i.} \quad \sum \sum x_{ij}^2$$

$$I=5, J=6, n=30$$

$$\bar{x}_{..} = \frac{1}{J} \sum \bar{x}_{i.} = 2.448$$

$$\begin{aligned} SST &= \sum_i \sum_j x_{ij}^2 - \frac{(\sum_i \sum_j x_{ij})^2}{n} \\ &= 183.4 - \frac{(2.448 \times 30)^2}{30} \\ &= 3.6189 \end{aligned}$$

$$\begin{aligned} SSE &= \sum_i \sum_j x_{ij}^2 - \sum_i \frac{(\sum_j x_{ij})^2}{J_i} \\ &= 183.4 - \sum_i \frac{(\bar{x}_{i.} \cdot 6)^2}{6} \\ &= 2.6896 \end{aligned}$$

$$SSTr = SST - SSE = 0.9293$$

$$MSTr = \frac{SSTr}{I-1} = 0.2323$$

$$df_{Tr} = I-1 = 4$$

$$MSE = \frac{SSE}{n-I} = 0.1076$$

$$df_E = n-I = 25$$

$$f = \frac{MSTr}{MSE} = 2.1594$$

Source	df	SS	MS	f
Tr	4	0.9293	0.2323	2.1594
Env	25	2.6896	0.1076	
Total	29	3.6189		

$$\text{at } \alpha = 0.05,$$

$$f_{0.05, 4, 25} = 2.7587 > 2.1594. \text{ Not enough evidence to reject } H_0.$$

True average DNA content is unaffected by type of carbohydrates in the diet.

10.39

$$\theta = \mu_1 - (\mu_2 + \mu_3 + \mu_4 + \mu_5)/4$$

$$\hat{\theta} = \bar{x}_1 - (\bar{x}_2 + \bar{x}_3 + \bar{x}_4 + \bar{x}_5)/4 = 0.1650$$

$$\sum \frac{c_i^2}{J_i} = \frac{1}{6} + \frac{1}{4}^2 \left(\frac{1}{6} \cdot 4 \right) = 0.2083$$

for $\alpha = 0.05$, $t_{0.025, 25} = 2.0595$

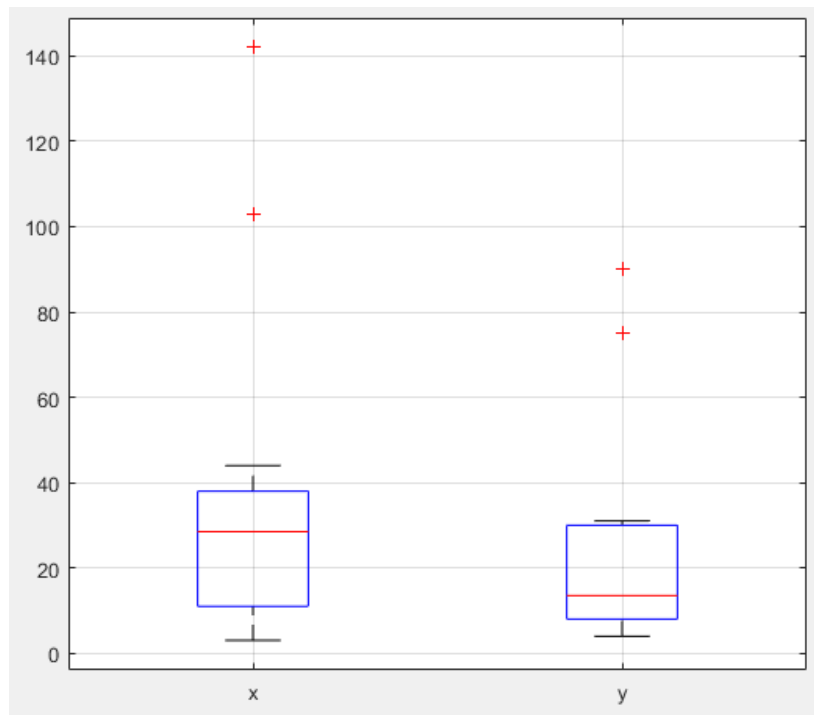
$$CI: 0.1650 \pm 2.0595 \cdot \sqrt{0.1076 \cdot 0.2083}$$

$$= (-0.1433, 0.4733)$$

the interval include zero.

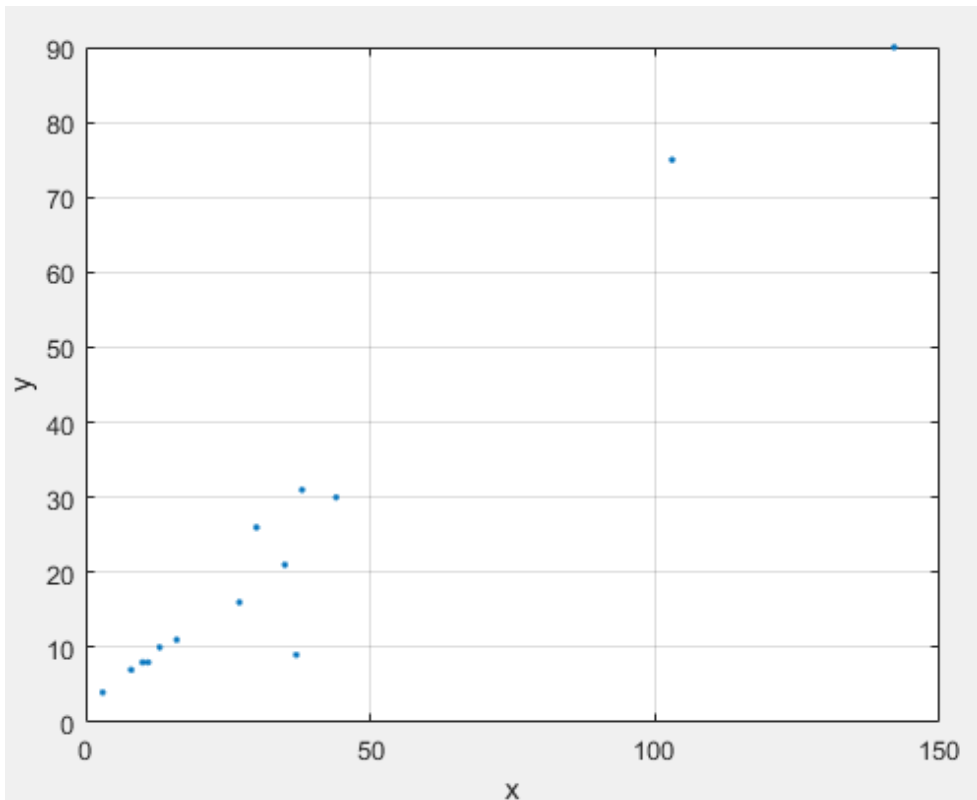
12.4

a)



The IQR of both x & y are quite similar, but the median of x is larger than that of y . Both sets of data have the same number of outliers on the top/larger side.

b)



x & y show a roughly linear correlation, with some outliers.

$$12.9 \quad y = -0.12 + 0.095x$$

$$a) \Delta x = 1 \text{ in}$$

$$\Delta y = 0.095 \text{ m}^3/\text{min}. \quad \text{It is the slope of the regression line.}$$

$$b) \Delta x = 5 \text{ in}$$

$$\Delta y = 5 \times 0.095 = -0.475 \text{ m}^3/\text{min}$$

$$c) \quad x=10, \quad x=15$$

$$My = 0.83$$

$$My = 1.305$$

$$d) \sigma = 0.025$$

$$\begin{aligned} P(y > 0.835) &= 1 - \Phi\left(\frac{0.835 - 0.83}{0.025}\right) \\ &= 1 - 0.5793 \\ &= 0.4207 \end{aligned}$$

$$\begin{aligned} P(y > 0.84) &= 1 - \Phi\left(\frac{0.01}{0.025}\right) \\ &= 0.3446 \end{aligned}$$

$$\begin{aligned} e) P(y_{10} > y_{11}) &= P(y_{10} - y_{11} > 0) \\ &= 1 - \Phi\left(\frac{0.095}{0.0354}\right) \\ &= 0.0036 \end{aligned}$$

$$\mu_{y/11} = 0.925$$

$$E(y_1, y_2) = \mu_{y/10} - \mu_{y/11} = -0.095$$

$$V(y_1, y_2) = 0.025^2 \cdot 2 = 0.00125$$

$$\sigma' = \sqrt{0.00125} = 0.0354$$