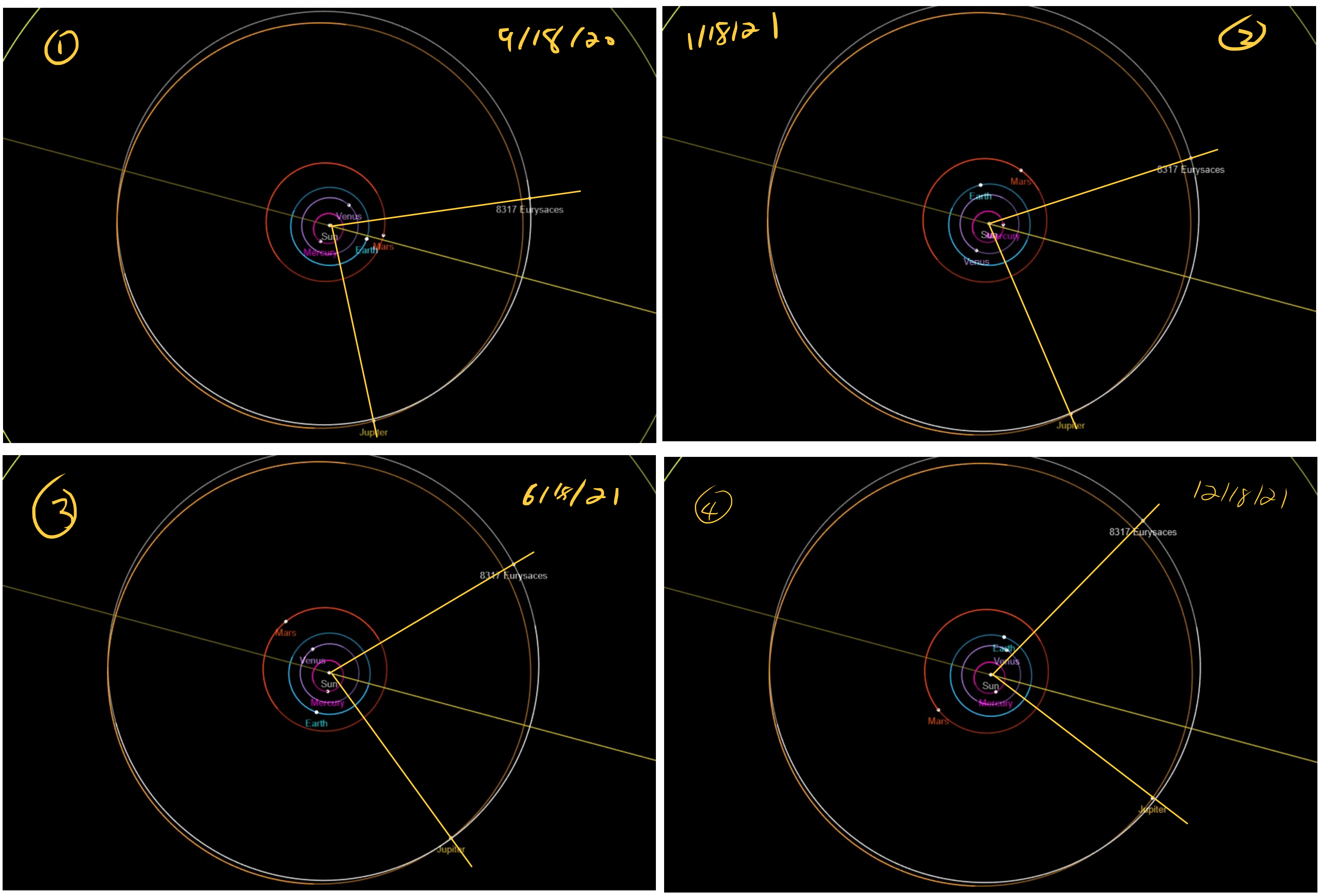
AAE 532 HW-3 Zhampeng Yang

1. a) Eurysaces is in the Greek camp as it in front of the Jupiter.

Its orbit has a period of 12.7 years around the Sun, which is slightly longer that of Jupiter, 11.86 years.



① : 86.5°

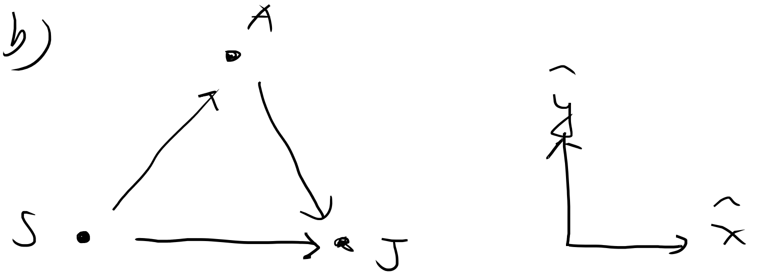
② : 85°

③ : 84°

④ : 82.5°

The angle decreases approximatly 1 degress every 6 months

1. b)



i)

$$\ddot{\bar{r}}_{SA} + \underbrace{G\frac{(m_S+m_A)}{r_{SA}^3}\bar{r}_{SA}}_{\text{Dominant}} = G m_J\Big(\underbrace{\frac{\bar{r}_{AJ}}{r_{AJ}^3}}_{\text{Direct}} - \underbrace{\frac{\bar{r}_{SJ}}{r_{SJ}^3}}_{\text{Indirect}}\Big)$$

$$\bar{r}_{SJ} = r_{SJ}\,\hat{x} \quad . \quad \bar{r}_{SA} = \begin{bmatrix}\cos\frac{\pi}{3} & 0\\ 0 & \sin\frac{\pi}{3}\end{bmatrix}\bar{r}_{SJ}$$

$$\bar{r}_{AJ} = \bar{r}_{SJ} - \bar{r}_{SA}$$

Dominant term: $\|0.1095\,\hat{x} + 0.1897\,\hat{y}\| \times 10^{-6} = \underline{2.1910 \times 10^{-7}\ km/s^2}$

Direct term: $\|0.1046\,\hat{x} - 0.1812\,\hat{y}\| \times 10^{-9} = \underline{2.0919 \times 10^{-10}\ km/s^2}$

Indirect term: $\|0.2092\,\hat{x}\| \times 10^{-9} = \underline{2.0919 \times 10^{-10}\ km/s}$

net perturbing acceleration: $\|\text{Direct} - \text{Indirect}\| = \underline{2.0919 \times 10^{-10}\ km/s^2}$

total acceleration: $\|\text{Direct} - \text{Indirect} - \text{dominant}\| = \underline{2.1931 \times 10^{-7}\ km/s^2}$

1. b)

ii) $$\ddot{\bar{r}}_{JA} + \underbrace{\frac{G(m_J + m_A)}{r_{JA}^3}\bar{r}_{JA}}_{\text{Dominant}} = Gm_s\left(\underbrace{\frac{\bar{r}_{AS}}{r_{AS}^3}}_{\text{Direct}} - \underbrace{\frac{\bar{r}_{JS}}{r_{JS}^3}}_{\text{Indirect}}\right)$$

Dominant : $\| -0.1646\hat{x} + 0.1812\hat{y} \| \times 10^{-9} = \underline{2.0919 \times 10^{-10}\ km/s^2}$

Direct : $\| -0.1095\hat{x} - 0.1897\hat{y} \| \times 10^{-6} = \underline{2.1910 \times 10^{-7}\ km/s^2}$

Indirect : $\| -0.2191\hat{x} \| \times 10^{-6} = \underline{2.1910 \times 10^{-7}\ km/s^2}$

Net perturbing acceleration : $\underline{2.1910 \times 10^{-7}\ km/s}$

Total acceleration : $\underline{2.1931 \times 10^{-7}\ km/s}$

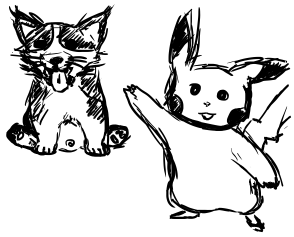
iii) For formulation in part (i), the dominant term is the largest. for formulation in part (ii), the direct & indirect terms, perterbing terms, are equally the largest.

Comparing net perturbations, the Sun has the largest impact.

The net accelerations on the asteroid are the same, and they should be the same. It is because the system remains the same and only reference frame has been shifted. The magnitude should be the same but the direction vector will be different.

Both formulations are correct, because this is simply change of reference frame.

We can model the system as a Sun-Asteroid two-body problem, as the acceleration provided by the Jupiter is significantly small in the order of 10^{-10} km/s.



2 a)

Coordinate System: EarthMJ2000Eq

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****** Changes made to the mission will not be reflected ******
****** in the data displayed until the mission is rerun  ******

        Propagate Command: Propagate1
        Spacecraft       : Sat
        Coordinate System: EarthMJ2000Eq

        Time System   Gregorian                     Modified Julian
        ----------------------------------------------------------------------
        UTC Epoch:    19 Sep 2020 16:40:37.256      29112.1948756433
        TAI Epoch:    19 Sep 2020 16:41:14.256      29112.1953038840
        TT  Epoch:    19 Sep 2020 16:41:46.440      29112.1956763840
        TDB Epoch:    19 Sep 2020 16:41:46.438      29112.1956763655

        Cartesian State                       Keplerian State
        ---------------------------           --------------------------------
        X  =   18030.291531473 km             SMA  =   60005.551177493 km
        Y  =   22.781816125395 km             ECC  =   0.6995224279922
        Z  =   24.319029367737 km             INC  =   44.962697100807 deg
        VX =  -0.0113222174795 km/sec         RAAN =   359.99501442565 deg
        VY =   4.3370803782779 km/sec         AOP  =   0.1093612816155 deg
        VZ =   4.3314356662285 km/sec         TA   =   360.00000000000 deg
                                              MA   =   360.00000000000 deg
                                              EA   =   360.00000000000 deg

        Spherical State                       Other Orbit Data
        ---------------------------           --------------------------------
        RMAG =    18030.322324802 km          Mean Motion        =   4.295183709e-05 deg/sec
        RA   =    0.0723949036935 deg         Orbit Energy       =  -3.3213630545694 km^2/s^2
        DEC  =    0.0772797148050 deg         C3                 =  -6.6427261091389 km^2/s^2
        VMAG =    6.1295782343427 km/s        Semilatus Rectum   =   30642.937174930 km
        AZI  =    45.037355084042 deg         Angular Momentum   =   110518.27128029 km^2/s
        VFPA =    90.000000001591 deg         Beta Angle         =  -1.1272751567757 deg
        RAV  =    90.149573848317 deg         Periapsis Altitude =   11652.186024802 km
        DECV =    44.962592867022 deg         VelPeriapsis       =   6.1295782343427 km/s
                                              VelApoapsis        =   1.0837166694310 km/s
                                              Orbit Period       =   146284.43699959 s

        Planetodetic Properties
        ---------------------------
        LST       =   0.3378541124879 deg
        MHA       =   249.20061968500 deg
        Latitude  =   0.1912789599837 deg
        Longitude =   111.13723442749 deg
        Altitude  =   11652.186262165 km

        Spacecraft Properties
        ------------------------------
        Cd                    =   2.200000
        Drag area             =   15.00000 m^2
        Cr                    =   1.800000
        Reflective (SRP) area =   1.000000 m^2
        Dry mass              =   850.00000000000 kg
        Total mass            =   850.00000000000 kg
        SPADDragScaleFactor   =   1.000000
        SPADSRPScaleFactor    =   1.000000
```

i) Periapsis : 18030.32232 km

ii) Apoapsis : 101980.6451 km

iii) Energy : -3.32136305 MJ/kg

iv) Semi-major axis : 60005.5518 km

v) semi-latus rectum : 30642.93717 km

vi) Angular momentum : 110518.2713 km^2/s

v) X : 18000 km

Y : 0 km

Z : 0 km

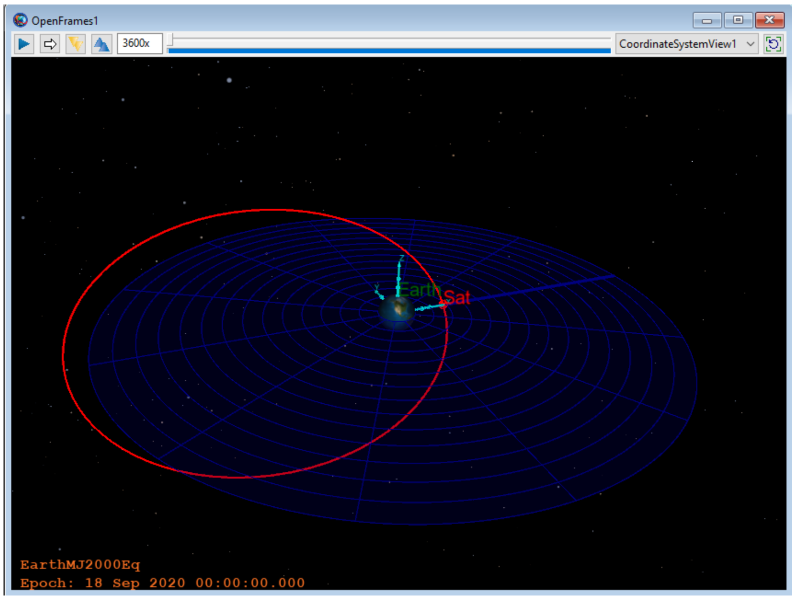
VX = 0 km/s

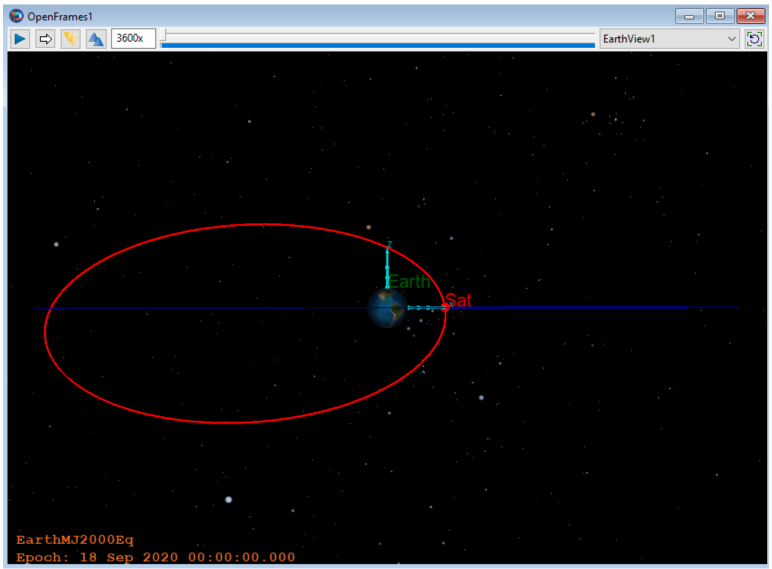
VY = 4.338524937 km/s

VZ = 4.338524937 km/s

These values are associated with the EarthMJ2000Eq frame

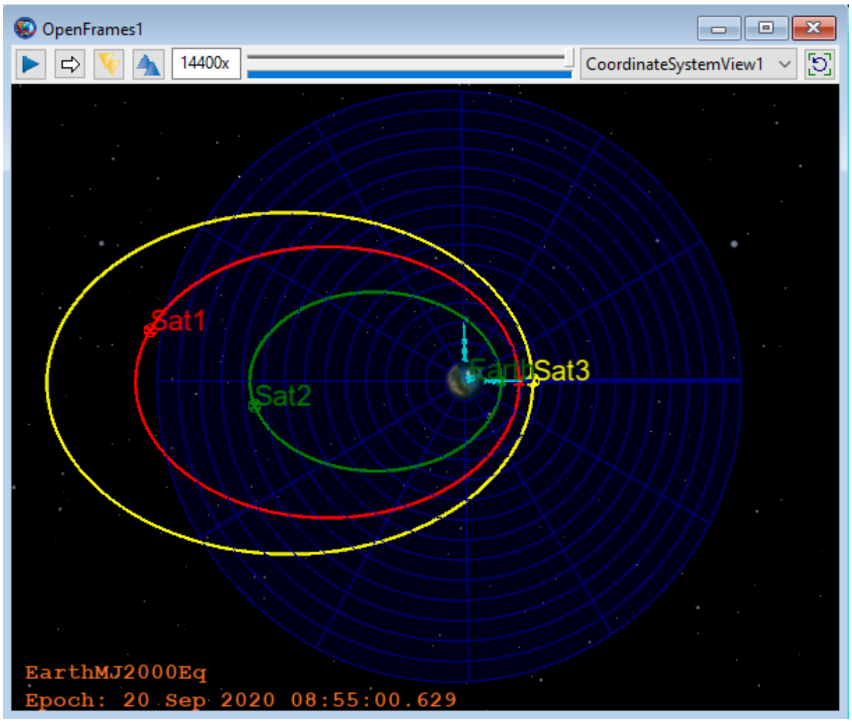
Name	Origin	Axes	Description
EarthMJ2000Eq	Earth	MJ2000Eq	An Earth equator inertial system based on IAU-1976/FK5 theory with 1980 update to nutation.





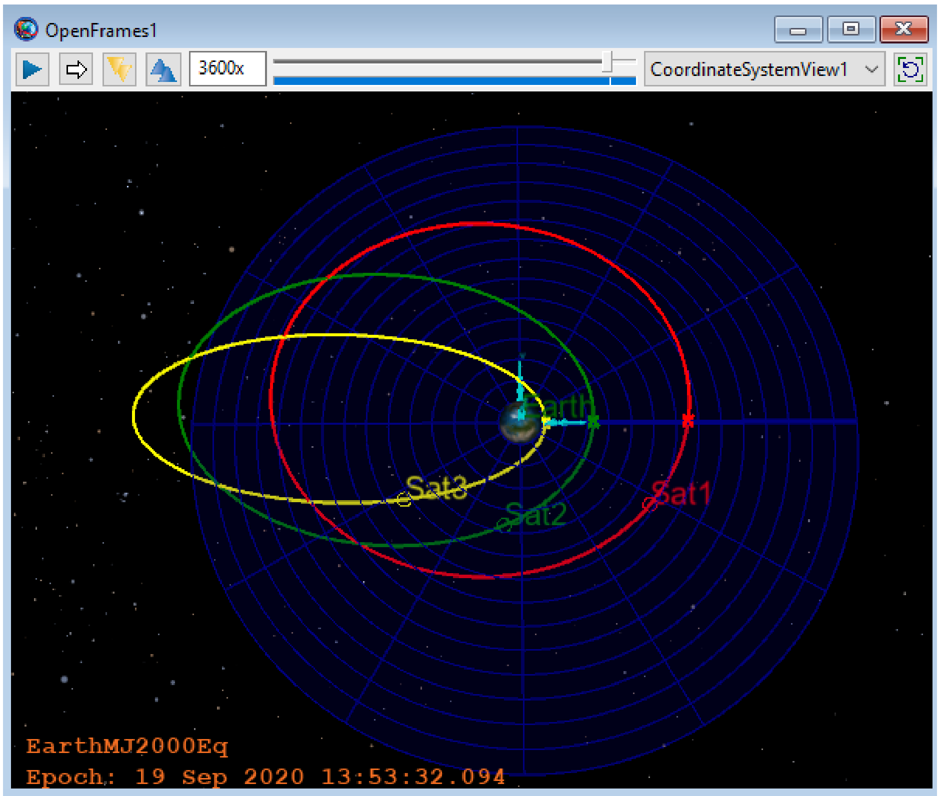
2 b)

case 1: $e = 0.7$, $i = 0^\circ$, $a = 60000, 40000, 75000$



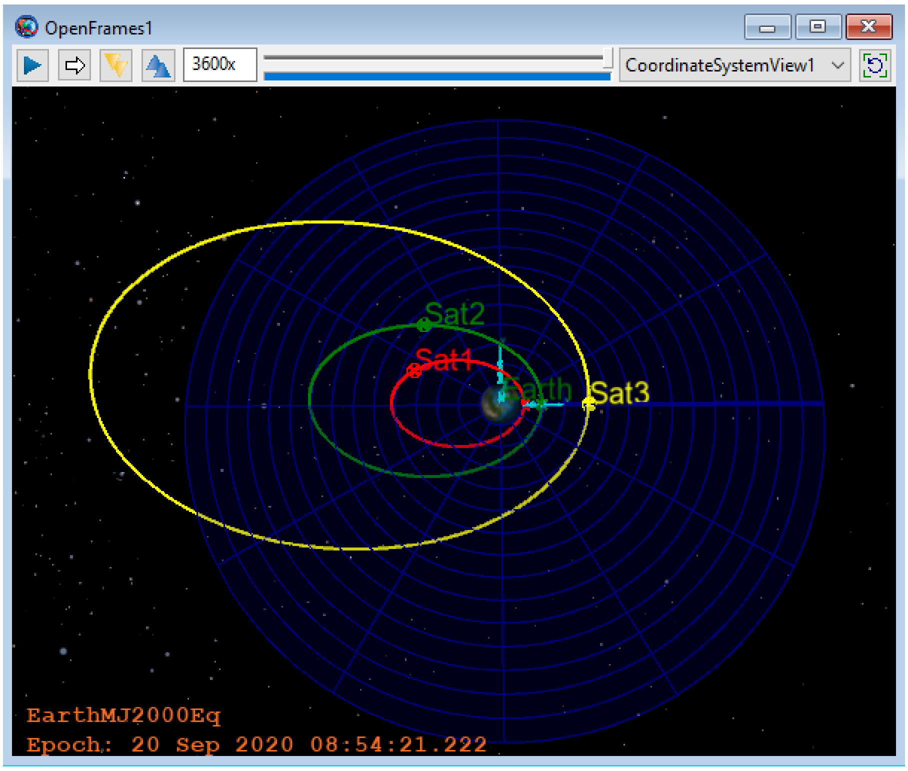
2 b)

Case 2: $a = 60{,}000$ km, $i = 45°$, $e = 0.2, 0.65, 0.88$

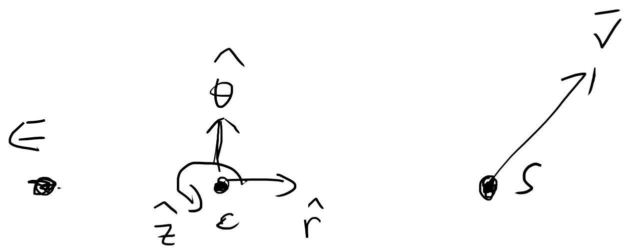


2 b)

case 3: $e = 0.65$, $i = 45°$, $a = 20000, 35000, 75000$ km



3.



$m_s = 600$ kg

$h = 8560$ km

$\dot{\bar{r}}_{SE} = 2.11\,\hat{r} + 4.89\,\hat{\theta}$ km/s

radius of the Earth : 6357 km.

a) Find $\bar{C}_3$, $\bar{h}$, T, C_4, $\mathcal{E}$, $\dot{A}$

$$\bar{r}_{ES} = 14917\,\hat{r} \text{ km}.$$

$$\bar{r}_{EC} \cdot (m_E + m_s) = \bar{r}_{ES} \cdot m_s$$

$$\bar{r}_{EC} = \bar{r}_{ES}\,\frac{m_s}{m_E + m_s}$$

$$\bar{r}_{EC} = 0.1499 \times 10^{-17} \text{ km}.$$

This value is small enough that we can approximate the center of mass of the system is at the center of the Earth.

Let $\bar{r}_{ES} = \bar{r}$

$$\bar{C}_3 = \frac{m_E m_s}{m_E + m_s}(\bar{r} \times \dot{\bar{r}}) = \underline{4.3766 \times 10^7\,\hat{z} \text{ kg km}^2/\text{s}}$$

$$\bar{h} = \bar{r} \times \dot{\bar{r}} = \underline{7.2944 \times 10^4\,\hat{z} \text{ km}^2/\text{s}}$$

$$T = \frac{1}{2}\frac{m_E m_s}{m_E + m_s}(\dot{\bar{r}} \cdot \dot{\bar{r}}) = 8.5093 \times 10^3 \text{ kg km}^2/\text{s}^2 \text{ (MJ)}$$

$$U = \frac{G m_E m_s}{\|\vec{r}\|} = \underline{1.6033 \times 10^4 \ kg\, km^2/s^2 \ (MJ)}$$

$$C_4 = T - U = \underline{-7.5235 \times 10^3 \ MJ}$$

$$\xi = C_4 \frac{(m_s + m_E)}{m_s \cdot m_E} = \underline{-12.5391 \ km^2/s^2}$$

$$\dot{A} = \frac{h}{2} = \underline{3.6472 \times 10^4 \ km^2/s}$$

b) The coefficient is $\frac{m_s + m_E}{m_s \cdot m_E} = 0.0017 \ kg^{-1}$

c) $\mu = G(m_s + m_E) = \underline{3.9860 \times 10^5 \ km^3/s^2}$

$$p = \frac{h^2}{\mu} = \underline{1.3349 \times 10^4 \ km}$$

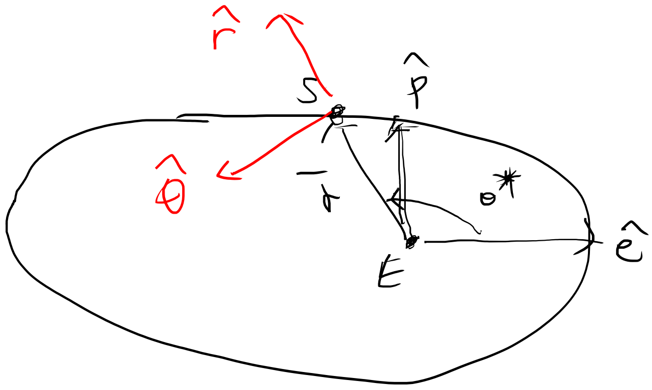
$$e = \sqrt{1 + \frac{2\xi h^2}{\mu^2}} = \underline{0.4002}$$

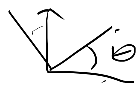
$$a = \frac{p}{(1-e^2)} = \underline{1.5894 \times 10^4 \ km}$$

$$\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = \underline{1.9942 \times 10^4 \ s}$$

$$\theta^* = \cos^{-1} \frac{\frac{1}{r} - \frac{\mu}{h^2}}{\sqrt{\frac{\mu^2}{h^4} + \frac{2\xi}{h^2}}} = \underline{1.6761 \ rad = 96.0349^\circ}$$

$$\gamma = \tan^{-1}\left(\frac{4.89}{2.11}\right) = \underline{1.4742 \ rad = 84.4642^\circ}$$





$$\hat{r} = \cos\theta^* \hat{e} + \sin\theta^* \hat{p} \qquad {}^rR^i = \begin{bmatrix} c\theta^* & s\theta^* & 0 \\ -s\theta^* & c\theta^* & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\theta} = -\sin\theta^* \hat{e} + \cos\theta^* \hat{p}$$

$${}^i\bar{r}_{SE} = {}^rR^{iT} \cdot {}^r\bar{r}_{SE}$$

$$= (-0.1568\,\hat{e} + 1.4834\,\hat{p}) \times 10^4 \text{ km}$$

d) $V_c = \sqrt{\dfrac{\mu}{r}} = 5.1693$ km/s

$V = \|\dot{\bar{r}}\| = 5.3258$ km/s

V is bigger than V_c, but not bigger than $\sqrt{2}\,V_c$, thus the spacecraft doesn't have the speed to escape the gravity pull from Earth.