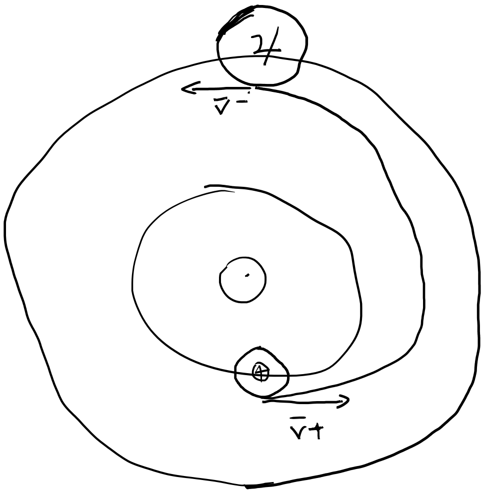
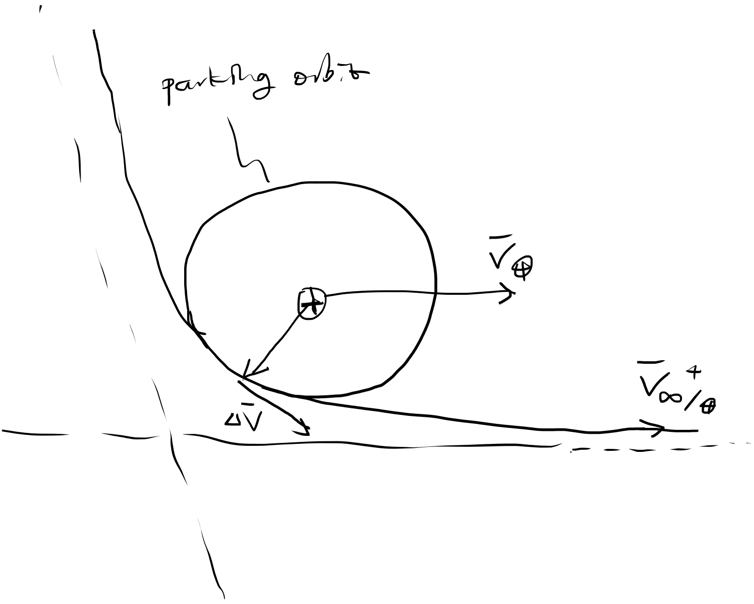
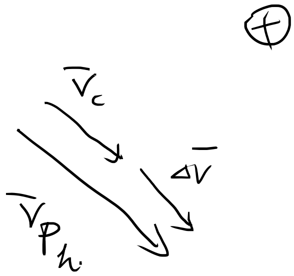
1.



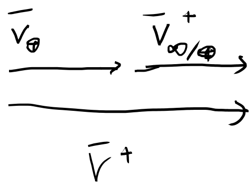
♃
$\bar{v}^-$
⊕
$\bar{v}^+$



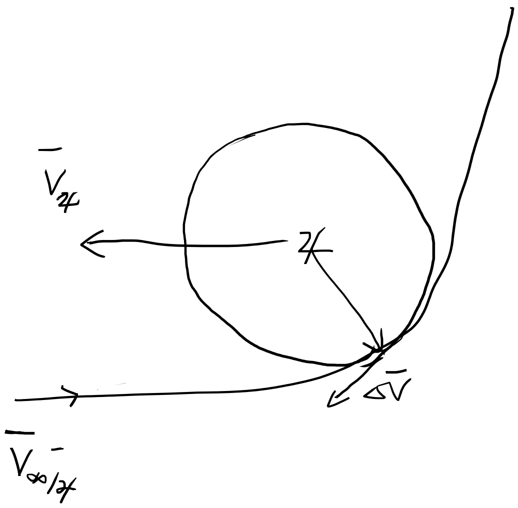
parking orbit
$\bar{V}_⊕$
$\Delta\bar{V}$
$\bar{V}^+_{\infty/⊕}$



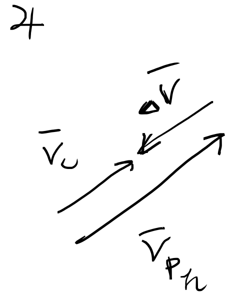
⊕
$\bar{V}_c$
$\Delta\bar{V}$
$\bar{V}_{P_h}$



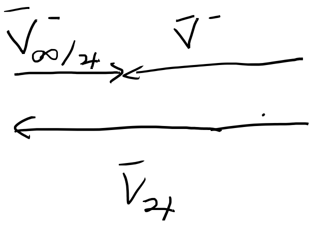
$\bar{V}_⊕$
$\bar{V}^+_{\infty/⊕}$
$\bar{V}^+$



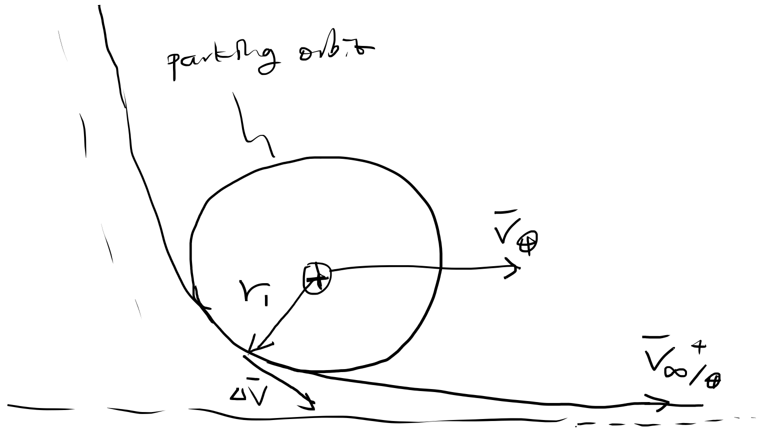
$\bar{V}_♃$
♃
$\Delta\bar{V}$
$\bar{V}^-_{\infty/♃}$

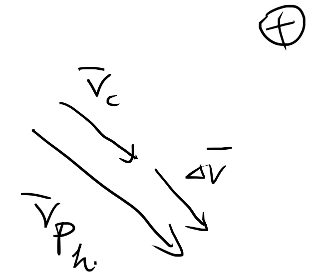


♃
$\Delta\bar{V}$
$\bar{V}_c$
$\bar{V}_{P_h}$

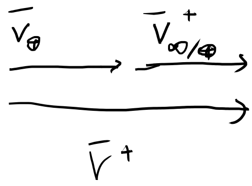


$\bar{V}^-_{\infty/♃}$
$\bar{V}^-$
$\bar{V}_♃$





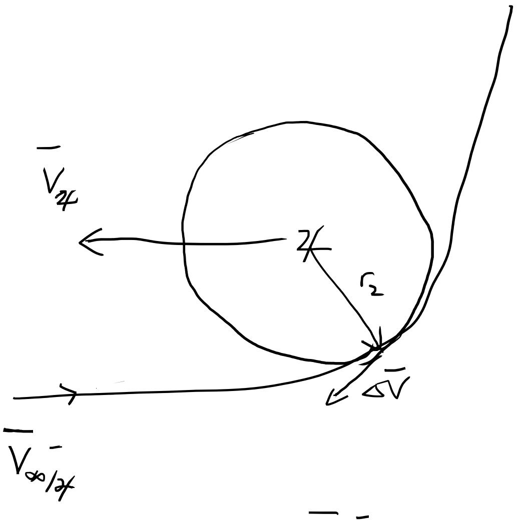
$V_{ph} = V_c + \Delta V$

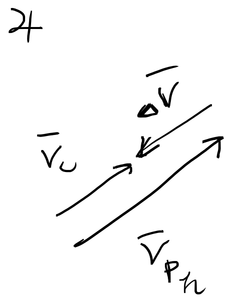


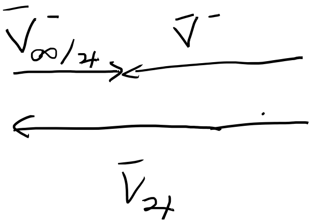
$$V^+_{\infty/\oplus} = V^+ - V_\oplus = 8.7925 \text{ km/s}$$

$$\frac{V^{+\,2}_{\infty/\oplus}}{2} = \frac{V_{ph}^2}{2} - \frac{\mu_\oplus}{r_1} \qquad V_c = \sqrt{\frac{\mu_\oplus}{r_1}} = 7.7548 \text{ km/s}$$

$$\Delta V_{dep} = \Delta V = \sqrt{V^{+\,2}_{\infty/\oplus} + \frac{2\mu_\oplus}{r}} - V_c = \boxed{6.3016 \text{ km/s}}$$







$$\bar{V}^-_{\infty/24} = V_{24} - V^- = 5.6432 \text{ km/s}$$

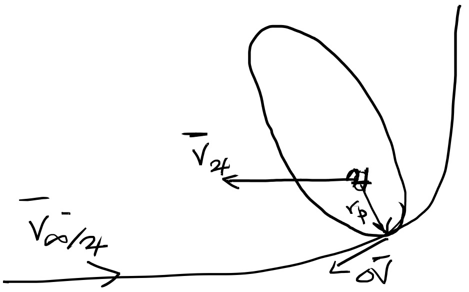
$$V_c = \sqrt{\frac{\mu_{24}}{r_2}} = 25.1595 \text{ km/s}$$

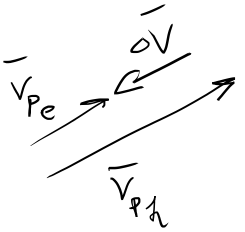
$$\Delta V_{arr} = \Delta V = \sqrt{V^{-\,2}_{\infty/24} + \frac{2\mu_{24}}{r_2}} - V_c = \boxed{10.5006 \text{ km/s}}$$

$$\Delta V_{total} = \Delta V_{dep} + \Delta V_{arr} = \boxed{16.8022 \text{ km/s}} > 14.436 \text{ km/s}.$$

The total cost is increased when considering local field. The cost in ΔV is decreased at departure, but increased at arrival. TOF of both scenario should be about the same, as it significantly depends on transfer path.

1 b)





$r_p = 2.8 R_J$
$= r_2$
$|\bar{V}_{\infty/J}|$ is unchanged.

$$a = \frac{r_p}{1-e} = 2.0018 \times 10^6 \text{ km}.$$

$$V_{pe} = \sqrt{\frac{2\mu_J}{r_p} - \frac{\mu_J}{a}} = 34.6800 \text{ km/s}$$

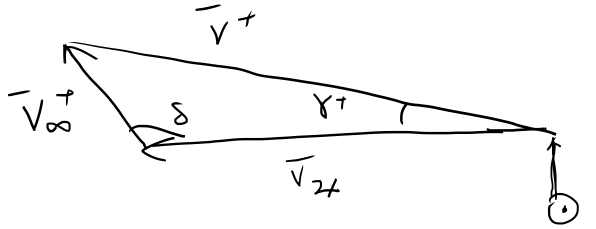
$$\Delta V_{arr} = \Delta V = \sqrt{V_{\infty/J}^2 + \frac{2\mu_J}{r_p}} - V_{pe} = \boxed{0.9801 \text{ km/s}}$$

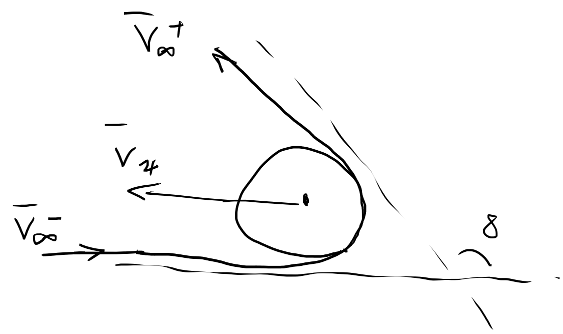
ΔV_{arr} is significantly smaller, thus reducing ΔV_{total}.

On a high eccentric orbit, the velocity at the periapsis is the highest. This ensures that velocities at periapsis of the hyperbola and the ellipse have a small difference, resulting in a smaller ΔV, thus reducing overall cost.

The eccentric orbit is used to reduce transfer cost from the Earth to Jupiter. Carrying less fuel, the Juno can carry more instrument on board for scientific investigation, or reduce launch cost.

c)





$$|a_h| = \frac{\mu_J}{V_{\infty/J}^2} = 3.9790 \times 10^6 \text{ km}$$

$$e_h = \frac{r_p}{|a|} + 1 = 1.0503$$

$$\delta = 2 \cdot \sin^{-1}\left(\frac{1}{e_h}\right) = 144.3894^\circ$$

$$v^+ = \sqrt{V_\infty^2 + V_J^2 - 2V_\infty V_J \cos\delta}$$

$$v^+ = \boxed{17.9495 \text{ km/s}}$$

$$r^+ = r^- = r_a \approx a_{24} = 7.7828\times10^8 \text{ km}$$

$$\gamma^+ = \sin^{-1}\left(\frac{V_\infty^+}{V^+}\cdot\sin\delta\right) = \boxed{10.5482^\circ}$$

ascending according to vector diagram

$$\theta^{*+} = \tan^{-1}\left(\frac{\frac{r^+v^{+2}}{\mu_\odot}\cdot\sin\gamma^+\cdot\cos\gamma^+}{\left(\frac{r^+v^{+2}}{\mu_\odot}\right)\cos^2\gamma^+ - 1}\right) = \boxed{22.3727^\circ},\ 202.3727^\circ$$

ascending

$$e^+ = \sqrt{\left(\frac{r^+v^{+2}}{\mu_\odot}-1\right)^2\cos^2\gamma^+ + \sin^2\gamma^+} = \boxed{0.8934}$$

$$p^+ = r^+\cdot(1+e^+\cdot\cos\theta^{*+}) = 1.4212\times10^9 \text{ km}$$

$$a^+ = p^+/(1-e^{+2}) = \boxed{7.0386\times10^9 \text{ km}}$$

$$r_p^+ = a^+(1-e^+) = \boxed{7.5064\times10^8 \text{ km}}$$

$$r_a^+ = a^+(1+e^+) = \boxed{1.3327\times10^{10} \text{ km}}$$

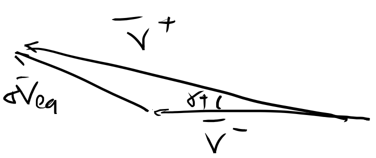
$$P = 2\pi\sqrt{\frac{a^3}{\mu_\odot}} = \boxed{1.0285\times10^{10} \text{ sec}} \approx 322.9609 \text{ years} \ ?$$

$$\xi = -\frac{\mu}{2a^+} = \boxed{-9.4274 \text{ km}^2/\text{sec}^2}$$

$$\Delta\omega = \theta^{*-} - \theta^{*+} = 180^\circ - 22.3727^\circ = \boxed{157.6273^\circ}$$

The spacecraft gain energy. as $|\bar{v}^+| > |\bar{V}^-|$

d).

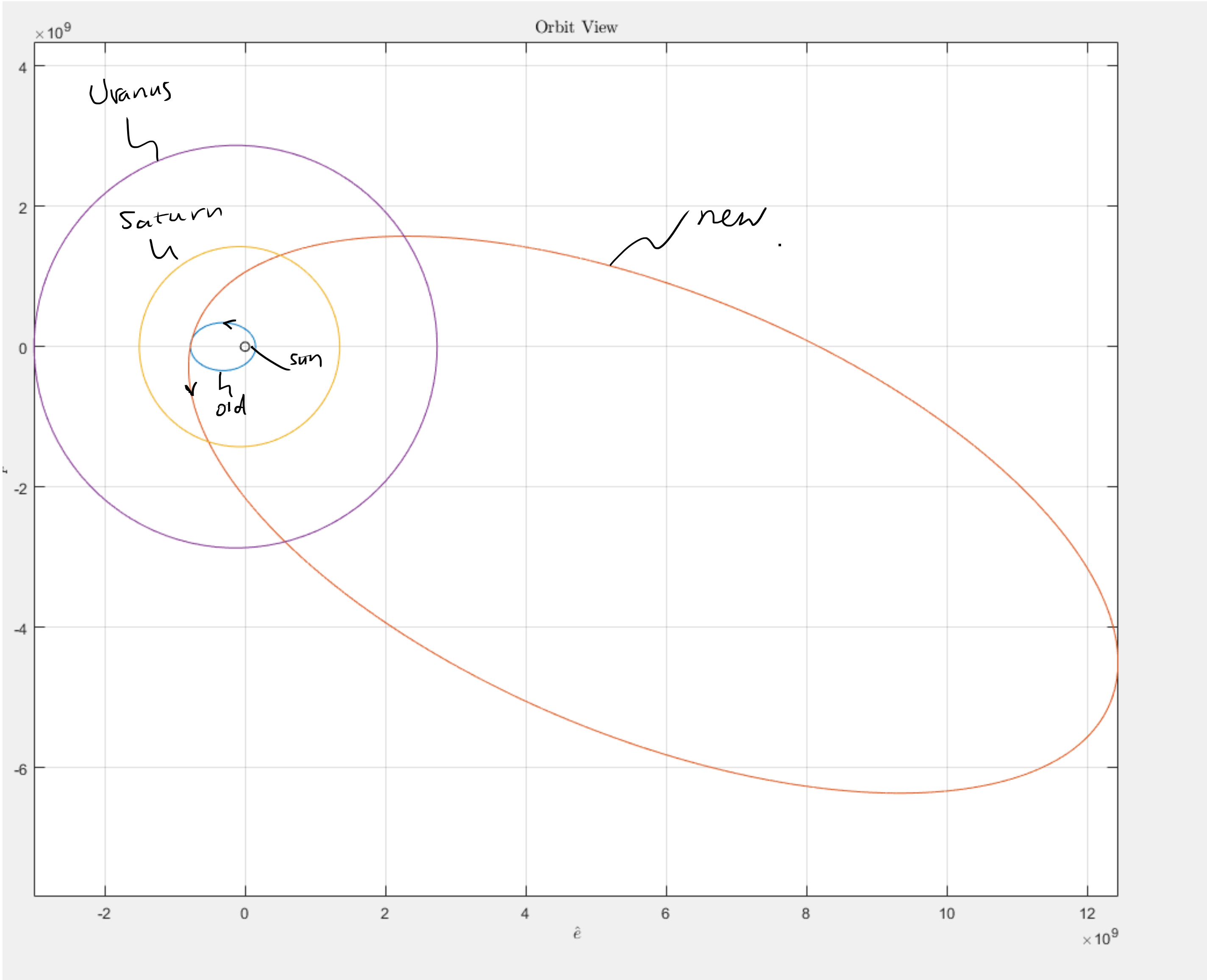


$$\Delta V_{eq} = \sqrt{v^{+2}+v^{-2}-2v^+v^-\cos\gamma^+} = \boxed{10.7458 \text{ km/s}}$$

$$\beta = \sin^{-1}\left(\frac{V_n}{\Delta V_{eq}}\cdot\sin\gamma^+\right) = 162.1947^\circ$$

$$\boxed{\alpha = 17.8053^\circ}$$

d).

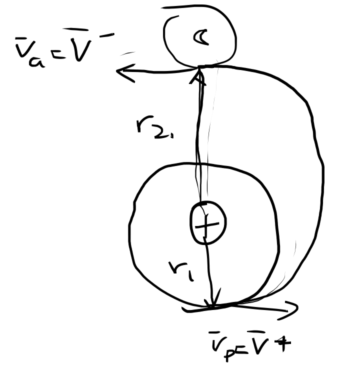


The spacecraft will likely reach the orbits of Saturn and Uranus, as its SMA is longer than SMA of both of these planets, and this new orbit is highly elliptical. $a_{Saturn} = 1.4274 \times 10^9$ km

$a_{uranus} = 2.8705 \times 10^9$ km

Encounters with both planets could occur, if timed properly.

2.



$r_1 = R_\oplus + 190$

$r_2 = a_{☾} - 200$

a) $r_p^- = r_1 = R_\oplus + 190 = \boxed{6568.1 \text{ km}}$

$r_a^- = r_2 = a_{☾} - 200 = \boxed{384200 \text{ km}}$

$a_T = \frac{1}{2}(r_p + r_a) = \boxed{1.9538 \times 10^5 \text{ km}}$

$e_T = 1 - \frac{r_1}{a_T} = \boxed{0.9664}$

at lunar arrival.

$r^- = r_a = \boxed{384200 \text{ km}}$

$v^- = \sqrt{\frac{2\mu_\oplus}{r^-} - \frac{\mu_\oplus}{a_T}} = \boxed{0.1868 \text{ km/s}}$

$\boxed{\gamma^- = 0^\circ \quad \theta^* = 180^\circ}$ at apoapsis of transfer orbit.

$\mathbb{P}^- = 2\pi\sqrt{\frac{a_T^{-3}}{\mu_\oplus}} = \boxed{8.5950 \times 10^5 \text{ sec}}$ TOF $= \frac{1}{2}\mathbb{P} = \boxed{4.2975 \times 10^5 \text{ sec}}$

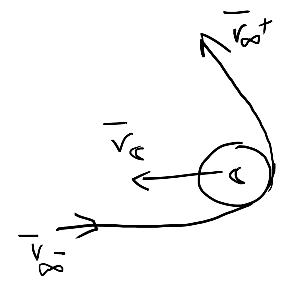
$\mathcal{E}^- = -\frac{\mu_\oplus}{2a_T^-} = \boxed{-1.0200 \text{ km}^2/\text{s}^2}$

$n_{☾} = \frac{2\pi}{\mathbb{P}_{☾}} = 2.6617 \times 10^{-6}$ rad/s

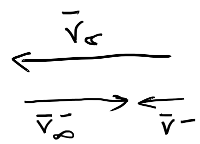
at departure from the Earth.

$\phi = \pi - n_{☾} \cdot \text{TOF} = \boxed{114.4615^\circ}$

b)



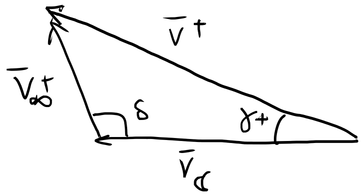
$$v_☾ = \sqrt{\frac{2\mu_\oplus}{r_2} - \frac{\mu_\oplus}{a_☾}} = 1.0188 \text{ km/s}$$



$r_{p,h} = R_☾ + 200 = 1938.2$ km.

$V_\infty = V_☾ - V^- = 0.8321$ km/s

$|a_h| = 7081.3$ km



$$e_h = \frac{r_{p,h}}{|a_h|} + 1 = 1.2737$$

$$\delta = 2 \cdot \sin^{-1}\left(\frac{1}{e_h}\right) = 103.4617^\circ$$

$$v^+ = \sqrt{V_\infty^2 + V_☾^2 - 2V_\infty V_☾ \cos\delta} = \boxed{1.4578 \text{ km/s}}$$

$$\gamma^+ = \sin^{-1}\left(\frac{V_\infty}{v^+} \cdot \sin\delta\right) = \boxed{33.7182^\circ}$$

$$r^+ = r^- = r_a = r_2 = \boxed{384200 \text{ km}}$$

$$\theta^{*+} = \tan^{-1}\left(\frac{\frac{r^+ v^{+2}}{\mu_\oplus}\sin\gamma^+ \cdot \cos\gamma^+}{\frac{r^+ v^{+2}}{\mu_\oplus} \cdot \cos^2\gamma^+ - 1}\right) = \boxed{66.1999^\circ}$$

$$e^+ = \sqrt{\left(\frac{r^+ v^{+2}}{\mu_E} - 1\right)^2 \cdot \cos^2\gamma^+ + \sin^2\gamma^+} = \boxed{1.0337}$$

$$p^+ = r^+ \cdot (1 + e^+ \cdot \cos\theta^{*+}) = \boxed{5.4946 \times 10^5 \text{ km}}$$

$$a^+ = p^+/(1 - e^{+2}) = \boxed{-7.9525 \times 10^6 \text{ km}}$$

$$r_p^+ = a^+(1 - e^+) = \boxed{2.6772 \times 10^5 \text{ km}}$$

$$r_a^+ = \boxed{\infty}$$

$\mathbb{P}^+ \Rightarrow$ not applicable in a hyperbolic orbit

$$\mathcal{E}^+ = -\frac{\mu_\oplus}{2a^+} = \boxed{0.0251 \text{ km}^2/\text{s}^2}$$

$$\Delta\omega = \theta^{*-} - \theta^{*+} = 180^\circ - 66.1999^\circ = \boxed{113.8001^\circ}$$

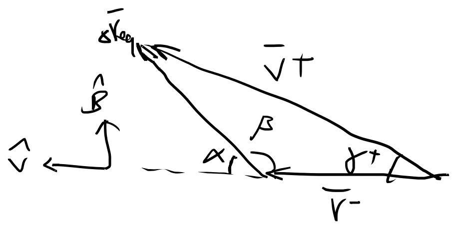
The spacecraft gains energy as it shifts from elliptical to hyperbolic orbit.

Since the new orbit is hyperbolic, it won't come close to the Earth.

The crew won't return to Earth without more maneuvers.

c) $|\Delta \bar{V}_{eq}| = \sqrt{v^{-2} + v^{+2} - 2v^- v^+ \cos\gamma} = 1.3066$

$\frac{\Delta V_{eq}}{\sin\gamma^+} = \frac{v^+}{\sin\beta} \Rightarrow \beta = 38.2691°, \underline{141.7309°}$



$\alpha = 180° - \beta = 38.2691°$

$\Delta \bar{V}^{VNB} = \Delta V_{eq} \cdot (\cos\alpha \, \hat{V} + \sin\alpha \, \hat{B})$

$= \boxed{1.0258 \, \hat{V} + 0.8092 \, \hat{B} \ \text{km/s}}$

d) From GMAT report.

$e^+ = 1.0336$

$a^+ = -7973463 \text{ km}$

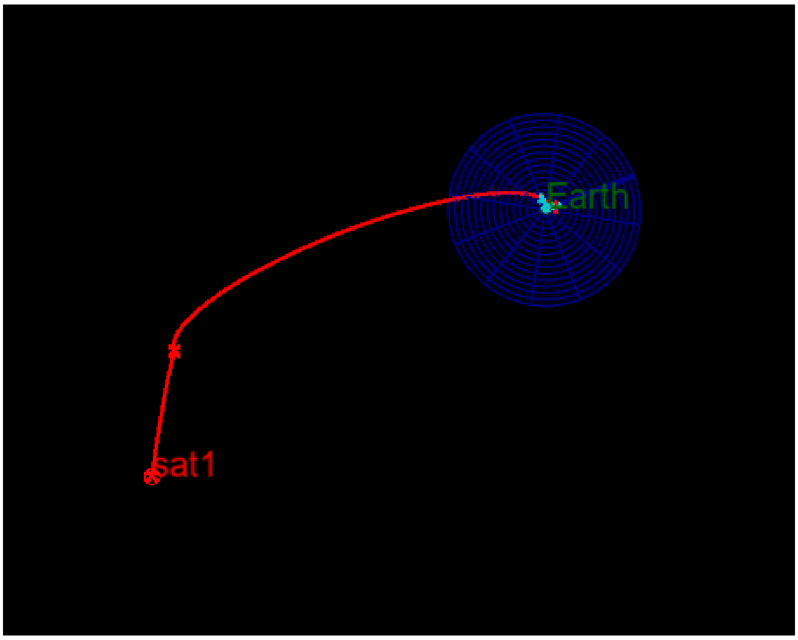
$p^+ = 544421$

$\theta^{*+} = 66.21°$

$\gamma^+ = 90° - 56.2762 = 33.724°$

$v^+ = 1.4577 \text{ km/s}$

These values are very close to the values obtained in b).
The slight difference can be attributed to the SMA values in GMAT database being inconsistent with the values in the table.



3. a)

$r_p = r_1 = a_♀ = 1.0821 \times 10^8$ km $\qquad a_T = \frac{1}{2}(r_1 + r_2) = 1.2890 \times 10^8$ km

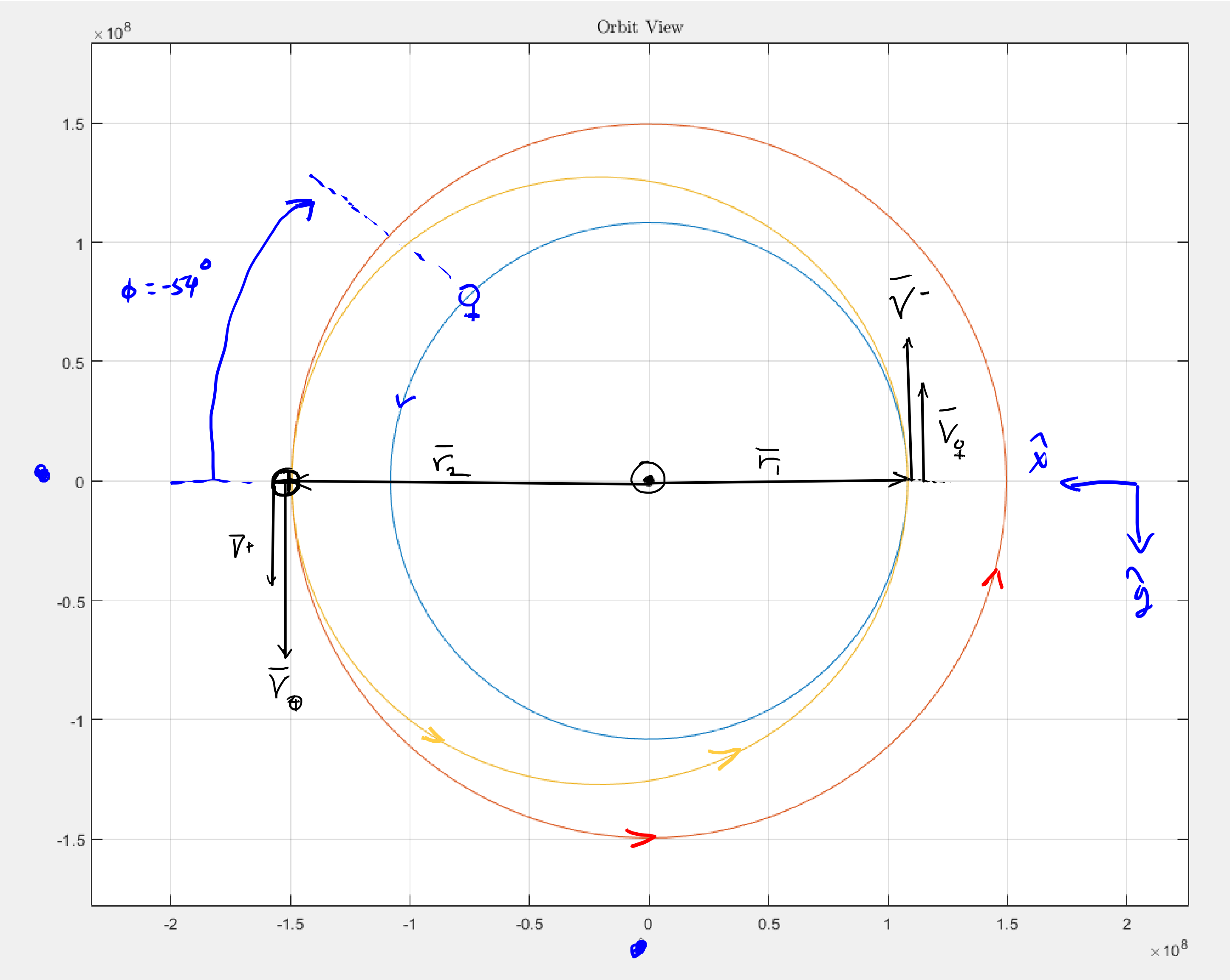
$r_a = r_2 = a_⊕ = 1.4960 \times 10^8$ km $\qquad e_T = 1 - r_2/a_T = 0.1605$

$$\mathbb{P} = 2\pi\sqrt{\frac{a_T^3}{\mu}} = \boxed{2.5242 \times 10^7 \text{ sec}}$$

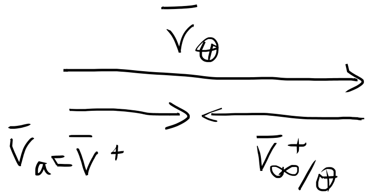
$$TOF = \frac{1}{2}\mathbb{P} = \boxed{1.2621 \times 10^7 \text{ sec}}$$

$$n_V = \sqrt{\frac{\mu_s}{a_V^3}} = 3.2365 \times 10^7 \text{ rad/s}$$

$$\phi = \pi - n_V \cdot TOF = \boxed{-54.0347^\circ}$$



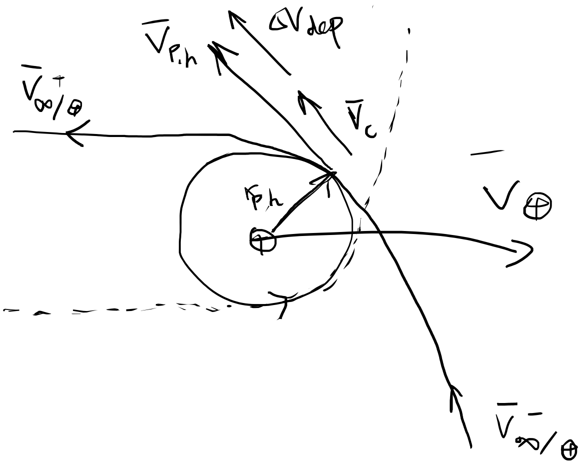
b)



$$V_{\oplus} = \sqrt{\frac{\mu_{\odot}}{a_E}} = 29.7847 \text{ km/s}$$

$$V_a = \sqrt{\frac{2\mu_{\odot}}{r_2} - \frac{\mu_{\odot}}{a_T}} = 27.2892 \text{ km/s}$$

$$V_{\infty/\oplus} = V_{\oplus} - V_a = \boxed{2.4955 \text{ km/s}}$$



$$r_{p_h} = R_{\oplus} + 210$$

$$V_c = \sqrt{\frac{\mu_{\oplus}}{r_{p.h}}} = 7.7784 \text{ km/s}$$

$$\Delta V_{dep} = \underbrace{\sqrt{V_{\infty/\oplus}^2 + \frac{2\mu_{\oplus}}{r_{p.h}}}}_{V_{p.h}} - V_c = \boxed{3.5014 \text{ km/s}}$$

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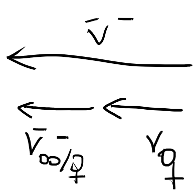
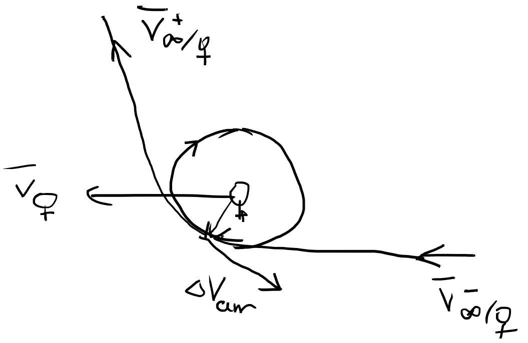
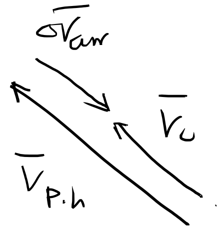
c)







$$r_{p.h} = R_{♀} + 2000 = 8051.9 \text{ km}$$

$$\boxed{\begin{aligned} \gamma^- &= 0^\circ \\ \theta^{*-} &= 0^\circ \end{aligned}}$$ at periapsis of transfer orbit.

$$r_p^- = r_1 = \boxed{1.0821 \times 10^8 \text{ km}}$$

$$r_a^- = r_2 = \boxed{1.4960 \times 10^8 \text{ km}}$$

$$\mathcal{E}^- = -\frac{\mu_{\odot}}{2a_T} = \boxed{-514.778 \text{ km}^2/\text{s}^2}$$

$$r^- = r_p^- = \boxed{1.0821 \times 10^8 \text{ km}}$$

$$V^- = \sqrt{\frac{2\mu_{\odot}}{r^-} - \frac{\mu_{\odot}}{a_T}} = \boxed{37.7276 \text{ km/s}}$$

$$V_c = \sqrt{\frac{\mu_{♀}}{r_{p.h}}} = 6.3518 \text{ km/s}$$

$$V_{♀} = \sqrt{\frac{\mu_{\odot}}{a_{♀}}} = 35.0209 \text{ km/s}$$

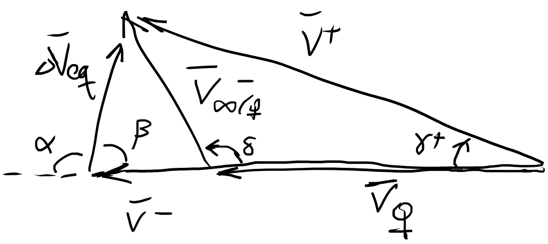
$$V_{\infty/♀} = V^- - V_{♀} = \boxed{2.7067 \text{ km/s}}$$

$$\Delta V_{arr} = \sqrt{V_{\infty/♀}^2 + \frac{2\mu_{♀}}{r_{p.h}}} - V_c = \boxed{3.0299 \text{ km/s}}$$

$$\Delta V_{total} = \Delta V_{dep} + \Delta V_{arr} = \boxed{6.5314 \text{ km/s}}$$

d) $|\bar{V}^+_{\infty/♀}| = |\bar{V}^-_{\infty/♀}| = \boxed{2.7067 \text{ km/s}}$

$|a_h| = \dfrac{\mu_♀}{V_{\infty/♀}^2} = 4.4342 \times 10^4 \text{ km}$



$e_h = \dfrac{r_{p,h}}{|a_h|} + 1 = 1.1876$

$\delta = 2\sin^{-1}\left(\dfrac{1}{e_h}\right) = \boxed{115.6271°}$

$r^+ = r^- = \boxed{1.0821 \times 10^8 \text{ km}}$

$v^+ = \sqrt{V_{\infty/♀}^2 + V_♀^2 - 2V_{\infty/♀} \cdot V_♀ \cdot \cos\delta} = \boxed{36.2738 \text{ km/s}}$

$\gamma^+ = \sin^{-1}\left(\dfrac{V_{\infty/♀}}{v^+} \cdot \sin\delta\right) = \boxed{3.8577°}$ ascending . from vector diagram.

$\theta^{*+} = \tan^{-1}\left(\dfrac{\frac{r^+ v^{+2}}{\mu_\odot} \cdot \sin\gamma^+ \cdot \cos\gamma^+}{\frac{r^+ v^{+2}}{\mu_\odot} \cdot \cos^2\gamma^+ - 1}\right) = \boxed{46.6535°}$

$\Delta V_{eq} = \sqrt{v^{+2} + v^{-2} - 2v^+ v^- \cos\gamma^+} = \boxed{2.8836 \text{ km/s}}$

$\beta = \sin^{-1}\left(\dfrac{v^+}{\Delta V_{eq}} \cdot \sin\gamma^+\right) = 57.8135°$ $\qquad \alpha = 180° - \beta = \boxed{122.1865°}$

$e^+ = \sqrt{\left(\dfrac{r^+ v^{+2}}{\mu_\odot} - 1\right)^2 \cdot \cos^2\gamma^+ + \sin^2\gamma^+} = \boxed{0.0990}$

$a^+ = \dfrac{p^+}{1 - e^{+2}} = \dfrac{r^+ \cdot (1 + e^+ \cdot \cos\theta^{*+})}{1 - e^{+2}} = 1.1671 \times 10^8 \text{ km}$

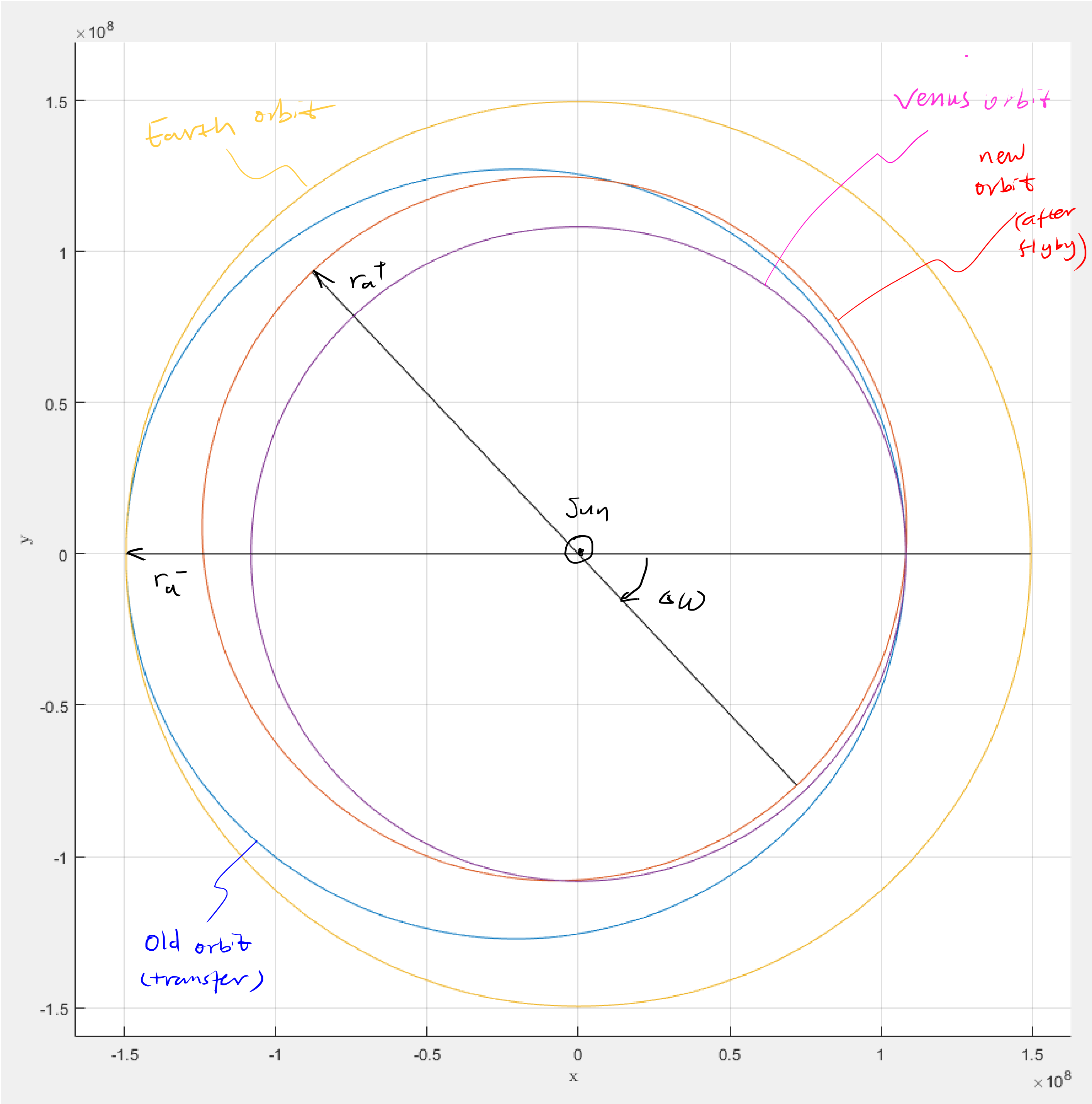
$r_p^+ = a^+ \cdot (1 - e^+) = \boxed{1.0515 \times 10^8 \text{ km}}$

$r_a^+ = a^+(1 + e^+) = \boxed{1.2826 \times 10^8 \text{ km}}$

$\mathcal{E} = -\dfrac{\mu_\odot}{2a^+} = \boxed{-568.5710 \text{ km}^2/\text{s}^2}$

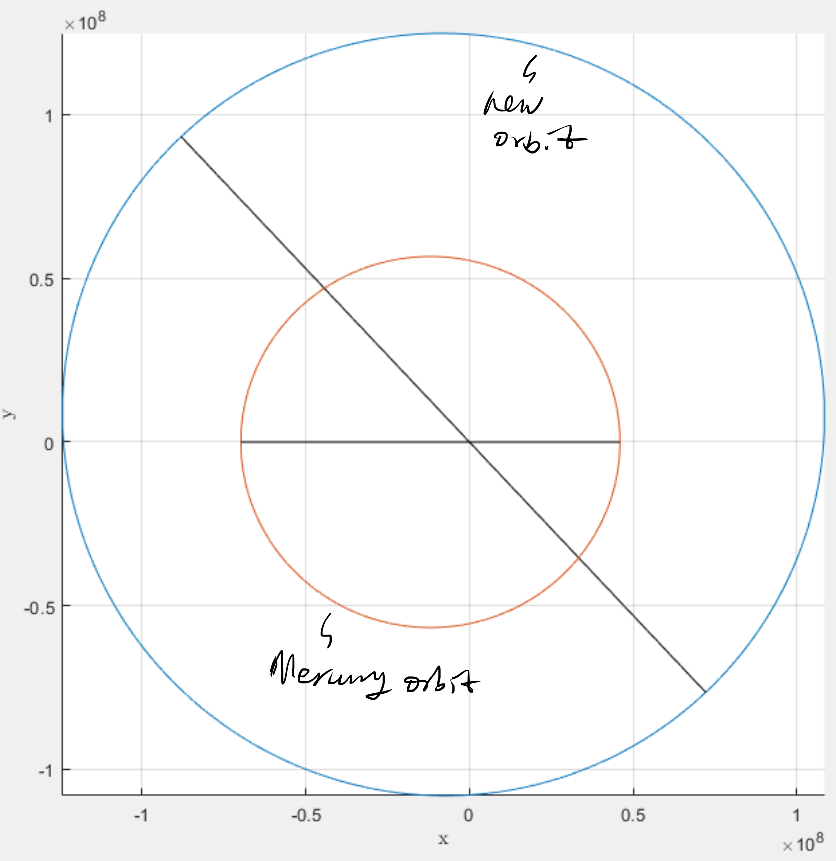
$\Delta\omega = \theta^{*-} - \theta^{*+} = 0 - 46.6535° = \boxed{-46.6535°}$

e)



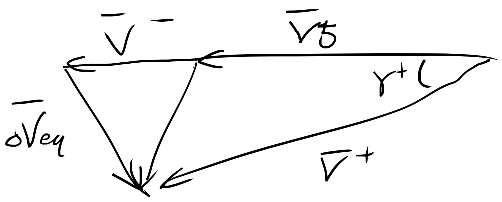
The spacecraft is ascending on the new orbit,

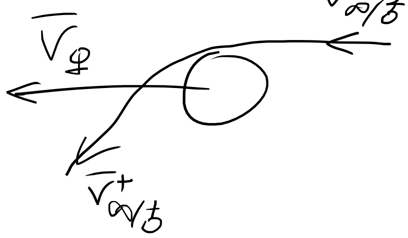
It will not reach Earth's orbit after the encounter.



As you can see, the spacecraft will not encounter the Mercury, or reach its orbit.

If it were a dark side passage.





γ^+ will be negative. The spacecraft will descend on the orbit

$|\Delta\bar{V}_{eq}|$ & $|\bar{V}^+|$ are unchanged.

Orbital parameters should remain the same, except for AOP. In this case, $\Delta\omega$ will be positive instead of negative on a light side passage.