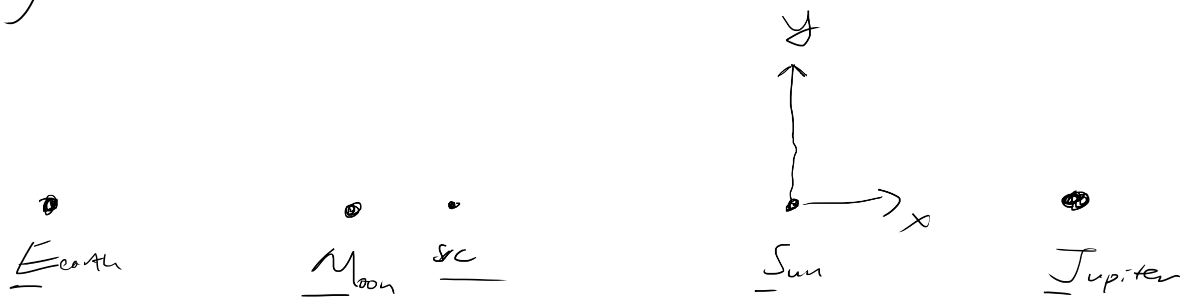
AAE532 HW-2 Zhanpeng Yang

1. a)




$$l_{SE} = 149597898 \text{ km}$$

$$l_{EM} = 384400 \text{ km} \qquad l_{SM} = l_{SE} - l_{EM}$$

$$l_{SJ} = 778299359 \text{ km}$$

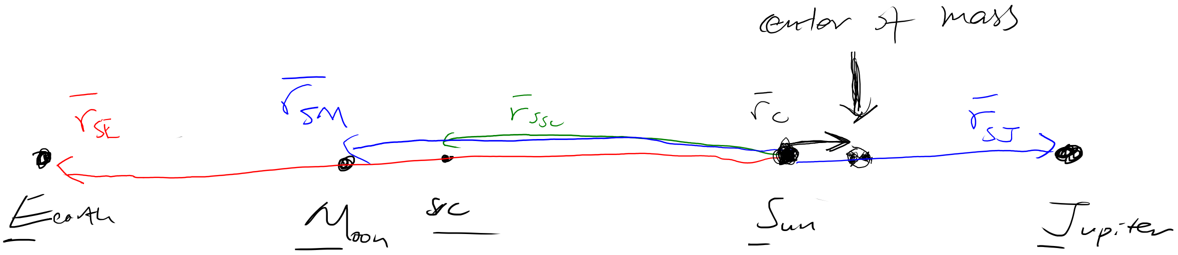
$$l_{MS/C} = 140000 \text{ km} \qquad l_{SS/C} = l_{SM} - l_{MS/C}$$

$$m_{total} \cdot \bar{r}_C = \cdot \bar{r}_{SE} + m_M \cdot \bar{r}_{SM} + m_{S/C} \cdot \bar{r}_{SS/C} + m_J \cdot \bar{r}_{SJ}$$

$$\bar{r}_c = \frac{m_E \bar{r}_{SE} + m_M \bar{r}_{SM} + m_{S/C} \bar{r}_{SS/C} + m_J \bar{r}_{SJ}}{m_{total}}$$

Matlab magic

$\bar{r}_c = 7.4193 \times 10^5$ km (between the Sun & the Jupiter)



1.b)

$$\cancel{m_{s/c}}\ddot{\bar{r}}_{s/c} = -\frac{G\cancel{m_{s/c}}m_S}{r_{S\,s/c}^3}\bar{r}_{S\,s/c} - \frac{G m_{s/c} m_E}{r_{E\,s/c}^3}\bar{r}_{E\,s/c}$$

$$-\frac{G\cancel{m_{s/c}}m_M}{r_{M\,s/c}^3}\bar{r}_{M\,s/c} - \frac{G m_{s/c} m_J}{r_{J\,s/c}^3}\bar{r}_{J\,s/c}$$

$$\boxed{\ddot{\bar{r}}_{s/c} = -\frac{\mu_S}{r_{S\,s/c}^3}\bar{r}_{S\,s/c} - \frac{\mu_E}{r_{E\,s/c}^3}\bar{r}_{E\,s/c} - \frac{\mu_M}{r_{M\,s/c}^3}\bar{r}_{M\,s/c} - \frac{\mu_J}{r_{J\,s/c}^3}\bar{r}_{J\,s/c}}$$

Since all bodies are colinear at this moment, we can analyze the problem in 1-D. ($x$_ direction)

☐ Fat bird
☐ Wide cat

$$\bar{r}_{S\,s/c} = \bar{r}_{s/c} - \bar{r}_S = -l_{S\,s/c}$$

$$\bar{r}_{E\,s/c} = \bar{r}_{s/c} - \bar{r}_E = l_{E\,s/c} = l_{EM} + l_{M\,s/c}$$

$$\bar{r}_{M\,s/c} = l_{M\,s/c}$$

$$\bar{r}_{J\,s/c} = \bar{r}_{JS} + \bar{r}_{S\,s/c} = -l_{SJ} - l_{S\,s/c}$$

Plug in values & calculate in matlab

$$\boxed{\begin{aligned}\ddot{\bar{r}}_{S\,s/c} &= 5.9719\times10^{-6}\ \text{km/s}^2\\ \ddot{\bar{r}}_{E\,s/c} &= -1.4495\times10^{-6}\ \text{km/s}^2\\ \ddot{\bar{r}}_{M\,s/c} &= -2.5014\times10^{-7}\ \text{km/s}^2\\ \ddot{\bar{r}}_{J\,s/c} &= 1.4734\times10^{-6}\ \text{km/s}^2\end{aligned}}$$

Accel due to the Sun is the largest, due the Jupiter is the smallest
Descending order:

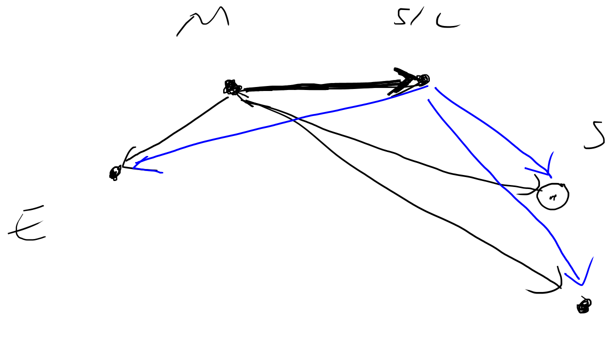
Sun
Earth
Moon
Jupiter

$$\underline{\ddot{\bar{r}}_{net} = 4.2724\times10^{-6}\ \text{km/s}^2}$$

1 (c) Since the Earth & the moon are on the negative x-direction (in our definition), their influence on the acceleration is negative, vice versa, the influence from the Sun & the Jupiter is positive. Sizewise, the acceleration due to the Jupiter being the smallest is mainly because of the long distance between it and the spacecraft. The largest acceleration from the sun can be attributed to its mass. These terms appear to be consistent with my expectations.

2. a)

Exagerated to show position vectors.




$$\ddot{\bar r}_{M\,s/c} + \frac{G(m_M+m_{s/c})}{r_{M\,s/c}^3}\,\bar r_{M\,s/c} = G\sum_{j=1}^{n} m_j\left(\frac{\bar r_{s/c\,j}}{r_{s/c\,j}^3} - \frac{\bar r_{Mj}}{r_{Mj}^3}\right)$$

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$$\ddot{\bar r}_{M\,s/c} = G m_E\left(\frac{\bar r_{s/c\,E}}{r_{s/c\,E}^3} - \frac{\bar r_{ME}}{r_{ME}^3}\right) + G M_S\left(\frac{\bar r_{s/c\,S}}{r_{s/c\,S}^3} - \frac{\bar r_{MS}}{r_{MS}^3}\right) + G M_J\left(\frac{\bar r_{s/c\,J}}{r_{s/c\,J}^3} - \frac{\bar r_{MJ}}{r_{MJ}^3}\right) \underbrace{- \frac{G(m_M+m_{s/c})}{r_{M\,s/c}^3}\,\bar r_{M\,s/c}}_{\text{Dominant acceleration}}$$

direct perturbation

Indirect perturbation

+

net perturbation

$$\ddot{\bar r}_{M\,s/c} = \underbrace{\overset{\text{direct}}{\boxed{G\left(m_E\frac{\bar r_{s/c\,E}}{r_{s/c\,E}^3} + m_S\frac{\bar r_{s/c\,S}}{r_{s/c\,S}^3} + m_J\frac{\bar r_{s/c\,J}}{r_{s/c\,J}^3}\right)}}\overset{\text{indirect}}{\boxed{-G\left(m_E\frac{\bar r_{ME}}{r_{ME}^3} + m_S\frac{\bar r_{MS}}{r_{MS}^3} + m_J\frac{\bar r_{MJ}}{r_{MJ}^3}\right)}}}_{\text{net}}$$

$$\underbrace{\boxed{-\frac{G(m_M+m_{s/c})}{r_{M\,s/c}^3}\,\bar r_{M\,s/c}}}_{\text{Dominant}}$$

2 b) Plug in values & matlab calc

Total direct perturbation : $4.5225 \times 10^{-6}$ km/s$^2$

Total Indirect perturbation : $-3.2633 \times 10^{-6}$ km/s$^2$

Dominant acceleration : $-2.5014 \times 10^{-7}$ km/s$^2$

net perturbing acceleration : indirect + direct = $1.2593 \times 10^{-6}$ km/s$^2$

total net acceleration : Dominant accel + net perturbation
$= 1.0091 \times 10^{-6}$ km/s$^2$

The dominant acceleration is not the largest in magnitude

The Sun provides the largest magnitude of $5.9719 \times 10^{-6}$ km/s, which is a direct perturbing acceleration.

From calculation, the Earth has the largest impact on the perturbation of the spacecraft, with a magnitude of 1.2481e-6 km/s$^2$, whereas the Sun & Jupiter provides 1.1201e-8 & 4.4478e-14 respectively.

If indirect perturbations were neglected, the Sun would have the largest impact of 5.9719e-6 km/s$^2$.

Indirect perturbations are important because they allow us to account for the interactions of all the bodies in a system.

2 c) The dominant acceleration (-2.5014e-7) is significantly smaller than the net perturbing acceleration (1.2593e-6) thus we cannot simply model the problem as a 2-body system.

A three-body problem may be considered for Moon-Earth-SC, as the net perturbations from the Sun & Jupiter are quite small.

Four-body problem including the Sun can provide more accurate simulations, thus I would probably consider it if resources allow.

2 d)

$$\ddot{\bar{r}}_{Es/c} + \frac{G(M_E + M_{s/c})}{r_{Es/c}^3}\bar{r}_{Es/c} = G m_S\left(\frac{\bar{r}_{s/cS}}{r_{s/cS}^3} - \frac{\bar{r}_{ES}}{r_{ES}^3}\right) + G m_M\left(\frac{\bar{r}_{s/cM}}{r_{s/cM}^3} - \frac{\bar{r}_{EM}}{r_{EM}^3}\right)$$

$$+ G m_J\left(\frac{\bar{r}_{s/cJ}}{r_{s/cJ}^3} - \frac{\bar{r}_{EJ}}{r_{EJ}^3}\right)$$

From matlab calculation.

Dominant acceleration: $-1.6495 \times 10^{-6}$ km/s$^2$

Total direct perturbing acceleration: $5.7219 \times 10^{-6}$ km/s$^2$

Total indirect perturbing acceleration: $-5.9634 \times 10^{-6}$ km/s$^2$

net perturbing acceleration: $-2.4153 \times 10^{-7}$ km/s$^2$

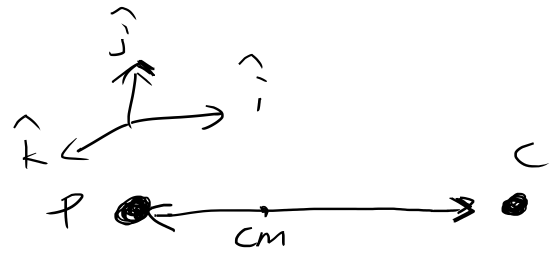
net acceleration: $-1.6910 \times 10^{-6}$ km/s$^2$

This model should be equally valid, only the reference point changed, thus changing the relative acceleration.

Some terms are different in magnitudes and directions,

but both formulation are correct, as only the reference point has been changed.

3. a)




$$m_{total} \cdot r_{pcm} = m_c \cdot r_{pc}$$

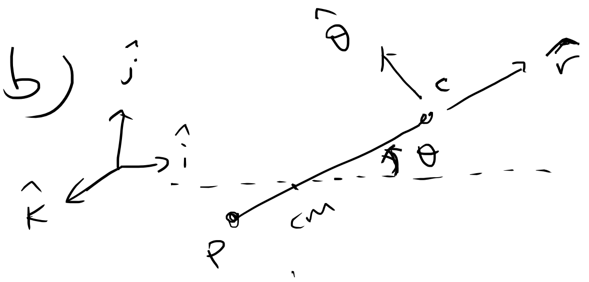
$$r_{pcm} = \frac{m_c}{m_{total}} r_{pc} = \frac{119.480}{981.601+119.480} \cdot 19596 = 2126.4 \text{ km}$$

$$\bar{r}_{cmp} = -2126.4\,\hat{i} \text{ km}$$

$$\bar{r}_{cmC} = \bar{r}_{pc} + \bar{r}_{cmp} = (19596 - 2126.4)\,\hat{i} = 17470\,\hat{i} \text{ km}$$

C.m is outside of the radius of Pluto, $r_{pcm} > \frac{d_p}{2}$

b)




$$\bar{r} = r\,\hat{r}$$

$$\dot{\bar{r}} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta}$$

$$\dot{\bar{r}}_p = -\frac{m_p}{m_p+m_c}\dot{\bar{r}} \qquad \text{at } t=0 \ \hat{\theta} = \hat{j} \ ; \ \hat{r} = \hat{i}$$

$$\dot{\bar{r}} = \frac{\dot{\bar{r}}_p (m_p+m_c)}{-m_p} = -0.025717\,\hat{j}\,\frac{m_p+m_c}{m_p}$$

$$= -0.0288 \text{ km/s } \hat{j}$$

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$$\bar{h} = \bar{r} \times \dot{\bar{r}}$$

$$= 19596\,\hat{i} \times -0.0288\,\hat{j}$$

$$= -564.3648\,\hat{k} \text{ km}^2/\text{s}$$

$$|\bar{h}| = r^2\dot{\theta}$$

$$\dot{\theta} = \frac{|\bar{h}|}{r^2} = \frac{564.3648}{19596^2} = \boxed{1.4697\text{e-}6 \text{ rad/s}}$$

3 c) $\bar{p} = m_p \cdot \dot{\bar{r}}_p + m_c \cdot \dot{\bar{r}}_c$

$= (981.601 \cdot (-0.025717)\hat{j} + 119.480 \cdot (0.211319)\hat{j}) / G$

$\boxed{= 6.8353e16 \ kg \cdot km/s}$

$\bar{p} = (m_p + m_c) \cdot \dot{\bar{r}}_{cm}$ conservation of linear momentum

$\dot{\bar{r}}_{cm} = \frac{\bar{p}}{(m_p + m_c)} = \boxed{4.1425e-6 \ km/s}$

The result makes sense

3 d) $\bar{c}_3 = \bar{h} \cdot \frac{m_p m_c}{m_p + m_c}$

$= -564.3648 \cdot \frac{m_p m_c}{m_p + m_c} = \boxed{-9.0084e23 \ kg \cdot km^2/s}$

e) $\frac{1}{2}(\dot{\bar{r}} \cdot \dot{\bar{r}}) - \frac{G(m_p + m_c)}{r} = C_4 \frac{(m_p + m_c)}{m_p m_c}$

$C_4 = \left[\frac{1}{2}(\dot{\bar{r}} \cdot \dot{\bar{r}}) - \frac{\mu_p + \mu_c}{r}\right] \cdot \frac{m_p m_c}{m_p + m_c}$

$\boxed{C_4 = -9.0351e19 \ kg \cdot km^2/s^2}$