HW5.

1.

x_n			y_n	g	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0	0	0	○	○	○	○	○	○	○	○	○	○
0	0	1	●	●	●	●	●	●	●	●	●	●
0	1	0	●	●	●	●	●	●	●	●	●	●
0	1	1	○	○	○	○	○	○	○	○	○	○
1	0	0	●	●	●	●	●	●	●	●	●	●
1	0	1		?	○	○	○	○	●	●	●	●
1	1	0		?	○	○	●	●	○	○	●	●
1	1	1		?	○	●	○	●	○	●	○	●

○

•

a) i) $g = [\bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet \ \bullet] = h_1.$

ii) Match 3 : $\{f_8.\}$

2 : $\{f_4, f_6, f_7\}$

1 : $\{f_2, f_3, f_5\}$

0 : $\{f_1\}$

b) i) $g = [\circ \ \circ \ \circ \ \circ \ \circ \ \circ \ \circ \ \circ] = h_2.$

ii) Match 3 : $\{f_1\}$

2 : $\{f_2, f_3, f_5\}$

1 : $\{f_4, f_6, f_7\}$

0 : $\{f_8\}$

1. c) i).

$g = [\circ \ \bullet \ \bullet \ \circ \ \bullet \ | \ \circ \ \circ \ \bullet]$.

ii). Match 3 : (f_2)

2 : (f_1, f_4, f_6)

1 : (f_3, f_5, f_8)

0 : (f_7)

d) i).

$g = [\circ \ \bullet \ \bullet \ \circ \ \bullet \ | \ \bullet \ \bullet \ \circ]$

ii) Match 3 : (f_7).

2 : (f_3, f_5, f_8)

1 : (f_1, f_4, f_6)

0 : (f_2).

3. $X_n \sim \text{Bernoulli}(0.5)$, $\bar{X}_N = (\frac{1}{N}) \sum_{n=1}^{N} X_n$

a) Show $\mathbb{P}(\bar{X}_N - \mu \geq \varepsilon) \leq 2^{-\beta N}$.

$\beta = 1 + (\frac{1}{2} + \varepsilon) \log_2 (\frac{1}{2} + \varepsilon) + (\frac{1}{2} - \varepsilon) \log_2 (\frac{1}{2} - \varepsilon)$. $\mu = 0.5$.

Chernoff bound states that:

$$\mathbb{P}(\bar{X}_N - \mu \geq \varepsilon) = \mathbb{P}\left(\frac{1}{N} \sum_{n=1} X_n - \mu \geq \varepsilon\right)$$

$$= \mathbb{P}\left(\sum_{n=1}^{N} X_n \geq (\varepsilon + \mu) N\right)$$

$$= \mathbb{P}\left(e^{s \sum_1^N X_n} \geq e^{s(\varepsilon+\mu)N}\right)$$

$$\leq \frac{\mathbb{E}\left[e^{s \sum_1^N X_n}\right]}{e^{s(\varepsilon+\mu)N}}$$

$$= \frac{\mathbb{E}[e^{sX_n}]^N}{e^{s(\varepsilon+\mu)N}}$$

$$= \left[\frac{\mathbb{E}[e^{sX_n}]}{e^{s(\varepsilon+\mu)}}\right]^N.$$

$$\mathbb{E}[e^{sX_n}] = e^s \mu + 1 - \mu = M_{X_n}(s)$$

$$\mathbb{P}(\bar{X}_N - \mu \geq \varepsilon) \leq e^{(-s(\varepsilon+\mu) + \log(e^s \mu + 1 - \mu))N}.$$

$$\frac{d}{ds}\left\{e^{-s(\varepsilon+\mu) + \log(e^s \mu + 1 - \mu)}\right\} = 0.$$

$$\frac{d}{ds}\left\{-s(\varepsilon+\mu) + \log(e^s \mu + 1 - \mu)\right\} = 0.$$

$$\varepsilon + \mu - \frac{\mu e^s}{\mu e^s + 1 - \mu} = 0.$$

$$\frac{\mu e^s}{\varepsilon + \mu} = \mu e^s + 1 - \mu.$$

$$\mu e^s \left(\frac{1}{\varepsilon + \mu} - 1 \right) = 1 - \mu$$

$$1 - \mu = \mu.$$

$$\mu e^s \left(\frac{1 - \varepsilon - \mu}{\varepsilon + \mu} \right) = 1 - \mu$$

$$e^{s_{min}} = \frac{(\varepsilon + \mu)}{(\mu - \varepsilon)}, \qquad s_{min} = \log\left[\frac{\varepsilon + \mu}{\mu - \varepsilon} \right].$$

sub s_{min} back.

$$\mathbb{P}(\bar{X}_N - \mu \geq \varepsilon) \leq \exp\left\{ -\log\left[\frac{(\varepsilon + \mu)}{(\mu - \varepsilon)} \right] (\varepsilon + \mu) + \log\left[\frac{(\varepsilon + \mu)\mu}{(\mu - \varepsilon)} + \mu \right] \right\}^N$$

$$= \exp\left\{ -\log\left[\frac{\varepsilon + \mu}{\mu - \varepsilon} \right] (\varepsilon + \mu) + \log\left[\frac{\mu \cdot 2\mu}{\mu - \varepsilon} \right] \right\}^N.$$

$$= \exp\left\{ -\log\left[\frac{\varepsilon + \mu}{\mu - \varepsilon} \right] (\varepsilon + \mu) + \log\left[\frac{\mu}{\mu - \varepsilon} \right] \right\}^N.$$

= I don't know where you pulled the 2^ out of, but ok.

Project : still trying to figure out the code for N2N.