Study Notes of Numerical Optimization

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Contents

Chapter 1

Basic notions in optimization

1.1 Types of optimization problems

Unconstrained Optimization problem:

$$\begin{aligned} \text{(P)} \quad & \min_{\boldsymbol{x}} f(\boldsymbol{x}) \\ & s.t.\ \boldsymbol{x} \in X, \end{aligned}$$

where $\boldsymbol{x} = (x_1, \dots, x_n)^T \in \mathbb{R}^n$, $f(x) : \mathbb{R}^n \to \mathbb{R}$, and X is an open set (usually $X = \mathbb{R}^n$).

1.2 Reference

- IOE 511/Math 652: Continuous Optimization Methods ch3

Part I

Unconstrained Optimization

Chapter 2

Optimality conditions for unconstrained problems

The definitions of global and local solutions of optimization problems are intuitive, but usually impossible to check directly. Hence, we will derive easily verifiable conditions that are either necessary for a point to be a local minimizer (thus helping us to identify candidates for minimizers), or sufficient (thus allowing us to confirm that the point being considered is a local minimizer), or, sometimes, both.

2.1 Optimality conditions: the necessary and the sufficient

$$
\begin{aligned}
\text{(P)} \quad & \min_{\boldsymbol{x}} f(\boldsymbol{x}) \\
& s.t.\ \boldsymbol{x} \in X,
\end{aligned}
$$

where $\boldsymbol{x} = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $f(x) : \mathbb{R}^n \to \mathbb{R}$, and X is an open set (usually $X = \mathbb{R}^n$).

Necessary condition for local optimality: "if $\overline{x}$ is a local minimizer of (P), then $\overline{x}$ must satisfy…" Such conditions help us identify all candidates for local optimizers.

Theorem 2.1

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2.2 Reference

- IOE 511/Math 652: Continuous Optimization Methods ch4

Part II

Duality

Part III

Application