

THE SOUTH CHINA NORMAL UNIVERSITY
School of Mathematical Sciences
Numerical Analysis (2023-2024 The Second Term)

Homework 11

Due Date: June 5, 2024 (Wednesday)

Name: _____ Student No.: _____ Date: May 30, 2024

§7.3 Recursive Rules and Romberg Integration

Exercise 1.

- (a) Start with $T(0) = (h/2)(f(a) + f(b))$. Then a sequence of trapezoidal rules $\{T(J)\}$ is generated by the recursive formula

$$T(J) = \frac{T(J-1)}{2} + h \sum_{k=1}^M f(x_{2k-1}) \quad \text{for } J = 1, 2, \dots, \quad (1)$$

where $h = (b-a)/2^J$, $M = 2^{J-1}$ and $\{x_k = a + kh\}$.

Show that the sequential trapezoidal rule converges to L (i.e., $\lim_{J \rightarrow \infty} T(J) = L$).

- (b) Suppose that $\{T(J)\}$ is the sequence of trapezoidal rules generated by (1). If $J \geq 1$ and $S(J)$ is Simpson's rule for 2^J subintervals of $[a, b]$, then $S(J)$ and the trapezoidal rules $T(J-1)$ and $T(J)$ obey the relationship

$$S(J) = \frac{4T(J) - T(J-1)}{3} \quad \text{for } J = 1, 2, \dots. \quad (2)$$

Show that the sequential Simpson rule converges to L (i.e., $\lim_{J \rightarrow \infty} S(J) = L$).

Solve 1

□