THE SOUTH CHINA NORMAL UNIVERSITY

School of Mathematical Sciences

Numerical Analysis (2022–2023 The Second Term)

Homework 3

Due Date: March 26, 2024 (Tuesday)

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§2.4 Exercises for Newton-Raphson and Secant Methods

Exercise 1. Let $f(x) = x^2 - x - 3$.

(a) Find the Newton-Raphson formula $p_k = g(p_{k-1})$.

(b) Start with $p_0 = 1.6$ and find p_1, p_2, and p_3.

(c) Start with $p_0 = 0.0$ and find p_1, p_2, p_3, and p_4. What do you conjecture about this sequence?

Solve (a) In this case, the Newton-Raphson iterative function is

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 - x - 3}{2x - 1} = \frac{(2x^2 - x) - x^2 + x + 3}{2x - 1} = \frac{x^2 + 3}{2x - 1}.$$

Hence, the Newton-Raphson formula is

$$p_k = \frac{P_{k-1}^2 + 3}{2p_{k-1} - 1}. \tag{1}$$

(b) Start with $p_0 = 1.6$, one can find

$$\begin{aligned} p_1 &= \frac{1.6^2 + 3}{2 \times 1.6 - 1} = 2.52727, \\ p_2 &= \frac{2.52727^2 + 3}{2 \times 2.52727 - 1} = 2.31521, \\ p_3 &= \frac{2.31521^2 + 3}{2 \times 2.31521 - 1} = 2.30282. \end{aligned}$$

(c) Start with $p_0 = 0.0$, one can find

$$
\begin{aligned}
p_1 &= \frac{0.0^2 + 3}{2 \times 1.6 - 1} = -3.0,\\
p_2 &= \frac{(-3.0)^2 + 3}{2 \times (-3.0) - 1} = -1.71429,\\
p_3 &= \frac{(-1.71429)^2 + 3}{2 \times (-1.71429) - 1} = -1.34101,\\
p_4 &= \frac{(-1.34101)^2 + 3}{2 \times (-1.34101) - 1} = -1.30317.
\end{aligned}
$$

we can conjecture that the convergence of the sequence is affected by the selection of the initial point p_0. □

Exercise 2. Let $f(x) = x^3 - 3x - 2$.

(a) Find the Newton-Raphson formula $p_k = g(p_{k-1})$.

(b) Start with $p_0 = 2.1$ and find p_1, p_2, p_3, and p_4.

(c) Is the sequence converging quadratically or linearly?

Solve (a) In this case, the Newton-Raphson iterative function is

$$
g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^3 - 3x - 2}{3x^2 - 3} = \frac{(3x^3 - 3x) - x^3 + 3x + 2}{3x^2 - 3} = \frac{2x^3 + 2}{3x^2 - 3}.
$$

Hence, the Newton-Raphson formula is

$$
p_k = \frac{2P_{k-1}^3 + 2}{3p_{k-1}^2 - 3}. \tag{2}
$$

(b) Start with $p_0 = 2.1$, one can find

$$
\begin{aligned}
p_1 &= \frac{2 \times 2.1^3 + 2}{3 \times 2.1^2 - 3} = 2.00606,\\
p_2 &= \frac{2 \times 2.00606^3 + 2}{3 \times 2.00606^2 - 3} = 2.00002,\\
p_3 &= \frac{2 \times 2.00002^3 + 2}{3 \times 2.00002^2 - 3} = 2.00000,\\
p_4 &= \frac{2 \times 2.00000^3 + 2}{3 \times 2.00000^2 - 3} = 2.0.
\end{aligned}
$$

(c) Since $f(2) = 0$, $f'(2) = (3 \times 2^2 - 3) = 9$, 2 is the simple root of f. Hence, the sequence converges quadratically. □