THE SOUTH CHINA NORMAL UNIVERSITY

School of Mathematical Sciences

Numerical Analysis (2023-2024 The Second Term)

Homework 8:

Due Date: May 15, 2022 (Wednesday)

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§5.1 Exercises for Least-squares Line

Exercise 1. Derive the normal equations for finding the least-squares parabola

$$y = Ax^2 + B.$$

Solve Suppose that $\{(x_k, y_k)\}_{k=1}^N$ are N points, where the abscissas $\{x_k\}_{k=1}^N$ are distinct. The coefficients A and B will minimize the quantity:

$$E(A, B) = \sum_{k=1}^{N} (Ax_k^2 + B - y_k)^2.$$

The partial derivatives $\partial E/\partial A$ and $\partial E/\partial B$ must all be zero. This results in

$$0 = \frac{\partial E(A, B)}{\partial A} = 2\sum_{k=1}^{N} (Ax_k^2 + B - y_k)x_k^2, \tag{1}$$

$$0 = \frac{\partial E(A, B)}{\partial B} = 2\sum_{k=1}^{N} (Ax_k^2 + B - y_k). \tag{2}$$

Equations (1) and (2) can be rearranged in the standard form for a system and result in the normal equations:

$$(\sum_{k=1}^{N} x_k^4)A + (\sum_{k=1}^{N} x_k^2)B = \sum_{k=1}^{N} x_k^2 y_k, \tag{3}$$

$$(\sum_{k=1}^{N} x_k^2)A + NB = \sum_{k=1}^{N} y_k. \tag{4}$$

□

§5.2 Exercises for Curve Fitting

Exercise 1. Carry out the indicated change of variables in Table 5.6, and derive the linearized form for the following functions:

$$y = \frac{D}{x+C}.$$

Solve If $C \neq 0$, then

$$y = \frac{D}{x+C} \Leftrightarrow xy + Cy = D \Leftrightarrow y = -\frac{1}{C}xy + \frac{D}{C}.$$

Let $X = xy$, $Y = y$, $A = -\frac{1}{C}$, $B = \frac{D}{C}$. This results in a linear relation between the new variables X and Y: $Y = AX + B$. If $C = 0$, then

$$y = \frac{D}{x+C} = \frac{D}{x}.$$

Let $X = \frac{1}{x}$, $Y = y$, $A = D$, $B = 0$. This results in a linear relation between the new variables X and Y: $Y = AX + B$. $\square$