Chapter6

P259 7

7. *Exponential population growth.* The population of a certain species grows at a rate that is proportional to the current population and obeys the I.V.P.

$$y' = 0.02y \quad \text{over } [0, 5] \text{ with } y(0) = 5000.$$

(a) Apply formula (6.19) to find Euler's approximation to $y(5)$ using the step sizes $h = 1$, $\frac{1}{12}$, and $\frac{1}{360}$.

(b) What is the limit in part (a) when h goes to zero?

(a) formula(6.19) $t_k = kh, y_k = y_0(1 + hR)^k$, for $k = 0, 1, \ldots, M.$

h	Number of iterations,M	Approximation to y(5)
1	5	$5000(1 + 0.02)^5 = 5520.40$
$\frac{1}{12}$	60	$5000(1 + \frac{0.02}{12})^{60} = 5525.39$
$\frac{1}{360}$	1800	$5000(1 + \frac{0.02}{360})^{1800} = 5525.83$

(b) Beause, $f(t, y) = 0.02y$ satisfies Lipschitz condition in $[0, 5]$. So the solution of the differential equation satisfying the above initial conditions is $y = 5000e^{0.02x}$, so if $h \to 0, y_k \to 5000e^{0.1} = 5525.85.$

P259 8

8. A skydiver jumps from a plane, and up to the moment he opens the parachute the air resistance is proportional to $v^{3/2}$ (v represents velocity). Assume that the time interval is $[0, 6]$ and that the differential equation for the downward direction is

$$v' = 32 - 0.032v^{3/2} \quad \text{over } [0, 6] \text{ with } v(0) = 0.$$

Use Euler's method with $h = 0.05$ and estimate $v(6)$.

```
>> f=@(x,v)32-0.032*v^(3/2);
E = euler(f,0,6,0,120);
E(end)

ans =

  92.497908568731191
```

Using Euler's method with h=0.5 and estimate $v(6) = 92.4979.$

P259 9

9. *Epidemic model.* The mathematical model for epidemics is described as follows. Assume

that there is a community of L members that contains P infected individuals and Q uninfected individuals. Let $y(t)$ denote the number of infected individuals at time t. For a mild illness, such as the common cold, everyone continues to be active, and the epidemic spreads from those who are infected to those uninfected. Since there are PQ possible contacts between these two groups, the rate of change of $y(t)$ is proportional to PQ. Hence the problem can be stated as the I.V.P.

$$y' = ky(L-y) \quad \text{with} \quad y(0) = y_0.$$

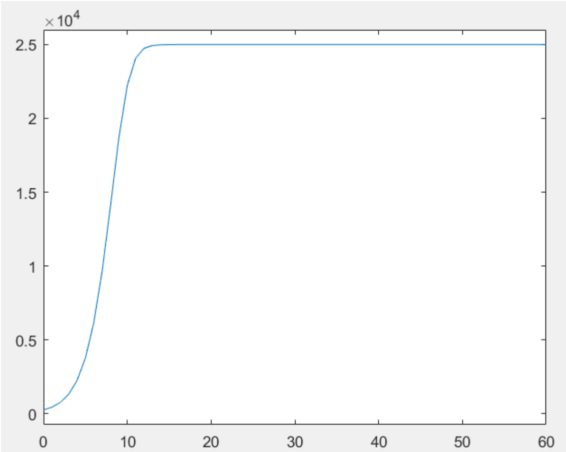
(a) Use $L = 25{,}000$, $k = 0.00003$, and $h = \overset{1}{\cancel{0.2}}$ with the initial condition $y(0) = 250$, and use Program 6.1 to compute Euler's approximate solution over $[0, 60]$.
(b) Plot the graph of the approximate solution from part (a).
(c) Estimate the average number of individuals infected by finding the average of the ordinates from Euler's method in part (a).
(d) Estimate the average number of individuals infected by fitting a curve to the data from part (a) and using the mean value theorem for integrals).

$(a) - (b)$ As shown below:

```
>> clear
f=@(t,y)0.00003*y*(25000-y);
E = euler(f,0,60,250,60);
```

```
>> format long
>> E

E =

   1.0e+04 *

                   0   0.025000000000000
   0.000100000000000   0.043562500000000
   0.000200000000000   0.075665067578125
   0.000300000000000   0.130696307526238
   0.000400000000000   0.223594080730619
   0.000500000000000   0.376291347397252
   0.000600000000000   0.616031304507379
   0.000700000000000   0.964206412447994
   0.000800000000000   1.408453020042240
   0.000900000000000   1.869670812174089
   0.001000000000000   2.223223237535940
   0.001100000000000   2.407824196511959
   0.001200000000000   2.474407135503390
   0.001300000000000   2.493405285461905
   0.001400000000000   2.498338274287525
   0.001500000000000   2.499583740172178
   0.001600000000000   2.499895883061371
   0.001700000000000   2.499973967513242
   0.001800000000000   2.499993491675003
   0.001900000000000   2.499998372906044
   0.002000000000000   2.499999593225717
   0.002100000000000   2.499999898306380
   0.002200000000000   2.499999974576592
   0.002300000000000   2.499999993644148
   0.002400000000000   2.499999998411037
   0.002500000000000   2.499999999602759
   0.002600000000000   2.499999999900690
   0.002700000000000   2.499999999975172
   0.002800000000000   2.499999999993793
   0.002900000000000   2.499999999998448
   0.003000000000000   2.499999999999612
   0.003100000000000   2.499999999999903
   0.003200000000000   2.499999999999976
   0.003300000000000   2.499999999999994
   0.003400000000000   2.499999999999999
   0.003500000000000   2.500000000000000
   0.003600000000000   2.500000000000000
   0.003700000000000   2.500000000000000
   0.003800000000000   2.500000000000000
   0.003900000000000   2.500000000000000
   0.004000000000000   2.500000000000000
   0.004100000000000   2.500000000000000
   0.004200000000000   2.500000000000000
   0.004300000000000   2.500000000000000
   0.004400000000000   2.500000000000000
   0.004500000000000   2.500000000000000
   0.004600000000000   2.500000000000000
   0.004700000000000   2.500000000000000
   0.004800000000000   2.500000000000000
   0.004900000000000   2.500000000000000
   0.005000000000000   2.500000000000000
   0.005100000000000   2.500000000000000
   0.005200000000000   2.500000000000000
   0.005300000000000   2.500000000000000
   0.005400000000000   2.500000000000000
   0.005500000000000   2.500000000000000
   0.005600000000000   2.500000000000000
   0.005700000000000   2.500000000000000
   0.005800000000000   2.500000000000000
   0.005900000000000   2.500000000000000
   0.006000000000000   2.500000000000000
```

```
plot(E(:,1),E(:,2))
set(gca,'YLim',[min(E(:,2))-1000 max(E(:,2))+1000]);
```



(c) the average of ordinates from Euler's method in part(a) is 21775.

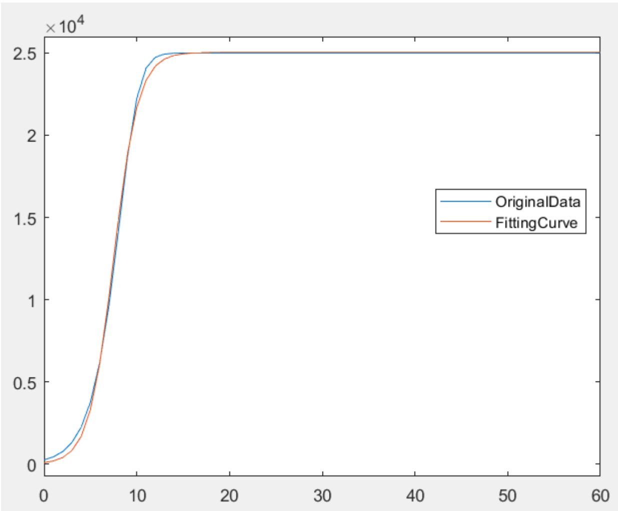
```
>> mean(E(:,2))

ans =

   2.1775e+04
```

(d) Beause, $f(t,y) = ky(L-y), L = 25,000, k = 0.00003$ satisfies Lipschitz condition in $[0,60]$. So the differential equation satisfying the above initial conditions has solutions like $y = A\frac{e^{Cx}}{B+e^{Cx}}$, we using fminsearch to find the fitting curve.

```
function z=Ea(u)
A=u(1);
B=u(2);
C=u(3);
f=@(t,y)0.00003*y*(25000-y);
E = euler(f,0,60,250,60);
s=size(E);
z=0;
for i=1:s(1)
    z=z+(A*exp(C*E(i,1))./(B+exp(C*E(i,1)))-E(i,2)).^2;
end
end

>> x=fminsearch(@Ea,[2500,0.02,0.8])

x =

   1.0e+04 *

   2.504108529150004   0.028064769646206   0.000074886999177
```

So we get the best fitting curve $y = 25041\frac{e^{0.75x}}{280+e^{0.75x}}$.

```
>> plot(E(:,1),E(:,2))
set(gca,'YLim',[min(E(:,2))-1000 max(E(:,2))+1000]);
hold on
y=25041*(exp(0.75*E(:,1)))./(280+exp(0.75*E(:,1)));
plot(E(:,1),y)
legend('OriginalData','FittingCurve')
```




```
>> g=@(x)25041*(exp(0.75*x))./(280+exp(0.75*x));
>> s = simprl(g,0,60,100);
>> s/60

ans =

     2.190344354432439e+04
```

the average number of individuals infected by fitting a curve to the data from part(a) and using the mean value theorem for integrals is 21904.

P267 6

6. Consider a projectile that is fired straight up and falls straight down. If air resistance is proportional to the velocity, the I.V.P. for the velocity $v(t)$ is

$$v' = -32 - \frac{K}{M}v \quad \text{with} \quad v(0) = v_0,$$

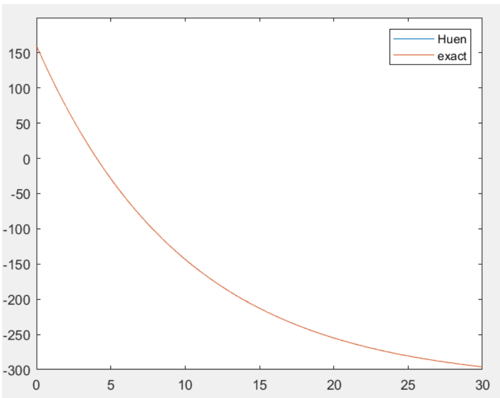
where v_0 is the initial velocity, M is the mass, and K the coefficient of air resistance. Suppose that $v_0 = 160$ ft/sec and $K/M = 0.1$. Use Heun's method with $h = 0.5$ to solve

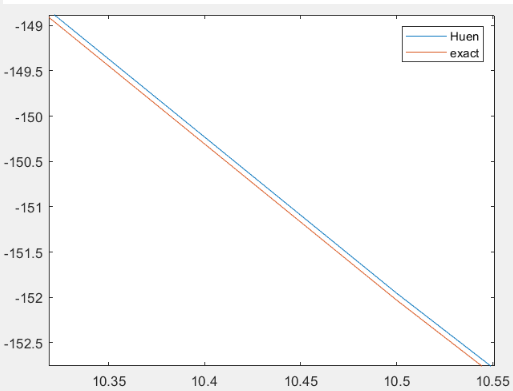
$$v' = -32 - 0.1v \quad \text{over } [0, 30] \text{ with } v(0) = 160.$$

Graph your computer solution and the exact solution $v(t) = 480e^{-t/10} - 320$ on the same coordinate system. Observe that the limiting velocity is -320 ft/sec.

```
>> clear
f=@(t,v)-32-0.1*v;
H = huen(f,0,30,160,60);
x=0:0.5:30;
y=480*exp(-x/10)-320;
plot(x,H(:,2),x,y)
legend('Huen','exact')
```

As shown below:





P267 7

7. In psychology, the Wever-Fechner law for stimulus-response states that the rate of change dR/dS of the reaction R is inversely proportional to the stimulus. The threshold value is the lowest level of the stimulus that can be detected consistently. The I.V.P. for this model is

$$R' = \frac{k}{S} \quad \text{with } R(S_0) = 0.$$

Suppose that $S_0 = 0.1$ and $R(0.1) = 0$. Use Heun's method with $h = 0.1$ to solve

$$R' = \frac{1}{S} \quad \text{over } [0.1, 5.1] \text{ with } R(0.1) = 0.$$

```
>> clear
R=@(S,R)1/S;
H = huen(R,0.1,5.1,0,50);
plot(H(:,1),H(:,2))
```

```
>> H

H =

   0.100000000000000                   0
   0.200000000000000   0.750000000000000
   0.300000000000000   1.166666666666667
   0.400000000000000   1.458333333333333
   0.500000000000000   1.683333333333333
   0.600000000000000   1.866666666666667
   0.700000000000000   2.021428571428571
   0.800000000000000   2.155357142857143
   0.900000000000000   2.273412698412698
   1.000000000000000   2.378968253968254
   1.100000000000000   2.474422799422800
   1.200000000000000   2.561544011544012
   1.300000000000000   2.641672216672217
   1.400000000000000   2.715848040848041
   1.500000000000000   2.784895659895660
   1.600000000000000   2.849478993228994
   1.700000000000000   2.910140757934876
   1.800000000000000   2.967330300418536
   1.900000000000000   3.021423867669998
   2.000000000000000   3.072739657143682
   2.100000000000000   3.121549180953206
   2.200000000000000   3.168085977490002
   2.300000000000000   3.212552380652058
   2.400000000000000   3.255124844420173
   2.500000000000000   3.295958177753507
   2.600000000000000   3.335188946984276
   2.699999999999999   3.372938234733564
   2.799999999999999   3.409313896109225
   2.899999999999999   3.444412418276713
   3.000000000000000   3.478320464253724
   3.100000000000000   3.511116163178455
   3.199999999999999   3.542870195436520
   3.300000000000000   3.573646710588035
   3.399999999999999   3.603504108092491
   3.500000000000000   3.632495704731146
   3.600000000000000   3.660670307905749
   3.699999999999999   3.688072710308152
   3.800000000000000   3.714744118558508
   3.899999999999999   3.740722526115862
   4.000000000000000   3.766043038936375
   4.100000000000000   3.790738160887595
   4.199999999999999   3.814838044743576
   4.300000000000000   3.838370713625082
   4.399999999999999   3.861362256965463
   4.500000000000000   3.883837004440210
   4.600000000000000   3.905817680768712
   4.699999999999999   3.927325543858444
   4.800000000000000   3.948380508397451
   4.899999999999999   3.969001256696771
   5.000000000000000   3.989205338329425
   5.100000000000000   4.009009259898052
```

the solution of the differential equation satisfying the above initial conditions is shown in the picture: