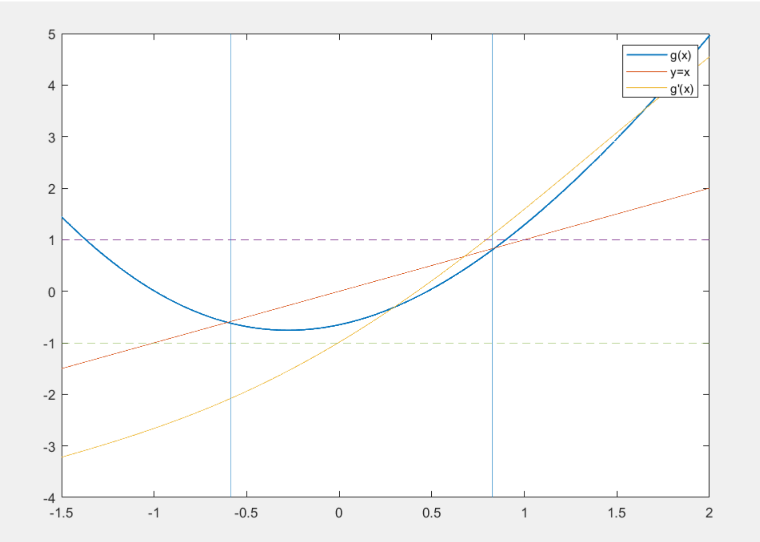
P11 1.

Use Program 1.1 to approximate the fixed points (if any) of each function. Answers should be accurate to 12 decimal places. Produce a graph of each function and the line $y = x$ that clearly show any fixed points.

(c) $g(x) = x^2 - sin(x + 0.15)$
additional Requirement: Produce a graph of function $g(x), g'(x)$ and the line $y = x$ that clearly shows any fixed points.

```
%plotfuc.m
figure;
x=-1.5:0.001:2;
g=x.^2-sin(x+15);
dg=2*x-cos(x+0.15);
y=x;
plot(x,g,'LineWidth',1)
hold on
plot(x,y)
hold on
plot(x,dg)
hold on
plot([-1.5,2],[1,1],'--')
hold on
plot([-1.5,2],[-1,-1],'--')
hold on
line([-0.588,-0.588],[-4,5]);
hold on
line([0.825,0.825],[-4,5]);
legend('g(x)','y=x',"g'(x)")
```



We can see from the graph that $g(x)$ has two fixed points within interval $[-1, 1]$. But the derivatives of these two fixed points are greater than 1. Thus, they are repelling fixed points, we cannot use fixed-point iteration to find the solutions to the equation $x = g(x)$.

P22 1.

Find an approximation (accurate to 10 decimal places) for the interest rate I that will yield a total annuity value of $\$500,000$ if 240 monthly payments of $\$300$ are made.

Solution:
Let P be the monthly savings and I be the annual interest rate .

Month	Principal	Money After Depositing
1	P	$P(1+\frac{I}{12})$
2	$P(1+\frac{I}{12})+P$	$P(1+\frac{I}{12})^2+P(1+\frac{I}{12})$
3	$P(1+\frac{I}{12})^2+P(1+\frac{I}{12})+P$	$P(1+\frac{I}{12})^3+P(1+\frac{I}{12})^2+$

After n month， the total amount of money A is
$A = P((1+\frac{I}{12})+(1+\frac{I}{12})^2+(1+\frac{I}{12})^3+...+(1+\frac{I}{12})^n)$.
Let $r = 1+\frac{I}{12}$， $A = P(r+r^2+r^3+...+r^n) = P\cdot r\frac{1-r^n}{1-r}$.
now, $P = 300$, $A = 500,000$, $n = 240$,
$A(I) = 300r\frac{1-r^{240}}{1-r} = \frac{300}{I/12}((1+\frac{I}{12})^{240}-1)(1+\frac{I}{12})$.

```
%moneyf.m
function [output] = moneyf(x)
    output=(300/(x/12))*((1+(x/12))^240-1)*(1+x/12);
end

%Command line window
>> moneyf(0.15)
ans =
   4.5479e+05

>> moneyf(0.16)
ans =
   5.2484e+05
```

Thus, the root of equation $500,000 = \frac{300}{I/12}((1+\frac{I}{12})^{240}-1)(1+\frac{I}{12})$ is within interval [0.15,0.16]. Use the False Position Method to find the solution of equation $f(x) = \frac{300}{I/12}((1+\frac{I}{12})^{240}-1)(1+\frac{I}{12}) - 500000 = 0$.

```
%f.m
function [output] = f(x)
output=(300/(x/12))*((1+(x/12))^240-1)*(1+x/12)-500000;
end

%regula.m
function [n,c,yc] = regula(f,a,b,delta,epsilon,max1)
digits(10)
ya=feval(f,a);
yb=feval(f,b);
if ya*yb>0
    disp('Note:f(a)*f(b)>0');
    return,
end
for k=1:max1
    dx=yb*(b-a)/(yb-ya);
    c=b-dx;
    ac=c-a;
    yc=feval(f,c);
    if yc==0
        break;
    elseif yb*yc>0
        b=c;
        yb=yc;
    else
        a=c;
        ya=yc;
    end
    dx=min(abs(dx),ac);
    if abs(dx)<delta,break,end
    if abs(yc)<epsilon,break,end
end
n=k;
c=vpa(c);
yc=feval(f,c);

%Command line window
>> [n,c,yc] = regula(@f,0.15,0.16,1e-10,1e-10,100)
n =
     6

c =
```

```
    0.1566

yc =
  -1.6076e-05
```

Thus , we find an approximation (accurate to 10 decimal places) for the interest rate $I = 0.1566 = 15.66\%$ that will yield a total annuity value of $\$500,000$ if 240 monthly payments of $\$300$ are made.

P39 7.

Consider the function $f(x) = xe^{-x}$.
(a) Find the Newton-Raphson formula $p_k = g(p_{k-1})$.
(b) If $p_0 = 0.2$, then find p_1, p_2, p_3 and p_4. What is $lim_{n\to\infty} p_k$?
(c) If $p_0 = 2.0$, then find p_1, p_2, p_3 and p_4. What is $lim_{n\to\infty} p_k$?
(d) What is the value of $f(p_4)$ in part (c) ?

Solution:
(a) The first derivate of $f(x) = xe^{-x}$ is $f'(x) = (1-x)e^{-x}$.
The Newton-Raphson iterative function is $g(x) = x - \frac{x}{1-x} = \frac{x^2}{x-1}$.
The Newton-Raphson formula is $p_k = \frac{p_{k-1}^2}{p_{k-1}-1}, k = 1, 2, \ldots$.

(b)

```
%f.m
function [output] = f(x)
output=x*exp(-x);
end

%df.m
function [output] = df(x)
syms k
f(k)=k*exp(-k);
df=diff(f(k));
k=x;
output=eval(df);
end

%newton.m
function [P,err,k,y] = newton(f,df,p0,delta,epsilon,max1)
digits(7)
P(1)=vpa(p0);
for k=1:max1
    p1=p0-feval(f,p0)/feval(df,p0);
    y(k)=vpa(feval(f,p0));
```

```
    err=abs(p1-p0);
    relerr=2*err/(abs(p1)+delta);
    p0=p1;
    P(k+1)=vpa(p0);
    if(err<delta)|(relerr<delta)|(abs(y)<epsilon)
        break
    end
end
```

```
%Command line window
>> [P,err,k,y] = newton(@f,@df,0.2,1e-12,1e-12,100)
```

The results are listed in table with starting value $p_0 = 0.2$.

k	p_k	$f(p_k)$
0	0.200000	0.163746
1	-0.050000	-0.052563
2	-0.002380	-0.002387
3	-0.000006	-0.000006
4	-0.000000	-0.000000

Thus, the fixed point is $x = 0$, $lim_{n\to\infty} p_k = 0$.

(c)-(d)

```
%Command line window
>> [P,err,k,y] = newton(@f,@df,2,1e-12,1e-12,100)
```

The results are listed in table with starting value $p_0 = 2.0$.

k	p_k	$f(p_k)$
0	2.000000	0.270671
1	4.000000	0.073263
2	5.333333	0.025749
3	6.564103	0.009256
4	7.743826	0.003356

$f(p_4) = 7.743826$, this time $lim_{n\to\infty} p_k = +\infty$.