Chapter 3

P102 2

(a) Use the plot command to plot $cos(x), P_4(x), P_6(x), and\ P_8(x)$ from Exercise 2 on the same graph using the interval $-1 \leq x \leq 1$
(b) Create a table with columns that consist of $cos(x), P_4(x), P_6(x), andP_8(x)$ evaluated at 19 equally spaced values of x from the interval $[-1, 1]$.

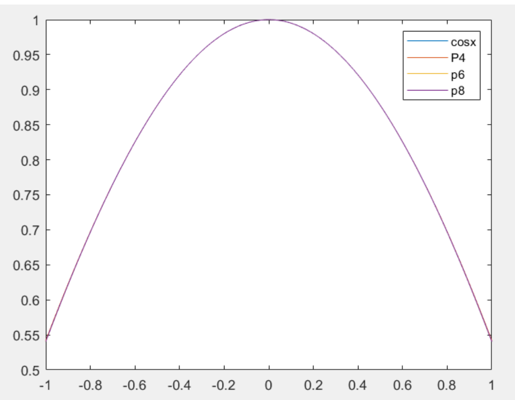
Taylor Polynomial Approximation:

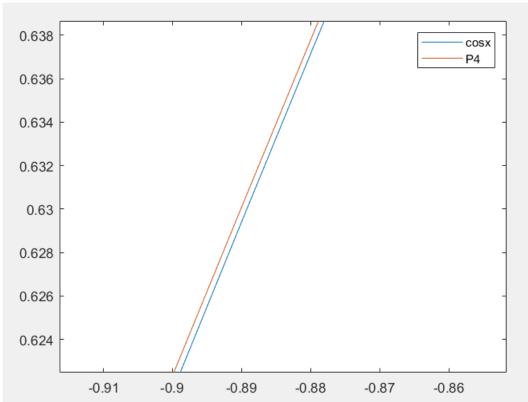
$$f(x) = P_N(x) + E_N(x),\ \ P_N(x) = \sum_{k=0}^{N} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

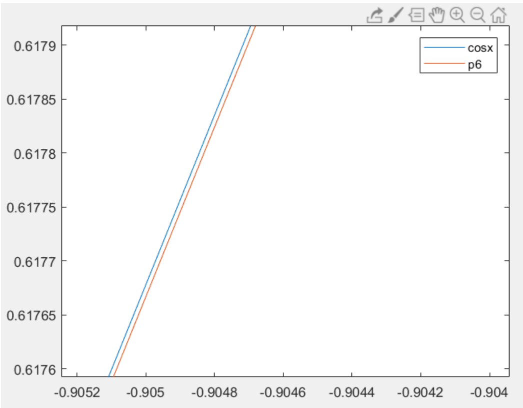
let $x_0 = 0$,
$P_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4$,
$P_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6$,
$P_8(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8$.

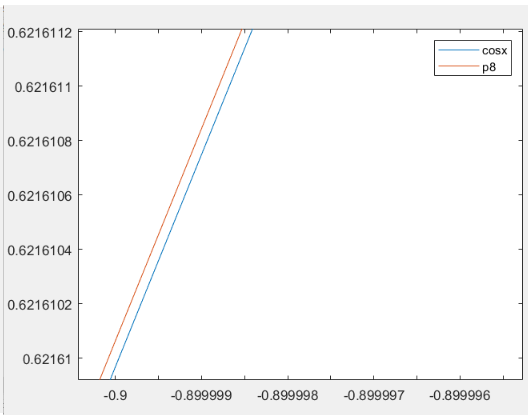
```
x=-1:0.01:1;
plot(x,cos(x))
hold on
plot(x,polyval([1/prod(1:4),0,-1/2,0,1],x))
hold on
plot(x,polyval([-1/prod(1:6),0,1/prod(1:4),0,-1/2,0,1],x))
hold on
plot(x,polyval([1/prod(1:8),0,-1/prod(1:6),0,1/prod(1:4),0,-1/2,0,1],x))
legend('cosx','P4','p6','p8')
```

Draw the graph, and compare the fitting degree of different order approximation curve near the end point, it can be seen that the polynomial of order 8 has a better fitting degree at the end point, so it has a better fitting degree for the function curve.









	$P_4(x)$	$P_6(x)$	$P_8(x)$	$cos(x)$
----	------------------	------------------	------------------	------------------
-0.9	0.622337500000000	0.621599387500000	0.621610063770089	0.621609968270664
-0.8	0.697066666666667	0.696702577777778	0.696706738793651	0.696706709347165
-0.7	0.765004166666667	0.764840765277778	0.764842195039931	0.764842187284488
-0.6	0.825400000000000	0.825335200000000	0.825335616571429	0.825335614909678
-0.5	0.877604166666667	0.877582465277778	0.877582562158978	0.877582561890373
-0.4	0.921066666666667	0.921060977777778	0.921060994031746	0.921060994002885
-0.3	0.955337500000000	0.955336487500000	0.955336489127232	0.955336489125606
-0.2	0.980066666666667	0.980066577777778	0.980066577841270	0.980066577841242
-0.1	0.995004166666667	0.995004165277778	0.995004165278026	0.995004165278026
0	1.000000000000000	1.000000000000000	1.000000000000000	1.000000000000000
0.1	0.995004166666667	0.995004165277778	0.995004165278026	0.995004165278026
0.2	0.980066666666667	0.980066577777778	0.980066577841270	0.980066577841242
0.3	0.955337500000000	0.955336487500000	0.955336489127232	0.955336489125606
0.4	0.921066666666667	0.921060977777778	0.921060994031746	0.921060994002885
0.5	0.877604166666667	0.877582465277778	0.877582562158978	0.877582561890373
0.6	0.825400000000000	0.825335200000000	0.825335616571429	0.825335614909678
0.7	0.765004166666667	0.764840765277778	0.764842195039931	0.764842187284488
0.8	0.697066666666667	0.696702577777778	0.696706738793651	0.696706709347165
0.9	0.622337500000000	0.621599387500000	0.621610063770089	0.621609968270664

P109 1, 2

1.Write a program in MATLAB that will implement Algorithm 3.1. The program should accept the coefficients of the polynomial

$P(x) = a_N x^N + a_{N-1} x^{N-1} + \ldots + a_2 x^2 + a_1 x + a_0$ as an $1 \times N$ matrix:

$P = [a_N \; a_{N-1} \; \ldots \; a_2 \; a_1 \; a_0]$.

$P(x) = 3x^2 + 2x + 1, \;\; P(1) = 6$

$P'(x) = 6x + 2, \;\; P'(1) = 8$

$I(x) = \int P(x) = x^3 + x^2 + x + C,$ let $C = 1,$ thus $I(1) = 4$.

```
%Horner1.m
function [v] = Horner1(A,X)
%Input  - A: Coefficients of P(x)
%         - X: Independent variable
%Output - the values of P(x)
A=flip(A);
N=length(A)-1;
B=zeros(N+1,1);
B(N+1)=A(N+1);
for k=N:-1:1
    B(k)=A(k)+B(k+1)*X;
end
```

```
v=B(1);
end

%test
>> A=[3,2,1];
>> X=1;
>> [v] = Horner1(A,X)
v =
     6
```

```
%Horner2.m
function [v] = Horner2(A,X)
%input  - N: Degree of P(x)
%       - A: Coefficients of P(x)
%       - C: Constant of integration
%       - X: Independent variable
%Output - the values of P'(x)
A=flip(A);
N=length(A)-1;
D=zeros(N,1);
D(N)=N*A(N+1);
for k=N:-1:2
    D(k-1)=(k-1)*A(k)+D(k)*X;
end
v=D(1);
end

%test
>> A=[3,2,1];
>> X=1;
>> [v] = Horner2(A,X)
v =
     8
```

```
%Horner3.m
function [v] = Horner3(A,X,C)
%input  - N: Degree of P(x)
%       - A: Coefficients of P(x)
%       - X: Independent variable
%output - \int P(x)
A=flip(A);
N=length(A)-1;
I=zeros(N+2,1);
I(N+2)=A(N+1)/(N+1);
for k=N+1:-1:2
    I(k)=A(k-1)/(k-1)+I(k+1)*X;
end
I(1)=C+I(2)*X;
v=I(1);
end
```

```
%test
>> A=[3,2,1];
>> X=1;
>> C=1;
>> [v] = Horner3(A,X,C)

v =
     4
```

2. For each of the given functions, the fifth-degree polynomial $P(x)$ passes through the six points $(0, f(0))$, $(0.2, f(0.2))$, $(0.4, f(0.4))$, $(0.6, f(0.6))$, $(0.8, f(0.8))$, $(1, f(1))$. The six coefficients of $P(x)$ are $a_0, a_1, \ldots, a_5$, where

$$P(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0.$$

(i) Find the coefficients of $P(x)$ by solving the 6×6 system of linear equations

$$a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 = f(x_j)$$

using $x_j = (j-1)/5$ and $j = 1, 2, 3, 4, 5, 6$ for the six unknowns $\{a_k\}_{k=0}^{5}$.
(ii) Use your MATLAB program from Problem 1 to compute the interpolated values $P(0.3)$, $P(0.4)$, and $P(0.5)$ and compare with $f(0.3)$, $f(0.4)$, and $f(0.5)$, respectively.
(iii) Use your MATLAB program to compute the extrapolated values $P(-0.1)$ and $P(1.1)$ and compare with $f(-0.1)$ and $f(1.1)$, respectively.
(iv) Use your MATLAB program to find the integral of $P(x)$ taken over $[0, 1]$ and compare with the integral of $f(x)$ taken over $[0, 1]$. Plot $f(x)$ and $P(x)$ over $[0, 1]$ on the same graph.
(v) Make a table of values for $P(x_k)$, $f(x_k)$, and $E(x_k) = f(x_k) - P(x_k)$, where $x_k = \cancel{k/100}$ k/10 for $k = 0, 1, \ldots, 100$.
(a) $f(x) = e^x$
(b) $f(x) = \sin(x)$
(c) $f(x) = (x+1)^{(x+1)}$

```
%Horner1.m
function [v] = Horner1(A,X)
le=length(X);
A=flip(A);
N=length(A)-1;
for i=1:le
    B=zeros(N+1,1);
    B(N+1)=A(N+1);
    for k=N:-1:1
        B(k)=A(k)+B(k+1)*X(i);
    end
    v(i)=B(1);
end
```

$(a)\ f(x)=e^x$

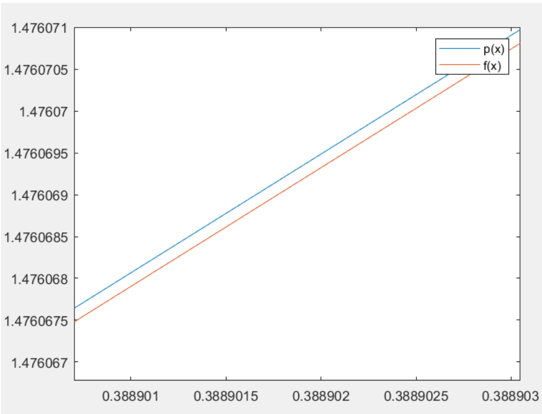
$x_1=0, x_2=0.5, x_3=0.4, x_4=0.6, x_5=0.8, x_6=1$

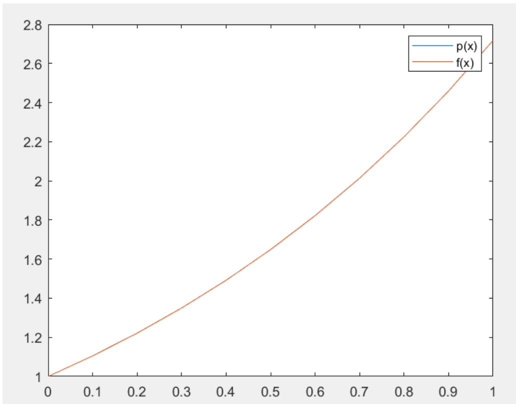
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5^2 & 0.5^3 & 0.5^4 & 0.5^5 \\ 1 & 0.4 & 0.4^2 & 0.4^3 & 0.4^4 & 0.4^5 \\ 1 & 0.6 & 0.6^2 & 0.6^3 & 0.6^4 & 0.6^5 \\ 1 & 0.8 & 0.8^2 & 0.8^3 & 0.8^4 & 0.8^5 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} 1 \\ e^{0.5} \\ e^{0.4} \\ e^{0.6} \\ e^{0.8} \\ e \end{pmatrix}$$

```
%The following commands are completed in the command line window
%(i)
X=[0,0.5,0.4,0.6,0.8,1]';
A=[X.^0,X.^1,X.^2,X.^3,X.^4,X.^5];
B=exp(X);
X = uptrbk(A,B)
X =
    1.0000
    1.0002
    0.4982
    0.1724
    0.0329
    0.0145

%(ii)
X=flip(X);
format long
[v] = Horner1(X,[0.3,0.4,0.5]) %The horner1 function used here has been improved, and the code is shown above
v =
   1.349860279454741   1.491824697641270   1.648721270700128
exp([0.3,0.4,0.5])
ans =
   1.349858807576003   1.491824697641270   1.648721270700128
% we can see that P(0.4)=f(0.4),P(0.5)=f(0.5)

%(iii)
[v] = Horner1(X,[-0.1,1.1])
v =
   0.904791419021796   3.004147840781791
exp([-0.1,1.1])
ans =
   0.904837418035960   3.004166023946433

%(iv)
Horner3(X,1,1)-Horner3(X,0,1)
ans =
   1.718283695538349
exp(1)-exp(0)
ans =
   1.718281828459046

```

```
x=0:0.1:1;
f=exp(x);
plot(x,polyval(X,x),x,f)
legend('p(x)','f(x)')

%(v)
point=0:0.1:1;
[v] = Horner1(X,point);
fp=exp(point);
[v',fp',fp'-v']
ans =
   1.000000000000000   1.000000000000000                   0
   1.105179509812279   1.105170918075648  -0.000008591736631
   1.221408067143980   1.221402758160170  -0.000005308983811
   1.349860279454741   1.349858807576003  -0.000001471878738
   1.491824697641270   1.491824697641270                   0
   1.648721270700128   1.648721270700128                   0
   1.822118800390509   1.822118800390509   0.000000000000000
   2.013752395897021   2.013752707470477   0.000000311573456
   2.225540928492468   2.225540928492468                   0
   2.459604486200630   2.459603111156950  -0.000001375043680
   2.718281828459046   2.718281828459046                   0
```

The graph of (a) (iv)





$(b)\ f(x) = e^x$

$x_1 = 0, x_2 = 0.5, x_3 = 0.4, x_4 = 0.6, x_5 = 0.8, x_6 = 1$

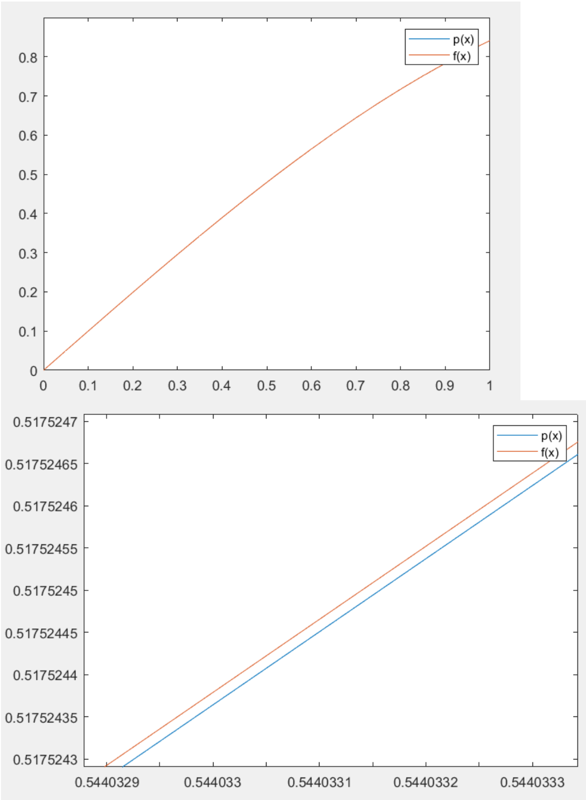
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5^2 & 0.5^3 & 0.5^4 & 0.5^5 \\ 1 & 0.4 & 0.4^2 & 0.4^3 & 0.4^4 & 0.4^5 \\ 1 & 0.6 & 0.6^2 & 0.6^3 & 0.6^4 & 0.6^5 \\ 1 & 0.8 & 0.8^2 & 0.8^3 & 0.8^4 & 0.8^5 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} 0 \\ sin(0.5) \\ sin(0.4) \\ sin(0.6) \\ sin(0.8) \\ sin(1) \end{pmatrix}$$

```
%The following commands are completed in the command line window
%(i)
>> X=[0,0.5,0.4,0.6,0.8,1]';
>> A=[X.^0,X.^1,X.^2,X.^3,X.^4,X.^5];
```

```
>> B=sin(X);
>> X = uptrbk(A,B)
X =
         0
    0.9999
    0.0005
   -0.1682
    0.0022
    0.0071

%(ii)
>> X=flip(X);
>> format long
>> [v] = Horner1(X,[0.3,0.4,0.5]) %The horner1 function used here has been improved, and the code is shown above
v =
   0.295519778718137   0.389418342308651   0.479425538604203
>> sin([0.3,0.4,0.5])
ans =
   0.295520206661340   0.389418342308651   0.479425538604203
% we can see that P(0.4)=f(0.4),P(0.5)=f(0.5)

%(iii)
>> [v] = Horner1(X,[-0.1,1.1])
v =
  -0.099820778985218   0.891212976980436
>> sin([-0.1,1.1])
ans =
  -0.099833416646828   0.891207360061435

%(iv)
>> Horner3(X,1,1)-Horner3(X,0,1)
ans =
   0.459697158952903
>> -cos(1)-(-cos(0))
ans =
   0.459697694131860

>> x=0:0.05:1;
f=sin(x);
plot(x,polyval(X,x),x,f)
legend('p(x)','f(x)')

%(v)
>> point=0:0.1:1;
[v] = Horner1(X,point);
fp=sin(point);
[v',fp',fp'-v']
ans =
                   0                   0                   0
   0.099830982487393   0.099833416646828   0.000002434159435
   0.198667806192501   0.198669330795061   0.000001524602560
```

```
    0.295519778718137    0.295520206661340    0.000000427943203
    0.389418342308651    0.389418342308651                    0
    0.479425538604203    0.479425538604203                    0
    0.564642473395035    0.564642473395035                    0
    0.644217781375738    0.644217687237691   -0.000000094138047
    0.717356090899523    0.717356090899523                    0
    0.783326488732489    0.783326909627483    0.000000420894995
    0.841470984807897    0.841470984807897                    0
```

The graph of (b) (iv)



$(c)\ f(x) = (x+1)^{(x+1)}$

$x_1 = 0, x_2 = 0.5, x_3 = 0.4, x_4 = 0.6, x_5 = 0.8, x_6 = 1$

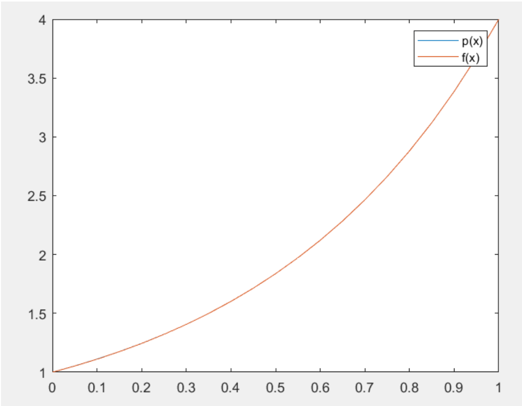
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5^2 & 0.5^3 & 0.5^4 & 0.5^5 \\ 1 & 0.4 & 0.4^2 & 0.4^3 & 0.4^4 & 0.4^5 \\ 1 & 0.6 & 0.6^2 & 0.6^3 & 0.6^4 & 0.6^5 \\ 1 & 0.8 & 0.8^2 & 0.8^3 & 0.8^4 & 0.8^5 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1.5^{1.5} \\ 1.4^{1.4} \\ 1.6^{1.6} \\ 1.8^{1.8} \\ 2^2 \end{pmatrix}$$

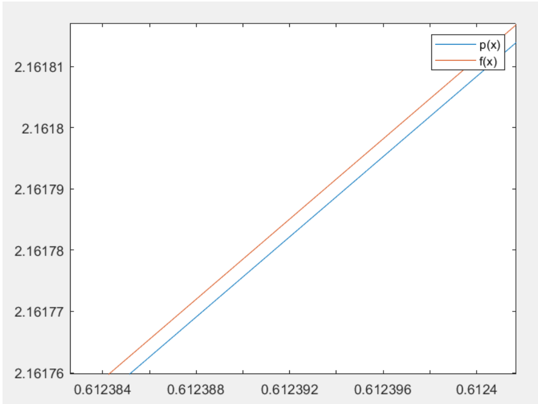
```
%The following commands are completed in the command line window
%(i)
>> X=[0,0.5,0.4,0.6,0.8,1]';
>> A=[X.^0,X.^1,X.^2,X.^3,X.^4,X.^5];
>> B=(X+1).^(X+1);
>> X = uptrbk(A,B)
X =
     1.0000
     1.0144
```

```
     0.8830
     0.8653
    -0.2047
     0.4420

%(ii)
>> X=flip(X);
>> format long
>> [v] = Horner1(X,[0.3,0.4,0.5]) %此处使用的Horner1函数是上面Horner1函数的改进，代码见上方
v =
   1.406559443634434   1.601692898202212   1.837117307087384
>> ([0.3,0.4,0.5]+1).^([0.3,0.4,0.5]+1)
ans =
   1.406456673237886   1.601692898202212   1.837117307087384
% we can see that P(0.4)=f(0.4),P(0.5)=f(0.5)

%(iii)
>> [v] = Horner1(X,[-0.1,1.1])
v =
   0.906503957590380   4.748141857254273
>> ([-0.1,1.1]+1).^([-0.1,1.1]+1)
ans =
   0.909532576082962   4.749638091742242

%(iv)
>> Horner3(X,1,1)-Horner3(X,0,1)
ans =
   2.050575148634692
>> syms x
f=inline((x+1)^(x+1));
g=quad(f,0,1)
g =
   2.050446241747296

>> x=0:0.05:1;
f=(x+1).^(x+1);
plot(x,polyval(X,x),x,f)
legend('p(x)','f(x)')

%(v)
>> point=0:0.1:1;
[v] = Horner1(X,point);
fp=(point+1).^(point+1);
[v',fp',fp'-v']
ans =
   1.000000000000000   1.000000000000000                   0
   1.111115623542183   1.110534241054576  -0.000581382487607
   1.244929494155687   1.244564747203978  -0.000364746951709
   1.406559443634434   1.406456673237886  -0.000102770396548
   1.601692898202212   1.601692898202212                   0
   1.837117307087384   1.837117307087384                   0
```

```
    2.121250571097592   2.121250571097592  -0.000000000000000
    2.464671471194466   2.464694899484870   0.000023428290404
    2.880650097068328   2.880650097068328                   0
    3.385678275712898   3.385570343918481  -0.000107931794417
    4.000000000000000   4.000000000000000                   0
```

The graph of (c) (iv)





P122 2

2.

The measured temperatures during a 5-hour period in a suburb of Los Angeles on November 8 are given in the following table.

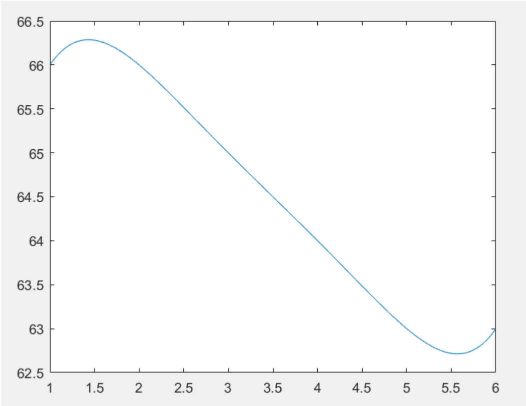
(a) Use Program 3.1 to construct a Lagrange interpolatory polynomial for the data in the table.

(b) Use Algorithm 3.1(iii) to estimate the average temperature during the given 5-hour period.

(c) Graph the data in the table and the polynomial from part (a) on the same coordinate system. Discuss the possible error that can result from using the polynomial in part (a) to estimate the average temperature.

Time, P.M.	Degrees Fahrenheit
1	66
2	66
3	65
4	64
5	63
6	63

```
%The following commands are completed in the command line window
X=[1,2,3,4,5,6];
Y=[66,66,65,64,63,63];
format long
[C] = lagran(X,Y);
C'
ans =
   0.016666666666667
  -0.291666666666657
   2.000000000000000
  -6.708333333333371
   9.983333333333121
  61.000000000000000
x=1:0.01:6;
plot(x,polyval(C,x))
```



If $f(x)$ is a continuous function on the closed, bounded interval$[a, b]$,then there is at least one number ε in (a, b), for which

$$\int_a^b f(x)dx = f(\varepsilon)(b-a)$$

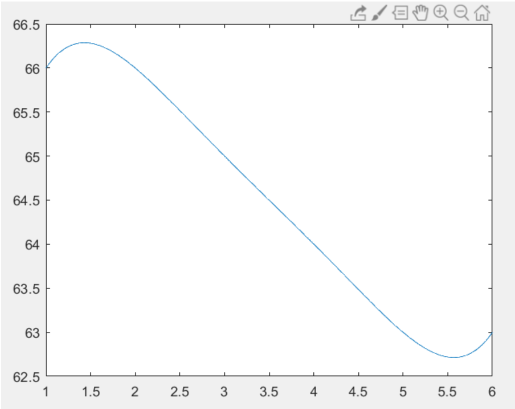
```
%The following commands are completed in the command line window
(Horner3(C,6,1)-Horner3(C,1,1))/(6-1);
ans =
  64.500000000002260
```

According to Mean Value Theorem for Integrals， the average-temperature is about 64.500000000002260.

P132 1

1.Use Program 3.2 and repeat Problem 2 in Algorithms and Programs from Section 3.3.

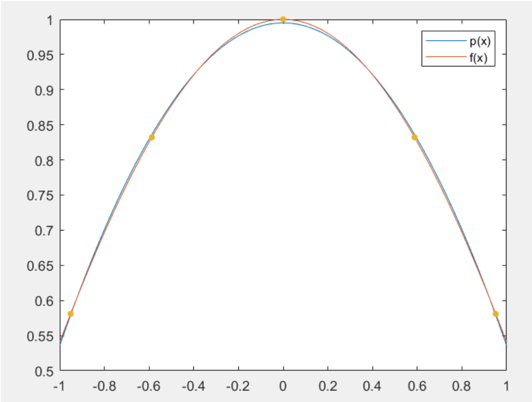
```
X=[1,2,3,4,5,6];
Y=[66,66,65,64,63,63];
format long
[C] = newpoly(X,Y);
C'
ans =
   0.016666666666667
  -0.291666666666667
   2.000000000000000
  -6.708333333333334
   9.983333333333334
  61.000000000000000
x=1:0.01:6;
plot(x,polyval(C,x))
(Horner3(C,6,1)-Horner3(C,1,1))/(6-1)
ans =
  64.500000000000028
```



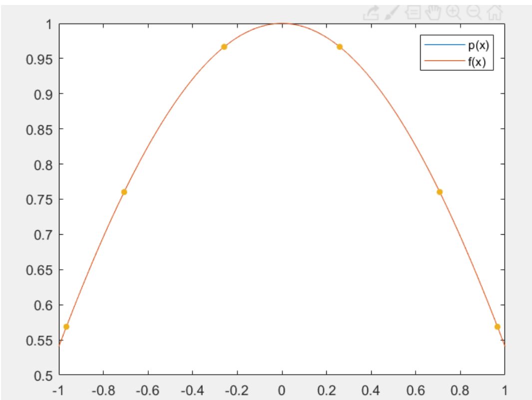
According to Mean Value Theorem for Integrals， the average-temperature is about 64.50000000000028.

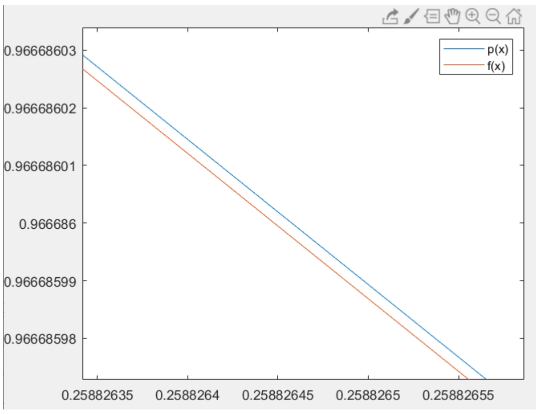
P144 3， 7
In Problems 3, use Program 3.3 to compute the coefficients $\{c_k\}$ for the Chebyshev polynomial approximation $P_N(x)$ to $f(x)$ over $[-1,1]$, when (a) $N=4$, (b) $N=5$, (c) $N=6$, and (d) $N=7$. In each case, plot $f(x)$ and $P_N(x)$ on the same coordinate system.
$3.$ $f(x)=cos(x)$
$7.$ Use Program 3.3 ($N=5$) to obtain an approximation for $\int_0^1 cos(x^2)dx$.
Solution:
3.

```
%N=4
[C,X,Y] = cheby('cos(x)',4,-1,1);
C'

ans =
    0.765197687084090
   -0.000000000000000
   -0.229807158311684
    0.000000000000000
    0.004995154604226

P=C(1)*poly2sym([1])+C(2)*poly2sym([1,0])+C(3)*poly2sym([2,0,-1])+C(
4)*poly2sym([4,0,-3,0]);

x=-1:0.01:1;
plot(x,polyval(sym2poly(P),x),x,cos(x));
hold on
scatter(X,Y,20,'filled');
legend('p(x)','f(x)')
```



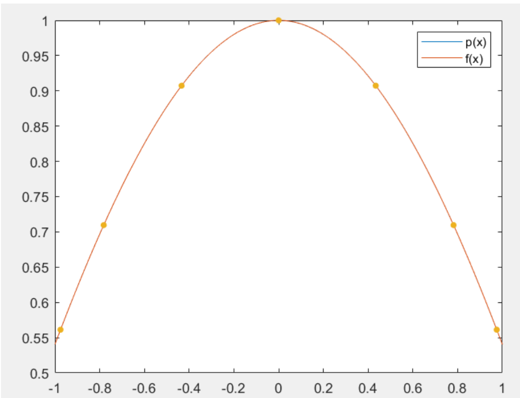

```
%N=5
[C,X,Y] = cheby('cos(x)',5,-1,1);
C'

ans =
    0.765197686556967
    0.000000000000000
   -0.229806969337677
    0.000000000000000
    0.004953089481336
   -0.000000000000000

P=C(1)*poly2sym([1])+C(2)*poly2sym([1,0])+C(3)*poly2sym([2,0,-1])+C(4)*poly2sym([4,0,-3,0])+C(5)*poly2sym([8,0,-8,0,1]);

x=-1:0.01:1;
plot(x,polyval(sym2poly(P),x),x,cos(x));
hold on
scatter(X,Y,20,'filled');
legend('p(x)','f(x)')
```

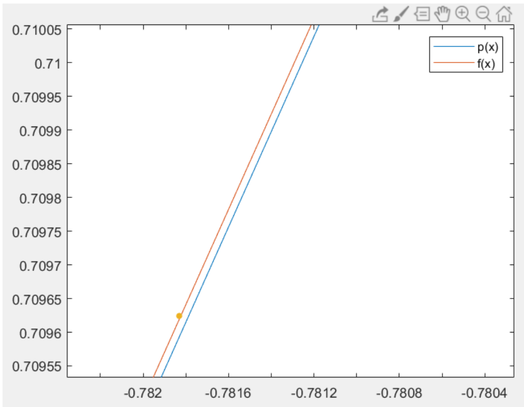




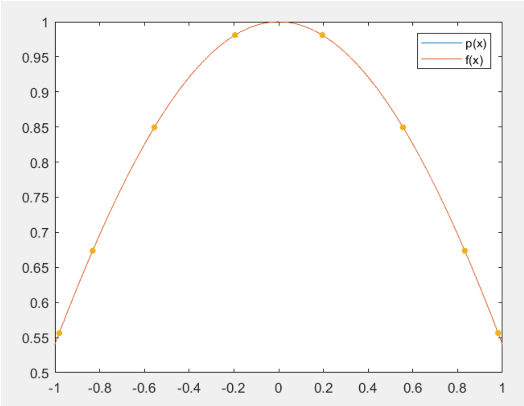

```
%N=6
[C,X,Y] = cheby('cos(x)',6,-1,1);
C'

ans =
    0.765197686557968
    0.000000000000000
```

```
  -0.229806969864801
   0.000000000000000
   0.004953278454343
   0.000000000000000
  -0.000042065122888

P=C(1)*poly2sym([1])+C(2)*poly2sym([1,0])+C(3)*poly2sym([2,0,-1])+C(4)*poly2sym([4,0,-3,0])+C(5)*poly2sym([8,0,-8,0,1])+C(6)*poly2sym([16,0,-20,0,5,0]);

x=-1:0.01:1;
plot(x,polyval(sym2poly(P),x),x,cos(x));
hold on
scatter(X,Y,20,'filled');
legend('p(x)','f(x)')
```






```
%N=7
[C,X,Y] = cheby('cos(x)',7,-1,1);
C'

ans =
   0.765197686557966
  -0.000000000000000
  -0.229806969863800
   0.000000000000000
   0.004953277927220
  -0.000000000000000
  -0.000041876149882
   0.000000000000001

P=C(1)*poly2sym([1])+C(2)*poly2sym([1,0])+C(3)*poly2sym([2,0,-1])+C(4)*poly2sym([4,0,-3,0])+C(5)*poly2sym([8,0,-8,0,1])+C(6)*poly2sym([16,0,-20,0,5,0])+C(7)*poly2sym([32,0,-48,0,18,0,-1]);

x=-1:0.01:1;
plot(x,polyval(sym2poly(P),x),x,cos(x));
hold on
scatter(X,Y,20,'filled');
legend('p(x)','f(x)')
```



7.

```
[C,X,Y] = cheby('cos(x.^2)',5,-1,1);
C'

ans =
   0.823585327550802
   0.000000000000000
  -0.232291660907438
   0.000000000000000
  -0.053997234339571
  -0.000000000000000

P=C(1)*poly2sym([1])+C(2)*poly2sym([1,0])+C(3)*poly2sym([2,0,-1])+C(4)*poly2sym([4,0,-3,0])+C(5)*poly2sym([8,0,-8,0,1]);

Horner3(sym2poly(P),1,1)-Horner3(sym2poly(P),0,1)
ans =
   0.904615696809253

syms x
f=inline(cos(x^2));
g=quad(f,0,1)
g =
   0.904524260466284
```

We obtain an approximation value 0.904524260466284 for $\int_0^1 cos(x^2)dx$.
P149 3

3. Compare the following approximations to $f(x) = \tan(x)$.

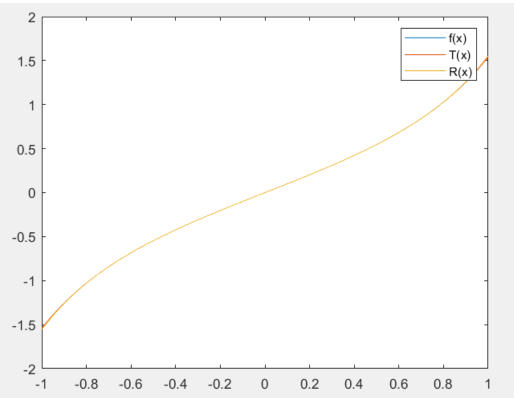
$$\text{Taylor:} \quad T_9(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835}$$

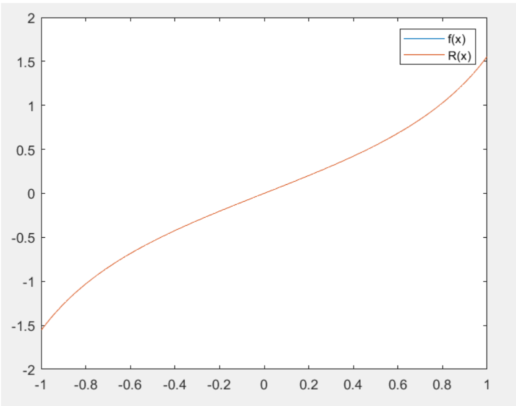
$$\text{Padé:} \quad R_{5,4}(x) = \frac{945x - 105x^3 + x^5}{945 - 420x^2 + 15x^4}$$

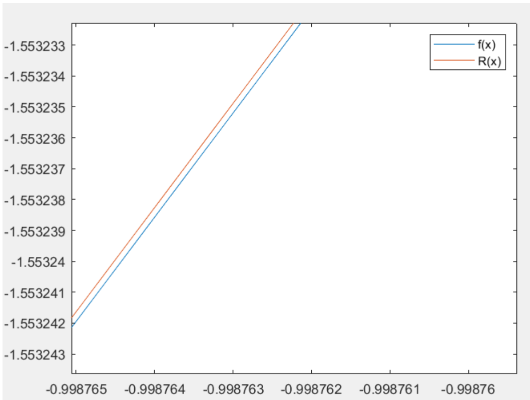
(a) Plot $f(x)$, $T_9(x)$, and $R_{5,4}(x)$ on the same coordinate system.
(b) Determine the maximum error that occurs when $f(x)$ is approximated with $T_9(x)$ and $R_{5,4}(x)$, respectively, over the interval $[-1, 1]$.

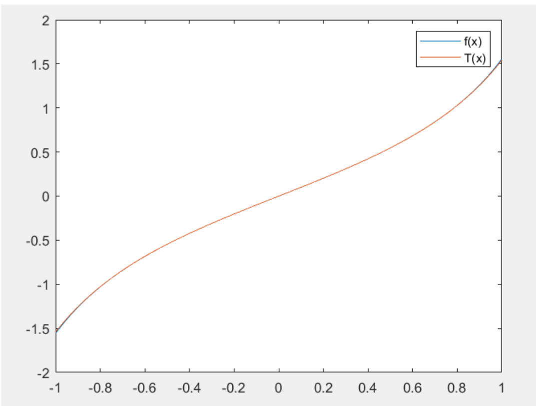
(a)

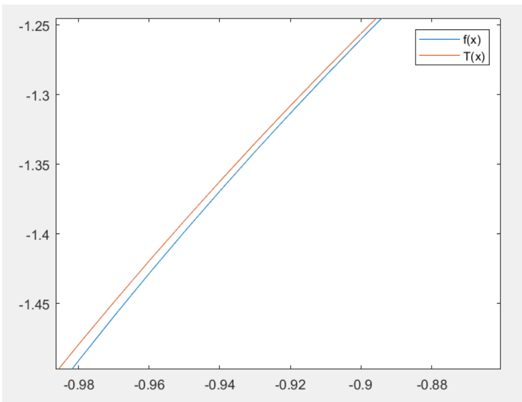
```
x=-1:0.01:1;
T=x+x.^3/3+x.^5*(2/15)+x.^7*(17/315)+x.^9*(62/2835);
R=(945*x-105*x.^3+x.^5)./(945-420*x.^2+15*x.^4);
f=tan(x);
plot(x,f,x,T,x,R)
legend('f(x)','T(x)','R(x)')
plot(x,f,x,R)
legend('f(x)','R(x)')
plot(x,f,x,T)
legend('f(x)','T(x)')
```





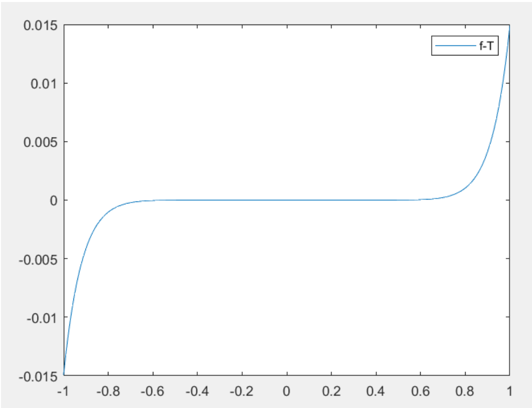


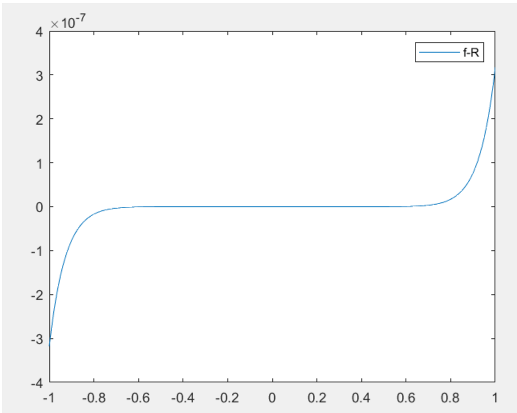




It can be seen that the fitting effect of Taylor approximation at the endpoint is not as good as that of Pade approximation

```
plot(x,f-T)
legend('f-T')
plot(x,f-R)
legend('f-R')

ER=(f-R)';
ER(1)
ans =
    -3.172474949408866e-07
ET=(f-T)';
ET(1)
ans =
  -0.014903315483827
```





It can be seen from the image that the maximum error is obtained at the end point. $max\ E_R(x) = 3.172474949408866 * 10^{-7}$, $max\ E_T(x) = 0.014903315483827$.