Chapter 5

P222 3,4

Note: Modify the number of the known points in Exercise 3 and 4 to 13, that means k is from 0 to 12.

3. The composite trapezoidal rule can be adapted to integrate a function known only at a set of points. Adapt Program 5.1 to approximate the integral of a function over an interval $[a, b]$ that passes through M given points. (*Note.* The nodes need not be equally spaced.) Use this program to approximate the integral of a function that passes through the points $\left\{\left(\sqrt{k^2+1}, k^{1/3}\right)\right\}_{k=0}^{13}$.

4. The composite Simpson's rule can be adapted to integrate a function known only at a set of points. Adapt Program 5.2 to approximate the integral of a function over an interval $[a, b]$ that passes through M given points. (*Note.* The nodes need not be equally spaced.) Use this program to approximate the integral of a function that passes through

the points $\left\{\left(\sqrt{k^2+1}, k^{1/3}\right)\right\}_{k=0}^{13}$.

$x = \sqrt{k^2+1}, y = k^{\frac{1}{3}}$, thus $k = \sqrt{x^2-1}, y = (x^2-1)^{\frac{1}{6}}$.

```
%traprlpoint.m
function s = traprlpoint(x,y,n)
%Output - s is the trapezoidal rule sum
%Input  - X is the abscissa of the point
%       - Y is the ordinate of the point
%       - n is the number of points
s=0;
for k=2:n
    s=s+(x(k)-x(k-1))*(y(k)+y(k-1))/2;
end
end
```

```
%simprlpoint.m
function s = simprlpoint(x,y,midpointy,n)
%Output - s is the Simpson's rule sum
%Input  - X is the abscissa of the point
%       - midpointy is the ordinate of the midpoint of each two points
%       - Y is the ordinate of the point
%       - n is the number of points
s=0;
for k=2:n
    h=x(k)-x(k-1);
    s=s+h*(y(k-1)+4*midpointy(k-1)+y(k))/6;
end
end
```

```
>> k=0:12;
x=sqrt(k.^2+1);
y=k.^(1/3);
>> s = traprlpoint(x,y,length(k))
s =
   19.5281
>>f=@(x)(x.^2-1).^(1/6);
for i=1:length(k)-1
midpointy(i)=f((x(i)+x(i+1))/2);
end
s=simprlpoint(x,y,midpointy,length(k))
s =
   19.6571
```

3. Adapting the composite trapezoidal rule ,we can get approximated integral of the function is 19.5281.
4. Adapting the composite Simpson's rule, we can get approximated integral of the function is 19.6571;

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6. Obtain approximations to each of the following definite integrals with an accuracy of ten decimal places. Use any of the programs from this section.

(a) $\displaystyle\int_{1/7\pi}^{1/4\pi} \sin(1/x)dx$ **(b)** $\displaystyle\int_{\frac{1}{5\pi}+10^{-5}}^{\frac{1}{4\pi}-10^{-5}} \frac{1}{\sin(1/x)}dx$

```
%aa.m
function S = aa(f,a,b)
%Output - S=M^4,M=(b-a)/h, h is the step size
%Input  - f
%       - a and b are the end points of the integral interval
syms x
ff=diff(f,x,4);
t=a:(b-a)/1000:b;
m=abs(ceil(max(subs(ff,t))));
syms K;
eqn=((b-a)^5*m)/(180*K)==1e-10;%K=M^4
S=ceil(solve(eqn));
end
```

```
>> syms x;
f1=sin(1/x);
S1 = aa(f1,pi/7,pi/4)
S1 =
32406099

>> ceil(S1^(1/4))
ans =
76
```

(a) We get M is equal to 76.

```
>> format long
f=@(x)sin(1./x);
s = simprl(f,pi/7,pi/4,76)

s =

   0.323152343496973
```

Using the composite Simpson's rule, the approximation to the definite integral with an accuracy of ten decimal places is 0.323152343496973.

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2. Use Program 5.4 to approximate the following two definite integrals with an accuracy of 10 decimal places. The exact value of each definite integral is π. Explain any apparent differences in the rates of convergence of the two Romberg sequences.

(a) $\int_0^2 \sqrt{4x - x^2}\,dx$ **(b)** $\int_0^1 \frac{4}{1+x^2}\,dx$

(a) the approximation of the definite integral is 3.141592653228606.

```
>> clear
f=@(x)(4*x-x.^2)^(1/2);
format long
[R,quad,err,h] = romber(f,0,2,20,1e-10)
```

```
R =

  列 1 至 10

   2.000000000000000                   0                   0                   0                   0                   0                   0                   0                   0                   0
   2.732050807568877   2.976067743425169                   0                   0                   0                   0                   0                   0                   0                   0
   2.995709068102441   3.083595154946962   3.090763649048414                   0                   0                   0                   0                   0                   0                   0
   3.089819144357174   3.121189169775418   3.123695437430648   3.124218164230366                   0                   0                   0                   0                   0                   0
   3.123253037827741   3.134397668984597   3.135278235598542   3.135462089537715   3.135506183362450                   0                   0                   0                   0                   0
   3.135102422877131   3.139052217893595   3.139362521154195   3.139427351083650   3.139442901128928   3.139446749337888                   0                   0                   0                   0
   3.139296912779685   3.140695076080536   3.140804599959665   3.140827490099435   3.140832980840673   3.140834339667371   3.140834678517268                   0                   0                   0
   3.140780792396614   3.141275418935591   3.141314108459261   3.141322195895762   3.141324135918493   3.141324616030984   3.141324735756592   3.141324765669137                   0                   0
   3.141305582957230   3.141480513144101   3.141494186091335   3.141497044466448   3.141497730147117   3.141497899838445   3.141497942154393   3.141497952726718   3.141497955369383                   0
   3.141491152719653   3.141553009307128   3.141557842384663   3.141558852802017   3.141559095187647   3.141559155173023   3.141559170131591   3.141559173868878   3.141559174803053   3.141559175036588
   3.141556766539018   3.141578637812139   3.141580346379140   3.141580703585401   3.141580789274748   3.141580810481089   3.141580815769321   3.141580817090546   3.141580817420801   3.141580817503361
   3.141579965411446   3.141587698368923   3.141588302406042   3.141588428692183   3.141588458986719   3.141588466483994   3.141588468353592   3.141588468820697   3.141588468937454   3.141588468966642
   3.141588167607754   3.141590901673190   3.141591115226808   3.141591159874756   3.141591170585276   3.141591173235910   3.141591173896900   3.141591174062043   3.141591174103322   3.141591174113642
   3.141591067549705   3.141592034197022   3.141592109698611   3.141592125483878   3.141592129270580   3.141592130207711   3.141592130441404   3.141592130499791   3.141592130514385   3.141592130518033
   3.141592092838892   3.141592434601954   3.141592461295616   3.141592466876521   3.141592468215316   3.141592468546640   3.141592468629262   3.141592468649905   3.141592468655065   3.141592468656355
   3.141592455334224   3.141592576166001   3.141592585603604   3.141592587576747   3.141592588050081   3.141592588167222   3.141592588196433   3.141592588203731   3.141592588205556   3.141592588206012
   3.141592583495826   3.141592626216360   3.141592629553051   3.141592630250661   3.141592630418010   3.141592630459425   3.141592630469753   3.141592630472333   3.141592630472978   3.141592630473139
   3.141592628807825   3.141592643911825   3.141592645091523   3.141592645338165   3.141592645397332   3.141592645411974   3.141592645415626   3.141592645416538   3.141592645416766   3.141592645416822
   3.141592644828032   3.141592650168101   3.141592650585186   3.141592650672387   3.141592650693305   3.141592650698482   3.141592650699773   3.141592650700095   3.141592650700176   3.141592650700196
   3.141592650492053   3.141592652380060   3.141592652527524   3.141592652558355   3.141592652565751   3.141592652567581   3.141592652568038   3.141592652568152   3.141592652568181   3.141592652568188
   3.141592652494583   3.141592653162093   3.141592653214229   3.141592653225129   3.141592653227744   3.141592653228391   3.141592653228552   3.141592653228593   3.141592653228603   3.141592653228606

  列 11 至 20

                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
                   0                   0                   0                   0                   0                   0                   0                   0                   0                   0
   3.141580817524001                   0                   0                   0                   0                   0                   0                   0                   0                   0
   3.141588468973939   3.141588468975764                   0                   0                   0                   0                   0                   0                   0                   0
   3.141591174116221   3.141591174116866   3.141591174117027                   0                   0                   0                   0                   0                   0                   0
   3.141592130518946   3.141592130519173   3.141592130519230   3.141592130519244                   0                   0                   0                   0                   0                   0
   3.141592468656677   3.141592468656758   3.141592468656778   3.141592468656783   3.141592468656784                   0                   0                   0                   0                   0
   3.141592588206126   3.141592588206154   3.141592588206161   3.141592588206163   3.141592588206163   3.141592588206163                   0                   0                   0                   0
   3.141592630473180   3.141592630473190   3.141592630473192   3.141592630473193   3.141592630473193   3.141592630473193   3.141592630473193                   0                   0                   0
   3.141592645416837   3.141592645416840   3.141592645416841   3.141592645416841   3.141592645416841   3.141592645416841   3.141592645416841   3.141592645416841                   0                   0
   3.141592650700201   3.141592650700202   3.141592650700203   3.141592650700203   3.141592650700203   3.141592650700203   3.141592650700203   3.141592650700203   3.141592650700203                   0
   3.141592652568189   3.141592652568190   3.141592652568190   3.141592652568190   3.141592652568190   3.141592652568190   3.141592652568190   3.141592652568190   3.141592652568190   3.141592652568190
   3.141592653228606   3.141592653228606   3.141592653228606   3.141592653228606   3.141592653228606   3.141592653228606   3.141592653228606   3.141592653228606   3.141592653228606   3.141592653228606

  列 21

                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
                   0
   3.141592653228606

quad =

   3.141592653228606

err =

     6.604161661982744e-10
```

(b) the approximation of the definite integral is 3.141592653589723.

```
>> clear
f=@(x)(4/(1+x.^2));
format long
[R,quad,err,h] = romber(f,0,1,6,1e-10)
```

```
R =

   3.000000000000000                   0                   0                   0                   0                   0                   0
   3.100000000000000   3.133333333333333                   0                   0                   0                   0                   0
   3.131176470588235   3.141568627450980   3.142117647058823                   0                   0                   0                   0
   3.138988494491089   3.141592502458707   3.141594094125888   3.141585783761874                   0                   0                   0
   3.140941612041389   3.141592651224822   3.141592661142563   3.141592638396796   3.141592665277717                   0                   0
   3.141429893174974   3.141592653552836   3.141592653708037   3.141592653590029   3.141592653649610   3.141592653638244                   0
   3.141551963485656   3.141592653589216   3.141592653591641   3.141592653589794   3.141592653589793   3.141592653589734   3.141592653589723


quad =

   3.141592653589723


err =

     4.852118706821784e-11


h =

   0.015625000000000
```

Let's see why the convergence rates of the two functions are different. Let's take a look at the derivatives of the two functions. Here we only take to order 5 :

```
>> syms x
f=(4*x-x^2)^(1/2);
for i=1:5
    t=0+eps:(2-0)/1000:2;
    ff=diff(f,x,i);
    m(i)=double(max(subs(ff,t)));
end
```

	1	2	3	4	5
1	6.7109e+07	-0.5000	1.0208e+39	-0.3750	1.8117e+71
2					

```
>> syms x
f=4/(1+x^2);
for i=1:5
    t=0+eps:(2-0)/1000:2;
    ff=diff(f,x,i);
    m(i)=double(max(subs(ff,t)));
end
```

1	2	3	4	5
-1.7764e-15	2	18.6741	96	85.7101

we can see that the odd derivative of $\sqrt{4x-x^2}$ on interval $[0,2]$ tends to infinity at some points， so the convergence speed for this function of Romberg integral is slow.