Chapter 4

P160 4

4. (a) Find the least-squares line for the data points $\{(x_k, y_k)\}_{k=1}^{50}$, where $x_k = (0.1)k$ and $y_k = x_k + \cos(k^{1/2})$.
(b) Calculate $E_2(f)$.
(c) Plot the set of data points and the least-squares line on the same coordinate system.

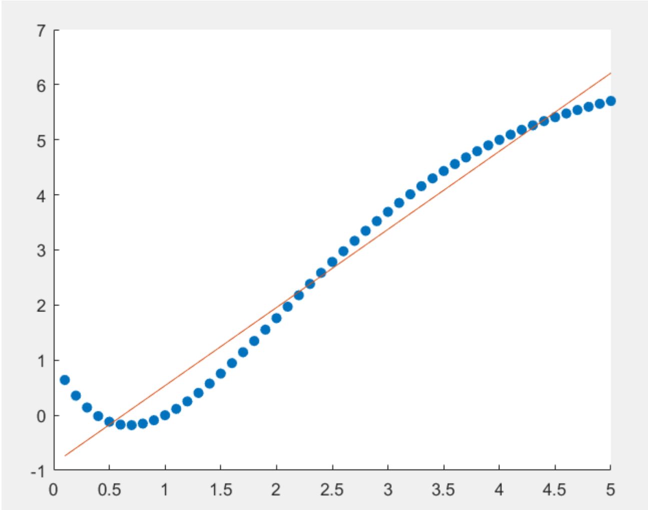
```
%isline.m
function [A,B,err] = isline(X,Y)
%Input  - X is the 1xn abscissa vector
%       - Y is the 1xn ordinate vector
%       - Fx is the 1xn function value vector
%Output - A is the coefficient of x in Ax+B
%       - B is the constant coefficient in Ax+B
xmean=mean(X);
ymean=mean(Y);
sumx2=(X-xmean)*(X-xmean)';%1x1
sumxy=(Y-ymean)*(X-xmean)';
A=sumxy/sumx2;
B=ymean-A*xmean;
Fx=A*X+B;
N=length(Fx);
err=(((Fx-Y)*(Fx-Y)')/N)^(1/2);
end
```

```
>> k=1:50;
X=0.1*k;
Y=X+cos(k.^(1/2));
[A,B,err] = isline(X,Y)

A =
    1.4186

B =
   -0.8808

err =
    0.4032

>> scatter(X,Y,'filled')
hold on
plot(X,A*X+B)
```

(a) the least-squares line is $f(x) = 1.1486x - 0.8808.$

(b) $E_2(f) = \sqrt{\frac{\sum_{k=1}^{N}(f(x_k)-y_k))^2}{N}} = 0.4032.$

(c) As shown below:



P174 1

1. The temperature cycle in a suburb of Los Angeles on November 8 is given in the accompanying table. There are 24 data points.

   (a) Follow the procedure outlined in Example 4.5 (use the fmins command) to find the least-squares curve of the form $f(x) = A\cos(Bx) + C\sin(Dx) + E$ for the given set of data.

   (b) Determine $E_2(f)$.

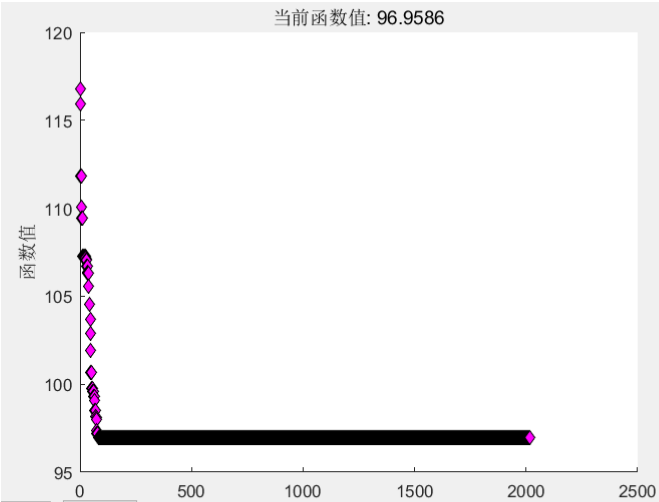
   (c) Plot the data and the least-squares curve from part (a) on the same coordinate system.

Time, p.m.	Degrees	Time, a.m.	Degrees
1	66	1	58
2	66	2	58
3	65	3	58
4	64	4	58
5	63	5	57
6	63	6	57
7	62	7	57
8	61	8	58
9	60	9	60
10	60	10	64
11	59	11	67
Midnight	58	Noon	68

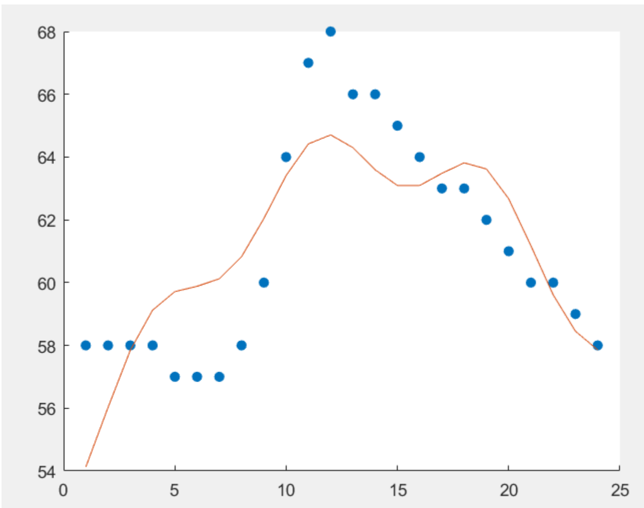
```
function z=E(u)
A=u(1);
B=u(2);
C=u(3);
D=u(4);
E=u(5);
t=1:24;
d=[58,58,58,58,57,57,57,58,60,64,67,68,66,66,65,64,63,63,62,61,60,60,59,58];
z=0
for i=1:24
    z=z+(A*cos(B*t(i))+C*sin(D*t(i))+E-d(i)).^2;
end
end

```

```
>>
options=optimset('PlotFcns',@optimplotfval,'MaxIter',5000,'MaxFunEvals',5000,'TolFun',1e-12,'Tolx',1e-12);
>> x=fminsearch(@E,[-0.9924,338,10.4823,0.1111,53.6353],options)
x =
   -0.9932   346.4002    10.4821     0.1111    53.6355

scatter(t,d,'filled')
hold on
plot(t,x(1)*cos(x(2).*t)+x(3)*sin(x(4).*t)+x(5))
```

(a) The fitting curve obtained by fminsearh is
$f(t) = -0.9932cos(346.4002t) + 10.4821sin(0.1111t) + 53.6355.$
(b) It can be seen $E_2(f) = 96.9586$ from the figure below:



(c) As shown below:



P191 5

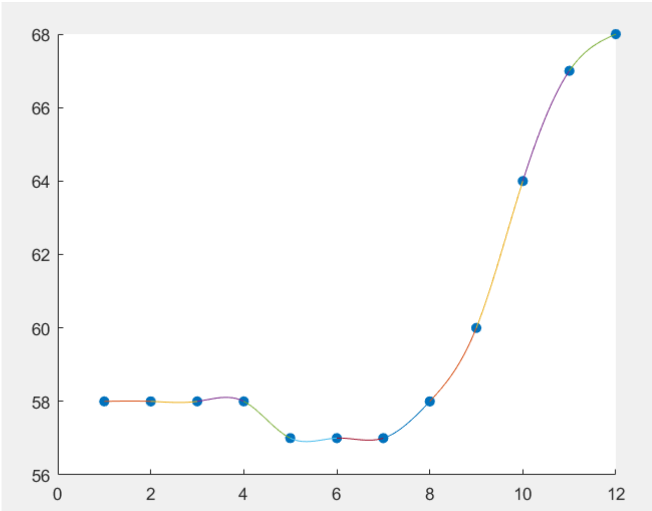
5. The accompanying table gives the hourly temperature readings (Fahrenheit) during a 12-hour period in a suburb of Los Angeles. Find the natural cubic spline for the data. Graph the natural cubic spline and the data on the same coordinate system. Use the natural cubic spline and the results of part (a) of Exercise 12 to approximate the average temperature during the 12-hour period.

Time, a.m.	Degrees	Time, a.m.	Degrees
1	58	7	57
2	58	8	58
3	58	9	60
4	58	10	64
5	57	11	67
6	57	Noon	68

```
%Natural Spline.m
function [S,M]=nsfit(X,Y)
%Input  -X is the 1xn abscissa vector
%       -Y is the 1xn ordinate vector
%       -dx0=S'(x0) first derivative boundary condition
%       -dxn=S'(xn) first derivative boundary condition
%Output -S: rows of S are the coefficients,in descending
%           order,for the cubic interpolants
N=length(X)-1;
H=diff(X);
D=diff(Y)./H;
A=H(2:N-1);
B=2*(H(1:N-1)+H(2:N));
C=H(2:N);
U=6*diff(D);

B(1)=B(1);
U(1)=U(1);
B(N-1)=B(N-1);
U(N-1)=U(N-1);
for k=2:N-1
    temp=A(k-1)/B(k-1);
    B(k)=B(k)-temp*C(k-1);
    U(k)=U(k)-temp*U(k-1);
end
M(N)=U(N-1)/B(N-1);
for k=N-2:-1:1
    M(k+1)=(U(k)-C(k)*M(k+2))/B(k);
end
M(1)=0;
M(N+1)=0;
for k=0:N-1
    S(k+1,1)=(M(k+2)-M(k+1))/(6*H(k+1));
    S(k+1,2)=M(k+1)/2;
    S(k+1,3)=D(k+1)-H(k+1)*(2*M(k+1)+M(k+2))/6;
    S(k+1,4)=Y(k+1);
end
end
```

```
>> X=1:12;
Y=[58,58,58,58,57,57,57,58,60,64,67,68];
[S,M]=nsfit(X,Y);
scatter(X,Y,'filled')
for i=1:11
x=i:0.01:i+1;
y=polyval(S(i,:),x-X(i));
hold on
plot(x,y)
end
```

The natural cubic spline interpolation is shown in the figure:



$\int_{x_k}^{x_{k+1}} S_k(x)\, dx = \int_{x_k}^{x_{k+1}} [a_k(x-x_k)^3 + b_k(x-x_k)^2 + c_k(x-x_k) + d_k]\, dx,$

$= \frac{a_k}{4}(x-x_k)^4 + \frac{b_k}{3}(x-x_k)^3 + \frac{c_k}{2}(x-x_k)^2 + d_k x|_{x=x_k}^{x=x_{k+1}}$

$= \frac{a_k}{4}(x_{k+1}-x_k)^4 + \frac{b_k}{3}(x_{k+1}-x_k)^3 + \frac{c_k}{2}(x_{k+1}-x_k)^2 + d_k(x_{k+1}-x_k)$

$= (((\frac{a_k}{4}h_k + \frac{b_k}{3})h_k + \frac{c_k}{2})h_k + d_k)h_k.$

$h_k = x_{k+1} - x_k.$

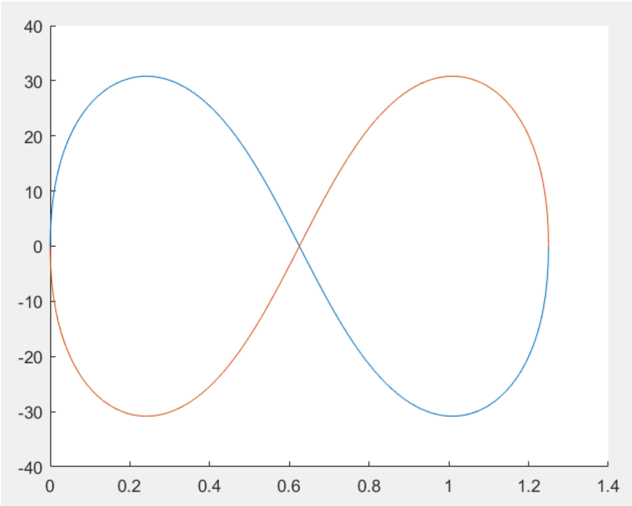
```
>> integ=0;
h=1;
for i=1:11
integ=integ+Horner1(S(i,:),h);
end

>> integ/(12-1)

ans =

   60.1818
```

accroding to $\int_a^b f(x)\, dx = f(c)(b-a)$, the average temperature is 60.1818.

P200 4(a)

4. Use the programs from Problem 1 and 3 to create
   (a) an infinity symbol: ∞.

(a) As shown below:




```
>> M1=[0,0;0,40;(0.125/2)+0.25,40;(0.125/2)+0.25*2,40;0.625,0;0.625+0.125/2,-40;0.625+0.125/2+0.25,-40;1.25,-40;1.25,0];
M2=[1.25,0;1.25,40;1.25-(0.125/2)-0.25,40;1.25-(0.125/2)-0.25*2,40;0.625,0;0.625-0.125/2,-40;0.625-0.125/2-0.25,-40;0,-40;0,0];
beziercurve(M1)
hold on
beziercurve(M2)
```

```
% bezier.m
function B = bezier(i,N,t)
B=(factorial(N)/(factorial(i)*factorial(N-i)))*(t.^(i)).*((1-t).^(N-i));
end
```

```
% beziercurve.m
function beziercurve(M)
N=length(M)-1;
t=0:0.01:1;
xp=M(:,1);
yp=M(:,2);
x=xp(1)*bezier(0,N,t);
y=yp(1)*bezier(0,N,t);
for i=1:N
    x=x+xp(i+1)*bezier(i,N,t);
    y=y+yp(i+1)*bezier(i,N,t);
end
plot(x,y)
end
```