Characteristics of undulatory locomotion in granular media

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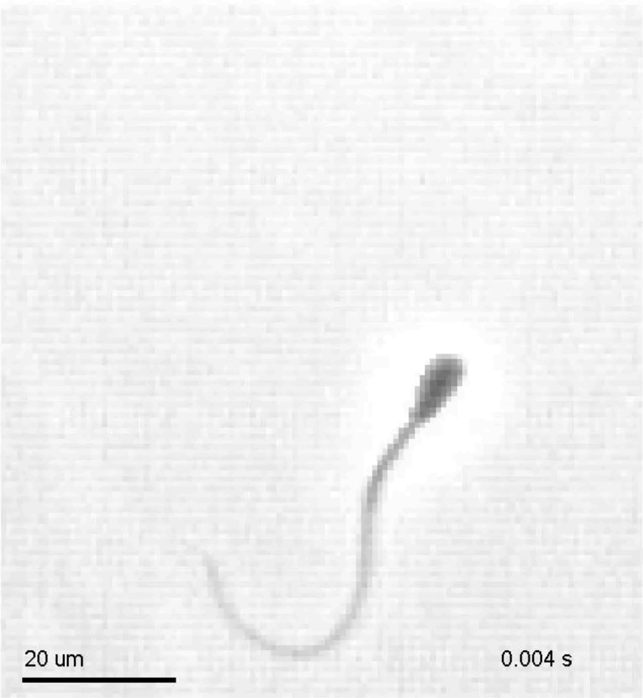
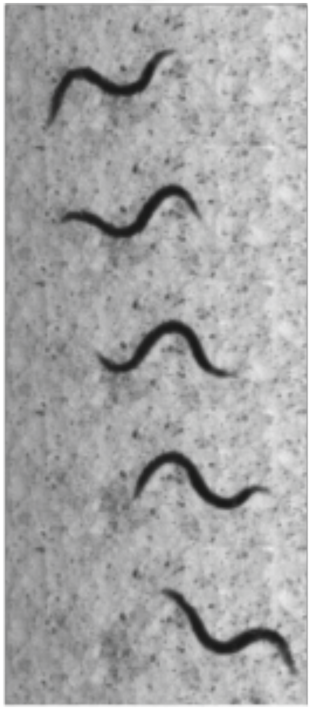
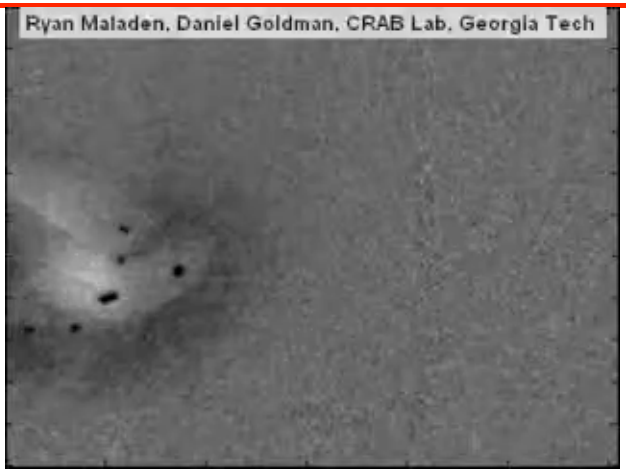
Undulatory locomotion



Friedrich, B. M., et al., J. Exp. Biol. 213, 1226–1234 (2010)



Dorgan, K. M., Law, C. J., Rouse, G. W., Proc. R. Soc. B 280(1757), 20122948.



Maladen, Ryan D., et al., *Science* 325.5938 (2009): 314-318.

Granular media

- Conglomeration of discrete macroscopic particles
- Fluid-like & solid-like
- Anisotropic response to intrusion
- Volume fraction: 0.58 (LP) - 0.62 (CP)

Assumptions:

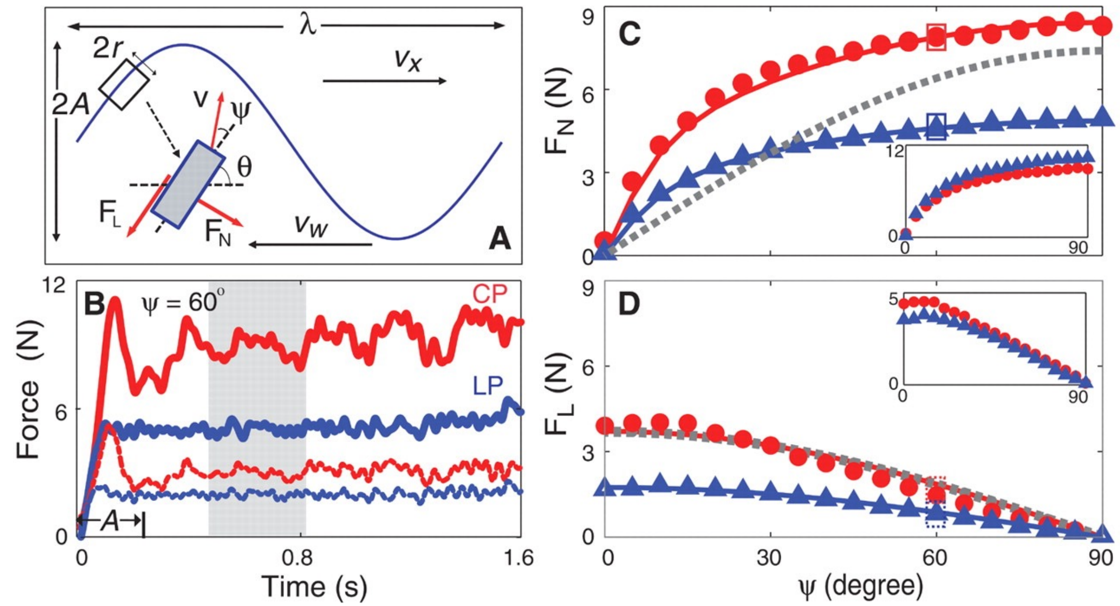
- dry GM
- slow motion: inertialess

How to characterize particle-body/particle-particle interaction?

Slender body dynamics

- Localized interaction

- Force measurements
  (Goldman's group)



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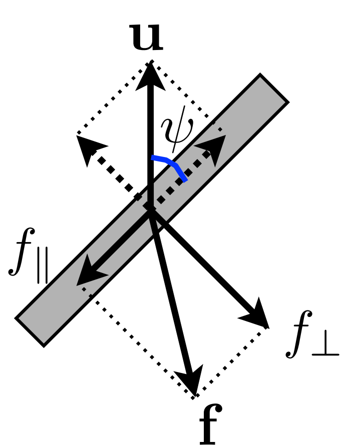


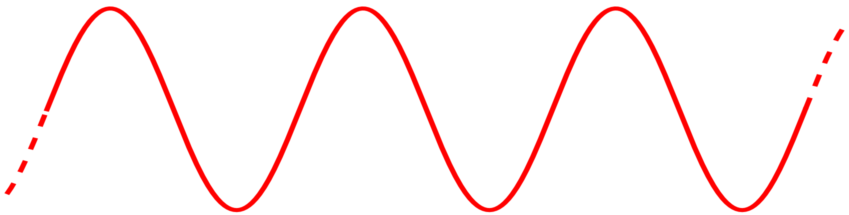
$$f_{\|} = C_{\|} \cos\psi$$

$$f_{\perp} = \boxed{C_{\perp}(\psi)} \sin\psi$$

$$C_{\perp}(\psi) > C_{\|}$$

Infinite swimmer

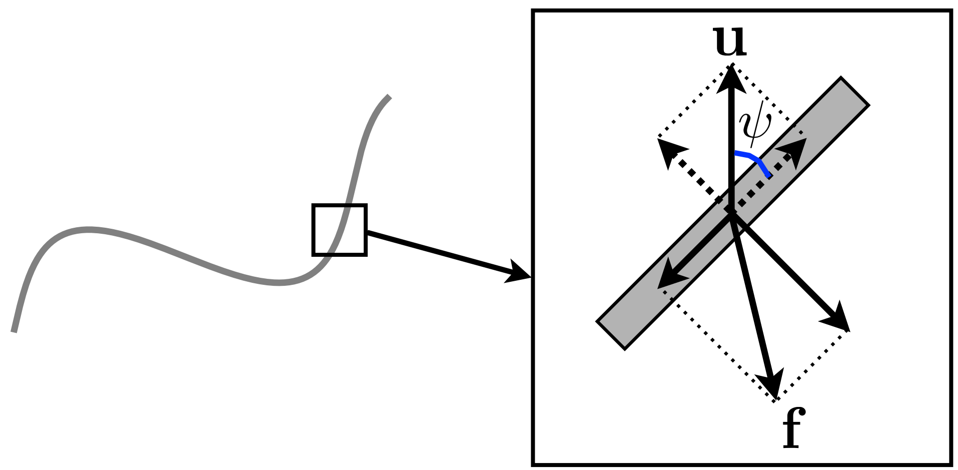
- Infinitely many number of sinusoidal waves
- 1D force balance



- Optimal deformation?
- Finite-size effect?

Swimmer model

- Prescribed planar waveform
- Nonlinear resistive force theory



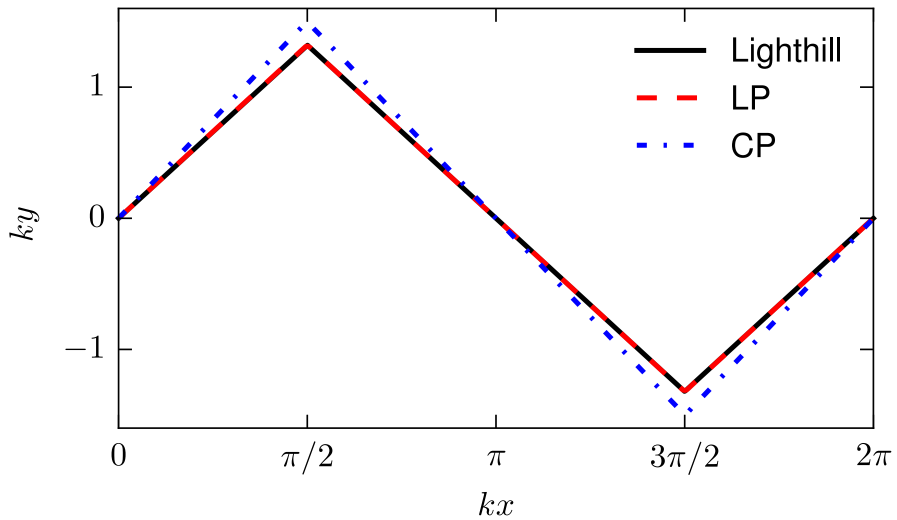
$$\mathbf{f} = -C_{||}\,\hat{\mathbf{u}}\cdot\mathbf{t}\mathbf{t} - C_{\perp}(\psi)\,(\hat{\mathbf{u}} - \hat{\mathbf{u}}\cdot\mathbf{t}\mathbf{t})$$

- Force-free and torque-free: $\int_0^L \mathbf{f}\,\mathrm{d}s = \mathbf{0}, \quad \int_0^L \mathbf{x}(s,t)\times\mathbf{f}\,\mathrm{d}s = \mathbf{0}$

Optimal swimming

Optimal shape: sawtooth

$$\eta = \frac{C_{\parallel} L U}{\left\langle \int_0^L \mathbf{f} \cdot \mathbf{u} \mathrm{d}s \right\rangle}$$



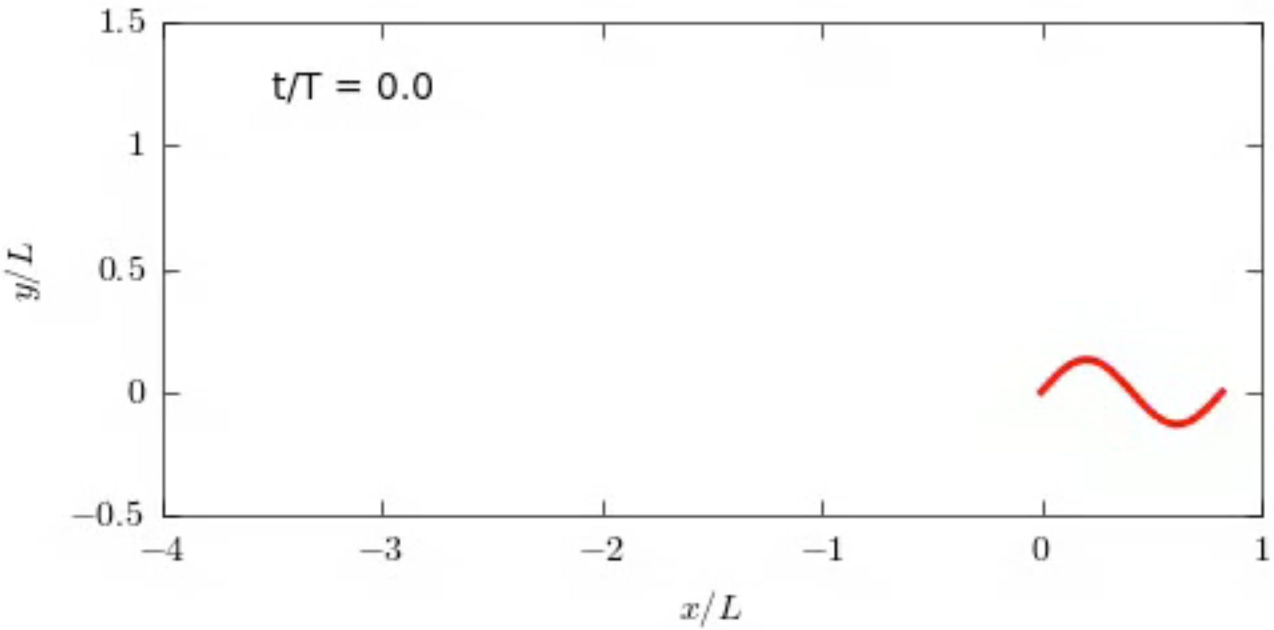
Local optimality

- low energy expenditure
- high propulsive force

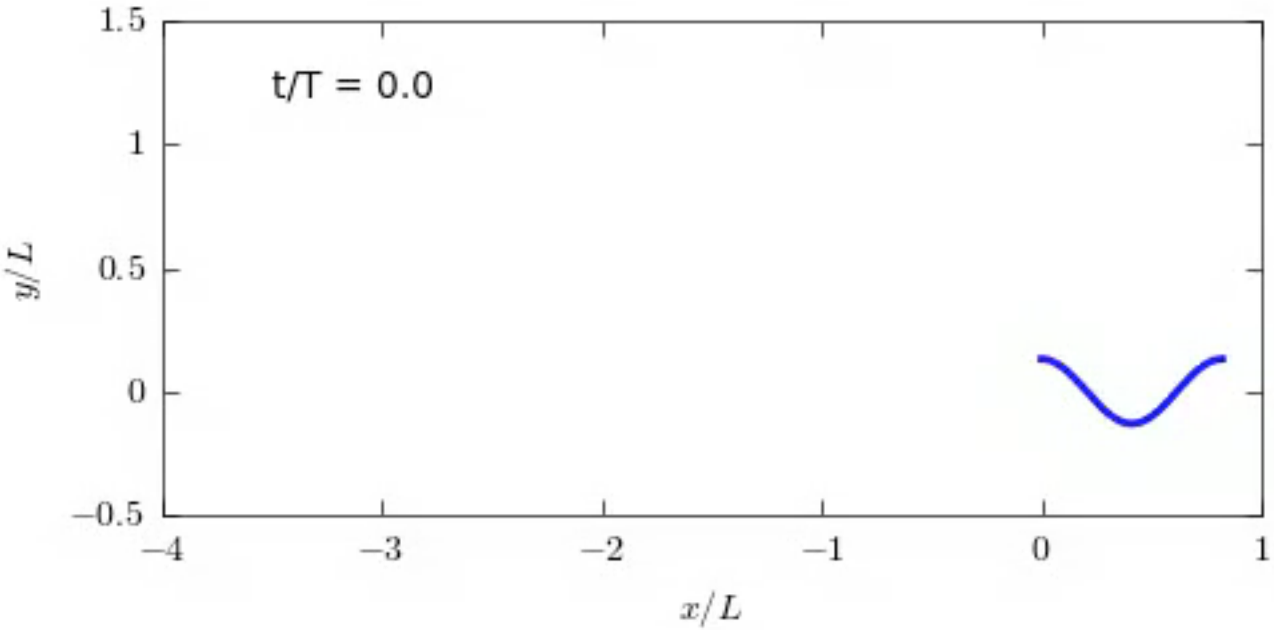
Sawtooth: global extension of a local optimum

Finite sinusoidal swimmers

Odd

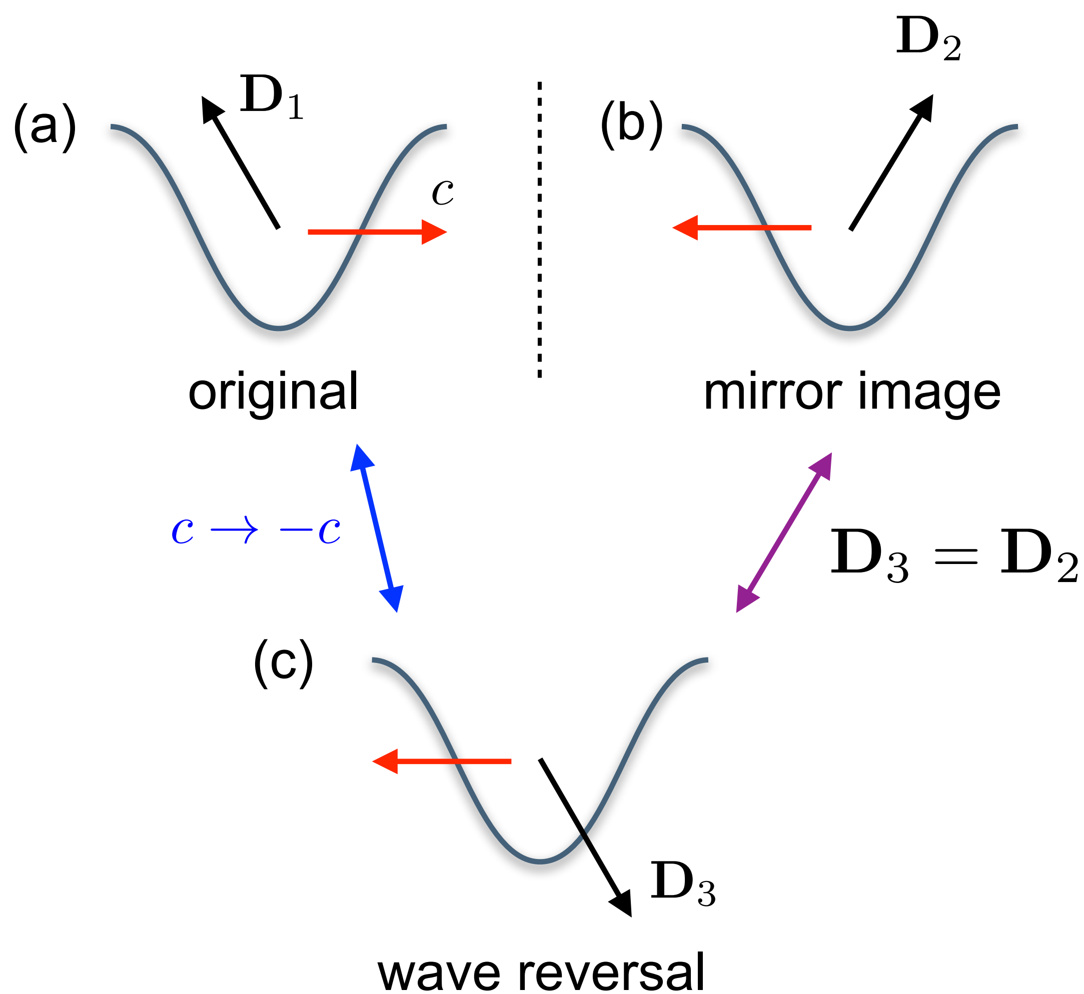


Even



Symmetry

$$\mathbf{u} \to -\mathbf{u} \quad \Longrightarrow \quad \mathbf{f} \to -\mathbf{f}$$

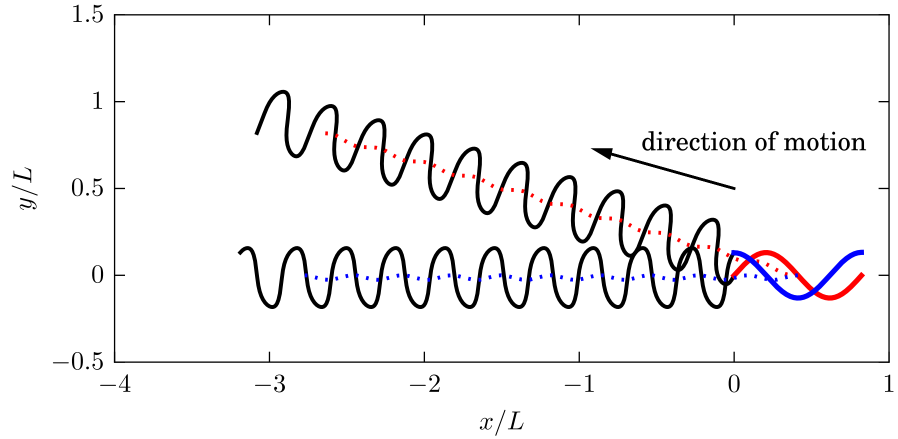


Koehler, S., Spoor, T., & Tilley, B. S. (2012). *Phys. Fluids*, 24(9), 091901.

Swimming characteristics

Reorientation

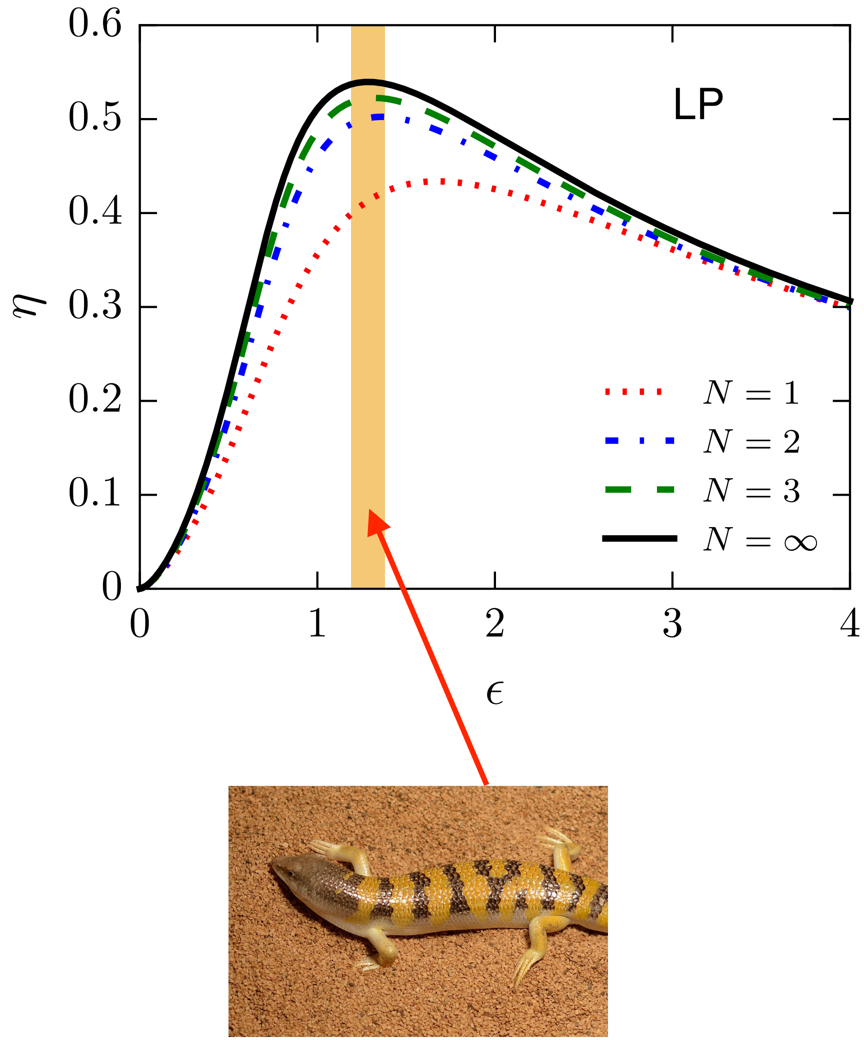
- Direction of swimming



Pitching

- Diminishes performance

Swimming efficiency



Conclusion

1. Sandfish swimming in nature is closely tuned for optimality

2. Distinct similarity: GM & Newtonian

- Local resistive force theory
- Kinematic reversibility

3. Rich dynamics: the effective design of artificial swimmers