Ibn Tofail University

Analysis II — Normal Exam

Year: 21-22

Exercise 1:

Calculate the following limits:

1. $\lim_{x \to 0} \frac{\sin x - \arcsin x}{\sin^2 x}$.
2. $\lim_{x \to 0} \frac{4^x - 2^x}{5^x - 3^x}$.
3. $\lim_{x \to +\infty} \left(a^{\frac{1}{x}} + b^{\frac{1}{x}} + c^{\frac{1}{x^3}} \right)^x$, where $a, b, c \in \mathbb{R}_+^*$ are fixed.

Answer Area

Exercise 2:

Course questions and applications:

1. State Leibniz's formula.
2. Let f be the function defined by $f(x) = x^3 e^{3x}$. For all $n \in \mathbb{N}$, determine the n-th derivative of f.
3. State Taylor's formula with integral remainder.

   For the remainder of this exercise, assume that $x \geq 0$.
4. Show that $\forall x \geq 0 : \left| e^{-x} - \sum_{k=0}^{n} \frac{(-1)^k x^k}{k!} \right| \leq \frac{x^{n+1}}{(n+1)!}$
5. Show that $\forall n \in \mathbb{N}^* : \lim_{n \to +\infty} \frac{x^n}{n!} = 0$.
6. Deduce $\lim_{n \to +\infty} \sum_{k=0}^{n} \frac{(-1)^k x^k}{k!}$.

Answer Area

Exercise 3:

Let f be the function defined by $f(x) = \frac{e^x - \ln(1+2x)}{1+\sin(x)}$.

1. Why can we state that this function has a Taylor expansion of any order around 0?
2. Determine a third-order Taylor expansion of f around 0. What are the values of $f''(0)$ and $f^{(3)}(0)$?
3. Give the equation of the tangent line to the curve of f at the point with x-coordinate 0.
4. What is the relative position of the curve of f with respect to this tangent line?

Answer Area

Exercise 4:

Let f be the function defined by: $f(x) = e^{\frac{1}{x}}\sqrt{x^2 + x + 1}$.

1. Study the asymptote of the curve representing f in the neighborhood of $+\infty$.
2. Study the relative position of this asymptote with respect to the curve representing f.

Answer Area