UNIVERSITY IBN TOFAIL

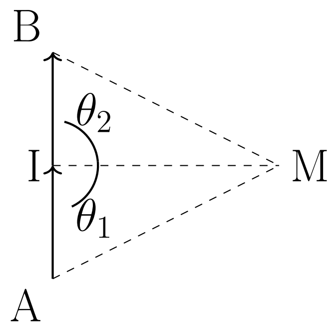
Electricity I

Problem Set IV

Exercise 1:

Calculate the magnetic field created by a segment carrying a current of intensity I at a point M located at distance a from the segment. We will denote θ_1 and θ_2 the angles between the perpendicular to the segment through M and the lines connecting M to the segment endpoints.

1. Express the magnetic field in the case of a finite straight wire.
2. Find the answer to the previous question (magnetic field created by an infinite straight wire) by applying Ampère's theorem.



Correction

1. **Magnetic field due to a finite straight wire:**
   Using the Biot-Savart law, the magnetic field at point M is:
   $$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$
   where θ_1 and θ_2 are the angles subtended by the segment endpoints at M, and a is the perpendicular distance from M to the wire. This result is derived by integrating the contributions from infinitesimal current elements along the wire.

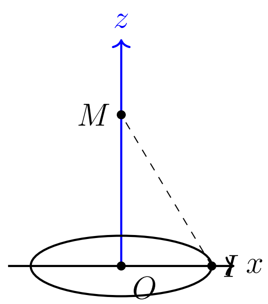
2. **Infinite straight wire via Ampère's theorem:**
   For an infinite wire, apply Ampère's law to a circular loop of radius r centered on the wire:
   $$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enc}} \implies B(2\pi r) = \mu_0 I \implies B = \frac{\mu_0 I}{2\pi r}.$$
   This matches the limit of the finite wire formula when $\theta_1, \theta_2 \to \frac{\pi}{2}$ (since $\sin\theta_1 + \sin\theta_2 \to 2$), and $a = r$ (the perpendicular distance becomes the radial coordinate).

Exercise 2:

Consider a filiform coil of radius R carrying a current of intensity I.

1. Determine the magnetic field created at a point on the axis of the coil located at a distance z from the center of the coil.

2. Plot the curve $B(z)$.



Correction

1. **Magnetic field on the axis of a circular coil:**
For a circular coil of radius R carrying current I, the magnetic field at a point M on its axis at distance z from the center is:

$$B(z) = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}.$$

This result is derived by integrating the Biot-Savart law over the loop, exploiting cylindrical symmetry. Each infinitesimal current element contributes a magnetic field component, and only the axial components survive due to symmetry.

2. **Plot of $B(z)$:**
The curve is symmetric about $z = 0$, peaking at:

$$B(0) = \frac{\mu_0 I}{2R}.$$

As $|z|$ increases, $B(z)$ decreases monotonically, approaching zero as $z \to \pm\infty$. The graph resembles a bell-shaped curve (similar to a Lorentzian profile) centered at $z = 0$, with inflection points at $z = \pm\frac{R}{\sqrt{2}}$.

Exercise 3:

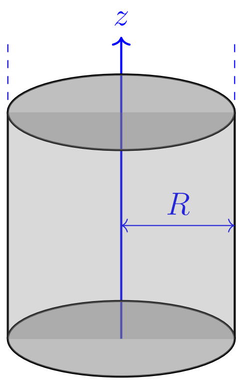
Consider an infinite cylinder of radius R carrying a current with uniform current density vector:

$$\vec{j} = j_0 \vec{k} \quad \text{with} \quad j_0 > 0$$

We are interested in the magnetostatic field $\vec{B}(M)$ created by this current distribution at any point M in space.

1. Using symmetry considerations, determine the direction of the magnetostatic field vector $\vec{B}(M)$ and the variables on which it depends.

2. By applying Ampère's theorem, determine the field $\vec{B}(M)$ at any point M in space.



Correction

1. **Symmetry analysis for an infinite cylinder:**
   Given the infinite cylinder with current density $\vec{j} = j_0\hat{z}$, the system exhibits cylindrical symmetry:

   - No dependence on the z-coordinate (infinite cylinder).

   - No dependence on the azimuthal angle ϕ (uniform current density).

   - The magnetic field $\vec{B}(M)$ must be azimuthal (i.e., $\vec{B} \parallel \hat{\phi}$) due to the right-hand rule.

   Thus, $\vec{B}(r)$ depends only on the radial distance r from the cylinder's axis.

2. **Magnetic field via Ampère's theorem:**
   Apply Ampère's law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$ to a circular loop of radius r:

   - **Inside the cylinder ($r \leq R$):** Enclosed current:

   $$I_{\text{enc}} = j_0 \cdot \pi r^2$$

   Ampère's law gives:

   $$B(2\pi r) = \mu_0 j_0 \pi r^2 \implies B = \frac{\mu_0 j_0 r}{2}.$$

   - **Outside the cylinder ($r > R$):** Enclosed current:

   $$I_{\text{enc}} = j_0 \cdot \pi R^2$$

   Ampère's law gives:(result: The field increases linearly inside the cylinder and decays inversely outside.)

   $$B(2\pi r) = \mu_0 j_0 \pi R^2 \implies B = \frac{\mu_0 j_0 R^2}{2r}.$$