Ibn Tofail University

Electricity I — Normal Exam

Year: 21-22

Exercise 1:

Consider two point charges at rest q and $-q$ placed respectively at points A and B on an Ox axis (Figure 1).

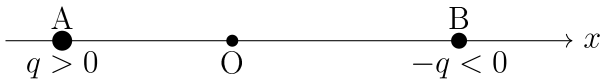

Figure 1:

1. Give the vector expression of the electrostatic force $\vec{F}_{q/-q}$ created at B.
2. Give the vector expression of the electrostatic field $\vec{E}$ created at B.
3. Give the expression of the electrostatic potential V created at B.

Answer Area

1. **Vector expression of the electrostatic force $\vec{F}_{q\to -q}$ created at B:**

   Using Coulomb's law:

$$\vec{F}_{q\to -q} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q(-q)}{|\vec{r}_B - \vec{r}_A|^2} \cdot \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|}$$

   Compute the vector $\vec{r}_B - \vec{r}_A = d\vec{i} - (-d\vec{i}) = 2d\vec{i}$, so:

$$\vec{F}_{q\to -q} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q^2}{(2d)^2} \cdot \vec{i} = -\frac{q^2}{16\pi\varepsilon_0 d^2}\vec{i}$$

   Final expression:

$$\boxed{\vec{F}_{q\to -q} = -\frac{q^2}{16\pi\varepsilon_0 d^2}\vec{i}}$$

2. **Vector expression of the electrostatic field $\vec{E}$ created at B:**

   Electric field due to charge q at A:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\vec{r}_B - \vec{r}_A|^2} \cdot \frac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(2d)^2} \cdot \vec{i} = \frac{q}{16\pi\varepsilon_0 d^2}\vec{i}$$

   Final expression:

$$\boxed{\vec{E} = \frac{q}{16\pi\varepsilon_0 d^2}\vec{i}}$$

3. **Expression of the electrostatic potential V created at B:**

   Electrostatic potential due to charge q at distance $2d$:

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2d} = \frac{q}{8\pi\varepsilon_0 d}$$

   Final expression:

$$\boxed{V = \frac{q}{8\pi\varepsilon_0 d}}$$

Exercise 2:

Consider two point charges at rest q and $-q$ placed respectively at points A and B (Figure 2). Given $OA = OB = a$.

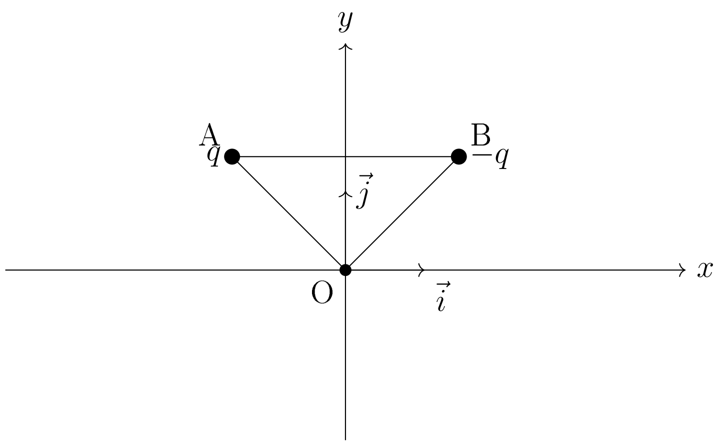

Figure 2:

1. Give the vector expression of the electrostatic field $\vec{E} = E_x\vec{i} + E_y\vec{j}$ created at O.
2. Give the expression of the electrostatic potential V created at O.

Answer Area

1. **Vector expression of the electrostatic field $\vec{E} = E_x\vec{i} + E_y\vec{j}$ created at point O:**

   Given: - Charge q is located at point A, with coordinates $(-a, 0)$ - Charge $-q$ is located at point B, with coordinates $(a, 0)$ - Point O is the origin $(0, 0)$

   The electric field at O is the vector sum of the fields due to both charges.

   - Electric field due to charge q at A:

   $$\vec{E}_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2} \cdot \vec{i}$$

   (since it points toward the right)

   - Electric field due to charge $-q$ at B:

   $$\vec{E}_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{a^2} \cdot (-\vec{i}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2} \cdot \vec{i}$$

   (also pointing toward the right)

   Total electric field at O:

   $$\vec{E} = \vec{E}_A + \vec{E}_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2}\vec{i} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a^2}\vec{i} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{q}{a^2}\vec{i}$$

   Final expression:

   $$\boxed{\vec{E} = \frac{q}{2\pi\varepsilon_0 a^2}\vec{i}}$$

2. **Expression of the electrostatic potential V created at point O:**

   Electrostatic potential is a scalar quantity. So we simply add the potentials from both charges:

   $$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} + \frac{-q}{a}\right) = \frac{1}{4\pi\varepsilon_0} \cdot 0 = 0$$

   Final expression:

   $$\boxed{V = 0}$$

Exercise 3:

A distribution of positive charges with linear density λ is uniformly distributed along an infinitely long wire. Give the expression of the electrostatic field $\vec{E}$ created by this wire using Gauss's theorem.

Answer Area

Expression of the electrostatic field $\vec{E}$ created by an infinitely long wire with linear charge density λ:
We are given a uniformly charged, infinitely long wire with linear charge density $\lambda > 0$. To find the electrostatic field $\vec{E}$ at a distance r from the wire, we use **Gauss's theorem**.

1. **Choose a Gaussian surface** Use a cylindrical surface of radius r and length L, coaxial with the wire.

2. **Apply Gauss's Law**

$$\Phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

   Since the electric field is radial and uniform in magnitude on the curved surface:

$$E \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

3. **Solve for E:**

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

   The direction of the field is radial. So the vector expression is:

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{e}_r$$

   Final expression:

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{e}_r}$$

Exercise 4:

Consider a cylindrical capacitor consisting of two coaxial cylindrical plates of infinite length, with radii R_1 and R_2, separated by vacuum ($R_2 > R_1$) (Figure 3). Let σ be the charge per unit area of the inner cylinder.

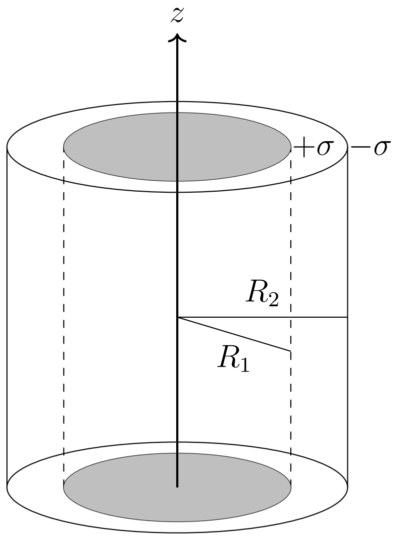

Figure 3:

1. Give the expression for the capacitance C of this cylindrical capacitor, knowing that, according to Gauss's theorem, the electrostatic field $\vec{E}$ between the two plates is written as:

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\frac{R_1}{r}\vec{e_r}$$

2. Give the expression for the capacitance C of this cylindrical capacitor if $e = R_2 - R_1 \ll R_1$.

Answer Area

1. **Expression for the capacitance C of the cylindrical capacitor:**

   Given: - Two coaxial cylindrical plates with radii R_1 and R_2, with $R_2 > R_1$ - The electric field between the plates is given by:

$$\vec{E} = \frac{\sigma R_1}{\varepsilon_0 r}\vec{e}_r$$

   To find the capacitance C, we first compute the potential difference V between the two cylinders:

$$V = \int_{R_1}^{R_2} \vec{E} \cdot d\vec{r} = \int_{R_1}^{R_2} \frac{\sigma R_1}{\varepsilon_0 r} dr = \frac{\sigma R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right)$$

   The charge per unit length on the inner cylinder is $Q = \sigma \cdot 2\pi R_1$

   Then the capacitance per unit length is:

$$C = \frac{Q}{V} = \frac{\sigma 2\pi R_1}{\frac{\sigma R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}$$

   Final expression for the capacitance per unit length:

$$\boxed{C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}}$$

2. **Expression for the capacitance C when $e = R_2 - R_1 \ll R_1$:**

   In this case, the separation e is very small compared to R_1. So we can approximate:

$$\ln\left(\frac{R_2}{R_1}\right) = \ln\left(1 + \frac{e}{R_1}\right) \approx \frac{e}{R_1} \quad (\text{for small } \frac{e}{R_1})$$

   Substituting into the previous result:

$$C \approx \frac{2\pi\varepsilon_0 R_1}{e}$$

   Alternatively, since the geometry resembles a parallel-plate capacitor when $e \ll R_1$, the result also makes sense as analogous to:

$$C_{\text{parallel plate}} = \frac{\varepsilon_0 A}{e}$$

   where $A = 2\pi R_1 L$ is the area for a cylinder of length L.

   Final approximated expression:

$$\boxed{C \approx \frac{2\pi\varepsilon_0 R_1}{e}}$$