Ibn Tofail University

Algebra II — Make-up Exam

Year: 21-22

Exercise 1:

Consider the matrix $C(r)$ defined by $\begin{pmatrix} r+1 & 3r+1 & 2r+1 \\ r+2 & r+2 & 3r+2 \\ 3r+3 & 2r+3 & r+3 \end{pmatrix}$ where r is a real number.

a) Calculate $\det(C(r))$ as a function of r.

b) Give the values of r for which $C(r)$ is invertible.

Answer Area

Exercise 2:

In $\mathbb{R}_2[X]$, the vector space of polynomials with real coefficients of degree less than or equal to 2, we define the linear map $f : \mathbb{R}^2 \to \mathbb{R}_2[X]$ by

$$f(a, b) = (a + b) + (2a - b)x + (3a - b)x^2$$

1. Using the rank theorem, show that f is not surjective.
2. Show that f is injective.
3. Determine $\operatorname{Im} f$, the image of f (give a basis of $\operatorname{Im} f$).

Answer Area

Exercise 3:

Let $B = \{e_1, e_2, e_3\}$ be the canonical basis of $\mathbb{R}^3$ and $B' = \{e'_1, e'_2, e'_3\}$ a family of vectors of $\mathbb{R}^3$ with $e'_1 = (2, 3, 2)$, $e'_2 = (1, 2, 1)$ and $e'_3 = (1, 1, 2)$.

1. a) Show that B' is a basis of $\mathbb{R}^3$.

   b) Determine the coordinates of the vector $v = (4, 6, 5)$ in the basis B'.

2. a) Determine $P = P_B^{B'}$, the transition matrix from basis B to basis B'.

   b) Using the comatrix of P, show that the transition matrix from basis B' to basis B is

$$\begin{pmatrix} 5 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

3. Let $g : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map defined by

$$g(x, y, z) = (-x + 2y - z, -6x + 5y, 2y - 2z)$$

   a) Determine A, the matrix of g in basis B.

   b) Determine A', the matrix of g in basis B'.

Answer Area