Ibn Tofail University

Analysis II — Make-up Exam

Year: 21-22

Exercise 1:

The three questions are independent:

1. Show that: $\forall x \in \mathbb{R}^+ : x - \frac{x^2}{2} \leq \ln(1+x) \leq x - \frac{x^2}{2} + \frac{x^3}{3}$.
2. Calculate the following limit: $\lim_{x\to 0} \left(\cot^2(3x) - \frac{1}{9x^2}\right)$
3. Find an equivalent near 0 of $2\exp u - \sqrt{1+4u} - \sqrt{1+6u^2}$.

Answer Area

Exercise 2:

1. Using concavity:

   (a) Show that: $\forall x \in [0; \frac{\pi}{2}] : \sin(x) \leq x$.

   (b) Also show that: $\forall u \in [0; 1] : \sin(\frac{\pi}{2}u) \geq u$. Deduce that $\forall x \in [0; \frac{\pi}{2}] : \sin(x) \geq \frac{2}{\pi}x$ and give the resulting bounds for $\sin x$ on $[0; \frac{\pi}{2}]$.

2. Let $n \in \mathbb{N}^*$ and $a_1, \cdots, a_n \in \mathbb{R}_+^*$. Show that: $\frac{n}{\frac{1}{a_1}+\cdots+\frac{1}{a_n}} \leq (a_1 \cdots a_n)^{\frac{1}{n}} \leq \frac{a_1+\cdots+a_n}{n}$

Answer Area

Exercise 3:

Show that the function below is of class C^1 on $\mathbb{R}$. Using a Taylor expansion, give the equation of the tangent to the curve C_f at the point with abscissa 0, as well as the position of the curve, near 0, with respect to the tangent: $f(x) = \frac{1}{x} \ln \frac{\exp(2x)-1}{2x}$.

Answer Area

Exercise 4:

Let f be the function defined by: $f(x) = (x-2)\exp\left(\frac{x-1}{x+1}\right)$.

1. Does the curve C_f, representing f, have a vertical asymptote? Justify.
2. Study the equation of the asymptote to the representative curve of f in the neighborhood of $+\infty$ and $-\infty$.
3. Study the relative position of this asymptote with respect to the curve C_f.

Answer Area