UNIVERSITY IBN TOFAIL

Optics

Problem Set I

Exercise 1: Descartes' Law – Gauss Conditions

Two homogeneous, transparent, and isotropic media with refractive indices n and n', where $n > n'$, are separated by a plane surface (Σ) perpendicular to the optical axis at point H (figure 1). A luminous point A' located on the optical axis in the homogeneous medium of refractive index n', constitutes the image of an object A located on the axis in a medium of refractive index n. We define $x = \overrightarrow{HA}$ and $x' = \overrightarrow{HA'}$

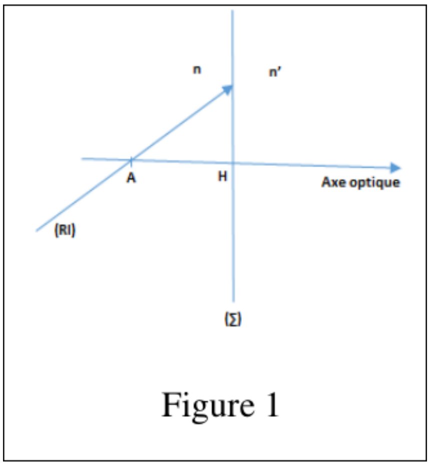

Figure 1: Plane interface between two media

1. Determine the position of point A' on the diagram.
2. Show that under Gauss approximation conditions, we obtain the relation: $\frac{n}{x} + \frac{n'}{x'} = 0$

Correction

Exercise 2: Refractometer

A refractometer consists of a glass cylinder with refractive index n, whose upper face is flat and perpendicular to its axis. A cross-section of the device is shown in figure 2. A drop of liquid with unknown refractive index is placed on this face. The device is illuminated through its entry face AB with a monochromatic ray at an incidence angle i. We are in conditions where total reflection occurs at point J.

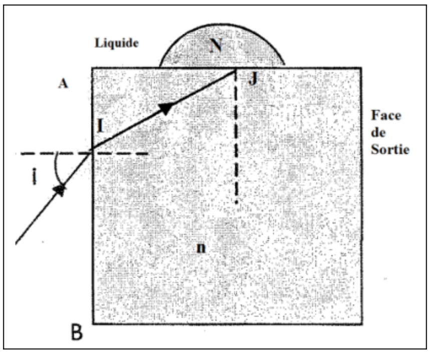

Figure 2: Refractometer

1. Trace the ray reflected at J and the ray emerging at K through the exit face.
2. Let i' be the emergence angle of this ray measured with respect to the normal at the interface. We vary the angle i up to the limit i_m of total reflection. We measure i'_m.
   (a) Express i'_m as a function of n and N.
   (b) Numerical application: $n = 1.5$. We measure $i'_m = 4555'$. What is the value of N?

Correction

Exercise 3: Brewster's Angle

A plane interface separates a transparent homogeneous medium with refractive index n_1 from a transparent homogeneous medium with refractive index n_2. An incident light ray in the medium with index n_1 (incidence angle i) is partly reflected (reflection angle $i' = i$) and partly transmitted into the medium with index n_2 (refraction angle r).

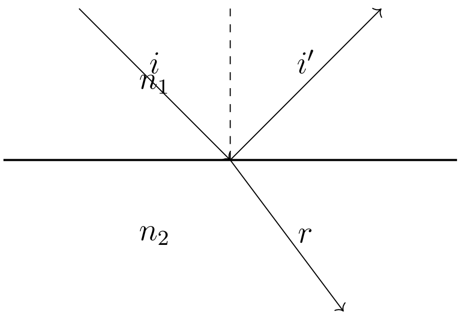

Figure 3: Reflection and refraction at a plane interface

1. For what value of the incidence angle i are the reflected and refracted rays perpendicular to each other? Numerical application: given $n_1 = 1$ and $n_2 = 1.33$, calculate i.

Correction

Exercise 4: Optical Fiber

An optical fiber consists of a cladding with refractive index $n_2 = 1.495$ surrounding a cylindrical core with refractive index $n_1 = 1.510$. The optical fiber is immersed in air ($n_{air} \approx 1$). We will assume that the entry face is a perpendicular section of the fiber and that the fiber is not curved. A light beam strikes the entry face at point I (see figure) with an incidence angle i. It enters the fiber with a refraction angle r and is reflected on the faces of the fiber with an angle α. The first reflection point is denoted as J. The objective is to transmit the maximum amount of light to the other end of the fiber, thus avoiding light that has entered the core from penetrating into the cladding.

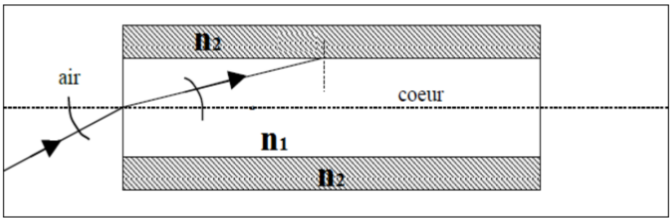

Figure 4: Light propagation in an optical fiber

1. Trace the path of the considered ray inside the fiber core.
2. What relationship exists between r and α?
3. What is the critical angle α_c for total internal reflection of the beam inside the fiber?
4. What is the value i_m of the incidence angle corresponding to the entry of the fiber? The numerical aperture N.A. is defined as the quantity $\sin(i_m)$.
5. Express N.A. in terms of n_1 and n_2. What condition must i satisfy for the beam to propagate along the fiber without leaving the core?

Suppose we send into the fiber a light pulse in the form of a converging conical beam with a half-angle at the vertex $i_s < i_m$.

6. Calculate the time t_0 taken to travel a distance L for a ray with angle $i_0 = 0$, then the time t_1 for a ray with angle i_s. What do you observe?

Correction

Exercise 5: Parallel-Faced Plate

A light beam strikes the entry face of a parallel-faced plate of thickness e, immersed in air and with refractive index n, at an incidence angle i. The path of the incident ray is shown in the figure below.

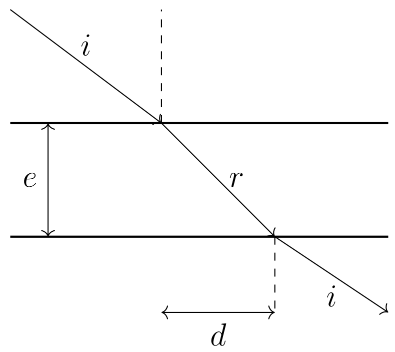

Figure 5: Light through a parallel-faced plate

1. Show that the incident ray and the emerging ray are indeed parallel.

2. Establish the expression for the displacement d in terms of e, i, and r. What is the value of d when $i = 0$ and $i = 90$?

3. Show that d can be expressed in terms of e, i, and n, in the following form:

$$d = e \cdot sin\left(1 - \frac{\sqrt{1 - sin^2 i}}{\sqrt{n^2 - sin^2 i}}\right)$$

4. Now assume that the angle i is very small. Show that d can be written in the simple form: $d = e \cdot i \cdot \left(1 - \frac{1}{n}\right)$ Comment on this relationship.
Numerical application: Calculate the value of d for $i = 5$, $n = 1.5$, and $e = 10$ cm.

Correction

Exercise 6: Prism

Consider a prism with apex angle A made of glass with refractive index n. It is placed in air with index $n_0 = 1$.

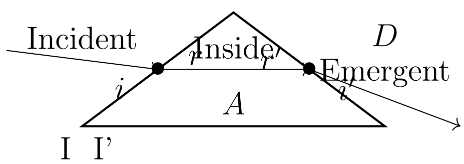

Figure 6: Light through a prism

1. Establish the four prism formulas. What do they become when A and i are small?

2. We want to determine the refractive index of the prism: Experience shows that for a given monochromatic radiation, the deviation D passes through a minimum value. Let D_m be the value of this minimum deviation angle.

   (a) Determine the condition on i and i' for $D = D_m$.

(b) Then deduce the condition on r and r'.

(c) Deduce the value of i as a function of A and D_m, and finally the value of the prism's refractive index.

(d) Calculate the value of the critical angle of incidence at point I'.

(e) Deduce that there exists a value A_M of A beyond which there will be no emergent ray, regardless of the incidence angle i. Calculate A_M for $n = 1.5$.

Correction