Ibn Tofail University

Algebra II — Make-up Exam

Year: 19-20

Exercise 1:

Let E be a K-vector space, $p \in \mathbb{N}^*$, and $u_1, u_2, ..., u_p, u_{p+1}$ be vectors in E.

1) Assume that $\{u_1, u_2, ..., u_p\}$ is linearly independent. Show the equivalence:
$u_{p+1} \notin \text{span}(\{u_1, u_2, ..., u_p\}) \Leftrightarrow \{u_1, u_2, ..., u_p, u_{p+1}\}$ is linearly independent.

2) Assume that $\{u_1, u_2, ..., u_p, u_{p+1}\}$ is a generating set of E and that
$u_{p+1} \in \text{span}(\{u_1, u_2, ..., u_p\})$. Show that $\{u_1, u_2, ..., u_p\}$ is a generating set of E.

Answer Area

Exercise 2:

In the $\mathbb{R}$-vector space $\mathbb{R}^4$, consider the following vector subspaces:
$F = \text{span}(\{(1,0,1,0),(0,1,0,1)\})$
$G = \{(x,y,z,t) \in \mathbb{R}^4 \mid x+y=0 \text{ and } z+t=0\}$
$H = \{(x,y,z,t) \in \mathbb{R}^4 \mid x-y+z-t=0\}$

1) Determine a basis and the dimension of G.

2) Determine a basis and the dimension of H.

3) Determine a basis of the vector subspace $F \cap G$.

4) Show that $\mathbb{R}^4 = (F \cap G) \oplus H$.

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Exercise 3:

Let $E = \mathbb{R}_2[X]$ be the vector space of polynomials with real coefficients of degree less than or equal to 2. And let $f : E \to E$ be the linear map defined by:
for all $P(X) \in E$, $f(P(X)) = 2P(X) - (X-1)P'(X)$
(where $P'(X)$ denotes the derivative of the polynomial $P(X)$).

1) Determine $\mathrm{Ker} f$.

2) Determine a complement of $\mathrm{Ker} f$ in E.

Answer Area