Матлог2 Подсказки

Определение 18.14. Следующие функции называются **клиниевски-ми скобками**: $[x, y] = c\,(l(x),\ c(r(x), y))$.

Тогда

$$[x_1, \ldots, x_{n+1}] = [[x_1, \ldots, x_n], x_{n+1}];$$
$$[k]_{21} = c(l(k), l(r(k)));$$
$$[k]_{22} = r(r(k));$$
$$[k]_{n,1} = [[k]21]_{n-1,1};$$
$$\ldots$$
$$[k]_{n,n-1} = [[k]21]_{n-1,n-1};$$
$$[k]_{n,n} = [k]_{2,2}\ .$$

Определение 8.9. *Базовые Машины Тьюринга:*

А. Перенос нуля: $q_1 001^x0 \ \mid\Rightarrow\ q_0 01^x00$;

Б$^+$. Правый сдвиг: $q_1 01^x0 \ \mid\Rightarrow\ 01^x q_0 0$;

Б$^-$. Левый сдвиг: $01^x q_1 0 \ \mid\Rightarrow\ q_0 01^x 0$;

В. Транспозиция: $01^x q_1 01^y 0 \ \mid\Rightarrow\ 01^y q_0 01^x 0$;

Г. Удвоение: $q_1 01^x 0^{x+2} \ \mid\Rightarrow\ q_0 01^x 01^x 0$;

Ц$_n$. Циклический сдвиг:

$$q_1 01^{x_1} 01^{x_2} 0 \ldots 01^{x_n} 0 \ \mid\Rightarrow\ q_0 01^{x_2} 0 \ldots 01^{x_n} 01^{x_1} 0;$$

K_n. Копирование:

$$q_1 01^{x_1} \ldots 01^{x_n} 0^{x_1 + \ldots + x_n + n + 1} \ \mid\Rightarrow$$
$$q_0 01^{x_1} \ldots 01^{x_n}\ 01^{x_1} \ldots 01^{x_n} 0;$$

Л. Ликвидация: $q_1 01^x 0 \ \mid\Rightarrow\ q_0 0^{x+2}$;

R. Вычитание единицы: $q_1 01^{x+1} 0 \ \mid\Rightarrow\ q_0 01^x 00$;

S. Добавление единицы: $q_1 01^x 00 \ \mid\Rightarrow\ q_0 01^{x+1} 0$.

Правила вывода:

1) $\dfrac{\Gamma \vdash \varphi;\ \Gamma \vdash \psi}{\Gamma \vdash (\varphi \,\&\, \psi)}$ 2) $\dfrac{\Gamma \vdash (\varphi \,\&\, \psi)}{\Gamma \vdash \varphi}$

3) $\dfrac{\Gamma \vdash (\varphi \,\&\, \psi)}{\Gamma \vdash \psi}$ 4) $\dfrac{\Gamma \vdash \varphi}{\Gamma \vdash (\varphi \vee \psi)}$

5) $\dfrac{\Gamma \vdash \psi}{\Gamma \vdash (\varphi \vee \psi)}$ 6) $\dfrac{\Gamma, \varphi \vdash \xi;\ \Gamma, \psi \vdash \xi;\ \Gamma \vdash (\varphi \vee \psi)}{\Gamma \vdash \xi}$

7) $\dfrac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash (\varphi \to \psi)}$ 8) $\dfrac{\Gamma \vdash \varphi;\ \Gamma \vdash (\varphi \to \psi)}{\Gamma \vdash \psi}$ (modus ponens)

9) $\dfrac{\Gamma, \neg\varphi \vdash}{\Gamma \vdash \varphi}$ 10) $\dfrac{\Gamma \vdash \neg\varphi;\ \Gamma \vdash \varphi}{\Gamma \vdash}$

11) $\dfrac{\Gamma, \varphi, \psi, \Gamma_1 \vdash \xi}{\Gamma, \psi, \varphi, \Gamma_1 \vdash \xi}$ 12) $\dfrac{\Gamma \vdash \varphi}{\Gamma, \psi \vdash \varphi}$

Замечание 10.10. Следующие правила вывода являются допустимы-ми:

а) $\dfrac{\psi_1, \ldots, \psi_n \vdash \varphi}{\xi_1, \ldots, \xi_k \vdash \varphi}$, $\{\psi_1, \ldots, \psi_n\} \subseteq \{\xi_1, \ldots, \xi_k\}$

б) $\dfrac{\psi_1, \ldots, \psi_n \vdash}{\xi_1, \ldots, \xi_k \vdash}$, $\{\psi_1, \ldots, \psi_n\} \subseteq \{\xi_1, \ldots, \xi_k\}$

в) $\dfrac{\Gamma \vdash \varphi;\ \Gamma, \varphi \vdash \psi}{\Gamma \vdash \psi}$ е) $\dfrac{\Gamma \vdash}{\Gamma \vdash \varphi}$ и) $\dfrac{\Gamma, \varphi \vdash \psi}{\Gamma, \neg\psi \vdash \neg\varphi}$

г) $\dfrac{\Gamma_1,\ \varphi,\ \psi,\ \Gamma_2 \vdash \xi}{\Gamma_1, (\varphi \,\&\, \psi), \Gamma_2 \vdash \xi}$ ж) $\dfrac{\Gamma,\ \varphi \vdash}{\Gamma \vdash \neg\varphi}$ к) $\dfrac{\Gamma, \neg\varphi \vdash \neg\psi}{\Gamma, \psi \vdash \varphi}$

д) $\dfrac{\Gamma \vdash (\varphi \,\&\, \neg\varphi)}{\Gamma \vdash}$ з) $\dfrac{\Gamma \vdash \varphi}{\Gamma, \neg\varphi \vdash}$

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Домашняя работа

③ стр 66 (сечение)

а) $$\dfrac{\dfrac{\Gamma_1 \vdash A;\quad \Gamma_2, A \vdash B}{\Gamma_1 \vdash A;\quad \Gamma_2 \vdash A \to B}\ (7)}{\Gamma_1, \Gamma_2 \vdash B}\ (8)$$

б) $$\dfrac{\Gamma, A, B \vdash C}{\Gamma, (A \& B) \vdash C}$$ объединение посылок

(3) $$\dfrac{\Gamma, A, B \vdash C;\quad (A \& B) \vdash (A \& B)}{\Gamma, A, B \vdash C;\quad A \& B \vdash B}$$

(7) $$\dfrac{\Gamma, A, B \vdash C;\quad A \& B \vdash B}{\Gamma, A, \vdash (B \to C);\quad A \& B \vdash B}$$

(8) $$\dfrac{\Gamma, A, \vdash (B \to C);\quad A \& B \vdash B}{\Gamma, A, A \& B \vdash C}$$ $A \& B \vdash A \& B$

(3) $$\dfrac{\Gamma, A, A \& B \vdash C}{\Gamma, A, A \& B \vdash C;\quad A \& B \vdash A}$$

(7) $$\dfrac{\Gamma, A, A \& B \vdash C;\quad A \& B \vdash A}{\Gamma, A \& B \vdash A \to C;\quad A \& B \vdash A}$$

(8) $$\dfrac{\Gamma, A \& B \vdash A \to C;\quad A \& B \vdash A}{\Gamma, A \& B, A \& B \vdash C}$$

(15) $$\dfrac{\Gamma, A \& B, A \& B \vdash C}{\Gamma, A \& B \vdash C}$$

в) $$\dfrac{\Gamma, A \& B \vdash C}{\Gamma, A, B \vdash C}$$ расщепление посылок

$A \vdash A,\ B \vdash B$

$A, B \vdash A \& B$

(1) $$\dfrac{\Gamma, (A \& B) \vdash C}{\Gamma, (A \& B) \vdash C}$$

(7) $$\dfrac{\Gamma, (A \& B) \vdash C}{\Gamma, \vdash (A \& B) \to C;\quad A, B \vdash (A \& B)}$$

(8) $$\dfrac{\Gamma, \vdash (A \& B) \to C;\quad A, B \vdash (A \& B)}{\Gamma, A, B \to C}$$

г) $$\dfrac{\Gamma, A \vdash C;\quad \Gamma, B \vdash C}{\Gamma, (A \vee B) \vdash C}$$ разбор случаев

$\overset{\Gamma_1}{(A \vee B)} \vdash (A \vee B)$

(6) $$\dfrac{\overset{\Gamma_2}{\Gamma}, A \vdash C;\quad \overset{\Gamma_3}{\Gamma}, B \vdash C}{(A \vee B), \Gamma, \Gamma \vdash C}$$

(15) $$\dfrac{(A \vee B), \Gamma, \Gamma \vdash C}{(A \vee B), \Gamma \vdash C}$$

(14) $$\dfrac{(A \vee B), \Gamma \vdash C}{\Gamma, (A \vee B) \vdash C}$$

д) $$\dfrac{\Gamma, A \vdash B}{\Gamma, \neg B \vdash \neg A}$$ контрапозиция

(10) $$\dfrac{\Gamma, A \vdash B \qquad \neg B \vdash \neg B}{\Gamma, A, \neg B \vdash}$$

(14) $$\dfrac{\Gamma, A, \neg B \vdash}{\Gamma, \neg B, A \vdash}$$

(11) $$\dfrac{\Gamma, \neg B, A \vdash}{\Gamma, \neg B \vdash \neg A}$$

Предложение 19 Равносильность является конгруэнцией на модели $\langle F; \vee, \&, \to, \neg \rangle$, т.е. если $\varphi \equiv \varphi_1$, $\psi \equiv \psi_1$, то:

1) $(\varphi \vee \psi) \equiv (\varphi_1 \vee \psi_1)$
2) $(\varphi \& \psi) \equiv (\varphi_1 \& \psi_1)$
3) $(\varphi \to \psi) \equiv (\varphi_1 \to \psi_1)$
4) $\neg\varphi \equiv \neg\varphi_1$

$\langle F/\equiv; \vee, \&, \to, \neg \rangle$

Док-во:

1) $$\dfrac{\dfrac{\varphi \vdash \varphi_1}{\varphi \vdash (\varphi_1 \vee \psi_1)} \qquad \dfrac{\psi \vdash \psi_1}{\psi \vdash (\varphi_1 \vee \psi_1)}}{(\varphi \vee \psi) \vdash (\varphi_1 \vee \psi_1)}$$ (разбор случаев)

$$\dfrac{\dfrac{\varphi_1 \vdash \varphi}{\varphi_1 \vdash (\varphi \vee \psi)} \qquad \dfrac{\psi_1 \vdash \psi}{\psi_1 \vdash (\varphi \vee \psi)}}{(\varphi_1 \vee \psi_1) \vdash (\varphi \vee \psi)}$$

2) $$\dfrac{\dfrac{\dfrac{\varphi \vdash \varphi_1}{\varphi, \psi \vdash \varphi_1} \qquad \dfrac{\psi \vdash \psi_1}{\varphi, \psi \vdash \psi_1}}{\varphi, \psi \vdash \varphi_1 \& \psi_1}\ (1)}{\varphi \& \psi \vdash \varphi_1 \& \psi_1}$$ (объед. посылок)

3) $$\dfrac{\varphi \vdash \varphi;\quad \varphi \to \psi \vdash \varphi \to \psi}{\varphi, \varphi \to \psi \vdash \psi \quad;\quad \psi \vdash \psi_1}$$

(сечение) $$\dfrac{\varphi, \varphi \to \psi \vdash \psi \quad;\quad \psi \vdash \psi_1}{\varphi, \varphi \to \psi \vdash \psi_1}$$ $\varphi_1 \vdash \varphi$

(сечение) $$\dfrac{\varphi, \varphi \to \psi \vdash \psi_1 \qquad \varphi_1 \vdash \varphi}{\varphi_1, \varphi \to \psi \vdash \psi_1}$$

$$\dfrac{\varphi_1, \varphi \to \psi \vdash \psi_1}{\varphi \to \psi \vdash \varphi_1 \to \psi_1}$$

4) $$\dfrac{\dfrac{\varphi \vdash \varphi_1}{\neg\varphi_1 \vdash \neg\varphi}\ (\text{контр.}) \qquad \dfrac{\varphi_1 \vdash \varphi}{\neg\varphi \vdash \neg\varphi_1}\ (\text{контрпозиц.})}{\varphi_1 \equiv \varphi_1}$$

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