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Spin axis behavior of the LAGEOS satellites

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[1] The satellites LAGEOS-I and LAGEOS-II are essential for the scientific study of various (geo)physical phenomena, such as geocenter motion and absolute scale. The high quality of such science products strongly depends on the absolute quality of the SLR observations and that of the orbit description. Therefore all accelerations experienced by the spacecraft need to be modeled as accurately as possible, the thermal radiation forces being one of them. Traditionally, this is done by estimating so-called empirical accelerations. However, the rotational dynamics of LAGEOS-I in particular no longer allows such a simple approach: a full modeling of the spin behavior, the temperature distribution over the spacecraft surface and the resulting net force prove necessary to achieve the best results. As a first step, a new model, Lageos Spin Axis Model (LOSSAM) has been developed. It is unique in its combination of analytical theory and empirical observations. Its mathematics is taken after previous investigators, although flaws have been corrected. LOSSAM describes the full spin behavior of LOSSAM based on the following phenomena: (1) the geomagnetic field, (2) the Earth's gravity field, (3) the satellite center of pressure offset, and (4) the effective difference in reflectivity between the satellite hemispheres. Its accuracy has been demonstrated by an improvement of about a 50% in the RMS residual of the Yarkovsky-Schach effect signal (as shown by *Lucchesi et al.* [2004]). Such a high-quality model for rotational behavior is indispensable for a proper force modeling, and hence also for the quality of typical LAGEOS science products. *INDEX TERMS:* 0699 Electromagnetics: General or miscellaneous; 1241 Geodesy and Gravity: Satellite orbits; 1299 Geodesy and Gravity: General or miscellaneous; *KEYWORDS:* LAGEOS, SLR, spin axis behavior

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1. Introduction

[2] The LAGEOS mission began on 4 May 1976, with the launch of LAGEOS-I, a fully passive spherical satellite for the purpose of accurately measuring geophysical phenomena like plate tectonics and Earth rotation by using the Satellite Laser Ranging (SLR) technique [*NASA*, 1975]; [*Cohen and Smith*, 1990]. Its mission was complemented by its twin, LAGEOS-II, launched on 22 October 1992, possibly followed by its successor LARES, which is expected to yield crucial information on relativistic phenomena like the Lense-Thirring effect [*Ciufolini et al.*, 1998].

[3] In order to extract the maximum amount of geodetic and geophysical information from the mm-accuracy observations, the orbits of the spacecraft need to be reconstructed as accurately as possible. Here, one has seen significant progress with time, with current (radial) orbit accuracies of about 1 cm. The orbit computations have yielded the existence of a decrease of the semi-major axis of LAGEOS-I (a similar observation can be made for LAGEOS-II). This decay in the orbit, at an average rate of 1.3 mm/d, can be modeled by an empirical along-track acceleration with a mean value of about -3.4 pm/s^2 [e.g., *Rubincam*, 1987; *Afonso et al.*, 1989]. These accelerations have been the subject of study for a long time and were explained at first as being caused by neutral and charged particle drag. However, later on these effects proved to be responsible only for 30% of the observed along-track acceleration [*Rubincam*, 1990]. Other attempts to explain this acceleration and its variation focused on aspects of solar, terrestrial infrared and albedo radiation [e.g., *Anselmo et al.*, 1983; *Rubincam and Weiss*, 1985]. [*Boudon et al.*, 1979] showed that photon thrust plays a major role in explaining the observed accelerations. This is caused by the unequally spatially distributed thermal radiation of the satellite due to a temperature gradient over its surface. On the basis of this crucial effect, two phenomena can be differentiated: the Yarkovsky effect (diurnal, seasonal), due to delayed (re)emission of energy coming from the

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Sun or the Earth, and the Yarkovsky-Schach effect, due to the temperature variations over the surface of the satellite when getting into and out of Earth's shadow. A further substantial contribution appeared to be induced by the effective difference in optical properties between the two hemispheres of a satellite [e.g., *Scharroo et al.*, 1991], which has been proposed to be due to the re-emission contributions of the 4 germanium retroreflectors [*Lucchesi*, 2004]. Since phenomena based on the thermal radiation of the satellite are strongly dependant on the evolution of the spin axis of the spacecraft [*Rubincam*, 1980, 1982], a successful description of the latter would result into a better definition of the acceleration pattern and thus better definition of the geophysical parameters obtained from LAGEOS orbits.

[4] For the purpose of modeling such forces and explaining or understanding the observed empirical accelerations, the orientation of the satellite axis of rotation proved to play a crucial role. Several authors have tried to explain the evolution of the LAGEOS spin axis, beginning with *Rubincam* [1987], who was the first to discuss the assumption of a constrained spin axis behavior, and to mention the possibility that the magnetic field of the Earth influenced the orientation of the spin axis. A few years later he inverted this line of reasoning and solved for the orientation of the spin axis by matching his model predictions for the along-track acceleration to the estimated empirical values, concluding that an almost fixed spin axis orientation could explain the data well [*Rubincam*, 1990]. A significant step further was made by *Bertotti and Iess* [1991], who applied the theory describing the motion of a conductor in a magnetic field to LAGEOS-I. This theory, known as the *BI* theory since [*Farinella et al.*, 1996], is the basis of all subsequent studies, including this one. A very important conclusion in *Bertotti and Iess* [1991] was the suggestion of an alternative solution for the initial orientation of the spin axis: the so-called Southern Solution, which is related to the solution of Rubincam (called the Northern Solution since then) by a simple spatial inversion. In 1996, *Farinella et al.* [1996] further developed the BI theory by correcting some errors which appeared in *Bertotti and Iess* [1991]. Their results confirmed the validity of the Southern Solution, and an orientation model was obtained for both LAGEOS-I and LAGEOS-II, based on perturbations caused by magnetic and gravitational torques. This model has been a reference since. A direct confirmation of the Southern Solution from solar glint observations was shown by *Avizonis* [1997]. The most recent further development of this theory came from *Vokrouhlický* [1996], who added two other torques suggested but not elaborated in *Farinella et al.* [1996] and not considered in *Bertotti and Iess* [1991], obtaining interesting new results. However, these results showed a chaotic pattern of evolution after a specific point in time. The Vokrouhlický study lacked "physical proof", i.e., a link to independent observations, but clearly pointed out the irrelevance of earlier concepts to describe the thermal forces on LAGEOS-I, in particular after that date. Here we will start from that model situation, and refine and further develop the various descriptions and use independent spin axis observations to make the results as realistic as possible. Other attempts to solve the spin axis problem come from *Ries et al.* [1993], *Habib et al.* [1994], *Kolenkiewicz et al.* [1995], and *Métris et al.* [1999].

[5] In order to have a proper background on the topic, the mathematical model and its foundation will be described first. Next, an overview of the available observations will be given. This will be followed by a discussion on the treatment of the aforementioned observations for the model refinement, including unknown parameters. This is followed by a discussion and interpretation of the results. Finally, conclusions will be given as well as suggestions for future improvements of the model, application of which will be done to further calculations concerning thermal forces.

2. LAGEOS Spin Axis Model

[6] The evolution of the LAGEOS spin axis orientation and spin rate can be obtained by applying Euler's equation for the motion of a solid body:

$$\frac{d\mathbf{L}}{dt} = \mathbf{M}_{magnetic} + \mathbf{M}_{gravitational} + \mathbf{M}_{offset} + \mathbf{M}_{anisotropy} \tag{1}$$

With this formulation all main contributions to the disturbing torque (magnetic, gravitational, center of pressure offset and reflective anisotropy) are taken into account. According to *Landau and Lifshitz* [1984] and *Bertotti and Iess* [1991], the magnetic torque of an axial symmetrical conductor rotating along its axis of symmetry with angular velocity $\boldsymbol{\omega}$ can be expressed as

$$\mathbf{M}_{magnetic} = \mathbf{m} \times \mathbf{B} = -V\alpha'(\hat{\boldsymbol{\omega}} \cdot \mathbf{B})(\hat{\boldsymbol{\omega}} \times \mathbf{B}) - V\alpha''(\mathbf{B} \times \hat{\boldsymbol{\omega}}) \times \mathbf{B} \tag{2}$$

where the expression appearing in *Bertotti and Iess* [1991] has been corrected after comparing with *Landau and Lifshitz* [1984], and the minus sign in the first term has been added (J. R. Sanmartín, personal communication, 2003). The vector $\mathbf{m}$ is the satellite's magnetic moment, $\mathbf{B}$ is the (local) magnetic field strength vector, $\hat{\boldsymbol{\omega}}$ is the unit vector directed along the satellite's angular velocity vector, and α' and α'' are the real and imaginary parts of the polarizability parameter per unit volume V of the conductor. The polarizability depends on ω, the modulus of the angular velocity, and on the penetration depth of the external magnetic field into the satellite. This depth δ can be expressed as [*Landau and Lifshitz*, 1984]

$$\delta = \frac{c}{\sqrt{2\pi\omega\sigma}} \tag{3}$$

and depends on the speed of light c, the electrical conductivity σ of the conductor, and the frequency of the external field, i.e., ω.

[7] The expressions for the polarizability parameters α' and α'' in the low frequency limit for a perfect sphere are contained in *Landau and Lifshitz* [1984] and *Bertotti and Iess* [1991]:

$$\alpha' = -\frac{1}{105\pi}\left(\frac{\rho}{\delta}\right)^4 \beta'\left[\left(\frac{\rho}{\delta}\right)^2\right] = -\frac{1}{105\pi}\left(\frac{\rho}{\delta}\right)^4 \beta'(0) + \mathcal{O}\left[\left(\frac{\rho}{\delta}\right)^6\right] \tag{4}$$

$$\alpha'' = \frac{1}{20\pi}\left(\frac{\rho}{\delta}\right)^2\beta''\left[\left(\frac{\rho}{\delta}\right)^2\right] = \frac{1}{20\pi}\left(\frac{\rho}{\delta}\right)^2\beta''(0) + \frac{1}{20\pi}\left(\frac{\rho}{\delta}\right)^4\beta'''(0) + \mathcal{O}\left[\left(\frac{\rho}{\delta}\right)^6\right] \quad (5)$$

with ρ being the radius of the satellite. Here, the suggestion by *Farinella et al.* [1996] for a possible improvement of the theory has been followed, i.e., taking more terms in the Taylor's expansion of these magnetization parameters, with coefficients $\beta'(0)$, $\beta''(0)$ and $\beta'''(0)$.

[8] Using these parameters, the expression of a component of the magnetic torque becomes [*Farinella et al.*, 1996]

$$M_i^{magnetic} = M_i^{(1)} + M_i^{(2)} \quad (6)$$

with

$$M_i^{(1)} = -V\frac{\alpha'}{\omega^2}\epsilon_{ijk}\,\omega_j\,\langle B_k\,B_l\rangle\,\omega_l = -V\frac{\alpha'}{\omega^2}\frac{d^2}{a^6}\,\epsilon_{ijk}\,\omega_j\,\alpha_{kl}\,\omega_l = \frac{4}{21}\nu_m\left(\frac{\rho}{c}\right)^2(2\pi\sigma)\frac{\beta'(0)}{\beta''(0)}\epsilon_{ijk}\,\omega_j\,\alpha_{kl}\,\omega_l \quad (7)$$

$$M_i^{(2)} = V\frac{\alpha''}{\omega}\omega_j\,\langle B^2\delta_{ij} - B_iB_j\rangle = -V\frac{\alpha''}{\omega}B_0^2\left(\frac{R_e}{a}\right)^6\omega_j\,\beta_{ij} = -V\frac{\alpha''}{\omega}\frac{d^2}{a^6}\omega_j\,\beta_{ij} = -\nu_m\omega_j\,\beta_{ij} \quad (8)$$

Here, d is the Earth's magnetic dipole strength, a is the orbital radius, R_e is the Earth's equatorial radius, B_0 is the intensity of the magnetic field at the Earth's geomagnetic equator, $\langle\cdots\rangle$ refers to the averaging process over satellite spin period, orbital revolution and day, as used in *Farinella et al.* [1996], ϵ_{ijk} is a component of the fully antisymmetric Levi-Civita tensor, ω_j a component of the angular velocity and α_{kl} and β_{ij} components of the tensor which comes from the calculation of $\langle B_kB_l\rangle$ and $\langle B^2\delta_{ij} - B_iB_j\rangle$, as done in *Farinella et al.* [1996]. The characteristic timescale ν_m of the magnetic despinning process is given by

$$\nu_m = V\frac{\alpha''}{\omega}\frac{d^2}{a^6} = V\frac{\alpha''}{\omega}B_0^2\left(\frac{R_e}{a}\right)^6 \quad (9)$$

Using the expansion of the polarization parameters (equation (5)), this equation becomes

$$\nu_m = \nu_{m_0}\left[1 + \left(\frac{\rho}{c}\right)^2(2\pi\sigma)\frac{\beta'''(0)}{\beta''(0)}\,\omega + \mathcal{O}\left[\left(\frac{\rho}{\delta}\right)^4\right]\right] \quad (10)$$

with $\nu_{m0} = (0.34)^{-1}$ yr^{-1}, a value different from the one used by *Farinella et al.* [1996] and *Bertotti and Iess* [1991].

[9] It must be remarked that the values for the magnetization parameters were adjusted a posteriori by *Bertotti and Iess* [1991] after the process of best-fitting the spin rate, as were the initial orientation and the initial spin rate. The BI values were used in *Farinella et al.* [1996] as well.

[10] The expression for the gravitational torque is given in *Farinella et al.* [1996]:

$$\mathbf{M}_{gravitational} = -\frac{3n^2}{4L^2}(C - A)\left(3\cos^2\vartheta - 1\right)(\mathbf{n}\cdot\mathbf{L})(\mathbf{n}\times\mathbf{L}) \quad (11)$$

where ϑ is the tilt angle, i.e., the angle between the angular velocity and the body axis of symmetry, n is the mean motion of the satellite in its orbit, **L** is the angular momentum, C and A the principal moments of inertia and **n** is the unit vector normal to the orbital plane. The typo which appeared in equation (9) of *Farinella et al.* [1996] for this torque has been corrected here by including the minus sign.

[11] In case of an offset h between the center of mass and the center of pressure, the solar radiation pressure causes a torque which can be modeled as [*Vokrouhlický*, 1996]

$$\mathbf{M}_{offset} = \frac{I_0hC_RA_{cross}}{c}(\mathbf{s}\times\mathbf{r}_{Sun}) \quad (12)$$

where I_0 is the irradiation power from the Sun, C_R the reflectivity coefficient, A_{cross} the cross-sectional area, **s** a unit vector in the direction of the satellite axis of symmetry (positive to the northern hemisphere of the satellite) and $\mathbf{r}_{Sun}$ a unit vector pointing from the satellite to the Sun. The exact value of the offset h is unknown, but for LAGEOS-I it is expected to be within the range [−0.79, +0.79] mm and [−0.1, +0.1] mm for LAGEOS-I and LAGEOS-II, respectively [*NASA*, 1975], [*Minott et al.*, 1993]. Although undetermined, the parameter h is important in scaling this offset and it will be considered later.

[12] The possible offset is considered to be along the axis of symmetry only, since this direction yields the torque perpendicular to the axis of symmetry (in this line of reasoning considered as the spin axis, i.e., neglecting ϑ) with the largest absolute value, therefore being the most effective cause of changing the orientation of the spin axis. Offset vectors pointing in a more general direction, with components in a plane perpendicular to the axis of symmetry as well, would also yield torques parallel to the axis of symmetry and would be responsible of a precession around this axis and a lesser change in orientation.

[13] *Scharroo et al.* [1991] concluded that a difference in reflectivity between the northern and southern hemisphere of LAGEOS-I exists. This torque can be expressed as

$$\mathbf{M}_{anisotropic\ ref.} = -\frac{2R_L}{3\pi}A_{cross}\frac{I_0}{c}\sin\vartheta_r(\mathbf{r}_{Sun}\times\mathbf{s}) \quad (13)$$

Here, ϑ_r is the angle between the vector pointing to the Sun $\mathbf{r}_{Sun}$ and the axis of symmetry **s**, and R_L is the radius of the satellite. The expression for this torque has been derived for an idealized satellite with the northern hemisphere acting as a perfect mirror and the southern hemisphere acting as a perfect absorber. Hence for a more realistic satellite, with a difference between the reflectivity coefficients equal to $\Delta\rho$

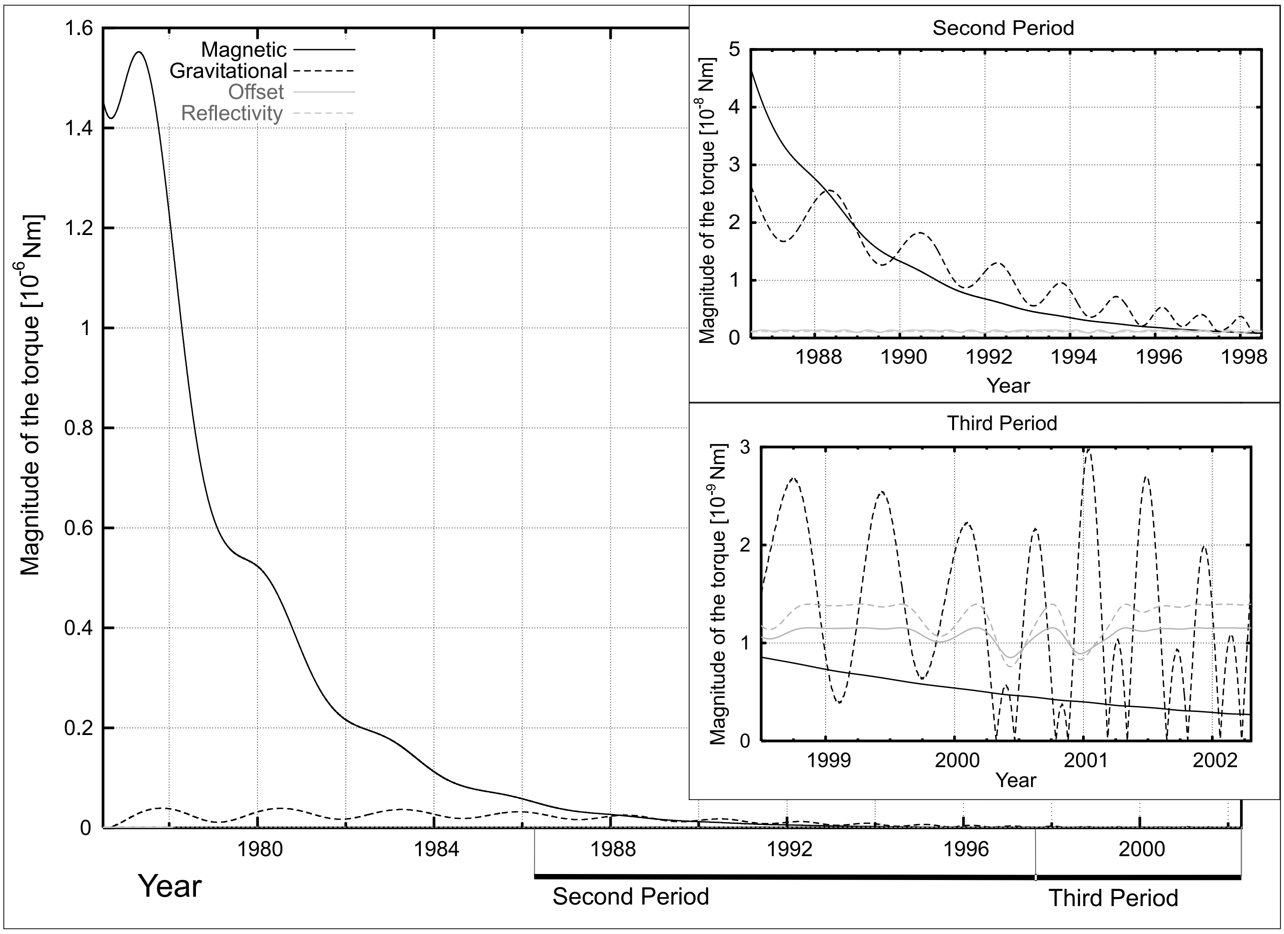

Figure 1. Time history of the magnitude of the different torques acting on LAGEOS-I. Three different characteristic periods can be distinguished, depending on the magnitude of the various torques: a first one where the magnetic torque is dominant, a second one where the order of magnitude of the magnetic and gravitational torques is comparable, and a third one where all torques have the same order of magnitude.

and a value C_R for the reflection coefficient, the expression becomes:

$$\mathbf{M}_{anisotropic\ ref.} = -\frac{2R_L}{3\pi} A_{cross} \frac{I_0}{c} C_R \Delta\rho \sin\vartheta_r (\mathbf{r}_{Sun} \times \mathbf{s}) \qquad (14)$$

Two equations remain that must be included to complete the description of the problem. The first one is the equation for the evolution of the tilt angle [*Farinella et al.*, 1996]:

$$\frac{d(\cos\vartheta)}{dt} = \Delta\beta'_{12}\nu_m \cos\vartheta \sin^2\vartheta \qquad (15)$$

where Δ is the nondimensional flattening parameter in terms of the principal moments of inertia, $\Delta = \frac{(C-A)}{C}$, and β'_{12} is the value of the component (1, 2) of the matrix β expressed in the $\mathcal{R}'$ frame defined in *Farinella et al.* [1996]. Considering as constant the new parameter $\zeta = \Delta\beta'_{12}\nu_m$ appearing in equation (15) for one step of integration τ_{step} (provided that $\tau_{step} \ll \tau_{characteristic}$, where $\tau_{characteristic}$ is the characteristic time of change of the parameters considered in ζ), the differential equation can be solved analytically for each integration step i:

$$(\cos\vartheta)_{i+1} = \frac{(\cos\vartheta)_i e^{\zeta\tau_{step}}}{\sqrt{1 + (\cos^2\vartheta)_i (e^{2\zeta\tau_{step}} - 1)}} \qquad (16)$$

[14] The second missing relation links the evolution of the angular momentum $\mathbf{L}$ to the evolution of the angular velocity $\boldsymbol{\omega}$, which for a solid object is [*Farinella et al.*, 1996]

$$\mathbf{L} = \bar{\mathbf{I}}\,\boldsymbol{\omega} \simeq C\left(1 - \Delta\sin^2\vartheta\right)\boldsymbol{\omega} \text{ with}$$
$$\bar{\mathbf{I}} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{bmatrix} \text{ for LAGEOS} \qquad (17)$$

The approximation can be made because of the inequality $\Delta \ll 1$ and the small value of the angle ϑ (which should be less than 0.1 rad [*NASA*, 1975]). With equations (1), (16) and (17) the description of the evolution of the spin axis is completed and the problem can be solved numerically.

[15] To complete this section on the spin axis model, Figure 1 illustrates the temporal behavior of the various torques acting on LAGEOS-I. It is clearly visible that three different characteristic periods can be distinguished: a first one where the magnetic torque is dominant, a second one where the order of magnitude of the magnetic and gravitational torques is comparable, and a third one

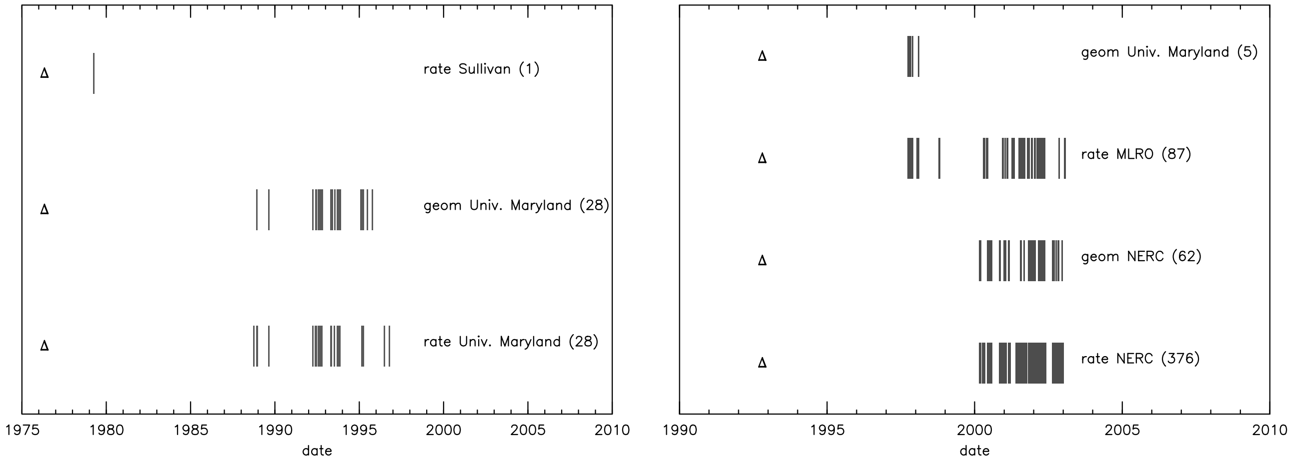

Figure 2. LAGEOS observations from the sources mentioned: *Sullivan* [1980] and Maryland for (left) LAGEOS-I and Maryland, NERC and MLRO (Matera Laser Ranging Observatory) for (right) LAGEOS-II. The triangles represent the launch dates.

where all torques have the same order of magnitude. It is not until this third period that the center-of pressure and reflective anisotropy come into play.

3. Observations

[16] The independent observations form a crucial element of this study. Measurements of the spin axis orientation and/or rate have been taken by *Sullivan* [1980], the Astro-Metrology Group from the Department of Physics, University of Maryland, USA; the NERC Space Geodesy Facility at Herstmonceux, UK; and the Centro di Geodesia Spaziale 'G. Colombo' in Matera, Italy.

[17] All sources basically use the same frequency analysis technique in order to measure the spin rate of the satellite. The technique is explained in detail in *Currie et al.* [1997], *Avizonis* [1997], *Bianco et al.* [2001] and *Otsubo et al.* [2004]. In essence, the light of a certain source (Sun, laser beam) is reflected by the Corner Cube Retroreflectors (CCRs) of the satellite, which are oriented in rows. By making a frequency analysis, the number of spikes can be assessed and an estimate of the spin rate is obtained. To get estimates for the orientation of the spin axis, the returns are combined with the geometry of the CCRs and that of the light source, satellite and observatory. Hence the orientation of the rotation axis can be identified. The temporal distribution of the observations is given in Figure 2.

3.1. LAGEOS-I

[18] For LAGEOS-I, direct orientation observations and spin rate measurements are only available from *Sullivan* [1980] with a single observation in 1979, and the Astro-Metrology Group. Direct observations of the spin axis orientation are given in *Currie* [1994], *Currie et al.* [1995], *Currie et al.* [1996] and *Rubincam et al.* [1997]. The data consist of values for the colatitude and longitude angles for a series of epochs, and cover the same time span, the years 1988 until 1995. The spin rate observations extend a little bit further, until 1997. Both type of observations have not been used in the development of any attitude model for LAGEOS-I until now.

3.2. LAGEOS-II

[19] Spin axis orientation data for LAGEOS-II have been reported in *Otsubo et al.* [2004]. These data are based on an analysis of sunlight reflections by the CCRs. In the same study, also spin rate observations are reported. *Bianco et al.* [2001] contains independent observations of the LAGEOS-II spin rate, based on a frequency analysis of SLR observations, the results being in perfect agreement with the spin rate observations reported in *Otsubo et al.* [2004]. These data sets have been expanded with observations taken throughout 2002. In summary, the available data cover a time span running from the end of 1998 until December 2002 (compare Figure 2). This is the first work combining model and observations for LAGEOS-II.

4. Analysis Strategy

[20] In the previous paragraphs, several remarks have been made to help the reader become aware of the elements that will be improved with respect to previous studies: model expansions, errors and typos, and observations. Among the most important ones is the treatment of the model parameters. The numerical values of some of them are known exactly from prelaunch tests, whereas others are "certain" within a range of values. Consequently, a number of parameters had to be adjusted in the development of the LOSSAM model. The general strategy of the development of LOSSAM will consist of confronting model equivalents with actual observations: the spin axis behavior following from a numerical integration of Euler's equation, for various combinations of parameters, will be confronted with the independent observations on spin rate and orientation. The (RMS) difference between the observations and their model equivalents will be used to judge the quality of the specific parameter combination. The parameters that were considered for estimation are as follows.

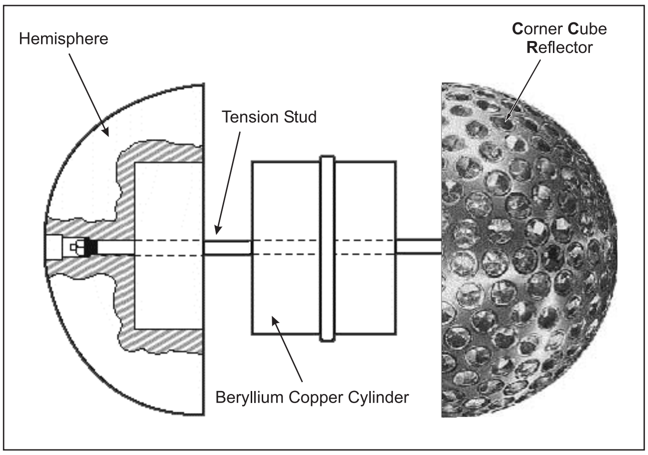

Figure 3. Structure of the LAGEOS satellites [*Cohen and Smith*, 1990].

[21] The magnetization parameters $\beta'(0)$, $\beta''(0)$ and $\beta'''(0)$ quantify how much the magnetic properties differ from that of a perfect sphere. Since the satellite is constructed from two hemispheres of the aluminum alloy UNI 6061T6, a central beryllium copper cylinder, and a tension stud of the same material (see Figure 3), the approximation as a perfectly homogeneous aluminum sphere is not very realistic. Thus deviations must be absorbed by the magnetization parameters. Values for these parameters were obtained by Bertotti and Iess from a best fit result with spin rate observations from several sources.

[22] The initial orientation of the spin axis (θ_0, λ_0) at the moment of apogee kick has an implicit uncertainty in the values derived from prelaunch and postlaunch data, which could be caused by the existence of an angle ϑ between the axis of rotation and the axis of symmetry. For LAGEOS-I the initial orientation values first obtained in *Rubincam* [1990] were modified, not only with the aforementioned spatial inversion, but also with the results of the best-fitting process of *Bertotti and Iess* [1991]. A similar best-fitting process was followed by *Farinella et al.* [1996] for LAGEOS-I and LAGEOS-II, modifying the initial conditions to reduce the residuals w.r.t. the observed accelerations.

[23] The initial rotational period T_0 was also estimated in the aforementioned process which yielded the initial conditions. The launch information mentioned a period of 0.6 s [*Ordahl*, 1975] and 1.0 s [*Anselmo and Pardini*, 1992], for LAGEOS-I and LAGEOS-II, respectively.

[24] The tilt angle at launch ϑ_0 was first included in *Farinella et al.* [1996]. It has a value of less than 0.1 rad [*NASA*, 1975], but its exact value is unknown (for both satellites).

[25] The center of pressure offset h is expected to be within the range $[-0.79, 0.79]$ mm and $[-0.1, 0.1]$ mm for LAGEOS I and II, respectively [*NASA*, 1975; *Johnson et al.*, 1976; *Minott et al.*, 1993]. The value of this parameter directly scales the offset torque and its influence on the spin axis evolution. As has been shown in section 2, this torque is very small if not negligible during the first two phases of the LAGEOS mission. Therefore, this torque scaling factor, will only be considered for LAGEOS-I, and will be ignored altogether for LAGEOS-II.

[26] The effective difference in reflectivity between the northern and the southern hemisphere $\Delta\rho$ is another parameter directly related to the additional torques included here. Based on its orbital behavior, a value close to 0.015 has been obtained for LAGEOS-I (i.e., 1.5% of difference) [*Scharroo et al.*, 1991], but which still allows a percentage of uncertainty. Therefore $\Delta\rho = 0.015 \pm 0.002$ for LAGEOS-I. As for LAGEOS-II, recognizing the symmetrical distribution of the germanium retroreflectors, only the existence of a misalignment of the retroreflector, together with an asymmetrical distribution of temperatures due to $T \sim \tau$ (rotational period in the order of magnitude of the thermal response time), would cause such torque. For LAGEOS-II, presently, $T/\tau \sim 1/40$ and therefore, it is ignored.

[27] A gravitational scaling parameter due to uncertainties in the value of the moments of inertia, or the relation between them, is also considered. Should all parameters appearing in equation (11) be perfectly known, there would be no need for a gravitational scaling parameter. However, uncertainties arise from literature; therefore a scaling of the gravitational torque is introduced by means of a general parameter k_g which multiplies the gravitational component of the torques.

[28] In Table 1, the various parameters are summarized. The nominal values come from a range of different sources. The LOSSAM entries represent the search space for the parameters. The criterium to judge the quality of a model as described by the mathematical formulation (section 2) and a specific combination of parameters is the agreement between the actual observations and their model equivalents, expressed in the usual form of an rms-of-fit. This RMS will be weighted for LAGEOS-I and unweighted for LAGEOS-II, since no statistical information is available for the geometry observations for the second version of the satellite.

[29] The strategy to be followed for both LAGEOS-I and LAGEOS-II is identical, although differences in some physical properties will have to be allowed. Thus in addition to obvious differences in initial conditions, the possibility of

Table 1. Numerical Values for the Nominal (i.e., A Priori, or Expected) Values for the Parameters in LOSSAM as Well as the Corresponding Search Space[a]

	LAGEOS-I		LAGEOS-II	
Cases	Nominal	LOSSAM	Nominal	LOSSAM
$\beta'(0)$, -	7.10^{-3}	$[10^{-4}, 10^{-1}]$	7.10^{-3}	$[10^{-4}, 0.05]$
$\beta''(0)$, -	0.213	[0.18, 0.30]	0.213	[0.18, 0.30]
$\beta'''(0)$, -	$<10^{-4}$	$[10^{-6}, 10^{-1}]$	$<10^{-4}$	$[10^{-6}, 10^{-1}]$
θ_0, deg	158	[145, 180]	173.1	[165, 180]
λ_0, deg	133	[120, 145]	219.1	[210, 235]
T_0, sec	0.6	[0.40, 0.65]	1	[0.9, 1.1]
ϑ_0, deg	<5	[0, 6]	<5	[0, 6]
h, mm	$[-0.79, 0.79]$	$[-0.79, 0.79]$	$[-0.1, 0.1]$	na
$\Delta\rho$, -	0.015	[0.013, 0.017]	0	na
k_g, -	1	[0.8, 1.1]	1	[0.95, 1.06]

[a]For both satellites, the values for the magnetization parameters, the initial colatitude and longitude, the initial spin rate, the initial tilt angle, the center of pressure offset, the difference in effective reflectivity and the gravitational torque scaling parameter are given.

Table 2. Solutions of the Parameters Considered in LOSSAM for Both Satellites[a]

	LAGEOS-I			LAGEOS-II		
Parameters	Total	Geom. Only	Rate Only	Total	Geom. Only	Rate Only
$\beta'(0)$, -	$[10^{-4}, 10^{-2}]$	$[10^{-4}, 10^{-1}]$	$[10^{-4}, 10^{-2}]$	[0.05, 0.20]	[0.02, 0.05]	[0.05, 0.20]
$\beta''(0)$, -	0.227 ± 5.10^{-3}	0.228 ± 5.10^{-3}	0.227 ± 5.10^{-3}	0.239 ± 5.10^{-3}	0.240 ± 5.10^{-3}	0.239 ± 5.10^{-3}
$\beta'''(0)$, -	$[10^{-6}, 10^{-4}]$	$[10^{-6}, 10^{-2}]$	$[10^{-6}, 10^{-4}]$	$[10^{-5}, 10^{-3}]$	$[10^{-6}, 10^{-3}]$	$[10^{-5}, 10^{-3}]$
θ_0, deg	165 ± 1	160 ± 2.5	165 ± 1	177.5 ± 0.5	172 ± 1	177.5 ± 0.5
λ_0, deg	125 ± 5	125 ± 5	125 ± 5	225 ± 10	225 ± 10	225 ± 10
T_0, sec	$0.430 \pm 5 \times 10^{-3}$	$0.490 \pm 5 \times 10^{-3}$	$0.430 \pm 5 \times 10^{-3}$	$0.92 \pm 5 \times 10^{-3}$	$0.98 \pm 5 \times 10^{-3}$	$0.92 \pm 5 \times 10^{-3}$
ϑ_0, deg	[0, 6]	3 ± 3	3 ± 3	0	0	0
h, mm	0 ± 0.79	0.79	0 ± 0.79	na	na	na
$\Delta\rho$, -	[0.013, 0.017]	[0.013, 0.017]	[0.013, 0.017]	na	na	na
k_g, -	$0.89 \pm 5 \times 10^{-3}$	$0.89 \pm 5 \times 10^{-3}$	$0.89 \pm 5 \times 10^{-3}$	$1.000 \pm 5 \times 10^{-4}$	$1.000 \pm 5 \times 10^{-4}$	$1.000 \pm 5 \times 10^{-4}$
RMS	30 deg and 0.53 s	8 deg	0.9 s	26 deg and 0.46 s	0.58 deg	0.46 s
WRMS	0.52	0.11	0.73	na	na	na

[a]The best solutions appearing here refer to the RMS for the cases: total, geometry only and rate only in an unweighted (RMS) or weighted (WRMS) form.

differences in magnetization parameters will be left open, contrary to *Farinella et al.* [1996].

5. Discussion and Results

[30] This section will present the results obtained for both LAGEOS-I and LAGEOS-II, for the evolution of the spin rate and the spin axis orientation. First of all a computation with all possible combinations of parameters was done for each satellite, yielding the satellite-specific set of parameters that gives the best agreement with the (geometric and spin rate) observations as presented in section 3. This typically resulted in an average rms-of-fit of 10 deg and 1 s, respectively. As can be expected, an even better agreement between observations and model equivalents can be achieved if the spin axis orientation and rotation period observations are treated separately (with a possible disagreement between the parameter solutions). This course will be taken in this section. The (more precise) outcome of such computations may be necessary for follow-up calculations, e.g., on the thermal forces. A demonstration of its accuracy is include in *Lucchesi et al.* [2004], which concludes that an improvement of about a 50% can be obtained when analyzing the real component of the eccentricity vector excitations of LAGEOS-II, adopting LOSSAM for the modeling of the Yarkovsky-Schach effect.

[31] This section is organized as follows: first, the results obtained for LAGEOS-I will be presented, for the combined geometry and spin rate case as well as for the cases addressing the individual measurement types. In the next section, a similar thing will be done for LAGEOS-II.

5.1. LAGEOS-I

[32] The optimization procedure for LAGEOS-I was aimed at getting a minimum value for the weighted rms-of-fit of the observations of "physical truth" w.r.t. their model equivalents. In doing so, the full covariance matrix of the geometric observations was used. This was necessitated by the nature of the observations themselves, which have a small and nearly circular error ellipse in some cases, while in some others there is a real uncertainty in one of the directions and therefore the orientation of the error ellipse should be considered. In the early days of the mission the rotation of the vehicle was fast enough to provide reliable and easily obtained solutions for the spin axis orientation. As for the data on the spin periods, the nominal uncertainties were used.

[33] The solutions for the parameters estimated for LAGEOS-I are summarized in Table 2. They are accompanied by a realistic uncertainty estimate, which for obvious reasons is not based on a formal standard deviation but which reflects the sensitivity of the fit values to changes in these parameters. When looking at the results obtained for the 'Total' case in column 1, Table 2 the total weighted RMS is of 0.52. The contributions to this value from the geometry observations is 0.15 (weighted RMS), which corresponds to about 10 deg, and 0.73 (weighted) for the spin period, which corresponds to 1 s.

[34] Table 2 also shows the results that have been obtained for the cases where only the orientation data have been used to obtain the model parameters (LAGEOS-I, column 2) or where the spin rate observations were used only (column 3). In the former case (which may be more relevant when actual thermal forces are computed) an rms-of-fit of 8 deg is obtained, whereas the case with spin rate observations results in a fit of 0.9 s. When expressed as a percentage of the instantaneous rotational period, this value corresponds with 2% and 7% for the initial and the most recent observations respectively. As an illustration, Figures 4 and 5 show the spin axis behavior in terms of geometry and spin period. The sensitivity of the rms-of-fit for (some of) the parameters is shown in Figure 6, with contour lines for the fit values when varying the initial colatitude and the initial longitude of the spin axis. All other parameters were kept fixed at the values shown in column 1 of Table 2, i.e., the best overall case. Figure 6 clearly shows that the region including the minimum is strongly driven by the colatitude, whereas the value of the longitude is rather irrelevant. The parameter solutions will be briefly discussed below.

[35] The spin axis behavior of LAGEOS-I appears to be sensitive to the magnetization parameters, more precisely to the exact value of $\beta''(0)$, as can be seen in Table 2: the solutions are very stable and well-determined, irrespective of the analysis procedure. The solutions deviate significantly w.r.t. the a priori value of 0.213 which was recovered by *Bertotti and Iess* [1991]. As for the other two parameters, the solutions are less sensitive: instead of accurately pinpointing one particular value, a range of options is possible here (Table 2).

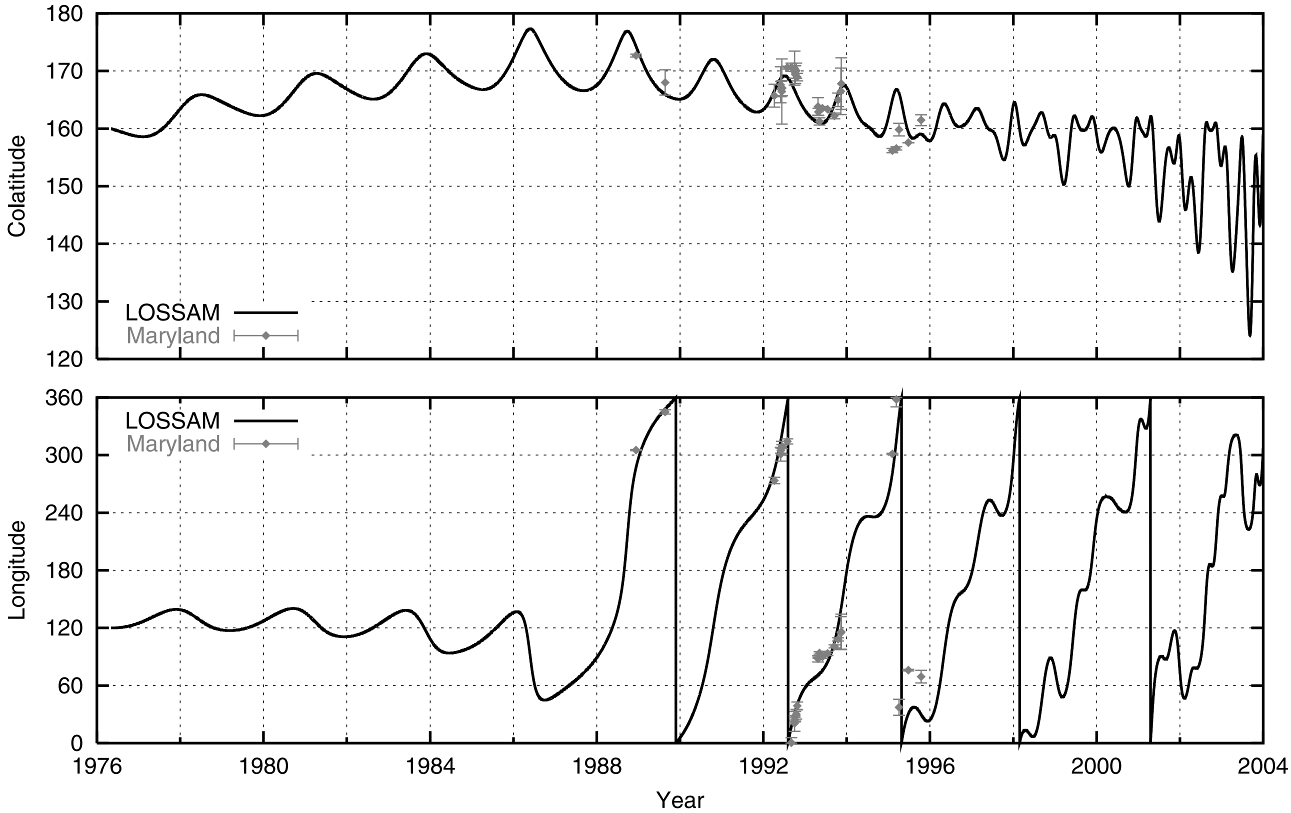

Figure 4. The spin axis orientation for LAGEOS-I in terms of longitude and colatitude of a unit vector in the direction of the spin axis, confronted with the whole set of observations. The time span is from launch until 2004, thus giving a prediction. The colatitude and longitude are measured w.r.t. the J2000 inertial reference frame.

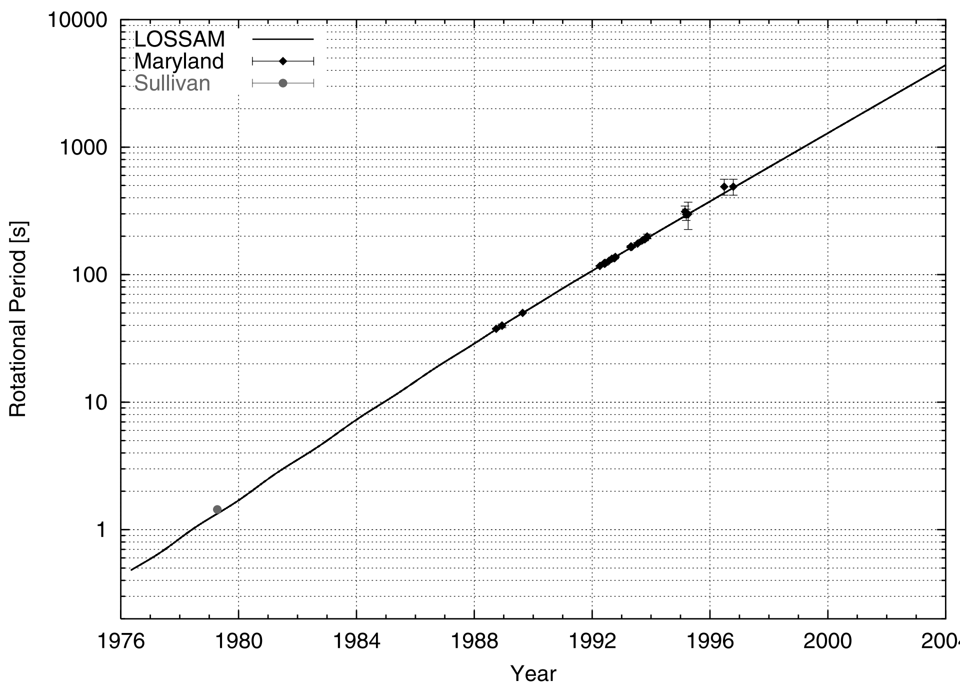

Figure 5. The rotational period for LAGEOS-I. The time span is from launch until 2004, thus giving a prediction. A logarithmic scale has been used to show the approximately linear (i.e., exponential) trend.

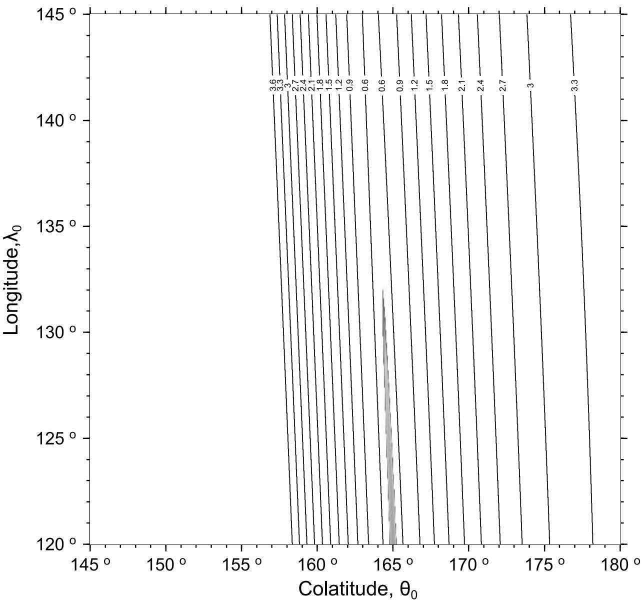

Figure 6. Total RMS value for LAGEOS-I as a function of the initial colatitude θ_0 and longitude λ_0. Other parameters are fixed at the values included in Column 1, Table 2. The grey zone indicates the location of the minimum RMS zone.

[36] Variations in the initial colatitude reasonably affect the spin axis behavior, meaning that the actual value could be established well (compare Figure 6). The solution (165 deg) is in fair agreement with the values reported in literature: 158 deg [*Bertotti and Iess*, 1991] or 155 deg [*Farinella et al.*, 1996].

[37] The initial longitude is more difficult to determine exactly. For one since the pattern of spin orientation observations easily allows a small rotation around the $\mathcal{Z}$ axis (i.e., the axis of symmetry), but also because the first geometric observations are not available until 1988, or 12 years after the launch of the spacecraft. As a result, the uncertainty of this parameter is estimated at a relatively large 5 deg (compare Figure 6). In spite of this inherent weakness, the recovered value (125 deg) has a reasonable agreement with other solutions ([*Bertotti and Iess*, 1991]: 104 deg; [*Farinella et al.*, 1996]: 133.5 deg).

[38] The initial spin period proved to have a strong link to the behavior of the spin period over time, as can be expected, and hence could be established with good confidence. The result 0.43 ± 0.005 s (compare Table 2) shows a difference of 20% w.r.t. previous values: 0.55 s [*Bertotti and Iess*, 1991]. Again, it must be emphasized that the initial epoch is 12 years away from the start of observations (when the single observation by [*Sullivan*, 1980] is ignored). Table 2 clearly shows that the spin rate observations have a major influence on the outcome for the initial spin period: when the rate observations are ignored (Table 2, column 2) the solution for T_0 changes by an amount of about 10 times the claimed accuracy.

[39] As for the initial tilt angle, this is another parameter which proved very difficult to establish exactly: making variations between 0 and 6 degrees hardly had any measurable effect. As a consequence, solutions slightly differ when considering values in between. For the situation when spin geometry observations are ignored (Table 2, column 3), the tilt angle is kept fixed at a zero value because the reduced observational database lacks relevant information.

[40] In the case of a possible center-of-pressure offset, it has to be realized that its effect is very small, and only comes into play in the third part of the mission (compare Figure 1). The results obtained when rate observations were available did not reveal any unique value, whereas the computations with geometry data only favored a positive value (meaning that the center of pressure is located above the center of mass). However the uncertainty of the solutions still leaves a reasonable range of values.

[41] The difference in reflectivity effect is caused by the reflective properties of the germanium CCRs [*Lucchesi*, 2004]. On the basis of an interpretation of the empirical along-track accelerations [*Scharroo et al.*, 1991] and uncertainty consideration, a range between 0.013 and 0.017 was tested in the computations. It turned out that the results obtained with any of these values were insensitive to the exact value used for the time span considered. It is for this reason that the LOSSAM solution basically reflects the input values (Table 2).

[42] A contradiction exists concerning the value of the oblateness, thus necessitating a gravitational torque scaling parameter. According to *Johnson et al.* [1976] the oblateness has a value of $\Delta = 0.035$, however [*Scharroo et al.*, 1991] concludes that a smaller value should be taken, based on geometrical analysis of the structure of LAGEOS-I. The solution that has been recovered in this work ($k_g = 0.89$) points into that direction. The value that can be recovered for the flattening of the satellite is thus: $\Delta = 0.0298$.

[43] Table 2 shows that the agreement between the real observations and the model equivalents can be improved if one particular data type is emphasized, for instance by fitting to the spin orientation observation only. This of course will be accompanied by a degradation in the "fit" of the other data type.

[44] Discomforting as this may seem initially, the parameter estimates show that this is not necessarily the case (Table 2). Individual parameter solutions may differ (w.r.t. each other and w.r.t. the common solution, reported in Table 2), but the differences are typically within the uncertainty estimates that accompany the solutions. However, it does suggest that there is still room for improvement of the overall model, since the three-dimensional satellite spin axis (i.e., length and orientation) of course follows just one path through "physical truth", and the results of the integration of Euler's equation should predict both aspects of the spin behavior at the same time. It is suspected that the averaging of some of the model elements is one of the most likely elements where improvements can be found: the magnetic torque, in particular, is known to show significant variations over one single orbital revolution, with accompanying effects on the spin axis behavior; each observation of course represents a mere snap-shot during flight. This aspect will be investigated in a follow-up of the current study.

5.2. LAGEOS-II

[45] In the case of LAGEOS-II, the process has been identical but without taking into account the torques due to center-of-figure offset and reflection anisotropy, because of

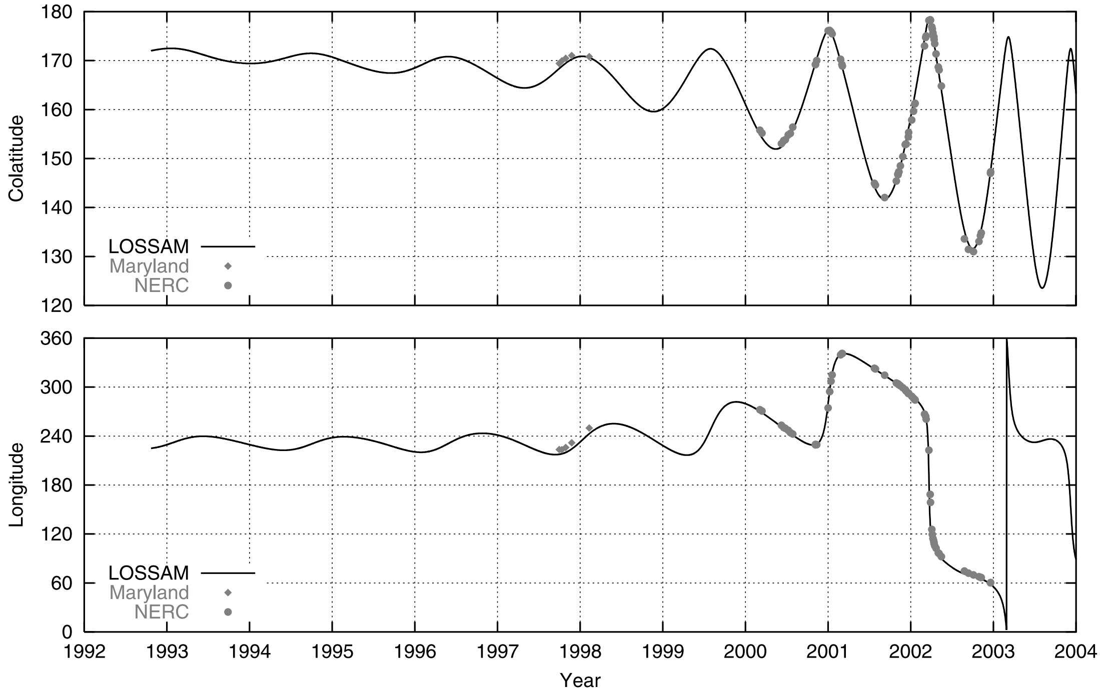

Figure 7. The spin axis orientation for LAGEOS-II in terms of longitude and colatitude of a unit vector in the direction of the spin axis, confronted with the whole set of observations. The time span is from launch until 2004, thus giving a prediction. The colatitude and longitude are measured w.r.t. the J2000 inertial reference frame.

the phase that this satellite is undergoing: for LAGEOS-II, the magnetic and gravitational torques are still predominant, causing the results (the rms-of-fit values) to be practically independent of the values of the offset h and the difference in reflectivity $\Delta\rho$.

[46] The best results obtained for LAGEOS-II, when best-fitting the geometric or spin period observations, are summarized in Table 2: 0.01 rad for the geometry only case and 0.46 s for the rate only case, which when expressed as a percentage of the instantaneous rotational period, corresponds with 5% and 1% for the initial and the most recent observations respectively. The resulting trends are plotted in Figures 7 and 8. Again, the parameter solutions are accompanied by realistic error estimates. The discussion will follow that of LAGEOS-I.

[47] The solutions for the magnetization parameters are given in Table 2. Although the satellite should be identical to LAGEOS-I, a deviation in some parameters can be expected. Because of the different orbital inclination and different value of the magnetic field strength, LAGEOS-II is facing a different regime for the $\beta^{(n)}(0)$ parameters. As was the case for LAGEOS-I, the $\beta''(0)$ parameter solution appears more stable (though different from the LAGEOS-I solutions). This applies to $\beta'(0)$ parameter as well, although very consistent over the various scenarios. The $\beta'''(0)$ parameter is difficult to assess exactly (Table 2).

[48] As with LAGEOS-I, the solution for the initial colatitude parameter turns out to be quite stable: 177.5 deg with an uncertainty of less than a degree. The solution is reasonably consistent with information available in literature: the launch report specifies a value of 173.1 deg, and *Farinella et al.* [1996] obtains a value of 169 ± 2.6 deg.

[49] In the case of the initial longitude, we are faced with a somewhat smaller sensitivity to the exact value of this second parameter for the initial orientation. Clearly, the objection of having observations available long after launch does not apply here, but the (to a certain extent) arbitrary orientation around the $\mathcal{Z}$ axis (compare Figure 7) still holds. The solution of 225 deg has an uncertainty of 10 deg. Literature provides a value of 219.1 deg from postlaunch data [*Robbins et al.*, 1994] and 260 ± 15.3 deg [*Farinella et al.*, 1996].

[50] Again, the initial spin period is strongly related to the individual spin period observations, in particular since these come relatively shortly after launch. The recovered value (0.92 s compare Table 2) compares favorably with the value reported in literature: 1.00 s [*Anselmo and Pardini*, 1992] and 0.98 s; [*Bianco et al.*, 2001].

[51] The initial tilt angle proved difficult to estimate. The best results show a common value of 0 deg.

[52] As for the gravitational torque scaling parameter, we recover a unit value, which is in perfect agreement with the satellite's specifications [*Minott et al.*, 1993].

[53] As it happened in the case of LAGEOS-I, a number of parameter solutions obtained for the various cases (total, geometry only and period only) show a good consistency (Table 2). Others however, appear to be quite sensitive to the data type considered. As for LAGEOS-II this hold for initial colatitude and initial spin period in particular.

6. Conclusions and Recommendations

[54] The life of the LAGEOS satellites can be divided into a number of different phases. Each phase is character-

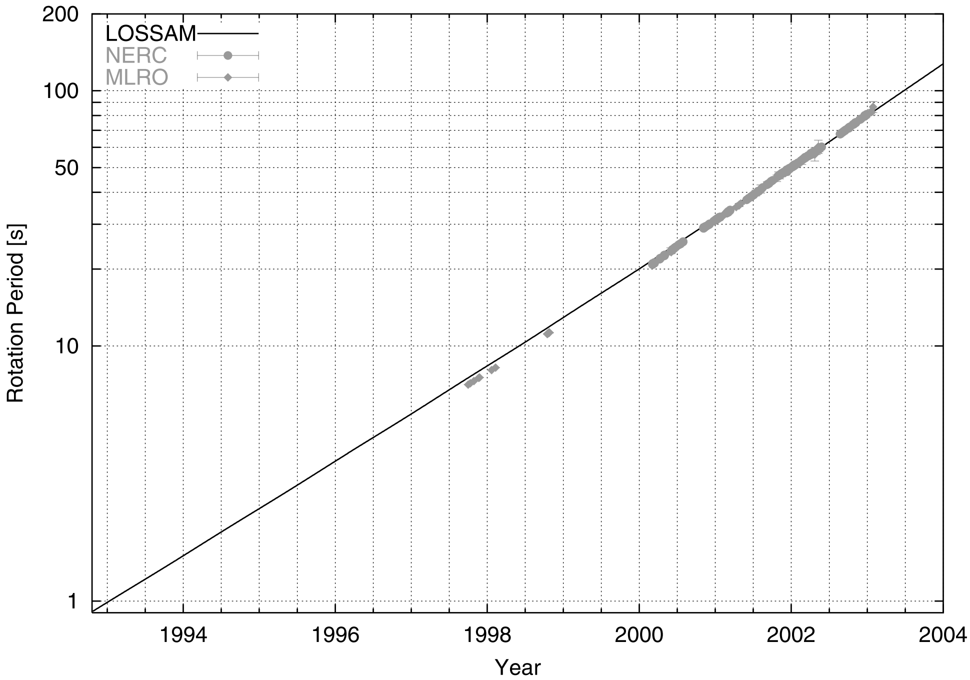

Figure 8. The rotational period solution for LAGEOS-II. The time span is from launch until 2004, thus giving a prediction. A logarithmic scale has been used to show the approximately linear (i.e., exponential) trend.

ized by representative values for the torques that act on the spacecraft, which in particular comes true for LAGEOS-I: a first period in which the torque due to the Earth's magnetic field is predominant, a second one in which torques due to the Earth's magnetic and gravitational field are comparable, and a third one in which all torques have the same order of magnitude. In this paper the full spectrum of torques and life cycle phases has been addressed.

[55] On the basis of independent observations on spin axis geometry and spin axis period, a new model LOSSAM has been developed which describes the rotational behavior of the two LAGEOS satellites with unprecedented accuracy: 9 deg and 0.6 deg in geometry and 1 s and 0.46 s in spin period for LAGEOS-I and LAGEOS-II respectively. To improve the quality of LOSSAM, two alternative computations have been performed that aimed at best-fitting the geometry and spin period observations, independently. The results, also included in this paper, may be used for subsequent investigations that will focus on the computation of thermal forces. Furthermore, tests have been made to assess the correction of the expression of the magnetic torque, causing an improvement of the total fit of about 20%.

[56] Most of the results, in terms of (initial) parameters, show a good internal consistency and a good agreement with results reported by previous investigators. An exception appears to be the initial colatitude and initial spin period for LAGEOS-II, which illustrates a clear conflict between the geometric and spin period components of the LOSSAM model. The current study will be continued with a more refined model of the various elements that play a role in the rotational dynamics behavior of the two satellites.

[57] The mass properties of LAGEOS-I prove to be a difficult issue. The present work suggests that the value of the oblateness of LAGEOS-I is smaller than values previously mentioned in literature, which corroborates the study reported in *Scharroo et al.* [1991].

[58] The differences in the parameter solutions observed between the three cases considered (i.e., total, geometry only and rate only) suggest that the dynamic model that is used to propagate the initial conditions is not perfect: depending on the type of observations (spin period, geometry) certain components or directions of the torques in the model appear to play a predominant role w.r.t. others (compare *Bertotti and Iess* [1991]). This is possibly related to the low-frequency approximation for the magnetization parameters that has been used in the mathematical development of the model. Another issue that has to be solved, concerns the resonance condition to which LAGEOS-I is approaching, the theory should be revisited as to account for it.

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