Naman Taggar

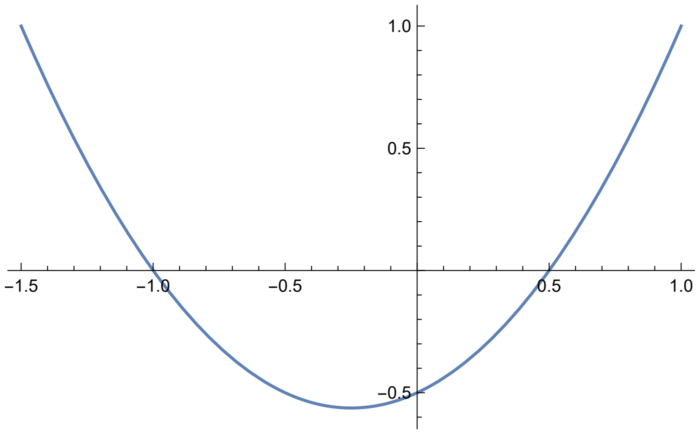
Day 2 @ Numerical Optimisation Workshop

1. Bisection Method

In[ ]:=
```
bisectionMethod[func_, var_, l_List, n_ : 10, eps_ : 0.0000005] := Module[
  {xmin, xmax, f, fv, mid},
  Catch[
   xmin = l[[1]]; xmax = l[[2]];
   f[fv_] := func /. var → fv;
   For[i = 1, i ≤ n, i++,
    mid = (xmin + xmax) / 2;
    If[f[mid] == 0, Throw[{i, N[mid, 10], N[f[mid], 10]}],
     If[f[xmin] × f[mid] < 0, xmax = mid, xmin = mid]
    ]
   ];
   Throw[{n, N[mid, 10], N[f[mid], 10]}]
  ]
 ]
```

In[ ]:=
```
f[x_] := (x - 0.5) (x + 1);
Plot[f[x], {x, -1.5, 1}]
bisectionMethod[f[x], x, {0.3, 0.6}, 10]
```

Out[ ]=



Out[ ]= {10, 0.500098, 0.000146494}

So, at the tenth iteration, x = 0.500098 and f(x) = 0.000146494.

2. Gradient Descent Method

In[•]:=
```
gDMethod[func_, var_, init_, iterations_ : 10, learningRate_ : 0.1] := Module[
  {f, ivar, grad, new, old},
  Catch[
   old = init;
   f[ivar_] := func /. var → ivar;
   grad[ivar_] := f'[ivar];
   For[i = 1, i ≤ iterations, i++,
    new = old - learningRate * grad[old];
    old = new
   ];
   Throw[{iterations, new, f[new]}]
  ]
 ]
```

In[•]:=
```
gDMethod[x^2 - 6 x + 9, x, 2]
```

Out[•]=
```
{10, 2.89263, 0.0115292}
```

3. Newton Method

In[•]:=
```
newtonMethod[expr_, var_, l_List, n_ : 10, eps_ : 0.0000005] := Module[
  {f, fv, xc, init, deriv},
  Catch[
   init = (l[[1]] + l[[2]]) / 2;
   f[fv_] := expr /. var → fv;
   deriv = f'[var];
   For[i = 1, i ≤ n, i++,
    xc = init - f[init] / (deriv /. var → init);
    If[Abs[f[xc]] ≤ eps, Throw[{i, N[xc, 10], N[f[xc], 10]}]];
    init = xc
   ];
   Throw[{n, N[xc, 10], N[f[xc], 10]}]
  ]
 ]
```

In[•]:=
```
newtonMethod[Cos[x] - x Exp[x], x, {0, 1}, 20]
```

Out[•]=
$\{2, 0.5177574241, -1.837841910 \times 10^{-7}\}$

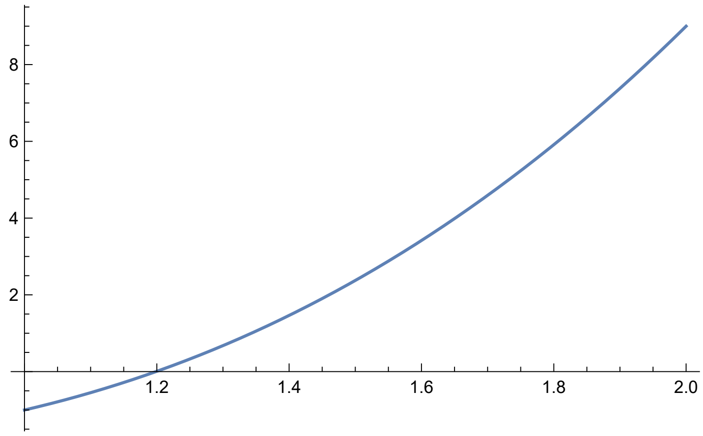
4. Secant Method

In[∘]:=
```
secantMethod[expr_, var_, l_List, n_ : 10, eps_ : 0.0000005] := Module[
  {f, fv, init, p1, p2},
  {p1, p2} = {l⟦1⟧, l⟦2⟧};
  Catch[
   init = (p1 + p2) / 2;
   f[fv_] := expr /. var → fv;
   For[i = 1, i ≤ n, i++,
    p2 = p1 - f[p1] (p1 - init) / (f[p1] - f[init]);
    If[Abs[f[p2]] ≤ eps, Throw[{i, N[p2, 10], N[f[p2], 10]}]];
    {init, p1} = {p1, p2};
   ];
   Throw[{n, N[p2, 10], N[f[p2], 10]}]
  ]
 ]
```

In[∘]:=
```
secantMethod[x^3 + 2 x^2 - 3 x - 1, x, {1, 2}]
```

Out[∘]= $\{5, 1.198691246, 1.660120746 \times 10^{-8}\}$

In[∘]:=
```
Plot[x^3 + 2 x^2 - 3 x - 1, {x, 1, 2}]
```

Out[∘]=


Perfect!

5. Gauss-Jacobi Method

```
(*My code*)
gsMat1 = {{8, 3, 4, 15}, {-2, 5, -2, 1}, {1, 1, 3, 5}}; (*Augmented matrix*)
```

```
In[ ]:= gJMethod[A_List, maxiters_ : 10] := Module[
          {n, tuple},
          n = Dimensions[A][[1]];
          Catch[
           If[Dimensions[A][[1]] + 1 == Dimensions[A][[2]],
            Nothing[], Throw["Invalid matrix supplied."]];
           tuple = Table[0, {i, n}];
           For[i = 1, i ≤ maxiters, i++,
            For[ni = 1, ni ≤ n, ni++,
              tuple[[ni]] = N[1 / A[[ni]][[ni]] (A[[ni, n + 1]] -
                    Sum[A[[ni, r]] × tuple[[r]], {r, Complement[Table[1, n], {ni}]}]), 10];
             ];
           ];
           Throw[{i - 1, tuple}]
          ]
         ]

In[ ]:= gJMethod[gsMat1]

Out[ ]= {10, {1.875000000, 0.9500000000, 1.041666667}}
```

6. Gauss-Seidal Method

```
(*Sir's Code*)

gSMethod[a0_, b0_, x0_, e0_, m0_] := Module[{}, A = a0;
  B = b0;
  X = x0;
  n = Length[X];
  e = N[e0];
  m = N[m0];
  k = 0;
  X1 = X;
  While[k < m, For[i = 1, i ≤ n, i++,
    X1[[i]] = N[(1 / A[[i, i]]) * (B[[i]] - Sum[A[[i, j]] * X1[[j]], {j, 1, i - 1}] -
          Sum[A[[i, j]] * X[[j]], {j, i + 1, n}])];];
   If[Norm[X1 - X] < e, Break[]];
   X = X1;]; k++ ;
  Print[NumberForm[X1, 10]]]
A = {{5, 1, 2}, {-3, 9, 4}, {1, 2, -7}};
B = {10, -14, -33};
X = {0, 0, 0};
gSMethod[A, B, X, 10^(-5), 25]

{0.9999989607, -3.000000895, 3.999999596}
```

SOR Method

```
(*Sir's Code*)
```

In[ ]:=
```
sORMethod[a0_, b0_, x0_, w0_, e0_, m0_] := Module[{}, A = a0;
  B = b0;
  X = x0;
  w = N[w0];
  e = N[e0];
  m = N[m0];
  n = Length[X];
  k = 0;
  X1 = X;
  While[k < m,
   For[i = 1, i ≤ n, i++, X1[[i]] = N[(1 - w) * X[[i]] + w * (1 / A[[i, i]]) * (B[[i]] - Sum[
          A[[i, j]] * X1[[j]], {j, 1, i - 1}] - Sum[A[[i, j]] * X[[j]], {j, i + 1, n}])];];
   If[Norm[X1 - X] < e, Break[]];
   X = X1;]; k++ ;
  Print[NumberForm[X1, 10]];]
A = {{5, 1, 2}, {-3, 9, 4}, {1, 2, -7}};
B = {10, -14, -33};
X = {0, 0, 0};
sORMethod[A, B, X, 0.9, 10^(-5), 25]
```

```
{0.9999992301, -2.999999652, 3.99999992}
```

End of file namantaggar_NAWorkshop_day2practical.pdf