Homework 4

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October 10, 2024

Code for this assignment is at: `https://github.com/zpm-bu/cs511-formal-methods/blob/assignments/lean/Homework/hw5.lean`

Exercise 1a. Prove the validity of the following sequent:

$$(y \approx 0) \land (y \approx x) \vdash 0 \approx x$$

1	$(y \approx 0) \land (y \approx x)$	premise
2	$y \approx 0$	$\land \text{e}_1$ 1
3	$y \approx x$	$\land \text{e}_2$ 1
4	$0 \approx x$	$\approx$e 2, 3 ($\varphi := y \approx x; t_1 := y; t_2 := 0$)

Exercise 1b. Prove the validity of the following sequent:

$$t_1 = t_2 \vdash (t + t_2) = (t + t_1)$$

I'm definitely not sure about this one. Can we introduce an arbitrary symbol like this?

	1	$t_1 \approx t_2$	premise
$t + t_2$	2	let	
	3	$t + t_2 \approx t + t_2$	$\approx$i 2
	4	$t + t_1 \approx t + t_2$	$\approx$e $1, 3$ $(\varphi := t + t_2 \approx t + t_2; t_1 := t_2; t_2 : t_1)$

Exercise 2a. Prove the validity of the following sequent:

$$(\exists x(S \to Q(x)) \vdash S \to \exists x Q(x)$$

	1	$\exists x(S \to Q(x))$	premise
	2	S	assume
x_0	3	let	
	4	$S \to Q(x_0)$	assume
	5	$Q(x_0)$	$\to$e 2, 4
	6	$\exists x(Q(x))$	$\exists$i 5
	7	$\exists x(Q(x))$	$\exists$e 1, 3 : 6
	8	$S \to \exists x(Q(x))$	$\to$i 2 : 7

Exercise 2d. Prove the validity of the following sequent:

$$\forall x(P(x)) \to S \vdash \exists x(P(x) \to S)$$

	1	$\forall x(P(x)) \to S$	premise
	2	$\neg\exists x(P(x) \to S)$	assume
x_0	3	let	
	4	$\neg P(x_0)$	assume
	5	$P(x_0)$	assume
	6	$\bot$	$\bot$i 4, 5
	7	S	$\bot$e
	8	$P(x_0) \to S$	$\to$i 5 : 7
	9	$\exists x(P(x) \to S)$	$\exists$i 8
	10	$\bot$	$\bot$i 2, 8
	11	$\neg\neg P(x_0)$	$\neg$i 4 : 10
	12	$P(x_0)$	$\neg\neg$e
	13	$\forall x(P(x))$	$\forall$i 3 : 12 ($\varphi := P(x_0)$)
	14	$P(x_0)$	$\forall$e 13
	15	S	$\to$e 1, 13
	16	$P(x_0) \to S$	$\to$i 14 : 15
	17	$\exists x(P(x) \to S)$	$\exists$i 16
	18	$\bot$	$\bot$i 2 : 17
	19	$\neg\neg\exists x(P(x) \to S)$	$\neg$i 2 : 18
	20	$\exists x(P(x) \to S)$	$\neg\neg$e 19