PROBLEMS FOR INTRO TO DATABASES AND INTRO TO CATEGORY THEORY

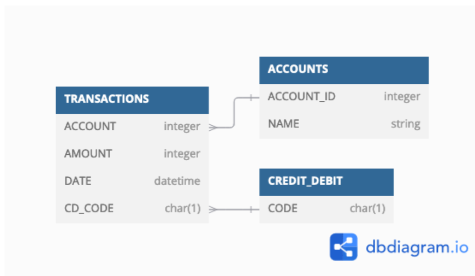
Exercise 1. Some exercises related to database methods.

Part (a) The SQL standard provides an operation `EXISTS`, which can be used as an existential quantifier. For example,

```
SELECT ... FROM ... WHERE EXISTS (<subquery>)
```

So to express a database, we certainly need at least first-order logic. Argue as to whether or not *second*-order logic or higher is needed for any SQL operations you are familiar with. Is first-order logic sufficient for all SQL operations?

Part (b) Consider the following schema:



Draw a diagram representing this schema, using the diagram language from the lecture. Use filled circles for table vertices and empty circles for type vertices.

Exercise 2. In 1763, Leonhard Euler created a very famous graph representing the islands of the Pregel River and the seven bridges across it. (To understand the very simple question he wanted to solve which motivated one of the first problems answered by graph theory, you may read the Wikipedia article on "The Seven Bridges of Königsberg.") Here is the graph K which he drew, with some arrows added:

$$\begin{tikzcd}
& B \arrow[dr, "k"] \arrow[dl, bend left, "g"] & \\
A \arrow[ur, bend left, "f"] \arrow[rr, "h"] \arrow[dr, bend left, "i"] & & C \\
& D \arrow[ur, "\ell"] \arrow[ul, bend left, "j"] &
\end{tikzcd}$$

Let $\mathcal{K}$ be the free category on K.

Part (a) List the 15 morphisms of $\mathcal{K}$ along with their domains and codomains.

Part (b) A diagram is called a "commutative" diagram if all paths with the same start and end point are equal; that is, if all "parallel" paths through the diagram produce the same result. Let $\mathcal{K}'$ be the *commutative* free category on K; list the morphisms of $\mathcal{K}'$. Using the answers of part (a) should make this a trivial exercise.

Part (c) Consider the following category $\mathcal{V}$:

$$U \xrightarrow{p} V \xleftarrow{q} W$$

Define a functor $F : \mathcal{V} \to \mathcal{K}$.

Pard (d) Imagine a category that looks like this:

$$\begin{tikzcd}
\bullet \arrow[r] & \bullet \arrow[d] \\
\bullet \arrow[u] & \bullet \arrow[l]
\end{tikzcd}$$

Argue why there cannot be a functor from this category to $\mathcal{K}$.