Homework 7

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Code for this assignment is at: `https://github.com/zpm-bu/cs511-formal-methods/blob/assignments/lean/Homework/hw7.lean`

Exercise 1. Use first-order ground tableaux to show that $\Gamma \vDash \varphi$ where

$$\Gamma \overset{\text{def}}{=} \{\forall x \forall y \forall z \, (P(x,y) \wedge P(y,z) \to P(x,z)), \forall x \forall y \, (P(x,y) \to P(y,x))\}$$

and
$$\varphi \overset{\text{def}}{=} \forall x \forall y \forall z \, (P(x,y) \wedge P(z,y) \to P(x,z))$$

and P is an arbitrary binary predicate.

$$\Gamma \cup \{\varphi\}$$
$$|$$
$$\forall x \forall y \forall z (P(x,y) \wedge P(y,z) \to P(x,z))$$
$$|$$
$$\forall y \forall z (P(t,y) \wedge P(y,z) \to P(t,z))$$
$$|$$
$$\forall z (P(t,u) \wedge P(u,z) \to P(t,z))$$
$$|$$
$$P(t,u) \wedge P(u,v) \to P(t,v)$$
$$|$$
$$\neg \forall x \forall y \forall z (P(x,y) \wedge P(z,y) \to P(x,z))$$
$$|$$
$$\neg \forall y \forall z (P(t,y) \wedge P(z,y) \to P(t,z))$$
$$|$$
$$\neg \forall z P(t,u) \wedge P(z,u) \to P(t,z)$$
$$|$$
$$\neg (P(t,u) \wedge P(v,u) \to P(t,v))$$
$$|$$
$$P(t,u) \wedge P(v,u)$$
$$|$$
$$\neg P(t,v)$$
$$|$$
$$P(t,u)$$
$$|$$
$$P(v,u)$$
$$|$$
$$\forall x \forall y P(x,y) \to P(y,x)$$
$$|$$
$$\forall y P(v,y) \to P(y,v)$$
$$|$$
$$P(v,u) \to P(u,v)$$

$$\diagup \qquad \diagdown$$

$$\neg (P(t,u) \wedge P(u,v)) \qquad P(t,v)\times$$

$$\diagup \qquad \diagdown$$

$$\neg P(t,u)\times \qquad \neg P(u,v)$$

$$\diagup \qquad \diagdown$$

$$\neg P(v,u)\times \qquad P(u,v)\times$$

Exercise 2. Use first-order ground tableaux to show...

$$\Gamma \cup \{\varphi\}$$
|
$$\forall x Q(x, a, a)$$
|
$$Q(a, s^2(a), s^2(a))$$
|
$$\forall x \forall y \forall z (Q(x, y, z) \to Q(x, s(y), s(z)))$$
|
$$\forall x \forall y \forall z (Q(x, y, z) \to Q(y, x, z))$$
|
$$\neg \exists x Q(s^2(a), s^3(a), x)$$
|
$$\neg Q(s^2(a), s^3(a), s^5(a))$$
|
$$\forall y \forall z (Q(a, y, z) \to Q(y, a, z))$$
|
$$\forall z (Q(a, s^2(a), z) \to Q(s^2(a), a, z))$$
|
$$Q(a, s^2(a), s^2(a)) \to Q(s^2(a), a, s^2(a))$$
|
$$\forall y \forall z Q(s^2(a), y, z) \to Q(s^2(a), s(y), s(z))$$
|
$$\forall z Q(s^2(a), a, z) \to q(s^2(a), s(a), s(z))$$
|
$$Q(s^2(a), a, s^2(a)) \to Q(s^2(a), s(a), s^3(a))$$
|
$$Q(s^2(a), s(a), s^3(a)) \to Q(s^2(a), s^2(a), s^4(a))$$
|
$$Q(s^2(a), s^2(a), s^4(a)) \to Q(s^2(a), s^3(a), s^5(a))$$

$$\neg Q(a, s^2(a), s^2(a)) \times \qquad Q(s^2(a), a, s^2(a))$$

$$\neg Q(s^2(a), a, s^2(a)) \times \qquad Q(s^2(a), s(a), s^3(a))$$

$$\neg Q(s^2(a), s(a), s^3(a)) \times \qquad Q(s^2(a), s^2(a), s^4(a))$$

$$\neg Q(s^2(a), s^2(a), s^4(a)) \times \qquad Q(s^2(a), s^3(a), s^5(a)) \times$$