Program Transformation and Analysis

Assignment 3

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1 Introduction

This the fourth of five weekly assignment in the course Program Transformation and Analysis (PAT) at Copenhagen University. The course professor is Robert Glück. The course is held in block 4, 2019.

In this assignment the focus is on partial evaluation.

2 Partial Evaluation

Partial evaluation is a method for program optimization by specialization[2]. To specialize an interpreter with respect to a given program and input. The specialization is to execute the program as far as possible with the known variables (static/dynamic) and ending with a new residual program [1, p. 69].

An example of partial evaluation is given by William and Ralf in their paper 'Tutorial on Online Partial Evaluation'[3]. Let

$$\textbf{if } x > y \textbf{ then } (10 + y)/x \textbf{ else } y \tag{1}$$

be an expression with two dynamic variables in a simple functional language. If y was to be known as the value 0 the expression could be specialized to the following residual expression[3, p. 2]:

$$\textbf{if } x > 0 \textbf{ then } 10/x \textbf{ else } 0 \tag{2}$$

3 Online Partial Evaluation

Partial evaluation in three steps:

- Collect all reachable states

  see which blocks of the program we need and what data will arrive at each block.

- Program point specialization

  for each reachable state (l,σ), create a version of l, that is specialized wrt σ.

- Transition compression

  remove trivial jumps (gotos)

  'Definition 4.5 Let pp be a label occuring in program p, and consider a jump goto pp. The replacement of this goto by a copy of the basic block labelled pp is transition compression.'[1, p. 81]

4 Assignment

Exercise 1

Show the main steps of specializing the Turing-machine interpreter with respect to the Turing-program into the target program. Use online or offline FCL-partial evaluator.

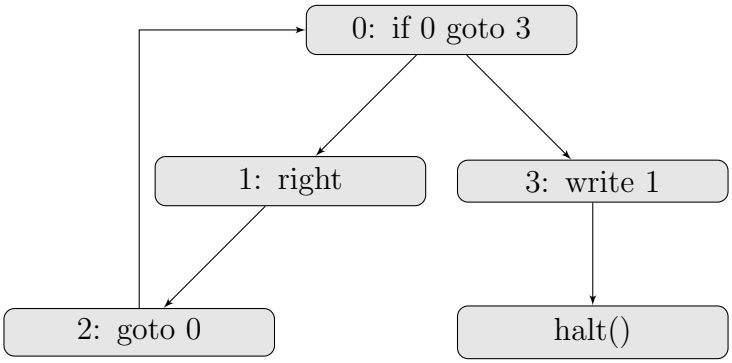
```
prg(String s):
        0: if 0 goto 3
        1: right
        2: goto 0
        3: write 1
```

We use the online FCL-partial evaluation method to specialize the program above. A description can be found in **section 5** where i show the process of the algorithm used on above program. **First we find all reachable states.**

Table over states.

Initial state	Successor states	
0	{1,3}	
1	{2}	
2	{0}	
3	{halt()}	

Above table shows which states are reachable from another. Ex. State 0, can reach 1 or 3. 1 can reach 2 and 2 can reach 0.



Second we specialize each block with respect to the program prg and the Turing-interpreter. The specialization is bluntly to replace dynamic values in the interpreter with static information known from program.

```
Original block:
  0: if 0 goto 3

Specialized wrt [input->S, symbol->0, nextlabel->3]:
  (0, [input->S, symbol->0, nextlabel->3]) :
        if 0 = firstsym(Right) goto 3 else 1;


Original block:
  1: right

Specialized wrt [input->S]:
  (1, [input->S]) :
        Left := cons(firstsym(Right), Left);
        Right:= tl(Right);
        Goto 2;


Original block:
  2: goto 0

Specialized wrt [input->S, nextlabel->0]:
  (2, [input->S, nextlabel->0]) :
        Goto 0;


Original block:
  3: write 1

Specialized wrt [input->S, symbol->1, nextlabel->stop]:
  (3, [input->S, symbol->1]) :
        Right:= cons(1, tl(Right));
        return right;
```

Ending with a specialized program as below:

(0, [input→ S, symbol→ 0, nextlabel→ 3]) :	if 0 = firstsym(Right) goto 3 else 1;
(1, [input→S]) :	Left := cons(firstsym(Right), Left); Right:= tl(Right); Goto 2;
(2, [input→S, nextlabel→0]) :	Goto 0;
(3, [input→S, symbol→1]) :	Right:= cons(1, tl(Right)); return right;

As the third and last step, we use transition compression on trivial jumps. Its shown in above specialization that the only purpose of instruction 2 is to move the instruction pointer to state 0, and can therefore be removed. So instead of letting state 2 follow up on state 1 we extend state 1 with the combination of state 1 and 0.

(0, [input→ S, symbol→ 0, nextlabel→ 3]) :	if 0 = firstsym(Right) goto 3 else 1;
(1, [input→S,symbol→0]) :	Left := cons(firstsym(Right), Left); Right:= tl(Right); if 0 = firstsym(Right) goto 3 else 1;
(3, [input→S, symbol→1]) :	Right:= cons(1, tl(Right)); return right;

The transformation, performs The First Futamura projection, which takes an S-interpreter $[int]$ written in a language L and s as a S-program and specializes the interpreter with respect to s and returns a residual target program.

$$\begin{aligned}[s] &= [int]_L[s] \\ &= [([mix]_L, [int, s])]_L \\ &= [target]_L\end{aligned}$$

As shown above, the specialized program is written the same language as the interpreter.

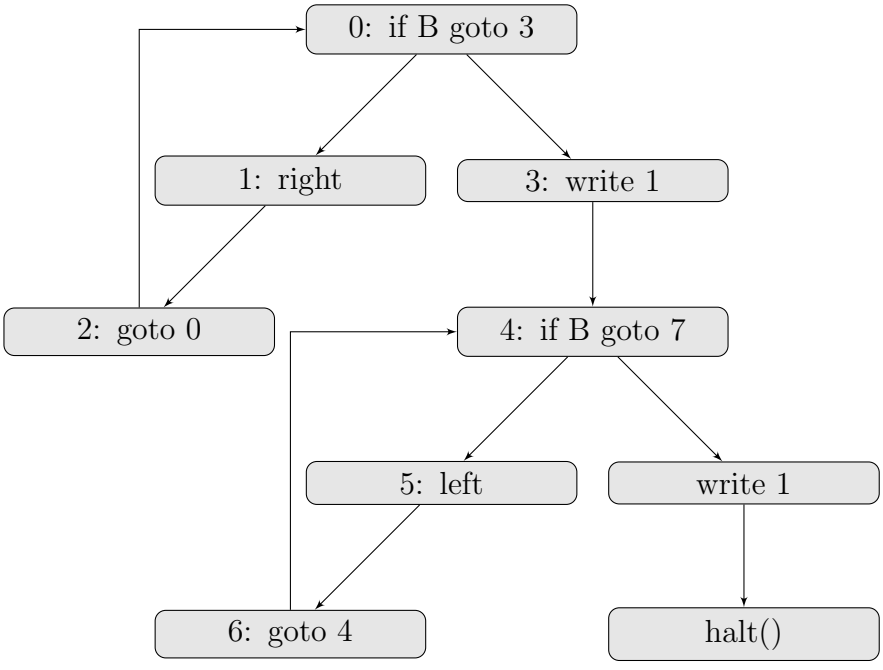
Exercise 2

Answer exercise 4.5[1]: Specialize the Turing interpreter with respect to the following program:

```
0: if B goto 3
1: right
2: goto 0
3: write 1
4: if B goto 7
5: left
6: goto 4
7: write 1
```

Find all reachable states.

	0 (goto loop/jump)		
1 (loop) 2 0 (goto loop/jump)	 5 (loop1) 6 4 (goto loop1/jump1)	3 (jump) 4 (goto loop1/jump1)	 7 (jump1) halt()



Second we specialize each block

```
Original block:
  0: if B goto 3

Specialized wrt [input->S, symbol->B, nextlabel->3]:
        if B = firstsym(Right) goto 3 else 1;


Original block:
  1: right

Specialized wrt [input->S]:
  (1, [input->S]) :
        Left := cons(firstsym(Right), Left);
        Right:= tl(Right);
        Goto 2;


Original block:
  2: goto 0

Specialized wrt [input->S, nextlabel->0]:
  (2, [input->S, nextlabel->0]) :
        new_tail(0, Q);
        Goto 0;


Original block:
  3: write 1

Specialized wrt [input->S, symbol->1, nextlabel->4]:
  (3, [input->S, symbol->1]) :
        Right:= cons(1, tl(Right));
        Goto 4
```

```
Original block:
  4: if B goto 7

Specialized wrt (4, [input->S, symbol->B, nextlabel->7]):
        if B = firstsym(Right) goto 7 else 5;


Original block:
  5: left

Specialized wrt [input->S]:
  (5, [input->S]) :
        Right := cons(firstsym(Right), Right);
        Left:= tl(Left);
        Goto 6;


Original block:
  6: goto 4

Specialized wrt [input->S, nextlabel->4]:
  (4, [input->S, nextlabel->4]) :
        Goto 4;


Original block:
  7: write 1

Specialized wrt [input->S, symbol->1, nextlabel->stop]:
  (7, [input->S, symbol->1]) :
        Right:= cons(1, tl(Right));
        return right;
```

Resulting in the following specialized program:

(0, [input→ S, symbol→ B, nextlabel→ 3]) :	if B = firstsym(Right) goto 3 else 1;
(1, [input→S]) :	Left := cons(firstsym(Right), Left); Right:= tl(Right); Goto 2;
(2, [input→S, nextlabel→0]) :	Goto 0;
(3, [input→S, symbol→1]) :	Right:= cons(1, tl(Right)); Goto 4;
(4, [input→ S, symbol→ B, nextlabel→ 7]) :	if B = firstsym(Right) goto 7 else 5;
(5, [input→S]) :	Right := cons(firstsym(Left), Right); Left:= tl(Left); Goto 6;
(6, [input→S, nextlabel→4]) :	Goto 4;
(7, [input→S, symbol→1]) :	Right:= cons(1, tl(Right)); return right;

Third we use transition compression to eliminate trivial jumps. Again state 2 can be replaced with the combination of state 1 + 0. And state 6 can be replaced by state 5 + 4. We see that state 3 and 4 can be merged as an initial state before the second loop. Which makes the specialization of state 1, 3 and 5 the following:

(0, [input→S, symbol→B, nextlabel→ 3]) :	if B = firstsym(Right) goto 3 else 1;
(1, [input→S, symbol→B]) :	Left := cons(firstsym(Right), Left); Right:= tl(Right); if B = firstsym(Right) goto 3 else 1;
(3, [input→S, symbol→{1,B}]) :	Right:= cons(1, tl(Right)); if B = firstsym(Right) goto 7 else 5;
(5, [input→S, symbol→B, nextlabel→ 7]) :	Right := cons(firstsym(Left), Right); Left:= tl(Left); if B = firstsym(Right) goto 7 else 5;
(7, [input→S, symbol→1]) :	Right:= cons(1, tl(Right)); return right;

Which produces the

Excercise 3

Answer exercise 4.7[1]: Will specialization of the Turing interpreter (Figure 4.4[1]) with respect to a program p terminate for all Turing programs p?

Well the process of specializing each block of a any given program p produced from the Turing interpreter will just replace some variables with static values, thus it must uphold the criteria, that if the program would terminate before it will after.

When we come to transition compression, i guess their could be events where the target program would not terminate. But that guess could be avoided be placing a rule on the compression. Lets say that goto's is residual if they don't stand alone. So only states where we jump directly with no other action involved is replaced. In that case we just move merge states which we otherwise would have reached anyway.

So my answer is, yes, any program p will terminate for all Turing program p if some restriction is set on the specialization.

5 Online PE ALgorithm

For exercise 1, we run the Online PE Algorithm on the given program. For this we need to know which states are reachable from another. A table over reachable states is first shown below. Second we need to set a symbol for a specialized block, in this case we'll mark it with bx, where x is replaced with the state-number. From here on we can run the algorithm.

Table over reachable states.

Initial state	Successor states	
0	{1,3}	
1	{2}	
2	{0}	
3	{halt()}	

Let below be a table over iterations of the while loop. And lets shortly describe the process:

Set P to the initial state (ex. 0).

- Set (l, σ) to one element in P.
- Remove (l, σ) from P
- if (l, σ) already in S, then continue to next element in P or return if P is empty.
- Mark it (put it in S)
- Specialize the block (replace block variables with known static values) and place it in R
- Set P to all reachable none-halt() states from (l, σ)
- loop over all the above bullets until P is empty.

While	(l, σ)	S	R	P_exit
first	(0, _)	{(0, _)}	{b0}	{(1, _), (3, _)}
second	(3, _)	{(0, _),(3, _)}	{b0, b3}	{(1, _)}
third	(1, _)	{(0, _),(3, _), (1,_)}	{b0, b3, b1}	{(2, _)}
fourth	(2, _)	{(0, _),(3, _), (1,_), (2,_)}	{b0, b3, b1, b2}	{(0, _)}
fith	(0, _)	{(0, _),(3, _), (1,_), (2,_)}	{b0, b3, b1, b2}	{}

References

[1] Jones N.D., Gomard C.K., Sestoft P., *Partial Evaluation and Automatic Program Generation.* Prentice Hall 1993. `https://www.itu.dk/people/sestoft/pebook/pebook.html` (Links to an external site.)

[2] `https://en.wikipedia.org/wiki/Partial_evaluation`

[3] William Cook, Ralf Lämmel, *Tutorial on Online Partial Evaluation.* `www.cs.utexas.edu/~wcook/tutorial/PEnotes.pdf`