

Program Transformation and Analysis

Assignment 3

Benjamin Brandt Ohrt, zpn492

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1 Introduction

This is the third of five weekly assignments in the course Program Transformation and Analysis (PAT) at Copenhagen University. The course professor is Robert Glück. The course is held in block 4, 2019.

In this assignment the focus is on the elimination of intermediate lists and trees and The Deforestation Algorithm of Philip Wadler.

1.1 Intermediate lists

"Intermediate lists has a cost at run time. If strict evaluation is used, the program requires space proportional to n ." "Even under lazy evaluation each list element requires time to be allocated, to be examined, and to be de-allocated." [1, p. 231]

Philip Wadler introduces a set of rules, which can eliminate the intermediate lists. Thus improving efficiency, because operations on list cells have been eliminated. Those rules can be found in fig. 4, 'Transition rules for the Deforestation Algorithm' [1, p. 238].

1.2 Language

Let the language below be one, where the Philip Wadler's rules can be used. [1, p. 233]

t	$::= v$	variable
	$::= c\ t_1 \dots t_k$	constructor application
	$::= f\ t_1 \dots t_k$	function application
	$::= \text{case } t_0 \text{ of } p_1 : t_1 \mid \dots \mid p_n : t_n$	case term
p	$::= c\ v_1 \dots v_k$	pattern

1.3 The Deforestation algorithm

The Deforestation algorithm is simply to apply the rules named in section 1.1 on possible terms of 1.2 [1, p. 236]

2 Assignment

Exercise 1

Show the transformation of function composition `append (append xs ys) zs` into the treeless program in Fig. 3 using the Deforestation Algorithm defined in Fig. 4. Present your transformation in the style of Fig. 5. Explain the steps and discuss shortly in what sense the composition is optimized. [1]

Let `append` be a function described in section 3

Rule	T[[append (append xs ys) zs]]	
	where f xs ys zs	
(6)	f Nil ys zs	= T[[append (append Nil ys) zs]]
(5)		= T[[append ys zs]]
		= append ys zs
(6)	f (Cons x xs) ys zs	= T[[append (append (Cons x xs) ys) zs]]
(5)		= T[[append ((Cons x) (append xs ys)) zs]]
(5)		= T[[append (append xs ys) (Cons x zs)]]
		= f xs ys (Cons x zs)
	f xs ys zs	append (append xs ys) zs transforms to:
		= case xs of
		Nil : append ys zs
		Cons x xs : Cons x (f xs ys zs)

Exercise 2

Apply the algorithm described in exercise 1 to `append xs (append ys zs)`. What is the difference to the result in exercise 1?

For the transformation of `append xs (append ys zs)` i use the Deforestation algorithm described in [1].

Rule	$T[\text{append } xs \text{ (append } ys \text{ zs)}]$		
(3)	$T[\text{case } xs \text{ of}$	Nil	$= (\text{append } ys \text{ zs})$
		$Cons \ x \ xs$	$= cons \ x \ (\text{append } xs \ (\text{append } ys \ zs))]$
(4)	$\text{case } xs \text{ of}$	Nil	$= T[\text{append } ys \ zs]$
		$Cons \ x \ xs$	$= T[Cons \ x \ (\text{append } xs \ (\text{append } ys \ zs))]$
(2)	$\text{case } xs \text{ of}$	Nil	$= T[\text{append } ys \ zs]$
		$Cons \ x \ xs$	$= Cons \ x \ T[(\text{append } xs \ (\text{append } ys \ zs))]$
(dec)		$h1 \ ys \ zs$	$= T[\text{append } ys \ zs]$
(dec)		$h0 \ xs \ ys \ zs$	$= T[\text{append } xs \ (h1 \ ys \ zs)]$
(sub)	$\text{case } xs \text{ of}$	Nil	$= h1 \ ys \ zs$
		$Cons \ x \ xs$	$= Cons \ x \ (h0 \ xs \ ys \ zs)$
$\text{append } xs \ (\text{append } ys \ zs) \text{ transforms to:}$			
	$h0 \ xs \ ys \ zs =$		
	$\text{case } xs \text{ of}$	Nil	$= (h1 \ ys \ zs)$
		$Cons \ x \ xs$	$= Cons \ x \ (h0 \ xs \ ys \ zs)$
	$h1 \ ys \ zs =$		
		$\text{case } ys \text{ of}$	
		Nil	$= zs$
		$Cons \ y \ ys$	$= Cons \ y \ (h1 \ ys \ zs)$

Exercise 3

Discuss the presentation structure of the article used by the author. What is the role of formalization and examples, what about the readability, and the like. In your opinion, can it be reason why the article is widely read and used?

I think the formalization and examples shows that these rules of elimination of intermediate lists and trees will work in general. Though is should be gvin that the functional programming language upholds the criteria of linear terms and tree less form [1].

For me it was a hard read. Well the introduction to intermediate lists and eliminating them was great and with somewhat good examples. But understanding the simple algorithms of flip and append took some time. Only because of knowledge of how they should work, helped understanding the general expressions of Cons and Branch. Another obstacle was my lack of understanding some of the transformation rules. For me it could have been nice with simple examples for each rule. That said the article both had explanation and full examples so i guess its my talents which is insufficient.

Again the reason why this article is widely read and used, is probably the use of a general language, which my guess is, it can fit in all functional languages.

3 Append

```
Append      : list a -> list a -> list a
Append xs ys =
  case xs of
    Nil      : ys
    Cons x xs : Cons x (append xs ys)
```

References

- [1] Wadler P., *Deforestation: transforming programs to eliminate trees*. *Theoretical Computer Science*, 73(2): 231-248, 1990. [https://doi.org/10.1016/0304-3975\(90\)90147-A](https://doi.org/10.1016/0304-3975(90)90147-A)