T1 $\nabla_A tr(AB) = B^T$

from $Y_{ij} = \sum_m A_{i,m} B_{m,j}$ when $Y = AB$

$$tr(AB) = tr(Y)$$

$$= \sum_m A_{i,m} B_{m,i} \quad \because \; i = j$$

$$\nabla_A tr(AB) = \begin{bmatrix} \frac{\partial tr(AB)}{\partial A_{11}} & \frac{\partial tr(AB)}{\partial A_{12}} & \cdots & \frac{\partial tr(AB)}{\partial A_{1m}} \\ \vdots & \vdots & & \vdots \end{bmatrix}$$

$$= \begin{bmatrix} B_{11} & B_{21} & \cdots & B_{m1} \\ B_{12} & & \ddots & \\ \vdots & & & \end{bmatrix}$$

$\therefore$ พจน์ที่คูณกับ A_{ij} คือ B_{ji}

$$= B^T \quad \#$$

T2 $\nabla_{A^T} f(A) = (\nabla_A f(A))^T$

$\nabla_{A^T} f(A)$

$$= \left(\begin{bmatrix} \frac{\partial f(A)}{\partial a_{11}} & \frac{\partial f(A)}{\partial a_{21}} & \cdots & \frac{\partial f(A)}{\partial a_{n1}} \\ \frac{\partial f(A)}{\partial a_{12}} & \cdots & \cdots & \cdots \\ \vdots & & & \\ \frac{\partial f(A)}{\partial a_{1m}} & \text{---} & & \frac{\partial f(A)}{\partial a_{nm}} \end{bmatrix}^T \right)^T$$

$$= \boxed{\begin{bmatrix} \frac{\partial f(A)}{\partial a_{11}} & \frac{\partial f(A)}{\partial a_{12}} & \cdots & \frac{\partial f(A)}{\partial a_{1n}} \\ \frac{\partial f(A)}{\partial a_{21}} & \cdots & \cdots & \text{---} \\ \vdots & & & \\ \frac{\partial f(A)}{\partial a_{m1}} & \text{---} & & \frac{\partial f(A)}{\partial a_{mn}} \end{bmatrix}}^T \Rightarrow \nabla_A f(A)$$

$= (\nabla_A f(A))^T$

T3. $\nabla_A tr ABA^T C = CAB + C^T AB^T$

let $BA^TC = X$

$\nabla_A tr(ABA^TC) = \nabla_A tr(AX)$

$tr(AX) = \Sigma_m (A_{i,m} X_{i,m})$ $\rightarrow$ $(BA^T)C$

$\hookrightarrow (B_{j,n} A^T_{n,k})\ C$

$A_{i,m}\ (B_{j,n} A_{k,n})\ C$

ปล. ทำไม่ได้ครับ (T v T)