P342 15

We can prove the proposition by strong induction. Let $P(n)$ denotes that the first player can win if there are n cookies in the top row and n cookies in the leftmost column and it's the second player's turn to eat cookies, then the first player can win.

Basic step:

When n=1, the second player can only eat the poisonous cookie. $P(1)$ is true. This completes the basic step.

Inductive step:

Fix k>1, and we assume that $P(j)$ is true for any positive integer $j \le k$. Firstly, the first player can eats all the cookies except the ones in the top row and the leftmost column. Then if the second player eats the poisonous cookie, the first player wins.

If the second player eats all the cookies to the right of the i^{th} ($1 \le i \le k$) cookie in the top row, then the first player can eat all the cookies below the i^{th} cookie in the leftmost column. By the inductive hypothesis we can know that $P(i)$ is true and so $P(k+1)$ is true.

If the second player eats all the cookies below the i^{th}

($1 \le i \le k$) cookie in the leftmost column, then the first player can eat all the cookies to the right of the i^{th} cookie in the top row. By the inductive hypothesis we can know that $P(i)$ is true and so $P(k+1)$ is true.

This completes the inductive step.

To conclude, the first player can always has a winning strategy.

P344 35

We can prove the proposition by strong induction. Let $P(n)$ denotes that n-1 multiplications are used to compute the product of n numbers no matter how parentheses are inserted into their product.

Basic step:

When n=1, we need no multiplication to find the product of a single real number. $P(1)$ is true. This completes the basic step.

Inductive step:

Fix k>1 and we assume that $P(j)$ is true for any positive integer $j \le k$. $a_1 \cdot a_2 \cdot \ldots \cdot a_k \cdot a_{k+1}$ can be seen as the multiplication of $a_1 \cdot a_2 \cdot \ldots \cdot a_k$ and a_{k+1}. If we do not insert parentheses, then we use (k-1)+1=k multiplications and $P(k+1)$ is true.

If we insert parentheses and they does not include a_{k+1}, by the inductive hypothesis we can know that we use (k-1)+1=k multiplications and $P(k+1)$ is true. If we insert parentheses and they include a_{k+1}, we can assume that there are m ($1<m<k+1$) distinct numbers between the leftmost parenthesis and the second rightmost parenthesis. By the inductive hypothesis we can know that we use (m-1)+(k-m+1)=k multiplications and $P(k+1)$ is true.

This completes the inductive step.

To conclude, n-1 multiplications are used to compute the product of n numbers no matter how parentheses are inserted into their product.

P358 15

We can prove the proposition by mathematical induction. Let $P(n)$ denote that $f_0 f_1 + f_1 f_2 + \cdots + f_{2n-1} f_{2n} = f_{2n}^2$.

Basic step:

When *n*=1, $f_0f_1+f_1f_2=0+1=f_2^2$. *P*(1) is true. This completes the basic step.

Inductive step:

Assume that for any integer *k*>1, *P*(*k*) is true. Then we have:

$f_0f_1+f_1f_2+\cdots+f_{2k+1}f_{2k+2}$

$=f_{2k}^2+f_{2k}f_{2k+1}+f_{2k+1}f_{2k+2}$

$=f_{2k}^2+f_{2k}(f_{2k+2}-f_{2k})+f_{2k+2}(f_{2k+2}-f_{2k})$

$=f_{2k+2}^2$

So *P*(*k*+1) is true. This completes the inductive step.

To conclude, $f_0f_1+f_1f_2+\cdots+f_{2n-1}f_{2n}=f_{2n}^2$ is true.

P359 47

a) $P_{m,m}=P_m$ is true because *m* is expressed as the sum of positive numbers, so a number exceeding *m* cannot be in a partition of *m*.

b)

If *m*=1, 1 can only be expressed as 1=1 then $P_{1,n}=1$.

If *n*=1, *m* can only be expressed as *m*=1+1+…+1 then $P_{m,1}=1$.

If *m*<*n*, a number exceeding *m* cannot be in a partition of m so $P_{m,n} = P_{m,m}$.

If *m*=*n*>1, *m* can only be expressed as *m*=*m* when *m* is allowed in the partition of *m* so $P_{m,n} = 1 + P_{m,m-1}$.

If *m*>*n*>1, the partitions of *m* can be classified into partitions using numbers not exceeding *n* and partitions using *n*. The former are counted in $P_{m,n-1}$ and the latter is counted in $P_{m-n,n}$, so $P_{m,n} = P_{m,n-1} + P_{m-n,n}$ is true.

c)

P_5

$=P_{5,5}$

$=1 + P_{5,4}$

$=1 + P_{5,3} + P_{1,4}$

$=2 + P_{5,1} + P_{3,2} + P_{2,2}$

$=4 + P_{3,1} + P_{1,2} + P_{2,1}$

$=7$

P_6

$=P_{6,6}$

$=P_{6,5} + P_{1,6}$

$=P_{6,4} + P_{1,5} + 1$

$=5 + P_{6,1} + P_{4,2} + P_{3,2}$

$=5 + 1 + P_{4,1} + P_{2,2} + P_{3,1} + P_{1,2}$

$=11$

So $P_5 = 7, P_6 = 11.$