

3.3 Given the grammar

$exp \rightarrow exp \text{ addop } term \mid term$
 $addop \rightarrow + \mid -$
 $term \rightarrow term \text{ mulop } factor \mid factor$
 $mulop \rightarrow *$
 $factor \rightarrow (exp) \mid \textit{number}$

write down leftmost derivations, parse trees, and abstract syntax trees for the following expressions.

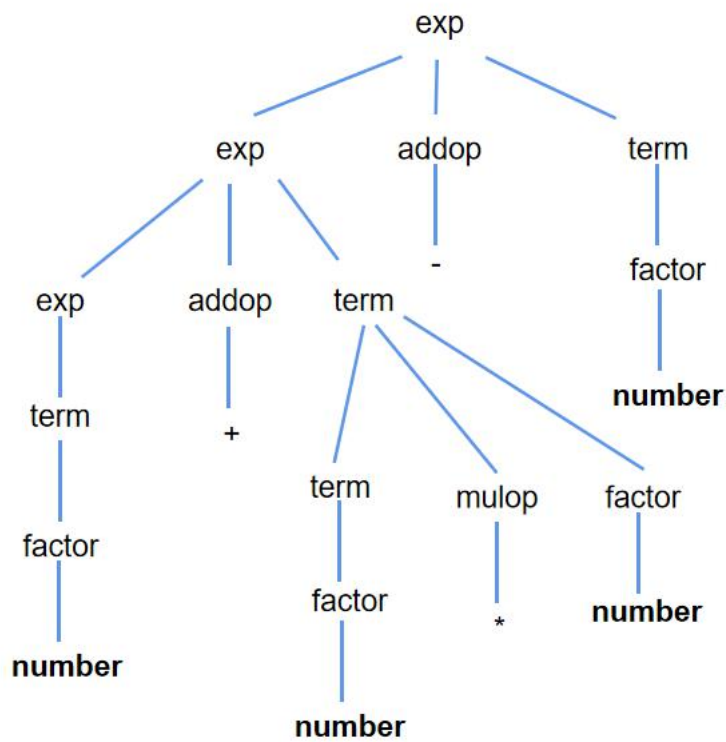
a) $3+4*5-6$ b) $3*(4-5+6)$ c) $3-(4+5*6)$

a)

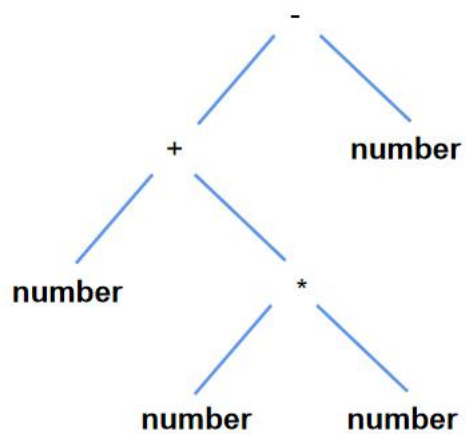
Leftmost derivations:

$exp \Rightarrow exp \text{ addop } term$
 $\Rightarrow exp \text{ addop } term \text{ addop } term$
 $\Rightarrow term \text{ addop } term \text{ addop } term$
 $\Rightarrow \textit{number} \text{ addop } term \text{ addop } term$
 $\Rightarrow \textit{number} + term \text{ addop } term$
 $\Rightarrow \textit{number} + term \text{ mulop } factor \text{ addop } term$
 $\Rightarrow \textit{number} + \textit{number} \text{ mulop } factor \text{ addop } term$
 $\Rightarrow \textit{number} + \textit{number} * factor \text{ addop } term$
 $\Rightarrow \textit{number} + \textit{number} * \textit{number} \text{ addop } term$
 $\Rightarrow \textit{number} + \textit{number} * \textit{number} + \textit{number}$
 $\Rightarrow \textit{number} + \textit{number} * \textit{number} + \textit{number}$

Parse trees:



AST:



b)

Leftmost derivations:

exp => *term*

=> *term mulop factor*

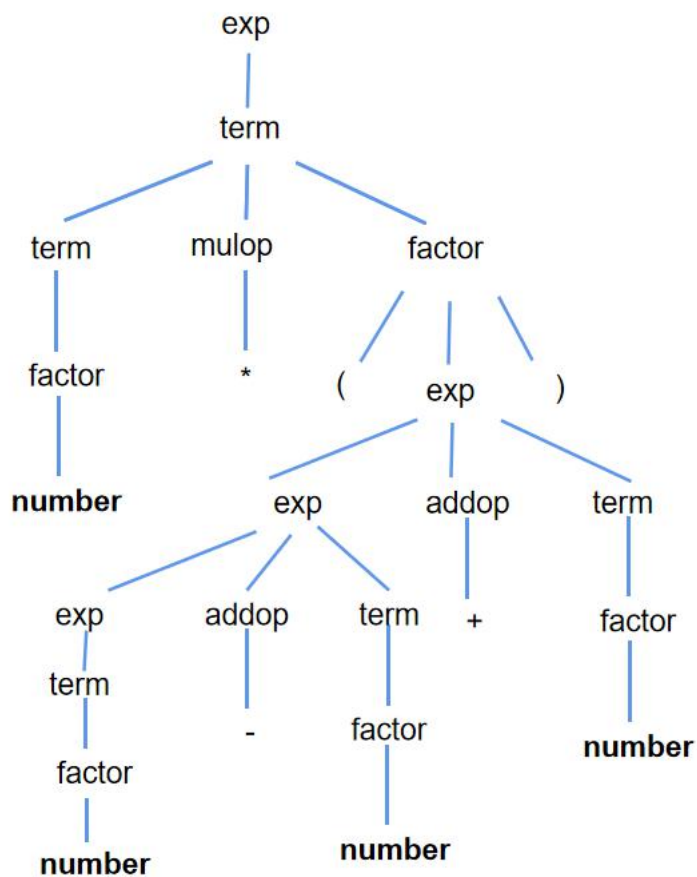
=> *factor mulop factor*

=> **number** *mulop factor*

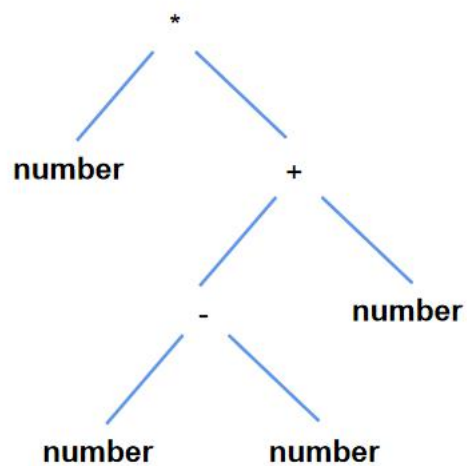
=> **number** * *factor*

=> **number** * (*exp*)
 => **number** * (*exp addop term*)
 => **number** * (*exp addop term addop term*)
 => **number** * (*term addop term addop term*)
 => **number** * (*factor addop term addop term*)
 => **number** * (**number** *addop term addop term*)
 => **number** * (**number** + *term addop term*)
 => **number** * (**number** + *factor addop term*)
 => **number** * (**number** + **number** *addop term*)
 => **number** * (**number** + **number** - *term*)
 => **number** * (**number** + **number** - *factor*)
 => **number** * (**number** + **number** - **number**)

Parse trees:



AST:



c)

Leftmost derivations:

exp => *exp addop term*

=> *term addop term*

=> *factor addop term*

=> **number** *addop term*

=> **number** - *term*

=> **number** - *factor*

=> **number**- (*exp*)

=> **number**- (*exp addop term*)

=> **number**- (*term addop term*)

=> **number**- (*factor addop term*)

=> **number**- (**number** *addop term*)

=> **number**- (**number** + *term*)

=> **number**- (**number** + *term mulop factor*)

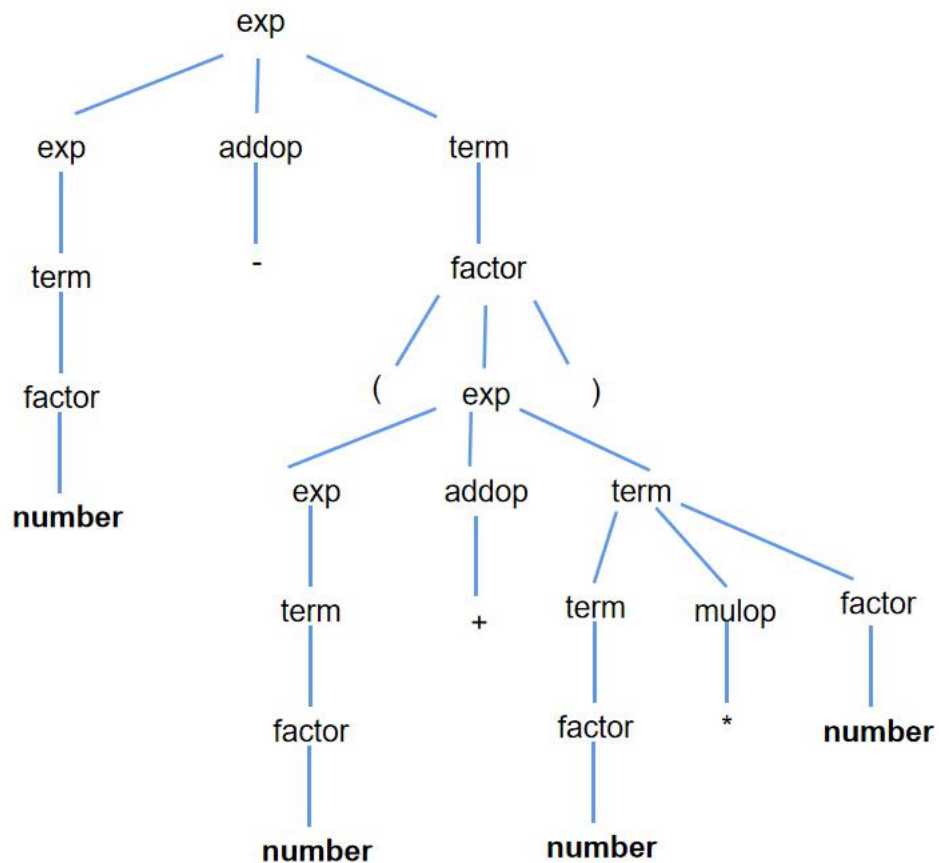
=> **number**- (**number** + *factor mulop factor*)

=> **number**- (**number** + **number** *mulop factor*)

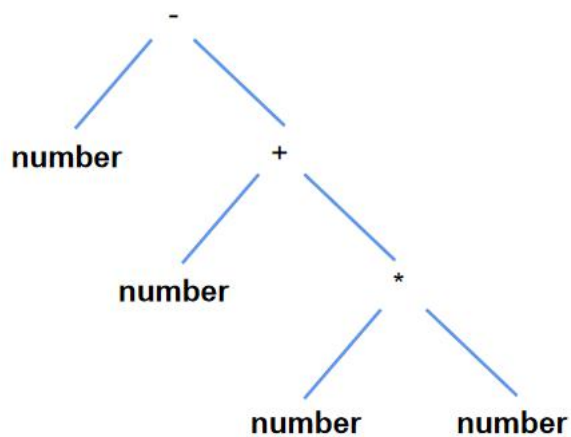
=> **number**- (**number** + **number** * *factor*)

=> **number**- (**number** + **number** * **number**)

Parse trees:



AST:



3.4 The following grammar generates all regular expressions over the alphabet of letters (we have used quotes to surround operators, since the vertical bar is an operator as well as a metasymbol) :

$$rex \rightarrow rex \text{ " | " } rex$$

$| \text{ rexp rexp}$
 $| \text{ rexp } "*"$
 $| "(" \text{ rexp } ")"$
 $| \text{ letter}$

- Give a derivation for the regular expression $(ab|b)^*$ using this grammar
- Show that this grammar is ambiguous
- Rewrite this grammar to establish the correct precedence for the operations
- What associativity does your answer in part (c) give to the binary operators? Why?

a)

$\text{rexp} \Rightarrow \text{rexp } "*"$
 $\Rightarrow "(" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ rexp } "|" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ rexp rexp } "|" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ letter rexp } "|" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ letter letter } "|" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ letter letter } "|" \text{ letter } ")" \text{ } "*"$

b)

Another way:

$\text{rexp} \Rightarrow \text{rexp } "*"$
 $\Rightarrow "(" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ rexp rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ letter rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ letter rexp } "|" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ letter letter } "|" \text{ rexp } ")" \text{ } "*"$
 $\Rightarrow "(" \text{ letter letter } "|" \text{ letter } ")" \text{ } "*"$

c)

$\text{rexp} \rightarrow \text{rexp } "|" \text{ rnon } | \text{ rnon}$
 $\text{rnon} \rightarrow \text{rnon rterm } | \text{ rterm}$
 $\text{rterm} \rightarrow \text{rterm } "*" \text{ } | \text{ item}$
 $\text{item} \rightarrow \text{letter } | "(" \text{ rexp } ")"$

d)

Left associativity.