5. Filters

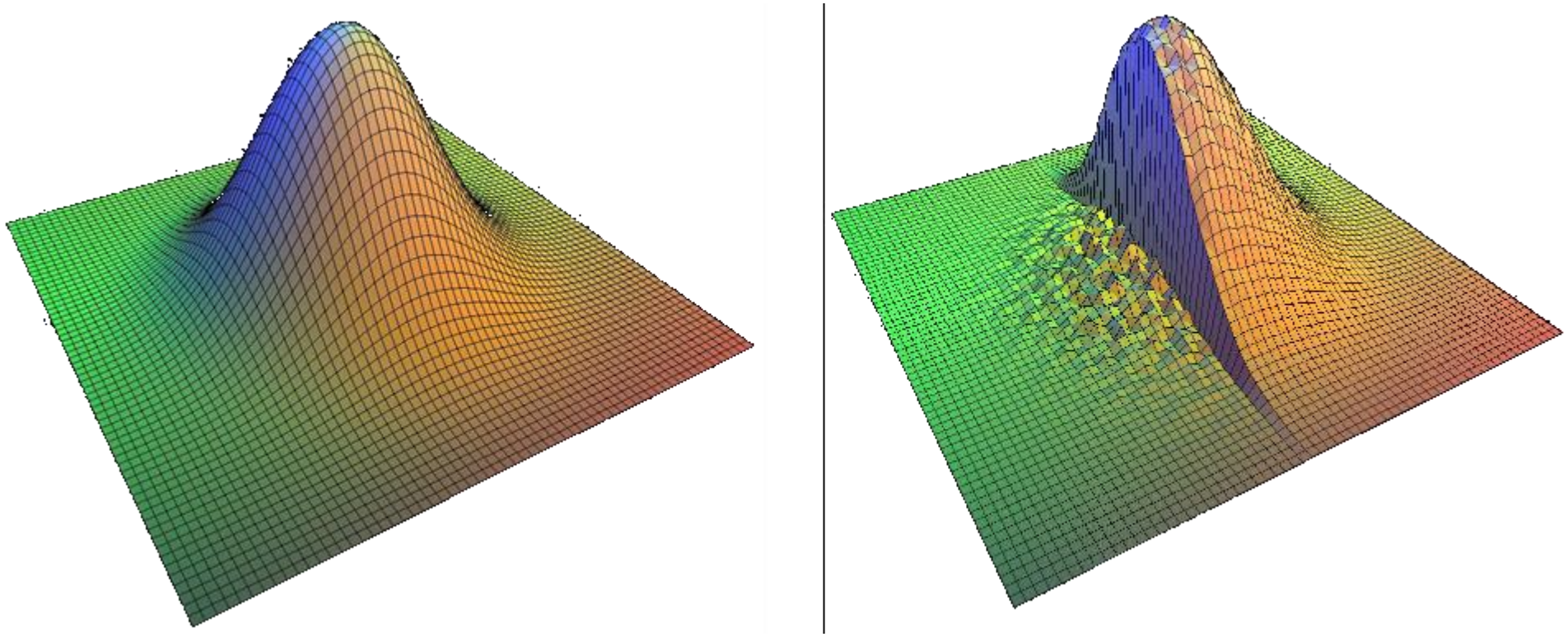
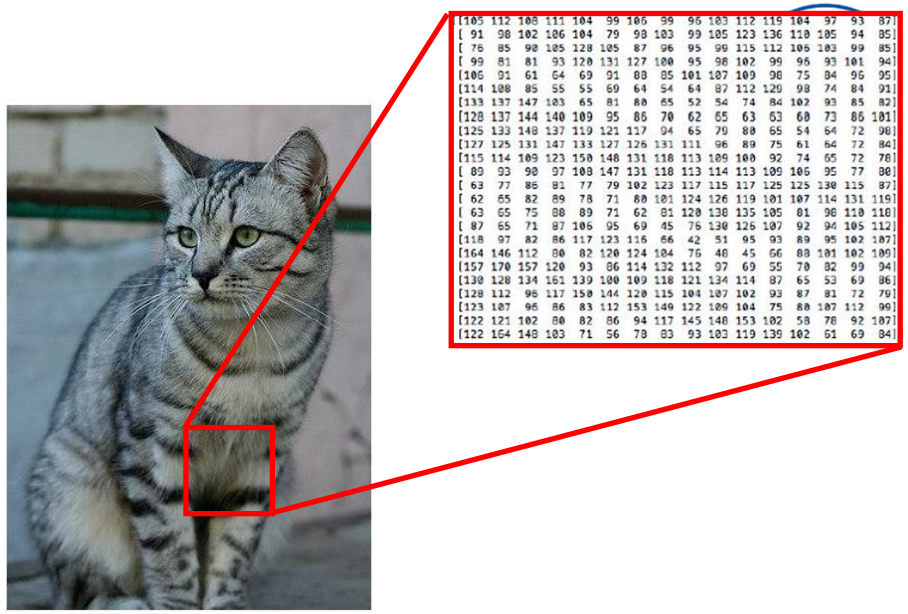


Image Filtering

- An image is a 2D array of pixel values



- Filtering:
  - Replace each pixel by a *linear* combination of its neighbors
  - The combination is determined by the filter's *kernel*
  - Often spatially-invariant, the same kernel is applied to all pixel locations

$$g[\cdot,\cdot] = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Gaussian Filter

$$g[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Box Filter

Box filter example

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[\cdot,\cdot]$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot,\cdot]$

output

	0	10	20						

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k, n+l]$$

output filter image (signal)

Box filter example

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[\cdot,\cdot]$

image

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[\cdot,\cdot]$

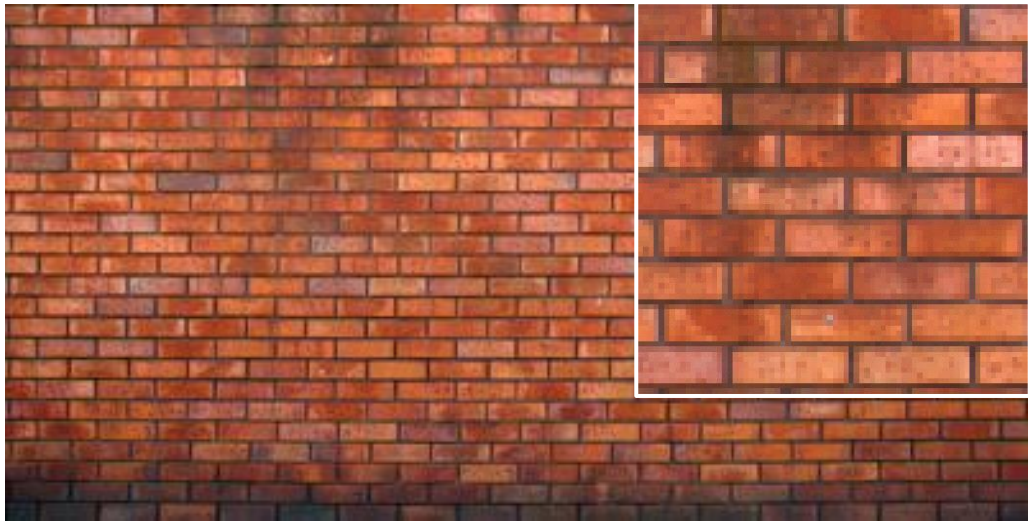
output

	0	10	20	30					

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k, n+l]$$

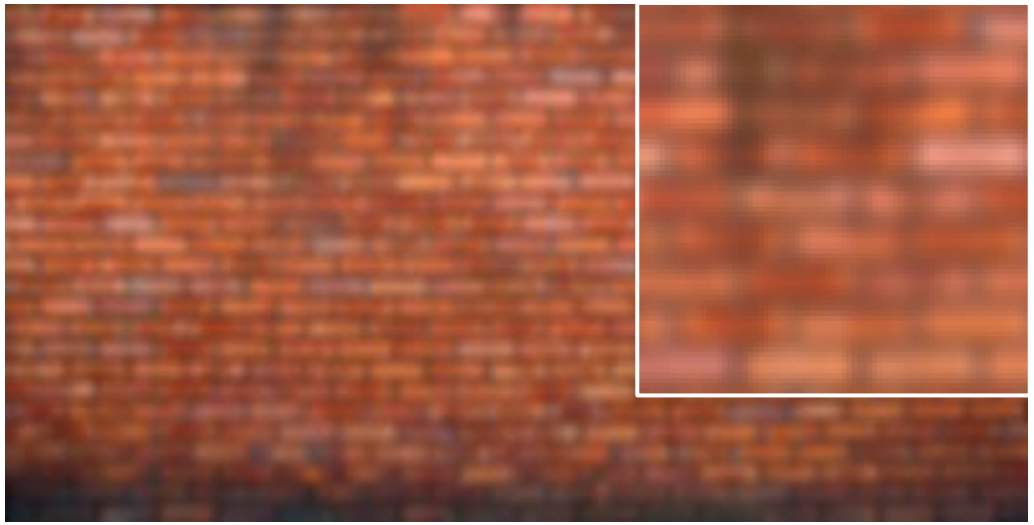
output filter image (signal)

Gaussian vs box filtering

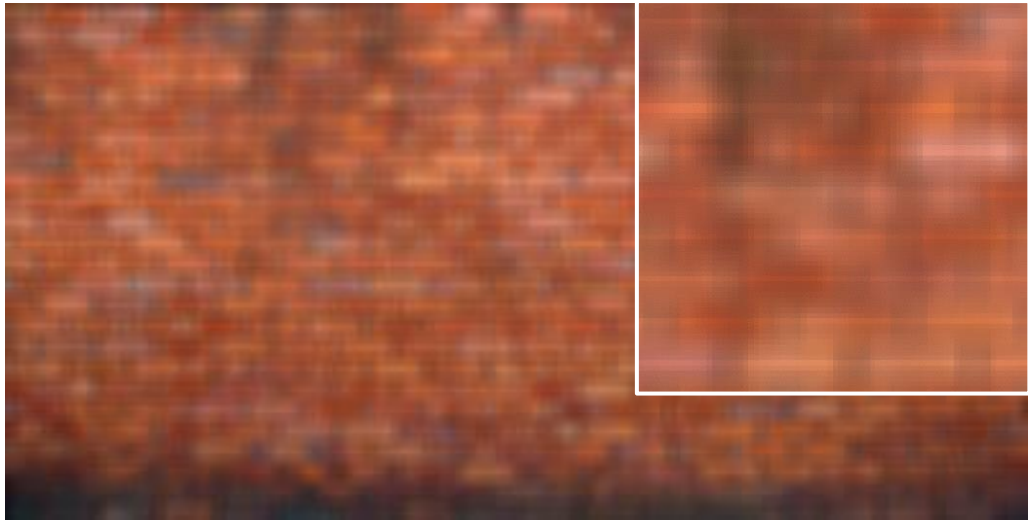


original

Which blur do you like better?

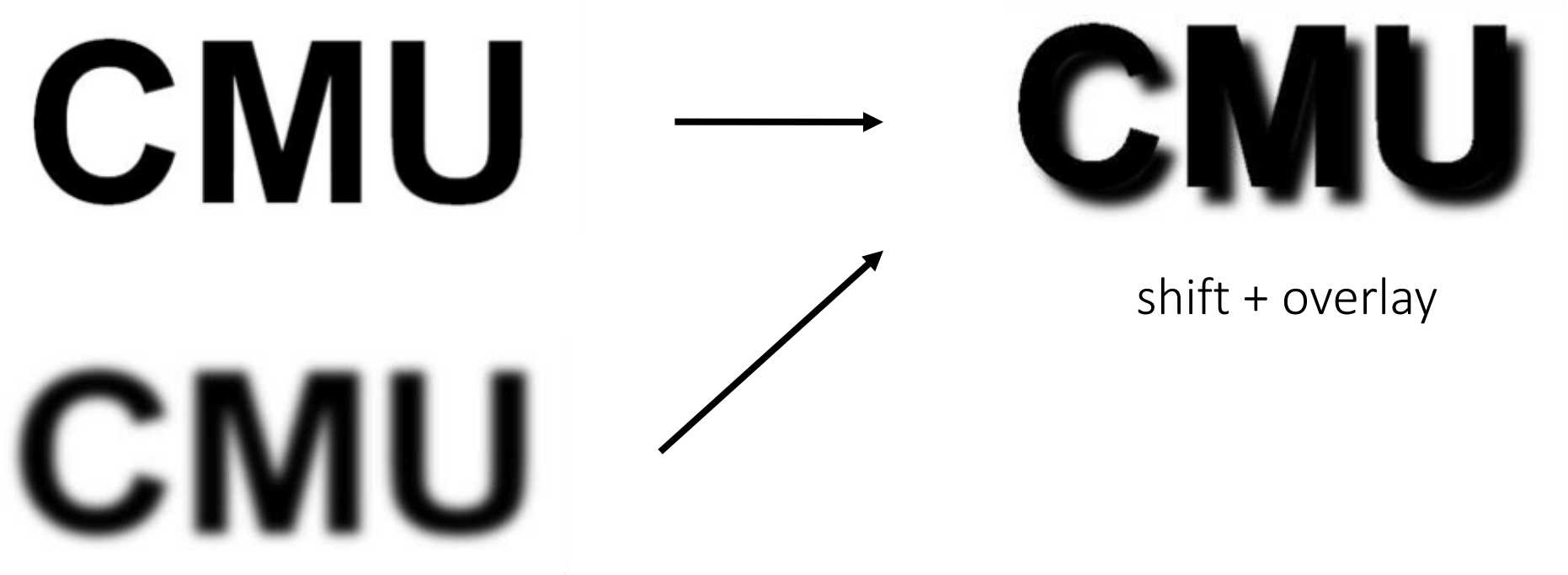


7x7 Gaussian



7x7 box

How to create a soft shadow effect?



Fun with filters

input

filter

0	0	0
0	1	0
0	0	0

output

unchanged

input

filter

0	0	0
0	0	1
0	0	0

output

shift to left
by one

Detecting Edges

- How would you detect edges in an image (i.e., discontinuities in a function)?
  - Take derivatives: derivatives are large at discontinuities

$$f'(x) = \lim_{h \to 0} \frac{f(x + 0.5h) - f(x - 0.5h)}{h}$$

- How do you differentiate a discrete image (or any other discrete signal)?
  - Use finite differences

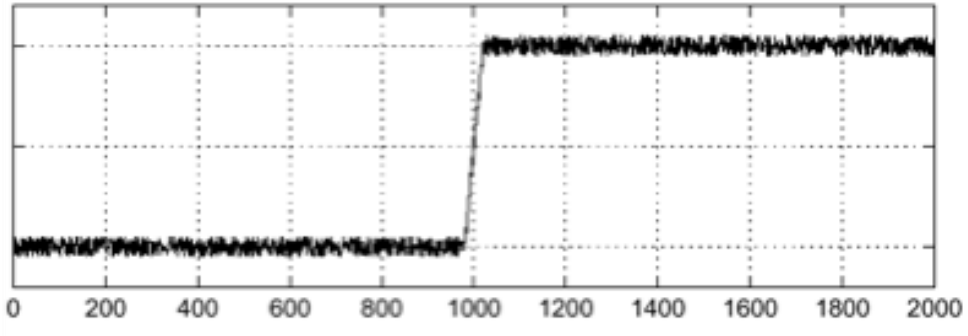
-1	0	1

1	0	-1

Remove limit and set $h = 2$

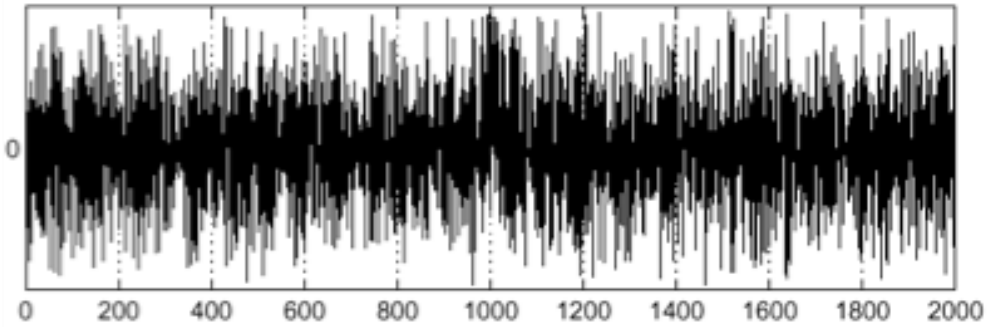
But Images are Noisy

intensity plot



Using a derivative filter:

derivative plot



Blur Before Taking Derivatives

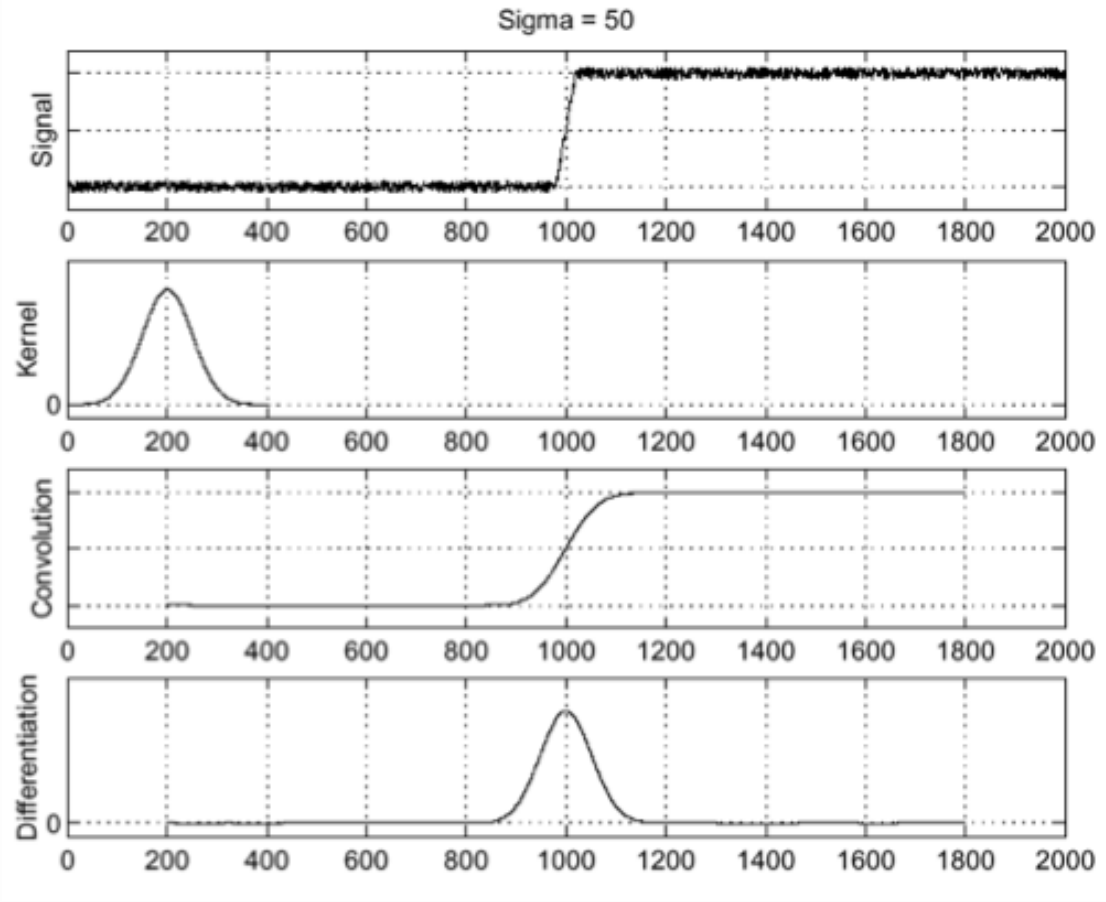
- When using derivative filters, it is critical to blur first!

input

Gaussian

blurred

derivative of blurred



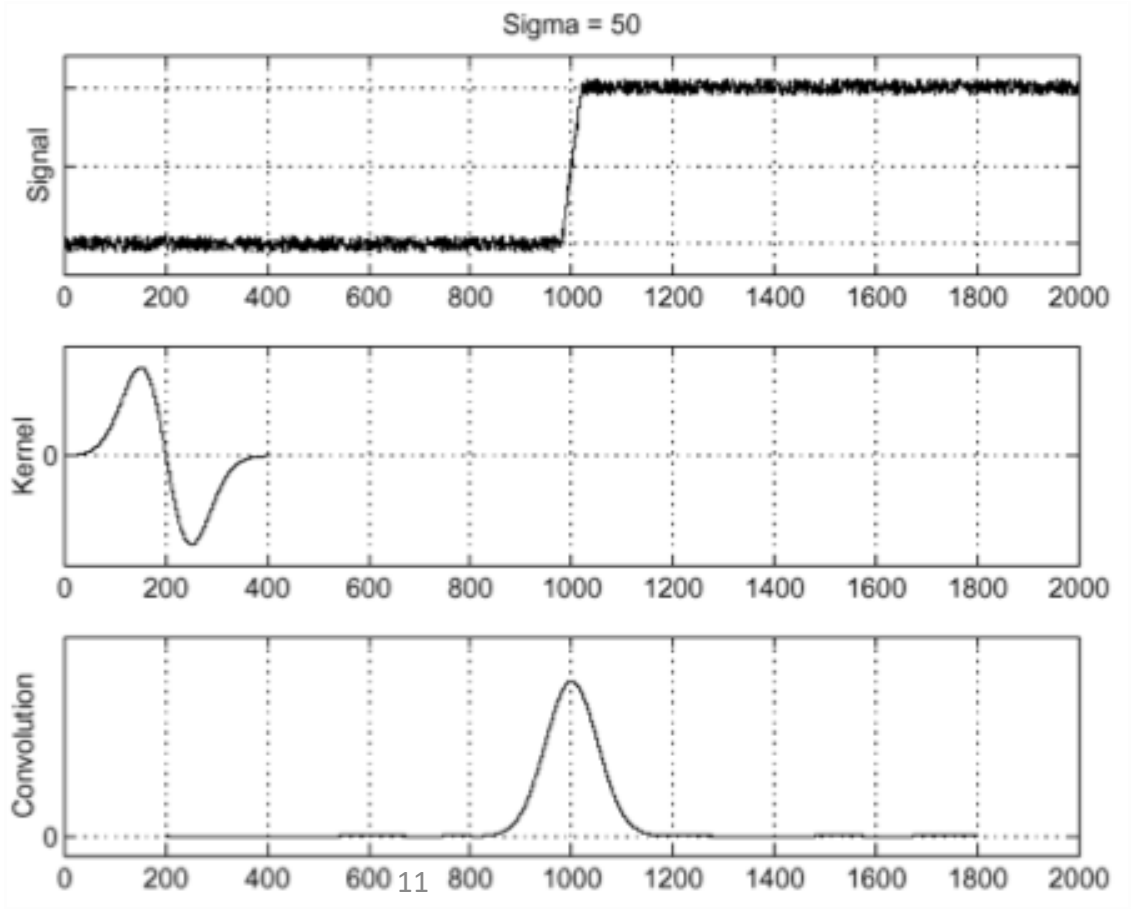
Derivative of Gaussian (DoG) filter

- Derivative theorem of convolution: $\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$
- Applying DoG = Applying blur first and then taking derivative

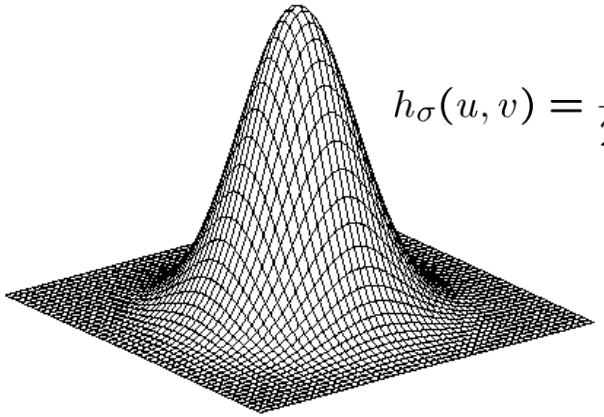
input

derivative of Gaussian

output (same as before)



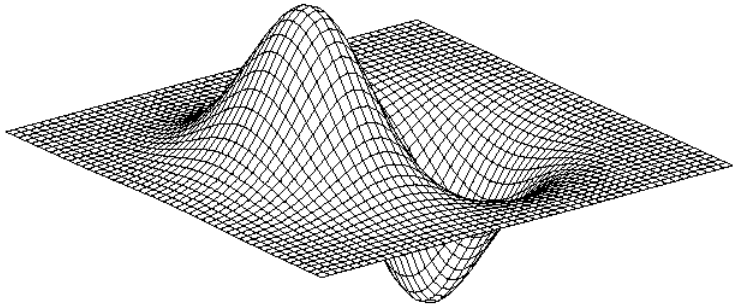
2D Gaussian filters



$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$



Derivative of Gaussian

Derivative of Gaussian filtering

The Sobel filter

Horizontal Sobel filter:

1	0	-1
2	0	-2
1	0	-1

Sobel filter

$=$

1
2
1

Blurring

$*$

1	0	-1

1D derivative filter

Vertical Sobel filter:

1	2	1
0	0	0
-1	-2	-1

$=$

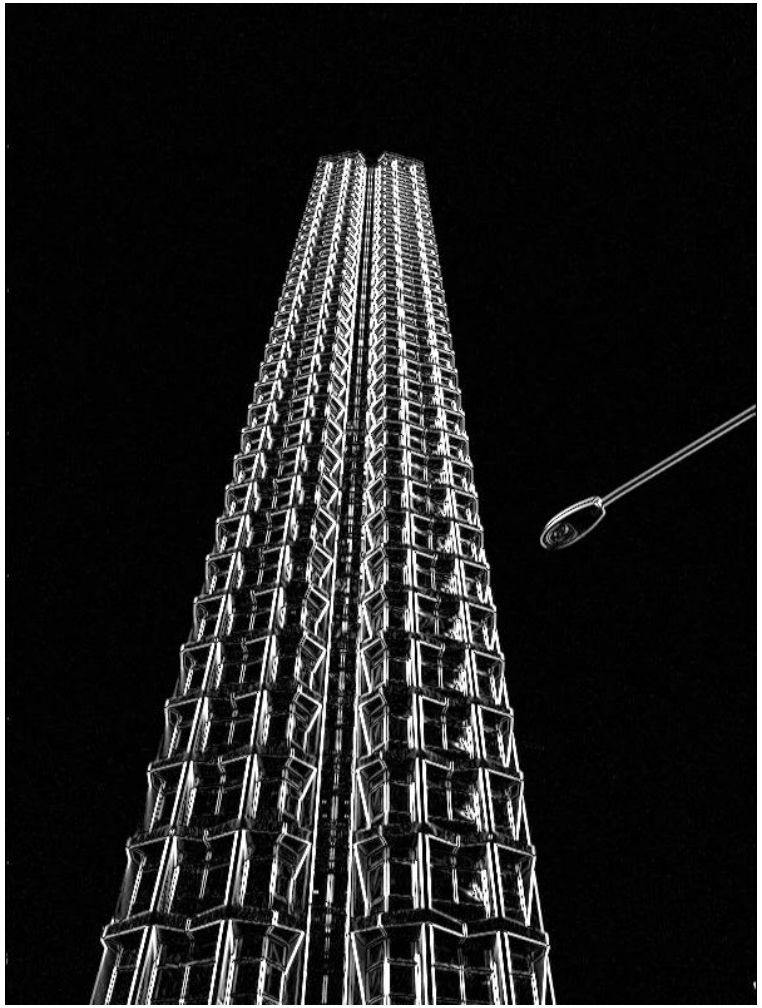
1
0
-1

$*$

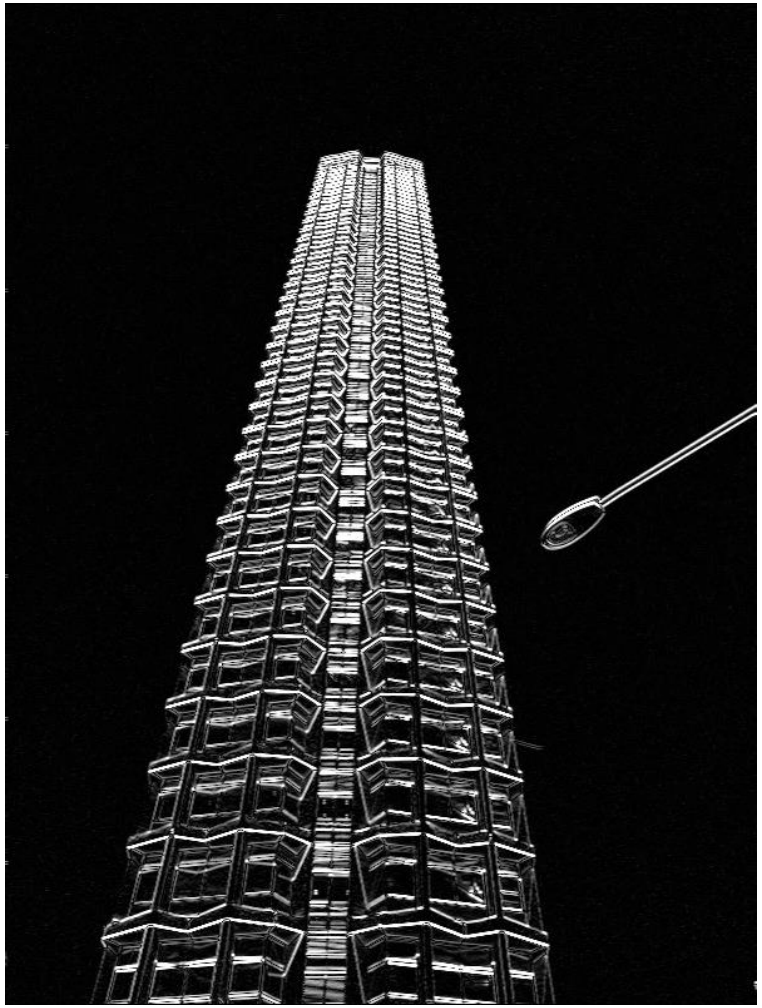
1	2	1

Sobel filter example

original

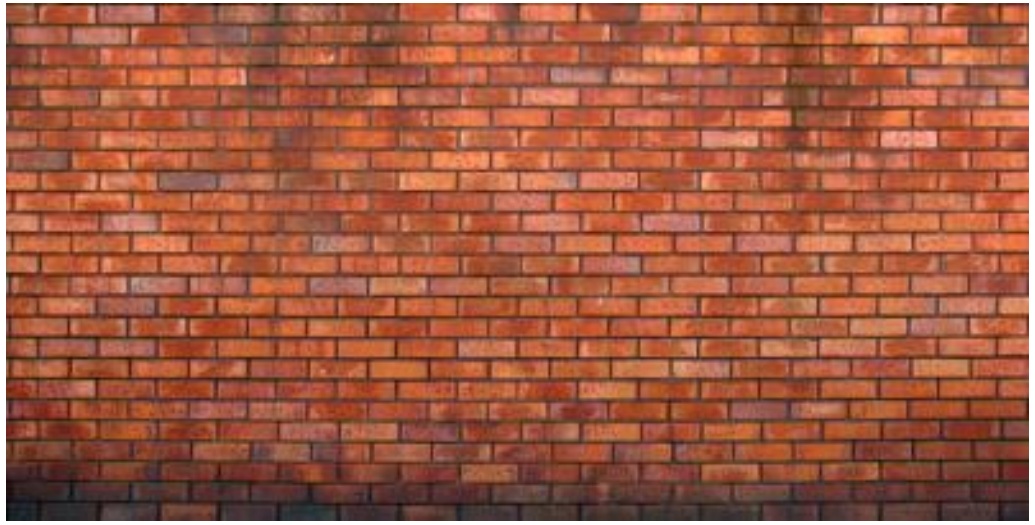


horizontal Sobel filter



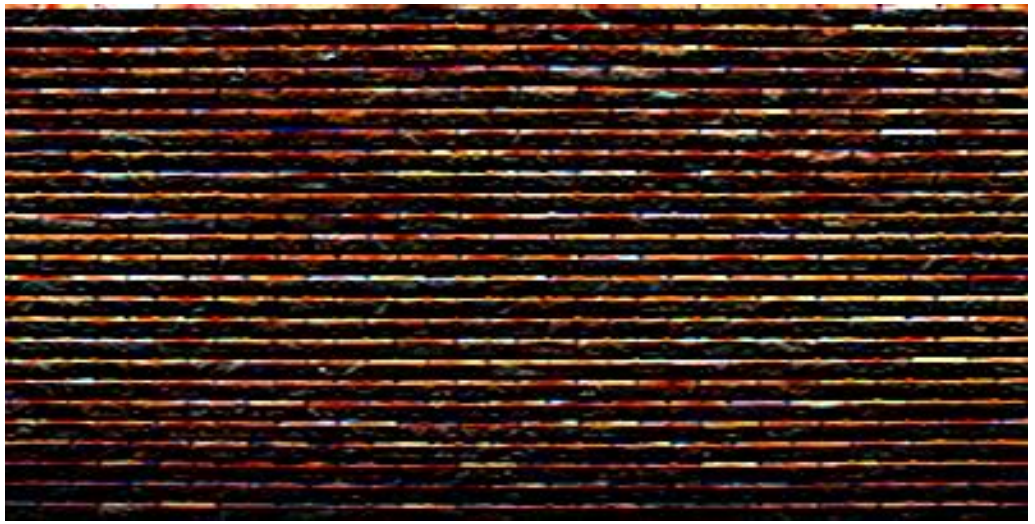
vertical Sobel filter

Sobel filter example



original

horizontal Sobel filter



vertical Sobel filter

Computing Image Gradients

1. Select a derivative filters (there are other similar filters, e.g. Scharr)

$$\boldsymbol{S}_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \qquad \boldsymbol{S}_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

2. Convolve with the image to compute derivatives.

$$\frac{\partial \boldsymbol{f}}{\partial x} = \boldsymbol{S}_x \otimes \boldsymbol{f} \qquad \frac{\partial \boldsymbol{f}}{\partial y} = \boldsymbol{S}_y \otimes \boldsymbol{f}$$

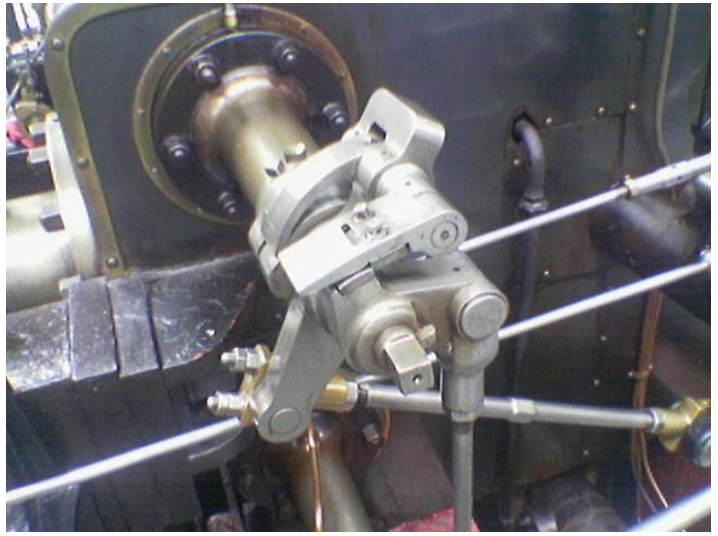
3. Form the image gradient, and compute its direction and amplitude.

$$\nabla \boldsymbol{f} = \left[\frac{\partial \boldsymbol{f}}{\partial x}, \frac{\partial \boldsymbol{f}}{\partial y}\right] \qquad \theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right) \qquad \|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

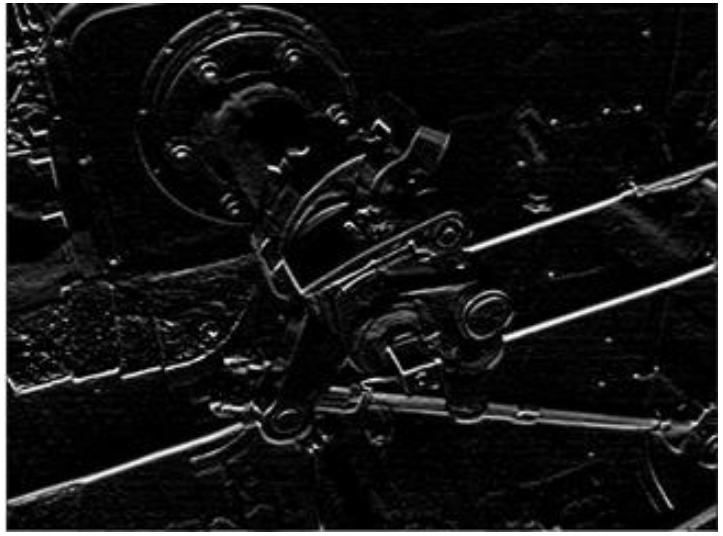
gradient direction amplitude

Image gradient example

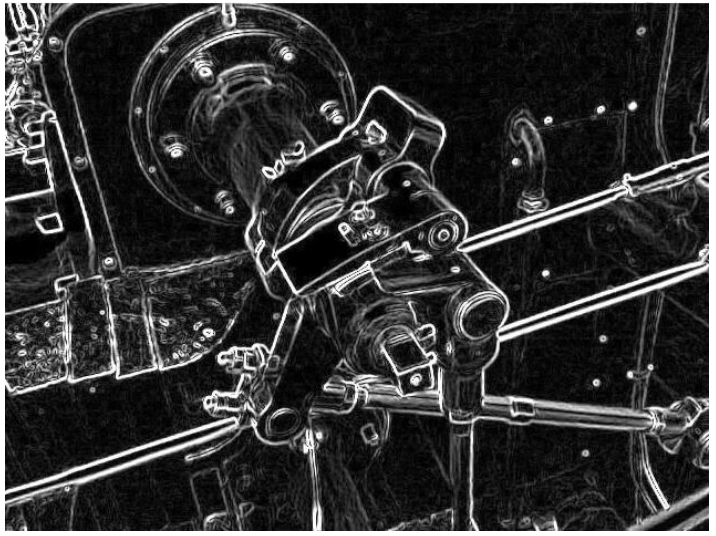
original



vertical derivative



gradient amplitude



horizontal derivative

Questions?

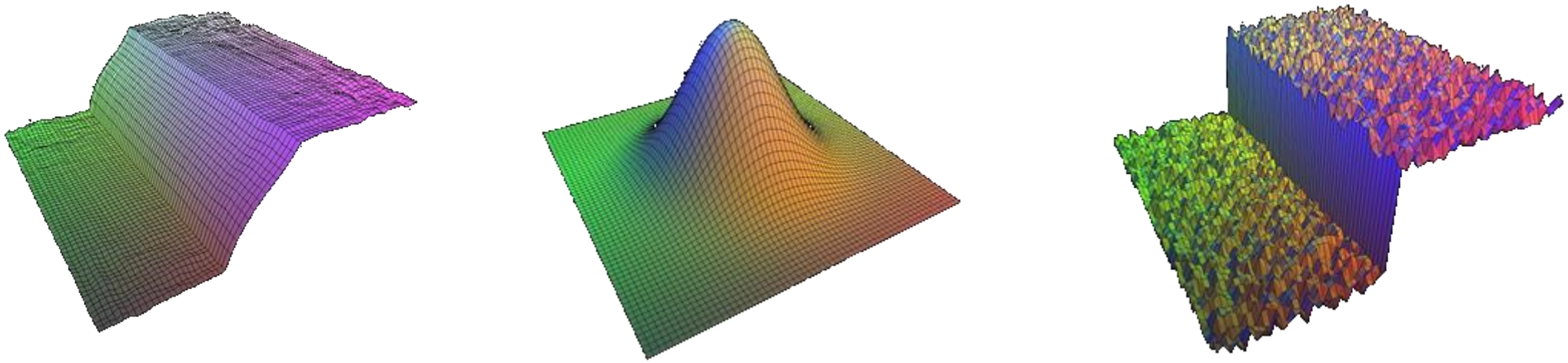
Bilateral filter

- Proposed by Tomasi and Manduci in ICCV 1998
  - “Bilateral Filtering for Gray and Color Images”
- A very good survey by Sylvain Paris et al. 2009
  - Published at Foundations and Trends in Computer Graphics and Vision
  - “Bilateral Filtering: Theory and Applications”
- Related to
  - SUSAN filter
    [Smith and Brady 95] http://citeseer.ist.psu.edu/smith95susan.html
  - Digital-TV [Chan, Osher and Chen 2001]
    http://citeseer.ist.psu.edu/chan01digital.html
  - sigma filter http://www.geogr.ku.dk/CHIPS/Manual/f187.htm

Start with Gaussian Filter

- Here, input is a step function + noise
- Spatial Gaussian filter f
- Output is blurred

$$J = f \otimes I$$

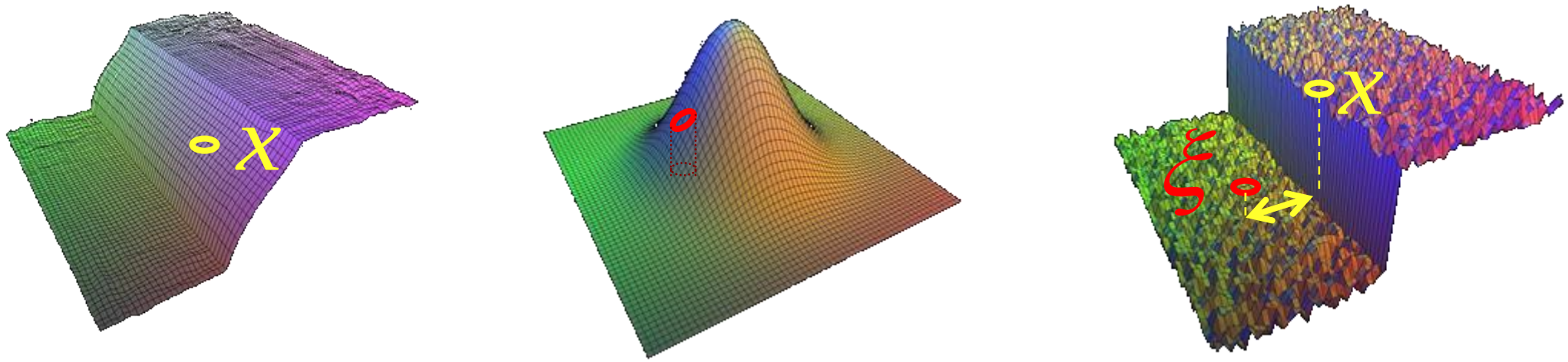


output ⟵ input

Gaussian Filter as Weighted Average

- Weight of ξ depends on its distance to x

$$J(x) = \sum_{\xi} f(x,\xi) I(\xi)$$

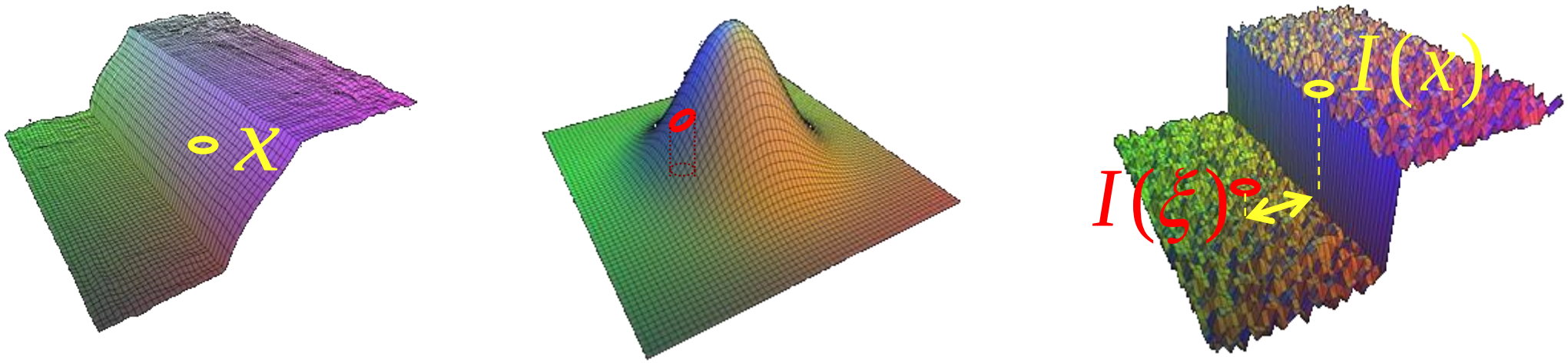


output ⟵ input

The Problem of Edges

- Here, $I(\xi)$ "pollutes" our estimated $J(x)$
- It is too different to be averaged together

$$J(x) = \sum_{\xi} f(x,\xi) I(\xi)$$



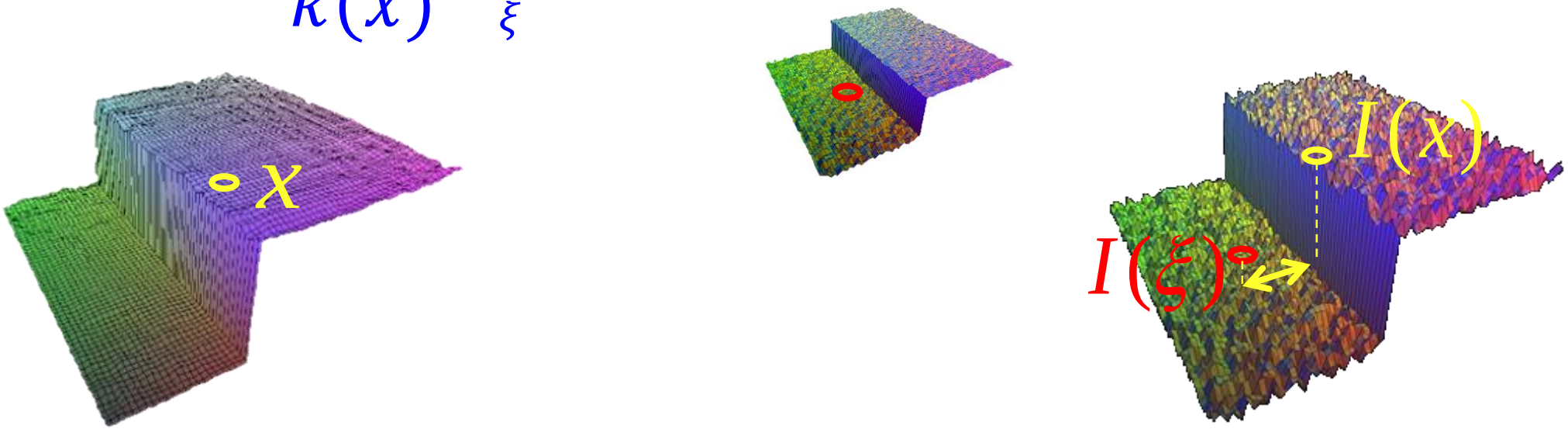
output ⟵ input

Principle of Bilateral Filter

[Tomasi and Manduchi 1998]

- Penalty Gaussian g on the intensity difference

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi)\, g(I(\xi) - I(x))\quad I(\xi)$$



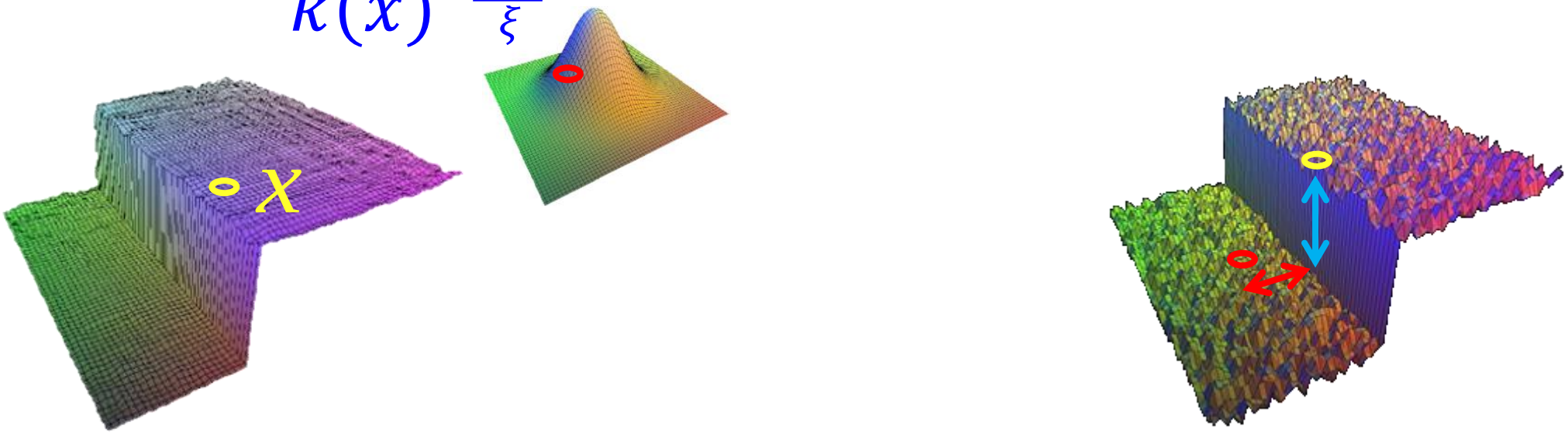
output ⟵ input

Bilateral Filter

[Tomasi and Manduchi 1998]

- Spatial Gaussian f

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi)\, g(I(\xi) - I(x))\quad I(\xi)$$

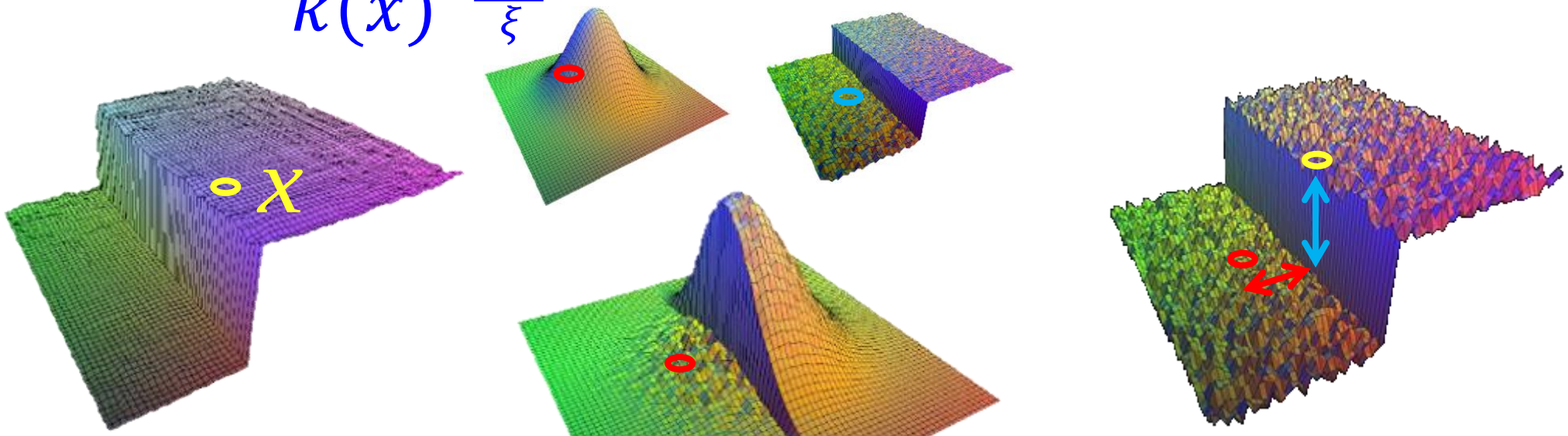


Bilateral Filter

[Tomasi and Manduchi 1998]

- Combined weight

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi)\, g(I(\xi) - I(x))\quad I(\xi)$$



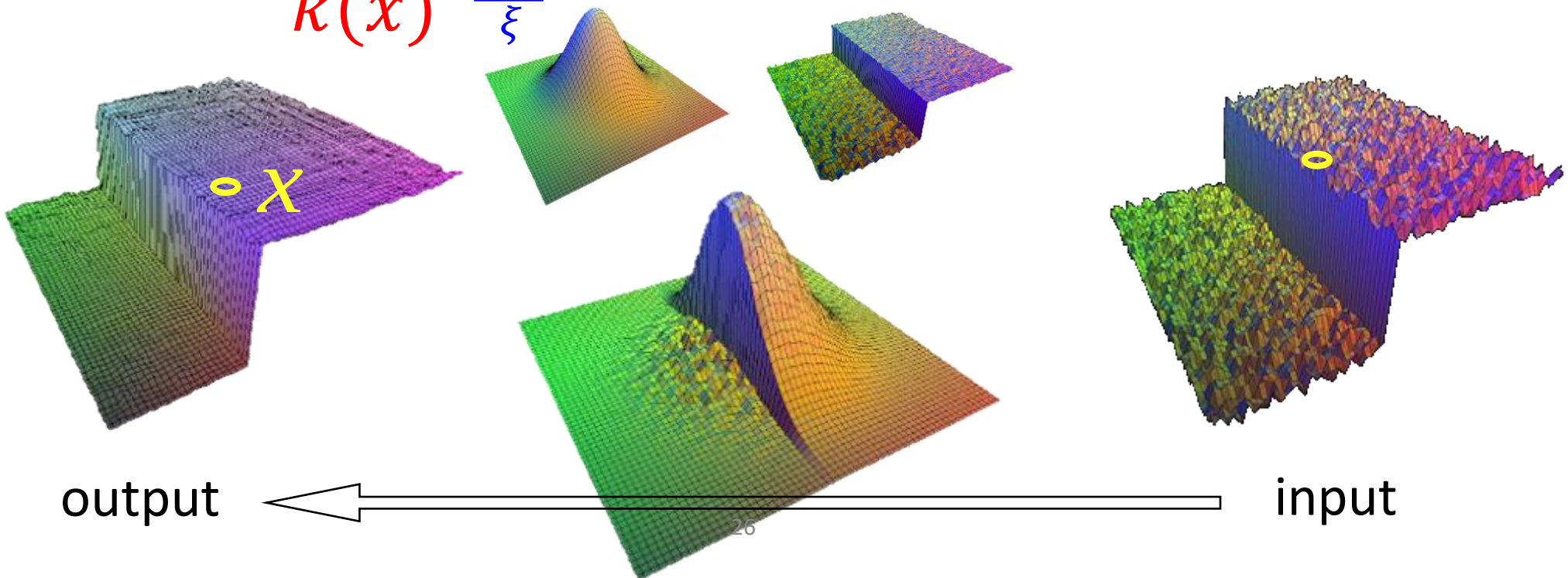
output ⟵ input

Normalization Factor

[Tomasi and Manduchi 1998]

$$k(x) = \sum_{\xi} f(x,\xi)\, g(I(\xi) - I(x))$$

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi)\, g(I(\xi) - I(x))\quad I(\xi)$$

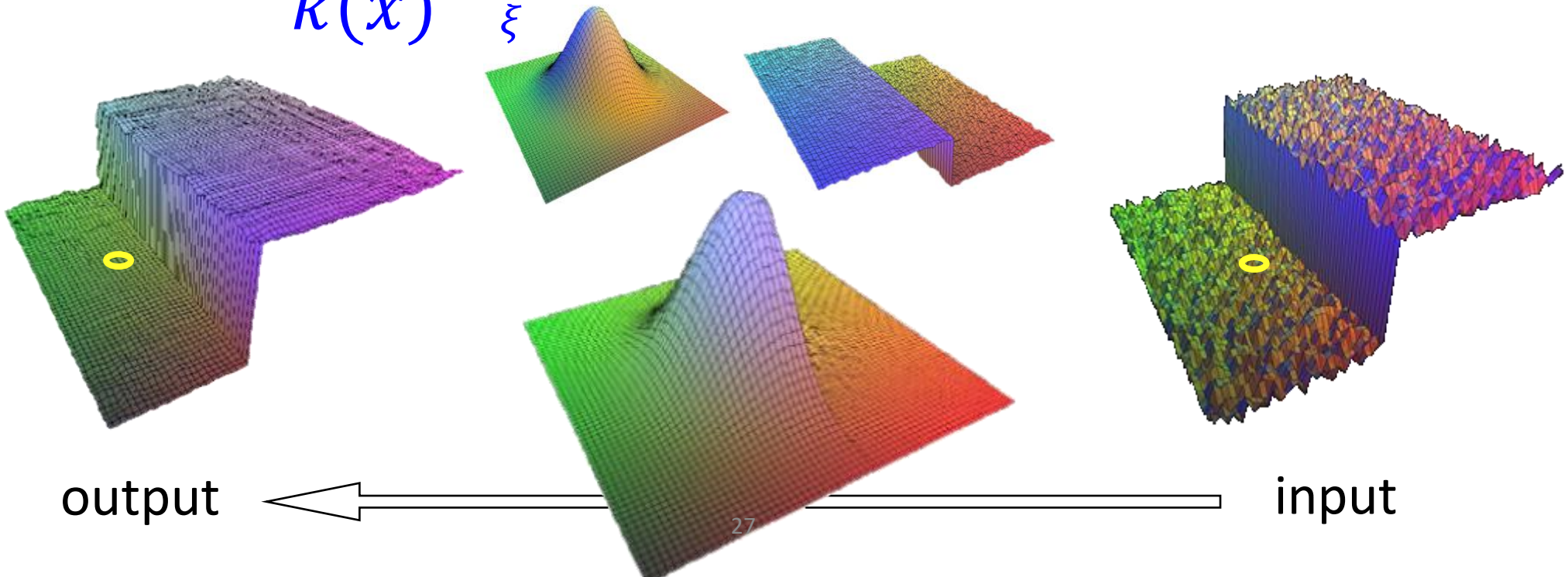


Bilateral Filter is Spatially-Variant

[Tomasi and Manduchi 1998]

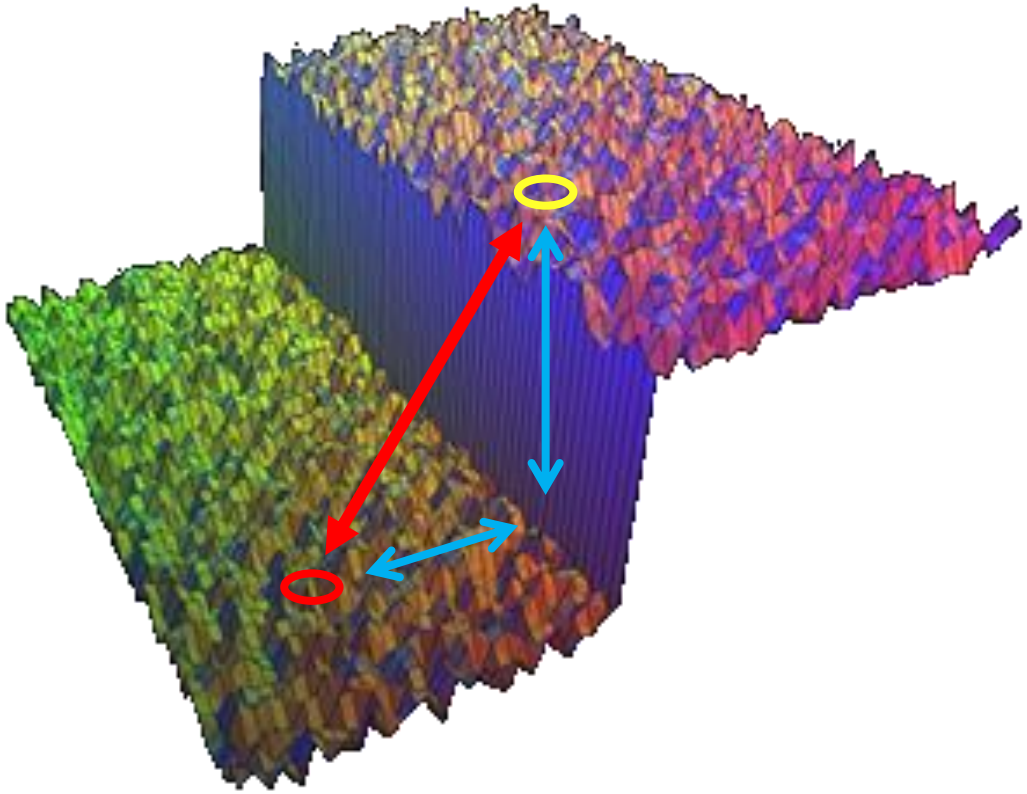
- The weights are different for each output pixel

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi)\, g(I(\xi) - I(x))\quad I(\xi)$$



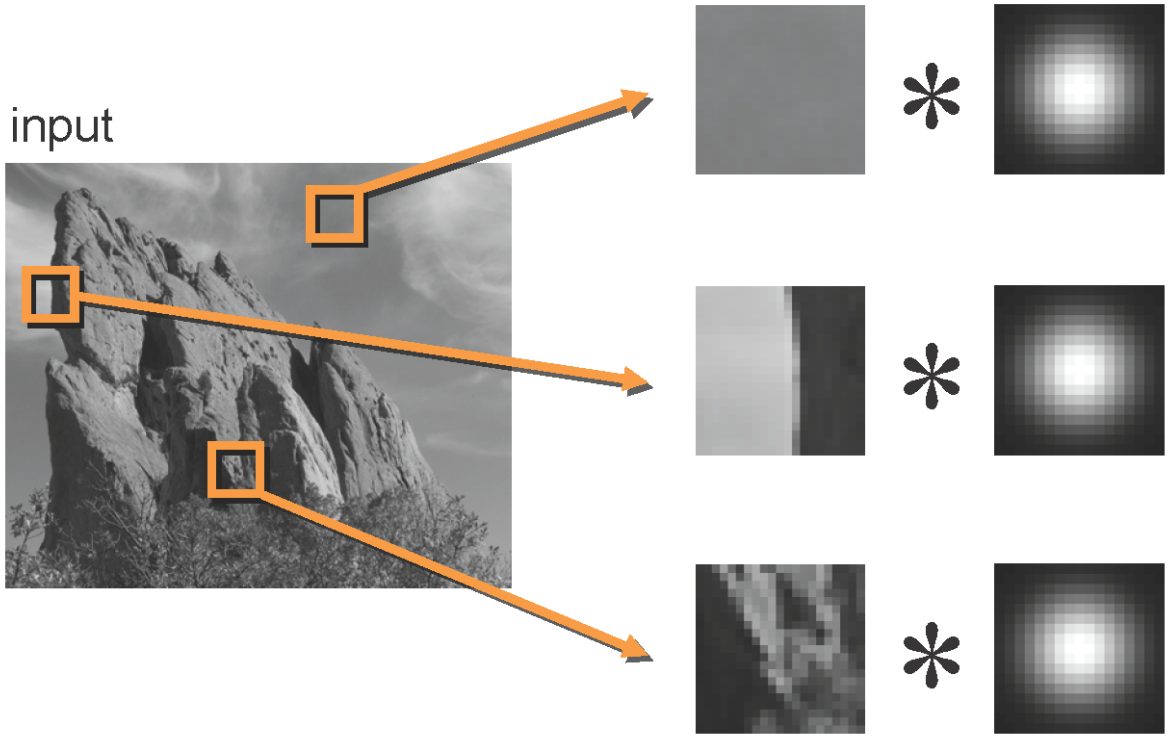
Other Explanation

- The bilateral filter uses the 3D distance to compute the weight



Bilateral Filter vs Gaussian Filter

- Bilateral Filter – fix to the Gaussian filter
- Gaussian filtering applies the same filter everywhere



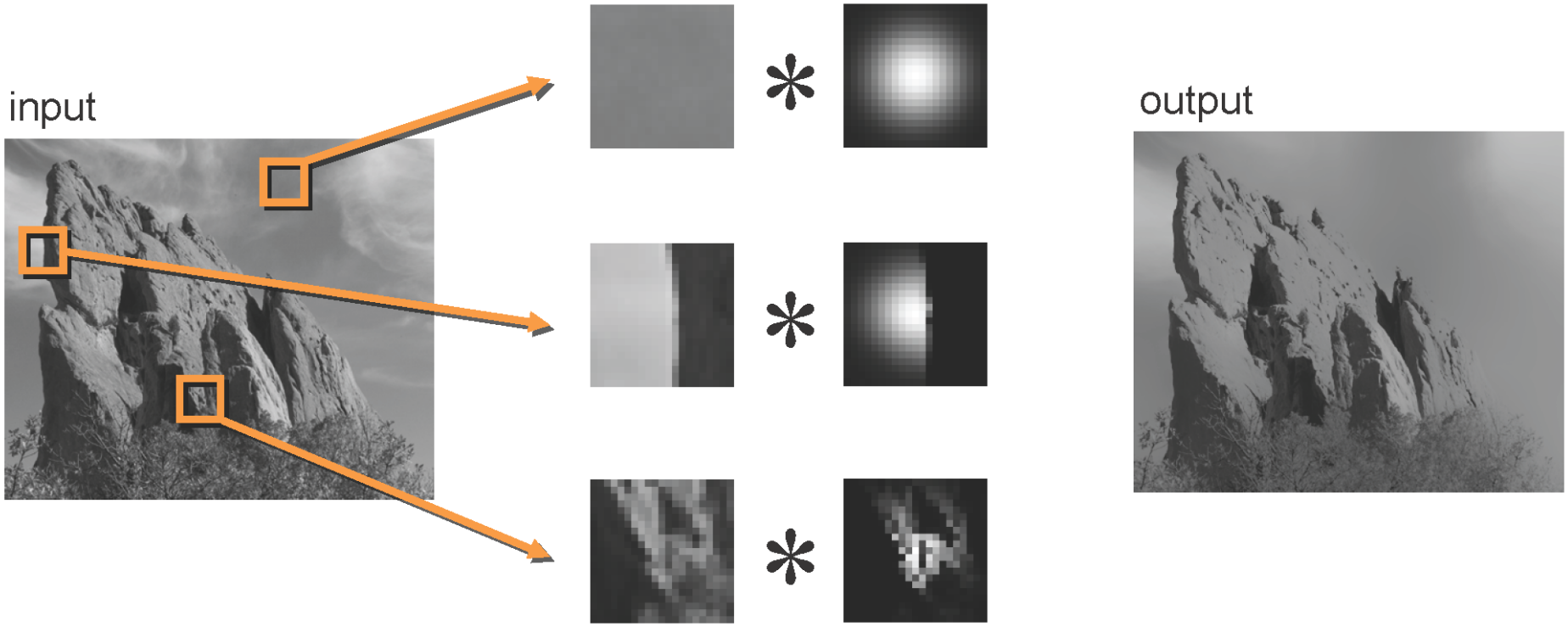
Same Gaussian kernel everywhere.

Sylvain Paris

[notes adapted from Sylvain Paris SIGGRAPH'07 course notes on Bilateral Filter]

Bilateral Filter vs Gaussian Filter

- Adjust kernel based on image content



The kernel shape depends on the image content.

31

Results: Denoise

noisy image

naïve denoising
Gaussian blur

better denoising
edge-preserving filter

Smoothing an image without blurring its edges.

More Examples

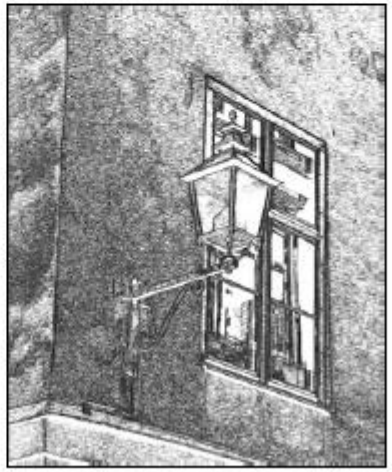
(a) *Photograph*

(b) *Edge-aware smoothing*

(c) *Detail enhancement*

(d) *Stylization*

(e) *Recoloring*



(f) *Pencil drawing*

(g) *Depth-of-field*

[Gastal and Oliveira 2011]

Questions?

Fast Bilateral Filtering for the Display of High-Dynamic-Range Images

Frédo Durand
Julie Dorsey

MIT

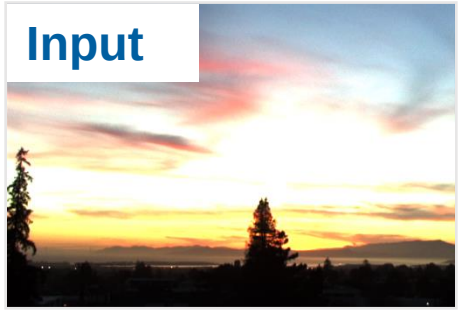
SIGGRAPH 2002

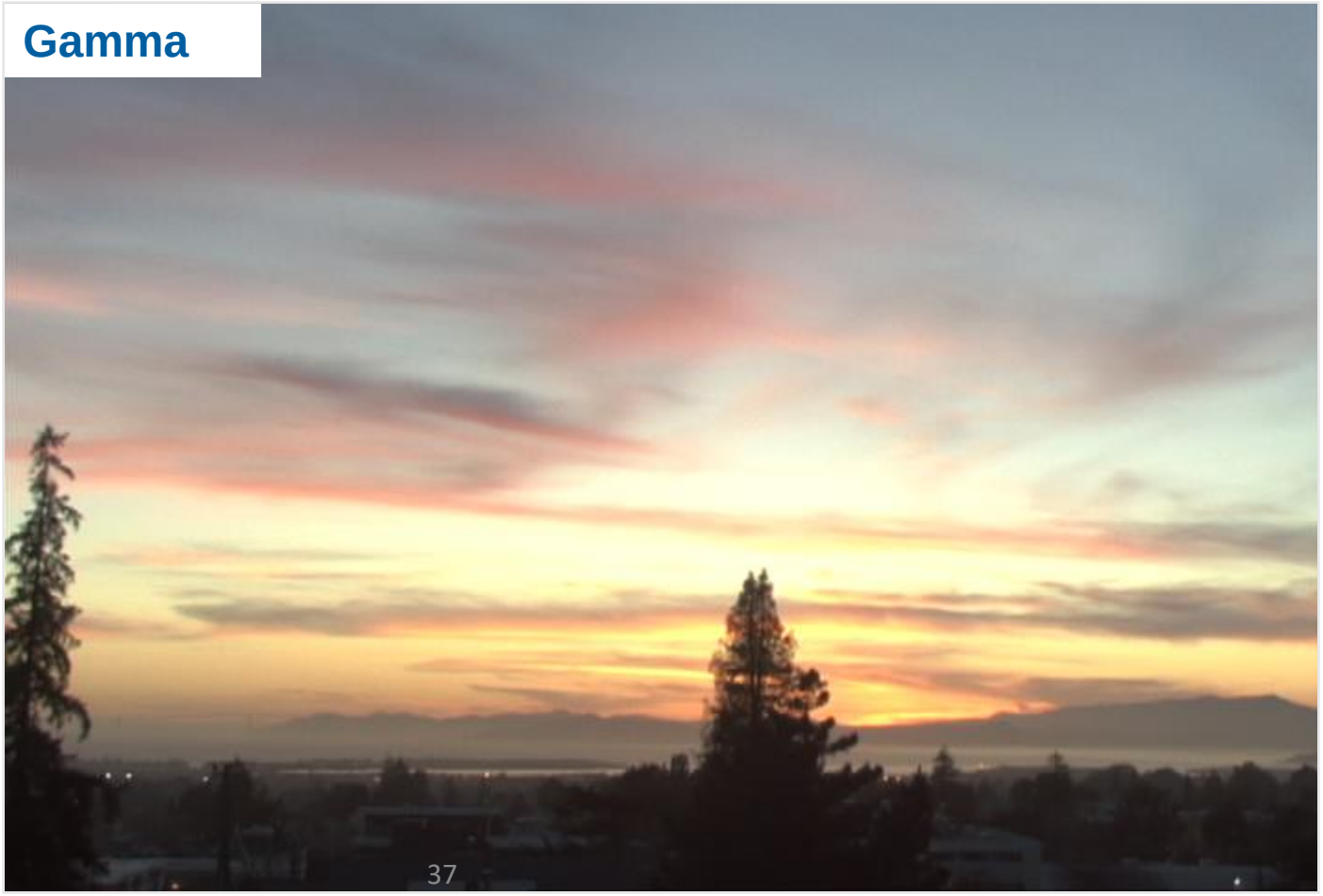
Show HDR Images in LDR Displayers

- Input: high-dynamic-range image
  - (floating point per pixel)

Naïve Method: Gamma Compression

- $X \rightarrow X^{\gamma}$ (where γ=0.5 in our case)
- But… colors are washed-out.



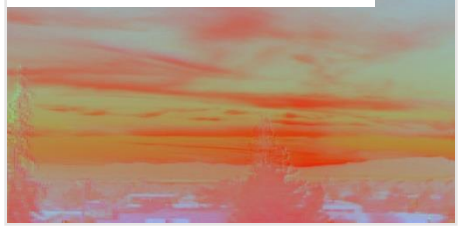


Gamma Compression on Intensity

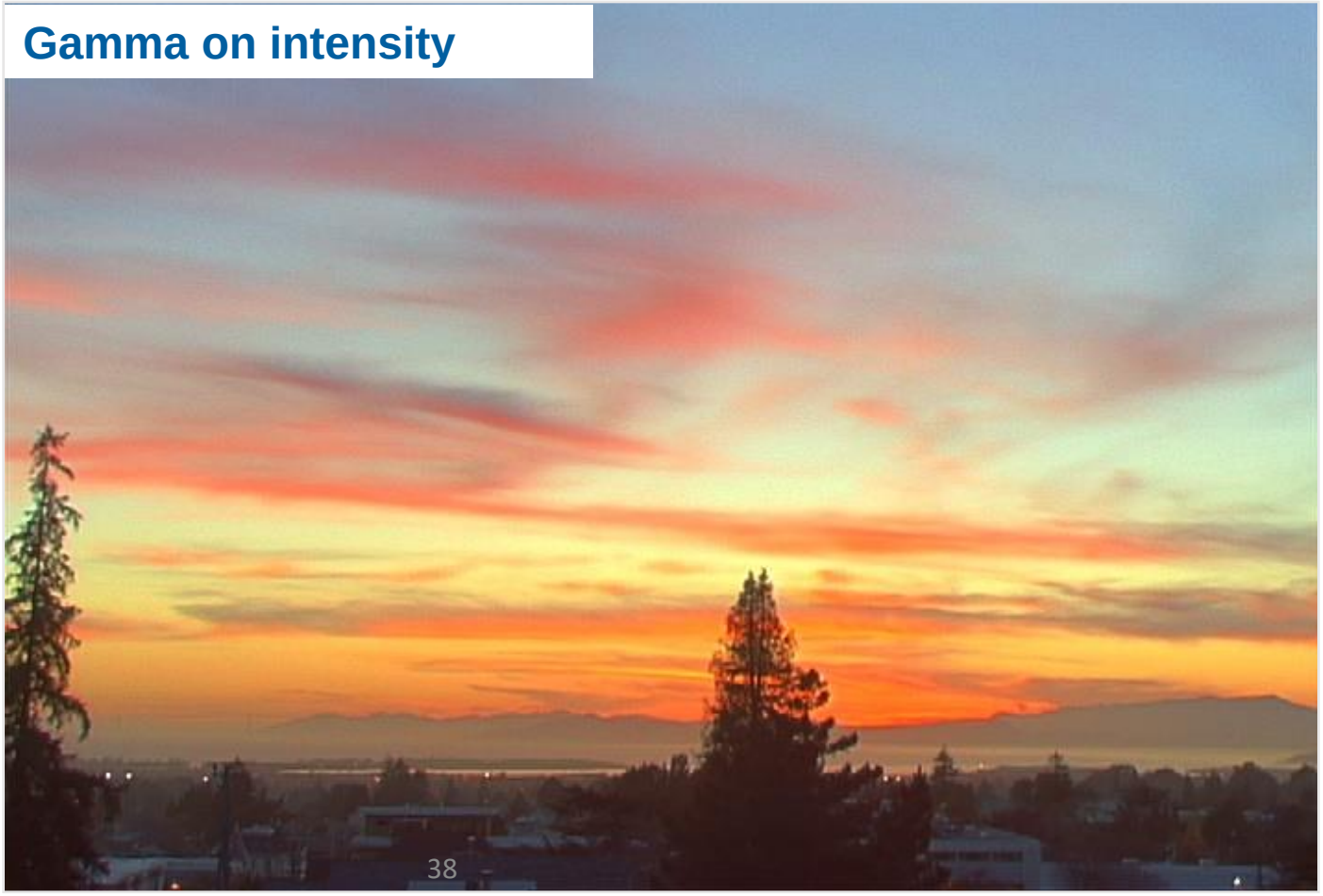
- Compress the L channel and keep UV unchanged
- Colors are OK, but details are blurred

Intensity

Color

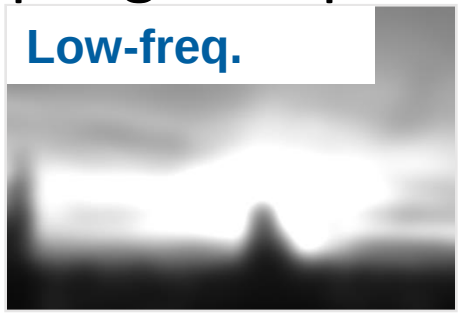


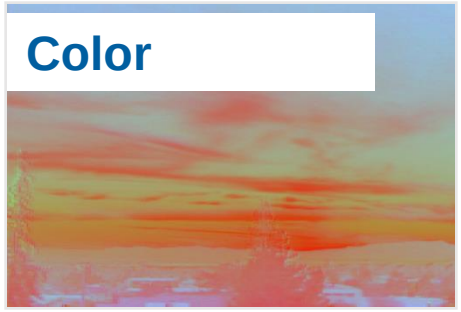
Gamma on intensity

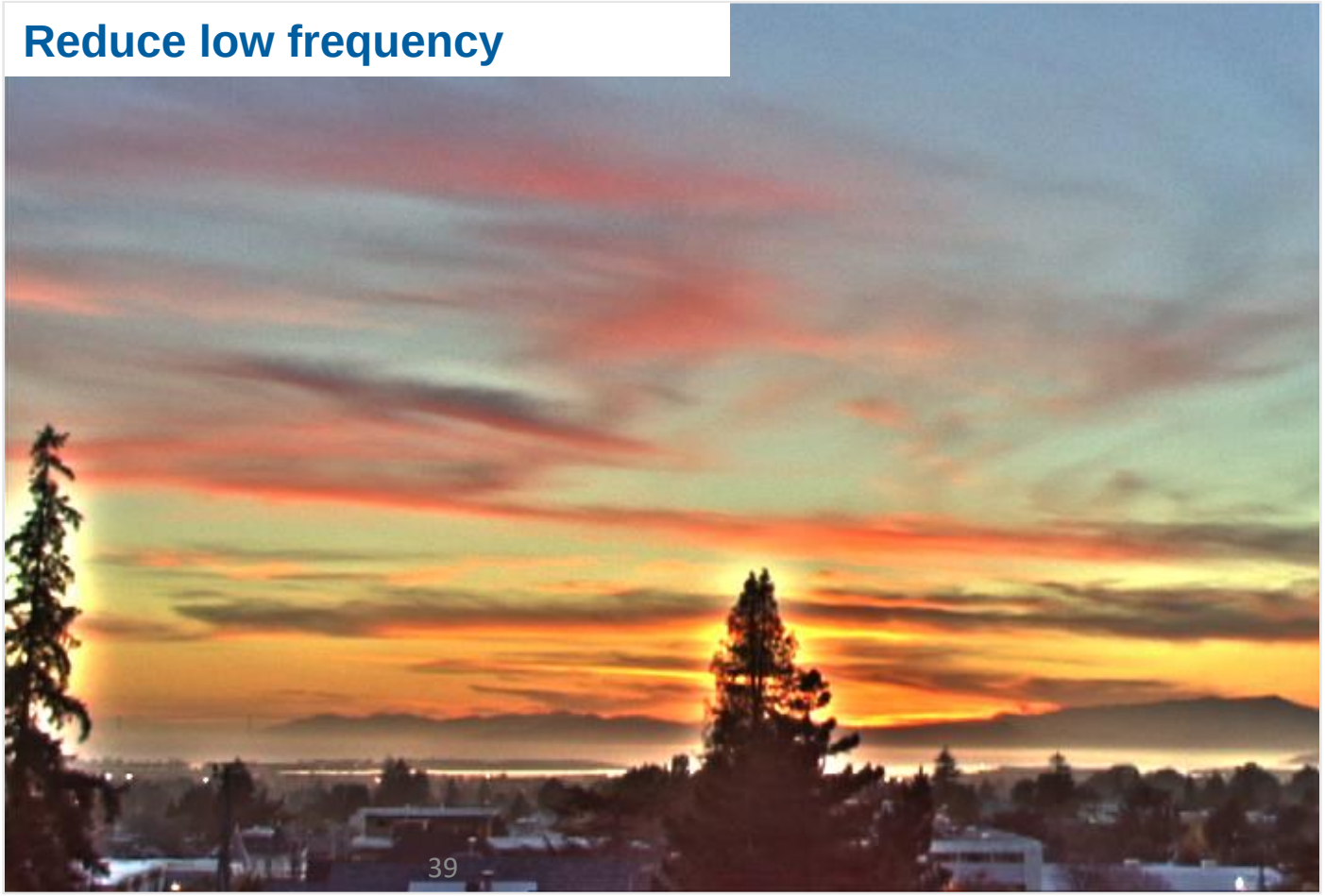


Oppenheim 1968, Chiu et al. 1993

- Separate frequencies in L, compress low frequency
- Keep high frequencies

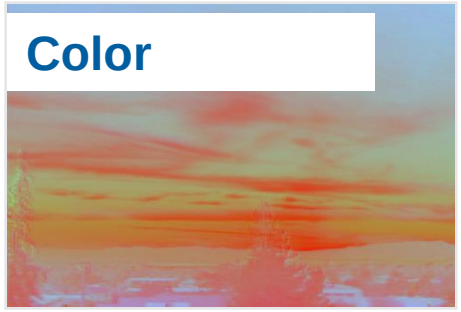


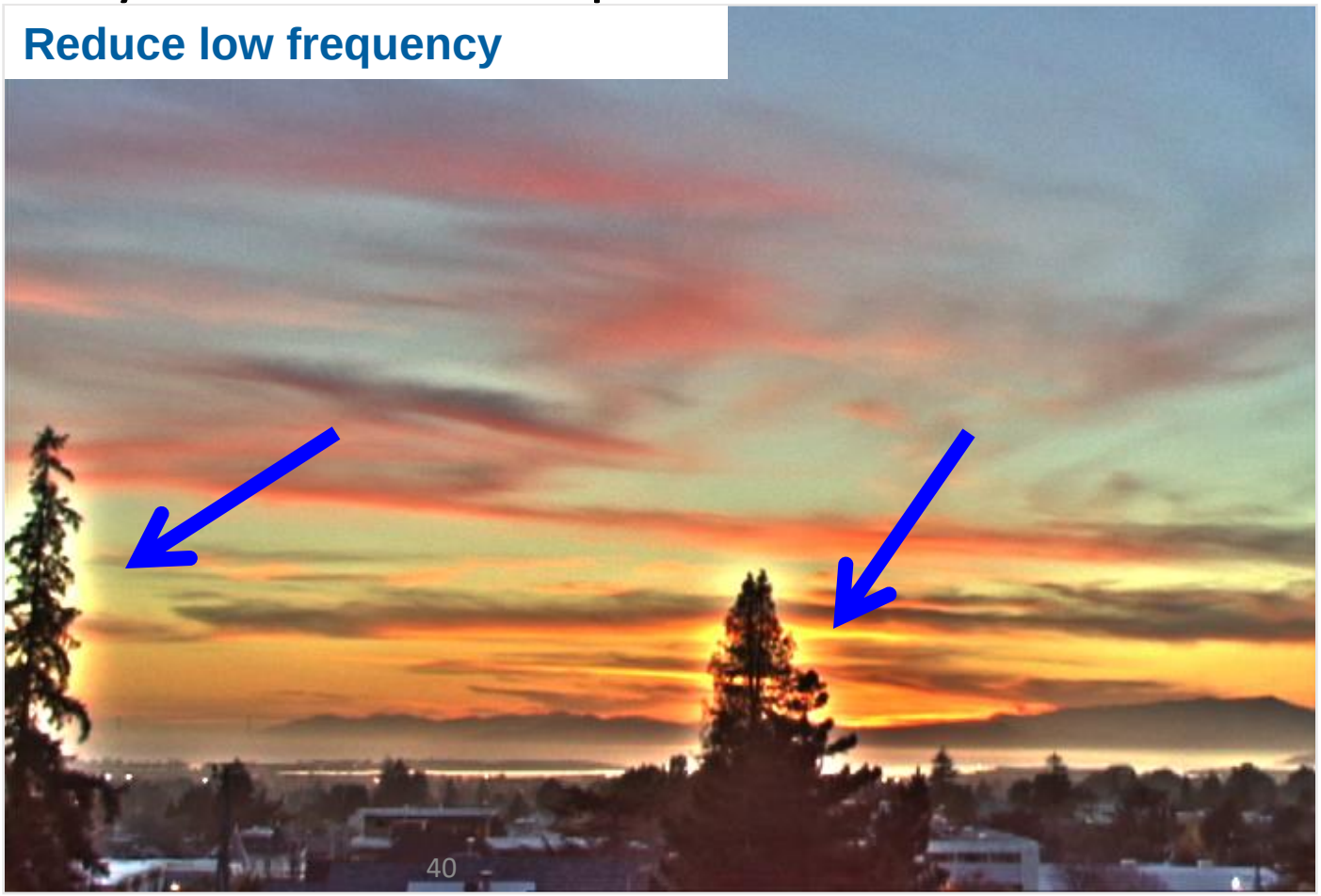




The Halo Nightmare

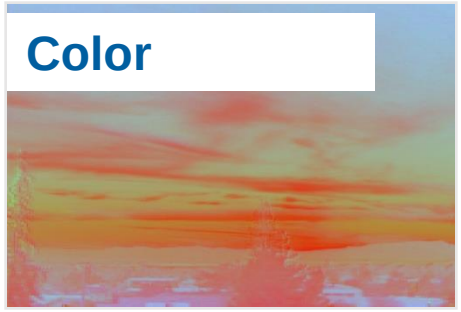
- Halo appears at strong edges
- Because high frequency is not well computed

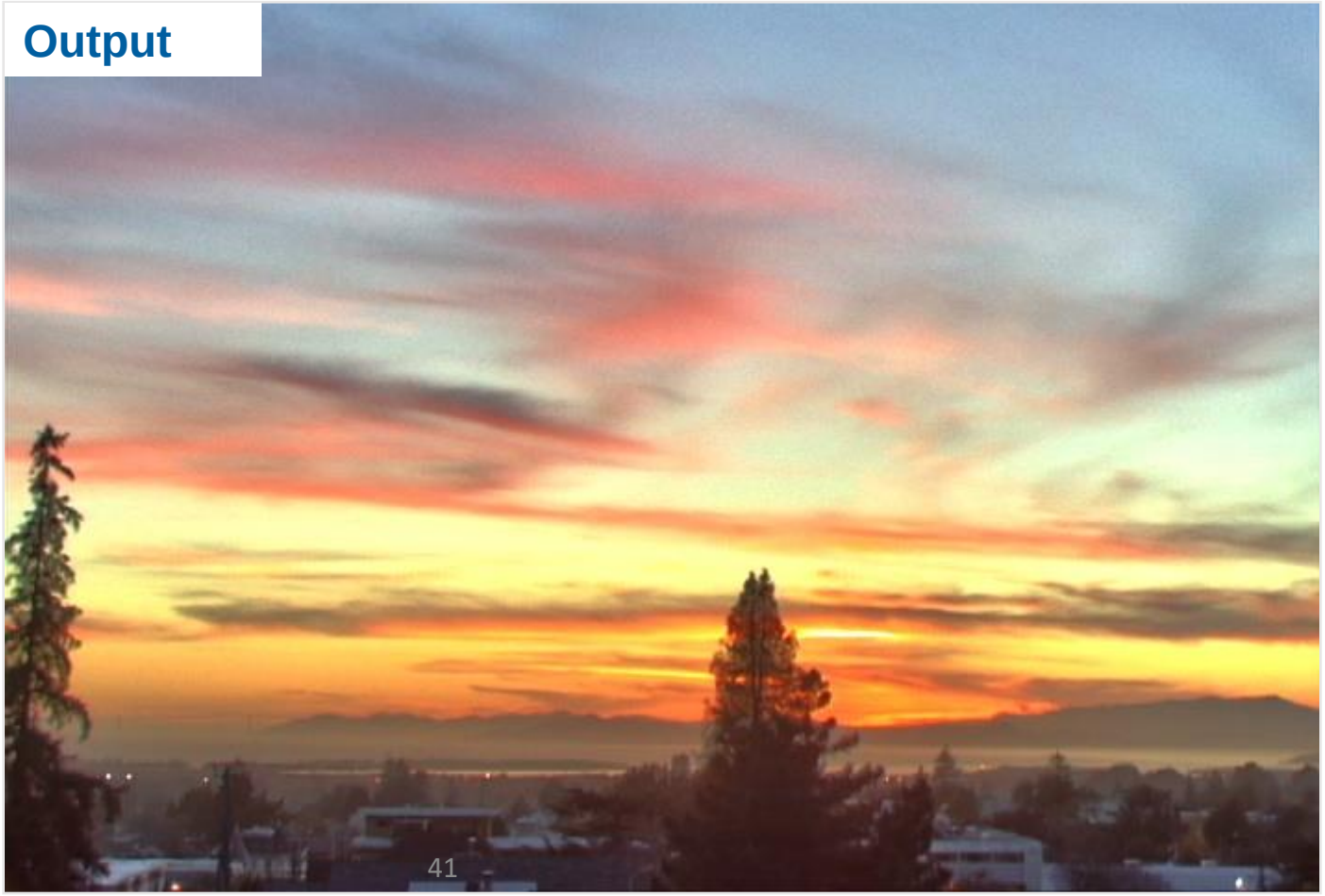




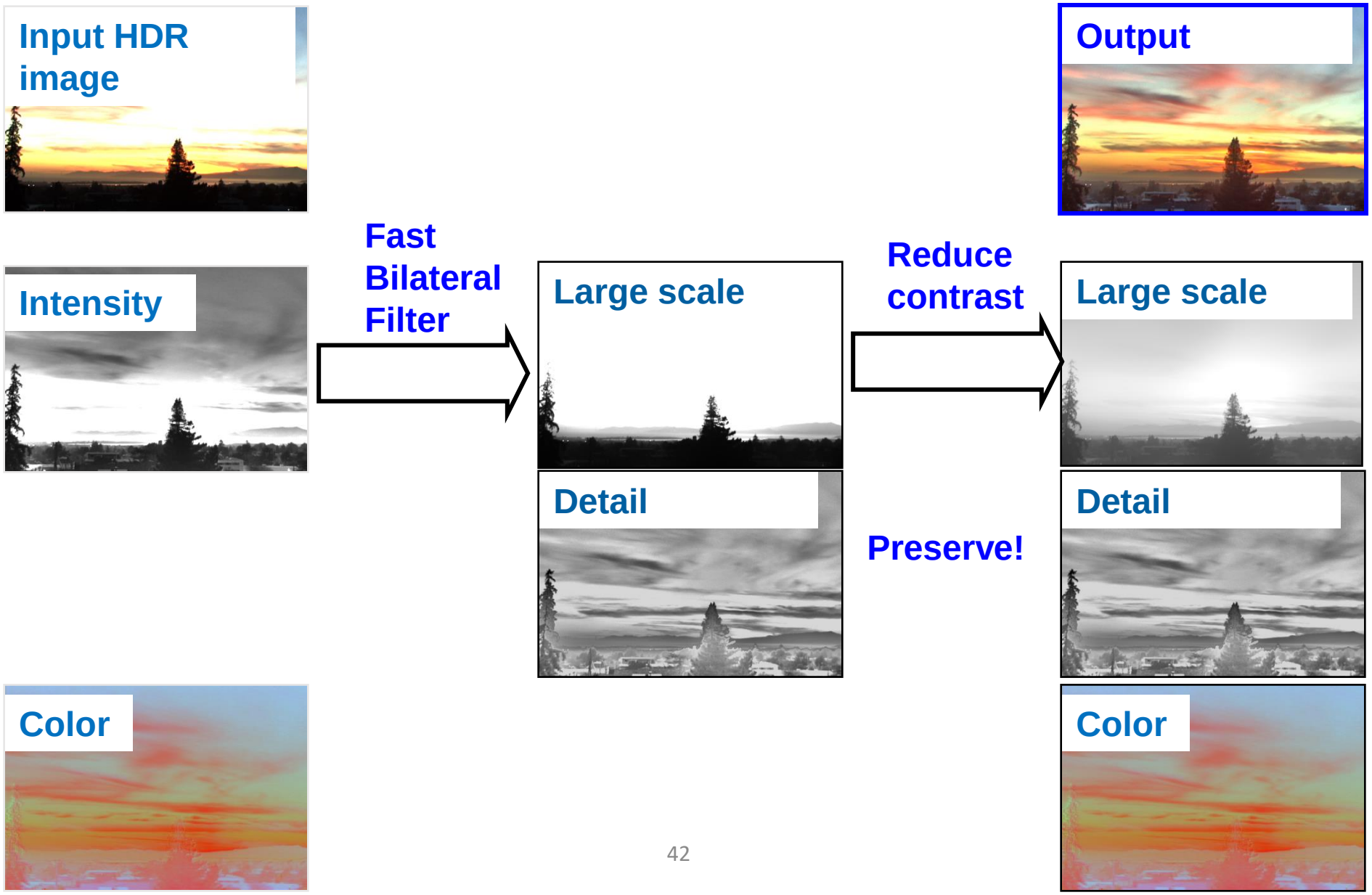
Separate Frequencies by Bilateral Filter

- Compute high frequency in a better way
- Such that there is no halo





Overall Pipeline



Questions?

Digital Photography with Flash and No-Flash Image Pairs

Georg Petschnigg
Richard Szeliski

Maneesh Agrawala
Michael Cohen

Hugues Hoppe
Kentaro Toyama

Microsoft Corporation

SIGGRAPH 2004

Idea

No-flash

Lighting is “warm and cozy”,
but image is noisy from sensor

Flash

Sharp and detailed,
no noise, but ambience is lost

Can the information be combined?

Goal

No-flash

Can you see the noise?
It is noticeable
(see original paper)

Flash

No noise, but color/lighting is completely different.

RESULT

Flash details with no flash lighting!

Bilateral and Joint Bilateral Filter

- Input: $\boldsymbol{A}$ is the ambient light image (no flash), $\boldsymbol{F}$ is the flash image
- Key to this paper, use two type of filters

Bilateral Filter

$$A_p^{Base} = \frac{1}{k(p)} \sum_{p' \in \Omega} g_d(p' - p)\, g_r(A_p - A_{p'})\, A_{p'}$$

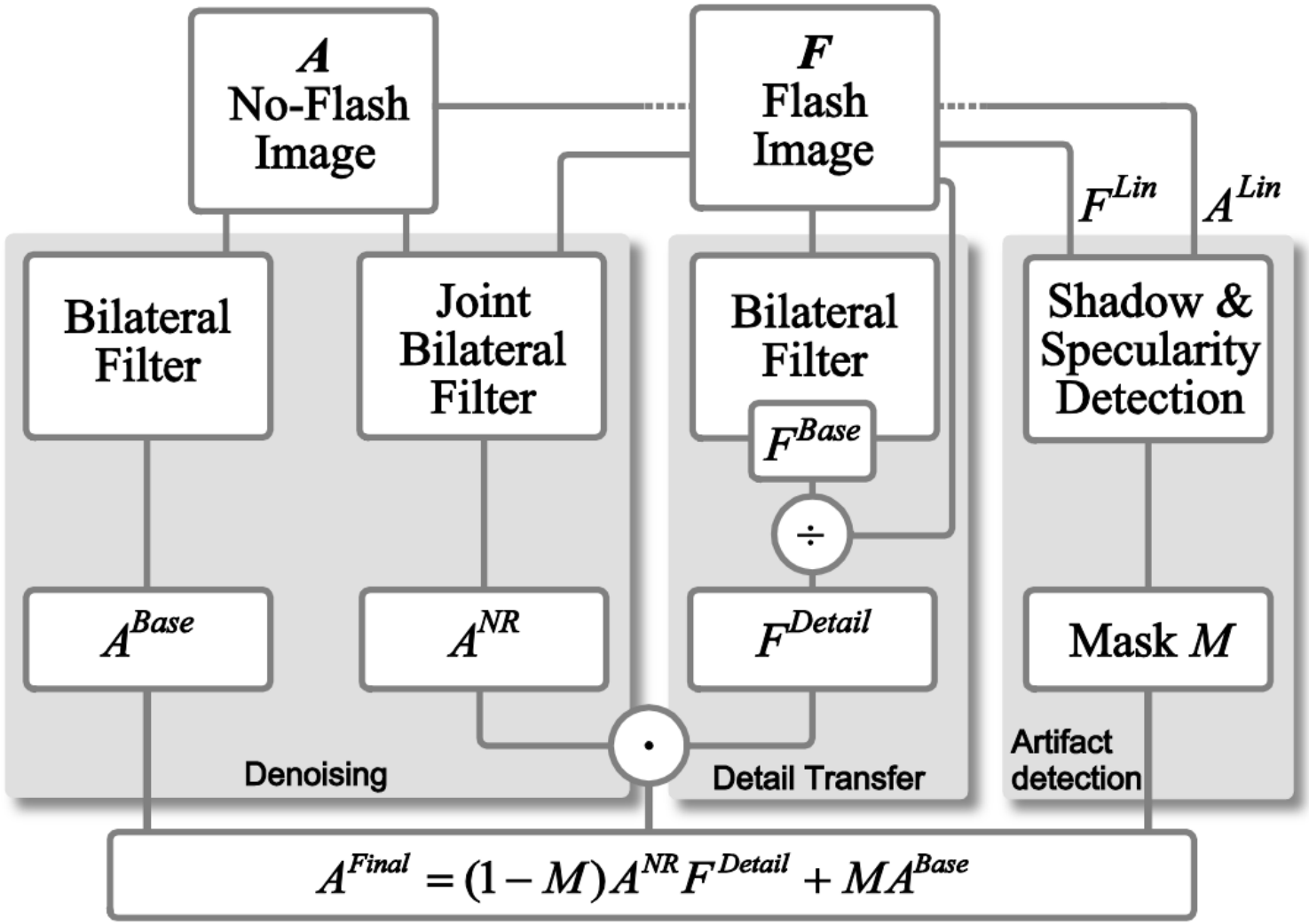
- Use flash image (with little noise) to provide range weight in the bilateral filter

Joint Bilateral Filter

$$A_p^{NR} = \frac{1}{k(p)} \sum_{p' \in \Omega} g_d(p' - p)\, \boxed{g_r(F_p - F_{p'})}\, A_{p'}$$

Flash image provides range scaling for BF for image A

Overall Procedure



$$F^{Detail} = \frac{F+\varepsilon}{F^{Base}+\varepsilon}$$

Putting it together

input

A F

A^{Final} = (1-M)

A^{NR}

Joint bilateral filter result

F^{detail}

Bilateral filter result on F, divided by original F (this is a high-pass filter)

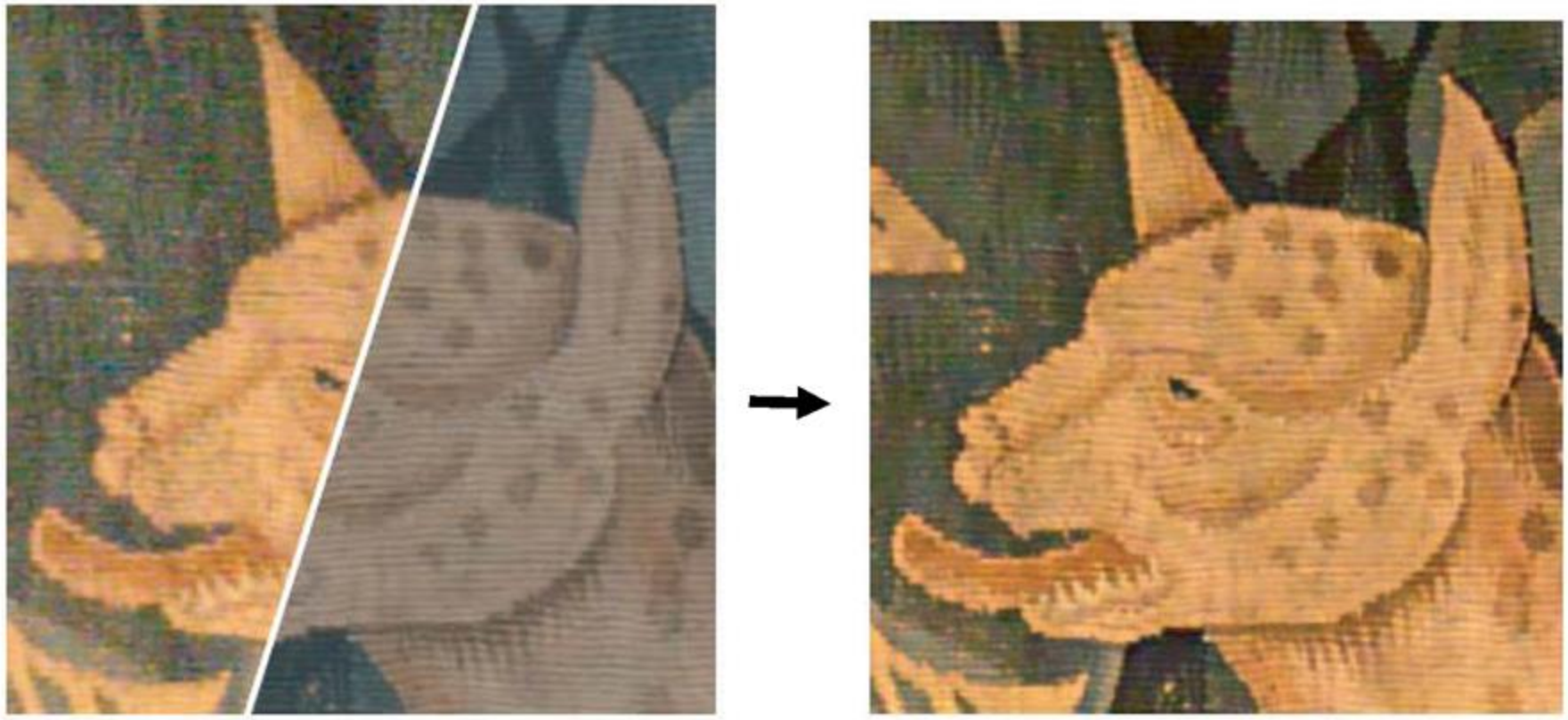
+ M

A^{Base}

Bilater filter on A

(this term is only used when M = 1, i.e. problem in flash image)

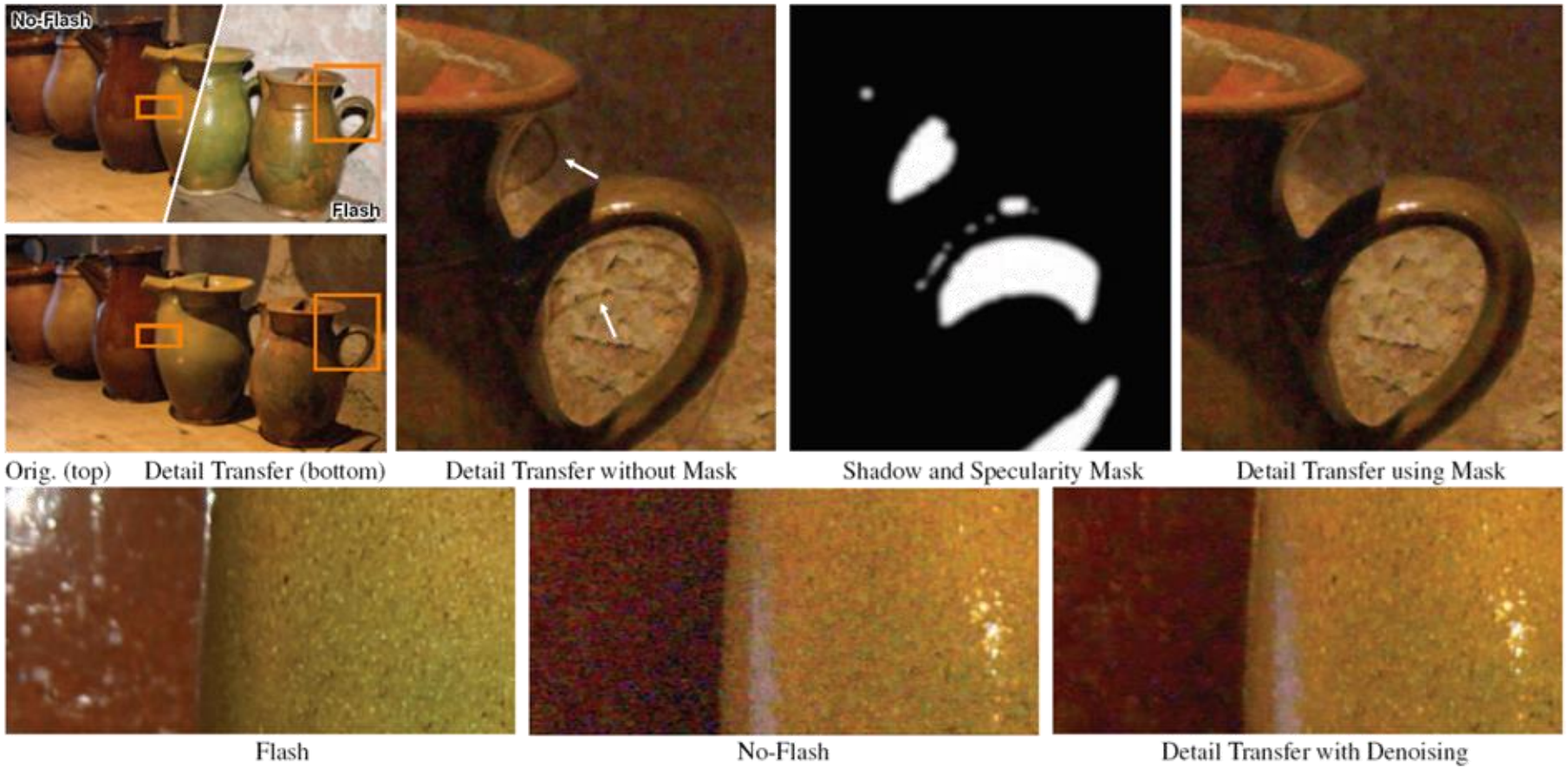
Example



Input, notice noise in
A image.

Output

Mask M



Orig. (top) Detail Transfer (bottom) | Detail Transfer without Mask | Shadow and Specularity Mask | Detail Transfer using Mask

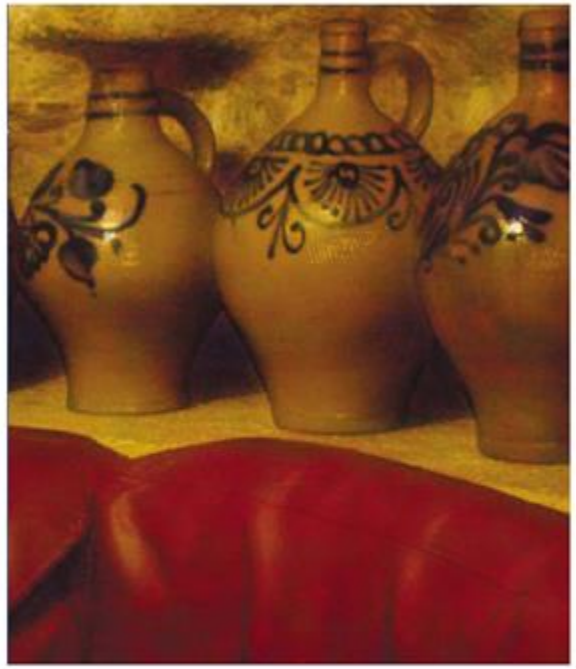
Flash | No-Flash | Detail Transfer with Denoising

Flash can result in secularities, these are avoided using a binary mask M. See equations.

Results

Flash

No-Flash



Result

More Reults

No-Flash Flash Red-Eye Corrected

Questions?

Guided Image Filtering

Kaiming He[1], Jian Sun[2], and Xiaoou Tang[1,3]

[1] Department of Information Engineering, The Chinese University of Hong Kong
[2] Microsoft Research Asia
[3] Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences, China

ECCV 2010

Formulation

- Given an input image P, the output image Q is computed as,

$$Q_i = \sum_j W_{ij}(I) P_j$$

- The weights are computed from the guide image I,

$$W_{ij}(I) = \frac{1}{|\omega|^2} \sum_{k:(i,j)\in\omega_k} \left(1 + \frac{(I_i - \mu_k)(I_j - \mu_k)}{\sigma_k^2 + \epsilon}\right)$$

the total number of such windows containing both i and j

the k-th local window containing both i and j

μ_k, σ_k are the mean and variance of pixel values in the window ω_k

Spatially-Variant Kernals

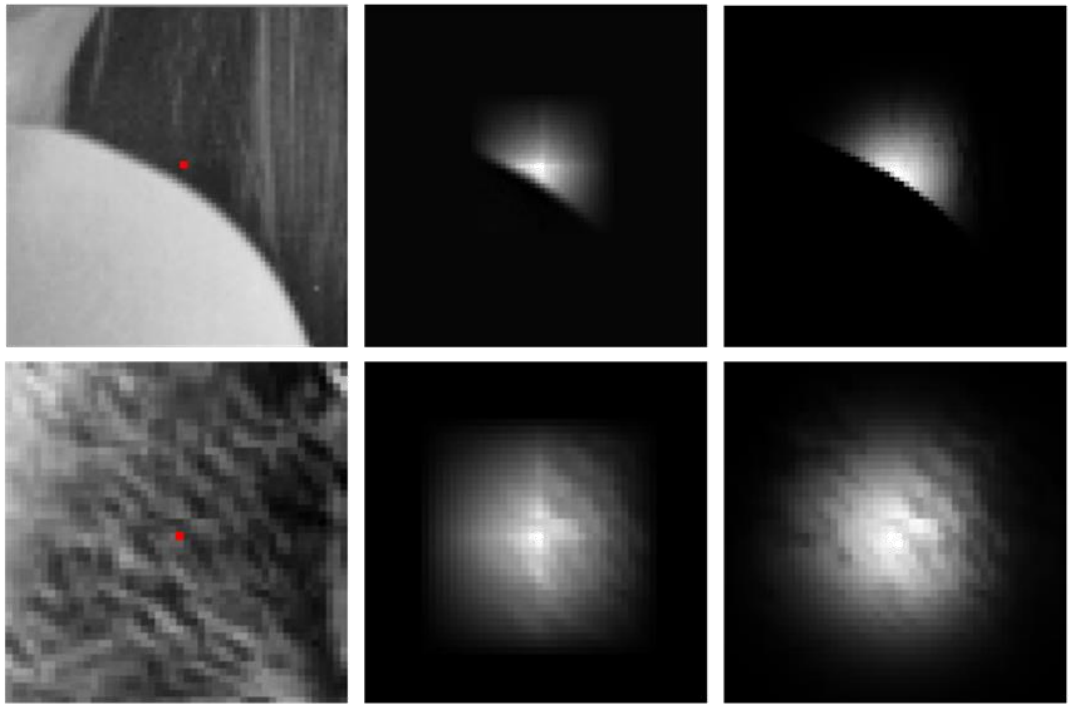
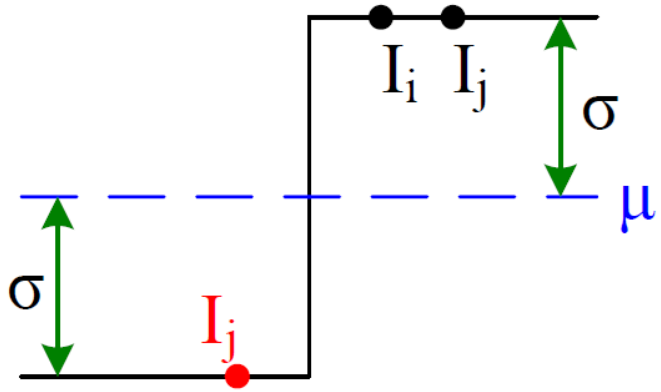


Fig. 3. Filter kernels. Top: a step edge (guided filter: $r = 7, \epsilon = 0.1^2$, bilateral filter: $\sigma_s = 7, \sigma_r = 0.1$). Bottom: a textured patch (guided filter: $r = 8, \epsilon = 0.2^2$, bilateral filter: $\sigma_s = 8, \sigma_r = 0.2$). The kernels are centered at the pixels denote by the red dots.

Explanation

- When i and j are from different sides of an edge,

$$\frac{(I_i - \mu_k)(I_j - \mu_k)}{\sigma_k^2} \sim -1$$



- When i and j are from the same side,

$$\frac{(I_i - \mu_k)(I_j - \mu_k)}{\sigma_k^2} \sim 1$$

$$W_{ij}(I) = \frac{1}{|\omega|^2} \sum_{k:(i,j)\in\omega_k} \left(1 + \frac{(I_i - \mu_k)(I_j - \mu_k)}{\sigma_k^2 + \epsilon}\right)$$

Justification

- How is the filter kernel designed??
- Inspired by the Matting Laplacian
  - Assuming the output Q is a linear scale of I within a local window
    i.e. $Q_i = a_k I_i + b_k, \forall\, i \in \omega_k$
  - Or, more intuitively, q has an edge only if I has an edge,
    i.e. $\nabla Q = \nabla I$
- More in the paper (one Jacobian iteration of solving the Matting Laplacian)

Better Than Bilateral Filter

- $\mathrm{O}(N)$ time, N is the number of pixels
  - Bilateral filter is $O(Nr^2)$, r is the local window size

$$Q_i = \sum_j W_{ij}(I) P_j$$

$$W_{ij}(I) = \frac{1}{|\omega|^2} \sum_{k:(i,j)\in\omega_k} \left(1 + \frac{(I_i - \mu_k)(I_j - \mu_k)}{\sigma_k^2 + \epsilon}\right)$$

μ_k, σ_k can be computed with box filter ($\mathrm{O}(N)$).

Homework 1: Photometric Stereo

- Deadline: 2023.12.17 23:59

Homework 1: Photometric Stereo

编辑

作业属性 | 作业批改 | 成绩统计

占成绩比例	10.0%	公布成绩时间	不公布
作业交付截止	2023.12.20 23:59	作业形式	个人作业（全部学生：120人）
计分规则	最后一次得分	完成指标	提交作业

评分方式（教师评阅 100.0%）

教师评阅

占成绩比例	100.0%

Please refer to the supplementary files (附件) for the detailed illustration of the homework (Homework 1.pdf) and the test data (Homework 1.zip).
Note that the deadline for the submission is 23:59, December 17th (2023). You can still upload it until December 20th (2023), but the penalty of 20% score reduction per 12 hours after ddl will be applied.

Submission & Late Policy

- Assignments are submitted via Learning in ZJU (学在浙大）.

- For a late submission, penalty of 20% score reduction per 12 hours is applied. If one submits an assignment within 12 hours after the deadline, the score becomes 80%. Within 24 hours, 60%, and so on.