P510 7

Let $f(n)$ denote the number of bit strings of length n that contain a pair of consecutive 0s.

a) By looking at the last two digits of bit strings of length n-1 we can conclude that $f(n) = f(n-1)+f(n-2)+2^{n-2}$ for $n \geq 2$.

b) $f(0)=0$ and $f(1)=0$.

c) From the recurrence relation we can have $f(4)=8$, $f(5)=19$, $f(6)=43$, and finally $f(7)=94$.

So there are 94 bit strings of length 7 that contain a pair of consecutive 0s.

P511 17

Let $f(n)$ denote the number of ternary strings of length n that do not contain consecutive symbols that are the same.

a) By looking at the last two digits of bit strings of length n-1 we can conclude that $f(n)=2*f(n-1)$.

b) $f(1)=3$.

c) We can know that $f(n)=3\cdot 2^{n-1}$ and so $f(6)=96$.

So there are 96 ternary strings of length 6 that do not contain consecutive symbols that are the same.

P525 13

We can have the characteristic equation $r^3-7r-6=0$ and we can have the roots $r_1=-1$, $r_2=3$ and $r_3=-2$.

Assume that $a_n=a\cdot(-1)^n+b\cdot 3^n+c\cdot(-2)^n$.

With the initial conditions we can have: $a+b+c=9$, $-a+3b-2c=10$ and $a+9b+4c=32$. Hence we can have $a=8$, $b=4$ and $c=-3$.

So $a_n=8\cdot(-1)^n+4\cdot 3^n-3\cdot(-2)^n$.

P526 35

The roots of the characteristic equation $r^2=4r-3$ are $r_1=1$, $r_2=3$.

For 2^n from $k\cdot 2^n=4k\cdot 2^{n-1}-3k\cdot 2^n-2+2^n$ we have $k=-4$. For $n+3$ from $a\cdot n^2+b\cdot n=4a\cdot(n-1)^2+4b\cdot(n-1)-3a\cdot(n-2)^2-3b\cdot(n-2)+n+3$ we have $a=-0.25$, $b=-2.5$. So the particular solution is $-4\cdot 2^n-\frac{n^2}{4}-\frac{5n}{2}$.

Then we have $1=a+b-4$ and $4=a+3b-43/4$, and $a=1/8$, $b=39/8$.

So the solution is $\frac{1}{8}+\frac{39}{8}\cdot 3^n-4\cdot 2^n-\frac{n^2}{4}-\frac{5n}{2}$.

P535 21

a) We simply have $f(4)=3$, and then $f(16)=7$.

So $f(16)=7$.

b) Given that n is a perfect square and $f(2)=1$, we can assume that $\mathrm{n}=2^{2^m}$. From the recurrence relation we can have $\mathrm{f(n)}=2^m f\left(\sqrt[2^m]{n}\right)+2^m-1=2^m f(2)+2^m-1$, and $2^m=\log n$.

So $f(n)=O(\log n)$.