

Theory of Computation, Fall 2021

Assignment 3 (Due October 16 Saturday 9:35am)

Q1. [1, Chapter 1.3] For any two regular expressions R_1 and R_2 , we say $R_1 = R_2$ if $L(R_1) = L(R_2)$. Let R be a regular expression. Are the following statements true or false? Provide counterexamples for false statements.

- (a) $R \cup \emptyset = R$ \top
- (b) $R\emptyset = R \setminus \phi$
- (c) $R \cup \emptyset^* = R$ false
- (d) $R\emptyset^* = R$ \top

Q2. Write a regular expression for the language $(a^*ba^*ba^*ba^*)^*$

$\{w \in \{a, b\}^* : \text{the number of } b\text{'s in } w \text{ is divisible by } 3\}$.

Q3. Consider the NFA N in Figure 1. Construct a regular expression R such that $L(R) = L(N)$. You should strictly follow the algorithm we used in the class, and show all the intermediate steps. More precisely, you should first convert N into an equivalent NFA that satisfies certain conditions, and then eliminate state q_1 and q_2 in order.

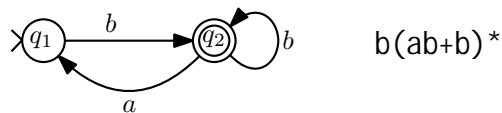


Figure 1: N

Q4. Let N be the NFA in Figure 1. Someone constructs another NFA N' as in Figure 2 and claims that $L(N') = (L(N))^*$. Prove that he/she is wrong. (Hint: It suffices to find a string that is in $L(N')$ but not in $(L(N))^*$).

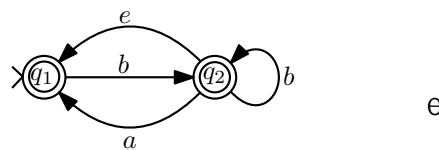
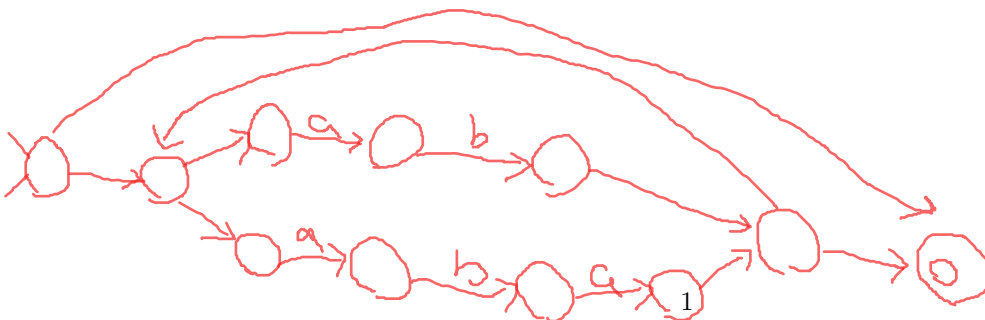


Figure 2: N'

Q5. Construct a NFA that accepts $(ab \cup aba)^*$.



Q6. Are the following statement true or false? Explain your answer. You may use the fact that the language $\{a^i b^i : i \geq 0\}$ is not regular (we will prove this using pumping theorem in next class).

- (a) Every subset of a regular language is regular. F. a^*b^* is regular but $a^i b^i$ is not
- (b) The union of a finite number of regular languages must be regular. True
- (c) Languages that contain only a finite number of strings must be regular. True
- (d) The union of an infinite number of regular languages must be regular. False. infinite states in the machine
- (e) The intersection of an infinite number of regular languages must be regular.
(Hint: Think about the De Morgan's Laws)

$$L \cup L' = (L' \cap L')'$$

References

- [1] Sipser M.. Introduction to the Theory of Computation. CENGAGE Learning (2013)
- [2] Lewis H., Papadimitriou C.. Elements of the Theory of Computation. Prentice-Hall (1998)