Theory of Computation, Fall 2021
Assignment 7 (Due December 17 Friday 9:35am)

1. Sigma^* is countable
2. subset of a countable set is also countable
3. every language is a subset of Sigma^*

Q1. Prove that every language is countable. You can use any theorem that we've proved in class.

Q2. Prove that there is an undecidable subset of $\{1\}^*$. {0,1}* \to {1}*

Q3. We have already proved that the following language is not recursive. Show that it is recursively enumerable.

1. for i=1,2,3,..
run M on each for i steps
2. halt if M halts within i steps

$$A = \{``M" : M \text{ is a Turing machine that halts on some string}\}$$

(Hint: You may use the fact that Turing machines are able to enumerate all the strings in Σ^* as $s_1, s_2, s_3, \ldots$)

Q4. Let A and B be two languages. Let f be a reduction from A to B. w\in A iff f(w)\in B

(a) Suppose that you have a Turing machine M_B that semidecides B. Try to construct a Turing machine M_A that semidecides A.

1. run M_B on f(w)
2. halt if halt

(b) What conclusion can you draw from (a)?

Q5. Consider the following language. Show that the following language is not recursive by showing a reduction from H to L.

$$A = \{``M" : M \text{ is a TM such that for any string } w,\ M \text{ halts on a string } w^R \text{ whenever it halts on } w\}.$$

(a) Show that the A is not recursive by showing a reduction from H to A.

(b) Use Rice's theorem to prove that A is not recursive.

L is RE and both w^R and w are in L

on input u
1.10, accept
2.not 01, reject
3.01, run M on w

References

[1] Sipser M.. Introduction to the Theory of Computation. CENGAGE Learning (2013)

[2] Lewis H., Papadimitriou C.. Elements of the Theory of Computation. Prentice-Gall (1998)