Theory of Computation, Fall 2021
Assignment 5 (Due November 12 Friday 9:35am)

Q1. Let a, b, c be three distinct symbols. Use pumping theorem to show that $A = \{wcw : w \in \{a, b\}^*\}$ is not context-free. `a^p b^p c a^p b^p`

Q2. Let A be a context-free language. Let B be a regular language. Prove that $A \cap B$ is context-free. You may assume that A and B are defined over the same alphabet Σ. (Hint: let P_A be a PDA accepting A. Let M_B be an NFA accepting B. Construct a PDA $P_\cap$ that conceptually runs P_A and M_B in parallel.)

Q3. Let $A = \{w \in \{a, b, c\}^* : w$ has same number of a's, b's, and c's$\}$.

(a) Prove that A is not context-free. (Hint: It is not necessary to use pumping thoerem. You may try the conclusion of Q2.) `a*b*c* is regular` `\bigcup: a^nb^nc^n is not cfl`

(b) Show that $\overline{A}$ is context-free. (Hint: it suffices to show that $\overline{A}$ is a union of serveral context-free languages) `i \neq j \neq k`

Q4. Fix an alphabet Σ contians $\triangleright$ and $\sqcup$. Given a precise definition for the head-moving machine $M_\rightarrow$, which, regardless of the symbol it reads, always moves its head to the right and then halt immediately. `\delta(s, a)=(h, \right) a\in \Sigma-\{\\}`

Q5. What happens if you run the machine $L_\sqcup$ on the tape $\triangleright ba\underline{\sqcup}$? `loop`

Q6. Design a right-shifting machine $S_\rightarrow$ that transforms $\triangleright \sqcup w\underline{\sqcup}$ into $\triangleright \sqcup \sqcup w\underline{\sqcup}$, where w is a string that contains no blank symbol. You may use the machines and the diagrams we presented in class. `L \to (a\neq\sqcup) \sqcupRaL_\sqcup`

References

[1] Sipser M.. Introduction to the Theory of Computation. CENGAGE Learning (2013)

[2] Lewis H., Papadimitriou C.. Elements of the Theory of Computation. Prentice-Gall (1998)