A Novel Nonlinearity Tolerant Super-Gaussian Distribution for Probabilistically Shaped Modulation

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Abstract *Probabilistically-shaped constellations are found to have larger than expected Kerr nonlinearity induced penalty in long-haul transmission. In this paper, a novel nonlinear tolerant super-Gaussian distribution has been proposed which outperforms the Maxwell-Boltzmann distribution. The simulation and experimental results both show a significant improvement.*

Introduction

In recent years, two important techniques emerged that have shown significant promise in achieving higher capacity or higher spectral efficiency (SE) for a given signal to noise ratio in the channel. Alternatively these techniques can improve signal to noise ratio (SNR) tolernace at a given SE. These techniques are namely geometric shaping (GS) and probabilistic shaping (PS). In geometric shaping, the position of constellation points will be optimized to reach higher SE or more tolerance to the fiber nonlinearity, while in probabilistic shaping the constellation points are on a uniform grid but with different probabilities at each point. Both techniques could reach up to 1.53dB improvement in SNR tolerance asymptotically for the Gaussian channel. These two methods have gained significant attention in coherent optical communication systems [1].

Probabilistic shaping is getting more attractive because the net system SE can be adjusted very finely over a wide range by just changing the probabilities of each transmitted symbol from a regular square QAM constellation set using a distribution matcher [2], meanwhile the underlying processing engine in the forward error correction (FEC) and digital signal processing (DSP) remains the same. In probabalistic shaping the labeling of the constellation points can use Gray code which gives best achievable rates for bit-wise decoders.

Although the PS modulation outperforms regular equiprobable QAM modulation in the linear regime, when propagated over long distances of fiber, the accumulated Kerr nonlinearity manifests itself stronger in PS modulation versus regular QAM modulation [6]. This nonlinear effect of the fiber can significantly offset the shaping gain obtained in the linear regime. The main reason for this degradation is that the Maxwell-Boltzmann distribution for the constellation points is optimized for the linear Additive White Gaussian Noise (AWGN) channel. For the long-haul fiber transmission this assumption is not entirely valid and the environment at optimum launch power can have a significant non-linear component. In this case there should be other considerations such as non-linear parameters of fiber to optimize the distribution of the points. In this paper, a novel super-Gaussian distribution has been proposed that can outperform the Maxwell-Boltzman distribution when subjected to long distance fiber transmission.

Problem definition

After propagating long distances, the signal in the fiber accumulates both amplified spontaneous emission (ASE) noise and nonlinear noise. For a dual-polarization transmission and adopting the approach in [4], the effective SNR of the channel, is given by:

$$\mathrm{SNR}_{\mathrm{eff}} = \frac{P_{tx}}{\sigma_{\mathrm{eff}}^2} = \frac{P_{tx}}{\sigma_{ASE}^2 + \sigma_{NLI}^2} \tag{1}$$

P_{tx} is the optical launch power, σ_{ASE}^2 is the noise variance of the ASE noise from optical amplifiers and σ_{NLI}^2 is the Non-Linear Interference (NLI) variance that includes both intra- and inter-channel nonlinear distortions. By rearranging the results in [4] and [5] the NLI variance σ_{NLI}^2 in (1) can be derived as:

$$\sigma_{NLI}^2 = P_{tx}^3[\chi_0 + (\hat{\mu}_4 - 2)\chi_4 + (\hat{\mu}_4 - 2)^2 \chi_4' + \hat{\mu}_6\chi_6] \tag{2}$$

where $\hat{\mu}_4$ and $\hat{\mu}_6$ are the standardized moments of the channel input X and χ_0, χ_4, χ_4', and χ_6 are real coefficients that represent the contributions of the fiber nonlinearities. Therefore, by combining (1) and (2), σ_{eff}^2 is:

$$\sigma_{\mathrm{eff}}^2 = \sigma_{ASE}^2 + \sigma_{NLI}^2 = \sigma_{ASE}^2 + P_{tx}^3\chi_0 + P_{tx}^3[(\hat{\mu}_4 - 2)\chi_4 + (\hat{\mu}_4 - 2)^2 \chi_4' + \hat{\mu}_6\chi_6] \tag{3}$$

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The first two parts of noise term are modulation independent and the rest is dependent to the modulation format. The K[th] standardized moment $\hat{\mu}_k$ of the channel input X is defined as:

$$\hat{\mu}_k = \frac{\mathrm{E}[|X - \mathrm{E}[X]|^k]}{(\mathrm{E}[|X - \mathrm{E}[X]|^2])^{\frac{k}{2}}} \quad (4)$$

As shown in (3) & (4), the $\hat{\mu}_4$ and $\hat{\mu}_6$ of the signal X is a function of the inherent modulation and they contribute to nonlinear distortion.

The well-known Maxwell-Boltzmann distribution has been used to characterize the distribution of X, and this has been proven to maximize capacity in the AWGN channel [6]. This distribution is shown below:

$$p(x) = e^{-\lambda x^2} / Z(\lambda) \quad , \lambda \geq 0 \quad (5)$$

where function $Z(\lambda)$ is chosen to normalize the distribution. By appropriate selection of λ, different SEs can be achieved. In (5), if we substitute the exponent power 2 with a parameter P, we have a general form of a super-Gaussian distribution.

$$p(x) = e^{-\lambda x^P} / Z(\lambda) \quad , \lambda \geq 0 \quad (6)$$

The advantage of the proposed distribution is that by modifying P, we could significantly reduce the $\hat{\mu}_4$ and $\hat{\mu}_6$ of the signal X. In fact, by changing P and λ, there are two degrees of freedom for achieving both the same SE, and at the same time, minimizing $\hat{\mu}_4$ and $\hat{\mu}_6$ of the signal X, thereby increasing nonlinear tolerance when propagating over fiber. In Table 1 below, we compare the $\hat{\mu}_4$ and $\hat{\mu}_6$ of Star-8QAM, PS-64QAM with Maxwell-Boltzmann distribution ($P = 2$, $\lambda = 0.11$, $\mathrm{SE} = 6$ b/s/Hz) and PS-64QAM with super-Gaussian distribution ($P = 3.5$, $\lambda = 0.17$, $\mathrm{SE} = 6$ b/s/Hz). All three modulations shown in the table have the same SE and dual polarization is assumed. The SE used here is the modulation SE and an FEC with 20% FEC is included for all 3 modulations.

Table. 1: 4th and 6th standardized moments of PS-64QAM and Star-8QAM, with SE = 6 b/s/Hz (dual-pol)

Modoulation format	$\hat{\mu}_4$	$\hat{\mu}_6$
PS-64QAM, $P = 2$, $\lambda = 0.11$	2	6
PS-64QAM, $P = 3.5$, $\lambda = 0.17$	1.61	3.29
Star-8QAM	1.36	2

To keep the SE of PS-64QAM modulation the same as Star-8QAM, the λ parameter in (6) should be different in each case. As it is shown, the $\hat{\mu}_4$ and $\hat{\mu}_6$ of the PS modulation with super-Gaussian distribution ($P = 3.5$) is significantly decreased compared to the Maxwell-Boltzmann distribution and it improves the performance of the PS modulation at optimum launch power. Figure 1 shows the distribution of constellation points on I/Q space for different exponent P values. It appears that by increasing P, the probability of the occurrence of the outer points are decreasing and this is the main reason that $\hat{\mu}_4$ and $\hat{\mu}_6$ are decreasing.

Simulation Results

The simulated Tx signal on each wave has a total baud rate of 33Gbaud constructed using 4 subcarriers each at 8.25Gbaud. The subcarriers are multiplexed at near Nyquist frequency spacing using an excess BW factor of 6%. 16 waves are multiplexed at near Nyquist channel spacing. The chosen modulations on each subcarrier has 6 b/s/Hz SE as shown in Table 1. The fiber used is an uncompensated large effective area fiber (TeraWave) at 67km spans. The total distance simulated is 9630km. The results are shown in Figure 2. The optimum exponent P value found is 3.5, and it provides 0.2dB higher Q at optimum launch power compared to P = 2, and the optimum launch power is also increased by a little less than 1dB.

The main advantage of the proposed distribution is that the Q improvement can be achieved just by updating the probability of the constellation points using super-Gaussian distribution. In nonlinear region, the super-Gaussian distribution is more tolerant. It can be seen in

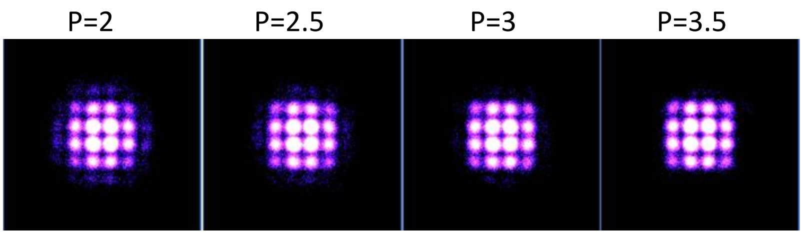

Fig. 1: Constellation points with super-Gaussian distribution for PS-64QAM with different exponent P, λ and $\mathrm{SE} = 6$ b/s/Hz.

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Figure 2 that the performance of the system with super-Gaussian distribution in linear region (low launch power) is ever slightly worse than the Maxwell-Boltzmann distribution. But this small degradation is negligible in that transmission systems generally operate close to the optimum launch power, and overall, the system with super-Gaussian shaping achieves longer reach.

In Figure 3, the ROSNR vs Launch power per wave has been shown for different modulation, at optimum launch power the PS-64QAM super-Gaussian has 0.7 dB ROSNR gain compare to the PS-64QAM with Maxwell-Boltzmann distribution over 151 spans of uncomensated TeraWave fiber with the length 9630 KM.

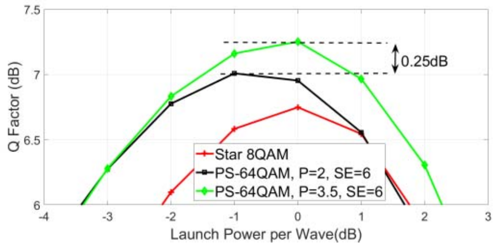

Fig. 2: Q vs Launch Power per wave for 6 b/s/Hz for 9360 Km uncompensated fiber

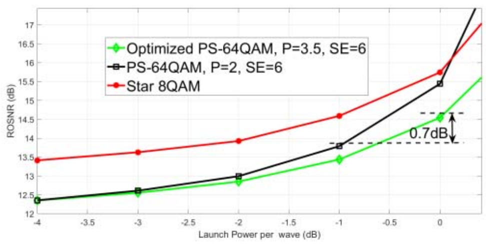

Fig. 3: ROSNR vs Launch power per wave for SE=6 b/s/Hz and different modulation formats

Experiment Results

The experiment was carried out using a recirculating loop with span length of 70km and TeraWave fiber. The total distance is ~4000km and uncompensated. 36 waves are bulk modulated using two sets of coherent dual-polarization transmitters. For high spectral efficiency, the waves are multiplexed at 6% excess BW factor. Each wave is constructed by digital multiplexing four 8.2Gbaud subcarriers with excess BW factor of 6%. The composite signal has electrical singled sided BW of about 17GHz. The four XI, XQ, YI, YQ RF signals at the output of four high speed 64GS/s DACs are driven into off-the-shelf dual-polarization MZ modulators. In the receiver off-the-shelf coherent receivers are used together with high speed sampling scope. The captured data post-processing is done using sophisticated DSP algorithms such as equalizer training using known symbols, and feed-forward carrier recovery using blind phase search algorithm. Figure 4 plots the measured relative Q versus launch power for different exponent P values. It can be seen that the best performance in the experiment would be at P = 4. It is shown that super-Gaussian distribution for PS-64QAM can have 0.25 dB Q gain at optimum lauch power compare to Maxwell-Boltzmann distribution.

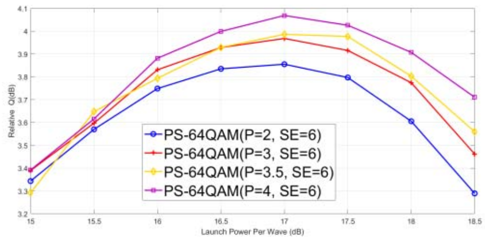

Fig. 4: Measured super-Gaussian nonlinear performance for 4000 km of uncompensated TeraWave fiber at PS-64QAM SE=6 b/s/Hz.

Conclusions

A super-Gaussian distribution has been proposed which outperforms Maxwell-Boltzmann distribution for probabilistically shaped signals. Both simulation and experiment results have shown that the proposed distribution have higher nonlinear tolerance while having negligible penalty in the linear region.

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