P292 13

a) $0+0+0+0+0+1+1+1+1+1+1 \equiv 0(\textbf{mod } 2)$

   This string is correct.

b) $1+0+1+0+1+0+1+0+1+0+1 \equiv 0(\textbf{mod } 2)$

   This string is correct.

c) $1+1+1+1+1+1+0+0+0+0+0 \equiv 0(\textbf{mod } 2)$

   This string is correct.

d) $1+0+1+1+1+1+0+1+1+1+1 \equiv 1(\textbf{mod } 2)$

   This string has an error.

P304 11

From the function we can know that $c-13 \equiv 15p$ ($\textbf{mod}$ 26). From Euclidean algorithm we can have 1+4*26=7*15, so the inverse of 15 modulo 26 is 7.

So we have $p \equiv 7(c-13) \equiv 7c+13(\textbf{mod } 26)$.

The decryption function is $p \equiv 7c+13(\textbf{mod } 26)$.

P305 27

From Euclidean algorithm we can know that 1+5*2436=937*13. The inverse of e=13 modulo 42·58 is 937.

Then we have 667^{937} **mod** 43·59 =1808, 1947^{937} **mod** 43·59 = 1121 and 671^{937}**mod** 43·59 = 0417.

It's clearly that 1808 1121 0417 means SILVER.

Then the plaintext is SILVER.

P330 13

Let *P*(*n*) denote $1^2-2^2+\cdots+(-1)^{n-1}n^2=(-1)^{n-1}\frac{n(n+1)}{2}$.

Basic step:

When *n*=1, $1^2=(-1)^0\frac{1(1+1)}{2}=1$ and *P*(1) is true.

Inductive step:

For any integer *k*>1, let's assume that *P*(*k*) is true. Then we have:

$$1^2-2^2+\cdots+(-1)^k(k+1)^2$$
$$=(-1)^{k-1}\frac{k(k+1)}{2}+(-1)^k\frac{(k+1)^2}{2}$$
$$=(-1)^k(k+1)(k+1-\frac{k}{2})$$
$$=(-1)^k\frac{(k+1)(k+2)}{2}$$

So *P*(*k*+1) is true. This completes the inductive step.

$1^2-2^2+\cdots+(-1)^{n-1}n^2=(-1)^{n-1}\frac{n(n+1)}{2}$ is true.

P330 27

Let $P(n)$ denote $1+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{n}}>2(\sqrt{n+1}-1)$.

Basic step:

When n=1, $1>2(\sqrt{2}-1)$. So $P(1)$ is true.

Inductive step:

For any integer k>1, assume that $P(k)$ holds. Then $1+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{\mathrm{k}}}+\frac{1}{\sqrt{\mathrm{k}+1}}>2\left(\sqrt{\mathrm{k}+1}-1\right)+\frac{1}{\sqrt{\mathrm{k}+1}}$. To prove $P(k+1)$ is true, we need to prove that $2\sqrt{k+1}+\frac{1}{\sqrt{\mathrm{k}+1}}>2\sqrt{k+2}$, which is equivalent to $\frac{1}{\sqrt{\mathrm{k}+1}}>2\sqrt{k+2}-2\sqrt{k+1}$. We can easily know that $2\sqrt{k+2}-2\sqrt{k+1}=\frac{2}{\sqrt{k+2}+\sqrt{k+1}}<\frac{2}{2\sqrt{\mathrm{k}+1}}=\frac{1}{\sqrt{\mathrm{k}+1}}$. So $P(k+1)$ is true.

This completes the inductive step.

So for every positive integer n, $1+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{n}}>2(\sqrt{n+1}-1)$ is true.