P550 23

a) $(x^2+\cdots+x^{k-2})(1+x^1+x^2+x^3)(x^2+x^3+x^4+x^5)=\frac{x^4(1+x+x^2+x^3)^2}{1-x}$.

So the generating function is $\frac{x^4(1+x+x^2+x^3)^2}{1-x}$.

b) 3+2+1=6.

So a_6 is 6.

P550 33

Let *G(x)* be the generating function of the sequence.

So $G(x)=\sum_{k=0}^{\infty} a_k\, x^k$.

So $G(x)-1=3\sum_{k=1}^{\infty} a_{k-1}\, x^k+2\sum_{k=1}^{\infty} x^k$

$=3x\sum_{k=0}^{\infty} a_k\, x^k+\frac{2x}{1-x}$

$=3xG(x)+\frac{2x}{1-x}$

So $G(x)=\frac{1+x}{(1-3x)(1-x)}$

$=\frac{2}{(1-3x)}-\frac{1}{1-x}$

$=2\sum_{k=0}^{\infty}(3x)^k-\sum_{k=0}^{\infty} x^k$

Consequently $a_k=2\cdot 3^k-1$.

P558 9

By the principle of inclusion-exclusion we have 507+ 292+312+344-14-213-211-43-0-0+0+0+0+0-0=974.

So there are 974 students.

P564 5

We only need to consider the multiples of 2, 3, 5, 7, 11 and 13. Let P_1 be the property that an integer is divisible by 2, let P_2 be the property that an integer is divisible by 3, let P_3 be the property that an integer is divisible by 5, let P_4 be the property that an integer is divisible by 7, let P_5 be the property that an integer is divisible by 11, let P_6 be the property that an integer is divisible by 13.

Thus the total number is $6 + N(P_1'P_2'P_3'P_4'P_5'P_6')$ and $N(P_1'P_2'P_3'P_4'P_5'P_6') = 199 - N(P_1) - N(P_2) - N(P_3) - N(P_4) - N(P_5) - N(P_6) + N(P_1P_2) + \cdots + \mathrm{N}(P_1P_2P_3P_4P_5P_6)$.

So $N(\mathrm{P}_1'\mathrm{P}_2'\mathrm{P}_3'\mathrm{P}_4'\mathrm{P}_5'\mathrm{P}_6') = 199 - 100 - 66 - 40 - 28 - 18 - 15 + 33 + 20 + 14 + 9 + 7 + 13 + 9 + 6 + 5 + 5 + 3 + 3 + 2 + 2 + 1 - 6 - 4 - 3 - 2 - 2 - 1 -$

$$1 - 1 - 1 - 0 - 1 - 1 - 1 - 0 - 0 - 0 - 0 - 0 - 0 - 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 - 0 - 0 - 0 - 0 - 0 - 0 + 0 = 40$$

So the number of primes less than 200 is 46.

P565 9

We can easily have 3^6-C(3,1)2^6+C(3,2)=540.

So there are 540 ways.

P581 7

a) Symmetric.

b) Symmetric and transitive.

c) Symmetric.

d) Reflexive symmetric and transitive.

e) Reflexive and transitive.

f) Reflexive, symmetric and transitive.

g) Antisymmetric.

h) Antisymmetric and transitive.

P582 31

a) $\{(a, b) | a$ is required to read or has read $b\}$

b) $\{(a, b) | a$ is required to read and has read $b\}$

c) $\{(a, b) |$ either a is required to read but has not read b or a is not required to read but has read $b\}$

d) $\{(a, b) | a$ is required to read but has not read $b\}$

e) $\{(a, b) | a$ is not required to read but has read $b\}$

P583 55

We can use mathematical induction.

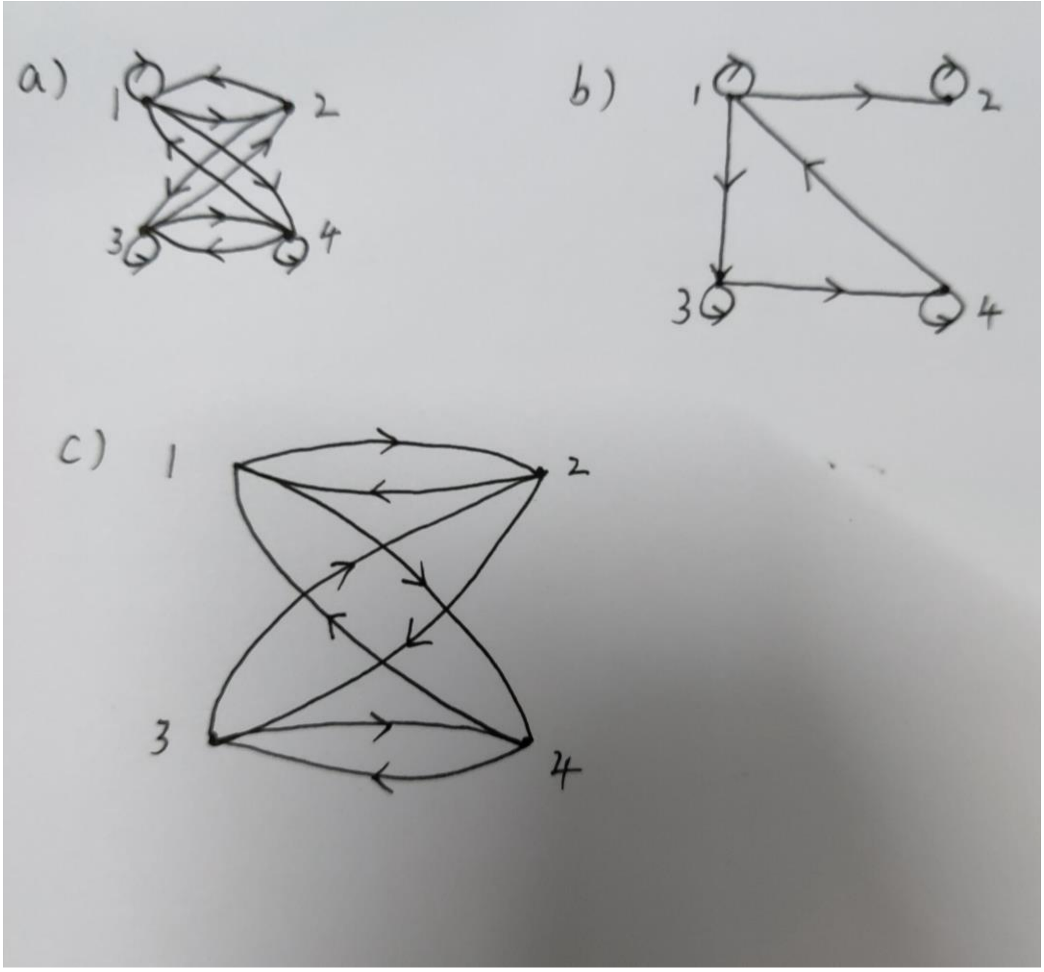
Basic step:

When k=1, R=R is obviously valid.

Inductive step:

Assume that for any integer k≥1， $R^k = R$ and so R^k is reflexive and transitive. Because R is transitive, we can easily have $R^{k+1} \subseteq R$. Then for any $(a, b) \in R$, we have $(b, b) \in R^k$ because R^k is reflexive. Thus $(a, b) \in (R^k \circ R) = R^{k+1}$. So $R \subseteq R^{k+1}$ and we have $R^{k+1} = R$. This completes the inductive step.

P597 21


a)
1
2
3
4
b)
1
2
3
4
c)
1
2
3
4