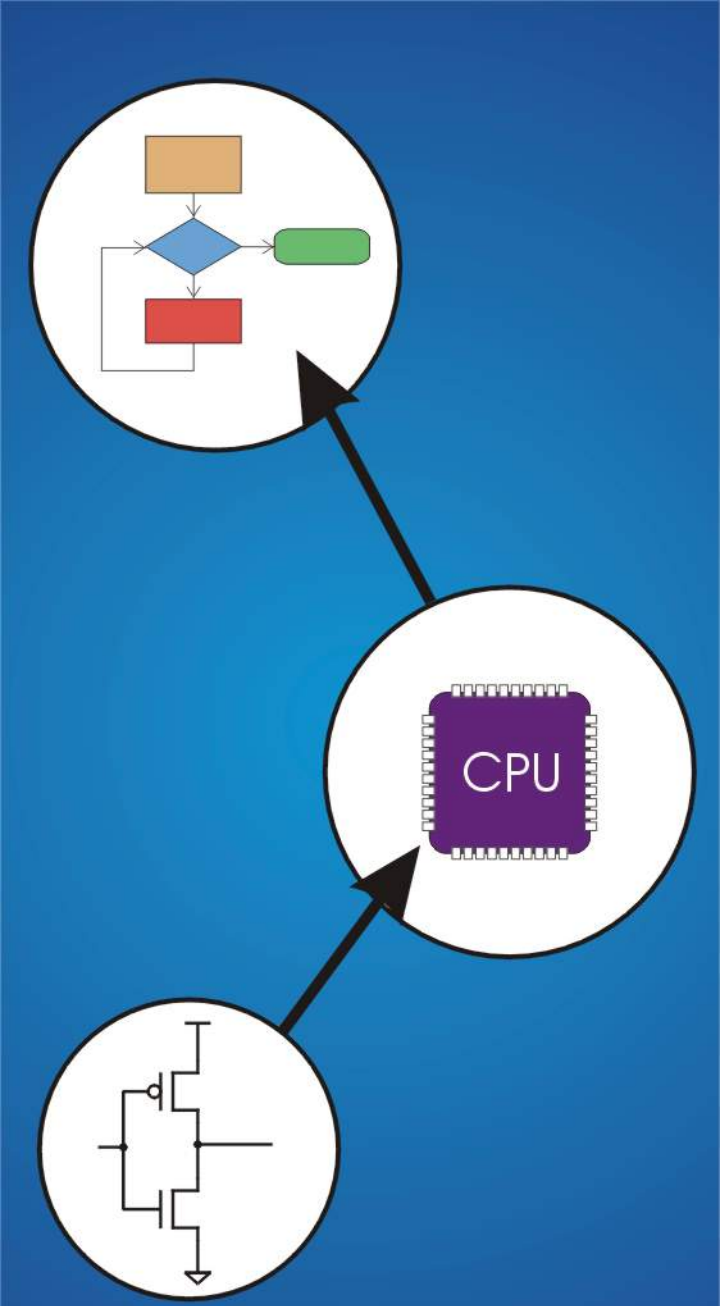


Chapter 17
Recursion

What is Recursion?

A recursive function is one that solves its task by calling itself on smaller pieces of data.

- **Similar to recurrence function in mathematics.**
- **Like iteration -- can be used interchangeably; sometimes recursion results in a simpler solution.**

Example: Running sum ($\sum_{1}^{n} i$)

Mathematical Definition:

RunningSum(1) = 1

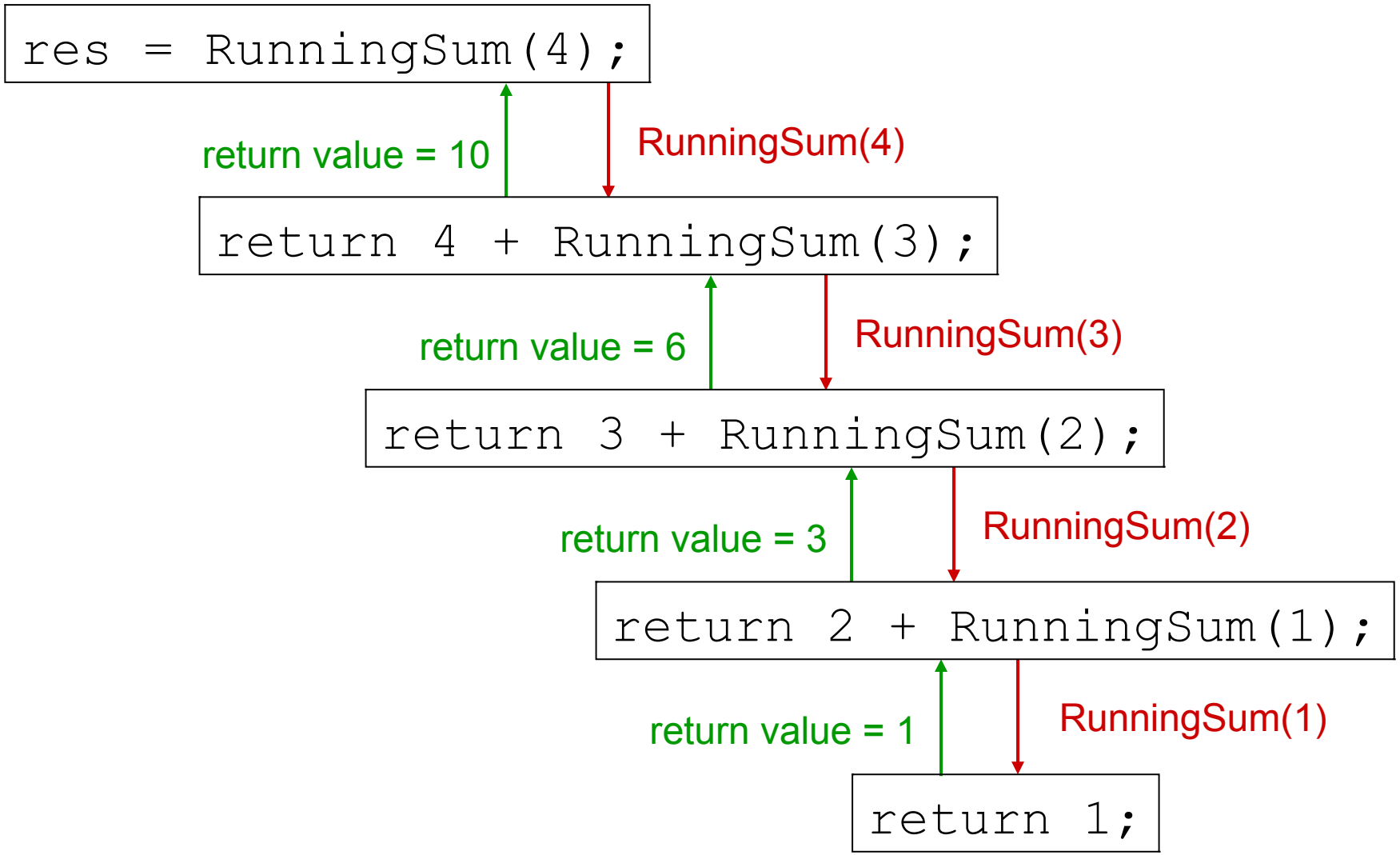
RunningSum(n) =

n + RunningSum(n-1)

Recursive Function:

```
int RunningSum(int n) {
  if (n == 1)
    return 1;
  else
    return n + RunningSum(n-1);
}
```

Executing RunningSum



High-Level Example: Binary Search

Given a sorted set of exams, in alphabetical order, find the exam for a particular student.

1. **Look at the exam halfway through the pile.**
2. **If it matches the name, we're done; if it does not match, then...**

3a. **If the name is greater (alphabetically), then search the upper half of the stack.**

3b. **If the name is less than the halfway point, then search the lower half of the stack.**

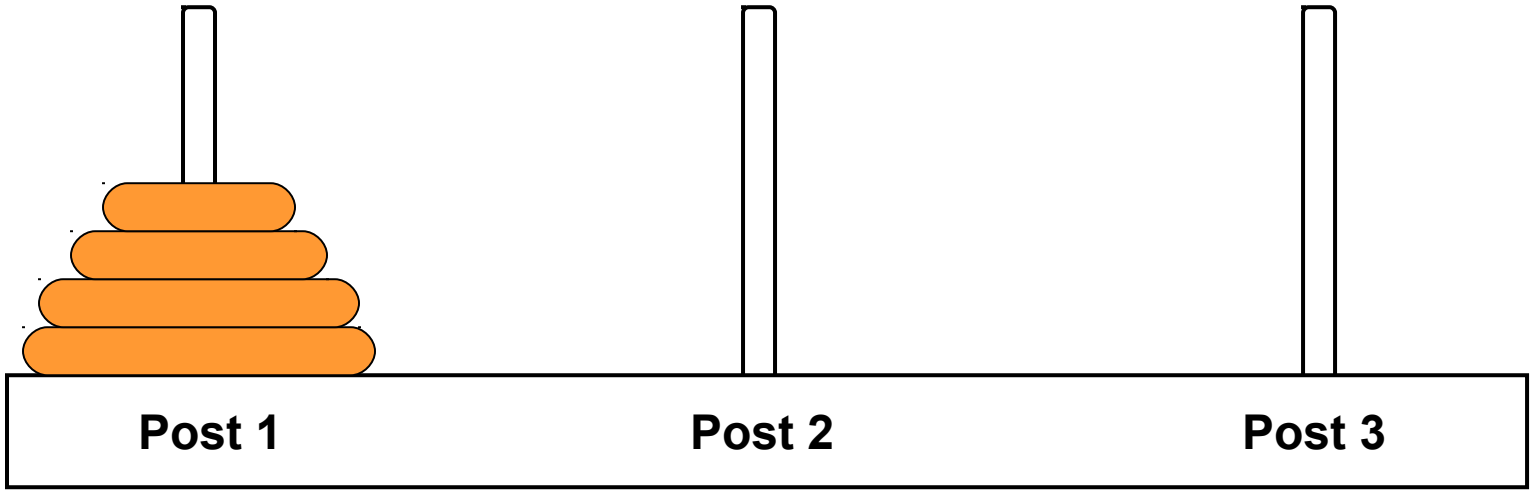
Binary Search: Pseudocode

Pseudocode is a way to describe algorithms without completely coding them in C.

```
FindExam(studentName, start, end)
{
  halfwayPoint = (end + start)/2;
  if (end < start)
    ExamNotFound();  /* exam not in stack */
  else if (studentName == NameOfExam(halfwayPoint))
    ExamFound(halfwayPoint); /* found exam! */
  else if (studentName < NameOfExam(halfwayPoint))
    /* search lower half */
    FindExam(studentName, start, halfwayPoint - 1);
  else /* search upper half */
    FindExam(studentName, halfwayPoint + 1, end);
}
```

High-Level Example: Towers of Hanoi

Task: Move all disks from current post to another post.



Rules:

(1) Can only move one disk at a time.

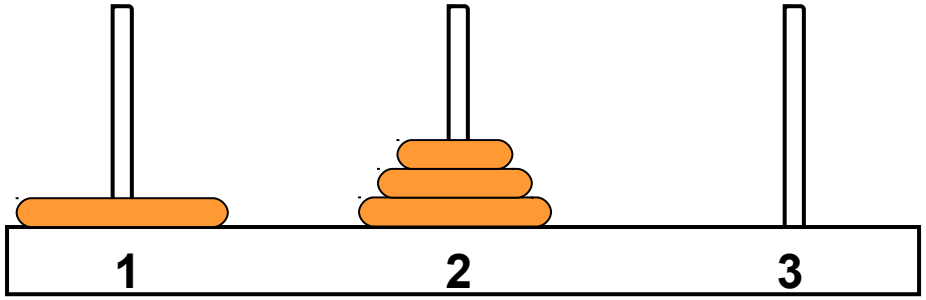
(2) A larger disk can never be placed on top of a smaller disk.

(3) May use third post for temporary storage.

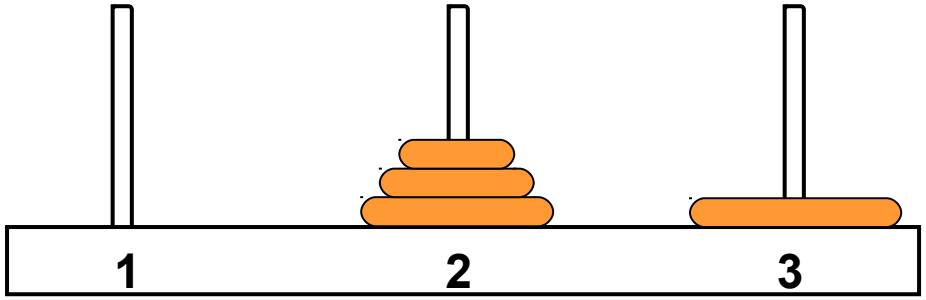
Task Decomposition

Suppose disks start on Post 1, and target is Post 3.

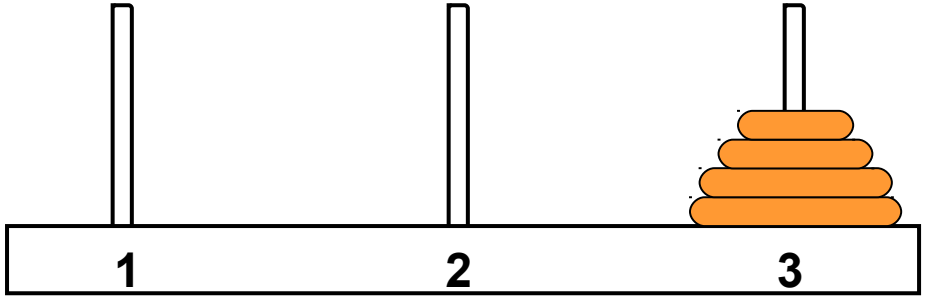
1. **Move top n-1 disks to Post 2.**



2. **Move largest disk to Post 3.**



3. **Move n-1 disks from Post 2 to Post 3.**



Task Decomposition (cont.)

Task 1 is really the same problem,
with fewer disks and a different target post.

- **"Move n-1 disks from Post 1 to Post 2."**

And Task 3 is also the same problem,
with fewer disks and different starting and target posts.

- **"Move n-1 disks from Post 2 to Post 3."**

So this is a recursive algorithm.

- **The terminal case is moving the smallest disk -- can move directly without using third post.**
- **Number disks from 1 (smallest) to n (largest).**

Towers of Hanoi: Pseudocode

```
MoveDisk(diskNumber, startPost, endPost, midPost)
{
  if (diskNumber > 1) {
    /* Move top n-1 disks to mid post */
    MoveDisk(diskNumber-1, startPost, midPost, endPost);

    printf("Move disk number %d from %d to %d.\n",
           diskNumber, startPost, endPost);

    /* Move n-1 disks from mid post to end post */
    MoveDisk(diskNumber-1, midPost, endPost, startPost);
  }
  else
    printf("Move disk number 1 from %d to %d.\n",
           startPost, endPost);
}
```

Detailed Example: Fibonacci Numbers

Mathematical Definition:

$$f(n) = f(n-1) + f(n-2)$$

$$f(1) = 1$$

$$f(0) = 1$$

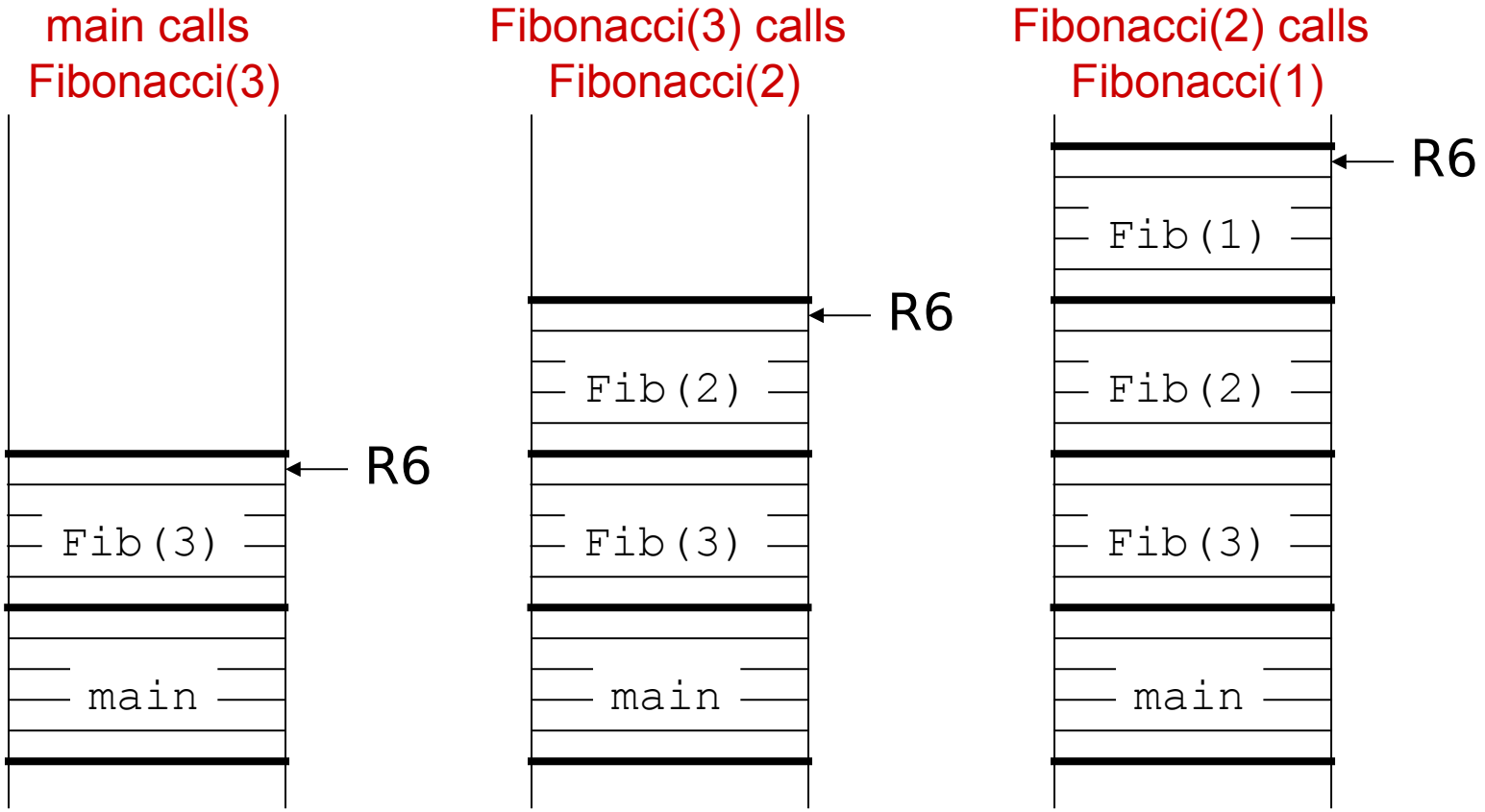
In other words, the n-th Fibonacci number is the sum of the previous two Fibonacci numbers.

Fibonacci: C Code

```
int Fibonacci(int n)
{
  if ((n == 0) || (n == 1))
    return 1;
  else
    return Fibonacci(n-1) + Fibonacci(n-2);
}
```

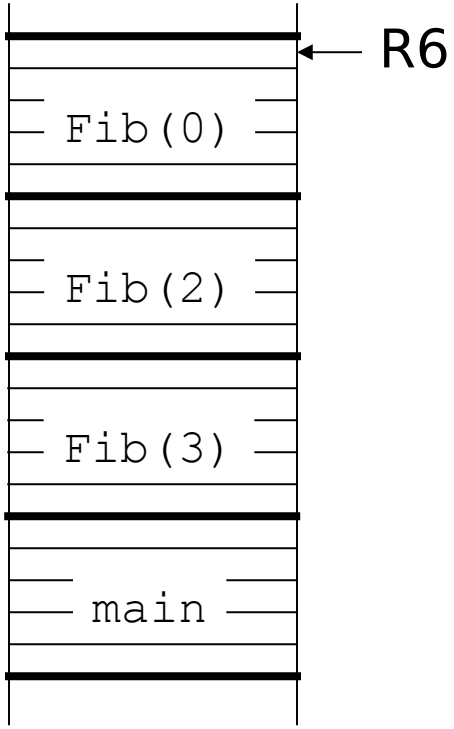
Activation Records

Whenever Fibonacci is invoked,
a new activation record is pushed onto the stack.

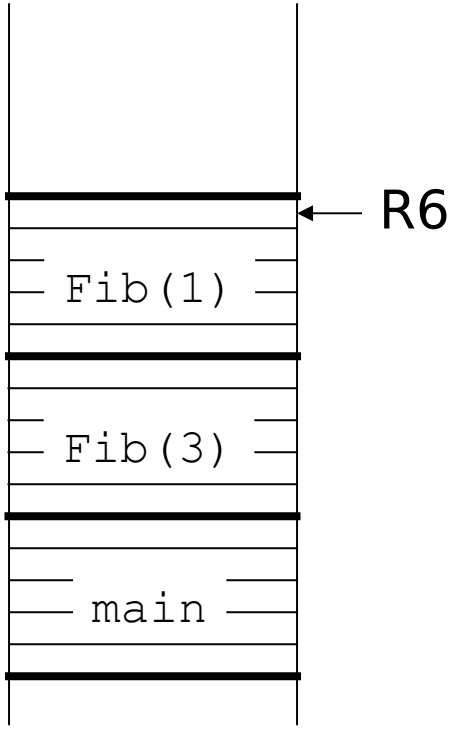


Activation Records (cont.)

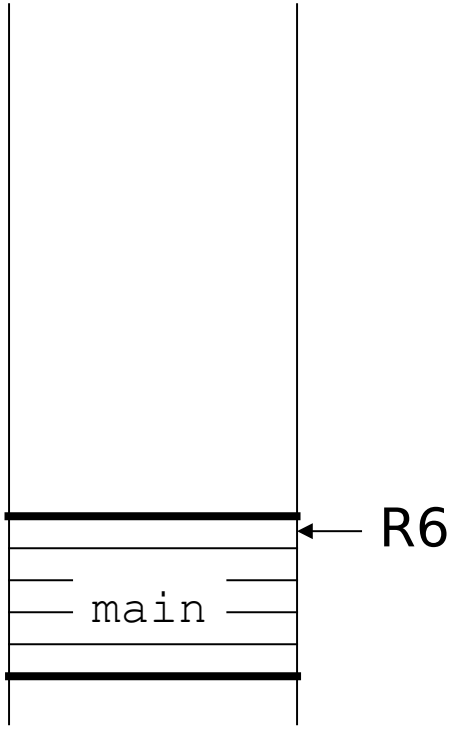
Fibonacci(1) returns, Fibonacci(2) calls Fibonacci(0)



Fibonacci(2) returns, Fibonacci(3) calls Fibonacci(1)



Fibonacci(3) returns



Tracing the Function Calls

If we are debugging this program, we might want to trace all the calls of Fibonacci.

- **Note: A trace will also contain the arguments passed into the function.**

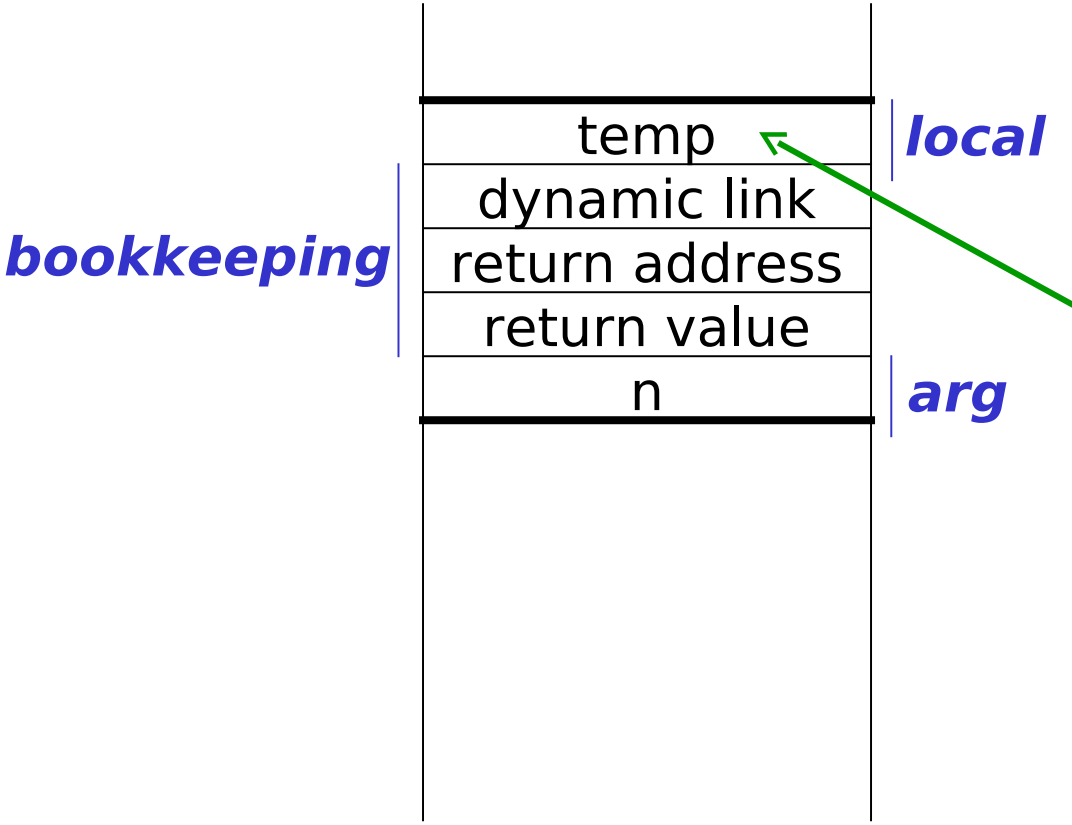
For Fibonacci(3), a trace looks like:

```
Fibonacci(3)
 Fibonacci(2)
  Fibonacci(1)
  Fibonacci(0)
 Fibonacci(1)
```

What would trace of Fibonacci(4) look like?

Fibonacci: LC-3 Code

Activation Record



LC-2 Code (part 1 of 3)

```
Fibonacci   ADD   R6, R6, #-2    ; skip ret val, push ret addr
            STR   R7, R6, #0
            ADD   R6, R6, #-1    ; push dynamic link
            STR   R5, R6, #0
            ADD   R5, R6, #-1    ; set frame pointer
            ADD   R6, R6, #-2    ; space for locals and temps

            LDR   R0, R5, #4     ; load n
            BRz   FIB_BASE       ; check for terminal cases
            ADD   R0, R0, #-1
            BRz   FIB_BASE
```

LC-3 Code (part 2 of 3)

```
LDR   R0, R5, #4   ; read parameter n
ADD   R0, R0, #-1  ; calculate n-1
ADD   R6, R6, #-1  ; push n-1
STR   R0, R6, #0
JSR   Fibonacci    ; call self

LDR   R0, R6, #0   ; pop return value
ADD   R6, R6, #1
STR   R0, R5, #-1  ; store in temp
LDR   R0, R5, #4   ; read parameter n
ADD   R0, R0, #-2  ; calculate n-2
ADD   R6, R6, #-1  ; push n-2
STR   R0, R6, #0
JSR   Fibonacci    ; call self
```

LC-3 Code (part 3 of 3)

```
            LDR   R0, R6, #0   ; pop return value
            ADD   R6, R6, #1
            LDR   R1, R5, #-1  ; read temp
            ADD   R0, R0, R1   ; Fibonacci(n-1) + Fibonacci(n-2)
            BRnzp  FIB_END     ; all done

FIB_BASE    AND   R0, R0, #0   ; base case – return 1
            ADD   R0, R0, #1

FIB_END     STR   R0, R5, #3   ; write return value (R0)
            ADD   R6, R5, #1   ; pop local variables
            LDR   R5, R6, #0   ; pop dynamic link
            ADD   R6, R6, #1
            LDR   R7, R6, #0   ; pop return address
            ADD   R6, R6, #1
            RET
```

A Final C Example: Printing an Integer

Recursively converts an unsigned integer as a string of ASCII characters.

- **If integer <10, convert to char and print.**
- **Else, call self on first (n-1) digits and then print last digit.**

```
void IntToAscii(int num) {
  int prefix, currDigit;
  if (num < 10)
    putchar(num + '0');  /* prints single char */
  else {
    prefix = num / 10;   /* shift right one digit */
    IntToAscii(prefix);  /* print shifted num */
    /* then print shifted digit */
    currDigit = num % 10;
    putchar(currDigit + '0');
  }
}
```

Trace of IntToAscii

Calling IntToAscii with parameter 12345: