



# LOGIC and Computer Design Fundamentals

# CHAPTER 6

Register & Register Transfers
Counters, Register Cells, Buses, & Serial Operations
(part II)

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### **Overview**



- □ Part 1 − Registers, Microoperations and Implementations
- □ Part 2 − Counters, register cells, buses, & serial operations
  - Microoperations on single register (continued)
    - Counters
  - Register cell design
  - Multiplexer and bus-based transfers for multiple registers
  - Serial transfers and microoperations
- □ Part 3 Control of Register Transfers

### **Course Outline**



Microoperations on a single register-2

Register cell design

Multiple registers transfers

Serial transfers and microoperations

### **Counters**



■ Counters are sequential circuits which "count" through a specific state sequence. They can count up, count down, or count through other fixed\_sequences. Two distinct types are in common usage:

#### **□** Ripple Counters

- Clock connected to the flip-flop clock input on the LSB bit flip-flop
- For all other bits, a flip-flop output is connected to the clock input, thus circuit is not truly synchronous!
- Output change is delayed more for each bit toward the MSB.
- Resurgent because of low power consumption

#### **■** Synchronous Counters

- Clock is directly connected to the flip-flop clock inputs
- Logic is used to implement the desired state sequencing

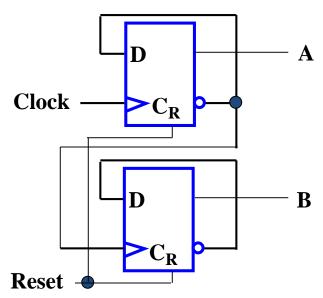


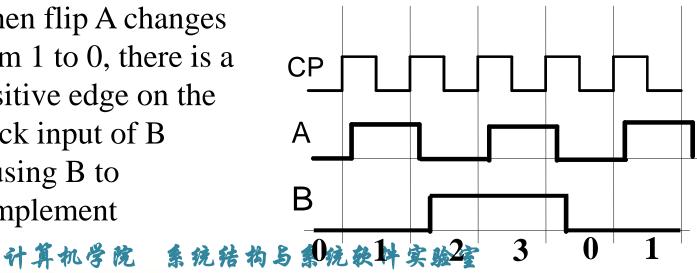
# **Ripple Counter**



#### **■** How does it work?

- When there is a positive edge on the clock input of A, A complements
- The clock input for flipflop B is the complemented output of flip-flop A
- When flip A changes from 1 to 0, there is a positive edge on the clock input of B causing B to complement

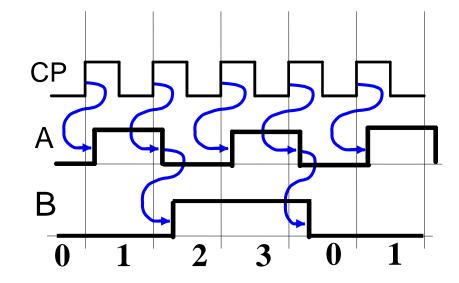




# Ripple Counter (continued)



- ☐ The arrows show the cause-effect relation-ship from the prior slide =>
- ☐ The corresponding sequence of states =>



$$(B,A) = (0,0), (0,1), (1,0), (1,1), (0,0), (0,1), ...$$

- Each additional bit, C, D, ...behaves like bit B, changing half as frequently as the bit before it.
- For 3 bits: (C,B,A) = (0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1), (0,0,0), ...



# Ripple Counter (continued)



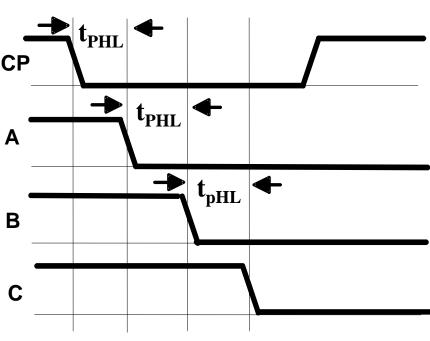
- □ These circuits are called *ripple counters* because each edge sensitive transition (positive in the example) causes a change in the next flip-flop's state.
- □ The changes "ripple" upward through the chain of flip-flops, i. e., each transition occurs after a clockto-output delay from the stage before.
- □ To see this effect in detail look at the waveforms on the next slide.

# Ripple Counter (continued)



 $\square$  Starting with C = B = A = 1, equivalent to (C,B,A) = 7 base 10, the next clock increments the count to (C,B,A) = 0 base 10. In fine timing detail:

- The clock to output delay t<sub>PHI</sub> causes an increasing delay from clock edge for each stage transition.
- Thus, the count "ripples" from least to most significant bit.
- For *n* bits, total worst case delay is  $n t_{PHI}$ .



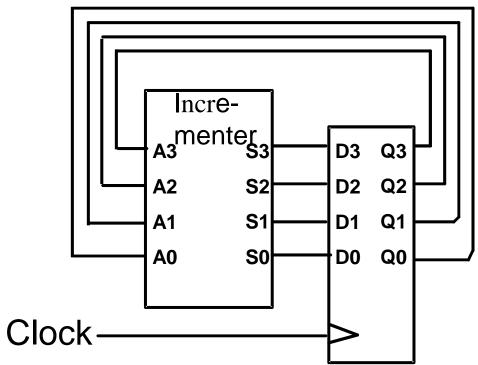


# Synchronous Counters

# **Synchronous Counters**



- To eliminate the 'ripple' effects, use a common clock for each flip-flop and a combinational circuit to generate the next state.
- For an up-counter, use an incrementer =>



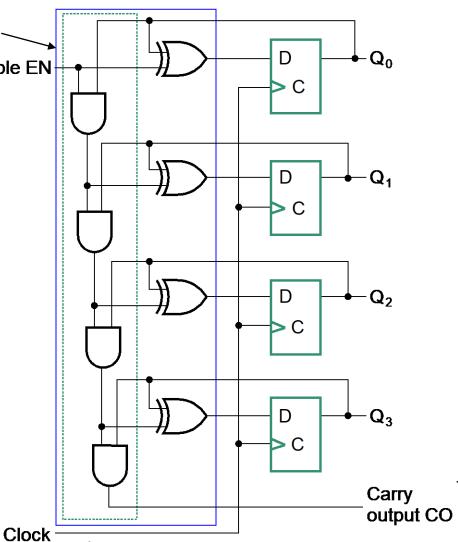
# Synchronous Counters Internal



- **Internal details** => Incrementer
- **□** Internal Logic

Count enable EN

- XOR complements each bit
- AND chain causes complement of a bit if all bits toward LSB from it equal 1
- **□** Count Enable
  - Forces all outputs of AND chain to 0 to "hold" the state
- □ Carry Out
  - Added as part of incrementer
  - Connect to Count Enable of additional 4-bit counters to form larger counters







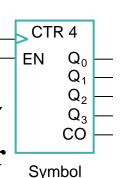


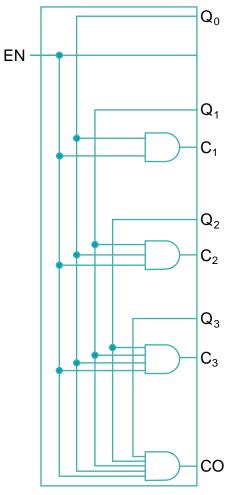
# Reduces delays Synchronous Counters



#### Carry chain

- series of AND gates through which the carry "ripples"
- Yields long path delays
- Called *serial gating*
- **Replace AND carry chain with ANDs =>** in parallel
  - Reduces path delays
  - Called *parallel gating*
  - Like carry lookahead
  - Lookahead can be used on COs and ENs to prevent long paths in large counters
- **■** Symbol for Synchronous Counter





Logic Diagram-Parallel Gating



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#### **Counter with Parallel Load**

Load

Count



#### Add path for input data

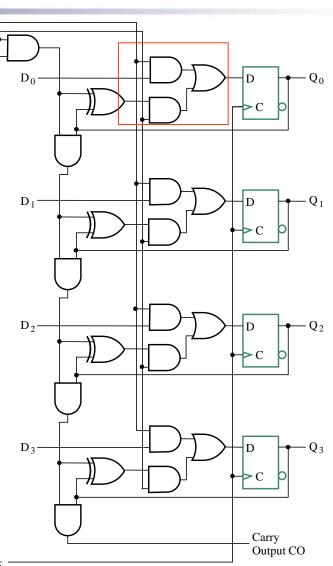
• enabled for Load = 1

#### ■ Add logic to:

- disable count logic for Load = 1
- disable feedback from outputs for Load = 1
- enable count logic for Load = 0and Count = 1

#### **■** The resulting function table:

Load	Count	Action
0	0	Hold Stored Value
0	1	<b>Count Up Stored Value</b>
1	X	Load D







# Common

# Counters



### **Other Counters**



#### □ See text for:

- Down Counter counts downward instead of upward
- *Up-Down Counter* counts up or down depending on value a control input such as Up/Down
- Parallel Load Counter Has parallel load of values available depending on control input such as Load

#### □ Divide-by-n (Modulo n) Counter

- $\blacksquare$  Count is remainder of division by n; n may not be a power of 2 or
- Count is arbitrary sequence of *n* states specifically designed state-by-state
- Includes modulo 10 which is the *BCD counter*



# Design Example: Synchronous BCD



- ☐ Use the sequential logic model to design a synchronous BCD counter with D flip-flops
- Input combinations 1010 through 1111 are don't cares

$Q_{2n+1}$	a	10	uuı	ııt	•
$Q_{2n+1}$ $Q_{8}$ $Q_{4}$	$\frac{Q_1}{00}$	01	11	10	
00	0/	1	0	1	
01	0	1	0	1	
11	X	X	X	X	
10	0	0	X	X	

$Q_{4n+1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{3}Q_{4}Q_{4}Q_{0}Q_{1}Q_{1}Q_{1}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{1}Q_{2}Q_{2}Q_{1}Q_{2}Q_{2}Q_{2}Q_{1}Q_{2}Q_{2}Q_{2}Q_{2}Q_{2}Q_{2}Q_{2}Q_{2$								
00	0	0	1	0				
01	1	1	0	1				
11	X	X	X	X				
10	0	0	X	X				

$$\begin{aligned} \overline{Q}_{I(n+1)} &= \overline{Q}_I \\ Q_{2(n+1)} &= Q_2 \oplus Q_1 \overline{Q}_8 \\ Q_{4(n+1)} &= Q_4 \oplus Q_1 Q_2 \\ Q_{8(n+1)} &= Q_8 \oplus (Q_1 Q_8 + Q_1 Q_2 Q_4) \end{aligned}$$

$$\begin{array}{c} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 &$$

#### State Table

<b>Current State</b>			Next State						
Q8 Q4 Q2 Q1			Q8 Q4 Q2 Q1				1		
0	0	0	0		0	0	0	1	
0	0	0	1		0	0	1	0	
0	0	1	0		0	0	1	1	
0	0	1	1		0	1	0	0	
0	1	0	0		0	1	0	1	
0	1	0	1		0	1	1	0	
0	1	1	0		0	1	1	1	
0	1	1	1		1	0	0	0	
1	0	0	0		1	0	0	1	
1	0	0	1		0	0	0	0	

# Input Equations for Synchronous BCD



**□** Use K-Maps to two-level optimize the next state equations and manipulate into forms containing XOR gates:

$$\mathbf{D}_{1} = \overline{\mathbf{Q}_{1}}$$

$$\mathbf{D}_{2} = \mathbf{Q}_{2} \oplus \mathbf{Q}_{1} \overline{\mathbf{Q}_{8}}$$

$$\mathbf{D}_{4} = \mathbf{Q}_{4} \oplus \mathbf{Q}_{1} \mathbf{Q}_{2}$$

$$\mathbf{D}_{8} = \mathbf{Q}_{8} \oplus (\mathbf{Q}_{1} \mathbf{Q}_{8} + \mathbf{Q}_{1} \mathbf{Q}_{2} \mathbf{Q}_{4})$$

- ☐ The logic diagram can be draw from these equations
  - An asynchronous or synchronous reset should be added
- **□** What happens if the counter is perturbed by a power disturbance or other interference and it enters a state other than 0000 through 1001?

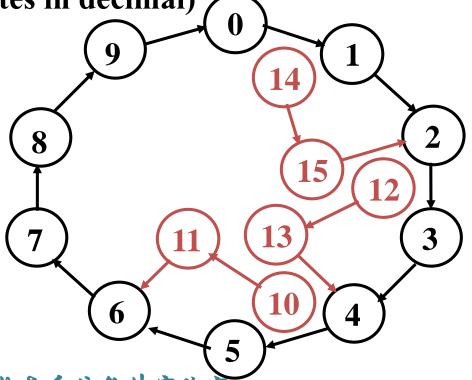
# The redundant state Synchronous BCD



□ Find the actual values of the six next states for the don't care combinations from the equations

■ Find the overall state diagram to assess behavior for the don't care states (states in decimal)

<b>Present State</b>				Next State			
Q8	Q4	Q2	Q1	Q8	Q4	Q2	Q1
1	0	1	0	1	0	1	1
1	0	1	1	0	1	1	0
1	1	0	0	1	1	0	1
1	1	0	1	0	1	0	0
1	1	1	0	1	1	1	1
1	1	1	1	0	0	1	0



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# **Invalid state handling**



- For the BCD counter design, if an invalid state is entered, return to a valid state occurs within two clock cycles
- **■** Is this adequate? If not:
  - Is a signal needed that indicates that an invalid state has been entered? What is the equation for such a signal?
  - Does the design need to be modified to return from an invalid state to a valid state in one clock cycle?
  - Does the design need to be modified to return from a invalid state to a specific state (such as 0)?
- **□** The action to be taken depends on:
  - the application of the circuit
  - design group policy
- **□** See pages 244 of the text.





# Modulo N

# Counters



# **Counting Modulo N**



- The following techniques use an *n*-bit binary counter with asynchronous or synchronous clear and/or parallel load:
  - Detect a *terminal count* of N in a Modulo-N count sequence to asynchronously Clear the count to 0 or asynchronously Load in value 0 (These lead to counts which are present for only a very short time and can fail to work for some timing conditions!)
  - Detect a terminal count of N 1 in a Modulo-N count sequence to Clear the count synchronously to 0
  - Detect a terminal count of N 1 in a Modulo-N count sequence to synchronously Load in value 0
  - Detect a terminal count and use Load to preset a count of the terminal count value minus (N - 1)
- □ Alternatively, custom design a modulo N counter as done for **BCD**

# Counting Modulo 7: Detect 7 and Asynchronously Clear

**D3** 

**D2** 

**D1** 

D<sub>0</sub>

>CP

**LOAD** 

**CLEAR** 

Clock-

Q3

Q2

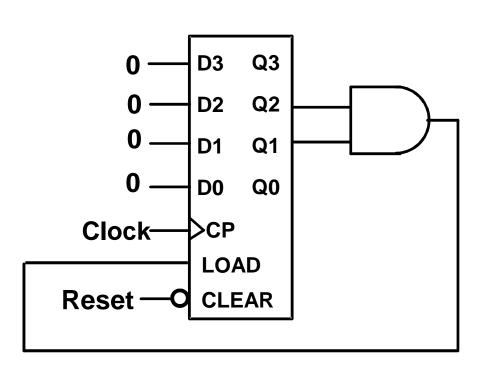
Q1

Q0

- A synchronous 4-bit binary counter with an asynchronous Clear is used to make a Modulo 7 counter.
- Use the Clear feature to detect the count 7 and clear the count to 0. This gives a count of 0, 1, 2, 3, 4, 5, 6, 7(short)0, 1, 2, 3, 4, 5, 6, 7(short)0, etc.
- DON'T DO THIS! Existence of state 7 may not be long enough to reliably reset all flip-flops to 0. Referred to as a "suicide" counter! (Count "7" is "killed," but the designer's job may be dead as well!)

#### **Counting Modulo 7:** (N-I)**Synchronously Load on Terminal Count of 6**

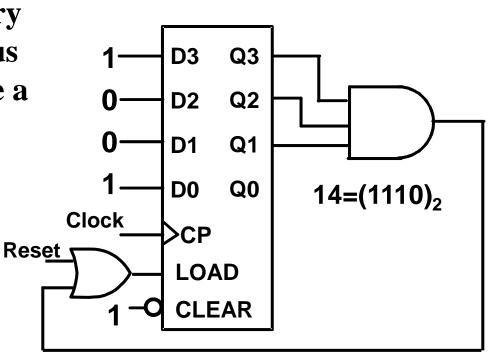
- A synchronous 4-bit binary counter with a synchronous load and an asynchronous clear is used to make a Modulo 7 counter
- Use the Load feature to detect the count "6" and load in "zero". This gives a count of 0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3, 4, 5, 6, 0, ...
- **□** Using don't cares for states above 0110, detection of 6 can be done with Load =  $Q_2 Q_1$



# Counting Modulo 6: Synchronously Preset 9 on Reset and Load 9 on Terminal Count 14



- A synchronous, 4-bit binary counter with a synchronous Load is to be used to make a Modulo 6 counter.
- Use the Load feature to preset the count to 9 on Reset and detection of count 14.

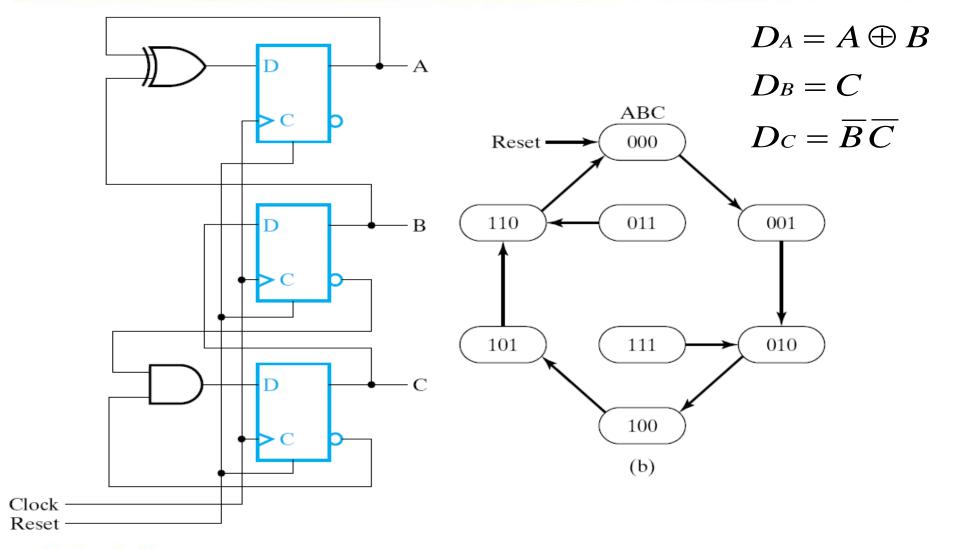


- □ This gives a count of 9, 10, 11, 12, 13, 14, 9, 10, 11, 12, 13, 14, 9, ...
- If the terminal count is 15 detection is usually built in as Carry Out (CO)



# Fixed Sequences Modulo 6





### **Course Outline**



Microoperations on a single register-2

Register cell design

Multiple registers transfers

Serial transfers and microoperations

# Register Cell Design



- **■** Assume that a register consists of identical cells
- Then register design can be approached as follows:
  - Design representative cell for the register
  - Connect copies of the cell together to form the register
  - Applying appropriate "boundary conditions" to cells that need to be different and contract if appropriate
- Register cell design is the first step of the above process

# **Register Cell Specifications**



- □ A register
- □ Data inputs to the register
- Control input combinations to the register
  - Example 1: Not encoded
    - Control inputs: Load, Shift, Add
    - At most, one of Load, Shift, Add is 1 for any clock cycle (0,0,0), (1,0,0), (0,1,0), (0,0,1)
  - Example 2: Encoded
    - □ Control inputs: S1, S0
    - All possible binary combinations on S1, S0 (0,0), (0,1), (1,0), (1,1)

# **Register Cell Specifications**



- A set of register functions (typically specified as register transfers)
  - Example:

Load:  $A \leftarrow B$ 

Shift:  $A \leftarrow sr B$ 

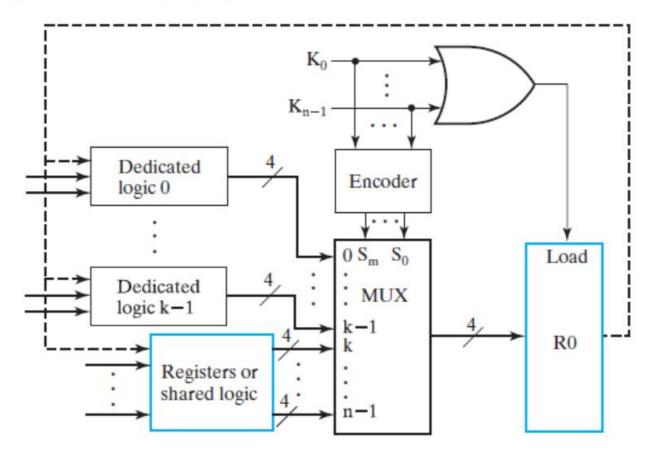
Add:  $A \leftarrow A + B$ 

- A hold state specification
  - Example:
    - Control inputs: Load, Shift, Add
    - □ If all control inputs are 0, hold the current register state

# Multiplexer Approach



Uses an n-input multiplexer with a variety of transfer sources and functions

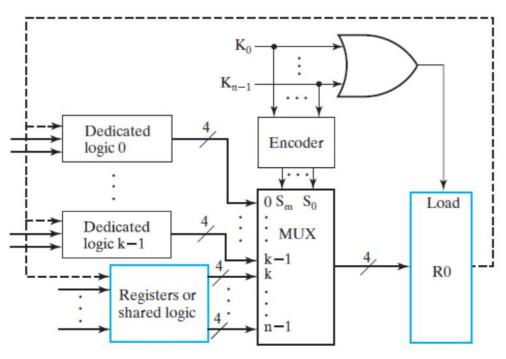


# Multiplexer Approach



- Load enable by OR of control signals  $K_0, K_1, ... K_{n-1}$ 
  - assumes no load for 00...0
- □ Use:
  - Encoder + Multiplexer (shown) or
  - n x 2 AND-OR

to select sources and/or transfer functions



# **Example 1: Register Cell Design**



### **■ Register A (m-bits) Specification:**

- Data input: B
- Control inputs (CX, CY)
- $\blacksquare$  Control input combinations (0,0), (0,1) (1,0)
- Register transfers:
- $\blacksquare$  CX: A  $\leftarrow$  B  $\lor$  A
- $\blacksquare$  CY :A  $\leftarrow$  B $\bigoplus$ A
- Hold state: (0,0)

# Register Cell Design



■ Load Control

$$Load = CX + CY$$

■ Since all control combinations appear as if encoded (0,0), (0,1), (1,0) can use multiplexer without encoder:

$$\begin{split} S1 &= CX \\ S0 &= CY \\ D0 &= A_i \\ D1 &= A_i \leftarrow B_i \oplus A_i \\ D2 &= A_i \leftarrow B_i \vee A_i \\ CX &= 1 \end{split}$$

■ Note that the decoder part of the 3-input multiplexer can be shared between bits if desired



# Sequential Circuit Design Approach



- ☐ Find a state diagram or state table
  - Note that there are only two states with the state assignment equal to the register cell output value
- □ Use the design procedure in Chapter 5 to complete the cell design
- **□** For optimization:
  - Use K-maps for up to 4 to 6 variables
  - Otherwise, use computer-aided or manual optimization

# Example 1 Again



#### **□** State Table:

	Hold	Ai v	/ Bi	Ai⊕Bi		
$A_{i}$	CX = 0	$\mathbf{CX} = 1 \qquad \mathbf{CX} = 1$		CY = 1	CY = 1	
	CY = 0	$\mathbf{B_i} = 0$	$B_i = 1$	$B_i = 0$	$\mathbf{B_i} = 1$	
0	0	0	1	0	1	
1	1	1	1	1	0	

- Four variables(CX, CY,  $A_i$ ,  $B_i$ ) give a total of 16 state table entries
- By using:
  - □ Combinations of variable names and values
  - $\square$  *Don't care* conditions (for CX = CY = 1)

only 8 entries are required to represent the 16 entries

# **Example 1 Again (continued)**



K-map - Use variable ordering CX, CY, A<sub>i</sub> B<sub>i</sub> and assume a D flip-flop

$\mathbf{D_i}(\mathbf{A}_{i(n+1)})$			A	ì	L
	0	0	1	1	
	0	1	0	1	
CXCY=11	X	X	X	X	CY
CX	0	1	1	1	
		F	i		,

## **Example 1 Again (continued)**



**□** The resulting SOP equation:

$$\mathbf{D_i} = \mathbf{CX} \mathbf{B_i} + \mathbf{CY} \mathbf{\overline{A_i}} \mathbf{B_i} + \mathbf{A_i} \mathbf{\overline{B_i}} + \mathbf{\overline{CY}} \mathbf{A_i}$$

■ Using factoring and DeMorgan's law:

$$\mathbf{D_i} = \mathbf{CX} \; \mathbf{B_i} + \overline{\mathbf{A_i}} \; (\mathbf{CY} \; \mathbf{B_i}) + \mathbf{A_i} (\overline{\mathbf{CY} \; \mathbf{B_i}})$$

$$\mathbf{D_i} = \mathbf{CX} \; \mathbf{B_i} + \mathbf{A_i} \bigoplus (\mathbf{CY} \; \mathbf{B_i})$$

The gate input cost per cell = 2 + 8 + 2 + 2 = 14

□ The gate input cost per cell for the previous version is:

Per cell: 19

**Shared decoder logic: 8** 

- Cost gain by sequential design > 5 per cell
- □ Also, no Enable on the flip-flop makes it cost less



### **Course Outline**



Microoperations on a single register-2

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## Multiplexer and Bus-Based Transfers for Multiple Registers



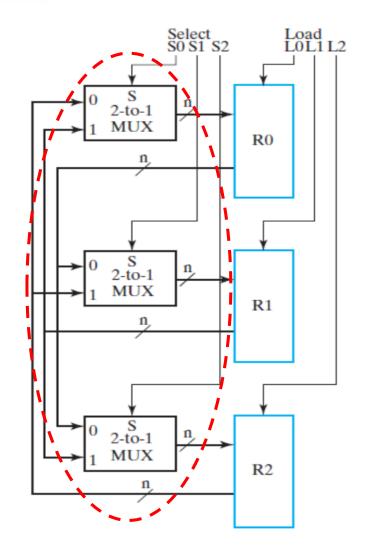
- □ Multiplexer dedicated to each register
- **□** *Shared* transfer paths for registers
  - A shared transfer object is a called a *bus* (Plural: buses)
- Bus implementation using:
  - multiplexers
  - three-state nodes and drivers
- □ In most cases, the number of bits is the length of the receiving register



### **Dedicated MUX-Based Transfers**



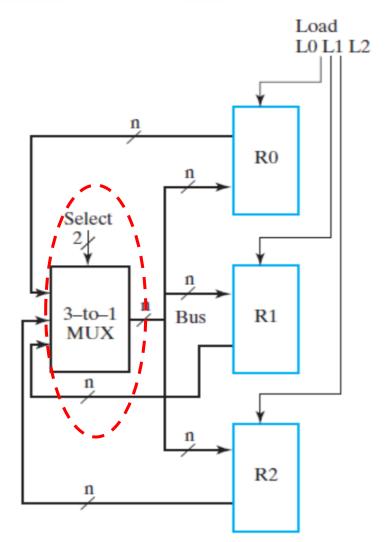
- Multiplexer connected to each register input produces a very flexible transfer structure =>
- □ Characterize the simultaneous transfers possible with this structure.



## Multiplexer Bus



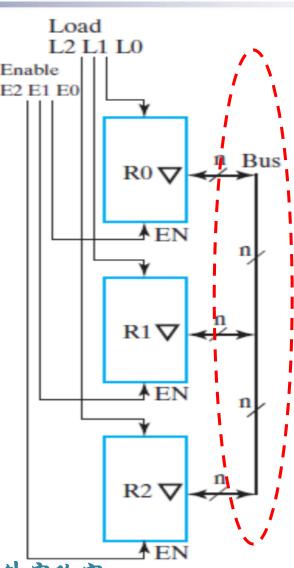
- A single bus driven by a multiplexer lowers cost, but limits the available transfers =>
- □ Characterize the simultaneous transfers no possible with this structure.
- □ Characterize the cost savings compared to dedicated multiplexers



### **Three-State Bus**



- □ The 3-input MUX can be replaced by a 3-state node (bus) and 3-state buffers.
- □ Cost is further reduced, but transfers are limited
- □ Characterize the simultaneous transfers no possible with this structure.
- Characterize the cost savings and compare
- □ Other advantages?



### **Course Outline**



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## **Serial Transfers and Microoperations**



#### Serial Transfers

- Used for "narrow" transfer paths
- Example 1: Telephone or cable line
  - □ Parallel-to-Serial conversion at source
  - □ Serial-to-Parallel conversion at destination
- Example 2: Initialization and Capture of the contents of many flip-flops for test purposes
  - □ Add shift function to all flip-flops and form large shift register
  - Use shifting for simultaneous Initialization and Capture operations

### Serial microoperations

- Example 1: Addition
- Example 2: Error-Correction for CDs

## **Serial Microoperations**

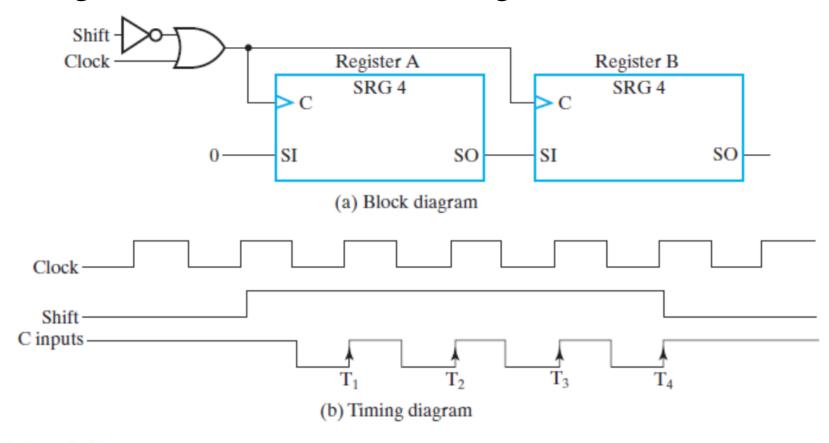


- By using two shift registers for operands, a full adder, and a flip flop (for the carry), we can add two numbers serially, starting at the least significant bit.
- □ Serial addition is a low cost way to add large numbers of operands, since a "tree" of full adder cells can be made to any depth, and each new level doubles the number of operands.
- Other operations can be performed serially as well, such as parity generation/checking or more complex error-check codes.
- □ Shifting a binary number <u>left</u> is equivalent to <u>multiplying</u> by 2.
- Shifting a binary number right is equivalent to dividing by 2.

# Synchronous serial transfer

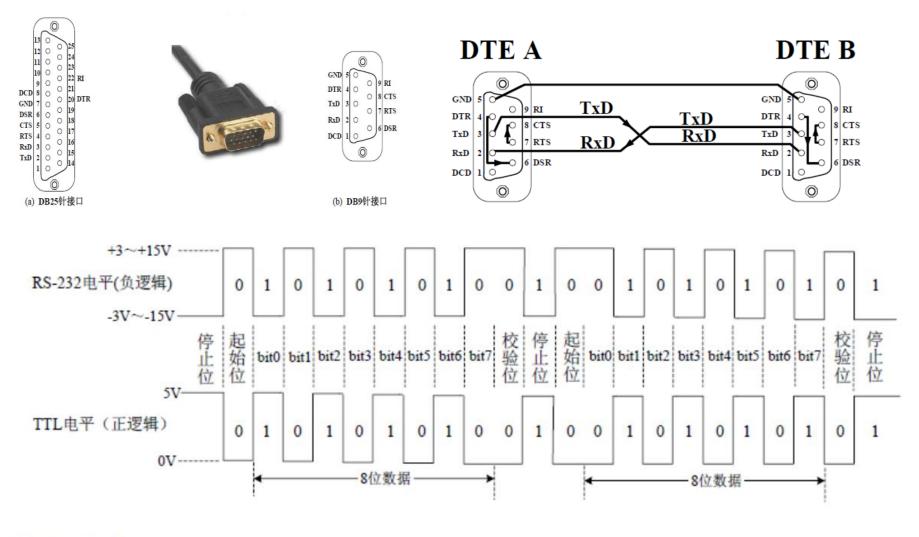


□ The serial transfer of information from register *A* to register *B* is done with shift register



## Asynchronous serial transfer





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### Serial Adder



- The circuit shown uses two shift registers for operands A(3:0)and B(3:0).
- **□** A full adder, and one more flip flop (for the carry) is used to compute the sum.
- **■** The result is stored in the A register and the final carry in the flip-flop
- **□** With the operands and the Control not shown) result in shift registers, a tree of full adders can be used to add a large number of operands. Used as a common digital signal processing technique.

**Load/Right Shift Registers** Full Adder A3 A2 A1 A0 Sum **Parallel Load** Cin **Serial** Cout Carry **B2 B1 B0 Parallel Load** Q (Clock and Load/Shift

In



Ch6-2

page392-393:

6-6, 6-13, 6-16, 6-17, 6-19

From Registers To Registers, On this matter!

Thank you!

## Quiz

- 1. A serial 2's complementer is to be designed. A binary integer of arbitrary length is presented to the serial 2's complementer, least significant bit first, on input X. When a given bit is presented on input X, the corresponding output bit is to appear during the same clock cycle on output Z. To indicate that a sequence is complete and that the circuit is to be initialized to receive another sequence, input Y becomes 1 for one clock cycle. Otherwise, Y is 0.
  - a) Find the state diagram for the serial 2's complementer.
  - b) Find the state table for the serial 2's complementer.
  - c) Find the output equations and next-state equations.
  - d) Draw the circuit diagram.