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- a) In the worst case, the number of the most comparisons is  $2n+2$ . When the size of the list doubles from  $n$  to  $2n$ , the number becomes  $4n+2$ .
- b) In the worst case, the number of the most comparisons is  $2\log_2 n+2$ . When the size of the list doubles from  $n$  to  $2n$ , the number becomes  $2\log_2 n+4$ .

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From  $a \equiv b \pmod{m}$  we can know that there exists an integer  $t$  such that  $a = b + tm$ .

We know that  $a^k - b^k = (a - b)(a^{k-1} + a^{k-2}b + \dots + b^{k-1}) = tm(a^{k-1} + a^{k-2}b + \dots + b^{k-1})$  ( $k \geq 2, k \in \mathbf{Z}$ ), so  $(a^k - b^k) \pmod{m} = 0$  and  $a^k \equiv b^k \pmod{m}$  hold.

So  $a^k \equiv b^k \pmod{m}$  holds.

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We can know that  $644 = (1010000100)_2$ . Let's set  $x=1$ ,  $i=0$  and  $power = 7 \bmod 645 = 7$ . Following Algorithm 5 we have the steps followed:

$i$	$x$	$power$
0	1	$7^2 \bmod 645=49$
1	1	$7^4 \bmod 645=466$
2	$(466 \cdot 1) \bmod 645=466$	$7^8 \bmod 645=436$
3	466	$7^{16} \bmod 645=466$
4	466	$7^{32} \bmod 645=436$
5	466	$7^{64} \bmod 645=466$
6	466	$7^{128} \bmod 645=436$
7	$(436 \cdot 466) \bmod 645=1$	$7^{256} \bmod 645=466$
8	1	$7^{512} \bmod 645=436$
9	$(436 \cdot 1) \bmod 645=436$	/

So  $7664 \bmod 645=436$ .

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For an integer  $k \geq 1$ ,  $(10^k - 1) \bmod 3 = 99 \dots 9 \bmod 3 = 0$ , so we know that  $10^k \equiv 1 \pmod{3}$ .

Let a positive integer be  $A = (a_n \dots a_1 a_0)_{10}$  as  $A = a_n \cdot 10^n + \dots + a_1 \cdot 10 + a_0$ . Then  $A \bmod 3 = \sum_{i=0}^n a_i \bmod 3$ .

So  $A \equiv \sum_{i=0}^n a_i \pmod{3}$  holds. If  $A \bmod 3 = 0$ , then  $\sum_{i=0}^n a_i \bmod 3 = 0$ . If  $\sum_{i=0}^n a_i \bmod 3 = 0$  then  $A \bmod 3 = 0$ .

So a positive integer is divisible by 3 iff the sum of its decimal digits is divisible by 3.