CPSC-354 Report

Zach Pratto
Chapman University

September 8, 2025

Abstract

This is the place to write an abstract. Not much has been abstracted yet.

Contents

1 Introduction

2 Week by Week

2.1 Week 1

Week 1 aligns with the first week of the semester.

2.1.1 Notes and Exploration

This section is optional.

2.1.2 Homework

The MU puzzle from the book asks us to transform MI into MU or $MI \Rightarrow MU$ using only the following rules:

Rule 1: If you possess a string whose last letter is I, you can add on a U at the end.
Rule schema: $XI \Rightarrow XIU$.

Rule 2: Suppose you have Mx. Then you may add Mxx to your collection.
Rule schema: $MX \Rightarrow MXX$.

Rule 3: If III occurs in one of the strings in your collection, you may make a new string with U in place of III.
Rule schema: $XIIIY \Rightarrow XUY$.

Rule 4: If UU occurs inside one of your strings, you can drop it.
Rule schema: $XUUY \Rightarrow XY$.

The puzzle does not have a solution. When experimenting with the rules given the initial condition, you quickly realize you can only start using rule 1 or rule 2. With rule 1, you realize once you have MIU, there's never a branch that allows you to get rid of the I after the M, so you look to rule 2, which seems more promising at first, as you are at least able to transform into strings in the form of MUX, which at least contains the MU. You might also try working backwards, trying to get $MIII$ or $MUUU$, as these would both immediately simplify to MU. But as you mess with it more, you see the same problems occurring over and over, where you can't get an odd number of sequential $U's$ without also getting other pieces of string between the initial M and the $U's$, and you can't get an odd number of sequential $I's$ at all. I don't have a proof, but it does seem as if the rules impose strict limits on the type of strings you can get, so it seems if you can't meet the strings you want to see (that'd lead to MU) to the strings you are actually able to transform into, then there is no path to the solution.

2.1.3 Questions

HW 1 Question: One way the MU puzzle seems impossible is by going backwards, where to get MU it seems like you'd have to get $MIII$ or $MUUU$ first, but getting $MIII$ or $MUUU$ feels impossible almost immediately with the limited rules. But would this way of looking at it make sense for a larger puzzle with more rules? I can imagine a similar puzzle with more rules where getting $MIII$ or $MUUU$ is still impossible, but other rules allow you to get to MU without using the $XIIIY \Rightarrow XUY$ or $XUUY \Rightarrow XY$ rules.

2.2 Week 2

2.2.1 Homework

Part 1 Draw a picture for each ARS. Determine if they are terminating, confluent, and whether they have unique normal forms.

ARS 1

$$A = \{\}, \quad R = \{\}$$

Picture: none

- Terminating? Yes
- Confluent? Yes
- Unique normal forms? Yes

ARS 2

$$A = \{a\}, \quad R = \{\}$$

Picture: a

- Terminating? Yes
- Confluent? Yes
- Unique normal forms? Yes

ARS 3

$$A = \{a\}, \quad R = \{(a, a)\}$$

Picture:
$$\begin{tikzcd} a \arrow[loop right] \end{tikzcd}$$

- Terminating? No
- Confluent? Yes
- Unique normal forms? No, because a looping final element can't really be final.

ARS 4

$$A = \{a, b, c\}, \quad R = \{(a, b), (a, c)\}$$

Picture:
$$\begin{tikzcd} & a \arrow[dl, dash] \arrow[dr, dash] & \\ b & & c \end{tikzcd}$$

- Terminating? Yes
- Confluent? No, a leads to two different elements which don't converge
- Unique normal forms? No, as the elements don't converge back to the same final element.

ARS 5

$$A = \{a, b\}, \quad R = \{(a, a), (a, b)\}$$

Picture:
$$\begin{tikzcd} a \arrow[loop above] \arrow[r, dash] & b \end{tikzcd}$$

- Terminating? No, a can be looped upon indefinitely.
- Confluent? Yes, because a has to end at b if ends at all.
- Unique normal forms? Yes

ARS 6

$$A = \{a, b, c\}, \quad R = \{(a, b), (b, b), (a, c)\}$$

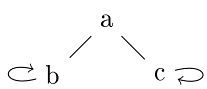
Picture:
$$\begin{tikzcd} & a \arrow[dl, dash] \arrow[dr, dash] & \\ b \arrow[loop left] & & c \end{tikzcd}$$

- Terminating? No, has a loop.
- Confluent? No
- Unique normal forms? No, loop makes b irreducible

ARS 7

$$A = \{a, b, c\}, \quad R = \{(a, b), (b, b), (a, c), (c, c)\}$$

Picture:



- Terminating? No, has loops.
- Confluent? No
- Unique normal forms? No, loops make b and c irreducible

Part 2 Find an example of an ARS for each of the possible 8 combinations.

Format: True or false (T or F) for (Confluent, Terminates, UNF)

1. **(T, T, T)** Example: ARS #2
2. **(T, T, F)** Example: impossible.
   Reason: if ARS is fully reducible and terminates, there must be multiple elements that converge into one element. Therefore, this ARS won't have more than 1 normal form, despite 3 elements, hence it doesn't have unique normal forms.
3. **(T, F, T)** Example: ARS #5
4. **(T, F, F)** Example: ARS #3
5. **(F, T, T)** Example: impossible. Reason: ARS is terminating and has unique normal forms, which implies confluence towards one element, which is supposed to be false, so impossible.
6. **(F, T, F)** Example: ARS #4
7. **(F, F, T)** Example: impossible. Reason: If an ARS has unique normal forms, then it implies multiple elements confluencing to one element, which is false here, so impossible.
8. **(F, F, F)** Example: ARS #6, ARS #7

2.2.2 Questions

HW 2 Question: So it seems like an ARS with unique normal forms should always reach the last element unless there's a non-terminating relation somewhere in the ARS. Is this true? if so, are there other uses for tracking terminability in ARS's?

3 Essay

4 Evidence of Participation

5 Conclusion

References

[BLA] Author, Title, Publisher, Year.