Corbyn Coffin	ME 342-02	Assignment 4.3	1/2

P2 Given: $I = \int_{-2}^{2}\int_{0}^{2}\int_{-3}^{1} (x^3 - 3yz)\,dx\,dy\,dz$

Find: 1.) Analytically, 2.) Single application simpson's 1/3 rule,
3.) Absolute error in simpson's integration
4.) Change in error if step-size were reduced 50%.

Solution:

1.) $I = \int_{-2}^{2}\int_{0}^{2}\left(\frac{x^4}{4} - 3xyz\right)\Big|_{-3}^{1}$

$\left(\frac{1}{4}x^4 - 3xyz\right)\Big|_{-3}^{1} = \left(\frac{1}{4}(1)^4 - 3(1)yz\right) - \left(\frac{1}{4}(-3)^4 - 3(-3)yz\right)$

$= \frac{1}{4} - 3yz - \frac{81}{4} - 9yz = -20 - 12yz$

$I = \int_{-2}^{2}\int_{0}^{2}(-20 - 12yz) = \int_{-2}^{2}(-20y - 6y^2z)\Big|_{0}^{2}$

$(-20y - 6y^2z)\Big|_{0}^{2} = (-20(2) - 6(2)^2z) - (\cancel{-20(0)} - \cancel{6(0)^2z})$

$= -40 - 24z$

$I = \int_{-2}^{2}(-40 - 24z) = -40z - 12z^2\Big|_{-2}^{2}$

$-40z - 12z^2\Big|_{-2}^{2} = (-40(2) - 12(2)^2) - (-40(-2) - 12(-2)^2)$

$= -80 - \cancel{48} - 80 + \cancel{48} = \underline{-160}$

$$\boxed{I = -160}$$

2.) $x^3 - 3yz$, $x = -3, -1, 1$ $y = 0, 1, 2$ $z = -2, 0, 2$

$\underline{y=0}$

$z=-2$, $I = [1-(-3)]\dfrac{-27+4(-1)+1}{6} = -20$

$z=0$, $I = [1-(-3)]\dfrac{-27+4(-1)+1}{6} = -20$

$z=2$, $I = [1-(-3)]\dfrac{-27+4(-1)+1}{6} = -20$

$I_{y=0} = [2-(-2)]\dfrac{-20+4(-20)+-20}{6} = -80$

$\underline{y=1}$

$z=-2$, $I = [1-(-3)]\dfrac{-21+4(5)+7}{6} = 4$

$z=0$, $I = [1-(-3)]\dfrac{-27+4(-1)+1}{6} = -20$

$z=2$, $I = [1-(-3)]\dfrac{-33+4(-7)+-5}{6} = -44$

$I_{y=1} = [2-(-2)]\dfrac{4+4(-20)+-44}{6} = -80$

$\underline{y=2}$

$z=-2$, $I = [1-(-3)]\dfrac{-15+4(11)+13}{6} = 28$

$z=0$, $I = [1-(-3)]\dfrac{-27+4(-1)+1}{6} = -20$

$z=2$, $I = [1-(-3)]\dfrac{-39+4(-13)+-11}{6} = -68$

$I_{y=2} = [2-(-2)]\dfrac{28+4(-20)+-68}{6} = -80$

$I = [2-(0)]\dfrac{-80+4(-80)+-80}{6} = -160$

$$\boxed{I = -160}$$

3.) 4.) Simpson's 1/3 Rule is exact for cubic functions so the absolute error is zero. This also means that changing the step-size would not change the absolute error.