ECON 468 Cheat Sheet
Zachary Probst Page:1

1 Basics

Distributions

Normal: $\phi(x) = (2\pi)^{-1/2}\exp(-\frac{1}{2}x^2)$

Normal CDF: $\mathbf{\Phi}(\mathbf{x}) = \int_{-\infty}^{x}\phi(y)dy$

Moments

Continuous: $m_k(X) \equiv \int_{-\infty}^{\infty} x^k f(x)dx$

Central:
$\mu_k \equiv E(X - E(X)^k) = \int_{-\infty}^{\infty}(x-\mu)^k f(x)dx$

Discrete Central:
$\mu_k \equiv E(X - E(X)^k) = \sum_{i=1}^{m} p(x)(x_i - \mu)^k$

Multivariate

Joint Density Func: $f(x_1, x_2) = \frac{\partial^2 F(x_1,x_2)}{\partial x_1 \partial x_2}$

Independence:
$F(x_1,x_2) = F(x_1,\infty)F(\infty,x_2)$
or $f(x_1,x_2) = f(x_1)f(x_2)$

Marginal Density:
$f(x_1) \equiv F_1(x_1,\infty) = \int_{-\infty}^{\infty} f(x_1,x_2)dx_2$

Conditional Density: $f(x_1|x_2) = \frac{f(x_1,x_2)}{f(x_2)}$

Law Iterate Expec: $E(E(X_1|X_2)) = E(X_1)$

Any Deterministic Func h:
$E(X_1h(X_2) \mid X_2) = h(X_2)E(X_1 \mid X_2)$

Matrix Algebra

Dot Product: $\mathbf{a}^\top\mathbf{b} = \sum_{i=1}^{n} a_i b_i$

Matrix Multiplication: $C_{ij} = \sum_{k=1}^{m} A_{ik}B_{ki}$

Transpose: $(\mathbf{AB})^\top = \mathbf{B}^\top\mathbf{A}^\top$

$||\mathbf{x}|| = (\mathbf{x}^\top\mathbf{y})^{1/2} = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}$

$\langle\mathbf{x},\mathbf{y}\rangle = ||\mathbf{x}||||\mathbf{y}||\cos(\theta)$

Cauchy-Schwartz: $\langle\mathbf{x},\mathbf{y}\rangle \le ||\mathbf{x}||||\mathbf{y}||$

Linear Indep: $\mathbf{X}^\top\mathbf{X}\mathbf{b} = \mathbf{0}$, $\mathbf{b} \neq \mathbf{0}$

2 Linear Regression

OLS

Estimator: $\hat{\beta} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y}$

$E(x_{ti}u_t) = 0 \implies \mathbf{X}^\top(\mathbf{y} - \mathbf{X}\beta) = \mathbf{0}$

$\text{SSR}(\beta) := \sum_{t=1}^{n}(y_i - \mathbf{X_t}\beta)^2$

$\mathbf{y}^\top\mathbf{y} = \hat{\beta}^\top\mathbf{X}^\top\mathbf{X}\hat{\beta} + (\mathbf{y}-\mathbf{X}\hat{\beta})^\top(\mathbf{y}-\mathbf{X}\hat{\beta})$

Projection: $\mathbf{P_X} = \mathbf{X}(\mathbf{X}^\top\mathbf{X})^{-\mathbf{1}}\mathbf{X}^\top$

$\mathbf{M_X} = \mathbf{I} - \mathbf{P_X} = \mathbf{I} - \mathbf{X}(\mathbf{X}^\top\mathbf{X})^{-\mathbf{1}}\mathbf{X}^\top$

$\mathbf{P_x y} = \mathbf{X}((\mathbf{X}^\top\mathbf{X})^{-\mathbf{1}}\mathbf{X}^\top\mathbf{y})$; $\mathbf{P_X P_X} = \mathbf{P_X}$

$\mathbf{M_X y} = \hat{\mathbf{u}}$; $\mathbf{M_X X} = \mathbf{0}$; $\mathbf{M_X M_X} = \mathbf{M_X}$

$\mathbf{P_X} + \mathbf{M_X} = \mathbf{I}$; $\mathbf{P_X M_X} = \mathbf{0}$; $\mathbf{P_X^\top} = \mathbf{P_X}$

$||\mathbf{y}||^2 = ||\mathbf{P_X y}||^2 + ||\mathbf{M_X y}||^2$; $||\mathbf{P_X y}|| \le ||\mathbf{y}||$

Centering: $\mathbf{M}_\iota\mathbf{x} = \mathbf{z} = \mathbf{x} - \bar{x}\iota$; $\iota^\top\mathbf{z} = 0$

$\mathbf{P_1} \equiv \mathbf{P_{X_1}}$; $\mathbf{P_1 P_X} = \mathbf{P_X P_1} = \mathbf{P_1}$

FWL: β_2 from $\mathbf{y} = \mathbf{X_1}\beta_1 + \mathbf{X}_2\beta_2 + \mathbf{u}$ and $\mathbf{M_1 y} = \mathbf{M_1 X}_2\beta_2 + \text{res}$ are the same. (+ res)

Seasonal w const: $\mathbf{s}_i' = \mathbf{s}_i - \mathbf{s_4}$, $i = 1,2,3$.

Avg is const coeff. $\mathbf{M_S y}$ is deseasonalized.

Bias

Bias: $E(\hat{\theta}) - \theta_0$